

Masks will Fall Off

Higher-Order Optimal Distinguishers

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Abstract. Higher-order side-channel attacks are able to break the security of cryptographic implementations even if they are protected with masking countermeasures. In this paper, we derive the best possible distinguishers (High-Order Optimal Distinguishers or HOOD) against masking schemes under the assumption that the attacker can profile. Our exact derivation admits simple approximate expressions for high and low noise and shows to which extent the optimal distinguishers reduce to known attacks in the case where no profiling is possible. From these results, we can explain theoretically the empirical outcome of recent works on second-order distinguishers. In addition, we extend our analysis to any order and to the application to masked tables precomputation. Our results give some insight on which distinguishers have to be considered in the security analysis of cryptographic devices.

Keywords: Side-channel analysis, higher-order masking, masking tables, higher-order optimal distinguisher (HOOD), template attack.

1 Introduction

In order to secure embedded devices against side-channel attacks, masking schemes have been introduced. Recent works have shown provable protections with a security parameter d , such that each sensitive variable is secured with d random masks [5]. The computation is carried out in such a way that the knowledge of any tuple of d intermediate variables does not disclose any information on any sensitive variable. Accordingly, all distinguishers using up to d leakages will fail to recover the correct key. A successful attack would be a $(d + 1)$ th-order CPA, which uses combination functions to transform the measured leakage and the prediction on each share into a single value in order to compute Pearson correlation coefficients [5, 11, 14, 15, 20, 21, 25].

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Using combination functions to fit to a known tool like CPA looks more like an engineering recipe than the optimal solution. Yet, it is shown by Prouff et al. [17] that, in the case of second-order attacks, using the normalized product combination function combined with an optimal prediction function is the most efficient solution among all known combination functions. Even more, Standaert et al. [22] showed that the information loss induced from the combination functions vanishes for high noise.

In [15] Oswald and Mangard introduced several template-based attacks on masking schemes. Among them is the so-called template-based DPA attack, which extends the traditional template attack from Chari et al. [6] to first-order masking schemes. Besides, their approach classifies measurements according to the key, and not according to sensitive variables.

A slightly different scenario has been analyzed by Tunstall et al. in [24], where the authors study the security of masking tables for software implementations as defined in [1]. The authors suggested a two-stage CPA: first, for each individual trace, extract the mask during the precomputation and, second, use this knowledge about the mask to reveal the secret key using a vertical attack.

In this article we tackle the questions “*what is the best possible distinguisher in case of profiling?*” and “*how far are they from known practical distinguishers?*”. In particular, we derive optimal higher-order distinguishers against higher-order masking schemes in case profiling is possible. Here, optimality means maximizing the success rate. Starting from second-order optimal distinguisher we derive approximations for high and low noise and recover known attacks. In particular, we show to what extent the optimal second-order distinguisher can be translated into a second-order CPA attack using combination functions. Given these results for second-order we extend our analysis to $(d + 1)$ th-order distinguisher against d th-order masking schemes.

Additionally, we investigate the scenario of masking tables as in [24]. We derive the optimal attack against masking tables and again derive approximations for the scenario of high and low noise which results in new attacks and compare it to the two-stage CPA.

2 Preliminaries

2.1 Masking Countermeasure & Notations

Even though many different masking schemes have been investigated so far, which clearly differ in their strength, the principle of attacking is equivalent. A masking scheme is characterized by the number of random masks that are used per sensitive variable. In the following we consider a d th-order masked implementation where we assume that the masks are uniformly distributed over a space \mathcal{M} . Calligraphic letters (e.g., \mathcal{X}) denote sets, capital letters (e.g., X) denote random variables taking values in these sets, and the corresponding lowercase letters (e.g., x) denote their realizations. Let k^* denote the secret cryptographic key, k any possible key hypothesis from the keyspace \mathcal{K} , and T be the input or

ciphertext of the cryptographic algorithm. The mapping $f : (\mathcal{T}, \mathcal{K}, \mathcal{M}) \rightarrow \mathbb{F}_2^n$ maps the input or ciphertext $t \in \mathcal{T}$, a key hypothesis $k \in \mathcal{K}$ and the mask $m \in \mathcal{M}$ to an internally processed variable in some space \mathbb{F}_2^n that is assumed to relate to the measured leakage X , where n is the number of bits. Generally it is assumed that f is known to the attacker. The measured leakage X can then be written as

$$X = \varphi(f(T, k^*, M)) + N, \quad (1)$$

where N denotes an independent—not necessarily Gaussian—additive noise with zero mean and where φ a device-specific deterministic function. In this paper we start by assuming that φ is known to the attacker due to profiling to consider the most powerful attack. We then show to which extend and scenarios this assumption can be relaxed while still achieving the same efficiency of the attack.

Specifically, in a d th-order masking scheme the implementation is protected with d masks with corresponding leakages

$$X^{(\omega)} = \varphi^{(\omega)}(f^{(\omega)}(T^{(\omega)}, k^*, M^{(\omega)})) + N^{(\omega)}, \quad (2)$$

with $\omega \in \{0, \dots, d\}$ and $M^{(\omega)} \in \mathcal{M}^{(\omega)}$ where the $\mathcal{M}^{(\omega)}$ does not need to be equal in general. Accordingly, a d th-order masking scheme can be broken using $(d + 1)$ th-order distinguishers by targeting $d + 1$ shares. For simplification we denote $Y(T^{(\omega)}, k, M^{(\omega)}) = \varphi^{(\omega)}(f^{(\omega)}(T^{(\omega)}, k, M^{(\omega)}))$.

Example 1 (First-order software masking). For example a first-order masking scheme ($d = 1$) might leak with

$$X^{(0)} = \text{HW}[M] + N^{(0)}, \quad (3)$$

$$X^{(1)} = \text{HW}[\text{Sbox}[T \oplus k^*] \oplus M] + N^{(1)}, \quad (4)$$

with $\text{Sbox} : \mathbb{F}_2^8 \rightarrow \mathbb{F}_2^8$ being the AES Substitution box and $T^{(1)} = T$ uniformly distributed over \mathbb{F}_2^8 (and $T^{(0)}$ is non-existent). Thus, $\varphi^{(0)}(\cdot) = \varphi^{(1)}(\cdot) = \text{HW}[\cdot]$ (the Hamming weight function), $M^{(0)} = M^{(1)} = M$, $f^{(0)}(T, k, M) = M$ and $f^{(1)}(T, k, M) = \text{Sbox}[T \oplus k] \oplus M$.

Example 2 (Tables pre-computation). Again when assuming a Hamming weight leakage model, a masking scheme using Sbox recomputation [12] might leak with

$$X^{(\omega)} = \text{HW}[\omega \oplus M] + N^{(\omega)}, \quad \forall \omega \in \{0, 1, \dots, 2^n - 1\} \cong \mathbb{F}_2^n \quad (5)$$

$$X^{(2^n)} = \text{HW}[T \oplus k^* \oplus M] + N^{(2^n)}. \quad (6)$$

A detailed description will be given in Sect. 5.

Definition 1 (Perfect masking (d th-order) [3]). Let us denote the random variables $F^{(\omega)}(t, k) = f^{(\omega)}(t, k, M^{(\omega)})$ for $\omega \in \{0, \dots, d\}$ and a fixed pair (t, k) . A masking scheme is perfect at d th-order if the joint distribution of maximum d of $F^{(0)}(t, k), \dots, F^{(d)}(t, k)$ is identically distributed of any pair $(t, k) \in \mathcal{T} \times \mathcal{K}$.

Note that d th-order security implies 1st, 2nd, \dots , $(d - 1)$ th-order security.

Proposition 1. *If a masking scheme is perfect, then whatever function ψ , $\sum_{m^{(\omega)} \in \mathcal{M}^{(\omega)}} \psi(f^{(\omega)}(t, k, m^{(\omega)}))$ is constant for any pair (t, k) for any $0 \leq \omega \leq d$.*

Proof. Let \tilde{t}, \tilde{k} be any value in \mathcal{T} and \mathcal{K} , respectively. As the masking scheme is perfect up to d th-order (which implies 1st-order) the distribution of $f^{(\omega)}(t, k, M^{(\omega)})$ is equivalent to $f^{(\omega)}(\tilde{t}, \tilde{k}, M^{(\omega)})$, hence $\psi(f^{(\omega)}(t, k, M^{(\omega)}))$ and $\psi(f^{(\omega)}(\tilde{t}, \tilde{k}, M^{(\omega)}))$ have the same distribution. In particular, the sum of realizations is identical. \square

In our setup we assume that the attacker is able to measure q i.i.d. measurements. All values indexed by $i \in \{1, \dots, q\}$ are in bold face (e.g. $\mathbf{a} = (a_1, \dots, a_q) \in \mathcal{A}^q$ for $a_i \in \mathcal{A}$). Values indexed by the intermediate variable index (ω) ($a^{(\omega)} \in \mathcal{A}$) are denoted by $\mathbf{a}^{(*)} = (a^{(0)}, \dots, a^{(d)}) \in \mathcal{A}^{d+1}$. Moreover, $a_i^{(\omega)} \in \mathcal{A}$ and $\mathbf{a}^{(*)} \in \mathcal{A}^{q \times (d+1)}$.

Note that contrary to \mathbf{a} , the vectors along (ω) can be linked, e.g., $\bigoplus_{\omega=0}^d M^{(\omega)} = 0$ in Example 1 or $\forall \omega \in \{0, \dots, 2^n - 1\} \cup \{2^n\}, M^{(\omega)} = M^{(0)}$ in Example 2. Thus the set of admissible masks, denoted by $\mathcal{M}^{(*)}$, is a subset of the Cartesian product over all $\mathcal{M}^{(\omega)}$. Additionally, regarding the noise, we have that $\forall (i, \omega) \neq (i', \omega')$, $N_i^{(\omega)}$ is independent of $N_{i'}^{(\omega')}$.

