Fully-Anonymous Short Dynamic Group Signatures Without Encryption

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Abstract. Group signatures are a central tool in privacy-enhancing crypto, which allow members of a group to anonymously sign on behalf of the group. Ideally, group signatures are dynamic and thus allow to dynamically enroll new members to a group. For such schemes Bellare et al. (CT-RSA'05) proposed a strong security model (BSZ model) that preserves anonymity of a group signature even if an adversary can see arbitrary key exposures or arbitrary openings of other group signatures. All previous constructions achieving this strong anonymity notion follow the so called sign-encrypt-prove (SEP) paradigm. In contrast, all known constructions which avoid this paradigm and follow the alternative "without encryption" paradigm introduced by Bichsel et al. (SCN'10), only provide a weaker notion of anonymity (which can be problematic in practice). Until now, it was not clear if constructions following this paradigm, while providing strong anonymity in the sense of BSZ, even exist.

In this paper we positively answer this question by providing a novel approach to dynamic group signature schemes following this paradigm, which is a composition of structure preserving signatures on equivalence classes (Asiacrypt'14) and other standard primitives. Our results are interesting for various reasons: We can prove our construction following this "without encryption" paradigm secure without requiring random oracles. Moreover, when opting for an instantiation in the ROM, the so obtained scheme is extremely efficient and outperforms existing fully anonymous constructions following the SEP paradigm regarding computational efficiency. Regarding constructions providing a weaker anonymity notion than BSZ, we surprisingly even outperform the popular short BBS group signature scheme (Crypto'04) and thereby obtain shorter signatures.

Keywords: group signatures ⋄ BSZ model ⋄ CCA2-full anonymity ⋄ efficiency ⋄ structure-preserving signatures on equivalence classes

1 Introduction

Group signatures, initially introduced by Chaum and van Heyst [CvH91], allow a group manager to set up a group so that every member of this group can

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later anonymously sign messages on behalf of the group. Thereby, a dedicated authority (called opening authority) can open a given group signature to determine the identity of the actual signer. Group signatures were first rigorously formalized for static groups by Bellare et al. in [BMW03] (denoted as the BMW model). In this setting, all members are fixed at setup and also receive their honestly generated keys at setup from the group manager. This model was later extended to the dynamic case by Bellare et al. in [BSZ05] (henceforth denoted as BSZ model), where new group members can be dynamically enrolled to the group. Further, it separates the role of the issuer and the opener so that they can operate independently. Moreover, the BSZ model requires a strong anonymity notion, where anonymity of a group signature is preserved even if the adversary can see arbitrary key exposures and arbitrary openings of other group signatures. A slightly weaker model, which is used to prove the security (and in particular anonymity) of the popular BBS group signature scheme was introduced by Boneh et al. [BBS04]. This model is a relaxation of the BSZ model, and in particular weakens anonymity so that the adversary can not request openings for signatures. As it is common, we refer to this anonymity notion as CPA-full anonymity, whereas we use CCA2-full anonymity to refer to anonymity in the sense of BSZ.

Group signatures have received significant attention from the cryptographic community and also get increasing practical relevance due to technological innovations in intelligent transportation systems (e.g., floating car data, toll systems) as well as public transportation systems (i.e., smart ticketing), where user privacy is considered to play an important role (cf. EU Directive 2010/40/EU). These developments make it important to have particularly efficient group signature candidates at hand.

Over the years, two main construction paradigms for group signatures have been established. The first one is the widely used sign-encrypt-prove (SEP) paradigm [CS97]. Here, a signature is essentially an encrypted membership certificate together with a signature of knowledge, where the signer demonstrates knowledge of some signed value in the ciphertext [ACJT00,BBS04,NS04,BSZ05, DP06,BW07,BW06,Gro07,LPY15,LLM+16,LMPY16]. As an alternative to this paradigm, Bichsel et al. in [BCN+10] proposed an elegant design paradigm for group signatures which does not require to encrypt the membership certificate to produce signatures. Essentially, they use a signature scheme which supports (1) randomization of signatures so that multiple randomized versions of the same signature are unlinkable, and (2) efficiently proving knowledge of a signed value. In their construction, on joining the group, the issuer uses such a signature scheme to sign a commitment to the user's secret key. The user can then produce a group signature for a message by randomizing the signature and computing a signature of knowledge for the message, which demonstrates knowledge of the signed secret key. To open signatures, in contrast to constructions following SEP which support constant time opening by means of decrypting the ciphertext in the signature, constructions in this paradigm require a linear scan, i.e., to check a given signature against each potential user. Bichsel et al. proposed an instantation based on the randomizable pairing-based Camensich-Lysyanskaya

(CL) signature scheme [CL04] (whose EUF-CMA security is based on the interactive LRSW assumption). Recently, Pointcheval and Sanders [PS16] proposed another randomizable signature scheme (whose EUF-CMA security is proven in the generic group model), which allows to instantiate the approach due to Bichsel et al. more efficiently. We note that while these two existing constructions do not explicitly use public key encryption, the required assumptions for the scheme imply public key encryption. Yet, it seems to be beneficial regarding performance to avoid to explicitly use public key encryption.

The main drawback of existing constructions following this paradigm is that they rely on a security model that is weaker than the BSZ model [BSZ05]. In particular, anonymity only holds for users whose keys do not leak. This essentially means that once a user key leaks, all previous signatures of this user can potentially be attributed to this user. Furthermore, the model in [BCN+10] assumes that the opening authority and the issuing authority are one entity, meaning that the issuer can identify all signers when seeing group signatures. Both weakenings can be highly problematic in practical applications of group signatures. It is a natural question to ask whether it is possible to prove that constructions following this paradigm provide CCA2- or CPA-full anonymity. Unfortunately, we have to answer this negatively. Even when allowing to modify the schemes to use explicit encryption upon joining the group (which might solve the separability issue regarding issuer and opener), it is easy to see that knowledge of the user secret key breaks CCA2- as well as CPA-full anonymity in both constructions [BCN+10, PS16].[1] This confirms the intuition that the anonymity notion used by existing constructions following this paradigm is weaker than CCA2-full anonymity. The notion of CPA-full anonymity is somewhat orthogonal to the anonymity notion used by [BCN+10, PS16]: it appropriately models the leakage of user secret keys, but restricts the open oracle access. Yet, in practice it seems that the risk that a user secret keys leaks is extremely hard to quantify, which is why we deem CPA-full anonymity to be more realistic. This is also underpinned by the fact that—to the best of our knowledge—no attacks arising from the restriction of the open oracle access in CPA-full anonymity are known.

In this work we target the following open question in this context, which is of both theoretical and practical interest:

> *Is it possible to come up with group signature schemes providing those more realistic* (CPA-*full and* CCA2-*full*) *anonymity notions, where (1) compelling efficiency is reached by avoiding the explicit encryption of the membership certificate upon signing, yet (2) allowing to make explicit usage of encryption during the joining of a group?*

We, henceforth, refer to such schemes as "without encryption".

[1] Each valid group signature contains a valid randomizable signature on the secret key of the user. While group signatures only contain a proof of knowledge of the signed secret key, being in possession of secret key candidates allows to simply test them using the verification algorithm of the randomizable signature scheme. This clearly provides a distinguisher against CCA2- as well as CPA-full anonymity.

Contribution. In this paper, we contribute a novel approach to construct group signatures "without encryption". In particular, our approach is a composition of structure preserving signatures on equivalence classes (SPS-EQ) [HS14], conventional digital signatures, public key encryption, non-interactive zero-knowledge proofs, and signatures of knowledge. Although these tools may sound quite heavy, we obtain surprisingly efficient group signatures, which provably provide CCA2-full anonymity in the strongest model for dynamic group signatures, i.e., the BSZ model. In doing so, we obtain the first construction which achieves this strong security notion without an encrypted membership certificate in the signature. Thus, we can positively answer the question posed above. In addition to that, we introduce an even more efficient CPA-fully anonymous variant of our scheme.

We proceed in showing how to instantiate our constructions in the random oracle model (ROM) to obtain particularly efficient schemes. When comparing to existing CCA2-fully anonymous constructions, we outperform them in terms of computational efficiency. When comparing to the popular BBS group signature scheme [BBS04] (which achieves CPA-full anonymity in the ROM), we surprisingly obtain significantly better computational efficiency and even shorter signatures. Finally, when comparing to existing instantiations in the vein of Bichsel et al. (which provide a less realistic anonymity notion), our instantiations provide very similar computational efficiency.

2 Preliminaries

In this section, we provide some preliminaries and recall the required primitives.

Notation. Let $x \xleftarrow{R} X$ denote the operation that picks an element uniformly at random from a finite set X and assign it to x. We assume that all algorithms run in polynomial time and use $y \leftarrow \mathsf{A}(x)$ to denote that y is assigned the output of the potentially probabilistic algorithm A on input x and fresh random coins and write $y \leftarrow \mathsf{A}(x;\ r)$ to make the random coins r of A explicit. We assume that every algorithm outputs a special symbol $\perp$ on error. We write $\Pr[\Omega : \mathcal{E}]$ to denote the probability of an event $\mathcal{E}$ over the probability space Ω. A function $\epsilon : \mathbb{N} \to \mathbb{R}^+$ is called negligible if for all $c > 0$ there is a k_0 such that $\epsilon(k) < 1/k^c$ for all $k > k_0$. In the remainder of this paper, we use ϵ to denote such a negligible function.

Let $\mathbb{G}_1 = \langle P \rangle$, $\mathbb{G}_2 = \langle \hat{P} \rangle$, and $\mathbb{G}_T$ be groups of prime order p. A bilinear map $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ is a map, where it holds for all $(P, \hat{Q}, a, b) \in \mathbb{G}_1 \times \mathbb{G}_2 \times \mathbb{Z}_p^2$ that $e(aP, b\hat{Q}) = e(P, \hat{Q})^{ab}$, and $e(P, \hat{P}) \neq 1$, and e is efficiently computable. We assume the Type-3 setting, where $\mathbb{G}_1 \neq \mathbb{G}_2$ and no efficiently computable isomorphism $\psi : \mathbb{G}_2 \to \mathbb{G}_1$ is known.

Definition 1 (Bilinear Group Generator). *Let* BGGen *be an algorithm which takes a security parameter* κ *and generates a bilinear group* $\mathsf{BG} = (p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, P, \hat{P})$ *in the Type-3 setting, where the common group order* p *of the groups* $\mathbb{G}_1, \mathbb{G}_2$ *and* $\mathbb{G}_T$ *is a prime of bitlength* κ*,* e *is a pairing and* P *and* $\hat{P}$ *are generators of* $\mathbb{G}_1$ *and* $\mathbb{G}_2$*, respectively.*

Based on this, we present the required cryptographic hardness assumptions.

Decisional Diffie-Hellman Assumption (DDH). Let $\mathbb{G} = \langle P \rangle$ be a group of prime order p, such that $\log_2 p = \kappa$. Then, for all PPT adversaries $\mathcal{A}$ there exists a negligible function $\epsilon(\cdot)$ such that:

$$\Pr\left[\begin{matrix} b \xleftarrow{R} \{0,1\},\ r,s,t \xleftarrow{R} \mathbb{Z}_p, \\ b^* \leftarrow \mathcal{A}(P, rP, sP, (b\cdot(rs) + (1-b)\cdot t)P \end{matrix} \ :\ b = b^*\right] \leq 1/2 + \epsilon(\kappa).$$

Symmetric External Diffie-Hellman Assumption (SXDH). Let BG be a bilinear group generated by BGGen. Then, the SXDH assumption states that the DDH assumption holds in $\mathbb{G}_1$ and $\mathbb{G}_2$.

Additionally, we introduce a plausible assumption in the Type-3 bilinear group setting.

Computational co-Diffie-Hellman Inversion Assumption (co-CDHI): Let $\mathsf{BG} \leftarrow \mathsf{BGGen}(1^\kappa)$. The co-CDHI assumption states that for all PPT adversaries $\mathcal{A}$ there exists a negligible function $\epsilon(\cdot)$ such that:

$$\Pr\left[a \xleftarrow{R} \mathbb{Z}_p,\ C \leftarrow \mathcal{A}(\mathsf{BG}, aP, 1/a\hat{P}) \ :\ C = 1/aP\right] \leq \epsilon(\kappa).$$

Digital Signature Schemes. Subsequently, we recall a definition of digital signature schemes.

Definition 2 (Digital Signatures). *A digital signature scheme Σ is a triple* $(\mathsf{KeyGen}, \mathsf{Sign}, \mathsf{Verify})$ *of PPT algorithms, which are defined as follows:*

$\mathsf{KeyGen}(1^\kappa)$: *This algorithm takes a security parameter κ as input and outputs a secret (signing) key sk and a public (verification) key pk with associated message space $\mathcal{M}$ (we may omit to mention the message space $\mathcal{M}$).*

$\mathsf{Sign}(\mathsf{sk}, m)$: *This algorithm takes a secret key sk and a message $m \in \mathcal{M}$ as input and outputs a signature σ.*

$\mathsf{Verify}(\mathsf{pk}, m, \sigma)$: *This algorithm takes a public key pk, a message $m \in \mathcal{M}$ and a signature σ as input and outputs a bit $b \in \{0,1\}$.*

Besides correctness we require existential unforgeability under adaptively chosen message attacks ($\mathsf{EUF\text{-}CMA}$) [GMR88]. Subsequently, we recall formal definitions of these properties.

