Promise Zero Knowledge and its Applications to Round Optimal MPC

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Abstract

We devise a new *partitioned simulation* technique for MPC where the simulator uses different strategies for simulating the view of aborting adversaries and non-aborting adversaries. The protagonist of this technique is a new notion of *promise zero knowledge* (ZK) where the ZK property only holds against non-aborting verifiers. We show how to realize promise ZK in three rounds in the simultaneous-message model assuming injective one way functions.

We demonstrate the following applications of our new technique:

- We construct the first round-optimal (i.e., four round) MPC protocol for general functions based on polynomially hard DDH (or QR or N^{th}-Residuosity).
- We further show how to overcome the four-round barrier for MPC by constructing a three-round protocol for "list coin-tossing" – a slight relaxation of coin-tossing that suffices for most conceivable applications – based on polynomially hard injective one-way functions. This result generalizes to randomized input-less functionalities also assuming DDH (or QR or N^{th}-Residuosity).

Previously, four round MPC protocols required sub-exponential-time hardness assumptions and no multi-party three-round protocols were known for any relaxed security notions with polynomial-time simulation against malicious adversaries.

In order to base security on polynomial-time standard assumptions, we also rely upon a *leveled rewinding security* technique that can be viewed as a polynomial-time alternative to leveled complexity leveraging for achieving "non-malleability" across different primitives.

Contents

1 Introduction

Provably secure protocols lie at the heart of the theory of cryptography. How can we design protocols, not only so that we cannot devise attacks against them, but so that we can *prove* that no such attacks exist (under well-studied complexity assumptions)? The goal of achieving a proof of security has presented many challenges and apparent trade-offs in secure protocol design. This is especially true with regards to the goal of minimizing rounds of interaction, which has been a long-standing driver of innovation in theoretical cryptography. We begin by focusing on one such challenge and apparent trade-off in the context of *zero-knowledge* (ZK) protocols [GMR89], one of the most fascinating and broadly applicable notions in cryptography.

Recall that in a ZK protocol, a prover should convince a verifier that some statement is true, without revealing to the verifier anything beyond the validity of the statement being proven. It is known that achieving zero knowledge with black-box simulation[1] is impossible with three or fewer rounds of simultaneous message exchange [GK96, GMPP16]. A curious fact emerges, however, when we take a closer look at the proof of this impossibility result. It turns out that three-round ZK is impossible when considering verifiers that essentially behave completely honestly, but that sometimes probabilistically refuse to finish the protocol. This is bizarre: ZK protocols are supposed to prevent the verifier from learning information from the prover; how can behaving honestly but aborting the protocol early possibly help the verifier learn additional information? Indeed, one might think that we can prove that such behavior cannot possibly help the verifier learn additional information. Counter-intuitively, however, it turns out that such early aborts are critical to the impossibility proofs of [GK96, GMPP16]. This observation is the starting point for our work; now that we have identified a key (but counter-intuitive) reason behind the impossibility results, we want to leverage this understanding to bypass the impossibility result in a new and useful way.

Promise Zero Knowledge. Our main idea is to consider adversarial verifiers that promise not to abort the protocol early with noticeable probability. However, we do not limit ourselves only to adversarial verifiers that behave honestly; we consider adversarial verifiers that may deviate from the prescribed protocol arbitrarily, as long as this deviation does not cause the protocol to abort. A *promise zero-knowledge* protocol is one that satisfies the correctness and soundness guarantees of ordinary zero-knowledge protocols, but only satisfies the zero knowledge guarantee against adversarial verifiers that "promise" not to abort with noticeable probability. The centerpiece of our work is a construction of three-round promise zero-knowledge protocol, in the simultaneous message model, for proving statements where the statement need not be decided until the last (third) round, but where such statements should come from a distribution such that both a statement and a witness for that statement can be sampled in the last round. We call this primitive a *distributional delayed-input promise zero-knowledge argument*. Our construction requires only the existence of injective one-way functions. Interestingly, in our construction, we rely upon information learned from the verifier in the third round, to simulate its view in the third round!

Partitioned Simulation, and Applications to MPC. But why should we care about promise ZK? Actual adversaries will not make any promise regarding what specific types of adversarial behavior they will or will not engage in. However, recall our initial insight – early aborting by an adversary should, generally speaking, only hurt the adversary, not help it. We know due to the

[1] In this work, we focus on black-box simulation. However, no solutions for three-round ZK from standard assumptions with non-black-box simulation [Bar01] are presently known either. [BKP17] showed how to construct 3 round ZK using non-black-box simulation from the non-standard assumption that keyless multi-collision resistant hash functions exist.

impossibility results of [GK96, GMPP16], that we cannot leverage this insight to achieve three-round standard ZK (with black-box simulation). Our goal instead, then, is to use our insight to replace ZK with promise ZK for the construction of other secure protocols. Specifically, we consider the most general goal of secure protocol design: secure multi-party computation (MPC), as we discuss further below.

To do so, we devise a novel *partitioned simulation* strategy for leveraging promise ZK. At a high-level, we split the simulation into two disjoint cases, *depending upon whether or not the adversary is an aborting adversary* (i.e., one who aborts with high probability). In one case, we will exploit promise ZK. In the other, we exploit the intuition that early aborting should only harm the adversary, to devise alternate simulation strategies that bypass the need for ZK altogether, and instead essentially rely on a weaker notion called strong witness indistinguishability, that was recently constructed in three rounds (in the "delayed-input" setting) in [JKKR17].

Secure Multi-Party Computation. The notion of secure multiparty computation (MPC) [Yao82, GMW87] is a unifying framework for general secure protocols. MPC allows mutually distrusting parties to jointly evaluate any efficiently computable function on their private inputs in such a manner that each party does not learn anything beyond the output of the function.

The round complexity of MPC has been extensively studied over the last three decades in a long sequence of works [GMW87, BMR90, KOS03, KO04, Pas04, PW10, Wee10, Goy11, GMPP16, ACJ17, BHP17, COSV17a, COSV17b]. In this work, we study the problem of *round-optimal* MPC against malicious adversaries who may corrupt an arbitrary subset of parties, in the plain model without any trust assumptions. The state-of-the-art results on round-optimal MPC for general functions are due to Ananth et al. [ACJ17] and Brakerski et al. [BHP17], both of which rely on sub-exponential-time hardness assumptions. (See Section 1.2 for a more elaborate discussion on related works.) Our goal, instead is to base security on standard, *polynomial-time* assumptions.

We now highlight the main challenge in basing security on polynomial-time assumptions. In the setting of four round protocols in the simultaneous-message model, a rushing adversary may always choose to abort after receiving the honest party messages in the last round. At this point, the adversary has already received enough information to obtain the purported output of the function being computed. This suggests that we must enforce "honest behavior" on the parties within the first three rounds in order to achieve security against malicious adversaries. As discussed above, three-round zero knowledge is impossible, and this is precisely why we look to our new notion of promise ZK and partitioned simulation to resolve this challenge.

However, this challenge is exacerbated in the setting of MPC as we must not only enforce honest behavior but also ensure non-malleability *across different cryptographic primitives* that are being executed in parallel within the first three rounds. We show how to combine our notions of promise ZK with new simulation ideas to overcome these challenges, relying only on polynomial-time assumptions.

Coin Tossing. Coin-tossing allows two or more participants to agree on an unbiased coin (or a sequence of unbiased coins). Fair multiparty coin-tossing is known to be impossible in the dishonest majority setting [Cle86]. Therefore, while current notions of *secure coin-tossing* require that the protocol have a (pseudo)-random outcome, the adversary is additionally allowed to abort depending on the outcome of the toss.

Presently, secure multiparty coin-tossing is known to require at least four rounds w.r.t. black-box simulation [GMPP16, KO04]. In this work, we seek to overcome this barrier.

Towards this, our key observation is that coin-tossing is perfectly suited for application of

partitioned simulation. The definition of secure coin-tossing roughly requires the existence of a simulator that successfully forces externally sampled random coin, and produces a distribution over adversary's views that is indistinguishable from a real execution. To account for the adversary aborting or misbehaving based on the outcome, the simulator is allowed to either force an external coin, or force an abort as long as the simulated distribution remains indistinguishable from the real one. Crucially, in the case of an adversary that always aborts before the end of the protocol, the prescribed output of any secure coin-tossing protocol is also abort: therefore, the simulator never needs to force *any external coin* against such an adversary! Simulating the view of such adversaries that always abort is thus completely trivial. This leaves the case of non-aborting adversaries, which is exactly the setting that promise ZK was designed for.

Using promise ZK, we design a three-round protocol for "list coin-tossing" – a notion that is slightly weaker that regular coin-tossing, but nevertheless, suffices for nearly all important applications of coin-tossing (see below for a discussion). Therefore, promise ZK gives us a way to overcome the four-round barrier for secure coin-tossing [GMPP16, KO04].

1.1 Our Results

We introduce the notion of promise ZK proof systems and devise a new partitioned simulation strategy for round-efficient MPC protocols. Our first result is a three-round distributional delayed-input promise ZK argument system based on injective one-way functions.

Theorem 1 (Informal)**.** *Assuming injective one-way functions, there exists a three round distributional delayed-input promise ZK argument system in the simultaneous-message model.*

Round-Optimal MPC. We present two applications of partitioned simulation to round-optimal MPC. We first devise a general compiler that converts any three-round semi-malicious MPC protocol, where the first round is public-coin (i.e., the honest parties simply send random strings in the first round), into a four-round malicious secure MPC protocol. Our compiler can be instantiated with standard assumptions such as DDH or Quadratic Residuosity or N^{th}-Residuosity. The resulting protocol is optimal in the number of rounds w.r.t. black-box simulation [GMPP16]. A three round semi-malicious protocol with the aforementioned property can be obtained based on DDH/QR/N^{th} Residuosity [GS18, BL18].

Theorem 2 (Informal)**.** *Assuming DDH/QR/N^{th}-Residuosity, there exists a four round MPC protocol for general functions with black-box simulation.*

List Coin-Tossing. We also study the feasibility of multiparty coin-tossing in only three rounds. While three round coin-tossing is known to be impossible [GMPP16], somewhat surprisingly, we show that a slightly relaxed variant that we refer to as *list coin-tossing* is, in fact, possible in only three rounds.

Very briefly, in list coin-tossing, the simulator is allowed to receive polynomially many random string samples from the ideal functionality (where the exact polynomial may depend upon the adversary), and it may choose any one of them as its output. It is not difficult to see that this notion already suffices for most conceivable applications of coin-tossing, such as implementing a common random string setup. For example, consider the setting where we want to generate a CRS in the setup algorithm of a non-interactive zero knowledge (NIZK) argument system. Now, in the ideal world, instead of running a simulator which "forces" one particular random string given by the ideal functionality, we can substitute it with the simulator of a list coin tossing protocol that

receives polynomially many random strings from the ideal functionality and "forces" one of them as the CRS. This would still suffice for the NIZK argument system.

We achieve the following result:

Theorem 3 (Informal)**.** *Assuming injective one-way functions, there exists a three round multiparty list coin-tossing protocol with black-box simulation. With the additional assumption of DDH/QR/N^{th}-Residuosity, this can be generalized to randomized inputless functionalities where security is defined analogously to list coin-tossing.*

Finally, we note that by applying the transformation[2] of [GMPP16] on the protocol from Theorem 3 for the two-party case, we can obtain a four round two-party list coin-tossing protocol in the *unidirectional-message* model. This result overcomes the barrier of five rounds for standard two-party coin-tossing established by [KO04].

Corollary 4 (Informal)**.** *Assuming injective one-way functions, there exists a four round two-party list coin-tossing protocol in the unidirectional-message model with black-box simulation.*

Leveled Rewinding Security. While promise ZK addresses the issue of proving honest behavior within three rounds, it does not address non-malleability issues that typically plague security proofs of constant-round protocols in the simultaneous-message model. In particular, when multiple primitives are being executed in parallel, we need to ensure that they are non-malleable w.r.t. each other. For example, we may require that a primitive A remains "secure" while the simulator (or a reduction) is (say) trying to extract adversary's input from primitive B via rewinding.

In the works of [ACJ17, BHP17], such issues are addressed by using complexity leveraging. In particular, they rely upon multiple levels of complexity leveraging to establish non-malleability relationships across primitives, e.g., by setting the security parameters such that primitive X is more secure than primitive Y that is more secure than primitive Z, and so on. Such a use a complexity leveraging is, in fact, quite common in the setting of limited rounds (see, e.g., [COSV16]).

We instead rely upon a *leveled rewinding security* technique to avoid the use of complexity leveraging and base security on polynomial-time assumptions. Roughly, in our constructions, primitives have various levels of "bounded rewinding" security that are carefully crafted so that they enable non-malleability relationships across primitives, while still enabling rewinding-based simulation and reductions. E.g., a primitive X may be insecure w.h.p. against 1 rewind, however, another primitive Y may be secure against 1 rewind but insecure against 2 rewinds. Yet another primitive Z may be secure against 2 rewinds but insecure against 3 rewinds, and so on. We remark that leveled rewinding security with a "single level" was previously used in [GRRV14]; here we extend this idea to "multiple levels".