We write $\mathbb{P}(m) = \mathbb{P}(M = m)$ for discrete probability distributions, p for densities, and when the random variable X is conditioned by the event $Y = y$, we use the notation $p_k(X|Y = y)$ to recall that y depends on a (fixed) key guess k . As the model is known by the attacker, we also have $p_k(X|Y = y) = p_k(X|T = t, M = m)$ when $y = y(t, k, m)$. Indeed, owing to Eq. (2), Y is a *sufficient statistic* for k [9]. We then use $p_k(x|t, m)$ to denote $p(X = x|Y = \varphi(f(t, k, m)))$. We denote the scalar product between \mathbf{x} and \mathbf{y} by $\langle \mathbf{x} | \mathbf{y} \rangle = \sum_{i=1}^q x_i y_i$, the Euclidean norm by $\|\mathbf{x}\|_2 = \sqrt{\langle \mathbf{x} | \mathbf{x} \rangle}$ and the componentwise product by $\mathbf{x} \cdot \mathbf{y} = (x_1 y_1, \dots, x_q y_q)$. Given a function $g(k)$, we use the notation $\arg \max_k g(k)$ to denote the value of k that maximizes $g(k)$. Finally, $\mathbb{1}_E : E \rightarrow \{0, 1\}$ denotes the indicator function of the set E .

2.2 Combination Functions for Higher-Order CPA Attacks

In order to conduct a second-order CPA attack, two kinds of *combination functions*, i.e., $c_X : \mathcal{X}^{d+1} \rightarrow \mathbb{R}$ and $c_Y : \mathcal{T}^{d+1} \rightarrow \mathbb{R}$, are required. However, this seems to be more inspired from an engineering perspective –an “act from necessity”– than a sound mathematical tool to maximize the success. That the use of combination functions comes with information loss was already pointed out by Mangard and Oswald and Mangard [15] and Standaert et al. [22]. The history and selection of combination functions is indeed epic, where the literature mostly concentrated on second-order CPA ($d = 1$).

The most prominent function to combine the leakages is the *product* combining function $c_X^{\text{prod}}(X^{(0)}, X^{(1)}) = X^{(0)} \cdot X^{(1)}$ introduced by Chari et al. in [5] and the *absolute difference* $c_X^{\text{diff}}(X^{(0)}, X^{(1)}) = |X^{(0)} - X^{(1)}|$ by Messerges in [14]. Oswald and Mangard [15] proposed (for e.g. the setting given in Example 1) an even

more exotic combination function and corresponding prediction function⁴:

$$c_X^{\text{sin}}(X^{(0)}, X^{(1)}) = \sin\left((X^{(0)} - X^{(1)})^2\right) \quad (7)$$

$$c_Y^{\text{sin}}(T) = -89.95 \sin(\text{HW}[Y])^3 - 7.82 \sin(\text{HW}[Y])^2 + 67.66 \sin(\text{HW}[Y]), \quad (8)$$

where $Y = Y(T, k, 0)$. Contrary to what was suggested in previous papers, Prouff et al. [17] showed that all these combination functions should be accompanied by $c_Y^{\text{opt}}(T^{(0)}, T^{(1)}) = \mathbb{E}\{c_X^*(Y^{(0)}, Y^{(1)})|T^{(0)}, T^{(1)}\}$ to maximize the absolute value of correlation, where the expectation is taken over the mask M and c_X^* denotes the same combination function as c_X but defined as a map $\mathcal{Y}^{d+1} \rightarrow \mathbb{R}$. Moreover, the *normalized product function*, i.e., $c_X^{\text{n-prod}}(X^{(0)}, X^{(1)}) = (X^{(0)} - \mathbb{E}\{X^{(0)}\}) \cdot (X^{(1)} - \mathbb{E}\{X^{(1)}\})$ is shown to be the most efficient of all known combination functions when considering a Hamming weight leakage model.

3 Optimal Distinguisher for Second-Order Attacks

3.1 Motivation

As highlighted in Subsect. 2.2 the introduction of combination functions for second-order CPA is more a necessary evil than an optimized procedure to maximize success. In [22] the authors empirically showed that a combination function always goes hand in hand with information loss. However, the authors depicted that for large noise the second-order CPA with the normalized product function $c_X^{\text{n-prod}}(X^{(0)}, X^{(1)})$ becomes (nearly) equivalent to the maximum likelihood distinguisher applied to the joint distribution.

This observation might not be obvious in theory since correlation is only an appropriate statistical tool when the underlying noise is Gaussian. Unfortunately, when multiplying two Gaussian distributions, as it is done for $c_X^{\text{n-prod}}(X^{(0)}, X^{(1)})$, does clearly *not* result in a Gaussian distribution.

Thus, our aim is to precisely state the higher-order optimal distinguisher (HOOD) expression for second-order when the attacker has full information about underlying the leakage and determine when this knowledge can be lessened to relate the expression to second-order CPA. This will help to understand known empirical results [10, 15, 17, 22]. In particular, we investigate low and high noise scenarios to see which combination function from the pool described in Subsect. 2.2 would be a reasonable choice.

3.2 Explicit Derivations

In [15] the authors state various template attacks against first-order masking schemes. The most efficient is a straightforward extension of the classical template attack [6] over all pairs (t, k) . Our approach goes in a similar direction: we utilize the joint distribution of both leakages $X^{(0)}$ and $X^{(1)}$ without using a combination function, which gives us the optimal second-order distinguisher maximizing the success rate.

⁴ Note that these are the corrected formulas given in [17].

Theorem 2 (Second-order HOOD). *If the model (i.e., $\varphi^{(\omega)}$) is known to the attacker for all ω , then the second-order HOOD is*

$$\mathcal{D}_{opt}^2(\mathbf{x}^{(*)}, \mathbf{t}^{(*)}) = \arg \max_{k \in \mathcal{K}} p_k(\mathbf{x}^{(*)} | \mathbf{t}^{(*)}) \quad (9)$$

$$= \arg \max_{k \in \mathcal{K}} \prod_{i=1}^q \sum_{m^{(*)} \in \mathcal{M}^{(*)}} \mathbb{P}(m^{(*)}) \prod_{\omega=0}^1 p_k(x_i^{(\omega)} | t_i^{(\omega)}, m^{(\omega)}). \quad (10)$$

Note that as the attacker knows the model he is able to compute the required probability distributions and densities.

Proof. Let us denote the key guess of any second-order distinguisher by $\hat{k} = \arg \max_{k \in \mathcal{K}} \mathcal{D}^2(\mathbf{x}^{(*)}, \mathbf{t}^{(*)})$. Then, using a frequentist approach we start from the success probability \mathbb{P}_S over all possible secret keys k

$$\mathbb{P}_S = \frac{1}{|\mathcal{K}|} \sum_{k \in \mathcal{K}} \mathbb{P}(\hat{k} = k) = \frac{1}{|\mathcal{K}|} \sum_{k \in \mathcal{K}} \sum_{\mathbf{t}^{(*)}} \mathbb{P}(\mathbf{t}^{(*)}) \mathbb{P}(\hat{k} = k | \mathbf{T}^{(*)} = \mathbf{t}^{(*)}) \quad (11)$$

$$= \frac{1}{|\mathcal{K}|} \sum_{k \in \mathcal{K}} \sum_{\mathbf{t}^{(*)} \in \mathcal{T}^{q \times (d+1)}} \mathbb{P}(\mathbf{t}^{(*)}) \int_{\mathcal{X}^{q \times (d+1)}} p_k(\mathbf{x}^{(*)} | \mathbf{t}^{(*)}) \mathbb{1}_{\hat{k} = k} d\mathbf{x}^{(*)} \quad (12)$$

$$= \frac{1}{|\mathcal{K}|} \sum_{\mathbf{t}^{(*)} \in \mathcal{T}^{q \times (d+1)}} \mathbb{P}(\mathbf{t}^{(*)}) \int_{\mathcal{X}^{q \times (d+1)}} p_{\hat{k}}(\mathbf{x}^{(*)} | \mathbf{t}^{(*)}) d\mathbf{x}^{(*)} . \quad (13)$$

In Eq. (11), we have to compute $\mathbb{P}(\hat{k} = k | \mathbf{t}^{(*)})$ where $\hat{k} = \hat{k}(\mathbf{x}^{(*)}, \mathbf{t}^{(*)}) = \arg \max_{k \in \mathcal{K}} \mathcal{D}^2(\mathbf{x}^{(*)}, \mathbf{t}^{(*)})$ is a function of $\mathbf{x}^{(*)}$ and $\mathbf{t}^{(*)}$. This is therefore a probability on the random variable $\mathbf{X}^{(*)}$ knowing $\mathbf{T}^{(*)} = \mathbf{t}^{(*)}$, which follows the density $p_k(\mathbf{x}^{(*)} | \mathbf{t}^{(*)})$. Like for every probability taken on a random variable with density, the required probability is the integral of density over the events. So $\mathbb{P}(\hat{k}(\mathbf{x}^{(*)}, \mathbf{t}^{(*)}) = k | \mathbf{t}^{(*)}) = \int p_k(\mathbf{x}^{(*)} | \mathbf{t}^{(*)}) d\mathbf{x}^{(*)}$, where the integral is taken over all $\mathbf{x}^{(*)}$ such that $\hat{k}(\mathbf{x}^{(*)}, \mathbf{t}^{(*)}) = k$; this is the indicator function inside the integral in Eq. (12).