Definition 3 (Correctness). *A digital signature scheme Σ is correct, if for all κ, all $(\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KeyGen}(1^\kappa)$ and all $m \in \mathcal{M}$ it holds that*

$$\Pr[\mathsf{Verify}(\mathsf{pk}, m, \mathsf{Sign}(\mathsf{sk}, m)) = 1] = 1.$$

Definition 4 ($\mathsf{EUF\text{-}CMA}$). *A digital signature scheme Σ is $\mathsf{EUF\text{-}CMA}$ secure, if for all PPT adversaries $\mathcal{A}$ there is a negligible function $\epsilon(\cdot)$ such that*

$$\left[\begin{matrix} (\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KeyGen}(1^\kappa), \\ (m^*, \sigma^*) \leftarrow \mathcal{A}^{\mathcal{O}^{\mathsf{Sign}(\mathsf{sk},\cdot)}}(\mathsf{pk}) \end{matrix} \ :\ \begin{matrix} \mathsf{Verify}(\mathsf{pk}, m^*, \sigma^*) = 1 \ \wedge \\ m^* \notin Q^{\mathsf{Sign}} \end{matrix}\right] \leq \epsilon(\kappa)\ ,$$

where $\mathcal{A}$ has access to an oracle $\mathcal{O}^{\mathsf{Sign}}$ that allows to execute the Sign algorithm and the environment keeps track of all message queried to $\mathcal{O}^{\mathsf{Sign}}$ via Q^{Sign}.

Public Key Encryption. We also require public key encryption, which we recall below.

Definition 5. *A public key encryption scheme* Ω *is a triple* $(\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ *of PPT algorithms, which are defined as follows:*

$\mathsf{KeyGen}(1^\kappa)$: *This algorithm takes a security parameter* κ *as input and outputs a secret decryption key* sk *and a public encryption key* pk *(and we assume that the message space* $\mathcal{M}$ *is implicitly defined by* pk*).*

$\mathsf{Enc}(\mathsf{pk}, m)$: *This algorithm takes a public key* pk *and a message* $m \in \mathcal{M}$ *as input and outputs a ciphertext* c*.*

$\mathsf{Dec}(\mathsf{sk}, c)$: *This algorithm takes a secret key* sk *and a ciphertext* c *as input and outputs a message* $m \in \mathcal{M}$ *or* $\perp$*.*

We require a public key encryption scheme to be correct and $\mathsf{IND\text{-}T}$ secure and recall the formal definitions below.

Definition 6 (Correctness). *A public key encryption scheme* Ω *is correct if it holds for all* κ*, for all* $(\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KeyGen}(1^\kappa)$*, and for all messages* $m \in \mathcal{M}$ *that*

$$\Pr[\mathsf{Dec}(\mathsf{sk}, \mathsf{Enc}(\mathsf{pk}, m)) = m] = 1.$$

Definition 7 ($\mathsf{IND\text{-}T}$ Security). *Let* $\mathsf{T} \in \{\mathsf{CPA}, \mathsf{CCA2}\}$*. A public key encryption scheme* Ω *is* $\mathsf{IND\text{-}T}$ *secure, if for all PPT adversaries* $\mathcal{A}$ *there exists a negligible function* $\epsilon(\cdot)$ *such that*

$$\Pr\left[\begin{array}{l} (\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KeyGen}(1^\kappa), \\ (m_0, m_1, \mathsf{st}) \leftarrow \mathcal{A}^{\mathcal{O}_\mathsf{T}}(\mathsf{pk}), \\ b \xleftarrow{R} \{0,1\},\ c \leftarrow \mathsf{Enc}(\mathsf{pk}, m_b), \\ b^* \leftarrow \mathcal{A}^{\mathcal{O}_\mathsf{T}}(c, \mathsf{st}) \end{array} : \begin{array}{r} b = b^* \ \wedge \\ c \notin Q^{\mathsf{Dec}} \ \wedge \ |m_0| = |m_1| \end{array}\right] \leq 1/2 + \epsilon(\kappa),$$

where the adversary runs in two stages,

$$\mathcal{O}_\mathsf{T} \leftarrow \begin{cases} \emptyset & \text{if } \mathsf{T} = \mathsf{CPA}, \text{ and} \\ \{\mathcal{O}^{\mathsf{Dec}}(\mathsf{sk}, \cdot)\} & \text{if } \mathsf{T} = \mathsf{CCA2}, \end{cases}$$

and Q^{Dec} *denotes the list of queries to* $\mathcal{O}^{\mathsf{Dec}}$ *and we set* $Q^{\mathsf{Dec}} \leftarrow \emptyset$ *if* $\mathsf{T} = \mathsf{CPA}$*.*

Non-Interactive Zero-Knowledge Proof Systems. Now, we recall a standard definition of non-interactive zero-knowledge proof systems. Therefore, let L_R be an **NP**-language with witness relation $R : L_R = \{x \mid \exists\, w : R(x, w) = 1\}$.

Definition 8 (Non-Interactive Zero-Knowledge Proof System). *A non-interactive proof system* Π *is a tuple of algorithms* $(\mathsf{Setup}, \mathsf{Proof}, \mathsf{Verify})$*, which are defined as follows:*

$\mathsf{Setup}(1^\kappa)$: *This algorithm takes a security parameter* κ *as input, and outputs a common reference string* crs*.*

$\mathsf{Proof}(\mathsf{crs}, x, w)$: *This algorithm takes a common reference string* crs*, a statement* x*, and a witness* w *as input, and outputs a proof* π*.*

$\mathsf{Verify}(\mathsf{crs}, x, \pi)$: *This algorithm takes a common reference string* crs*, a statement* x*, and a proof* π *as input, and outputs a bit* $b \in \{0,1\}$*.*

Subsequently, we recall formal definition of those properties (adapted from [BGI14]).

Definition 9 (Completeness). *A non-interactive proof system* Π *is* complete*, if for every adversary* $\mathcal{A}$ *it holds that*

$$\Pr\left[\begin{array}{l}\mathsf{crs} \leftarrow \mathsf{Setup}(1^\kappa),\ (x,w) \leftarrow \mathcal{A}(\mathsf{crs}),\\ \pi \leftarrow \mathsf{Proof}(\mathsf{crs}, x, w)\end{array} : \begin{array}{r}\mathsf{Verify}(\mathsf{crs}, x, \pi) = 1\\ \wedge\ (x,w) \in R\end{array}\right] \approx 1.$$

Definition 10 (Soundness). *A non-interactive proof system* Π *is* sound*, if for every PPT adversary* $\mathcal{A}$ *there is a negligible function* $\epsilon(\cdot)$ *such that*

$$\Pr\left[\mathsf{crs} \leftarrow \mathsf{Setup}(1^\kappa),\ (x,\pi) \leftarrow \mathcal{A}(\mathsf{crs})\ :\ \mathsf{Verify}(\mathsf{crs}, x, \pi) = 1\ \wedge\ x \notin L_R\right] \leq \epsilon(\kappa).$$

If we quantify over all adversaries $\mathcal{A}$ and require $\epsilon = 0$, we have perfect soundness, but we present the definition for computationally sound proofs (arguments).

Definition 11 (Adaptive Zero-Knowledge). *A non-interactive proof system* Π *is* adaptively zero-knowledge*, if there exists a PPT simulator* $S = (\mathcal{S}_1, \mathcal{S}_2)$ *such that for every PPT adversary* $\mathcal{A}$ *there is a negligible function* $\epsilon(\cdot)$ *such that*

$$\left|\begin{array}{l}\Pr\left[\mathsf{crs} \leftarrow \mathsf{Setup}(1^\kappa)\ :\ \mathcal{A}^{\mathcal{P}(\mathsf{crs},\cdot,\cdot)}(\mathsf{crs}) = 1\right]\ -\\ \Pr\left[(\mathsf{crs}, \tau) \leftarrow \mathcal{S}_1(1^\kappa)\ :\ \mathcal{A}^{\mathcal{S}(\mathsf{crs},\tau,\cdot,\cdot)}(\mathsf{crs}) = 1\right]\end{array}\right| \leq \epsilon(\kappa),$$

where, τ *denotes a simulation trapdoor. Thereby,* $\mathcal{P}$ *and* $\mathcal{S}$ *return* $\perp$ *if* $(x,w) \notin R$ *or* $\pi \leftarrow \mathsf{Proof}(\mathsf{crs}, x, w)$ *and* $\pi \leftarrow \mathcal{S}_2(\mathsf{crs}, \tau, x)$*, respectively, otherwise.*

If $\epsilon = 0$, we have perfect adaptive zero-knowledge.

Signatures of Knowledge. Below we recall signatures of knowledge (SoKs) [CL06], where L_R is as above. For the formal notions we follow [BCC+15] and use a stronger generalization of the original extraction property termed f-extractability. A signature of knowledge (SoK) for L_R is defined as follows.

Definition 12. *A* SoK *is a tuple of PPT algorithms* $(\mathsf{Setup}, \mathsf{Sign}, \mathsf{Verify})$*, which are defined as follows:*

$\mathsf{Setup}(1^\kappa)$: *This algorithm takes a security parameter* κ *as input and outputs a common reference string* crs*. We assume that the message space* $\mathcal{M}$ *is implicitly defined by* crs*.*

$\mathsf{Sign}(\mathsf{crs}, x, w, m)$: *This algorithm takes a common reference string* crs*, a word* x*, a witness* w*, and a message* m *as input and outputs a signature* σ*.*

$\mathsf{Verify}(\mathsf{crs}, x, m, \sigma)$: *This algorithm takes a common reference string* crs*, a word* x*, a message* m*, and a signature* σ *as input and outputs a bit* $b \in \{0,1\}$*.*

Definition 13 (Correctness). *A* SoK *with respect to* L_R *is correct, if there exists a negligible function* $\epsilon(\cdot)$ *such that for all* $x \in L_R$*, for all* w *such that* $(x,w) \in R$*, and for all* $m \in \mathcal{M}$ *it holds that*

$$\Pr\left[\mathsf{crs} \leftarrow \mathsf{Setup}(1^\kappa),\ \sigma \leftarrow \mathsf{Sign}(\mathsf{crs}, x, w, m) : \mathsf{Verify}(\mathsf{crs}, x, m, \sigma) = 1\right] \geq 1 - \epsilon(\kappa).$$

Definition 14 (Simulatability). *A* SoK *with respect to* L_R *is simulatable, if there exists a PPT simulator* $\mathcal{S} = (\mathsf{SimSetup}, \mathsf{SimSign})$ *such that for all PPT adversaries* $\mathcal{A}$ *there exists a negligible function* $\epsilon(\cdot)$ *such that it holds that*

$$\left| \begin{array}{l} \Pr\left[\mathsf{crs} \leftarrow \mathsf{Setup}(1^\kappa),\ b \leftarrow \mathcal{A}^{\mathsf{Sign}(\mathsf{crs},\cdot,\cdot,\cdot)}(\mathsf{crs}) \ :\ b = 1\right] \ - \\ \Pr\left[(\mathsf{crs}, \tau) \leftarrow \mathsf{SimSetup}(1^\kappa),\ b \leftarrow \mathcal{A}^{\mathsf{Sim}(\mathsf{crs},\tau,\cdot,\cdot,\cdot)}(\mathsf{crs}) \ :\ b = 1\right] \end{array} \right| \leq \epsilon(\kappa),$$

where $\mathsf{Sim}(\mathsf{crs}, \tau, x, w, m) \coloneqq \mathsf{SimSign}(\mathsf{crs}, \tau, x, m)$ *and* Sim *only responds if* $(x,w) \in R$*.*

Definition 15 (f-Extractability). *A* SoK *with respect to* L_R *is* f*-extractable, if in addition to* $\mathcal{S}$ *there exists a PPT extractor* $\mathsf{Extract}$*, such that for all PPT adversaries* $\mathcal{A}$ *there exists a negligible function* $\epsilon(\cdot)$ *such that it holds that*

$$\Pr\left[\begin{array}{l} (\mathsf{crs}, \tau) \leftarrow \mathsf{SimSetup}(1^\kappa), \\ (x, m, \sigma) \leftarrow \mathcal{A}^{\mathsf{Sim}(\mathsf{crs},\tau,\cdot,\cdot,\cdot)}(\mathsf{crs}), \\ y \leftarrow \mathsf{Extract}(\mathsf{crs}, \tau, x, m, \sigma) \end{array} : \begin{array}{r} \mathsf{Verify}(\mathsf{crs}, x, m, \sigma) = 0 \ \vee \\ (x, m, \sigma) \in Q^{\mathsf{Sim}} \ \vee \\ (\exists\ w : (x,w) \in R \ \wedge \\ y = f(w)) \end{array}\right] \geq 1 - \epsilon(\kappa),$$

where Q^{Sim} *denotes the queries (resp. answers) of* Sim*.*

We note that, as illustrated in [BCC+15], this notion is a strengthening of the original extractability notion from [CL06] which implies the original extractability notion if f is the identity. In this case, we simply call the f-extractability property *extractability*. Analogous to [BCC+15], we require the used SoK to be at the same time extractable and *straight-line* f*-extractable* with respect to some f other than the identity, where straight-line as usual says that the extractor runs without rewinding the adversary [Fis05].

Structure Preserving Signatures on Equivalence Classes. Subsequently, we briefly recall structure-preserving signatures on equivalence classes (SPS-EQ) as presented in [HS14, FHS14]. Therefore, let p be a prime and $\ell > 1$; then $\mathbb{Z}_p^\ell$ is a vector space and one can define a projective equivalence relation on it, which propagates to $\mathbb{G}_i^\ell$ and partitions $\mathbb{G}_i^\ell$ into equivalence classes. Let $\sim_{\mathcal{R}}$ be this relation, i.e., for $M, N \in \mathbb{G}_i^\ell : M \sim_{\mathcal{R}} N \Leftrightarrow \exists\, s \in \mathbb{Z}_p^* : M = sN$. An SPS-EQ scheme now signs an equivalence class $[M]_{\mathcal{R}}$ for $M \in (\mathbb{G}_i^*)^\ell$ by signing a representative M of $[M]_{\mathcal{R}}$. One of the design goals of SPS-EQ is to guarantee that two message-signature pairs from the same equivalence class cannot be linked. Let us recall the formal definition of an SPS-EQ scheme subsequently.