1.2 Related Work

Concurrent Work. In a concurrent and independent work, Halevi et al. [HHPV17] construct a four round MPC protocol against malicious adversaries in the plain model based on different assumptions than ours. In particular, they rely upon enhanced trapdoor permutations and public-key encryption schemes that admit affine homomorphisms with equivocation (which in turn can be based on LWE/DDH/QR; see [HHPV17]). They do not consider the problems of promise ZK and list coin-tossing.

[2]The work of Garg et al. [GMPP16] establishes an impossibility result for three round multiparty coin-tossing by transforming any three round two-party coin-tossing protocol in the simultaneous-message model into a four round two-party coin-tossing protocol in the unidirectional-message model, and then invoking [KO04] who proved the impossibility of four round two-party coin-tossing.

Round Complexity of MPC. The study of constant-round protocols for MPC was initiated by Beaver et al. [BMR90]. They constructed constant-round MPC protocols in the presence of honest majority. Subsequently, a long sequence of works constructed constant-round MPC protocols against dishonest majority based on a variety of assumptions and techniques (see, e.g., [KOS03, Pas04, PW10, Wee10, Goy11]).

Garg et al. [GMPP16] initiated the study of the exact round complexity of MPC. They constructed five (resp., six) round MPC using indistinguishability obfuscation (resp., LWE) and three-round robust non-malleable commitments. Recently, in concurrent works, Ananth et al. [ACJ17] and Brakerski et al. [BHP17] constructed four round MPC protocol based on sub-exponential-time hardness assumptions. While [BHP17] require sub-exponential LWE and adaptive commitments [PPV08], the work of [ACJ17] relied on sub-exponential DDH. Furthermore, [ACJ17] also constructed a five round MPC protocol based on polynomially hard DDH. Recently, concurrent to our work, Benhamouda and Lin [BL18] constructed five round MPC based on any five round OT.[3]

Ciampi et al. [COSV17a] constructed four-round multiparty coin-tossing from polynomial-time assumptions. The same authors also construct four-round two-party computation in the simultaneous-message model from polynomial-time assumptions [COSV17b]. However, their results do not extend to general multiparty functionalities, which is the focus of our work.

In the setting of super-polynomial-time simulation, [BGI$^+$17, JKKR17] construct two round two-party computation from sub-exponential DDH. In the multi-party setting, [KS17] construct coin-flipping and [BGJ$^+$17] extend this result to input-less randomized functionalities, again in two rounds with super-polynomial simulation. Garg et al. [GKP17] construct five round concurrent two-party computation from quasi-poly hard assumptions and [BGJ$^+$17] construct three round concurrent MPC from sub-exponential LWE.

All of the above results are in the plain model where no trusted setup assumptions are available. Asharov et. al. [AJL$^+$12] constructed three round semi-malicious MPC protocols in the CRS model. Subsequently, two-round semi-malicious MPC protocols in the CRS model were constructed by Garg et al. [GGHR14] using indistinguishability obfuscation, and by Mukherjee and Wichs [MW16] using LWE assumption. Recently, concurrent to our work, two-round semi-malicious protocols in the plain model were constructed by Garg and Srinivasan [GS17, GS18] and Benhamouda and Lin [BL18] from two-round (semi-malicious) OT. All of these results can be transformed to malicious secure protocols in the CRS model using NIZKs.

2 Technical Overview

In this section, we provide an overview of the main ideas underlying our results.

2.1 Promise Zero Knowledge

Recall that the notion of promise ZK is defined in the simultaneous-message model, where in every round, both the prover and the verifier send a message simultaneously.[4] Crucially, the ZK property is only defined w.r.t. a set of admissible verifiers that promise to send a "valid" non-aborting message in the last round with some noticeable probability.

We construct a three round distributional promise ZK protocol with black-box simulation based on injective one-way functions. We work in the delayed-input setting where the statement being

[3] Benhamouda and Lin [BL18], in fact, give a more general transformation from any k-round OT to k-round MPC, for $k \geq 5$.

[4] An adversarial prover or verifier can be rushing, i.e., it may wait to receive a message from the honest party in any round before sending its own message in that round.

proven is revealed to the (adversarial) verifier only in the last round.[5] Further, we work in the distributional setting, where statements being proven are sampled from an efficiently sampleable public distribution, i.e., it is possible to efficiently sample a statement together with a witness.

For simplicity of presentation, here we describe our construction using an additional assumption of two-round WI proofs, a.k.a. Zaps [DN00]. In our actual construction of promise ZK, we replace the Zaps with three round delayed-input WI proofs with some additional security guarantees that we construct based on injective one-way functions.[6]

Our construction of promise ZK roughly follows the FLS paradigm [FLS90] for ZK:

- First, the prover and the verifier engage in a three round "trapdoor generation phase" that determines a secret "trapdoor" that is known to the verifier but not the prover.
- Next, in a proof phase, the prover commits to 0 in a (three round) delayed-input extractable commitment and proves via a Zap that either the purported statement is true or that it committed to the trapdoor (instead of 0).

By appropriately parallelizing both of these phases, we obtain a three round protocol in the simultaneous-message model. Below, we discuss the challenges in proving soundness and promise ZK properties.

Proving Soundness. In order to argue soundness, a natural strategy is to rewind the cheating prover in the second and third round to extract the value it has committed in the extractable commitment. If this value is the trapdoor, then we can (hopefully) break the hiding property of the trapdoor generation phase to obtain a contradiction. Unfortunately, this strategy doesn't work as is since the trapdoor generation phase is parallelized with the extractable commitment. Thus, while extracting from the extractable commitment, we may inadvertently also break the security of the trapdoor generation phase! Indeed, this is the key problem that arises in the construction of non-malleable protocols.

To address this, we observe that in order to prove soundness, it suffices to extract the trapdoor from the cheating prover with some noticeable probability (as opposed to overwhelming probability). Now, suppose that the extractable commitment scheme is such that it is possible to extract the committed value via k rewinds (for some small integer k) if the "main thread" of execution is non-aborting with noticeable probability. Then, we can still argue soundness if the trapdoor generation has a stronger hiding property, namely, security under k rewinds (but is insecure under more than k rewinds to enable simulation; see below). This is an example of leveled rewinding security technique with a single level; later we discuss its application with multiple levels.

We note that standard extractable commitment schemes such as [PRS02, Ros04] (as well as their delayed-input variants) achieve the above extraction property for $k = 1$. This means that we only require the trapdoor generation phase to maintain hiding property under 1 rewinding. Such a scheme can be easily constructed from one-way functions.

Proving Promise ZK. In order to prove the promise ZK property, we construct a simulator that learns information from the verifier in the third round, in order to simulate its view in the third round! Roughly, our simulator first creates multiple "look-ahead" execution threads[7] with

[5] In our actual construction, we consider a slightly more general setting where a statement x has two parts (x_1, x_2): the first part x_1 is revealed in the second round while the second part x_2 is revealed in the third round. This generalization is used in our applications of promise ZK, but we ignore it here for simplicity of presentation.

[6] In particular, replacing Zaps with delayed-input WI proofs relies on *leveled rewinding security* technique with multiple levels that we describe in Section 2.2. We do not discuss it here to avoid repetition.

[7] Throughout, whenever the simulator rewinds, we call each rewound execution a look-ahead thread. The messages that are eventually output by the simulator constitute the main thread.

the adversarial verifier in order to extract the trapdoor from the trapdoor generation phase. Note that unlike typical ZK protocols where such a look-ahead thread only consists of partial protocol transcript, in our case, each look-ahead thread must contain a full protocol execution since the trapdoor generation phase completes in the third round.

Now, since the adversarial verifier may be rushing, the simulator must first provide its third round message (namely, the second message of Zap) on each look-ahead thread in order to learn the verifier's third round message. Since the simulator does not have a trapdoor yet, the only possibility for the simulator to prepare a valid third round message is by behaving honestly. However, the simulator does not have a witness for the statement proven by the honest prover. Thus, it may seem that we have run into a circularity.

This is where the distributional aspect of our notion comes to the rescue. Specifically, on the look-ahead execution threads, the simulator simply samples a fresh statement together with a witness from the distribution and proves the validity of the statement like an honest prover. Once it has extracted the trapdoor, it uses its knowledge to cheat (only) on the main thread (but continues to behave honestly on each look-ahead thread).[8]

2.2 Four Round Secure Multiparty Computation

We now describe the main ideas underlying our compiler from any three round semi-malicious MPC protocol Π (where the first round is public coin) to a four round malicious-secure MPC protocol Σ. For simplicity of presentation, in the discussion below, we ignore the first round of Π, and simply treat it as a *two round* protocol.

Starting Ideas. Similar to several previous works, our starting idea is to follow the GMW paradigm [GMW87] for malicious security. This entails two main steps: (1) Enabling extraction of adversary's inputs, and (2) Forcing honest behavior on the adversary in each round of Π. A natural idea to implement the first step is to require each party to commit to its input and randomness via a three round extractable commitment protocol. To force honest behavior, we require each party to give a delayed-input ZK proof together with every message of Π to establish that it is "consistent" with the input and randomness committed in the extractable commitment.

In order to obtain a four-round protocol Σ, we need to parallelize all of these sub-protocols appropriately. This means that while the proof for the second message of Π can be given via a four round (delayed-input) regular ZK proof, we need a *three round* proof system to prove the well-formedness of the first message of Π. However, as discussed earlier, three-round ZK proofs are known to be impossible w.r.t. black-box simulation [GK96, GMPP16] and even with non-black box simulation, are not known from standard assumptions.

Promise ZK and Partitioned Simulation. While [ACJ17, BHP17] tackled this issue by using sub-exponential hardness, we address it via partitioned simulation to base security on polynomial-time assumptions. Specifically, we use different mechanisms for proving honest behavior depending upon whether or not the adversary is aborting in the third round. For now, let us focus on the case where the adversary does not abort in the third round of Σ; later we discuss the aborting adversary case.

For the non-aborting case, we rely upon a three-round (delayed-input) distributional promise ZK to prove well-formedness of the first message of Π. As we discuss below, however, integrating

[8] The idea of using a witness to continue simulation is an old one [BS05]. Most recently, [JKKR17] used this idea in the distributional setting.

promise ZK in our construction involves overcoming several technical challenges due to specific properties of the promise ZK simulator (in particular, its requirement to behave honestly in look-ahead threads).[9] We also remark that in our actual construction, to address non-malleability concerns [DDN91], the promise ZK and the standard ZK protocols that we use are suitably "hardened" using three-round non-malleable commitments [GPR16, Khu17] to achieve *simulation soundness* [Sah99] in order to ensure that the proofs given by the adversarial parties remain sound even when the proofs given by honest parties are simulated. For simplicity of discussion, however, here we largely ignore this point, and instead focus on the technical ideas that are more unique to our construction.

We now proceed to discuss the main technical challenges underlying our construction and its proof of security.

How to do "Non-Malleable" Input Extraction? Let us start with the issue of extraction of adversary's input and trapdoors (for simulation of ZK proofs). In the aforementioned protocol design, in order to extract adversary's input and trapdoors, the simulator rewinds the second and third rounds. Note, however, that this process also rewinds the input commitments of the honest parties since they are executed in parallel. This poses the following fundamental challenge: we must somehow maintain privacy of honest party's inputs *even under rewinds*, while still extracting the inputs of the adversarial parties.

A plausible strategy to address this issue is to cheat in the rewound executions by sending random third round messages in the input commitment protocol on behalf of each honest party. This effectively nullifies the effect of rewinding on the honest party input commitments. However, in order to implement such a strategy, we need the ability to cheat in the ZK proofs since they are proving "well-formedness" of the input commitments.[10]

Unfortunately, such a strategy is not viable in our setting. As discussed in the previous subsection, in order to simulate the promise ZK on the main thread, the simulator must behave "honestly" on the rewound execution threads. This suggests that we cannot simply "sidestep" the issue of rewinding and instead must somehow make the honest party input commitments immune to rewinding. Yet, we must do this while still keeping the adversary input commitments extractable. Thus, it may seem that we have reached an impasse.

Leveled Rewinding Security to the Rescue. In order to break the symmetry between input commitments of honest and adversarial parties, we use the following sequence of observations:

- The security of the honest party input commitments is only invoked when we switch from a hybrid experiment (say) H_i to another experiment H_{i+1} inside our security proof. In order to argue indistinguishability of H_i and H_{i+1} by contradiction, it suffices to build an adversary that breaks the security of honest party input commitments with some noticeable probability (as opposed to overwhelming probability).
- This means that the reduction only needs to generate the view of the adversary in hybrids H_i and H_{i+1} with some noticeable probability. This, in turn, means that the reduction only needs to successfully extract the adversary's inputs and trapdoor (for generating its view) with noticeable probability.
- Now, recall that the trapdoor generation phase used in our promise ZK construction is secure against one rewind. However, if we rewind two times, then we can extract the trapdoor with

[9] Our construction of four round MPC, in fact, uses promise ZK in a non-black-box manner for technical reasons. We ignore this point here as it is not important to the discussion.

[10] Indeed, [ACJ17] implement such a strategy in their security proof by relying on sub-exponential hardness assumptions.

noticeable probability.

- Now, suppose that we can construct an input commitment protocol that maintains hiding property even if it is rewound two times, but guarantees extraction with noticeable probability if it is rewound three times. Given such a commitment scheme, we resolve the above problem as follows: the reduction rewinds the adversary three times, which ensures that with noticeable probability, it can extract *both* the trapdoor and the inputs from the adversary. In the first two rewound executions, the reduction generates the third round messages of the honest party input commitments honestly. At this point, the reduction already has the trapdoor. Now, in the third rewound execution, it generates random third messages in the honest party input commitments and uses the knowledge of the trapdoor to cheat in the proof.