Now, $\mathbb{P}(\mathbf{t}^{(*)})$ is independent of the key. Thus, for each given sequence $\mathbf{x}^{(*)}, \mathbf{t}^{(*)}$ maximizing the success rate amounts to choose $k = \hat{k}$ such that $p_k(\mathbf{x}^{(*)} | \mathbf{t}^{(*)})$ is maximized. Moreover,

$$p_k(\mathbf{x}^{(*)} | \mathbf{t}^{(*)}) = \prod_{i=1}^q p_k(x_i^{(*)} | t_i^{(*)}) \quad (14)$$

$$= \prod_{i=1}^q \sum_{m^{(*)} \in \mathcal{M}^{(*)}} \mathbb{P}(m^{(*)}) p_k(x_i^{(*)} | t_i^{(*)}, m^{(*)}) \quad (15)$$

$$= \prod_{i=1}^q \sum_{m^{(*)} \in \mathcal{M}^{(*)}} \mathbb{P}(m^{(*)}) \prod_{\omega=0}^1 p_k(x_i^{(\omega)} | t_i^{(\omega)}, m^{(\omega)}). \quad (16)$$

We used from (15) to (16) that $N_i^{(\omega)}$ is i.i.d. across the values of $i = \{1, \dots, q\}$ and independent for $\omega = \{0, 1\}$. Accordingly, $\arg \max_{k \in \mathcal{K}}$ of Eq. (16) forms the optimal distinguisher $\mathcal{D}_{opt}^2(\mathbf{x}^{(*)}, \mathbf{t}^{(*)})$. \square

Remark 1. To simplify our notation, we assume in the following that the masks at each order are drawn from the same space \mathcal{M} , with uniform probability $\mathbb{P}(M = m) = 1/|\mathcal{M}|$ and only one text byte is manipulated with the masks, as in software implementations (cf. Ex. 1). That is, $\forall i \ t_i^{(0)} = t_i^{(1)} = t_i$ and, moreover, there is only one mask $m^{(0)} = m^{(1)} = m$. Accordingly, Eq. (10) simplifies to

$$\mathcal{D}_{opt}^2(\mathbf{x}^{(*)}, \mathbf{t}) = \arg \max_{k \in \mathcal{K}} \prod_{i=1}^q \sum_{m \in \mathcal{M}} p_k(x_i^{(0)} | t_i, m) \cdot p(x_i^{(1)} | t_i, m). \quad (17)$$

However, all our results hereafter can be easily extended to the scenario without simplifications.

As it is most often assumed that the noise distribution at the manipulation of each share is Gaussian (e.g., [15, 17]), we further deduce Eq. (17) for Gaussian noise.

Proposition 3 (Second-order HOOD for Gaussian noise). *Assuming that $N^{(\omega)} \sim \mathcal{N}(0, \sigma^{(\omega)2})$ then the second-order optimal distinguisher becomes*

$$\mathcal{D}_{opt}^{2,G}(\mathbf{x}^{(0)}, \mathbf{x}^{(1)}, \mathbf{t}) = \arg \max_{k \in \mathcal{K}} \prod_{i=1}^q \sum_{m \in \mathcal{M}} \exp \left\{ -\frac{1}{2} \left(\frac{-2x_i^{(0)}y^{(0)}(t_i, k, m) + y^{(0)}(t_i, k, m)^2}{\sigma^{(0)2}} + \frac{-2x_i^{(1)}y^{(1)}(t_i, k, m) + y^{(1)}(t_i, k, m)^2}{\sigma^{(1)2}} \right) \right\}. \quad (18)$$

Proof. In this case $p_k(x_i^{(\omega)} | t_i, m)$ is the 1D Gaussian density with mean $y^{(\omega)}(t_i, k, m)$ and standard deviation $\sigma^{(\omega)}$. Removing all constants gives us the required formula. \square

As a next step we give approximations for high noise and low noise.

Corollary 4 (Second-order HOOD for high Gaussian noise). *When considering that $\mathbb{E}\{y(T, m, k)\} = \mathbb{E}\{\varphi(f(T, m, k))\}$ is independent of the choice of $k \in \mathcal{K}$ (owing to Proposition 1)⁵, which is given in case of high noise since a large number of measurements q is considered, then the distinguishing rule simplifies to*

$$\mathcal{D}_{opt}^{2,G,\sigma^\uparrow}(\mathbf{x}^{(*)}, \mathbf{t}) = \arg \max_{k \in \mathcal{K}} \prod_{i=1}^q \sum_{m \in \mathcal{M}} \exp \left\{ \frac{x_i^{(0)}y^{(0)}(t_i, k, m)}{\sigma^{(0)2}} + \frac{x_i^{(1)}y^{(1)}(t_i, k, m)}{\sigma^{(1)2}} \right\}. \quad (19)$$

Proof. In Eq. (18) we can now remove the terms $y^{(\omega)}(t_i, k, m)^2$ for $\omega = \{0, 1\}$ because, $\mathbb{E}\{y(T, m, k)\} = \mathbb{E}\{\varphi(f(T, m, k))\}$ is independent of $k \in \mathcal{K}$. This gives Eq. (19). \square

⁵ This assumption has been also made in [23].

Remark 2. We see that the exact optimal distinguishing expression (Eq. (18)) operates in the *direct scale*, such that the function φ (including its scaling factor) and thus the exact relationship between X and Y has to be known. Whereas the distinguisher for high noise (Eq. (19)) operates in the *proportional scale*⁶, thus the relationship between X and Y has only to be known up to an irrelevant affine law. That is to say, the attacker shall know that $X^{(\omega)} = a^{(\omega)}\varphi^{(\omega)}(f^{(\omega)}(T^{(\omega)}, k, m)) + b^{(\omega)} + N^{(\omega)}$ with unknown $a^{(\omega)}, b^{(\omega)} \in \mathbb{R}$. For more information on direct and proportional scales we refer the reader to [26].

Remark 2 already gives a hint about a possible relationship between the second-order HOOD and second-order CPA for high noise, which we will discuss in the next subsection.

Proposition 5 (Second-order HOOD for low Gaussian noise). *Assuming that both shares have the same low noise standard deviation $\sigma = \sigma^{(0)} = \sigma^{(1)}$ then the optimal distinguisher reduces at first order to*

$$\mathcal{D}_{opt}^{2,G,\sigma\downarrow}(\mathbf{x}^{(*)}, \mathbf{t}) = \arg \min_{k \in \mathcal{K}} \sum_{i=1}^q \max_{m \in \mathbb{F}_2^n} (x_i^{(0)} - y_i^{(0)}(t_i, k, m))^2 + (x_i^{(1)} - y_i^{(1)}(t_i, k, m))^2. \quad (20)$$

Proof. Starting from Eq. (18) we have

$$\hat{k} = \arg \max_{k \in \mathcal{K}} \prod_{i=1}^q \sum_{m \in \mathbb{F}_2^n} \exp \left\{ -\frac{1}{\sigma^{(0)2}} (x_i^{(0)} - y_i^{(0)}(t_i, k, m))^2 - \frac{1}{\sigma^{(1)2}} (x_i^{(1)} - y_i^{(1)}(t_i, k, m))^2 \right\}. \quad (21)$$

Now as $\sigma = \sigma^{(0)} = \sigma^{(1)}$ and as the sum over exponential reduces at first order to the minimum we have the first order approximation for $\sigma \rightarrow 0$

$$= \arg \max_{k \in \mathcal{K}} \prod_{i=1}^q \min_{m \in \mathbb{F}_2^n} \exp \left\{ -(x_i^{(0)} - y_i^{(0)}(t_i, k, m))^2 - (x_i^{(1)} - y_i^{(1)}(t_i, k, m))^2 \right\}. \quad (22)$$

Applying the logarithm that is strictly monotonous increasing yields

$$\begin{aligned} &= \arg \max_{k \in \mathcal{K}} \sum_{i=1}^q \min_{m \in \mathbb{F}_2^n} \left(-(x_i^{(0)} - y_i^{(0)}(t_i, k, m))^2 - (x_i^{(1)} - y_i^{(1)}(t_i, k, m))^2 \right) \quad (23) \\ &= \arg \min_{k \in \mathcal{K}} \sum_{i=1}^q \max_{m \in \mathbb{F}_2^n} \left((x_i^{(0)} - y_i^{(0)}(t_i, k, m))^2 + (x_i^{(1)} - y_i^{(1)}(t_i, k, m))^2 \right). \quad \square \end{aligned}$$

Interestingly, one can see directly from Eq. (20) that the optimal distinguisher for low noise cannot be rewritten as correlation with any combination functions and moreover that it operates in the direct scale. Even more, the nature of distinguisher seems not very intuitive. In particular, deciding for the key with the minimal maximal distance between leakage and prediction seems unusual.

⁶ But not *anti-proportional* scale, or in other words, the “sign” has to be known.

3.3 Comparison with Second-Order CPA

Proposition 6 (Relationship between second-order HOOD for high noise and second-order CPA). *The second-order HOOD for high noise can be approximated as*

$$\mathcal{D}_{opt}^{2,G,\sigma\uparrow} \approx \arg \max_k \langle \mathbf{x}^{(0)} \cdot \mathbf{x}^{(1)} | \sum_{m \in \mathcal{M}} y^{(0)}(\mathbf{t}, k, m) y^{(1)}(\mathbf{t}, k, m) \rangle, \quad (24)$$

which is all the more equivalent as the noise gets larger. Accordingly, as the noise is larger, the closer the optimal distinguishing rule to second-order CPA with

$$c_X^{n\text{-prod}}(X^{(0)}, X^{(1)}) = (X^{(0)} - \mathbb{E}\{X^{(0)}\}) \cdot (X^{(1)} - \mathbb{E}\{X^{(1)}\}) \text{ and} \quad (25)$$

$$c_Y^{opt}(Y^{(*)}) = \mathbb{E}\{c_X^*(Y^{(0)}(T, k, M), Y^{(1)}(T, k, M)) | T\}. \quad (26)$$

Proof. We use the first order Taylor expansion $\exp\{\varepsilon\} = 1 + \varepsilon + \mathcal{O}(\varepsilon^2)$. Note that this approximation is all the better as ε is close to zero and thus as the argument of $\exp\{\cdot\}$ his high. Starting from Eq. (19) and using $y_i^{(\omega)} = y^{(\omega)}(t_i, k, m)$, we have

$$\mathcal{D}^{2,G,\sigma\uparrow}(\mathbf{x}^{(0)}, \mathbf{x}^{(1)}, \mathbf{t}) = \arg \max_k \prod_{i=1}^q \sum_{m \in \mathcal{M}} \exp\left\{\frac{1}{\sigma^{(0)^2} x_i^{(0)} y_i^{(0)}}\right\} \exp\left\{\frac{1}{\sigma^{(1)^2} x_i^{(1)} y_i^{(1)}}\right\} \quad (27)$$

$$\approx \arg \max_k \prod_{i=1}^q \sum_{m \in \mathcal{M}} \left(1 + \frac{1}{\sigma^{(0)^2} x_i^{(0)} y_i^{(0)}}\right) \left(1 + \frac{1}{\sigma^{(1)^2} x_i^{(1)} y_i^{(1)}}\right) \quad (28)$$

$$= \arg \max_k \prod_{i=1}^q \sum_{m \in \mathcal{M}} \left(1 + \frac{1}{\sigma^{(0)^2} \sigma^{(1)^2} x_i^{(0)} y_i^{(0)} x_i^{(1)} y_i^{(1)}} + \frac{1}{\sigma^{(0)^2} x_i^{(0)} y_i^{(0)}} + \frac{1}{\sigma^{(1)^2} x_i^{(1)} y_i^{(1)}}\right). \quad (29)$$