Definition 16. *An* SPS-EQ *on* $\mathbb{G}_i^*$ *(for* $i \in \{1,2\}$*) consists of the following PPT algorithms:*

$\mathsf{BGGen}_{\mathcal{R}}(1^\kappa)$*: This algorithm on input of a security parameter* κ *outputs a bilinear group* BG*.*

$\mathsf{KeyGen}_{\mathcal{R}}(\mathsf{BG}, \ell)$*: This algorithm on input of a bilinear group* BG *and a vector length* $\ell > 1$ *outputs a key pair* $(\mathsf{sk}, \mathsf{pk})$*.*

$\mathsf{Sign}_{\mathcal{R}}(M, \mathsf{sk})$*: This algorithm on input a representative* $M \in (\mathbb{G}_i^*)^\ell$ *and a secret key* sk *outputs a signature* σ *for the equivalence class* $[M]_{\mathcal{R}}$*.*

$\mathsf{ChgRep}_{\mathcal{R}}(M, \sigma, \rho, \mathsf{pk})$*: This algorithm on input of a representative* $M \in (\mathbb{G}_i^*)^\ell$ *of class* $[M]_{\mathcal{R}}$*, a signature* σ *for* M*, a scalar* ρ *and a public key* pk *returns an updated message-signature pair* (M', σ')*, where* $M' = \rho \cdot M$ *is the new representative and* σ' *its updated signature.*

$\mathsf{Verify}_{\mathcal{R}}(M, \sigma, \mathsf{pk})$*: This algorithm on input of a representative* $M \in (\mathbb{G}_i^*)^\ell$*, a signature* σ *and a public key* pk *outputs a bit* $b \in \{0,1\}$*.*

$\mathsf{VKey}_{\mathcal{R}}(\mathsf{sk}, \mathsf{pk})$ *This algorithm on input a secret key* sk *and a public key* pk *outputs a bit* $b \in \{0,1\}$*.*

For security, one requires the following properties.

Definition 17 (Correctness). *An* SPS-EQ *scheme on* $(\mathbb{G}_i^*)^\ell$ *is called* correct *if for all security parameters* $\kappa \in \mathbb{N}$*,* $\ell > 1$*,* $\mathsf{BG} \leftarrow \mathsf{BGGen}_{\mathcal{R}}(1^\kappa)$*,* $(\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KeyGen}_{\mathcal{R}}(\mathsf{BG}, \ell)$*,* $M \in (\mathbb{G}_i^*)^\ell$ *and* $\rho \in \mathbb{Z}_p^*$*:*

$$\mathsf{VKey}_{\mathcal{R}}(\mathsf{sk}, \mathsf{pk}) = 1 \quad \wedge \quad \Pr\left[\mathsf{Verify}_{\mathcal{R}}(M, \mathsf{Sign}_{\mathcal{R}}(M, \mathsf{sk}), \mathsf{pk}) = 1\right] = 1 \quad \wedge$$
$$\Pr\left[\mathsf{Verify}_{\mathcal{R}}(\mathsf{ChgRep}_{\mathcal{R}}(M, \mathsf{Sign}_{\mathcal{R}}(M, \mathsf{sk}), \rho, \mathsf{pk}), \mathsf{pk}) = 1\right] = 1.$$

For EUF-CMA security, outputting a valid message-signature pair, corresponding to an unqueried equivalence class, is considered to be a forgery:

Definition 18 (EUF-CMA). *An* SPS-EQ *over* $(\mathbb{G}_i^*)^\ell$ *is* existentially unforgeable under adaptively chosen-message attacks*, if for all PPT adversaries* $\mathcal{A}$ *with access to a signing oracle* $\mathcal{O}^{\mathsf{Sign}_{\mathcal{R}}}$*, there is a negligible function* $\epsilon(\cdot)$ *such that:*

$$\Pr\left[\begin{array}{l}\mathsf{BG} \leftarrow \mathsf{BGGen}_{\mathcal{R}}(1^\kappa), \\ (\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KeyGen}_{\mathcal{R}}(\mathsf{BG}, \ell), \\ (M^*, \sigma^*) \leftarrow \mathcal{A}^{\mathcal{O}^{\mathsf{Sign}_{\mathcal{R}}}(\mathsf{sk}, \cdot)}(\mathsf{pk})\end{array} : \begin{array}{r}[M^*]_{\mathcal{R}} \neq [M]_{\mathcal{R}} \quad \forall M \in Q^{\mathsf{Sign}_{\mathcal{R}}} \quad \wedge \\ \mathsf{Verify}_{\mathcal{R}}(M^*, \sigma^*, \mathsf{pk}) = 1\end{array}\right] \leq \epsilon(\kappa),$$

where $Q^{\mathsf{Sign}_{\mathcal{R}}}$ *is the set of queries that* $\mathcal{A}$ *has issued to the signing oracle* $\mathcal{O}^{\mathsf{Sign}_{\mathcal{R}}}$*.*

Besides EUF-CMA security, an additional security property for SPS-EQ was introduced in [FHS15].

Definition 19 (Perfect Adaption of Signatures). *An* SPS-EQ *scheme on* $(\mathbb{G}_i^*)^\ell$ perfectly adapts signatures *if for all tuples* $(\mathsf{sk}, \mathsf{pk}, M, \sigma, \rho)$ *where it holds that* $\mathsf{VKey}_{\mathcal{R}}(\mathsf{sk}, \mathsf{pk}) = 1$*,* $\mathsf{Verify}_{\mathcal{R}}(M, \sigma, \mathsf{pk}) = 1$*,* $M \in (\mathbb{G}_i^*)^\ell$*, and* $\rho \in \mathbb{Z}_p^*$*, the distributions* $(\rho M, \mathsf{Sign}_{\mathcal{R}}(\rho M, \mathsf{sk}))$ *and* $\mathsf{ChgRep}_{\mathcal{R}}(M, \sigma, \rho, \mathsf{pk})$ *are identical.*

An instantiation providing all above security properties is provided in [FHS14, FHS15]. Here, assuming the DDH assumption to hold on the message space yields that different message-signature pairs from the same equivalence class cannot be linked.

3 Dynamic Group Signatures

Subsequently, we recall the established model for dynamic group signatures. We follow Bellare et al. [BSZ05] (BSZ model), with the slight difference that we relax the perfect correctness to only require computational correctness. Furthermore, we also present the weaker anonymity notion of CPA-full anonymity from [BBS04] and the notion of opening soundness [SSE+12], which addresses issues regarding hijacking of signatures by malicious group members. In particular, we use the notion of weak opening soundness, where the opening authority is required to be honest, since we believe that this notion provides a good trade-off between computational efficiency of potential instantiations and expected security guarantees.

$\mathsf{GKeyGen}(1^\kappa)$: This algorithm takes a security parameter κ as input and outputs a triple $(\mathsf{gpk}, \mathsf{ik}, \mathsf{ok})$ containing the group public key gpk, the issuing key ik as well as the opening key ok.

$\mathsf{UKeyGen}(1^\kappa)$: This algorithm takes a security parameter κ as input and outputs a user key pair $(\mathsf{usk}_i, \mathsf{upk}_i)$.

$\mathsf{Join}(\mathsf{gpk}, \mathsf{usk}_i, \mathsf{upk}_i)$: This algorithm takes the group public key gpk and the user's key pair $(\mathsf{usk}_i, \mathsf{upk}_i)$ as input. It interacts with the Issue algorithm and outputs the group signing key gsk_i of user i on success.

$\mathsf{Issue}(\mathsf{gpk}, \mathsf{ik}, i, \mathsf{upk}_i, \mathsf{reg})$: This algorithm takes the group public key gpk, the issuing key ik, the index i of a user, user i's public key upk_i, and the registration table reg as input. It interacts with the Join algorithm and adds an entry for user i in reg on success. In the end, it returns reg.

$\mathsf{Sign}(\mathsf{gpk}, \mathsf{gsk}_i, m)$: This algorithm takes the group public key gpk, a group signing key gsk_i, and a message m as input and outputs a group signature σ.

$\mathsf{Verify}(\mathsf{gpk}, m, \sigma)$: This algorithm takes the group public key gpk, a message m and a signature σ as input and outputs a bit $b \in \{0, 1\}$.

$\mathsf{Open}(\mathsf{gpk}, \mathsf{ok}, \mathsf{reg}, m, \sigma)$: This algorithm takes the group public key gpk, the opening key ok, the registration table reg, a message m, and a valid signature σ on m under gpk as input. It extracts the identity of the signer and returns a pair (i, τ), where τ is a proof.

$\mathsf{Judge}(\mathsf{gpk}, m, \sigma, i, \mathsf{upk}_i, \tau)$: This algorithm takes the group public key gpk, a message m, a valid signature σ on m under gpk, an index i, user i's public key upk_i, and a proof τ. It returns a bit $b \in \{0, 1\}$.

3.1 Oracles

In the following we recall the definitions of the oracles required by the security model. We assume that the keys $(\mathsf{gpk}, \mathsf{ik}, \mathsf{ok})$ created in the experiments are implicitly available to the oracles. Furthermore, the environment maintains the sets $\mathtt{HU}, \mathtt{CU}$ of honest and corrupted users, the set $\mathtt{GS}$ of message-signature tuples returned by the challenge oracle, the lists $\mathsf{upk}, \mathsf{usk}, \mathsf{gsk}$ of user public keys, user private keys, and group signing keys. The list upk is publicly readable and the environment also maintains the registration table reg. Finally, $\mathtt{SI}$ represents a

variable that ensures the consistency of subsequent calls to CrptU and SndToI. All sets are initially empty and all list entries are initially set to $\perp$. In the context of lists, we use $\mathsf{upk}_i, \mathsf{usk}_i$, etc. as shorthand for $\mathsf{upk}[i], \mathsf{usk}[i]$, etc.

$\mathsf{AddU}(i)$: This oracle takes an index i as input. If $i \in \mathtt{CU} \cup \mathtt{HU}$ it returns $\perp$. Otherwise it runs $(\mathsf{usk}_i, \mathsf{upk}_i) \leftarrow \mathsf{UKeyGen}(1^\kappa)$ and

$$(\mathsf{reg}, \mathsf{gsk}_i) \leftarrow \langle \mathsf{Issue}(\mathsf{gpk}, \mathsf{ik}, i, \mathsf{upk}_i, \mathsf{reg}) \leftrightarrow \mathsf{Join}(\mathsf{gpk}, \mathsf{usk}_i, \mathsf{upk}_i) \rangle.$$

Finally, it sets $\mathtt{HU} \leftarrow \mathtt{HU} \cup \{i\}$ and returns upk_i.

$\mathsf{CrptU}(i, \mathsf{upk}_j)$: This oracle takes an index i and user public key upk_j as input. If $i \in \mathtt{CU} \cup \mathtt{HU}$ it returns $\perp$. Otherwise it sets $\mathtt{CU} \leftarrow \mathtt{CU} \cup \{i\}$, $\mathtt{SI} \leftarrow i$ and $\mathsf{upk}_i \leftarrow \mathsf{upk}_j$.

$\mathsf{SndToI}(i)$: This oracle takes an index i as input. If $i \neq \mathtt{SI}$ it returns $\perp$. Otherwise, it plays the role of an honest issuer when interacting with the corrupted user i. More precisely, it runs

$$\mathsf{reg} \leftarrow \langle \mathsf{Issue}(\mathsf{gpk}, \mathsf{ik}, i, \mathsf{upk}_i, \mathsf{reg}) \leftrightarrow \mathcal{A} \rangle$$

thereby interacting with the dishonest user who aims to join the group but does not necessarily follow the Join protocol.

$\mathsf{SndToU}(i)$: This oracle takes an index i as input. If $i \notin \mathtt{HU}$ it sets $\mathtt{HU} \leftarrow \mathtt{HU} \cup \{i\}$, runs $(\mathsf{usk}_i, \mathsf{upk}_i) \leftarrow \mathsf{UKeyGen}(1^\kappa)$. Then it plays the role of the honest user i when interacting with a corrupted issuer. More precisely, it runs

$$\mathsf{gsk}_i \leftarrow \langle \mathcal{A} \leftrightarrow \mathsf{Join}(\mathsf{gpk}, \mathsf{usk}_i, \mathsf{upk}_i) \rangle,$$

thereby interacting with the dishonest issuer who does not necessarily follow the Issue protocol.

$\mathsf{USK}(i)$: This oracle takes an index i as input and returns $(\mathsf{gsk}_i, \mathsf{usk}_i)$.

$\mathsf{RReg}(i)$: This oracle takes an index i as input and returns reg_i.

$\mathsf{WReg}(i, \rho)$: This oracle takes an index i and a registration table entry ρ as input and sets $\mathsf{reg}_i \leftarrow \rho$.

$\mathsf{GSig}(i, m)$: This oracle takes an index i and a message m as input. If $i \notin \mathtt{HU}$ or $\mathsf{gsk}_i = \perp$ it returns $\perp$ and $\sigma \leftarrow \mathsf{Sign}(\mathsf{gpk}, \mathsf{gsk}_i, m)$ otherwise.

$\mathsf{Ch}(b, i_0, i_1, m)$: This algorithm takes a bit b, two indexes i_0 and i_1, and a message m as input. If $\{i_0, i_1\} \not\subseteq \mathtt{HU} \ \vee \ \mathsf{gsk}_{i_0} = \perp \ \vee \ \mathsf{gsk}_{i_1} = \perp$ it returns $\perp$. Otherwise, it computes $\sigma \leftarrow \mathsf{Sign}(\mathsf{gpk}, \mathsf{gsk}_{i_b}, m)$, sets $\mathtt{GS} \leftarrow \mathtt{GS} \cup \{(m, \sigma)\}$ and returns σ.

$\mathsf{Open}(m, \sigma)$: This oracle takes a message m and a signature σ as input. If $(m, \sigma) \in \mathtt{GS}$ or $\mathsf{Verify}(\mathsf{gpk}, m, \sigma) = 0$ it returns $\perp$. Otherwise, it returns $(i, \tau) \leftarrow \mathsf{Open}(\mathsf{gpk}, \mathsf{ok}, \mathsf{reg}, m, \sigma)$.