The above strategy allows us to extract the adversary's inputs with noticeable probability while still maintaining privacy of honest party inputs. To complete this idea, we construct a new extractable commitment scheme from injective one-way functions that achieves the desired "bounded-rewinding security" property.

Taking a step back, note that in order to implement the above strategy, we created two levels of rewinding security: while the trapdoor generation phase is secure against one rewind (but insecure against two rewinds), the input commitment protocol is secure against two rewinds (but insecure against three rewinds). We refer to this technique as *leveled rewinding security* with multiple levels, and this is precisely what allows us to avoid the use of leveled complexity leveraging.

Using Promise ZK. In the works of [ACJ17, BHP17], the simulator behaves honestly in the first three rounds using random inputs for the honest parties. *We depart from this proof strategy, and instead, require our simulator to cheat even in the first three rounds on the main thread.*[11] Indeed, such a simulation strategy seems necessary for our case since the recent two-round semi-malicious MPC protocols of [GS18, BL18] – which we use to instantiate our compiler – require a cheating simulation strategy even in the first round.

To implement this proof strategy, we turn to promise ZK. However, recall that promise ZK simulator works by behaving honestly in the look-ahead threads. When applied to our MPC construction, this means that we must find a way to behave honestly on the look-ahead threads that are used for extracting inputs and trapdoors from the adversary. However, at first it is not immediately clear how to implement such a strategy. Clearly, our final simulator cannot use honest party inputs on the look-ahead threads to behave honestly.

Instead, our simulator uses random inputs to behave honestly on the look-ahead threads. The main challenge then is to argue that when we switch from using real honest inputs (in an intermediate hybrid) to random inputs on the look-ahead threads, the probability of extraction of adversary's inputs and trapdoors remains unchanged. Crucially, here, we do not need to consider a joint view across all the look-ahead threads, and instead, it suffices to argue the indistinguishability of adversary's view on each look-ahead thread (when we switch from real input to random input) one at a time. We rely upon techniques from the work of Jain et al. [JKKR17] for this indistinguishability argument. The same proof technique is also used to argue security in the case when the adversary aborts in the third round with overwhelming probability. We discuss this next.

Aborting Adversary Case. In the case where the adversary aborts in the third round with overwhelming probability, we cannot rely upon promise ZK since there is no hope for extraction from such an aborting adversary (which is necessary for simulating promise ZK). Therefore, in

[11] We emphasize that this strategy is only used in the case where the adversary does not abort in the third round. As we discuss below, we use a different strategy in the aborting adversary case.

this case, the simulator simply behave honestly on the main thread using random inputs (as in [ACJ17, BHP17]). The main challenge then is to switch in an indistinguishable manner from honest behavior in the first three rounds using real inputs to honest behavior using random inputs, while relying only on polynomial-time assumptions.

We address this case by relying upon techniques from [JKKR17]. We remark that we cannot directly use the three-round strong WI argument system of [JKKR17] since it requires the instance being proven to be disclosed to the verifier only in the third round of the protocol. This is not true in our case, since the instance also consists of the transcript of the three-round extractable commitment (and other sub-protocols like the trapdoor generation). Nevertheless, we are able to use ideas from [JKKR17] in a non-black-box manner to enable our security proof; we refer the reader to the technical sections for more details.

Other Issues. We note that since our partitioned simulation technique crucially relies upon identifying whether an adversary is aborting or not, we have to take precaution during simulation to avoid the possibility of the simulator running in exponential time. For this reason, we use ideas first developed in [GK96] and later used in many subsequent works, to ensure that the running time of our simulator is expected polynomial-time.

Finally, we note that the above discussion is oversimplified, and omits several technical points. We refer the reader to the technical sections for full details.

2.3 List Coin-Tossing

We now describe the main ideas underlying our construction of three round multiparty list coin-tossing. We start by describing the basic structure of our protocol:

- We start with a two-round semi-honest multiparty coin-tossing protocol based on injective one-way functions. Such a protocol can be constructed as follows: In the first round, each party i commits to a string r_i chosen uniformly at random, using a non-interactive commitment scheme. In the second round, each party reveals r_i without the decommitment information. The output is simply the XOR of all the r_i values.
- To achieve malicious security, we "compile" the above semi-honest protocol with a (delayed-input) distributional promise ZK protocol. Roughly speaking, in the third round, each party i now proves that the value r_i is the one it had committed earlier. By parallelizing the two sub-protocols appropriately, we obtain a three round protocol.

We first note that as in the case of our four round MPC protocol, here also we need to "harden" the promise ZK protocol with non-malleability properties. We do so by constructing a three-round simulation-extractable promise ZK based on injective one-way functions and then using it in the above compiler. Nevertheless, for simplicity of discussion, we do not dwell on this issue here, and refer the reader to the technical sections for further details.

We now describe the main ideas underlying our simulation technique. As in the case of four round MPC, we use partitioned simulation strategy to split the simulation into two cases, depending upon whether the adversary aborts or not in the third round.

Aborting Case. If the adversary aborts in the third round, then the simulator simply behaves honestly using a uniformly random string r_i on behalf of each honest party i. Unlike the four round MPC case, indistinguishability can be argued here in a straightforward manner since the simulated transcript is identically distributed as a real transcript. The main reason why such a strategy works is that since the parties do not have any input, there is no notion of "correct out-

put" that the simulator needs to enforce on the (aborting) adversary. This is also true for any randomized inputless functionality, and indeed for this reason, our result extends to such functionalities. Note, however, that this is not true for general functionalities where each party has an input.

Non-Aborting Case. We next consider the case where the adversary does not abort in the third round with noticeable probability. Note that in this case, when one execution thread completes, the simulator learns the random strings r_j committed to by the adversarial parties by simply observing the adversary's message in the third round.

At this point, the simulator queries the ideal functionality to obtain the random output (say) R and then attempts to "force" it on the adversary. This involves simulating the simulation-extractable promise ZK and sending a "programmed" value r_i' on behalf of one of the honest parties so that it leads to the desired output R. Now, since the adversary does not abort in the last round with noticeable probability, it would seem that after a polynomial number of trials, the simulator should succeed in forcing the output. At this point, it may seem that we have successfully constructed a three round multiparty coin-tossing protocol, which would contradict the lower bound of [GMPP16]!

We now explain the flaw in the above argument. As is typical to security with abort, an adversary's aborting behavior may depend upon the output it receives in the last round. For example, it may always choose to abort if it receives an output that starts with 00. Thus, if the simulator attempts to repeatedly force the same random output on the adversary, it may never succeed.

This is where list coin-tossing comes into the picture. In list coin-tossing, the simulator obtains a polynomial number of random strings from the ideal functionality, as opposed to a single string in regular coin-tossing. Our simulator attempts to force each of (polynomially many) random strings one-by-one on the adversary, in the manner as explained above. Now, each of the trials are independent, and therefore the simulator is guaranteed to succeed in forcing one of the random strings after a polynomial number of attempts.

Organization. We define some preliminaries in Section 3 and some building blocks for our protocols in Section 4. In Section 5, we define and construct Simulation-Extractable Promise ZK. Our three round List Coin Tossing protocol is described in Appendix **??**. Finally, the construction of our four round maliciously secure MPC protocol is in Appendix **??**.

3 Preliminaries

Here, we recall some preliminaries that will be useful in the rest of the paper. Throughout this paper, we will use λ to denote the security parameter, and $\mathsf{negl}(\lambda)$ to denote any function that is asymptotically smaller than $\frac{1}{\mathsf{poly}(\lambda)}$ for any polynomial $\mathsf{poly}(\cdot)$. We will use PPT to describe a probabilistic polynomial time machine. We will also use the words "rounds" and "messages" interchangeably, whenever clear from context.

3.1 Secure Multiparty Computation

In this work we follow the standard real/ideal world paradigm for defining secure multi-party computation. We refer the reader to [Gol04] for the precise definition.

Semi-malicious adversary. An adversary is said to be semi-malicious if it follows the protocol correctly, but with potentially maliciously chosen randomness.

3.2 Delayed-Input Interactive Arguments

In this section, we describe delayed-input interactive arguments.

Definition 1 (Delayed-Input Interactive Arguments)**.** *An n-round delayed-input interactive protocol (P, V) for deciding a language L is an argument system for L that satisfies the following properties:*

- **Delayed-Input Completeness.** *For every security parameter $\lambda \in \mathbb{N}$, and any $(x, w) \in R_L$ such that $|x| \leq 2^\lambda$,*

$$\Pr[(P, V)(1^\lambda, x, w) = 1] = 1 - \mathsf{negl}(\lambda).$$

  where the probability is over the randomness of P and V. Moreover, the prover's algorithm initially takes as input only 1^λ, and the pair (x, w) is given to P only in the beginning of the n'th round.

- **Delayed-Input Soundness.** *For any PPT cheating prover P^* that chooses x^* (adaptively)* after *the first $n-1$ messages, it holds that if $x^* \notin L$ then*

$$\Pr[(P^*, V)(1^\lambda, x^*) = 1] = \mathsf{negl}(\lambda).$$

  where the probability is over the random coins of V.

Remark 1. *We note that in a delayed-input interactive argument satisfying Definition 1, completeness and soundness also hold when (part of) the instance is available in the first $(n-1)$ rounds.*

We will also consider delayed-input interactive arguments in the simultaneous-message setting, that satisfy soundness against rushing adversaries.

3.3 Extractable Commitments

Here onwards until Section 5, we will discuss protocols where only one party sends a message in any round.

Definition 2 (Extractable Commitments)**.** *Consider any statistically binding, computationally hiding commitment scheme $\langle C, R\rangle$. Let $\mathsf{Trans}\langle C(m, r_C), R(r_R)\rangle$ denote a commitment transcript with committer input m, committer randomness r_C and receiver randomness r_R, and let $\mathsf{Decom}(\tau, m, r_C)$ denote the algorithm that on input a commitment transcript τ, committer message m and randomness r_C outputs 1 or 0 to denote whether or not the decommitment was accepted (we explicitly require the decommitment phase to not require receiver randomness r_R).*

Then $\langle C, R\rangle$ is said to be extractable if there exists an expected PPT oracle algorithm $\mathcal{E}$, such that for any PPT cheating committer C^* the following holds. Let $\mathsf{Trans}\langle C^*, R(r_R)\rangle$ denote a transcript of the interaction between C^* and R. Then $\mathcal{E}^{C^*}(\mathsf{Trans}\langle C^*, R(r_R)\rangle)$ outputs m, r_C such that over the randomness of $\mathcal{E}$ and of sampling $\mathsf{Trans}\langle C^*, R(r_R)\rangle$:

$$\Pr[(\exists \widetilde{m} \neq m, \widetilde{r}_C) \text{ such that } \mathsf{Decom}(\tau, \widetilde{m}, \widetilde{r}_C) = 1] = \mathsf{negl}(\lambda)$$

Remark 2. *The notion of extraction described in Definition 2 is often referred to as over-extraction. This is because the extractor $\mathcal{E}$ is allowed to output any arbitrary value if $\mathsf{Trans}\langle C^*, R(r_R)\rangle$ does not contain a commitment to any valid message. On the other hand, if $\mathsf{Trans}\langle C^*, R(r_R)\rangle$ is a valid commitment to some message m, $\mathcal{E}$ must output the correct committed message m.*

Definition 3 (k-Extractable Commitments)**.** *An extractable commitment satisfying Definition 2 is said to be k-extractable if there exists a polynomial $p(\cdot)$ such that the extractor $\mathcal{E}^{C^*}(\mathsf{Trans}\langle C^*, R(r_R)\rangle)$ with $k-1$ queries to C^*, outputs m, r_C such that over the randomness of $\mathcal{E}$ and of sampling $\mathsf{Trans}\langle C^*, R(r_R)\rangle$:*

$$\Pr[\mathsf{Decom}(\tau, m, r_C) = 1] \geq p(\lambda)$$

Delayed-Input Extractable Commitments. We say that an extractable commitment is *delayed-input* if the committer uses the input message m only in the last round of the protocol.

Theorem 5. *[PRS02, Ros04] For any constant $K > 0$, assuming injective one-way functions, there exists a three round delayed-input K-extractable commitment scheme satisfying Definition 3.*

3.4 Non-Malleable Commitments

We start with the definition of non-malleable commitments by Pass and Rosen [PR05] and further refined by Lin et al [LPV08] and Goyal [Goy11]. (All of these definitions build upon the original definition of Dwork et al. [DDN91]).

In the real experiment, a man-in-the-middle adversary MIM interacts with a committer C in the left session, and with a receiver R in the right session. Without loss of generality, we assume that each session has identities or tags, and require non-malleability only when the tag for the left session is different from the tag for the right session.

At the start of the experiment, the committer C receives an input val and MIM receives an auxiliary input z, which might contain a priori information about val. Let $\mathsf{MIM}_{\langle C,R\rangle}(\mathsf{val}, z)$ be a random variable that describes the value $\widetilde{\mathsf{val}}$ committed by MIM in the right session, jointly with the view of MIM in the real experiment.