In Eq. (29), owing to the perfect masking definition, the terms $\sum_{m \in \mathcal{M}} x_i^{(0)} y_i^{(0)}$ and $\sum_{m \in \mathcal{M}} x_i^{(1)} y_i^{(1)}$ are constant ($\text{const}^{(0)}$ and $\text{const}^{(1)}$). Additionally, as the logarithm function is increasing, we consider the logarithm of the product, and we use the approximation $\ln\{1 + \varepsilon\} = \varepsilon + \mathcal{O}(\varepsilon^2)$, (reciprocal of the previous Taylor's expansion of the exponential function), which is again all the better as ε is close to zero and thus for high noise. Accordingly,

$$\mathcal{D}^{2,G,\sigma\uparrow}(\mathbf{x}^{(0)}, \mathbf{x}^{(1)}, \mathbf{t}) \approx \arg \max_k \ln \prod_{i=1}^q \sum_{m \in \mathcal{M}} \left(1 + \frac{1}{\sigma^{(0)^2} \sigma^{(1)^2} x_i^{(0)} y_i^{(0)} x_i^{(1)} y_i^{(1)}} + \frac{1}{\sigma^{(0)^2} x_i^{(0)} y_i^{(0)}} + \frac{1}{\sigma^{(1)^2} x_i^{(1)} y_i^{(1)}}\right) \quad (30)$$

$$= \arg \max_k \sum_{i=1}^q \ln \left\{1 + \sum_{m \in \mathcal{M}} \frac{1}{\sigma^{(0)^2} \sigma^{(1)^2} x_i^{(0)} y_i^{(0)} x_i^{(1)} y_i^{(1)}} + \text{const}^{(0)} + \text{const}^{(1)}\right\} \quad (31)$$

$$\begin{aligned}
&\approx \arg \max_k \sum_{i=1}^q x_i^{(0)} x_i^{(1)} \sum_{m \in \mathcal{M}} y^{(0)}(t_i, k, m) y^{(1)}(t_i, k, m) + \text{const}^{(0)} + \text{const}^{(1)} \\
&= \arg \max_k \langle \mathbf{x}^{(0)} \cdot \mathbf{x}^{(1)} | \sum_{m \in \mathcal{M}} y^{(0)}(\mathbf{t}, k, m) y^{(1)}(\mathbf{t}, k, m) \rangle. \tag{32}
\end{aligned}$$

Note that we can remove the $\text{const}^{(0)}, \text{const}^{(1)}$ as they do not depend on the key guess. For large number of measurements (resulting from large noise) the $\arg \max_{k \in \mathcal{K}}$ of the correlation coefficient can be simplified as

$$\arg \max_{k \in \mathcal{K}} \frac{\langle \mathbf{x} - \bar{\mathbf{x}} | \mathbf{y}(k) \rangle}{\|\mathbf{x} - \bar{\mathbf{x}}\|_2 \cdot \|\mathbf{y}(k) - \bar{\mathbf{y}}(k)\|_2} \approx \arg \max_{k \in \mathcal{K}} \langle \mathbf{x} - \bar{\mathbf{x}} | \mathbf{y}(k) \rangle. \tag{33}$$

Accordingly, if $\mathbf{x}^{(0)}$ and $\mathbf{x}^{(1)}$ are centered, then in Eq. (32) $c_X^{\text{n-prod}} = \mathbf{x}^{(0)} \cdot \mathbf{x}^{(1)}$, and $c_Y^{\text{opt}} = \sum_{m \in \mathcal{M}} y^{(0)}(\mathbf{t}, k, m) y^{(1)}(\mathbf{t}, k, m)$ is the *optimal prediction function*. \square

As correlation is a measure in the proportional scale, we can relax our assumptions made about the knowledge of the attacker. More precisely, he does not need to know $y^{(0)}$ and $y^{(1)}$ exactly but any linear transformation $l^{(\omega)}(y^{(\omega)}) = ay^{(\omega)} + b$, as it is most often assumed in the literature [17, 22]. Yet, in Prop. 6 we do not recover the absolute value of the correlation, thus, for second-order CPA the “sign” must be known and taking the absolute value does not result in an equivalence for high noise, which is also empirically validated in our experiments in Subject. 3.4.

Remark 3. Prouff et al. illustrated in [17] that for large noise the improved (i.e., centered) product combining function has the best efficiency among the known combination functions, which is inline with our findings in Prop. 6. Moreover, we can claim that the improved product combining function is the most efficient among *all* combining functions for high noise as it becomes equivalent to the optimal second-order distinguisher. Moreover, our study is not restricted to a particular HW or HD leakage model scenario as in the previous studies.

Remark 4. The determination of optimal combination functions is a vivid research topic. As already mentioned, the optimality of the centered product amongst all combination functions has been conjectured by Prouff et al. in [17]. Afterwards, mathematical arguments for optimality were given by Carlet et al. [4], and independently by Ding et al. in [8].

Remark 5. As underlined in [22], the function to be maximized in Eq. (24) is a straightforward generalization of Pearson’s correlation coefficient to the case of three random variables: $X^{(0)}, X^{(1)}$, and $\mathbb{E}\{y^{(0)}(T, k, M) y^{(1)}(T, k, M) | T\}$, where the expectation is taken over M .

3.4 Experimental Validation

For our experimental validation we used simulations of a first-order masking scheme where each share is leaking in the Hamming weight model to be able

to directly compare our results to previous publications conducting the same setting [17, 22] (see Example 1). We simulated the noise arising from a Gaussian distributions $N \sim \mathcal{N}(0, \sigma^2)$ for $\sigma = \sigma^{(0)} = \sigma^{(1)} \in \{0.5, 1, 4\}$. To be reliable we conducted 500 independent experiments with uniformly distributed k^* to compute the empirical success rate. Moreover, when plotting the empirical success rate, we additionally highlight the standard deviation of the success rate by error bars. If the error bars do not overlap, we can unambiguously conclude that one distinguisher is better than the other [13].

For our simulations we calculated the second-order HOOD and second-order CPA with the normalized product and the absolute difference combination function as described in Subsect. 2.2. For a low value of σ , $\mathcal{D}_{opt}^{2,G}$ (HOOD) clearly outperforms 2nd-order CPA (2O-CPA) independent of the combination functions (see Figures 1a) and 1b, which is inline with our theoretical analysis and the empirical analysis in [22]. For high values of σ , the second-order HOOD and second-order CPA with the normalized product combining function become equivalently efficient (see Fig. 1c), which coincides with Prop. 6.

Note that as said before, taking the absolute value of the correlation (plots with label: “2O-CPA prod (absolut)” in Fig. 1) is not equivalent to HOOD, which is confirmed in Fig. 1c. The same remark applies for the absolute difference of absolute values of the leakage (of combination function $c_X(x^{(0)}, x^{(1)}) = ||x^{(0)}| - |x^{(1)}||$, plots with label: “2O-CPA abs diff (absolut)” in Fig. 1), which is indeed less efficient than the absolute difference of the genuine leakage (of combination function $c_X(x^{(0)}, x^{(1)}) = |x^{(0)} - x^{(1)}|$, plots with label: “2O-CPA abs diff” in Fig. 1).

4 Higher-Order Optimal Distinguisher (HOOD) for Any Order

The claim in [17] that the normalized product combining function $c_X^{n\text{-prod}}$ in combination with c_Y^{opt} is optimal⁷ was only done for $d = 1$. We now extend our investigation to $(d + 1)$ th-order distinguishers in order to analyze if the assumption can straightforwardly be generalized.

Theorem 7 (General HOOD). *When $\varphi^{(\omega)} : \mathbb{F}_2^n \rightarrow \mathbb{R}$ is known for all ω , $N_i^{(\omega)}$ i.i.d. across values of $i = \{1, \dots, q\}$ and independent across the values of $\omega = \{0, \dots, d\}$, then the general higher-order optimal distinguisher is*

$$\mathcal{D}_{opt}^d(\mathbf{x}^{(*)}, \mathbf{t}^{(*)}) = \arg \max_{k \in \mathcal{K}} \prod_{i=1}^q \sum_{m^{(*)} \in \mathcal{M}^{(*)}} \mathbb{P}(m^{(*)}) \prod_{\omega=0}^d p_k(x_i^{(\omega)} | t_i^{(\omega)}, m^{(\omega)}). \quad (34)$$

Proof. The proof is a straightforward extension of proof of Theorem 2. □

⁷ Note again, that the authors used the absolute correlation coefficient of the correct key as a measure of optimality; not the success rate.