3.2 Security Notions

We require dynamic group signatures to be correct, anonymous, traceable, non-frameable, and weakly opening sound. We recall the formal definitions below.

Correctness, informally requires that everything works correctly if everyone behaves honestly. Note that we relax perfect correctness to computational correctness.

Definition 20 (Correctness). *A* GSS *is correct, if for all PPT adversaries* $\mathcal{A}$ *there is a negligible function* $\epsilon(\cdot)$ *such that*

$$\Pr\left[\begin{array}{l|rl}(\mathsf{gpk},\mathsf{ik},\mathsf{ok}) \leftarrow \mathsf{GKeyGen}(1^\kappa), & & \\ \mathcal{O} \leftarrow \{\mathsf{AddU}(\cdot),\mathsf{RReg}(\cdot)\}, & & \mathsf{Verify}(\mathsf{gpk},m,\sigma)=1 \wedge \\ (i,m) \leftarrow \mathcal{A}^{\mathcal{O}}(\mathsf{gpk}), & : & i \in \mathtt{HU} \wedge \mathsf{gsk}_i \neq \bot \wedge i=j \wedge \\ \sigma \leftarrow \mathsf{Sign}(\mathsf{gpk},\mathsf{gsk}_i,m), & & \mathsf{Judge}(\mathsf{gpk},m,\sigma,i,\mathsf{upk}_i,\tau)=1 \\ (j,\tau) \leftarrow \mathsf{Open}(\mathsf{gpk},\mathsf{ok},\mathsf{reg},m,\sigma) & & \end{array}\right] \geq 1-\epsilon(\kappa).$$

Anonymity captures the intuition that group signers remain anonymous for everyone except the opening authority. Thereby, the adversary can see arbitrary key exposures. Furthermore, in the CCA2 case the adversary can even request arbitrary openings of other group signatures.

Definition 21 (T-Full Anonymity). *Let* $\mathsf{T} \in \{\mathsf{CPA},\mathsf{CCA2}\}$. *A* GSS *is* T*-fully anonymous, if for all PPT adversaries* $\mathcal{A}$ *there is a negligible function* $\epsilon(\cdot)$ *such that*

$$\Pr\left[\begin{array}{l|l}(\mathsf{gpk},\mathsf{ik},\mathsf{ok}) \leftarrow \mathsf{GKeyGen}(1^\kappa),\ b \xleftarrow{R} \{0,1\}, & b=b^* \\ b^* \leftarrow \mathcal{A}^{\mathcal{O}_\mathsf{T}}(\mathsf{gpk},\mathsf{ik}) & \end{array}\right] \leq 1/2+\epsilon(\kappa),$$

where

$$\mathcal{O}_\mathsf{T} \leftarrow \begin{cases} \left\{\begin{array}{l}\mathsf{Ch}(b,\cdot,\cdot,\cdot),\ \mathsf{SndToU}(\cdot),\mathsf{WReg}(\cdot,\cdot),\\ \mathsf{USK}(\cdot),\ \mathsf{CrptU}(\cdot,\cdot)\end{array}\right\} & \textit{if } \mathsf{T}=\mathsf{CPA}, \textit{ and} \\ \left\{\begin{array}{l}\mathsf{Ch}(b,\cdot,\cdot,\cdot),\ \mathsf{Open}(\cdot,\cdot),\ \mathsf{SndToU}(\cdot),\\ \mathsf{WReg}(\cdot,\cdot),\ \mathsf{USK}(\cdot),\ \mathsf{CrptU}(\cdot,\cdot)\end{array}\right\} & \textit{if } \mathsf{T}=\mathsf{CCA2}. \end{cases}$$

Traceability models the requirement that, as long as the issuer behaves honestly and its secret key remains secret, every valid signature can be traced back to a user. This must even hold if the opening authority colludes with malicious users.

Definition 22 (Traceability). *A* GSS *is traceable, if for all PPT adversaries* $\mathcal{A}$ *there is a negligible function* $\epsilon(\cdot)$ *such that*

$$\Pr\left[\begin{array}{l|rl}(\mathsf{gpk},\mathsf{ik},\mathsf{ok}) \leftarrow \mathsf{GKeyGen}(1^\kappa), & & \\ \mathcal{O} \leftarrow \{\mathsf{SndToI}(\cdot),\ \mathsf{AddU}(\cdot), & & \mathsf{Verify}(\mathsf{gpk},m,\sigma)=1 \wedge \\ \mathsf{RReg}(\cdot),\ \mathsf{USK}(\cdot),\ \mathsf{CrptU}(\cdot)\}, & : & (i=\bot \vee \\ (m,\sigma) \leftarrow \mathcal{A}^{\mathcal{O}}(\mathsf{gpk},\mathsf{ok}), & & \mathsf{Judge}(\mathsf{gpk},m,\sigma,i,\mathsf{upk}_i,\tau)=0) \\ (i,\tau) \leftarrow \mathsf{Open}(\mathsf{gpk},\mathsf{ok},\mathsf{reg},m,\sigma) & & \end{array}\right] \leq \epsilon(\kappa).$$

Non-frameability requires that no one can forge signatures for honest users. This must even hold if the issuing authority, the opening authority, and, other malicious users collude.

Definition 23 (Non-Frameability). *A* GSS *is non-frameable, if for all PPT adversaries* $\mathcal{A}$ *there is a negligible function* $\epsilon(\cdot)$ *such that*

$$\Pr\left[\begin{array}{l|rl}(\mathsf{gpk},\mathsf{ik},\mathsf{ok}) \leftarrow \mathsf{GKeyGen}(1^\kappa), & & \mathsf{Verify}(\mathsf{gpk},m,\sigma)=1 \wedge \\ \mathcal{O} \leftarrow \{\mathsf{SndToU}(\cdot),\ \mathsf{WReg}(\cdot,\cdot), & & i \in \mathtt{HU} \wedge \mathsf{gsk}_i \neq \bot \wedge \\ \mathsf{GSig}(\cdot,\cdot),\ \mathsf{USK}(\cdot),\ \mathsf{CrptU}(\cdot)\}, & : & i \notin \mathtt{USK} \wedge (i,m) \notin \mathtt{SIG} \wedge \\ (m,\sigma,i,\tau) \leftarrow \mathcal{A}^{\mathcal{O}}(\mathsf{gpk},\mathsf{ok},\mathsf{ik}) & & \mathsf{Judge}(\mathsf{gpk},m,\sigma,i,\mathsf{upk}_i,\tau)=1 \end{array}\right] \leq \epsilon(\kappa),$$

where $\mathtt{USK}$ *and* $\mathtt{SIG}$ *denote the queries to the oracles* USK *and* Sign*, respectively.*

Weak opening soundness [SSE+12] essentially requires that no malicious user can claim ownership of a signature issued by an honest user, as long as the opening authority behaves honestly.

Definition 24 (Weak Opening Soundness). *A* GSS *is weakly opening sound, if for all PPT adversaries* $\mathcal{A}$ *there is a negligible function* $\epsilon(\cdot)$ *such that*

$$\Pr\left[\begin{array}{l}(\mathsf{gpk},\mathsf{ik},\mathsf{ok}) \leftarrow \mathsf{GKeyGen}(1^\kappa),\\ \mathcal{O} \leftarrow \{\mathsf{AddU}(\cdot)\},\\ (m,i,j,\mathsf{st}) \leftarrow \mathcal{A}^{\mathcal{O}}(\mathsf{gpk}),\\ \sigma \leftarrow \mathsf{Sign}(\mathsf{gpk},\mathsf{gsk}_i,m),\\ \tau \leftarrow \mathcal{A}^{\mathcal{O}}(\mathsf{st},\sigma,\mathsf{gsk}_j)\end{array} : \begin{array}{r} i \neq j \ \wedge \ \{i,j\} \subseteq \mathtt{HU} \ \wedge \\ \mathsf{Judge}(\mathsf{gpk},m,\sigma,j,\mathsf{upk}_j,\tau) = 1\end{array}\right] \leq \epsilon(\kappa).$$

4 Construction

Our construction idea is inspired by [HS14], who use the "unlinkability" feature of $\mathsf{SPS\text{-}EQ}$ signatures to construct anonymous credentials. Essentially, a credential in their approach represents a signature for an equivalence class and to show a credential they always present a newly re-randomized signature to a random representative of this class. While, due to the intuitive relation of anonymous credentials and group signatures, it might seem straightforward to map this idea to group signatures, it turns out that there are various subtle, yet challenging issues which we need to solve.

First, the anonymity experiment does not put many restrictions on the Ch and the USK oracle. In particular, Ch can be called an arbitrary number of times and USK can be called for all users. Thus, the user secret keys must be of a form so that it is possible to embed decision problem instances into them upon simulation, while not influencing their distribution (as otherwise an adversary would be able to detect the simulation). More precisely, anonymity in our paradigm seems to require that the user keys contain no $\mathbb{Z}_p$ elements, which, in turn, renders the non-frameability proof more difficult. Second, if $\mathsf{CCA2}$-full anonymity is required, the simulatability of the open oracle needs to be ensured, while the reduction must not be aware of the opening information (as otherwise the reduction could trivially break anonymity on its own and would be meaningless). This seems to crucially require a proof system providing rather strong extractability properties. To maintain efficiency, it is important to find the mildest possible requirement which still allows the security proofs to go through. Finally, the non-frameability adversary is given the issuing key as well as the opening key. Thus, the reduction must be able to simulate the whole join process without knowledge of a user secret key in a way that the distribution change is not even detectable with the knowledge of these keys.

Now, before we present our full construction, we briefly revisit our basic idea. In our scheme, each group member chooses a secret vector $(R,P) \in (\mathbb{G}_1^*)^2$ representing an equivalence class where the second component P is identical for

all users. When joining the group, a blinded version $q \cdot (R, P)$ with $q \xleftarrow{R} \mathbb{Z}_p^*$ of this vector, i.e., another representative of the class, is signed by the issuer using an SPS-EQ, and, by the re-randomization property of SPS-EQ and the feature to publicly change representatives of classes, the user thus obtains a signature on the unblinded key (R, P) using $\mathsf{ChgRep}_{\mathcal{R}}$ with q^{-1}. To provide a means to open signatures, a user additionally has to provide an encryption of a value $\hat{R} \in \mathbb{G}_2$ such that $e(R, \hat{P}) = e(P, \hat{R})$ on joining (and has to sign the ciphertext as an identity proof). The group signing key of the user is then the pair consisting of the vector (R, P) and the SPS-EQ signature on this vector. A group member can sign a message m on behalf of the group by randomizing it's group signing key and computing a signature of knowledge (SoK) to the message m proving knowledge of the used randomizer.[2] The group signature is then the randomized group signing key and the SoK.

Very roughly, a signer then remains anonymous since it is infeasible to distinguish two randomized user secret keys under DDH in $\mathbb{G}_1$. The unforgeability of SPS-EQ ensures that each valid signature can be opened. Furthermore, it is hard to forge signatures of honest group members since it is hard to unblind a user secret key under co-CDHI and the signature of knowledge essentially ensures that we can extract such an unblinded user secret key from a successful adversary.

4.1 Detailed Construction

In our scheme, we require zero-knowledge proofs upon Join and Open. The **NP** relation R_{J} corresponding to the proof carried out in Join is defined as

$$((U_i, Q, \hat{C}_{\mathsf{J}_i}, \mathsf{pk}_{\mathsf{O}}), (r, \omega)) \in R_{\mathsf{J}} \iff \hat{C}_{\mathsf{J}_i} = \Omega.\mathsf{Enc}(\mathsf{pk}_{\mathsf{O}}, r\hat{P};\ \omega) \ \wedge\ U_i = r \cdot Q.$$

The **NP** relation R_{O} corresponding to the proof carried out upon Open is

$$((\hat{C}_{\mathsf{J}_i}, \mathsf{pk}_{\mathsf{O}}, \sigma), (\mathsf{sk}_{\mathsf{O}}, \hat{R})) \in R_{\mathsf{O}} \iff \hat{R} = \Omega.\mathsf{Dec}(\mathsf{sk}_{\mathsf{O}}, \hat{C}_{\mathsf{J}_i}) \ \wedge \\ \mathsf{pk}_{\mathsf{O}} \equiv \mathsf{sk}_{\mathsf{O}} \ \wedge\ e(\sigma_1[1][1], \hat{P}) = e(\sigma_1[1][2], \hat{R}).$$

Thereby, $\mathsf{pk} \equiv \mathsf{sk}$ denotes the consistency of pk and sk.

Furthermore, upon Sign we require a signature of knowledge which is with respect to the following **NP** relation R_{S}.

$$((P, Q), \rho) \in R_{\mathsf{S}} \iff Q = \rho \cdot P\ .$$

For the sake of compact presentation, we assume that the languages defined by R_{J}, R_{O}, R_{S} are implicit in the CRSs $\mathsf{crs}_{\mathsf{J}}$, $\mathsf{crs}_{\mathsf{O}}$, and $\mathsf{crs}_{\mathsf{S}}$, respectively. The full construction is presented as Scheme 1.

[2] For technical reasons and in particular for extractability, we actually require a signature of knowledge for message $m' = \sigma_1 || m$, where σ_1 contains the re-randomized user secret key and SPS-EQ signature.