In the ideal experiment, a PPT simulator $\mathcal{S}$ directly interacts with MIM. Let $\mathsf{Sim}_{\langle C,R\rangle}(1^\lambda, z)$ denote the random variable describing the value $\widetilde{\mathsf{val}}$ committed to by $\mathcal{S}$ and the output view of $\mathcal{S}$.

In either of the two experiments, if the tags in the left and right interaction are equal, then the value $\widetilde{\mathsf{val}}$ committed in the right interaction, is defined to be $\perp$.

We define a strengthened version of non-malleable commitments for use in this paper.

Definition 4 (Special Non-malleable Commitments)**.** *A three round commitment scheme $\langle C, R\rangle$ is said to be special non-malleable if:*

- *For every* synchronizing[12] *PPT* MIM*, there exists a PPT simulator $\mathcal{S}$ such that the following ensembles are computationally indistinguishable:*

$$\{\mathsf{MIM}_{\langle C,R\rangle}(\mathsf{val}, z)\}_{\lambda\in\mathbb{N},\mathsf{val}\in\{0,1\}^\lambda,z\in\{0,1\}^*} \text{ and } \{\mathsf{Sim}_{\langle C,R\rangle}(1^\lambda, z)\}_{\lambda\in\mathbb{N},\mathsf{val}\in\{0,1\}^\lambda,z\in\{0,1\}^*}$$

- *$\langle C, R\rangle$ is delayed-input, that is, correctness holds even when the committer obtains his input only in the last round.*

- *$\langle C, R\rangle$ satisfies* last-message pseudorandomness*, that is, for every non-uniform PPT receiver R^*, it holds that $\{\mathsf{REAL}_0^{R^*}(1^\lambda)\}_\lambda$ and $\{\mathsf{REAL}_1^{R^*}(1^\lambda)\}_\lambda$ are computationally indistinguishable, where for $b \in \{0,1\}$, the random variable $\mathsf{REAL}_b^{R^*}(1^\lambda)$ is defined via the following experiment.*

  1. *Run $C(1^\lambda)$ and denote its output by (com_1, σ), where σ is its secret state, and com_1 is the message to be sent to the receiver.*

[12] A synchronizing adversary is one that sends its message for every round before obtaining the honest party's message for the next round.

2. *Run the receiver* $R^*(1^\lambda, \mathsf{com}_1)$*, who outputs a message* com_2*.*
3. *If* $b = 0$*, run* $C(\sigma, \mathsf{com}_2)$ *and send its message* com_3 *to* R^**. Otherwise, if* $b = 1$*, compute* $\mathsf{com}_3 \xleftarrow{\$} \{0,1\}^m$ *and send it to* R^**. Here* $m = m(\lambda)$ *denotes* $|\mathsf{com}_3|$*.*
4. *The output of the experiment is the output of* R^**.*

- $\langle C, R \rangle$ *satisfies 2-extractability according to Definition 3.*

Goyal et al. [GPR16] construct three-round special non-malleable commitments satisfying Definition 4 based on injective OWFs.

Imported Theorem 1 ([GPR16])**.** *Assuming injective one-way functions, there exists a three round non-malleable commitment satisfying Definition 4.*

4 Building Blocks

We now describe some of the building blocks we use in our constructions.

4.1 Trapdoor Generation Protocol

In this section, we define and construct a primitive called Trapdoor Generation Protocol. In such a protocol, a sender S (a.k.a. trapdoor generator) communicates with a receiver R. The protocol satisfies two properties: (i) Sender security, i.e., no cheating PPT receiver can learn a valid trapdoor, and (ii) Extraction, i.e., there exists an expected PPT algorithm (a.k.a. extractor) that can extract a trapdoor from an adversarial sender via rewinding.

We construct a three-round trapdoor generation protocol where the first message sent by the sender determines the set of valid trapdoors, and in the next two rounds the sender proves that indeed it knows a valid trapdoor. Such schemes are known in the literature based on various assumptions [PRS02, Ros04, COSV17a]. Here, we consider trapdoor generation protocols with a stronger sender security requirement that we refer to as *1-rewinding security*. Below, we formally define this notion and then proceed to give a three-round construction based on one-way functions. Our construction is a minor variant of the trapdoor generation protocol from [COSV17a].

Syntax. A trapdoor generation protocol

$$\mathsf{TDGen} = (\mathsf{TDGen}_1, \mathsf{TDGen}_2, \mathsf{TDGen}_3, \mathsf{TDOut}, \mathsf{TDValid}, \mathsf{TDExt})$$

is a three round protocol between two parties - a sender (trapdoor generator) S and receiver R that proceeds as below.

1. **Round 1 -** $\mathsf{TDGen}_1(\cdot)$**:**
   S computes and sends $\mathsf{td}_1^{S\to R} \leftarrow \mathsf{TDGen}_1(\mathsf{r}_S)$ using a random string r_S.
2. **Round 2 -** $\mathsf{TDGen}_2(\cdot)$**:**
   R computes and sends $\mathsf{td}_2^{R\to S} \leftarrow \mathsf{TDGen}_2(\mathsf{td}_1^{S\to R}; \mathsf{r}_R)$ using randomness r_R.
3. **Round 3 -** $\mathsf{TDGen}_3(\cdot)$**:**
   S computes and sends $\mathsf{td}_3^{S\to R} \leftarrow \mathsf{TDGen}_3(\mathsf{td}_2^{R\to S}; \mathsf{r}_S)$
4. **Output -** $\mathsf{TDOut}(\cdot)$
   The receiver R outputs $\mathsf{TDOut}(\mathsf{td}_1^{S\to R}, \mathsf{td}_2^{R\to S}, \mathsf{td}_3^{S\to R})$.

5. **Trapdoor Validation Algorithm - $\mathsf{TDValid}(\cdot)$:**
   Given input $(\mathsf{t}, \mathsf{td}_1^{S\to R})$, output a single bit 0 or 1 that determines whether the value t is a valid trapdoor corresponding to the message td_1 sent in the first round of the trapdoor generation protocol.

In what follows, for brevity, we set td_1 to be $\mathsf{td}_1^{S\to R}$. Similarly we use td_2 and td_3 instead of $\mathsf{td}_2^{R\to S}$ and $\mathsf{td}_3^{S\to R}$, respectively. Note that the algorithm $\mathsf{TDValid}$ does not form a part of the interaction between the trapdoor generator and the receiver. It is, in fact, a public algorithm that enables public verification of whether a value t is a valid trapdoor for a first round message td_1.

Extraction. There exists a PPT extractor algorithm TDExt that, given a set of values[13] $(\mathsf{td}_1, \{\mathsf{td}_2^i, \mathsf{td}_3^i\}_{i=1}^3)$ such that $\mathsf{td}_2^1, \mathsf{td}_2^2, \mathsf{td}_2^3$ are distinct and $\mathsf{TDOut}(\mathsf{td}_1, \mathsf{td}_2^i, \mathsf{td}_3^i) = 1$ for all $i \in [3]$, outputs a trapdoor t such that $\mathsf{TDValid}(\mathsf{t}, \mathsf{td}_1) = 1$.

1-Rewinding Security. We define the notion of *1-rewinding security* for a trapdoor generation protocol TDGen. Consider the following experiment between a sender S and any (possibly cheating) receiver R^*.

Experiment E:

- R^* interacts with S and completes one execution of the protocol TDGen. R^* receives values $(\mathsf{td}_1, \mathsf{td}_3)$ in rounds 1 and 3 respectively.
- Then, R^* rewinds S to the beginning of round 2.
- R^* sends S a new second round message td_2^* and receives a message td_3^* in the third round.
- At the end of the experiment, R^* outputs a value t^*.

Definition 5 (1-Rewinding Security)**.** *A trapdoor generation protocol* $\mathsf{TDGen} = (\mathsf{TDGen}_1, \mathsf{TDGen}_2, \mathsf{TDGen}_3, \mathsf{TDOut}, \mathsf{TDValid})$ *achieves 1-rewinding security if, for every non-uniform PPT receiver* R^* *in the above experiment E,*

$$\Pr\big[\mathsf{TDValid}(\mathsf{t}^*, \mathsf{td}_1) = 1\big] = \mathsf{negl}(\lambda),$$

where the probability is over the random coins of S*, and where* t^* *is the output of* R^* *in the experiment E, and* td_1 *is the message from* S *in round 1.*

4.1.1 Construction

We now describe a three round trapdoor generation protocol based on one way functions.

Let S and R denote the sender and the receiver, respectively. Let λ denote the security parameter. Let $(\mathsf{Gen}, \mathsf{Sign}, \mathsf{Verify})$ be a signature scheme that is existentially unforgeable against chosen-message attacks. Such schemes are known based on one-way functions [GMR88].

Theorem 6. *Assuming the existence of one way functions, the protocol* Π^{TD} *described in Figure 1 is a 1-rewinding secure trapdoor generation protocol.*

[13] These values can be obtained from the malicious sender via an expected PPT rewinding procedure. The expected PPT simulator in our applications performs the necessary rewindings and then feeds these values to the extractor TDExt.

1. **Round 1 -** $\mathsf{TDGen}_1(\mathsf{r}_S)$**:**
   S does the following:
   - Generate $(\mathsf{sk}, \mathsf{vk}) \leftarrow \mathsf{Gen}(\mathsf{r}_S)$.
   - Send $\mathsf{td}_1^{S\to R} = \mathsf{vk}$ to R.
2. **Round 2 -** $\mathsf{TDGen}_2(\mathsf{td}_1^{S\to R})$**:**
   R sends a random string m as the message $\mathsf{td}_2^{R\to S}$ to S.
3. **Round 3 -** $\mathsf{TDGen}_3(\mathsf{td}_1^{S\to R}, \mathsf{td}_2^{R\to S}; \mathsf{r}_S)$**:**
   S computes and sends $\mathsf{td}_3^{S\to R} = \mathsf{Sign}(\mathsf{sk}, \mathsf{m}; \mathsf{r}_\mathsf{m})$ where r_m is randomly chosen.
4. **Output: -** $\mathsf{TDOut}(\mathsf{td}_1^{S\to R}, \mathsf{td}_2^{R\to S}, \mathsf{td}_3^{S\to R})$
   The receiver R outputs 1 if $\mathsf{Verify}(\mathsf{td}_1^{S\to R}, \mathsf{m}, \mathsf{td}_3^{S\to R}) = 1$.
5. **Trapdoor Validation Algorithm -** $\mathsf{TDValid}(\mathsf{t}, \mathsf{td}_1)$**:**
   Given input $(\mathsf{t}, \mathsf{td}_1)$, the algorithm does the following:
   - Let $\mathsf{t} = \{\mathsf{m}_i, \sigma_i\}_{i=1}^3$.
   - Output 1 if $\mathsf{m}_1, \mathsf{m}_2, \mathsf{m}_3$ are distinct and $V(\mathsf{td}_1, \mathsf{m}_i, \sigma_i) = 1$ for all $i \in [3]$.

Figure 1: Trapdoor Generation Protocol Π^{TD}.

Proof. Suppose the protocol Π^{TD} is not 1-rewinding secure. That is, there exists a malicious receiver R^* that breaks the 1-rewinding security. We will use R^* to design an adversary $\mathcal{A}_{\mathsf{Sign}}$ that breaks the unforgeability of the signature scheme. $\mathcal{A}_{\mathsf{Sign}}$, and upon receiving a verification key vk, it interacts with $\mathcal{C}_{\mathsf{Sign}}$ and with R^*, as follows: First, it sets $\mathsf{td}_1 = \mathsf{vk}$ and sends td_1 to R^* in round 1. Upon receiving a query $\mathsf{td}_2 = \mathsf{m}$ from R^*, $\mathcal{A}_{\mathsf{Sign}}$ forwards this to $\mathcal{C}_{\mathsf{Sign}}$ and receives a value σ_m from $\mathcal{C}_{\mathsf{Sign}}$ which it sends to R^* as the message td_3. Then, upon receiving a query $\mathsf{td}_2^* = \mathsf{m}^*$ from R^* in the rewound execution, $\mathcal{A}_{\mathsf{Sign}}$ once again does the same. That is, $\mathcal{A}_{\mathsf{Sign}}$ forwards this to $\mathcal{C}_{\mathsf{Sign}}$ and receives a value σ_{m^*} from $\mathcal{C}_{\mathsf{Sign}}$ which it sends to R^* as the message td_3^*.

Then, since R^* breaks the 1-rewinding security, it outputs a value t^* in experiment E such that $\mathsf{TDValid}(\mathsf{t}^*, \mathsf{td}_1) = 1$ with non-negligible probability p. Recall from the definition of the algorithm $\mathsf{TDValid}$, it must be the case that $\mathsf{t}^* = \{\mathsf{m}_i, \sigma_i\}_{i=1}^3$ such that $\mathsf{m}_1, \mathsf{m}_2, \mathsf{m}_3$ are distinct and $V(\mathsf{vk}, \mathsf{m}_i, \sigma_i) = 1$ for all i. $\mathcal{A}_{\mathsf{Sign}}$ picks the value $\mathsf{m}_i \notin \{\mathsf{m}, \mathsf{m}^*\}$ and outputs (m_i, σ_i) as a forgery. □

Extractor $\mathsf{TDExt}(\cdot)$**.** The extractor works as follows. It receives a verification key $\mathsf{vk} = \mathsf{td}_1$, and a set of values $\{\mathsf{m}_i, \sigma_i\}_{i=1}^3$ such that m_i are all distinct and $\mathsf{Verify}(\mathsf{vk}, \mathsf{m}_i, \sigma_i) = 1$ for every $i \in [3]$. Then, TDExt outputs $\mathsf{t} = \{\mathsf{m}_i, \sigma_i\}_{i=1}^3$ as a valid trapdoor. Correctness of the extraction is easy to see by inspection.