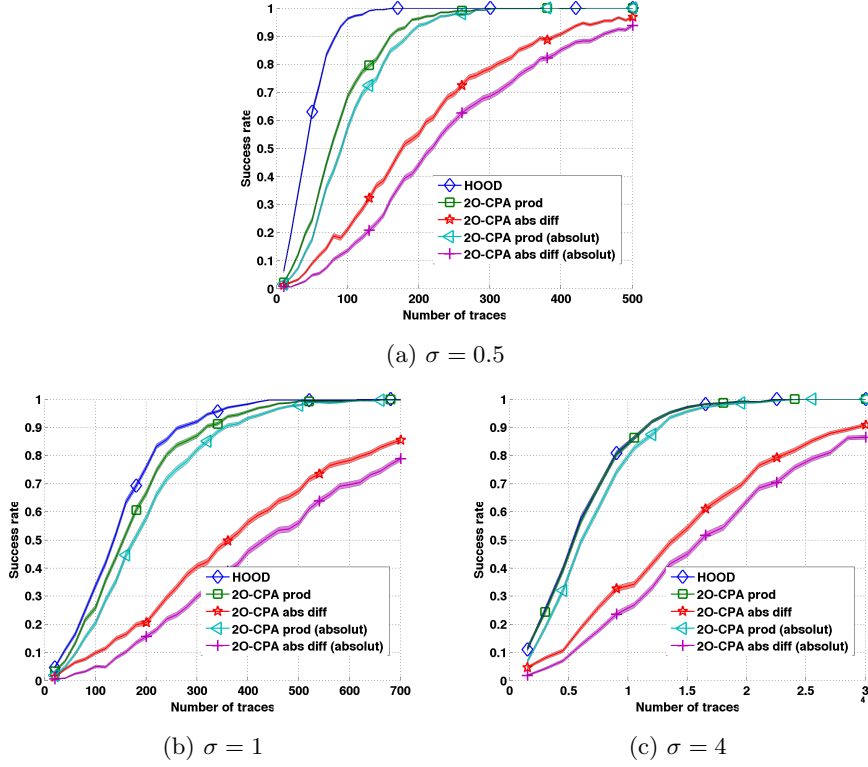


Fig. 1: Success rate for second-order attacks

Remark 6. The optimal distinguisher is *additive*, meaning that if new measurements $q < i \leq q'$ are taken, then $\mathcal{D}_{opt}^d(\mathbf{x}^{(*)}, \mathbf{t}^{(*)})$ can be re-evaluated without re-evaluating the already computed product (sum when considering the logarithm) on $1 \leq i \leq q$ in (34).

Proposition 8 (HOOD for Gaussian noise). *Under the same assumptions as in Theorem 7 and additionally assuming Gaussian noise, i.e., $N_i^{(\omega)} \sim \mathcal{N}(0, \sigma^{(\omega)^2})$, Eq. (34) becomes*

$$\mathcal{D}_{opt}^{d,G}(\mathbf{x}^{(*)}, \mathbf{t}) = \arg \max_{k \in \mathcal{K}} \sum_{i=1}^q \log \left\{ \sum_{m^{(*)} \in \mathcal{M}^{(*)}} \exp \left\{ \sum_{\omega=0}^d \frac{1}{\sigma^{(\omega)^2}} (x_i^{(\omega)} y_i^{(\omega)} - \frac{1}{2} y_i^{(\omega)^2}) \right\} \right\}. \quad (35)$$

Proof. As $p_k(x_i^{(\omega)} | t_i, m) = p_{k, N^{(\omega)}}(x_i^{(\omega)} - y^{(\omega)}(t_i, k, m))$ we have

$$\arg \max_{k \in \mathcal{K}} \prod_{i=1}^q \sum_{m^{(*)} \in \mathcal{M}^{(*)}} \mathbb{P}(m^{(*)}) \prod_{\omega=0}^d p_k(x_i^{(\omega)} | t_i, m^{(\omega)}) = \quad (36)$$

$$\arg \max_{k \in \mathcal{K}} \prod_{i=1}^q \sum_{m^{(*)} \in \mathcal{M}^{(*)}} \mathbb{P}(m^{(*)}) \prod_{\omega=0}^d \frac{1}{\sqrt{2\pi}\sigma^{(\omega)}} \exp\left\{-\frac{1}{2\sigma^{(\omega)^2}}(x_i^{(\omega)} - y^{(\omega)}(t_i, k, m))\right\}.$$

Now, removing all key-independent constants yields

$$\arg \max_{k \in \mathcal{K}} \prod_{i=1}^q \sum_{m \in \mathcal{M}} \prod_{\omega=0}^d \exp\left\{-\frac{1}{2\sigma^{(\omega)^2}}(x_i^{(\omega)} - y^{(\omega)}(t_i, k, m))^2\right\}.$$

Now, as the product of $\exp\{\cdot\}$ is the $\exp\{\cdot\}$ of the sum and expanding the square and removing the key-independent factor $x_i^{(\omega)^2}$ gives the required equation. \square

Proposition 9 (HOOD for high Gaussian noise). *For high Gaussian noise (low SNR) we can further approximate the HOOD to*

$$\mathcal{D}_{opt}^{d,G,\sigma\uparrow}(\mathbf{x}^{(*)}, \mathbf{t}) = \arg \max_k \prod_{i=1}^q \sum_{m \in \mathcal{M}} \exp\left\{\sum_{\omega=0}^d \frac{1}{\sigma^{(\omega)^2}} x_i^{(\omega)} y^{(\omega)}\right\}, \quad (37)$$

and as $\sigma^{(\omega)}$ becomes large Eq. (37) becomes closer to $(d+1)$ th-order CPA with

$$c_X^{n\text{-prod}}(X^{(*)}) = \prod_{\omega=0}^d (X^{(\omega)} - \mathbb{E}\{X^{(\omega)}\}) \quad \text{and} \quad c_Y^{opt}(Y^{(*)}) = \mathbb{E}\{c_X^*(Y^{(*)}(M, k))\}.$$

Proof. As in the case of $d = 1$, we use the first-order Taylor expansion $\exp\{\varepsilon\} = 1 + \varepsilon + O(\varepsilon^2)$. Starting from Eq. (35), we have

$$\mathcal{D}^{d,G,\sigma\uparrow}(\mathbf{x}^{(0)}, \mathbf{x}^{(1)}, \mathbf{t}) = \arg \max_k \prod_{i=1}^m \sum_{m \in \mathcal{M}} \prod_{\omega=0}^d \exp\left\{\frac{1}{2\sigma^{(\omega)^2}} x_i^{(\omega)} y^{(\omega)}\right\} \quad (38)$$

$$\approx \arg \max_k \prod_{i=1}^m \sum_{m \in \mathcal{M}} \prod_{\omega=0}^d \left(1 + \frac{1}{2\sigma^{(\omega)^2}} x_i^{(\omega)} y^{(\omega)}\right). \quad (39)$$

Now, in Eq. (39) when factorizing the product over ω all the terms not depending on all shares $1, \dots, d$ simultaneously, i.e., $\prod_{\omega=0}^d x_i^{(\omega)} y_i^{(\omega)}$, do not depend on the key due to the perfect masking definition. Moreover, following the same argumentation as for Prop. 6, we recover that if $\forall \omega \mathbf{x}^{(\omega)}$ are centered, then $c_X^{n\text{-prod}} = \prod_{\omega=0}^d \mathbf{x}^{(\omega)}$, and $c_Y^{opt} = \sum_{m \in \mathcal{M}} \prod_{\omega=0}^d y^{(\omega)}(\mathbf{t}, k, m)$ is the *optimal prediction function* for higher-order CPA. \square

Proposition 9 shows that the normalized production combination function combined with the optimal prediction function is therefore not only optimal for d th-order CPA in case of $d = 1$ but for any value of d .

Proposition 10 (HOOD for low Gaussian noise). *For low noise variance $\sigma = \sigma^{(0)} = \dots = \sigma^{(d)}$ the optimal distinguisher (Eq. (35)) is simplified to*

$$\mathcal{D}_{opt}^{d,G,\sigma\downarrow}(\mathbf{x}^{(*)}, \mathbf{t}) = \arg \min_{k \in \mathcal{K}} \sum_{i=1}^q \max_{m \in \mathcal{M}} \sum_{\omega=0}^d (x_i^{(\omega)} - y_i^{(\omega)})^2 \quad (40)$$

$$= \arg \min_{k \in \mathcal{K}} \sum_{i=1}^q \max_{m \in \mathcal{M}} \|x_i^{(*)} - y_i^{(*)}\|_2^2. \quad (41)$$

Proof. The proof is a straightforward extension of the proof for Prop. 5. \square

5 HOOD for Precomputation Masking Tables

5.1 Classical Attacks

We now consider the attack of a masking scheme using Sbox recomputation as described in [12]. Appendix A provides a description of the underlying algorithm.

It is noteworthy that the traditional approach to reduce the multiplicity of leakage samples by a combination $c_X : \mathcal{X}^d \rightarrow \mathbb{R}$ actually *would fail* in the setup of masking tables. Indeed, the combination functions are usually considered symmetric into its arguments, meaning that any swap of the inputs does not affect the combination. This (tacit) hypothesis has been made, for instance, for

- the absolute difference $c_X^{\text{diff}}(X^{(*)}) = |X^{(0)} - X^{(1)}| = |X^{(1)} - X^{(0)}|$, and
- the centered product $c_X^{\text{n-prod}}(X^{(*)}) = (X^{(0)} - \mathbb{E}\{X^{(0)}\})(X^{(1)} - \mathbb{E}\{X^{(1)}\}) = (X^{(1)} - \mathbb{E}\{X^{(1)}\})(X^{(0)} - \mathbb{E}\{X^{(0)}\})$.

We assume here that the attacker applies the combination function on the leakages occurring during the Sbox recomputation (see Alg. 1), i.e., the attacker gains 2^n leakages

$$X^{(0)} = \varphi^{(0)}(M) + N^{(0)} \quad (42)$$

$$X^{(1)} = \varphi^{(1)}(M \oplus 1) + N^{(1)} \quad (43)$$

\vdots

$$X^{(2^n-1)} = \varphi^{(2^n-1)}(M \oplus (2^n - 1)) + N^{(2^n-1)}, \quad (44)$$

and would apply⁸ e.g., $c_X^{\text{diff}}(X^{(*)})$ or $c_X^{\text{n-prod}}(X^{(*)})$. Additionally, he measures the leakage $X^{(2^n)} = \varphi^{(2^n)}(T \oplus k \oplus M) + N^{(2^n)}$ and finally combines it with the previous combined leakages as $\bar{c}_X(X^{(2^n)}, c_X(X^{(0)}, \dots, X^{(2^n-1)}))$.