$\mathsf{GKeyGen}(1^\kappa)$: Run $\mathsf{BG} \leftarrow \mathsf{BGGen}_{\mathcal{R}}(1^\kappa)$, $(\mathsf{sk}_{\mathcal{R}}, \mathsf{pk}_{\mathcal{R}}) \leftarrow \mathsf{KeyGen}_{\mathcal{R}}(\mathsf{BG}, 2)$, $(\mathsf{sk}_{\mathsf{O}}, \mathsf{pk}_{\mathsf{O}}) \leftarrow \Omega.\mathsf{KeyGen}(1^\kappa)$, $\mathsf{crs}_{\mathsf{J}} \leftarrow \Pi.\mathsf{Setup}(1^\kappa)$, $\mathsf{crs}_{\mathsf{O}} \leftarrow \Pi.\mathsf{Setup}(1^\kappa)$, $\mathsf{crs}_{\mathsf{S}} \leftarrow \mathsf{SoK.Setup}(1^\kappa)$, set $\mathsf{gpk} \leftarrow (\mathsf{pk}_{\mathcal{R}}, \mathsf{pk}_{\mathsf{O}}, \mathsf{crs}_{\mathsf{J}}, \mathsf{crs}_{\mathsf{O}}, \mathsf{crs}_{\mathsf{S}})$, $\mathsf{ik} \leftarrow \mathsf{sk}_{\mathcal{R}}$, $\mathsf{ok} \leftarrow \mathsf{sk}_{\mathsf{O}}$ and return $(\mathsf{gpk}, \mathsf{ik}, \mathsf{ok})$.

$\mathsf{UKeyGen}(1^\kappa)$: Return $(\mathsf{usk}_i, \mathsf{upk}_i) \leftarrow \Sigma.\mathsf{KeyGen}(1^\kappa)$.

$\mathsf{Join}^{(1)}(\mathsf{gpk}, \mathsf{usk}_i, \mathsf{upk}_i)$: Choose $q, r \xleftarrow{R} \mathbb{Z}_p^*$, set $(U_i, Q) \leftarrow (r \cdot qP, qP)$, and output $M_{\mathsf{J}} \leftarrow ((U_i, Q), \hat{C}_{\mathsf{J}_i}, \sigma_{\mathsf{J}_i}, \pi_{\mathsf{J}_i})$ and $\mathsf{st} \leftarrow (\mathsf{gpk}, q, U_i, Q)$, where

$$\hat{C}_{\mathsf{J}_i} \leftarrow \Omega.\mathsf{Enc}(\mathsf{pk}_{\mathsf{O}}, r\hat{P};\ \omega),\ \ \sigma_{\mathsf{J}_i} \leftarrow \Sigma.\mathsf{Sign}(\mathsf{usk}_i, \hat{C}_{\mathsf{J}_i}),$$

$$\pi_{\mathsf{J}_i} \leftarrow \Pi.\mathsf{Proof}(\mathsf{crs}_{\mathsf{J}}, (U_i, Q, \hat{C}_{\mathsf{J}_i}, \mathsf{pk}_{\mathsf{O}}), (r, \omega)).$$

$\mathsf{Issue}(\mathsf{gpk}, \mathsf{ik}, i, \mathsf{upk}_i, \mathsf{reg})$: Receive $M_{\mathsf{J}} = ((U_i, Q), \hat{C}_{\mathsf{J}_i}, \sigma_{\mathsf{J}_i}, \pi_{\mathsf{J}_i})$, return reg and send σ' to user i, where

$$\mathsf{reg}_i \leftarrow (\hat{C}_{\mathsf{J}_i}, \sigma_{\mathsf{J}_i}),\ \ \sigma' \leftarrow \mathsf{Sign}_{\mathcal{R}}((U_i, Q), \mathsf{sk}_{\mathcal{R}}),$$

if $\Pi.\mathsf{Verify}(\mathsf{crs}_{\mathsf{J}}, (U_i, Q, \hat{C}_{\mathsf{J}_i}, \mathsf{pk}_{\mathsf{O}}), \pi_{\mathsf{J}_i}) = 1 \ \wedge\ \Sigma.\mathsf{Verify}(\mathsf{upk}_i, \hat{C}_{\mathsf{J}_i}, \sigma_{\mathsf{J}_i}) = 1$, and return $\perp$ otherwise.

$\mathsf{Join}^{(2)}(\mathsf{st}, \sigma')$: Parse st as $(\mathsf{gpk}, q, U_i, Q)$ and return gsk_i, where

$$\mathsf{gsk}_i = ((rP, P), \sigma) \leftarrow \mathsf{ChgRep}_{\mathcal{R}}((U_i, Q), \sigma', q^{-1}, \mathsf{pk}_{\mathcal{R}}),$$

if $\mathsf{Verify}_{\mathcal{R}}((U_i, Q), \sigma', \mathsf{pk}_{\mathcal{R}}) = 1$, and return $\perp$ otherwise.

$\mathsf{Sign}(\mathsf{gpk}, \mathsf{gsk}_i, m)$: Choose $\rho \xleftarrow{R} \mathbb{Z}_p^*$, and return $\sigma \leftarrow (\sigma_1, \sigma_2)$, where

$$\sigma_1 \leftarrow \mathsf{ChgRep}_{\mathcal{R}}(\mathsf{gsk}_i, \rho, \mathsf{pk}_{\mathcal{R}}),\ \ \sigma_2 \leftarrow \mathsf{SoK.Sign}(\mathsf{crs}_{\mathsf{S}}, (P, \sigma_1[1][2]), \rho, \sigma_1||m).$$

$\mathsf{Verify}(\mathsf{gpk}, m, \sigma)$: Return 1 if the following holds, and 0 otherwise:

$$\mathsf{Verify}_{\mathcal{R}}(\sigma_1, \mathsf{pk}_{\mathcal{R}}) = 1 \quad \wedge \quad \mathsf{SoK.Verify}(\mathsf{crs}_{\mathsf{S}}, (P, \sigma_1[1][2]), \sigma_1||m, \sigma_2) = 1.$$

$\mathsf{Open}(\mathsf{gpk}, \mathsf{ok}, \mathsf{reg}, m, \sigma)$: Parse σ as (σ_1, σ_2), and ok as sk_{O}. Obtain the lowest index i,[a] so that it holds for $(\hat{C}_{\mathsf{J}_i}, \sigma_{\mathsf{J}_i}) \leftarrow \mathsf{reg}_i$ that $\hat{R} \leftarrow \Omega.\mathsf{Dec}(\mathsf{sk}_{\mathsf{O}}, \hat{C}_{\mathsf{J}_i})$ and $e(\sigma_1[1][1], \hat{P}) = e(\sigma_1[1][2], \hat{R})$. Return (i, τ) and $\perp$ if no such entry exists, where

$$\tau \leftarrow (\pi_{\mathsf{O}}, \hat{C}_{\mathsf{J}_i}, \sigma_{\mathsf{J}_i}),\ \text{and}\ \pi_{\mathsf{O}} \leftarrow \Pi.\mathsf{Proof}(\mathsf{crs}_{\mathsf{O}}, (\hat{C}_{\mathsf{J}_i}, \mathsf{pk}_{\mathsf{O}}, \sigma), (\mathsf{sk}_{\mathsf{O}}, \hat{R})).$$

$\mathsf{Judge}(\mathsf{gpk}, m, \sigma, i, \mathsf{upk}_i, \tau)$: Parse τ as $(\pi_{\mathsf{O}}, \hat{C}_{\mathsf{J}_i}, \sigma_{\mathsf{J}_i})$, and return 1 if the following holds and 0 otherwise:

$$\Sigma.\mathsf{Verify}(\mathsf{upk}_i, \hat{C}_{\mathsf{J}_i}, \sigma_{\mathsf{J}_i}) = 1 \quad \wedge \quad \Pi.\mathsf{Verify}(\mathsf{crs}_{\mathsf{O}}, (\hat{C}_{\mathsf{J}_i}, \mathsf{pk}_{\mathsf{O}}, \sigma), \pi_{\mathsf{O}}) = 1.$$

[a] We assume that the indexes are in ascending order w.r.t. the time of registration.

Scheme 1: Fully-Anonymous Dynamic Group Signature Scheme

Note that if multiple users collude and use the same value r upon $\mathsf{Join}^{(1)}$, we always return the first user who registered with this particular value r in Open. Then, Open always returns the signer who initiated the collusion by sharing the r value, which, we think, is the most reasonable choice. Note that this is in line

with the BSZ model: traceability only requires that every valid signature can be opened, while not requiring that it opens to one particular user out of the set of colluding users; correctness and non-frameability are defined with respect to honest users and are therefore clearly not influenced.

4.2 Security

Theorem 1. *If* SPS-EQ *is correct,* SoK *is correct, and* Π *is sound, then Scheme 1 is correct.*

Proof. Correctness is straight forward to verify by inspection. We only have to take care of one detail: There is the possibility that two honest executions of AddU yield the same value r (which is chosen uniformly at random upon $\mathsf{Join}^{(1)}$). Thus, the probability of two colliding r is negligible. □

Subsequently, we will formally prove the remaining security properties. In our proofs, we omit to make the negligible distribution switches which arise when sampling uniformly random from $\mathbb{Z}_p$ instead of $\mathbb{Z}_p^*$ explicit and instead treat them as conceptual changes for the sake of compactness.

Theorem 2. *If* Π *is adaptively zero-knowledge,* SoK *is simulatable,* Ω *is* IND-CPA *secure,* SPS-EQ *perfectly adapts signatures, and the DDH assumption holds in* $\mathbb{G}_1$*, then Scheme 1 is* CPA*-full anonymous.*

Theorem 3. *If* Π *is adaptively zero-knowledge,* SoK *is simulatable and straight-line* f*-extractable, where* $f : \mathbb{Z}_p \to \mathbb{G}_2$ *is defined as* $r \mapsto r \cdot \hat{P}$*,* Ω *is* IND-CCA2 *secure,* SPS-EQ *perfectly adapts signatures, and the DDH assumption holds in* $\mathbb{G}_1$*, then Scheme 1 is* CCA2*-full anonymous.*

Proof (Anonymity). We prove Theorem 2 and 3 by showing that the output distributions of the Ch oracle are (computationally) independent of the bit b, where we highlight the parts of the proof which are specific to Theorem 3 and can be omitted to prove Theorem 2. Therefore, let $q_{\mathsf{Ch}} \leq \mathsf{poly}(\kappa)$ be the number of queries to Ch, $q_{\mathsf{O}} \leq \mathsf{poly}(\kappa)$ be the number of queries to Open, and $q_{\mathsf{SndToU}} \leq \mathsf{poly}(\kappa)$ be the number of queries to SndToU.

Game 0: The original anonymity game.

Game 1: As Game 0, but we run $(\mathsf{crs}_{\mathsf{J}}, \tau_{\mathsf{J}}) \leftarrow \Pi.\mathcal{S}_1(1^\kappa)$ instead of $\mathsf{crs}_{\mathsf{J}} \leftarrow \Pi.\mathsf{Setup}(1^\kappa)$ upon running GKeyGen and store the trapdoor τ_{J}. Then, we simulate all calls to Π.Proof executed in Join using the simulator (without a witness).

Transition - Game 0 → Game 1: A distinguisher $\mathcal{D}^{0\to1}$ is an adversary against adaptive zero-knowledge of Π, and, therefore, the probability to distinguish Game 0 and Game 1 is negligible, i.e., $|\Pr[S_1] - \Pr[S_0]| \leq \epsilon_{\mathsf{ZK}_{\mathsf{J}}}(\kappa)$.

Game 2: As Game 1, but we run $(\mathsf{crs}_{\mathsf{O}}, \tau_{\mathsf{O}}) \leftarrow \Pi.\mathcal{S}_1(1^\kappa)$ instead of $\mathsf{crs}_{\mathsf{O}} \leftarrow \Pi.\mathsf{Setup}(1^\kappa)$ upon running GKeyGen and store the trapdoor τ_{O}. Then, we simulate all calls to Π.Proof in Open using the simulator (without a witness).

Transition - Game 1 → Game 2: A distinguisher $\mathcal{D}^{1\to 2}$ is an adversary against adaptive zero-knowledge of Π, and, therefore, the probability to distinguish Game 1 and Game 2 is negligible, i.e., $|\Pr[S_2] - \Pr[S_1]| \leq \epsilon_{\mathsf{ZK_O}}(\kappa)$.

Game 3: As Game 2, but we run $(\mathsf{crs_S}, \tau_\mathsf{S}) \leftarrow \mathsf{SoK.SimSetup}(1^\kappa)$ instead of $\mathsf{crs_S} \leftarrow \mathsf{SoK.Setup}(1^\kappa)$ upon running $\mathsf{GKeyGen}$ and store the trapdoor τ_S.

Transition - Game 2 → Game 3: A distinguisher $\mathcal{D}^{2\to 3}$ is an adversary against simulatability of SoK. Therefore, the distinguishing probability is negligible, i.e., $|\Pr[S_3] - \Pr[S_2]| \leq \epsilon_{\mathsf{SIM}}(\kappa)$.

Game 4: As Game 3, but instead of setting $(\mathsf{sk_O}, \mathsf{pk_O}) \leftarrow \Omega.\mathsf{KeyGen}(1^\kappa)$ in $\mathsf{GKeyGen}$, we obtain $\mathsf{pk_O}$ from an $\mathsf{IND\text{-}CPA}$ (resp. $\mathsf{IND\text{-}CCA2}$) challenger and set $\mathsf{sk_O} \leftarrow \perp$.

In the $\mathsf{CCA2}$ case, the environment additionally maintains a secret list $\mathtt{GSK}$ and upon each call to the SndToU oracle it sets $\mathtt{GSK}[i] \leftarrow \mathsf{gsk}_i$. Furthermore, we simulate the Open algorithm executed within the Open oracle as follows.

$\mathsf{Open}(\mathsf{gpk}, \mathsf{ok}, \mathsf{reg}, m, \sigma)$: Obtain $\hat{R}$ using the straight-line f-extractor, and obtain the lowest index i such that $e(\mathtt{GSK}[i][1][2], \hat{R}) = e(\sigma_1[1][2], \hat{P})$. If $i = \perp$ return $\perp$. If the entry for i in the registration table was not overwritten by the adversary, compute a simulated proof τ and return (i, τ). Otherwise, submit $\hat{C}_{\mathsf{J}_i}$ to the decryption oracle provided by the $\mathsf{IND\text{-}CCA2}$ challenger and return whatever the original open oracle would return (but with a simulated proof).

If the extractor fails at some point, we choose $b \xleftarrow{R} \{0,1\}$ and return b.

Transition - Game 3 → Game 4 (CPA*):* This change is conceptual, i.e., $\Pr[S_3] = \Pr[S_4]$.