Remark: In the application to our MPC protocol, one party is the sender and sends the first round message td_1. Each of the other $(n-1)$ parties send a second round message $\mathsf{td}_{2,i}$ and the sender now sets the concatenation of all of them as the second round message td_2 - that is, $\mathsf{td}_2 = (\mathsf{td}_{2,1}||\ldots||\mathsf{td}_{2,n-1})$. The sender then computes td_3 as before.

4.2 WI with Bounded Rewinding Security

We define the notion of three-round delayed-input witness indistinguishable (WI) argument with "bounded-rewinding security," and construct such a primitive assuming the existence of injective one-way functions. Such a primitive has been implicitly constructed and used in the literature previously in the non-delayed-input setting. For example, Goyal et al. [GRRV14] construct and use the notion of WI arguments with 1-rewinding security based on injective one-way functions.

We formally define three-round delayed-input WI with bounded-rewinding security here. In appendix Appendix A, we describe a construction for the same which is obtained by combining a delayed-input WI [LS90] that is not rewinding secure with a non-delayed-input WI that is bounded-rewinding secure [IKOS07]. For our applications, we instantiate the rewinding parameter B with the value 5.

Definition 6 (3-Round Delayed-Input WI with Bounded Rewinding Security)**.** *Fix a positive integer B. A delayed-input 3-round interactive argument (as defined in Definition 1) for an NP language L, with an NP relation R_L is said to be WI with B-*Rewinding Security *if for every non-uniform PPT interactive Turing Machine V^*, it holds that $\{\mathsf{REAL}_0^{V^*}(1^\lambda)\}_\lambda$ and $\{\mathsf{REAL}_1^{V^*}(1^\lambda)\}_\lambda$ are computationally indistinguishable, where for $b \in \{0,1\}$ the random variable $\mathsf{REAL}_b^{V^*}(1^\lambda)$ is defined via the following experiment. In what follows we denote by P_1 the prover's algorithm in the first round, and similarly we denote by P_3 his algorithm in the third round.*

Experiment $\mathsf{REAL}_b^{V^*}(1^\lambda)$:

1. *Run $P_1(1^\lambda)$ and denote its output by (rwi_1, σ), where σ is its secret state, and rwi_1 is the message to be sent to the verifier.*
2. *Run the verifier $V^*(1^\lambda, \mathsf{rwi}_1)$, who outputs (x, w_0, w_1) and a message rwi_2.*
3. *Run $P_3(\sigma, \mathsf{rwi}_2, x, w_b)$, where P_3 is the (honest) prover's algorithm for generating the third message of the WI protocol, and send its message rwi_3 to V^*.*
4. *Set a counter $i = 0$.*
5. *If $i < B$, then set $i = i + 1$, and V^* (given all the information so far) generates another tuple (x, w_0, w_1) and message rwi_2^i, and receives the (honest) prover's message $P_3(\sigma, \mathsf{rwi}_2^i, x, w_b)$. Repeat this step until $i = B$.*
6. *The output of the experiment is the output of V^*.*

In Appendix A, we prove the following theorem:

Theorem 7. *Assuming injective one-way functions, there exists a three round delayed-input witness-indistinguishable argument system with $(B = 5)$-rewinding security.*

5 Promise Zero Knowledge

In this section, we introduce our new notion of promise zero knowledge interactive arguments. Unlike the standard notion of zero knowledge interactive arguments that is defined in the unidirectional-message model of communication, promise ZK is defined in the simultaneous-message model, where in every round, both the prover and the verifier simultaneously send a message to each other. Crucially, in promise ZK, the zero knowledge property is only required to hold against a specific class of "valid" verifiers (that do not send invalid messages).

Validity Check. Before defining promise ZK, we enhance the syntax for simultaneous-message interactive arguments to include an additional algorithm Valid. That is, a simultaneous-message interactive argument is denoted by (P, V, Valid). The notions of completeness and soundness remain intact as before. Looking ahead, the intuition behind introducing the new algorithm is that we want to capture those verifiers who send a "valid" message in every round (including the last round). We do this by using the Valid algorithm.

This algorithm Valid is protocol specific. For example, if the honest verifier is instructed to prove knowledge of a trapdoor that he generated, and the proof fails, then his messages are not valid. Importantly, even if only the verifier's last message is invalid, and even though the prover does not need to explicitly respond to this message[14] we refer to this transcript as invalid. We denote by Valid the (public verification) algorithm which checks whether the transcript, including the verifier's last message, is valid or not, that is,

$$\mathsf{Valid}(\mathsf{Trans}(P(x,w), V^*)) = 1$$

if and only if all the messages sent by V^* appear to be valid, given the transcript. The correctness requirement of this algorithm is that if the verifier's messages are generated honestly according to the protocol, then

$$\Pr[\mathsf{Valid}(\mathsf{Trans}(P(x,w), V)) = 1] = 1.$$

Looking ahead, in our protocols, at the end of each execution of the ZK protocol, the prover will check whether the verifier sent "valid" messages, and if not, the prover will abort.

5.1 Definitions

We now proceed to describe our notion of promise zero knowledge. Roughly speaking, we define promise ZK similarly to standard ZK, with two notable differences: First, promise ZK is defined in the simultaneous-message model. Second, the zero knowledge property is only defined w.r.t. a special class of verifiers who generate a valid transcript, with some noticeable probability. In order to define this notion, we need to have an estimation of the probability that the cheating verifier sends an invalid message throughout the protocol.

Validity Approximation. Consider a delayed-input simultaneous message interactive argument system (P, V, Valid). Consider any verifier V^*, and any efficiently sampleable distribution $\mathcal{D} = \{\mathcal{D}_\lambda\}$, where $\mathcal{D}_\lambda$ samples pairs (x, w) such that $x \in \{0,1\}^\lambda$ and $(x, w) \in R_L$

In what follows we denote by $P = (P_1, P_2)$, a prover that is split into two parts. First, $(\mathsf{view}_{V^*,1}, \mathsf{st}) \leftarrow P_1(1^\lambda)$ is obtained, and then $P_2(x, w, \mathsf{st})$ continues the rest of the P algorithm with V^*. This is done primarily because we would like to approximate the the validity probability of V^* conditioned on $\mathsf{view}_{V^*,1}$.

Let $\mathsf{Trans}(P_2(x, w, \mathsf{st}), V^*)$ denote the protocol transcript between P_2 and V^*: that is, $\mathsf{Trans}(P, V^*) = (\mathsf{view}_{V^*,1}, \mathsf{Trans}(P_2(x, w, \mathsf{st}), V^*))$. Let

$$q_{\mathsf{view}_{V^*,1}} = \Pr[\mathsf{Valid}(\mathsf{view}_{V^*,1}, \mathsf{Trans}(P_2(x, w, \mathsf{st}), V^*)) = 1 | (\mathsf{view}_{V^*,1}, \mathsf{st}) \leftarrow P_1(1^\lambda)]$$

where the probability is over the generation of $(x, w) \leftarrow \mathcal{D}_\lambda$ and the coins of P_2. We emphasize that $q_{\mathsf{view}_{V^*,1}}$ depends on $\mathcal{D}$ and on V^*, we omit this dependence from the notation to avoid cluttering.

[14] We use this promise ZK protocol as a building block in our MPC protocols, and in these protocols, the party acting as prover does indeed read this last ZK message sent by the verifier, and based on its validity decides whether to abort the MPC protocol. See, for example, Section 6.

Definition 7. *For any constant $c \in \mathbb{N}$, a PPT oracle algorithm $\mathsf{pExtract}_c$ is said to be a* validity approximation algorithm*, if the following holds for all malicious verifiers V^* and for all efficiently sampleable distributions $\mathcal{D} = \{\mathcal{D}_\lambda\}$:*

- *If $\mathsf{pExtract}_c^{V^*,\mathcal{D}}(\mathsf{view}_{V^*,1}, \mathsf{st}) = 0$, then $q_{\mathsf{view}_{V^*,1}} < 2 \cdot \lambda^{-c}$.*
- *Otherwise, if $\mathsf{pExtract}_c^{V^*,\mathcal{D}}(\mathsf{view}_{V^*,1}, \mathsf{st}) = p$, then $p \geq \lambda^{-c}$ and $\frac{p}{2} < q_{\mathsf{view}_{V^*,1}} < 2 \cdot p$.*

We now formalize our notion of promise ZK. We note that this only considers the delayed-input distributional setting. For simplicity of exposition, we restrict ourselves to 3-round protocols since this work is only concerned with constructions and applications of 3-round promise zero-knowledge. We note that this definition can be extended naturally to any number of rounds.

Definition 8 (Promise Zero Knowledge)**.** *A 3-round distributional delayed-input simultaneous-message interactive argument (P, V, Valid) for a language L is said to be* promise zero knowledge against delayed-input verifiers *if there exists an oracle machine $\mathsf{Sim} = (\mathsf{Sim}_1, \mathsf{Sim}_2, \mathsf{Sim}_3)$ such that for every constant $c \in \mathbb{N}$, and any validity approximation algorithm $\mathsf{pExtract}_c$, for every polynomials $\nu = \nu(\lambda)$ and $\widetilde{\nu} = \widetilde{\nu}(\lambda)$, for every efficiently sampleable distribution $\mathcal{D} = \{\mathcal{D}_\lambda\}$ such that $\mathsf{Supp}(\mathcal{D}_\lambda) = \{(x, w) : x \in L \cap \{0,1\}^\lambda, w \in R_L(x)$ where $x = (x_2, x_3), w = (w_2, w_3)\}$, for any delayed-input PPT verifier V^* that obtains x_i in round i and any $z \in \{0,1\}^{\mathsf{poly}(\lambda)}$, conditions 1 and 2 (defined below) hold for REAL_{V^*} and IDEAL_{V^*} (defined below).*

- *REAL_{V^*} is computed as follows:*
  - *Sample $(\mathsf{view}_{V^*,1}, \mathsf{st}) \leftarrow P_1(1^\lambda)$.*
  - *Sample $(x, w) \leftarrow \mathcal{D}_\lambda$ where $x = (x_2, x_3)$.*
  - *Execute the interaction $(\mathsf{view}_{V^*,1}, \langle P_2(x, w, \mathsf{st}), V^*(\mathsf{view}_{V^*,1})\rangle)$, where V^* obtains x_i in round i.*
  - *The output of the experiment is the view of V^* in the execution $(x, \langle P(x, w), V^*(\mathsf{view}_{V^*,1})\rangle)$.*
- *IDEAL_{V^*} is computed as follows:*
  - *Sample $(\mathsf{view}_{V^*,1}, \mathsf{st}) \leftarrow P_1(1^\lambda)$.*
  - *Compute $p = \mathsf{pExtract}_c^{V^*}(\mathsf{view}_{V^*,1}, \mathsf{st})$.*
  - *Sample $(x, w) \leftarrow \mathcal{D}_\lambda$ where $x = (x_2, x_3)$.*
  - *If $p = 0$,*
    - *Execute the interaction $(\mathsf{view}_{V^*,1}, \langle P_2(x, w, \mathsf{st}), V^*(\mathsf{view}_{V^*,1})\rangle)$, where V^* obtains x_i in round i.*
    - *The output of the experiment is $(x, \langle P(x, w), V^*(\mathsf{view}_{V^*,1})\rangle)$.*
  - *Else, execute $\mathsf{Sim}^{V^*}(x, \mathsf{view}_{V^*,1}, \mathsf{st}, p) \to (\mathsf{view}_{V^*,2}, \mathsf{view}_{V^*,3})$, which operates as follows:*
    - *Compute $\mathsf{Sim}_1^{V^*}(\mathsf{view}_{V^*,1}, \mathsf{st}, p) \to \mathsf{st}_1$.*
    - *Then compute $\mathsf{Sim}_2^{V^*}(x_2, \mathsf{view}_{V^*,1}, \mathsf{st}_1) \to (\mathsf{view}_{V^*,2}, \mathsf{st}_2)$.*
    - *Finally, compute $\mathsf{Sim}_3^{V^*}(x_3, \mathsf{view}_{V^*,1}, \mathsf{view}_{V^*,2}, \mathsf{st}_2)$ to output $(\mathsf{view}_{V^*,3})$.*

Conditions 1 and 2 are defined as follows:

1. *No PPT distinguisher can distinguish REAL_{V^*} from IDEAL_{V^*} with advantage greater than λ^{-c}.*

2. *For any input $x = (x_2, x_3)$, the running time of $\mathsf{Sim}_1^{V^*}(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}, p)$ is polynomial in λ and linear in $\frac{1}{p}$, and the running times of $\mathsf{Sim}_2^{V^*}(x_2, \mathsf{view}_{V^*,1}, \mathsf{st})$ and $\mathsf{Sim}_3^{V^*}(x_3, \mathsf{view}_{V^*,1}, \mathsf{view}_{V^*,2}, \mathsf{st}_2)$ are polynomial in λ and independent of p.*

Going forward, we use *promise ZK argument* to refer to a distributional promise zero-knowledge simultaneous-message argument system, satisfying delayed-input completeness and soundness, as well as zero-knowledge against delayed-input verifiers according to Definition 8.