⁸ The centered product combination function naturally extends from two to any number of inputs. However, the absolute difference is inherently a binary combination function. A possible generalization in the context of arity d could be thought of as: $c_X^{\text{diff}}(X^{(*)}) = \sum_{\substack{0 \leq \omega < d \\ 0 \leq \omega' < d}} |X^{(\omega)} - X^{(\omega')}|$. Such expression remains unchanged under permutation of the inputs.

Following the methodology in [17] and assuming an equal leakage function on each share⁹, i.e., $\varphi = \varphi^{(0)} = \dots = \varphi^{(2^n)}$, the optimal function to combine the predictions would then be

$$c_Y^{\text{opt}} = \mathbb{E}\{\bar{c}_X^*(c_X^*(\varphi(M), \varphi(M \oplus 1), \dots, \varphi(M \oplus (2^n - 1))), \varphi(t \oplus k \oplus M))\} \quad (45)$$

$$= \frac{1}{2^n} \sum_{m \in \mathbb{F}_2^n} \bar{c}_X^*(c_X^*(\varphi(m), \varphi(m \oplus 1), \dots, \varphi(m \oplus (2^n - 1))), \varphi(t \oplus k \oplus m))$$

$$= \frac{1}{2^n} \sum_{m' \in \mathbb{F}_2^n} \bar{c}_X^*(c_X^*(\varphi(m' \oplus k), \dots, \varphi(m' \oplus k \oplus (2^n - 1))), \varphi(t \oplus m')) \quad (46)$$

$$= \frac{1}{2^n} \sum_{m' \in \mathbb{F}_2^n} \bar{c}_X^*(c_X^*(\varphi(M'), \varphi(m' \oplus 1), \dots, \varphi(M' \oplus (2^n - 1))), \varphi(t \oplus m')).$$

(47)

In Eq. (46), we change m for $m' = m \oplus k$ and in Eq. (47), the input terms at position ζ are replaced with those at position $\zeta \oplus k$ (because of the symmetry property of c). Accordingly, c_Y^{opt} does not depend on the key k and is even constant as the same operation can be done on $t \oplus k$, therefore higher-order CPA fails.

Of course, the Sbox precomputation masking scheme can be attacked by the classic means, that ignore the precomputation stage (i.e., lines 7 to 10 in Alg. 1 are already vulnerable alone). More precisely, if several Sboxes are computed with different plaintext bytes and different key bytes, then *collision attacks* are possible. Also a second-order attack with a combination function can be achieved, for instance, between the addition of the mask to the plaintext and the Sbox call, i.e., between lines 7 and 9 in Alg. 1 in Appendix A. Additionally, such second-order attack can also be achieved between one leakage from the Sbox recomputation (say line 3 in Alg. 1 in Appendix A) and line 8 (or preferably with line 9 for a better contrast [2]).

However, a better attack would consist in using altogether all the leakages from the Sbox recomputation with one (or more) of the samples used during the computation proper (starting from line 8, when the key is involved). One example of such strategy has been exposed in [24], which we label as 2-stage CPA attack.

Definition 2 (2-stage CPA attack [24]).

$$2 \times \text{CPA}^{mt}(\mathbf{x}, \mathbf{t}) = \arg \max_{k \in \mathcal{K}} \rho(\mathbf{x}^{(2^n)}, y^{(2^n)}(\mathbf{t}, k, \hat{\mathbf{m}})), \quad (48)$$

where $\forall i \hat{m}_i$ is the mask that maximizes the correlation between $x_i^{(\omega)}$ and $y_i^{(\omega)} = \omega \oplus m_i$ for $\omega \in [0, 2^n[$. This attack is a synergy between a horizontal and a vertical attack. For each trace (separately $\forall i$), the first attack in Eq. (48) consists in recovering the mask during the precomputation (lines 2 to 5 in Appendix A).

⁹ This assumption is reasonable for software implementation, which is the adequate scenario for masking tables.

Second, a regular CPA using a model in which both the plaintext t and the mask m are assumed as public knowledge is launched. Even if the mask \hat{m} is not recovered correctly for each trace (since 2^n leakage samples during the precomputation can be seen as small), it can be expected that the value of the mask is recovered by the first horizontal attack probabilistically well enough for it to be biased, i.e., better guessed than random. This gives a rough idea of the proof of soundness for this attack.

Nonetheless, this attack is probably not the most efficient, as it uses separately the information available from the Sbox precomputation and from the leakage of the AES algorithm proper. The next subsection investigates the optimal attack and gives approximation for high and low noise.

5.2 HOOD for Precomputation Masking Tables

When using masking tables (Alg. 1) the attacker first has all leaking samples during the precomputation, i.e., $y_i^{(\omega)} = \varphi(\omega \oplus m)$ that are independent of i for $0 \leq \omega \leq (2^n - 1)$, and, second, the leakage arising from the combination of the mask m , plaintext t_i and the key, i.e., $y_i^{(2^n)} = \varphi(t_i \oplus k \oplus m)$. Thus, all terms for $\omega \neq 2^n$ do not depend on the key and the higher-order optimal distinguisher from Eq. (35) can be further deduced.

Theorem 11 (HOOD for masking tables). *When $\varphi : \mathbb{F}_2^n \rightarrow \mathbb{R}$ is known, $N_i^{(\omega)} \sim \mathcal{N}(0, \sigma_\omega^2)$ and i.i.d. across values of $i = \{1, \dots, q\}$ and independent across the values of $\omega = \{0, \dots, 2^n\}$, then the higher-order optimal distinguisher against masking tables takes the form*

$$\begin{aligned} \mathcal{D}_{opt}^{mt,G}(\mathbf{x}^{(\star)}, \mathbf{t}) = \\ \arg \max_{k \in \mathcal{K}} \sum_{i=1}^q \log \left\{ \sum_{m \in \mathbb{F}_2^n} \exp \left\{ \sum_{\omega \in \mathbb{F}_2^n} \frac{1}{\sigma^{(\omega)^2}} \left(x_i^{(\omega)} \varphi(\omega \oplus m) - \frac{1}{2} \varphi^2(\omega \oplus m) \right) \right. \right. \\ \left. \left. + \frac{1}{\sigma^{(2^n)^2}} \left(x_i^{(2^n)} \varphi(t_i \oplus m \oplus k) - \frac{1}{2} \varphi^2(t_i \oplus m \oplus k) \right) \right\} \right\}. \end{aligned} \quad (49)$$

Proof. Straightforward computation from Eq. (35) yields

$$\arg \max_{k \in \mathcal{K}} \prod_{i=1}^q \sum_{m \in \mathbb{F}_2^n} \prod_{\omega \in \mathbb{F}_2^n} \exp \left\{ \frac{1}{\sigma^{(\omega)^2}} \left(x_i^{(\omega)} y_i^{(\omega)} - \frac{1}{2} y_i^{(\omega)^2} \right) \right\} \quad (50)$$

$$= \arg \max_{k \in \mathcal{K}} \sum_{i=1}^q \log \left\{ \sum_{m \in \mathbb{F}_2^n} \exp \left\{ \sum_{\omega \in \mathbb{F}_2^n} \frac{1}{\sigma^{(\omega)^2}} \left(x_i^{(\omega)} y_i^{(\omega)} - \frac{1}{2} y_i^{(\omega)^2} \right) \right\} \right\} \quad (51)$$

Now plugging the respective leakages as described in Subsect. 5.2 gives

$$= \arg \max_{k \in \mathcal{K}} \sum_{i=1}^q \log \left\{ \sum_{m \in \mathbb{F}_2^n} \exp \left\{ \sum_{\omega \in \mathbb{F}_2^n} \frac{1}{\sigma(\omega)^2} \left(x_i^{(\omega)} \varphi(\omega \oplus m) - \frac{1}{2} \varphi^2(\omega \oplus m) \right) \right. \right. \\ \left. \left. + \frac{1}{\sigma(2^n)^2} \left(x_i^{(2^n)} \varphi(t_i \oplus m \oplus k) - \frac{1}{2} \varphi^2(t_i \oplus m \oplus k) \right) \right\} \right\}. \quad \square \quad (52)$$

Proposition 12 (HOOD for masking tables for low SNR). *For large Gaussian noise (or low SNR) the distinguisher becomes*

$$\mathcal{D}_{opt}^{mt, G, \sigma \uparrow}(\mathbf{x}^{(*)}, \mathbf{t}) = \\ \arg \max_{k \in \mathcal{K}} \sum_{\omega \in \mathbb{F}_2^n} \frac{1}{\sigma(\omega)^2} \sum_{i=1}^q \left(\begin{array}{l} x_i^{(\omega)} x_i^{(2^n)} \sum_m \varphi(\omega \oplus m) \varphi(t_i \oplus k \oplus m) \\ - \frac{1}{2} x_i^{(2^n)} \sum_m \varphi(t_i \oplus k \oplus m) \varphi(\omega \oplus m)^2 \\ - \frac{1}{2} x_i^{(\omega)} \sum_m \varphi(\omega \oplus m) \varphi(t_i \oplus k \oplus m)^2 \\ + \frac{1}{4} \sum_m \varphi(\omega \oplus m)^2 \varphi(t_i \oplus k \oplus m)^2 \end{array} \right). \quad (53)$$

Proof. Due to the lack of space we neglect the term $\arg \max_{k \in \mathcal{K}}$ in front of each line. Starting from Eq. (35) we use again the first-order Taylor expansion $\exp\{\varepsilon\} = 1 + \varepsilon + O(\varepsilon^2)$. So,

$$\prod_{i=1}^q \sum_{m \in \mathbb{F}_2^n} \prod_{\omega=0}^{2^n} \left(1 + \frac{1}{\sigma(\omega)^2} (x_i^{(\omega)} y_i^{(\omega)} - \frac{1}{2} y_i^{(\omega)^2}) + \frac{1}{2\sigma(\omega)^4} (x_i^{(\omega)} y_i^{(\omega)} - \frac{1}{2} y_i^{(\omega)^2})^2 \right).$$