Transition - Game 3 → Game 4 ($\mathsf{CCA2}$*):* By the straight-line f-extractability of the SoK, one can extract a witness ρ in every call to Open with overwhelming probability $1 - \epsilon_{\mathsf{EXT}}(\kappa)$. Thus, we have that $\Pr[S_4] = 1/2 + (\Pr[S_3] - 1/2) \cdot (1 - \epsilon_{\mathsf{EXT}}(\kappa))^{q_\mathsf{O}}$.

Game 5: As Game 4, but we compute the ciphertext $\hat{C}_{\mathsf{J}_i}$ in the Join algorithm (executed within the SndToU oracle) as $\hat{C}_{\mathsf{J}_i} \leftarrow \Omega.\mathsf{Enc}(\mathsf{pk}, \hat{P})$, i.e., with a constant message that is independent of the user.

Transition - Game 4 → Game 5: A distinguisher $\mathcal{D}^{4\to 5}$ is a distinguisher for the $\mathsf{IND\text{-}CPA}$ (resp. $\mathsf{IND\text{-}CCA2}$) game of Ω, i.e., $|\Pr[S_5] - \Pr[S_4]| \leq q_{\mathsf{SndToU}} \cdot \epsilon_{\mathsf{CPA}}(\kappa)$ (resp. $|\Pr[S_5] - \Pr[S_4]| \leq q_{\mathsf{SndToU}} \cdot \epsilon_{\mathsf{CCA2}}(\kappa)$).[3]

Game 6: As Game 5, but we introduce a conceptual change which will make the following distribution changes easier to follow. The environment obtains and stores a DDH instance (aP, bP, cP) in $\mathbb{G}_1$. Additionally, we further modify the Join algorithm (executed within SndToU) as follows. Instead of choosing

[3] For compactness, we collapsed the q_{SndToU} game changes into a single game change and note that one can straight forwardly unroll this to q_{SndToU} game changes where a single ciphertext is exchanged in each game.

$r \xleftarrow{R} \mathbb{Z}_p$, we use the random self reducibility of DDH to obtain an independent DDH instance $(uP, vP, wP) \overset{R^{SR}}{\leftarrow} (aP, bP, cP)$, choose $q \xleftarrow{R} \mathbb{Z}_p^*$, compute $(U_i, Q_i) \leftarrow (q \cdot uP, q \cdot P)$. Furthermore, the environment maintains a secret list $\mathtt{DDH}$ and upon each Issue it sets $\mathtt{DDH}[i] \leftarrow (uP, vP, wP)$.

Transition - Game 5 → Game 6: The output distributions in Game 5 and Game 6 are identical, i.e., $\Pr[S_6] = \Pr[S_5]$. Note that at this point only the uP-parts of the instances are used.

Game 7: As Game 6, but all calls to $\mathsf{ChgRep}_{\mathcal{R}}(M, \rho, \mathsf{pk}_{\mathcal{R}})$ are replaced by $\mathsf{Sign}_{\mathcal{R}}(\rho \cdot M, \mathsf{sk}_{\mathcal{R}})$.

Transition - Game 6 → Game 7: Under perfect adaption of signatures, the output distributions in Game 6 and Game 7 are identical, i.e., $\Pr[S_7] = \Pr[S_6]$.

Game 8_j ($1 \leq j \leq q_{\mathsf{Ch}}$): As Game 7, but we modify the Ch oracle as follows. For the first j queries, instead of running $\sigma_1 \leftarrow \mathsf{Sign}_{\mathcal{R}}(\rho \cdot \mathsf{gsk}_{i_b}[1], \mathsf{sk}_{\mathcal{R}})$, we choose $R \xleftarrow{R} \mathbb{G}_1$, and compute $\sigma_1 \leftarrow \mathsf{Sign}_{\mathcal{R}}(\rho \cdot (R, P), \mathsf{sk}_{\mathcal{R}})$.

Transition - Game 7 → Game 8_1: A distinguisher $\mathcal{D}^{7 \to 8_1}$ is a DDH distinguisher. To show this, we present an implementation of the Ch oracle, that—depending on the validity of the DDH instance (aP, bP, cP)—interpolates between Game 7 and Game 8_1. That is, in the first query we obtain the tuple $(uP, vP, wP) \leftarrow \mathtt{DDH}[i_b]$ and compute σ_1 as $\sigma_1 \leftarrow \mathsf{Sign}_{\mathcal{R}}((wP, vP), \mathsf{sk}_{\mathcal{R}})$. Then, if the initial DDH instance (aP, bP, cP) is valid, we have a distribution as in Game 7, whereas we have a distribution as in Game 8_1 otherwise. The success probability of a distinguisher between Game 7 and Game 8_1 is thus negligible, i.e., $|\Pr[S_{8_1}] - \Pr[S_7]| \leq \epsilon_{\mathsf{DDH}}(\kappa)$.

Transitions - Game 8_j → Game 8_{j+1} ($1 \leq j \leq q_{\mathsf{Ch}}$): The answers of the Ch oracle for the first j queries are already random in Game 8_j. Then, it is easy to show that a distinguisher $\mathcal{D}^{8_j \to 8_{j+1}}$ is a DDH distinguisher, i.e., by embedding $(uP, vP, wP) \leftarrow \mathtt{DDH}[i_b]$ in the answer of the Ch query $j+1$ using the same strategy as above. Summing up, we have $|\Pr[S_{8_q}] - \Pr[S_{8_1}]| \leq (q_{\mathsf{Ch}} - 1) \cdot \epsilon_{\mathsf{DDH}}(\kappa)$.

In Game $8_{q_{\mathsf{Ch}}}$, the simulation is independent of the bit b, i.e., $\Pr[S_{8_q}] = 1/2$; what remains is to obtain a bound on the success probability in Game 0. In the CPA case, we have that $\Pr[S_0] \leq 1/2 + q_{\mathsf{SndToU}} \cdot \epsilon_{\mathsf{CPA}}(\kappa) + q_{\mathsf{Ch}} \cdot \epsilon_{\mathsf{DDH}}(\kappa) + \epsilon_{\mathsf{ZK_J}}(\kappa) + \epsilon_{\mathsf{ZK_O}}(\kappa) + \epsilon_{\mathsf{SIM}}(\kappa)$, which proves Theorem 2. In the $\mathsf{CCA2}$ case, we have that $\Pr[S_0] \leq \Pr[S_3] + \epsilon_{\mathsf{ZK_J}}(\kappa) + \epsilon_{\mathsf{ZK_O}} + \epsilon_{\mathsf{SIM}}(\kappa)$, that $\Pr[S_4] \leq 1/2 + q_{\mathsf{SndToU}} \cdot \epsilon_{\mathsf{CCA2}}(\kappa) + q_{\mathsf{Ch}} \cdot \epsilon_{\mathsf{DDH}}(\kappa)$, and that $\frac{\Pr[S_4] - 1/2}{(1 - \epsilon_{\mathsf{EXT}}(\kappa))^{q_{\mathsf{O}}}} + 1/2 = \Pr[S_3]$. This yields $\Pr[S_0] \leq \frac{\Pr[S_4] - 1/2}{(1 - \epsilon_{\mathsf{EXT}}(\kappa))^{q_{\mathsf{O}}}} + 1/2 + \epsilon_{\mathsf{ZK_J}}(\kappa) + \epsilon_{\mathsf{ZK_O}} + \epsilon_{\mathsf{SIM}}(\kappa) = \frac{q_{\mathsf{SndToU}} \cdot \epsilon_{\mathsf{CCA2}}(\kappa) + q_{\mathsf{Ch}} \cdot \epsilon_{\mathsf{DDH}}(\kappa)}{(1 - \epsilon_{\mathsf{EXT}}(\kappa))^{q_{\mathsf{O}}}} + 1/2 + \epsilon_{\mathsf{ZK_J}}(\kappa) + \epsilon_{\mathsf{ZK_O}}(\kappa) + \epsilon_{\mathsf{SIM}}(\kappa)$, which proves Theorem 3. □

We note that one can avoid the factor q_{Ch} in the proof using the fact that one can obtain q_{Ch} DDH instances $(uP, v_iP, w_iP)_{i \in [q_{\mathsf{Ch}}]}$ from a single DDH instance (uP, vP, wP) so that (in-)validity of the original instance carries over to each $(uP, v_iP, w_iP)_{i \in [q_{\mathsf{Ch}}]}$. To do so, one chooses $x_i, y_i \xleftarrow{R} \mathbb{Z}_p$ and computes $v_iP \leftarrow x_i \cdot vP + y_iP$ and $w_iP \leftarrow x_i \cdot wP + y_i \cdot uP$.

Theorem 4. *If* $\mathsf{SPS\text{-}EQ}$ *is* $\mathsf{EUF\text{-}CMA}$ *secure, and* Π *is sound, then Scheme 1 is traceable.*

Proof (Traceability). We show that traceability holds using a sequence of games, where we let $q \leq \mathsf{poly}(\kappa)$ be the number of queries to the SndToI oracle.

Game 0: The original traceability game.
Game 1: As Game 0, but we obtain crs_J from a soundness challenger of Π.
Transition - Game 0 → Game 1: This change is conceptual, i.e., $\Pr[S_0] = \Pr[S_1]$.
Game 2: As Game 1, but after every successful execution of SndToI, we obtain $\hat{R} \leftarrow \Omega.\mathsf{Dec}(\mathsf{sk}_\mathsf{O}, C_{\mathsf{J}_i})$ and abort if $e(U_i, \hat{P}) \neq e(Q, \hat{R})$.
Transition - Game 0 → Game 1: If we abort we have a valid proof π_{J_i} attesting that $(U_i, Q, \hat{C}_{\mathsf{J}_i}, \mathsf{pk}_\mathsf{O}) \in L_{\mathsf{R}_\mathsf{J}}$, but by the perfect correctness of Ω there exists no ω such that $C_{\mathsf{J}_i} = \Omega.\mathsf{Enc}(\mathsf{pk}_\mathsf{O}, r \cdot \hat{P};\ \omega) \ \wedge\ U_i = r \cdot Q$, i.e., $(U_i, Q, \hat{C}_{\mathsf{J}_i}, \mathsf{pk}_\mathsf{O})$ is actually not in L_{R_J}. Thus, we only abort if the adversary breaks the soundness of Π in one oracle query, i.e., $\Pr[S_2] = \Pr[S_1] \cdot (1 - \epsilon_\mathsf{S}(\kappa))^q$.
Game 3: As Game 2, but we obtain BG and a public key $\mathsf{pk}_\mathcal{R}$ from an $\mathsf{EUF\text{-}CMA}$ challenger of the $\mathsf{SPS\text{-}EQ}$. Whenever an $\mathsf{SPS\text{-}EQ}$ signature is required, $\mathcal{R}^\mathsf{f}$ forwards the message to be signed to the signing oracle provided by the $\mathsf{EUF\text{-}CMA}$ challenger.
Transition - Game 2 → Game 3: This change is conceptual, i.e., $\Pr[S_2] = \Pr[S_3]$.

If the adversary eventually outputs a valid forgery (m, σ), we know that σ contains an $\mathsf{SPS\text{-}EQ}$ signature σ_1 for some (rP, P) such that we have never seen a corresponding $r\hat{P}$, i.e., there is no entry i in the registration table where $\hat{C}_{\mathsf{J}_i}$ contains $r\hat{P}$ s.t. $e(\sigma_1[1][1], \hat{P}) = e(\sigma_1[1][2], r\hat{P})$ holds. Consequently, σ_1 is a valid $\mathsf{SPS\text{-}EQ}$ signature for an unqueried equivalence class and we have that $\Pr[S_3] \leq \epsilon_\mathsf{F}(\kappa)$. This yields $\Pr[S_0] \leq \frac{\epsilon_\mathsf{F}(\kappa)}{(1-\epsilon_\mathsf{S}(\kappa))^q}$, which proves the theorem. □

Theorem 5. *If Π is sound and adaptively zero-knowledge, SoK is simulatable and extractable, Σ is $\mathsf{EUF\text{-}CMA}$ secure, Ω is perfectly correct, and the co-CDHI assumption holds, then Scheme 1 is non-frameable.*

Proof (Non-frameability). We prove non-frameability using a sequence of games. Thereby we let the number of users in the system be $q \leq \mathsf{poly}(\kappa)$.

Game 0: The original non-frameability game.
Game 1: As Game 0, but we guess the index i that will be attacked by the adversary. If the adversary attacks another index, we abort.
Transition - Game 0 → Game 1: The winning probability in Game 1 is the same as in Game 0, unless an abort event happens, i.e., $\Pr[S_1] = \Pr[S_0] \cdot 1/q$.
Game 2: As Game 1, but we run $(\mathsf{crs}_\mathsf{J}, \tau_\mathsf{J}) \leftarrow \Pi.\mathcal{S}_1(1^\kappa)$ instead of $\mathsf{crs}_\mathsf{J} \leftarrow \Pi.\mathsf{Setup}(1^\kappa)$ upon running $\mathsf{GKeyGen}$ and store the trapdoor τ_J. Then, we simulate all calls to $\Pi.\mathsf{Proof}$ in Join using the simulator (without a witness).
Transition - Game 1 → Game 2: A distinguisher $\mathcal{D}^{1\to 2}$ is an adversary against adaptive zero-knowledge of Π, and, therefore, the probability to distinguish Game 1 and Game 2 is negligible, i.e., $|\Pr[S_2] - \Pr[S_1]| \leq \epsilon_{\mathsf{ZK}_\mathsf{J}}(\kappa)$.
Game 3: As Game 2, but we obtain crs_O from a soundness challenger upon running $\mathsf{GKeyGen}$.
Transition - Game 2 → Game 3: This change is conceptual, i.e., $\Pr[S_3] = \Pr[S_2]$.