Defining Simulation-Sound Promise ZK in the multi-party setting. We now consider a man-in-the-middle adversary that interacts in promise zero-knowledge protocols as follows: It opens polynomially many sessions where it plays the role of the verifier interacting with an honest prover; these are called "left" sessions, and we denote by ν the number of such left sessions. We note that in all left sessions, the honest prover proves the same statement with the same witness. It can simultaneously initiate polynomially many sessions where it plays the role of the prover interacting with an honest verifier: these are called "right" sessions, and we denote by $\widetilde{\nu}$ the number of such right sessions. We restrict ourselves to *synchronous* (rushing) adversaries, that for each round j, send all their j'th round messages (in all sessions), before observing any of the honest parties messages for the next round of the protocol.

We formalize the notion of simulation-soundness against a rushing man-in-the-middle adversary below, where we use $\widetilde{a}$ to denote any random variable a that corresponds to a right session.

Redefining Validity Approximation. Similarly to before, we need to approximate the probability that the messages sent by a man-in-the-middle adversary in the left execution are valid, conditioned on all messages in the first round of the protocol. We consider ν "left" sessions and $\widetilde{\nu}$ "right" sessions. Similar to the setting of promise ZK, we denote by $P = (P_1, P_2)$, an honest prover for the "left" sessions that is split into two parts, P_1 generates the first round message, and P_2 generates the messages of the second and third rounds. Below, we abuse notation and use P, P_1, P_2 not only to denote the interaction of the honest prover in a single session, but also to denote the interaction of the honest prover in all ν left sessions, using independent randomness for each such execution. Let $\mathsf{Trans}_{\mathrm{left}}(P_2(x, w, \mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}), \mathsf{MIM})$ denote all the transcripts in the "left" sessions between $P_2(x, w, \mathsf{view}_{\mathsf{MIM},1}, \mathsf{st})$ and MIM, which can be decomposed as follows: $\mathsf{Trans}_{\mathrm{left}}(P, \mathsf{MIM}) = (\mathsf{view}_{\mathsf{MIM},1}, \mathsf{Trans}_{\mathrm{left}}(P_2(x, w, \mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}), \mathsf{MIM}))$. For any $\mathsf{view}_{\mathsf{MIM},1}$ sampled according to honest prover and verifier strategy as described above, let

$$q_{\mathsf{view}_{\mathsf{MIM},1}} = \Pr[\mathsf{Valid}(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{Trans}_{\mathrm{left}}(P_2(x, w, \mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}), \mathsf{MIM})) = 1 | (\mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}) \leftarrow P_1(1^\lambda)]$$

where Valid above refers to the AND of all the validity tests for each of the ν left sessions, and the probability is over the generation of $(x, w) \leftarrow \mathcal{D}_\lambda$ and the coins of each of the ν instantiations of P_2. We emphasize that $q_{\mathsf{view}_{\mathsf{MIM},1}}$ depends on $\mathcal{D}$ and on MIM, we omit this dependence from the notation to avoid cluttering. We re-define the algorithm $\mathsf{pExtract}_c$ from Definition 8 to depend additionally on the honest verifier first messages in the right sessions.

Definition 9. *For any constant $c \in \mathbb{N}$, a PPT oracle algorithm $\mathsf{pExtract}_c$ is said to be a* validity approximation algorithm*, if the following holds for all MIM and for all efficiently sampleable distributions $\mathcal{D} = \{\mathcal{D}_\lambda\}$, with probability at least $1 - 2^{-\lambda}$ over the coins of the algorithm, we have that:*

- *If $\mathsf{pExtract}_c^{\mathsf{MIM},\mathcal{D}}(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}) = 0$, then $q_{\mathsf{view}_{\mathsf{MIM},1}} < 2 \cdot \lambda^{-c}$.*
- *Else, if $\mathsf{pExtract}_c^{\mathsf{MIM},\mathcal{D}}(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}) = p$, then $p \geq \lambda^{-c}$ and $\frac{p}{2} < q_{\mathsf{view}_{\mathsf{MIM},1}} < 2 \cdot p$.*

Remark 3. *We briefly describe a canonical polynomial-time validity approximation algorithm for any constant $c \in \mathbb{N}$:*

1. $\mathsf{pExtract}_c^{\mathsf{MIM},\mathcal{D}}(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{st})$ *executes $\lambda^2 \cdot \lambda^c$ independent executions of all sessions with* MIM*, using freshly sampled instance-witness pairs from the distribution $\mathcal{D}_\lambda$ to complete the left executions in the role of the honest provers, and acting as honest verifiers in the right sessions.*

2. *Let ρ be the number of these executions that resulted in* all *left executions begin valid. We call such executions successful trials.*

3. *If $\rho < \lambda^2$, output 0.*

4. *Otherwise, output $\rho/(\lambda^2 \cdot \lambda^c)$.*

We now informally analyze this algorithm:

- *Observe that if $\mathsf{pExtract}_c^{\mathsf{MIM},\mathcal{D}}(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{st})$ outputs zero, this means that fewer than λ^2 trials succeeded. On the other hand, if $q_{\mathsf{view}_{\mathsf{MIM},1}} \geq 2 \cdot \lambda^{-c}$, then the expected number of successful trials is at least $2\lambda^2$. By a Chernoff bound, except with probability at most $2^{-\lambda}$, at least λ^2 trials must succeed if $q_{\mathsf{view}_{\mathsf{MIM},1}} \geq 2 \cdot \lambda^{-c}$. Thus, the first condition is satisfied.*

- *Observe that if $\mathsf{pExtract}_c^{\mathsf{MIM},\mathcal{D}}(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{st})$ outputs a nonzero value, then this value must be at least λ^{-c} by construction. And again, the required condition on $q_{\mathsf{view}_{\mathsf{MIM},1}}$ follows immediately from a Chernoff bound.*

For simplicity, we restrict ourselves to 3 rounds in the definition below. This suffices for our construction and applications.

Definition 10 (Simulation-Sound Promise Zero Knowledge)**.** *A 3-round publicly-verifiable promise zero-knowledge argument against delayed-input verifiers (P, V, Valid) is said to be simulation-sound if there exists an oracle machine $\mathsf{Sim} = (\mathsf{Sim}_1, \mathsf{Sim}_2, \mathsf{Sim}_3)$ such that, for every constant $c \in \mathbb{N}$, and any validity approximation algorithm $\mathsf{pExtract}_c$, for every polynomials $\nu = \nu(\lambda)$ and $\widetilde{\nu} = \widetilde{\nu}(\lambda)$, for every efficiently sampleable distribution $\mathcal{D} = \{(\mathcal{X}_\lambda, \mathcal{W}_\lambda)\}$ such that $\mathsf{Supp}((\mathcal{X}, \mathcal{W})_\lambda) = \{(x, w) : x \in L \cap \{0,1\}^\lambda, w \in R_L(x) \text{ where } x = (x_2, x_3)\}$, and every distribution $\mathcal{X}'_\lambda$ such that $\mathcal{X}_\lambda$ and $\mathcal{X}'_\lambda$ are computationally indistinguishable, for any PPT synchronous* MIM *that initiates ν "left" sessions and $\widetilde{\nu}$ "right" sessions, we require the following to hold. Let*

$$\mathsf{Sim}^{\mathsf{MIM}}(x', \mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}, p) \to (\mathsf{view}_{\mathsf{MIM},2}, \mathsf{view}_{\mathsf{MIM},3}, \{\widetilde{x}_i\}_{i \in [\widetilde{\nu}]})$$

where $\mathsf{view}_{\mathsf{MIM},1}$ are all the messages sent in the first round (both left and right executions) with MIM*, and st denotes all the corresponding secret states of the honest parties, and $p = \mathsf{pExtract}_c^{\mathsf{MIM},\mathcal{D}}(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{st})$.*

- *For any input $x' = (x'_2, x'_3)$, we have that $\mathsf{Sim}^{\mathsf{MIM}}(x', \mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}, p)$ operates by first computing*

$$\mathsf{Sim}_1^{\mathsf{MIM}}(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}, p) \to \mathsf{st}_1$$

then computing

$$\mathsf{Sim}_2^{\mathsf{MIM}}(x'_2, \mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}_1) \to (\mathsf{view}_{\mathsf{MIM},2}, \mathsf{st}_2)$$

and then computing

$$\mathsf{Sim}_3^{\mathsf{MIM}}(x'_2, x'_3, \mathsf{view}_{\mathsf{MIM},1}, \mathsf{view}_{\mathsf{MIM},2}, \mathsf{st}_2) = (\mathsf{view}_{\mathsf{MIM},3}, \{\widetilde{x}_i\}_{i \in [\widetilde{\nu}]}).$$

Here, $\mathsf{view}_{\mathsf{MIM},2}$ *and* $\mathsf{view}_{\mathsf{MIM},3}$ *denotes the set of all messages sent in the second and third round (respectively) of the multi-party execution with* MIM*. We require that* $\{\widetilde{x}_i\}$ *(which is part of the output of* $\mathsf{Sim}^{\mathsf{MIM}}$*) is consistent*[15] *with* $(\mathsf{view}_{\mathsf{MIM},2}, \mathsf{view}_{\mathsf{MIM},3})$*.*

- *For any input* $x' = (x'_2, x'_3)$*, we require that the running time of* $\mathsf{Sim}_1^{\mathsf{MIM}}(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}, p)$ *is polynomial in* λ *and linear in* $\frac{1}{p}$*, while the running times of* $\mathsf{Sim}_2^{\mathsf{MIM}}(x'_2, \mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}_1)$ *and* $\mathsf{Sim}_3^{\mathsf{MIM}}(x'_2, x'_3, \mathsf{view}_{\mathsf{MIM},1}, \mathsf{view}_{\mathsf{MIM},2}, \mathsf{st}_2)$ *are polynomial in* λ*, independent of* p*.*
- *If* $\Pr\left[\mathsf{pExtract}_c^{\mathsf{MIM}}(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}) \geq \lambda^{-c}\right] \geq \lambda^{-c}$*, then we have:*

$$\left(x', \mathsf{view}_{\mathsf{MIM},1}, \mathsf{IDEAL}_{\mathsf{MIM}}(x', \mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}) \middle| \ \mathsf{pExtract}_c^{\mathsf{MIM}}(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}) \geq \lambda^{-c}\right) \approx$$
$$\left(x, \mathsf{view}_{\mathsf{MIM},1}, \mathsf{REAL}_{\mathsf{MIM}}(x, w, \mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}) \middle| \ \mathsf{pExtract}_c^{\mathsf{MIM}}(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}) \geq \lambda^{-c}\right)$$

where $(x, w) \leftarrow (\mathcal{X}, \mathcal{W})_\lambda$*,* $x' \leftarrow \mathcal{X}'_\lambda$*, and* $(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{st})$ *is generated by simulating all the messages sent in the first round of the execution with* MIM*,*[16] *where* $\mathsf{view}_{\mathsf{MIM},1}$ *denotes all the simulated messages and* st *denotes the secret states of all the honest parties, and*

$$\mathsf{IDEAL}_{\mathsf{MIM}}(x', \mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}) = (\mathsf{view}_{\mathsf{MIM},1}, \mathsf{view}_{\mathsf{MIM},2}, \mathsf{view}_{\mathsf{MIM},3}),$$

where the variables $(\mathsf{view}_{\mathsf{MIM},2}, \mathsf{view}_{\mathsf{MIM},3})$ *are computed by running* $\mathsf{Sim}^{\mathsf{MIM}}(x', \mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}, p)$ *for* $p = \mathsf{pExtract}_c^{\mathsf{MIM},\mathcal{D}}(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{st})$*. The experiment* $\mathsf{REAL}_{\mathsf{MIM}}(x, w, \mathsf{view}_{\mathsf{MIM},1})$ *is computed by running a real world execution with* MIM*, where the provers in the "left" sessions uses the input* (x, w) *and where the first round messages are* $\mathsf{view}_{\mathsf{MIM},1}$*, and by* $\mathsf{Valid}(\mathsf{Trans}_{\mathrm{left}}(P_2(x, w, \mathsf{st})))$ *we mean that all left sessions in the execution of* $\mathsf{REAL}_{\mathsf{MIM}}$ *are valid.*

- *Over the randomness of* Sim*, of generating* $(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{st})$ *and over* $x' \leftarrow \mathcal{X}'_\lambda$*,*

$$\Pr\left[\bigvee_{i \in [\widetilde{\nu}]} \left(\mathsf{Acc}(\widetilde{\mathsf{Trans}_i}) = 0\right) \bigvee \left(\bigwedge_{i \in [\widetilde{\nu}]} \widetilde{x}_i \in L\right)\right] \geq 1 - \lambda^{-c},$$

where $\{\widetilde{\mathsf{Trans}_i}\}$ *is the transcript of the* i*'th right execution when* $(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{view}_{\mathsf{MIM},2}, \mathsf{view}_{\mathsf{MIM},3})$ *are computed in* $\mathsf{IDEAL}_{\mathsf{MIM}}(x', \mathsf{view}_{\mathsf{MIM},1}, \mathsf{st})$ *as above, and* $\mathsf{Acc}(\widetilde{\mathsf{Trans}_i}) = 0$ *denotes the event that the (publicly verifiable) transcript* $\widetilde{\mathsf{Trans}_i}$ *causes an honest verifier to reject.*

5.2 Constructing Simulation Sound Promise ZK

In this section, we describe our construction of Simulation Sound Promise ZK. Formally, we prove the following theorem:

Theorem 8. *Assuming the existence of polynomially secure injective one way functions, there exists a three round simulation-sound promise ZK argument according to Definition 10.*

5.2.1 The Protocol

Let P and V denote the prover and verifier, respectively. Let L be any NP language with an associated relation R_L. Let $\mathcal{D}_\lambda = (\mathcal{X}_\lambda, \mathcal{W}_\lambda)$ be any efficiently sampleable distribution on R_L.