Furthermore, an expansion at second-order gives

$$\prod_{i=1}^q \sum_{m \in \mathbb{F}_2^n} \left(1 + \sum_{\omega=0}^{2^n} \frac{1}{\sigma(\omega)^2} (x_i^{(\omega)} y_i^{(\omega)} - \frac{1}{2} y_i^{(\omega)^2}) + \frac{1}{2\sigma(\omega)^4} (x_i^{(\omega)} y_i^{(\omega)} - \frac{1}{2} y_i^{(\omega)^2})^2 \right. \\ \left. + \sum_{\omega \neq \omega'} \frac{1}{\sigma(\omega)^2 \sigma(\omega')^2} (x_i^{(\omega)} y_i^{(\omega)} - \frac{1}{2} y_i^{(\omega)^2}) (x_i^{(\omega')} y_i^{(\omega')} - \frac{1}{2} y_i^{(\omega')^2}) \right). \quad (54)$$

From the perfect masking condition (see Prop. 1), the first-order term

$$\sum_{m \in \mathbb{F}_2^n} \sum_{\omega=0}^{2^n} \frac{1}{\sigma(\omega)^2} (x_i^{(\omega)} y_i^{(\omega)} - \frac{1}{2} y_i^{(\omega)^2}) = \sum_{\omega=0}^{2^n} \frac{1}{\sigma(\omega)^2} (x_i^{(\omega)} \sum_{m \in \mathbb{F}_2^n} y_i^{(\omega)} - \frac{1}{2} \sum_{m \in \mathbb{F}_2^n} y_i^{(\omega)^2})$$

is constant as well as

$$\sum_{m \in \mathbb{F}_2^n} \sum_{\omega=0}^{2^n} \frac{1}{2\sigma(\omega)^4} (x_i^{(\omega)} y_i^{(\omega)} - \frac{1}{2} y_i^{(\omega)^2})^2 \quad (55)$$

$$= \sum_{\omega=0}^{2^n} \frac{1}{2\sigma(\omega)^4} (x_i^{(\omega)^2} \sum_{m \in \mathbb{F}_2^n} y_i^{(\omega)^2} + \frac{1}{4} \sum_{m \in \mathbb{F}_2^n} y_i^{(\omega)^4} - x_i^{(\omega)} \sum_{m \in \mathbb{F}_2^n} y_i^{(\omega)^3}). \quad (56)$$

The other terms in ω, ω' can be written as

$$2 \sum_{\omega < \omega'} \frac{1}{\sigma(\omega)^2 \sigma(\omega')^2} \left(x_i^{(\omega)} x_i^{(\omega')} \sum_{m \in \mathbb{F}_2^n} y_i^{(\omega)} y_i^{(\omega')} - \frac{1}{2} x_i^{(\omega')} \sum_{m \in \mathbb{F}_2^n} y_i^{(\omega')} y_i^{(\omega)^2} \right. \\ \left. - \frac{1}{2} x_i^{(\omega)} \sum_{m \in \mathbb{F}_2^n} y_i^{(\omega)} y_i^{(\omega')^2} + \frac{1}{4} \sum_{m \in \mathbb{F}_2^n} y_i^{(\omega)^2} y_i^{(\omega')^2} \right). \quad (57)$$

Moreover, all terms involving only combinations of $\omega < d = 2^n$ do not depend on the key, thus we can further simplify to the required equation

$$\sum_{\omega \in \mathbb{F}_2^n} \frac{1}{\sigma(\omega)^2} \left(\sum_{i=1}^q x_i^{(\omega)} x_i^{(2^n)} \sum_{m \in \mathbb{F}_2^n} y^{(\omega)} y^{(2^n)} - \frac{1}{2} x_i^{(2^n)} \sum_{m \in \mathbb{F}_2^n} y^{(2^n)} y^{(\omega)^2} \right. \\ \left. - \frac{1}{2} x_i^{(\omega)} \sum_{m \in \mathbb{F}_2^n} y^{(\omega)} y^{(2^n)^2} + \frac{1}{4} \sum_{m \in \mathbb{F}_2^n} y^{(\omega)^2} y^{(2^n)^2} \right). \quad \square \quad (58)$$

Proposition 13 (Relationship between HOOD and CPA for masking tables). *When all noise variances are equal, i.e., $\sigma = \sigma^{(\omega)} \forall \omega$, Eq. (53) further simplifies to*

$$\mathcal{D}_{opt}^{mt, G, \sigma \uparrow}(\mathbf{x}^{(*)}, \mathbf{t}) = \arg \max_{k \in \mathcal{K}} \sum_{\omega \in \mathbb{F}_2^n} \sum_{i=1}^q \left(x_i^{(\omega)} x_i^{(2^n)} \sum_{m \in \mathbb{F}_2^n} \varphi(\omega \oplus m) \varphi(t_i \oplus k \oplus m) \right. \\ \left. - \frac{1}{2} x_i^{(\omega)} \sum_{m \in \mathbb{F}_2^n} \varphi(\omega \oplus m) \varphi^2(t_i \oplus k \oplus m) \right), \quad (59)$$

which becomes close to a combination of higher-order CPAs, i.e.,

$$\mathcal{D}_{C-CPA}^{mt, \sigma \uparrow}(\mathbf{x}^{(*)}, \mathbf{t}) = \arg \max_{k \in \mathcal{K}} \sum_{\omega \in \mathbb{F}_2^n} \rho(c_X^{n-prod}(\mathbf{x}^{(\omega)}, \mathbf{x}^{(2^n)}), c_Y^{opt}(\mathbf{y}^{(\omega)}, \mathbf{y}^{(2^n)})) \\ - \frac{1}{2} \rho(\mathbf{x}^{(\omega)}, c_Y^{opt}(\mathbf{y}^{(\omega)}, \mathbf{y}^{(2^n)^2})). \quad (60)$$

Proof. If all the variances are equal we have

$$\sum_{\omega \in \mathbb{F}_2^n} \frac{\varphi^2(\omega \oplus m)}{\sigma(\omega)} = \frac{1}{\sigma} \sum_{\omega \in \mathbb{F}_2^n} \varphi^2(\omega \oplus m) = \frac{1}{\sigma} \sum_{\omega \in \mathbb{F}_2^n} \varphi^2(\omega). \quad (61)$$

So, regarding the second term in Eq. (53) we have

$$\sum_{\omega \in \mathbb{F}_2^n} \frac{1}{\sigma(\omega)^2} \sum_{i=1}^q x_i^{(2^n)} \sum_{m \in \mathbb{F}_2^n} \varphi(t_i \oplus k \oplus m) \varphi(\omega \oplus m)^2 \quad (62)$$

$$= \sum_{i=1}^q x_i^{(2^n)} \sum_{m \in \mathbb{F}_2^n} \varphi(t_i \oplus k \oplus m) \sum_{\omega \in \mathbb{F}_2^n} \frac{1}{\sigma(\omega)^2} \varphi(\omega \oplus m)^2 \quad (63)$$

$$= \sum_{i=1}^q x_i^{(2^n)} \sum_{m \in \mathbb{F}_2^n} \varphi(t_i \oplus k \oplus m) \sum_{\omega \in \mathbb{F}_2^n} \frac{1}{\sigma^2} \varphi(\omega)^2 \quad (64)$$

$$= \sum_{i=1}^q x_i^{(2^n)} \sum_{m \in \mathbb{F}_2^n} \varphi(t_i \oplus m) \sum_{\omega \in \mathbb{F}_2^n} \frac{1}{\sigma^2} \varphi(\omega)^2, \quad (65)$$

which clearly does not depend on the key k . The same goes for the fourth term, which proves the first part. Now, rewriting Eq. (59) gives

$$\begin{aligned} \arg \max_{k \in \mathcal{K}} \sum_{\omega \in \mathbb{F}_2^n} \langle \mathbf{x}^{(\omega)} \mathbf{x}^{(2^n)} \mid \sum_{m \in \mathbb{F}_2^n} \varphi(\omega \oplus m) \varphi(\mathbf{t} \oplus k \oplus m) \rangle \\ - \langle \frac{1}{2} \mathbf{x}^{(\omega)} \mid \sum_{m \in \mathbb{F}_2^n} \varphi(\omega \oplus m) \varphi^2(\mathbf{t} \oplus k \oplus m) \rangle, \end{aligned} \quad (66)$$

and using the same argumentation as in the proof of Prop. 9 gives the required formula from the second part. \square

Interestingly, instead of using one CPA to recover the mask and one to recover the secret key (see Def. 2) we recover that the best methodology is to attack each share $\omega < 2^n$ with $\omega = 2^n$ (minus a regulation term) and then use a combination of all attacks. Note again that we can make the same relaxations about the leakage model as done in Subsect. 3.3.

Remark 7. For low noise, we can straightforwardly use Prop. 10, which is validated in our empirical results.

5.3 Experimental Validation

To empirically validate our theoretical results we use simulations of a first order masking scheme with precomputation tables. We target the xor operation in the precomputation phase and the AddRoundKey of the algorithm (see line 3 and line 8 of Alg. 1 in Appendix A). Thus, we have the same leakages as depicted in Examples 2, where for computationally reasons for all distinguishers we only target four bits ($n = 4$).