Game 4: As Game 3, but we setup the SoK in simulation mode, i.e., we run $(\mathsf{crs}_\mathsf{S}, \tau_\mathsf{S}) \leftarrow \mathsf{SoK.SimSetup}(1^\kappa)$ instead of $\mathsf{crs}_\mathsf{S} \leftarrow \mathsf{SoK.Setup}(1^\kappa)$ upon running $\mathsf{GKeyGen}$ and store the trapdoor τ_S. Then, we simulate all calls to $\mathsf{SoK.Sign}$ using the simulator, i.e., without a witness.

Transition - Game 3 → Game 4: A distinguisher $\mathcal{D}^{3\to4}$ is an adversary against simulatability of SoK. Therefore, the distinguishing probability is negligible, i.e., $|\Pr[S_4] - \Pr[S_3]| \leq \epsilon_{\mathsf{SIM}}(\kappa)$.

Game 5: As Game 4, but we modify the Join algorithm (executed within the SndToU oracle) when queried for user with index i as follows. We obtain a co-CDHI instance $(aP, {}^1\!/_a\hat{P})$ for BG, choose $r \xleftarrow{R} \mathbb{Z}_p$, set $(U_i, Q) \leftarrow (r \cdot P, aP)$, and compute $\hat{C}_{\mathsf{J}_i} \leftarrow \Omega.\mathsf{Enc}(\mathsf{pk}_\mathsf{O}, r \cdot {}^1\!/_a\hat{P})$ and store r. On successful execution we set $\mathsf{gsk}_i \leftarrow ((U_i, Q), \sigma')$ (note that π_{J_i} as well as the signatures in the GSig oracle are already simulated, i.e., the discrete log of Q is not required to be known to the environment).

Transition - Game 4 → Game 5: Since r is uniformly random, we can write it as $r = r'a$ for some $r' \in \mathbb{Z}_p$. Then it is easy to see that the game change is conceptual, i.e., $\Pr[S_5] = \Pr[S_4]$.

Game 6: As Game 5, but for every forgery output by the $\mathcal{A}$, we extract $\rho \leftarrow \mathsf{SoK.Extract}(\mathsf{crs}_\mathsf{S}, \tau_\mathsf{S}, (P, \sigma_1[1][2]), \sigma_1 || m, \sigma_2)$ and abort if the extraction fails.

Transition - Game 5 → Game 6: By the extractability of the SoK, one can extract a witness ρ with overwhelming probability $1 - \epsilon_{\mathsf{EXT}}(\kappa)$. Thus, we abort with probability $\epsilon_{\mathsf{EXT}}(\kappa)$ and $\Pr[S_6] = \Pr[S_5] \cdot (1 - \epsilon_{\mathsf{EXT}}(\kappa))$.

Game 7: As Game 6, but we further modify the Join algorithm when queried for user with index i (executed within the SndToU oracle) as follows. Instead of choosing $(\mathsf{usk}_i, \mathsf{upk}_i) \leftarrow \mathsf{UKeyGen}(1^\kappa)$, we engage with an $\mathsf{EUF\text{-}CMA}$ challenger, obtain upk_i and set $\mathsf{usk}_i \leftarrow \emptyset$. If any signature is required, we obtain it using the oracle provided by the $\mathsf{EUF\text{-}CMA}$ challenger.

Transition Game 6 → Game 7: This change is conceptual, i.e., $\Pr[S_7] = \Pr[S_6]$.

At this point we have three possibilities if $\mathcal{A}$ outputs a valid forgery.

1. If a signature for $\hat{C}_{\mathsf{J}_i}$ was never requested, $\mathcal{A}$ is an $\mathsf{EUF\text{-}CMA}$ forger for Σ and the forgery is $(\hat{C}_{\mathsf{J}_i}, \sigma_{\mathsf{J}_i})$. The probability for this to happen is upper bounded by $\epsilon_\mathsf{f}(\kappa)$.
2. Otherwise, we know that $\hat{C}_{\mathsf{J}_i}$ is honestly computed by the environment and—by the perfect correctness of Ω—thus contains ${}^r\!/_a\hat{P}$, which leaves us two possibilities:
   (a) If $e(\sigma[1][1], \hat{P}) = e(\sigma[1][2], {}^r\!/_a\hat{P})$, $\mathcal{A}$ is an adversary against co-CDHI, since we can obtain $(((r \cdot {}^1\!/_aP, P), \sigma')) \leftarrow \mathsf{ChgRep}_\mathcal{R}(\sigma_1, \rho^{-1}, \mathsf{pk}_\mathcal{R})$ and use r to output $r^{-1} \cdot (r \cdot {}^1\!/_aP) = {}^1\!/_aP$. The probability for this to happen is upper bounded by $\epsilon_{\mathsf{co\text{-}CDHI}}(\kappa)$.
   (b) If $e(\sigma[1][1], \hat{P}) \neq e(\sigma[1][2], {}^r\!/_a\hat{P})$, $\mathcal{A}$ has produced an opening proof for a statement which is actually not in L_{R_O}. The probability for this to happen is upper bounded by $\epsilon_\mathsf{S}(\kappa)$.

Taking the union bound we obtain $\epsilon_{\mathsf{nf7}}(\kappa) \leq \epsilon_\mathsf{f}(\kappa) + \epsilon_{\mathsf{co\text{-}CDHI}}(\kappa) + \epsilon_\mathsf{S}(\kappa)$, which yields the following bound for the success probability in Game 1: $\Pr[S_1] \leq$

$\Pr[S_5] + \epsilon_{\mathsf{ZK_J}}(\kappa) + \epsilon_{\mathsf{SIM}}(\kappa)$. Furthermore, we know that $\Pr[S_5] = \frac{\epsilon_{\mathsf{nf7}}(\kappa)}{1-\epsilon_{\mathsf{EXT}}(\kappa)}$ and $\Pr[S_0] = \Pr[S_1] \cdot q$. Taking all together we have that $\Pr[S_0] \leq q \cdot (\frac{\epsilon_{\mathsf{nf7}}(\kappa)}{1-\epsilon_{\mathsf{EXT}}(\kappa)} + \epsilon_{\mathsf{ZK_J}}(\kappa) + \epsilon_{\mathsf{SIM}}(\kappa))$, which is negligible. □

Theorem 6. *If Ω is perfectly correct, and Σ is* EUF-CMA *secure, then Scheme 1 is weakly opening sound.*

Proof. Upon honestly executing Join for users i and j, the probability that their r (resp. $\hat{R}$) values collide is negligible. The perfect correctness of Ω and the EUF-CMA security of Σ thus uniquely determine user i as the signer of σ with overwhelming probability. Then, it is easy to see that an adversary against weak opening soundness is an adversary against soundness of Π. □

5 Instantiation in the ROM

To compare our approach to existing schemes regarding signature size and computational effort upon signature generation and verification, we present the sign and verification algorithms for an instantiation of our scheme with the SPS-EQ from [FHS14, FHS15], whose security is shown to hold in the generic group model. For the instantiation of signatures of knowledge (SoKs) in the ROM, we apply the Fiat-Shamir (FS) [FS86] heuristic to Σ-protocols and further apply the transformation from [FKMV12] to obtain simulation soundness.

Before we introduce the approaches to obtain CPA-fully (resp. CCA2-fully) anonymous instantiations, we recall that the group signing key gsk_i consists of a vector of two group elements $(R, P) \in (\mathbb{G}_1^*)^2$ and an SPS-EQ signature $\sigma \in \mathbb{G}_1 \times \mathbb{G}_1^* \times \mathbb{G}_2^*$ on this vector. Randomization of a gsk_i with a random value $\rho \in \mathbb{Z}_p^*$, i.e., $\mathsf{ChgRep}_{\mathcal{R}}$, requires 4 multiplications in $\mathbb{G}_1$ and 1 multiplication in $\mathbb{G}_2$. Verification of an SPS-EQ signature on gsk_i requires 5 pairings.

We note that the proofs performed within Join and Open can straight forwardly be instantiated using standard techniques. Therefore, and since they are neither required within Sign nor Verify, we do not discuss instantiations here.

5.1 CPA-Full Anonymity

Subsequently, we show how Sign and Verify are instantiated in the CPA-full anonymity setting. Therefore, let $H : \{0,1\}^* \to \mathbb{Z}_p$ be a random oracle and let x be the proven statement (which is implicitly defined by the scheme):

$\mathsf{Sign}(\mathsf{gpk}, \mathsf{gsk}_i, m)$: Parse gsk_i as $((R, P), \sigma)$, choose $\rho \xleftarrow{R} \mathbb{Z}_p$, compute $\sigma_1 = ((R', P'), \sigma') \leftarrow \mathsf{ChgRep}_{\mathcal{R}}(\mathsf{gsk}_i, \rho, \mathsf{pk}_{\mathcal{R}})$. Choose $\nu \xleftarrow{R} \mathbb{Z}_p$, compute $N \leftarrow \nu P$, $c \leftarrow H(N||\sigma_1||m||x)$, $z \leftarrow \nu + c \cdot \rho$, set $\sigma_2 \leftarrow (c, z)$, and return $\sigma \leftarrow (\sigma_1, \sigma_2)$.

$\mathsf{Verify}(\mathsf{gpk}, m, \sigma)$: Parse σ as $(\sigma_1, \sigma_2) = (((R', P'), \sigma), (c, z))$, return 0 if $\mathsf{Verify}_{\mathcal{R}}(\sigma_1, \mathsf{pk}_{\mathcal{R}}) = 0$. Otherwise compute $N \leftarrow zP - cP'$ and check whether $c = H(N||\sigma_1||m||x)$ holds. If so return 1 and 0 otherwise.

Since the used Σ-protocol is a standard proof of knowledge of the discrete logarithm $\log_P P'$, it is easy to see that applying the transformations from [FKMV12] yields a SoK in the ROM with the properties we require. All in all, group signatures contain 4 elements in $\mathbb{G}_1$, 1 element in $\mathbb{G}_2$ and 2 elements in $\mathbb{Z}_p$. Counting only the expensive operations, signing costs 5 multiplications in $\mathbb{G}_1$ and 1 multiplication in $\mathbb{G}_2$, and verification costs 2 multiplications in $\mathbb{G}_1$ and 5 pairings.

5.2 CCA2-Full Anonymity

For CCA2-full anonymity, we require our signatures of knowledge to be straight-line extractable, since standard rewinding techniques would lead to an exponential blowup in the reduction (cf. [BFW15]). One posssibility would be to rely on the rather inefficient approach to straight-line extraction due to Fischlin [Fis05]. However, as we do not need to straight-line extract the full witness w, but it is sufficient for us to straight-line extract an image of w under a one-way function $f : \rho \mapsto \rho \cdot \hat{P}$, we can fortunately use the notion of straight-line f-extractable SoKs as recently proposed by Cerulli et al. [BCC+15]. This allows us to still use the FS paradigm with good efficiency. The construction builds upon the generic conversion in [FKMV12, BPW12] and the generic trick in [BCC+15] to obtain straight-line f-extractability is by computing an extractable commitment to the image of the witness w under a function f with respect to an extraction key in the CRS and proving consistency with the witness.[4]

For straight-line extractability, we let $\hat{Y}$ be a public key for the ElGamal variant in $\mathbb{G}_2$ from [BCC+15], which is generated upon $\mathsf{SoK.Setup}$ and represents the CRS of SoK. $\mathsf{SoK.SimSetup}$ additionally returns τ such that $\hat{Y} = \tau \cdot \hat{P}$. Furthermore, let x be the proven statement (implicitly defined by the scheme and the generic compiler). Subsequently, we show how Sign and Verify are instantiated in this setting, where $H : \{0,1\}^* \to \mathbb{Z}_p$ is modelled as a random oracle:

$\mathsf{Sign}(\mathsf{gpk}, \mathsf{gsk}_i, m)$: Parse gsk_i as $((R, P), \sigma)$, choose $\rho \xleftarrow{R} \mathbb{Z}_p$, compute $\sigma_1 = ((R', P'), \sigma') \leftarrow \mathsf{ChgRep}_{\mathcal{R}}(\mathsf{gsk}_i, \rho, \mathsf{pk}_{\mathcal{R}})$. Choose $u, \nu, \eta \xleftarrow{R} \mathbb{Z}_p$, compute $(\hat{C}_1, \hat{C}_2) = (u\hat{Y}, \rho\hat{P} + u\hat{P})$, $N \leftarrow \nu P$, $\hat{M}_1 \leftarrow \eta\hat{Y}$, $\hat{M}_2 \leftarrow (\nu+\eta)\hat{P}$, $c \leftarrow H(N||\hat{M}_1||\hat{M}_2||\sigma_1||m||x)$, $z_1 \leftarrow \nu + c \cdot \rho$, $z_2 \leftarrow \eta + c \cdot u$, set $\sigma_2 \leftarrow (\hat{C}_1, \hat{C}_2, c, z_1, z_2)$, and return $\sigma \leftarrow (\sigma_1, \sigma_2)$.

$\mathsf{Verify}(\mathsf{gpk}, m, \sigma)$: Parse σ as $(\sigma_1, \sigma_2) = (((R', P'), \sigma), (c, z_1, z_2))$, return 0 if $\mathsf{Verify}_{\mathcal{R}}(\sigma_1, \mathsf{pk}_{\mathcal{R}}) = 0$. Otherwise compute $N \leftarrow z_1 P - cP'$, $\hat{M}_1 \leftarrow z_2 \cdot \hat{Y} - c \cdot \hat{C}_1$, $\hat{M}_2 \leftarrow (z_1 + z_2) \cdot \hat{P} - c \cdot \hat{C}_2$, and check whether $c = H(N||\hat{M}_1||\hat{M}_2||\sigma_1||m||x)$ holds. If so return 1 and 0 otherwise.

We now show that the above instantiation provides the properties we require. That is, we show that the Σ-protocol provides perfect completeness, special honest-verifier zero-knowledge (SHVZK) and special soundness. We additionally require the Σ-protocol to provide quasi-unique responses [Fis05], i.e., given an accepting proof it should be computationally infeasible to find a new valid response for that proof, in order for the compiler in [BCC+15] to apply.