[15] Note that $(\mathsf{view}_{\mathsf{MIM},2}, \mathsf{view}_{\mathsf{MIM},3})$ includes the instances $\{\widetilde{x}_i\}$, and we add the instances explicitly to the output of $\mathsf{Sim}^{\mathsf{MIM}}$ only so that we will be able to refer to it later.

[16] Note that this can be simulated easily since the protocol is delayed-input which means that the parties do not use their private inputs to compute their first round message.

Building Blocks. Our construction relies on the following cryptographic primitives.

- $\mathsf{TDGen} = (\mathsf{TDGen}_1, \mathsf{TDGen}_2, \mathsf{TDGen}_3, \mathsf{TDOut})$ is the three-message trapdoor generation protocol from Section 4, that is 3-extractable according to Definition 3, with corresponding extractor TDExt.
- $\mathsf{RWI} = (\mathsf{RWI}_1, \mathsf{RWI}_2, \mathsf{RWI}_3, \mathsf{RWI}_4)$ is the three round delayed-input witness indistinguishable argument with bounded rewinding security for $L = 5$ from Definition 6. The fourth algorithm RWI_4 is the final verification algorithm.
- $\mathsf{NMCom} = (\mathsf{NMCom}_1, \mathsf{NMCom}_2, \mathsf{NMCom}_3)$ denotes a special non-malleable commitment according to Definition 4.

NP Languages. We define the following relation R' that will be useful in our construction. Parse instance $\mathsf{st} = (x, \mathsf{c}, \mathsf{td}_1)$, where $\mathsf{c} = (\mathsf{c}_1, \mathsf{c}_2, \mathsf{c}_3)$. Parse witness $\mathsf{w} = (w, \mathsf{t}, \mathsf{r})$. Then, $R'(\mathsf{st}, \mathsf{w}) = 1$ if and only if : $\left(R(x, w) = 1\right) \bigvee \left(\mathsf{TDValid}(\mathsf{td}_1, \mathsf{t}) = 1 \wedge \mathsf{c}_1 = \mathsf{NMCom}_1(\mathsf{r}) \wedge \mathsf{c}_3 = \mathsf{NMCom}_3(\mathsf{t}, \mathsf{c}_1, \mathsf{c}_2; \mathsf{r})\right)$. We denote the corresponding language by L'.

That is, either :

1. x is in the language L with witness w, OR,
2. the third non-malleable commitment $(\mathsf{c}_1, \mathsf{c}_2, \mathsf{c}_3)$ is to a value t that is a valid trapdoor for the message td_1 generated using the trapdoor generation algorithms.

We construct a three round protocol $\pi^{\mathsf{SE-PZK}} = (P, V, \mathsf{Valid})$ for L in Figure 2. The completeness of this protocol follows from the correctness of the underlying primitives.

5.3 Security Proof

We describe the simulator $\mathsf{Sim} = (\mathsf{Sim}_1, \mathsf{Sim}_2, \mathsf{Sim}_3)$ in Figure 3. We will denote messages generated in right sessions by $\widetilde{a}$, while corresponding messages in left sessions will be denoted by a. In Figure 3 we only describe a simulator in the one-one setting, this generalizes to the many-many setting of Definition 10 by a direct hybrid argument.

5.3.1 Analysis of the Simulator: Hybrids

Suppose that there exists a constant $c > 0$ and a distinguisher that distinguishes between the real and ideal games (with parameter c) with noticeable probability ϵ. We will now prove indistinguishability of real and ideal games by using a sequence of hybrids. We will also maintain an invariant that will help us prove the (main) simulation-extraction property.

The first hybrid Hyb_0 will correspond to the real world experiment where the adversary MIM interacts with an honest prover P on the left and honest verifier V on the right. The last hybrid Hyb_4 corresponds to the ideal world where MIM interacts with the simulator Sim.

- Hyb_0 **- Real World:** In this hybrid, the first round messages are all honestly executed with the adversary MIM. Then $\mathsf{pExtract}_c$ is called on this view, and it produces an output p. If $p = 0$, the experiment aborts and outputs $\perp$. If not, the challenger plays the role of the honest prover on the left session and honest verifier on the right session.

Inputs: Prover P with tag tag obtains input $(x = (x_2, x_3), w) \leftarrow (\mathcal{X}, \mathcal{W})$ in the second round.

1. **Round 1:**
   - **Prover message:**
     - Compute $\mathsf{rwi}_1 = \mathsf{RWI}_1(1^\lambda, \widehat{r})$.
     - Sample $r \leftarrow \{0,1\}^*$.
     - Compute $\mathsf{c}_1 \leftarrow \mathsf{NMCom}_1(\mathsf{r})$ using NMCom with tag tag and uniform randomness[a].
     - Send $(\mathsf{rwi}_1, \mathsf{c}_1)$.
   - **Verifier message:**
     Sample $\mathsf{r}_{\mathsf{td}} \xleftarrow{\$} \{0,1\}^*$, then compute and send $\mathsf{td}_1 \leftarrow \mathsf{TDGen}_1(\mathsf{r}_{\mathsf{td}})$.
2. **Round 2:**
   - **Prover message:**
     - Obtain input $(x = (x_2, x_3), w)$ which is a randomly chosen sample from $(\mathcal{X}_\lambda, \mathcal{W}_\lambda)$. Send x_2 to V.
     - Compute and send $\mathsf{td}_2 \leftarrow \mathsf{TDGen}_2(\mathsf{td}_1)$.
   - **Verifier message:**
     Compute and send $\mathsf{rwi}_2 \leftarrow \mathsf{RWI}_2(\mathsf{rwi}_1)$ and $\mathsf{c}_2 \leftarrow \mathsf{NMCom}_2(\mathsf{c}_1)$.
3. **Round 3:**
   - **Prover message:**
     - Compute $\mathsf{c}_3 \leftarrow \{0,1\}^m$ and let $\mathsf{c} = (\mathsf{c}_1, \mathsf{c}_2, \mathsf{c}_3)$.
     - Set $x' = (x, \mathsf{c}, \mathsf{td}_1)$ and $w' = (w, \perp, \perp)$. Compute $\mathsf{rwi}_3 \leftarrow \mathsf{RWI}_3(\mathsf{rwi}_1, \mathsf{rwi}_2, x', w')$ for $R'(x', w') = 1$ for R' defined above.
     - Send $(x_3, \mathsf{c}_3, \mathsf{rwi}_3)$.
   - **Verifier message:**
     Sample and send $\mathsf{td}_3 \leftarrow \mathsf{TDGen}_3(\mathsf{td}_1, \mathsf{td}_2, \mathsf{r}_{\mathsf{td}})$ using uniform randomness.
4. **Verifier Output:**
   Output $\mathsf{RWI}_4(\mathsf{rwi}_1, \mathsf{rwi}_2, \mathsf{rwi}_3, \mathsf{st})$.

Valid(Trans):
Given the transcript of the protocol execution, output 1 if $\mathsf{TDOut}(\mathsf{td}_1, \mathsf{td}_2, \mathsf{td}_3) = 1$.

[a] We omit explicit dependence of the algorithm on tag to avoid cluttering.

Figure 2: Three round Simulation-Sound Promise ZK argument.

- Hyb_1 **- Trapdoor Extraction:** In this hybrid, if the value $p > 0$, then the challenger $\mathcal{C}$ creates a fresh set of $(\lambda \cdot \frac{1}{p})$ look-ahead threads on the left session. In all the look-ahead threads, $\mathcal{C}$ performs each thread exactly as described in Hyb_0 using fresh randomness. Additionally, using the messages in the look-ahead threads, $\mathcal{C}$ also runs the "Trapdoor Extraction" phase described in Step 5 of the description of Sim to extract the trapdoor t_V. It outputs "Special Abort" if trapdoor extraction fails. Finally, $\mathcal{C}$ completes the main thread on the left session by running the honest prover strategy exactly as in Hyb_0.

  Remark 4. *Looking ahead, we will maintain the invariant that the* MIM *will not be able to commit to the trapdoor of the honest verifier in the right session, except with negligible*

The simulator $\mathsf{Sim} := (\mathsf{Sim}_1, \mathsf{Sim}_2, \mathsf{Sim}_3)$ generates all verifier messages for the right session according to honest verifier strategy. For the left session, Sim_1 generates messages as follows:

1. **Round 1:**
   - Sample $\mathsf{r} \xleftarrow{\$} \{0,1\}^*$, compute $\mathsf{rwi}_1 \leftarrow \mathsf{RWI}_1(1^\lambda)$, and $\mathsf{c}_1 \leftarrow \mathsf{NMCom}_1(\mathsf{r})$.
   - Send $\mathsf{msg} = (\mathsf{rwi}_1, \mathsf{c}_1)$ together with honest verifier message for the right session, and store the randomness used to generate it as st.
   - Obtain message td_1 from MIM for the left session.
2. **Look-ahead threads:**
   - Let $p = \mathsf{pExtract}_c^{\mathsf{MIM}}(\mathsf{msg}, \mathsf{st})$. If $p = 0$, output $\perp$.
   - Else, create a set of $(\lambda \cdot \frac{1}{p})$ look-ahead threads as follows, with Round 1 fixed as above.
3. **Round 2:** In thread i for $i \in [\lambda/p]$,
   - Sample independently $(x_i = (x_{2,i}, x_{3,i}), w_i) \leftarrow \mathcal{D}_\lambda$.
   - Sample independently $\mathsf{td}_{2,i} \leftarrow \mathsf{TDGen}_2(\mathsf{td}_1)$, send $(\mathsf{td}_{2,i}, x_{2,i})$.
   - Receive $\mathsf{rwi}_{2,i}$ and $\mathsf{c}_{2,i}$.
4. **Round 3:** In thread i for $i \in [\lambda/p]$,
   - Compute $\mathsf{c}_{3,i} \xleftarrow{\$} \{0,1\}^m$ and set $x'_i = (x_i, \mathsf{c}_i, \mathsf{td}_i)$, $w'_i = (\mathsf{r}_{b,i}, \perp, \perp)$, where $\mathsf{c}_i = (\mathsf{c}_1, \mathsf{c}_{2,i}, \mathsf{c}_{3,i}), \mathsf{td}_i = (\mathsf{td}_1, \mathsf{td}_{2,i}, \mathsf{td}_{3,i})$.
   - Generate $\mathsf{rwi}_{3,i} \leftarrow \mathsf{RWI}_3(\mathsf{rwi}_1, \mathsf{rwi}_2, x'_i, w'_i)$.
   - Send $(x_{3,i}, \mathsf{rwi}_{3,i})$ and receive $\mathsf{td}_{3,i}$ (which may or may not be Valid).
5. **Trapdoor Extraction:**
   - Extract trapdoor from look-aheads by computing $\mathsf{t}_V = \mathsf{TDExt}(\mathsf{td}_1, \{\mathsf{td}_{2,i}, \mathsf{td}_{3,i}\}_{i \in [\lambda/p]})$.
   - Output $\mathsf{t}_V, \mathsf{st}$ and "Special Abort" if TDExt fails.
6. For left session, $\mathsf{Sim}_2(x_2)$ samples $\mathsf{td}_2 \leftarrow \mathsf{TDGen}_2(\mathsf{td}_1)$, sends (td_2, x_2) and receives $\mathsf{rwi}_2, \mathsf{c}_2$.
7. For left session, $\mathsf{Sim}_3(x_3, \mathsf{t}_V, \mathsf{st})$ does the following: Compute $\mathsf{c}_3 \leftarrow \mathsf{NMCom}_3(\mathsf{c}_1, \mathsf{c}_2, \mathsf{t}_V)$ and $\mathsf{rwi}_3 \leftarrow \mathsf{RWI}_3(\mathsf{rwi}_1, \mathsf{rwi}_2, x', w')$, where $x' = (x, \mathsf{c}, \mathsf{td}), w' = (\perp, \mathsf{t}_V, \mathsf{r})$, $\mathsf{c} = (\mathsf{c}_1, \mathsf{c}_2, \mathsf{c}_3), \mathsf{td} = (\mathsf{td}_1, \mathsf{td}_2, \mathsf{td}_3)$. Send $(x_3, \mathsf{c}_3, \mathsf{rwi}_3)$ and output the view of the MIM.