Remark 8. Targeting the AddRoundKey phase has some advantages. First, it allows to perform the evaluation on only four bits without the loss of generality of using a four bits Sbox. Second, in the Sbox precomputation algorithm of

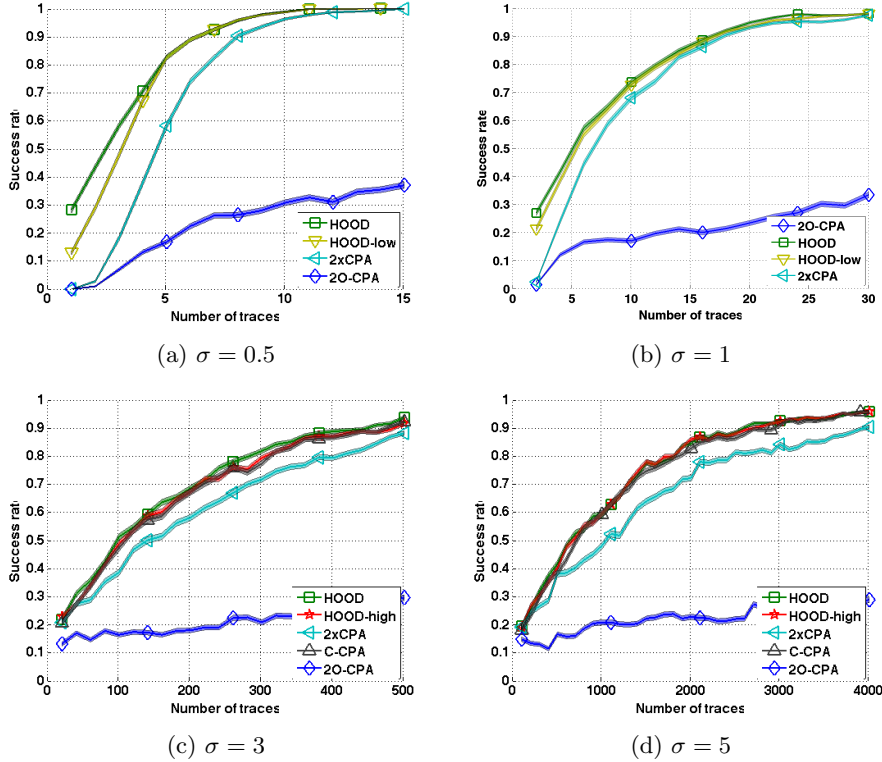


Fig. 2: Success rate for masking tables

Coron [7] the output masks are different for each entry of the Sbox and could therefore not be combined with the mask of the precomputation table. However, as in our analysis the attacker can still take advantage of the 2^n leakages of the masked inputs of the Sbox combined with the leakage of the AddRoundKey operation.

Similarly to the previous experiments, T is uniformly distributed over \mathbb{F}_2^4 and the noise is arising from a Gaussian distribution $N \sim \mathcal{N}(0, \sigma^2)$ for $\sigma = \sigma^{(0)} = \dots = \sigma^{(16)} \in \{0.5, 1, 3, 5\}$. Again to compute the success rate we conducted 500 independent experiments with uniformly distributed k^* and shaded the success rate with error bars.

Figure 2 shows the success rate. For low noise ($\sigma = 0.5$ and $\sigma = 1$) the optimal distinguisher (HOOD) and its approximation for low noise (HOOD-low) perform similar and better than the 2nd-order CPA (2O-CPA) with normalized product combination function and the 2-stage CPA in Eq. 48 (2xCPA). Naturally, all distinguisher outperform 2nd-order CPA as it only utilizes two leakages $X^{(0)}$ and $X^{(256)}$. For higher noise ($\sigma = 3$ and $\sigma = 5$) the HOOD and its approximation for high noise (HOOD-high) perform better than the 2-stage CPA (2xCPA)

and 2nd-order CPA. Moreover it can be noticed that the distinguisher based on combinations of CPA (Eq. (60)) (C-CPA) and the optimal ones are equally efficient. Accordingly, we have empirically validated that our new distinguisher approximated from the optimal distinguisher is valid for high noise and more efficient than the two-stage CPA. In particular, it requires around 1000 traces less to reach $\hat{\mathbb{P}}_S = 90\%$ for $\sigma = 5$.

6 More Case Studies

In the previous sections, it has been shown that HOOD is not only a successfully *substitute* to classical distinguishers, such as HO-CPA, (recall Sec. 4) but also is able to handle gracefully *highly multivariate* leakage settings (recall Sec. 5). In this section, we explore more examples which illustrates the advantages of the HOOD approach.

Let us consider for instance the setting of Alg. 7, that is of a masked AES. Let us further assume that the SBox recomputation (that is, lines 2 to 5) is protected (e.g., by shuffling, or because it is precomputed offline, etc.). As a further simplification, let us assume that the countermeasure is only first-order secure, i.e., $m' = m$. The attack can now target lines 1 (generation of masks), 7, 8 and 9 (masked computation). The variables manipulated on those lines are:

1. $f^{(1)}(T, k, M) = M$,
2. $f^{(2)}(T, k, M) = T$,
3. $f^{(3)}(T, k, M) = T \oplus M$,
4. $f^{(4)}(T, k, M) = k$,
5. $f^{(5)}(T, k, M) = T \oplus M \oplus k$,
6. $f^{(6)}(T, k, M) = S(T \oplus k) \oplus M$.

Let us denote $X^{(\omega)}$, for $1 \leq \omega \leq 6$, the leakages associated to manipulation of these variables.

The classical approach consists in selecting two leakages amongst those, and in computing a combination of them. However, the HOOD approach is able to simultaneously use all of them. Assuming a Gaussian noise of variance $\sigma^{(\omega)^2}$, the HOOD is:

$$\arg \max_{k \in \mathbb{F}_2^n} \prod_{i=1}^q \sum_{m \in \mathbb{F}_2^n} \exp - \sum_{\omega=1}^5 \left(\frac{x_i^{(\omega)} - \varphi^{(\omega)}(f^{(\omega)}(t_i, k, m))}{\sigma^{(\omega)}} \right)^2. \quad (67)$$

Clearly, the term $\left(\frac{x_i^{(2)} - \varphi^{(2)}(t)}{\sigma^{(2)}} \right)^2$ does not depend on the mask, and can thus be factored out from the sum over $m \in \mathbb{F}_2^n$. In addition, this term is not key dependent, hence does not impact the maximization over $k \in \mathbb{F}_2^n$. Therefore, the contribution of $X^{(2)}$ is not helping to distinguisher (but does not hurt either). On the contrary, all the remaining five leakages cannot be simplified from within Eqn. (67). We say that T is a *non-sensitive variable*; symmetrically, we can define a *sensitive variable*.

Definition 3. *A variable is said sensitive if it depends on some unknown, in our case the mask or the key.*

This definition extends previous ones (see for instance [18]). Intuitively, a sensitive variable can be used constructively (in conjunction with others), to build a successful attack. Therefore, one can claim that this 2nd-order 5-variate attack is more efficient, in terms of success rate, than any attack which would use only 2 leakages. Anyway, there was, so far, no consensus in the literature about the best strategy using two shares. It is known from the analysis of *confusion* that it is better to use $X^{(6)}$, which involves an SBox, than $X^{(5)}$; but is it wiser to combine $X^{(6)}$ with $X^{(1)}$ or $X^{(3)}$? This is very unclear. The correct answer is to prevent from selecting only two leakages when more are available, and instead exploit all of them simultaneously in a single attack (Eqn. (67)).

7 Conclusions and Perspectives

We have found the optimal distinguishers for higher-order masking, and especially, analyzed the application of second-order distinguisher and distinguisher against masking tables. This gives the first theoretical proof that for a high noise non-profiled second-order CPA becomes as efficient as the optimal distinguisher in terms of success rate. In particular, we explain that the normalized product combining function with the optimal prediction function [17] is sound and the optimal one among all (known and unknown) combination functions. We furthermore extended this result to $(d + 1)$ th-order distinguisher, which has not been analyzed before. For low noise, the optimal distinguisher does not reduce to any kind of correlation. In the application of masking tables we provide a new distinguisher based on correlation whose again is as efficient as the optimal distinguisher in case of high noise. Naturally, this new distinguisher outperforms all known (non-profiled) distinguisher for this application. Given all these results we theoretically and empirically show that for high noise the security analysis with non-profiled distinguisher is sufficient as it coincides with the optimal distinguisher.

These results raise various new perspectives. First of all, our methodology of starting from the optimal distinguisher and deriving approximated distinguisher could be applied to other scenarios. One application, for example, could be the scenario used in [19]. Moreover, future work should deal with the exact analysis of the impact of noise on the masking efficiency in a theoretical manner. This comes along with an analysis of the impact of the number of shares, in particular, with an investigation of the arguments done in [16, 25] about exponential attack complexity.

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A Algorithm of masking tables

The principle of masking tables is illustrated in Alg. 1. Instead of showing a complete masked AES, only the masked computation of AddRoundKey followed by SubBytes is shown.

We have indicated the words length of all data as n , typically, $n = 8$ bit for AES. Two random masks m and m' are drawn initially from \mathbb{F}_2^n and all the data manipulated by the algorithm will be exclusive-ored with one of the two masks.

Passing additively masked data through the Sbox is not obvious, as this operation is non-linear. Therefore, the Sbox is recomputed masked, as shown on lines 2 to 5: a new table S' , that has also size $2^n \times n$ bits, is required for this purpose. It can be seen that the addresses are accessed sequentially in the ordinal order. In the Sbox precomputation step, the key byte k is not manipulated. The leakage only concerns the mask.

Then, the masked computation which involves the masked table follows, in lines 7 to 10. Masking the plaintext is straightforward (see line 7). Key addition can be done safely as a second step, as the plaintext is already masked (see line 8). Of course, if the whole AES were to be evaluated, the demasking step (line 10) would be deferred till the end of the AES.

```

input :  $t$ , one byte of plaintext, and  $k$ , one byte of key
output: The application of AddRoundKey and SubBytes on  $t$ , i.e.,  $S(t \oplus k)$ 
1  $m \leftarrow_{\mathcal{R}} \mathbb{F}_2^n, m' \leftarrow_{\mathcal{R}} \mathbb{F}_2^n$  // Draw of random input and output masks
2 for  $\omega \in \{0, 1, \dots, 2^n - 1\}$  do // Sbox masking
3    $z \leftarrow \omega \oplus m$  // Masked input
4    $z' \leftarrow S[\omega] \oplus m'$  // Masked output
5    $S'[z] \leftarrow z'$  // Creating the masked Sbox entry
6 end
7  $t \leftarrow t \oplus m$  // Plaintext masking
8  $t \leftarrow t \oplus k$  // Masked AddRoundKey
9  $t \leftarrow S'[t]$  // Masked SubBytes

// ... normally, the full AES is computed here ...

10  $t \leftarrow t \oplus m'$  // Demasking
11 return  $t$ 

```

Algorithm 1: Beginning of a block cipher masked by Sbox precomputation