[4] Note that one can still obtain the full witness w using a rewinding extractor.

Lemma 1. *The above Σ-protocol is perfectly complete, SHVZK, special-sound and has quasi-unique responses.*

Proof. We investigate all the properties below.

Perfect completeness. Is straight forward to verify and omitted.

SHVZK. We describe a simulator which outputs transcripts being indistinguishable from real transcripts. First, it chooses $P' \xleftarrow{R} \mathbb{G}_1, \hat{C}_1 \xleftarrow{R} \mathbb{G}_2, \hat{C}_2 \xleftarrow{R} \mathbb{G}_2$. While P' and $\hat{C}_1$ are identically distributed as in a real transcript, the random choice of $\hat{C}_2$ is not detectable under DDH in $\mathbb{G}_2$ which holds in the SXDH setting (more generally under IND-CPA of the used encryption scheme). Then, the simulator chooses $z_1, z_2, c \xleftarrow{R} \mathbb{Z}_p$ and computes $N \leftarrow z_1 \cdot P - c \cdot P'$, $\hat{M}_1 \leftarrow z_2 \cdot \hat{Y} - c \cdot \hat{C}_1$, $\hat{M}_2 \leftarrow (z_1 + z_2) \cdot \hat{P} - c \cdot \hat{C}_2$. It is easy to see that the transcript $(P', \hat{C}_1, \hat{C}_2, N, \hat{M}_1, \hat{M}_2, z_1, z_2, c)$ represents a valid transcript and its distribution is computationally indistinguishable from a real transcript.

Special soundness. Let us consider that we have two accepting answers (z_1, z_2, c) and (z_1', z_2', c') from the prover for distinct challenges $c \neq c'$. Then we have that

$$z_1 - c \cdot \rho = z_1' - c' \cdot \rho \text{ and } z_2 - c \cdot u = z_2' - c' \cdot u,$$

and extract a witness as $\rho \leftarrow \frac{z_1 - z_1'}{c - c'}$, $u \leftarrow \frac{z_2 - z_2'}{c - c'}$.

Quasi-unique responses. The answers z_1 and z_2 are uniquely determined by the word $\hat{Y}$, P', $\hat{C}_1$, $\hat{C}_2$, the commitments N, $\hat{M}_1$, $\hat{M}_2$ as well as the challenge c (and thus the verification equation). □

Lemma 2. *Applying the generic conversions from [FKMV12] to the Fiat-Shamir transformed version of the above Σ-protocol with the setup* SoK.Setup *as described in Section 5.2 produces a signature of knowledge in the random oracle model, that is extractable and straight-line f-extractable.*

The proof is analogous to [BCC+15], but we re-state it for completeness.

Proof. For simulatability, we observe that the CRS output by SoK.SimSetup is identical to the CRS output by SoK.Setup and SoK.SimSign programs the random oracle to simulate proofs. Simulatability then follows from SHVZK. For extractability we rely on rewinding, special soundness and quasi-unique responses, using the results from [FKMV12]. For straight-line f-extractability, we use the trapdoor τ to decrypt $(\hat{C}_1, \hat{C}_2)$ in the proof transcript and obtain $\rho\hat{P} = f(\rho)$. □

Switching Groups. All in all, this instantiation yields signatures containing 4 elements in $\mathbb{G}_1$, 3 elements in $\mathbb{G}_2$, and 3 $\mathbb{Z}_p$ elements. Counting only the expensive operations, signing costs 5 multiplications in $\mathbb{G}_1$ and 6 multiplications in $\mathbb{G}_2$, and verification costs 2 multiplications in $\mathbb{G}_1$, 4 multiplications in $\mathbb{G}_2$, and 5 pairings. We observe that the protocol presented above now requires more operations in the more expensive group $\mathbb{G}_2$ than in $\mathbb{G}_1$. However, as we work in the SXDH setting, we can simply switch the roles of $\mathbb{G}_1$ and $\mathbb{G}_2$ and thus all elements in $\mathbb{G}_1$ to $\mathbb{G}_2$ and vice versa. This gives us improved computational performance at the expense of slightly larger signatures.

6 Evaluation and Discussion

Subsequently, we discuss our work in the light of some recent concurrent and independent work and discuss open issues. Moreover, we provide a performance evaluation with respect to other related schemes.

6.1 Relation to Recent Work

In independent and concurrent work, a new model for fully-dynamic group signatures was proposed by Bootle et al. in [BCC+16]. Bootle et al. address maliciously generated issuer and opener keys, include the notion of opening soundness from [SSE+12] and formally model revocation by means of epochs. Although our work is independent of theirs, we want to briefly put our construction in context of their recent model.

In our scheme, one can straight forwardly incorporate the requirement to support maliciously generated keys in the fashion of [BCC+16] by extending the actual public keys of issuer and opener by a (straight-line extractable) zero-knowledge proof of knowledge of the respective secret issuer and opener key.

Revocation for our scheme could be achieved using standard techniques from the literature. However, it is not studied in [BCC+16] if any of those approaches to revocation fits in their model. For a practical revocation approach, it seems to be reasonable to choose a re-issuing based approach, i.e., to set up a new group after every epoch, as also used in [BCC+16]. Their group signature construction being secure in their model builds upon accountable ring signatures [BCC+15]. It comes at the cost of a group public key size linear in the number of group members as well as a signature size logarithmic in the number of group members, and the revocation related re-issuing requires every group member to obtain the new group public key. Applying the same revocation approach to our scheme yields public keys as well as signatures of constant size, and re-issuing requires each group member which is still active to re-join the new group.

While our scheme provides weak opening soundness, achieving the stronger notion for our scheme (where the opening authority may be malicious) would require the opening authority to additionally prove that the opened index i corresponds to the lowest index in reg so that the respective entry together with the signature in question satisfies the relation R_{O}. Such a proof could efficiently be instantiated using non-interactive plaintext in-equality proofs [BDSS16].

6.2 Performance Evaluation and Comparison

To underline the practical efficiency of our approach, we provide a comparison of our ROM instantiation with other schemes in the ROM. In particular we use two schemes who follow the approach of Bichsel et al., i.e., [BCN+10, PS16], which provide less realistic anonymity guarantees (denoted CCA^-), and the well known BBS scheme [BBS04] (with and without precomputations) providing CPA-full anonymity. We note that we use the plain BBS scheme for comparison, which does not even provide non-frameability and the non-frameable version would be

even more expensive. Moreover, we use the group signature scheme with the shortest known signatures [DP06] (with and without precomputations) being secure in the strong BSZ model and thus providing CCA2-full anonymity. Finally, we also compare our scheme to the recent CCA2-fully anonymous scheme by Libert et al. [LMPY16] which is secure in the ROM under SXDH.

In Table 1 we provide a comparison of the estimated efficiency in a 254bit BN-pairing setting, based on performance values on an ARM-Cortex-M0+ with drop-in hardware accelerator [UW14]. This processor is small enough to be suited for smart cards or wireless sensor nodes [UW14]. Then, in Table 2, we provide an abstract comparison regarding signature size and computational costs, and, we also include the type of the underlying hardness assumption.

Computational Efficiency. When comparing our CPA-fully anonymous scheme as well as our CCA2-fully anonymous scheme to other schemes providing the same anonymity guarantees, ours are the by now fastest ones regarding signature generation and verification costs. While some of the schemes used for comparison use slightly less progressive assumptions, it seems that very good performance requires more progressive assumptions. When looking for instance at the most compact CCA2-fully anonymous group signatures in the standard model under standard assumptions (SXDH and XDLIN) by Libert et al. [LPY15], signature sizes in the best case will have 30 $\mathbb{G}_1$ and 14 $\mathbb{G}_2$ elements ($\approx$ 15000 bit when taking the setting in Table 1), large public keys and computation times that are far from being feasible for resource constrained devices.

Scheme	Anon.	Signature Size	Signature Cost	Verification Cost
[BCN+10]	CCA$^-$	1273bit	351ms	1105ms
[PS16]	CCA$^-$	1018bit	318ms	777ms
[BBS04]	CPA	2289bit	1545ms	2092ms
[BBS04] (prec.)	CPA	2289bit	1053ms	1600ms
This paper	CPA	2037bit	266ms	886ms
This paper	CCA2	3309bit	771ms	1290ms
This paper (switch)	CCA2	3563bit	703ms	1154ms
[DP06]	CCA2	2290bit	1380ms	2059ms
[DP06] (prec.)	CCA2	2290bit	1020ms	1353ms
[LMPY16]	CCA2	2547bit	1688ms	2299ms

Table 1. Estimated efficiency based on a BN-pairing implementation on an ARM-Cortex-M0+ with a drop-in hardware accelerator, operating at 48MHz [UW14]. Using 254-bit curves, this implementation delivers the performance values 33ms-101ms-252ms-164ms ($\mathbb{G}_1$-$\mathbb{G}_2$-$\mathbb{G}_T$-pairing). For the estimation of signature sizes, we use 255bit for elements in $\mathbb{G}_1$, 509bit for elements in $\mathbb{G}_2$ and 254bit for elements in $\mathbb{Z}_p$. The semantics of 'CCA^-' is the same as in Table 2. We note that [BBS04] is defined for a Type-2 pairing setting, which means that our performance estimation for this scheme is rather optimistic and likely to be worse in practice.

Regarding signature generation, we want to emphasize that our CPA-fully anonymous instantiation is the fastest among all schemes used for comparison (even among the ones providing CCA^- anonymity), and, to the best of our knowledge, the fastest among all existing schemes. This is of particular importance since signature generation is most likely to be executed on a constrained device. Regarding signature verification our CPA-fully anonymous instantiation is only outperformed by the CCA^- anonymous instantiation in [PS16].

Signature Size. Comparing schemes providing the same anonymity guarantees, our CPA-fully anonymous instantiation even provides shorter signature sizes than the popular BBS scheme [BBS04]. Regarding CCA2-fully anonymous schemes, it seems that gained efficiency in the "without encryption" paradigm comes at the cost of larger signatures compared to instantiations following the SEP paradigm. It is interesting to note in this context that the schemes in vein of Bichsel et al. providing only CCA^- anonymity have the smallest signatures among all schemes.

6.3 Interesting Properties and Observations

Firstly, we observe that our construction neither requires any pairing computations nor computations in the target group $\mathbb{G}_T$ upon signature creation, which makes it especially suitable for constrained devices. Secondly, it seems that one could exploit the $\hat{R}$ values in a relatively straight forward manner to obtain traceable signatures [KTY04, Cho09].

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Scheme	Anon.	Signature Size	Signature Cost	Verification Cost	Assumption Type
[BCN+10]	CCA$^-$	$3\mathbb{G}_1 + 2\mathbb{Z}_p$	$1\mathbb{G}_T + 3\mathbb{G}_1$	$5\text{P} + 1\mathbb{G}_T + 1\mathbb{G}_1$	Interactive
[PS16]	CCA$^-$	$2\mathbb{G}_1 + 2\mathbb{Z}_p$	$1\mathbb{G}_T + 2\mathbb{G}_1$	$3\text{P} + 1\mathbb{G}_T + 1\mathbb{G}_1$	GGM
[BBS04]	CPA	$3\mathbb{G}_1 + 6\mathbb{Z}_p$	$3\text{P} + 3\mathbb{G}_T + 9\mathbb{G}_1$	$5\text{P} + 4\mathbb{G}_T + 8\mathbb{G}_1$	q-Type (non-static)
[BBS04] (prec.)	CPA	$3\mathbb{G}_1 + 6\mathbb{Z}_p$	$3\mathbb{G}_T + 9\mathbb{G}_1$	$4\mathbb{G}_T + 8\mathbb{G}_1$	q-Type (non-static)
This paper	CPA	$1\mathbb{G}_2 + 4\mathbb{G}_1 + 2\mathbb{Z}_p$	$1\mathbb{G}_2 + 5\mathbb{G}_1$	$5\text{P} + 2\mathbb{G}_1$	GGM
This paper	CCA2	$3\mathbb{G}_2 + 4\mathbb{G}_1 + 3\mathbb{Z}_p$	$6\mathbb{G}_2 + 5\mathbb{G}_1$	$5\text{P} + 4\mathbb{G}_2 + 2\mathbb{G}_1$	GGM
This paper (switch)	CCA2	$4\mathbb{G}_2 + 3\mathbb{G}_1 + 3\mathbb{Z}_p$	$5\mathbb{G}_2 + 6\mathbb{G}_1$	$5\text{P} + 2\mathbb{G}_2 + 4\mathbb{G}_1$	GGM
[DP06]	CCA2	$4\mathbb{G}_1 + 5\mathbb{Z}_p$	$3\text{P} + 3\mathbb{G}_T + 4\mathbb{G}_1$	$5\text{P} + 4\mathbb{G}_T + 7\ \mathbb{G}_1$	q-Type (non-static)
[DP06] (prec.)	CCA2	$4\mathbb{G}_1 + 5\mathbb{Z}_p$	$3\mathbb{G}_T + 8\mathbb{G}_1$	$1\text{P} + 3\mathbb{G}_T + 2\mathbb{G}_2 + 7\mathbb{G}_1$	q-Type (non-static)
[LMPY16]	CCA2	$7\mathbb{G}_1 + 3\mathbb{Z}_p$	$4\text{P} + 2\mathbb{G}_T + 16\mathbb{G}_1$	$8\text{P} + 3\mathbb{G}_T + 7\mathbb{G}_1$	Standard

Table 2. Comparison of related group signature schemes in the ROM regarding signature size, signing and verification cost, and required hardness assumptions, where, in terms of computational costs, we only count the expensive operations in $\mathbb{G}_1$, $\mathbb{G}_2$, and $\mathbb{G}_T$ as well as the pairings. The values for [BCN+10] and [PS16] are taken from [PS16]. We use 'CCA^-' to denote anonymity in the sense of [BCN+10] and note that precomputation in [BBS04, DP06] requires to store extra elements in $\mathbb{G}_T$.

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