Figure 3: Simulator algorithm Sim.

probability. This will follow by various arguments, leveraging non-malleability of the NMCom *commitment (for* Hyb_2 *below), bounded-rewinding security of the WI arguments (for* Hyb_3 *below), and an extractability argument for the* NMCom *commitment (for* Hyb_4*) below.*

- Hyb_2 **- Changing Commitment to Trapdoor in Main Thread:** In the main thread, on the left sessions, $\mathcal{C}$ generates the NMCom commitment message $\mathsf{c}_{t,3}$ to commit to the extracted trapdoor t_V.
- Hyb_3 **- Switching bounded-rewinding secure WI arguments in Main Thread:** In

the main thread, on the left sessions, $\mathcal{C}$ uses the witness t_V committed in c_t in the bounded-rewinding secure WI arguments.

- Hyb_4 **- Switching instances in Main Thread:** In the main thread, on the left sessions, $\mathsf{Sim}_{\mathsf{Hyb}}$ on the main thread, obtains input an instance x' sampled from the distribution $\mathcal{X}'_\lambda$. The description of $\mathcal{C}$ in this hybrid matches the description of the simulator Sim.

5.3.2 Invariant

We now describe the invariant.

Definition 11 (Invariant T)**.** *Consider a right session between* MIM *and verifier* V*. Here,* td_1 *denotes the first message of the trapdoor generation protocol with* V *as the trapdoor generator. The values* $(\widetilde{\mathsf{c}}_{t,1}, \widetilde{\mathsf{c}}_{t,2}, \widetilde{\mathsf{c}}_{t,3})$ *denote the messages of the non-malleable commitment generated by the* MIM *in the right session. We say that the event* E_t *occurs if* $\exists(\widetilde{\mathsf{t}}, \widetilde{\mathsf{r}}_t, \widetilde{r})$ *such that:*

$$\big(\widetilde{\mathsf{c}}_{t,1} = \mathsf{NMCom}_1(\widetilde{\mathsf{r}}_t; \widetilde{r})\big) \wedge \big(\widetilde{\mathsf{c}}_{t,3} = \mathsf{NMCom}_3(\widetilde{\mathsf{t}}; \widetilde{\mathsf{r}}_t)\big) \wedge \big(\mathsf{TDValid}(\widetilde{\mathsf{td}}_1, \widetilde{\mathsf{t}}) = 1\big).$$

That is, the event E_t *occurs if, in the right session, the adversary* MIM*, using the non-malleable commitment, commits to a valid trapdoor* $\widetilde{\mathsf{t}}$ *for the trapdoor generation messages of* V*.*

The invariant is : $\mathsf{Pr}[\textit{Event } \mathsf{E}_t \textit{ occurs}] \leq \mathsf{negl}(\lambda)$.

5.3.3 Hybrid Security Argument

In this section, we will establish simulation soundness by analyzing the hybrids presented above. First, recall that we suppose that there exists a constant $c > 0$ for which an adversary $\mathcal{A}$ distinguishes between the real and ideal games (with parameter c) with noticeable probability. Moreover, since all hybrids are identical when $\mathsf{pExtract}_c = 0$, note that $\mathcal{A}$ must distinguish conditioned on $\mathsf{pExtract}_c^{\mathsf{MIM}}(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}) \geq \lambda^{-c}$.

We now prove that every pair of consecutive hybrids conditioned on $\mathsf{pExtract}_c^{\mathsf{MIM}}(\mathsf{view}_{\mathsf{MIM},1}, \mathsf{st}) \geq \lambda^{-c}$ is indistinguishable, and this completes the proof.

Claim 1. *Assuming "1-rewinding security" of the trapdoor generation protocol* TDGen *and the existence of an extractor* $\mathsf{Ext}_{\mathsf{NMCom}}$ *for the non-malleable commitment scheme* NMCom*, Invariant* T *holds in* Hyb_0.

Proof. We will prove this by contradiction. Assume that the invariant does not hold in Hyb_0. That is, there exists an MIM such that it causes event E_t to occur with non-negligible probability $q(\lambda) = \frac{1}{\mathsf{poly}(\lambda)}$ for some polynomial $\mathsf{poly}(\cdot)$. We will use this adversary to design an adversary $\mathcal{A}_{\mathsf{TDGen}}$ that breaks the "1-rewinding security" of the trapdoor generation protocol TDGen as defined in Section 4. Before describing $\mathcal{A}_{\mathsf{TDGen}}$, we recall from Imported Theorem 1 that the NMCom satisfies the 2-extractability property from Definition 3, and we denote the extraction algorithm that on input two transcripts outputs the extracted value with probability $\frac{1}{p(\lambda)}$ for some polynomial $p(\cdot)$, by NMExt. The adversary $\mathcal{A}_{\mathsf{TDGen}}$ behaves as follows.

- Obtain input first round message t_1 corresponding to the protocol TDGen.
- Interact with the MIM emulating the role of challenger $\mathcal{C}$ in Hyb_0. Set the verifier first message for right interaction to $\widetilde{\mathsf{td}}_1$, generate all other messages according to Hyb_0 for rounds 1 and 2.

- On obtaining the MIM's second round message, parse it to obtain value $\widetilde{\mathsf{td}}_2$. Forward $\widetilde{\mathsf{td}}_2$ as the second message of TDGen.
- On input t_3, set the third TDGen message of the verifier for the right session $\widetilde{\mathsf{td}}_3 = t_3$. Generate all other messages for this round according to Hyb_0, and record the non-malleable commitment $\widetilde{\mathsf{c}}_{t,1}, \widetilde{\mathsf{c}}_{t,2}, \widetilde{\mathsf{c}}_{t,3}$ of the MIM.
- Rewind MIM back to the end of round 1 (note that no rewinding occurs in the actual Hyb_0, we only rewind for the purpose of the reduction).
- Generate fresh messages for round 2 according to the strategy in Hyb_0.
- As in the main thread, on obtaining the MIM's second round message, parse it to obtain value $\widetilde{\mathsf{td}}'_2$. Forward $\widetilde{\mathsf{td}}'_2$ as the (rewinding) second message of TDGen.
- On input t'_3, set the third trapdoor generation message of the verifier for the right session $\widetilde{\mathsf{td}}'_3 = t'_3$. Generate all other messages for this round according to Hyb_0, and record the non-malleable commitment $\widetilde{\mathsf{c}}_{t,1}, \widetilde{\mathsf{c}}'_{t,2}, \widetilde{\mathsf{c}}'_{t,3}$ of the MIM.
- Run $\mathsf{NMExtract}(\widetilde{\mathsf{c}}_{t,1}, \widetilde{\mathsf{c}}_{t,2}, \widetilde{\mathsf{c}}_{t,3}, \widetilde{\mathsf{c}}'_{t,2}, \widetilde{\mathsf{c}}'_{t,3})$ and output the message m extracted from both (which could be $\perp$).

By the 2-extractability property, there exists a polynomial $\widetilde{p}(\cdot)$, such that in this experiment, if the MIM committed to the trapdoor in t_1 with probability q, $\mathcal{A}_{\mathsf{TDGen}}$ outputs t_1 with probability at least $\widetilde{p} \cdot q$. On the other hand, the 1-rewinding security of TDGen implies that no adversary has advantage greater than $\mathsf{negl}(\lambda)$ in guessing t_1: this gives a contradiction and completes the proof of the claim. □

Claim 2. *Assuming Claim 1, invariant T holds in Hyb_1.*

Proof. Since the main threads in Hyb_0 and Hyb_1 are identically distributed, the invariant continues to hold in Hyb_1. □

Claim 3. $\mathsf{Hyb}_0 \approx_c \mathsf{Hyb}_1$.

Proof. The only difference between Hyb_0 and Hyb_1 is statistical: the challenger outputs an additional "Special Abort" in Hyb_1 if trapdoor extraction fails in Step 5. By Definition 10, the adversary is promised to output valid transcripts in the left session with probability at least $\frac{p}{2}$. Thus, given $\frac{\lambda}{p}$ rewinding executions, the adversary outputs at least 2 valid transcripts with probability $1-(1-\frac{p}{2})^{\frac{\lambda}{p}} \geq 1-\exp^{-\lambda}$. This means that Hyb_0 and Hyb_1 are at most $\mathsf{negl}(\lambda)$-apart, proving the claim. □

Claim 4. *Assuming NMCom is a special non-malleable commitment scheme according to Definition 4, invariant T holds in Hyb_2.*

Proof. The only difference between Hyb_1 and Hyb_2 is that in Hyb_2, the simulator computes the non-malleable commitment in the main thread of the left session using the adversary's trapdoor. Assume for the sake of contradiction that there exists a MIM that causes event E_t to occur with probability $q = \frac{1}{\mathsf{poly}(\cdot)}$ in Hyb_2 for some polynomial $\mathsf{poly}(\cdot)$. We will use MIM to design an adversary $\mathcal{A}_{\mathsf{NMCom}}$ that breaks the security of the non-malleable commitment scheme. $\mathcal{A}_{\mathsf{NMCom}}$ performs the role of $\mathcal{C}$ in its interaction with MIM and behaves as follows.

- Obtain input the first round message of the left NMCom, and set $\mathsf{c}_{t,1}$ to this message. Generate all other messages for the first round same as Hyb_1, using honest prover and verifier strategy.
- Start creating lookahead threads exactly as in Hyb_1. Note that in each lookahead thread, the third message of NMCom for the left session is sampled uniformly at random and independent of the first two messages.
- Extract trapdoor t_V according to Step 5 from the look-ahead threads.
- Run the second round of the main thread exactly as in Hyb_1, and output the MIM's second round challenge $\mathsf{c}_{t,2}$.
- Sample r uniformly at random and output (t_V, r) to the NMCom challenger.
- On input the third message $\mathsf{c}_{t,3}$ of NMCom, generate the third round of the protocol using $\mathsf{c}_{t,3}$ in the left non-malleable commitment and generating other messages according to Hyb_1.

By the non-malleability of NMCom, the joint distribution of the view and value committed by the MIM remains indistinguishable between Hyb_1 and Hyb_2. Thus, invariant T holds in Hyb_2. □

Claim 5. $\mathsf{Hyb}_1 \approx_c \mathsf{Hyb}_2$.

Proof. The main difference between Hyb_1 and Hyb_2 is that in Hyb_2, the simulator computes the non-malleable commitment using the adversary's trapdoor value in the left session. Suppose there exists an adversary V^* that can distinguish the two hybrids with noticeable probability, we can use V^* to design an adversary $\mathcal{A}_{\mathsf{Hid}}$ (identical to $\mathcal{A}_{\mathsf{NMCom}}$ in Claim 4 above) that breaks the hiding of the non-malleable commitment scheme. □

Claim 6. *Assuming* RWI *is bounded rewinding witness indistinguishable according to Definition 6,* TDGen *is* 3*-extractable according to Definition 3, and the special non-malleable commitment* NMCom *is* 2*-extractable according to Definition 4, invariant* T *holds in* Hyb_3.

Proof. The only difference between Hyb_2 and Hyb_3 is that in Hyb_3, in the main thread of the left session, the simulator computes the WI proof using the trapdoor witness. Assume for the sake of contradiction that there exists an MIM adversary that causes event E_t to occur with probability q in Hyb_3, where $q = \frac{1}{\mathsf{poly}(\cdot)}$ for some polynomial $\mathsf{poly}(\cdot)$. We will use $\mathcal{A}$ to design an adversary $\mathcal{A}_{\mathsf{RWI}}$ that breaks the security of the bounded rewinding secure WI scheme.

$\mathcal{A}_{\mathsf{RWI}}$ interacts with the MIM as challenger, behaving as follows:

- Obtain input the first message rwi_1 of the bounded rewinding WI argument. Send this as the first round message on behalf of honest prover, and generate all other first round messages according to Hyb_2.
- Instead of executing λ/p lookahead threads, execute exactly 2 look-ahead threads. For both threads, obtain RWI messages externally using witness $(w, \perp, \perp)$.
- Set all the values $\{\mathsf{c}_{t,2,j}, \mathsf{c}_{t,3,j}\}_{j\in[3,\lambda/p]}$ to $\perp$, and run $\mathsf{TDGen}(\mathsf{c}_{t,1}, \{\mathsf{c}_{t,2,j}, \mathsf{c}_{t,3,j}\}_{j\in[\lambda/p])}$ to output t_V (which may be $\perp$).
- If $\mathsf{t}_V = \perp$, abort, else generate rounds 2 and 3 for the main thread according to Hyb_2 using t_V. Obtain the RWI message for use in the left execution, externally, to use witness $(w, \perp, \perp)$ or $(\perp, \mathsf{t}_V, r_t)$ depending on the verifier challenge. Record the MIM's non-malleable commitment $(\widetilde{\mathsf{c}}_{t,1}, \widetilde{\mathsf{c}}_{t,2}, \widetilde{\mathsf{c}}_{t,3})$.

- Next, rewind to the end of round 1 and again generate rounds 2 and 3 (with fresh uniform randomness) according to Hyb_2 using t_V. Obtain the RWI message for use in the left execution, externally, to use witness $(w, \bot, \bot)$ or $(\bot, \mathsf{t}_V, r_t)$ depending on the verifier challenge. Record the MIM's non-malleable commitment $(\widetilde{\mathsf{c}}_{t,1}, \widetilde{\mathsf{c}}'_{t,2}, \widetilde{\mathsf{c}}'_{t,3})$.

- Output $\mathsf{NMExtract}(\widetilde{\mathsf{c}}_{t,1}, \widetilde{\mathsf{c}}_{t,2}, \widetilde{\mathsf{c}}_{t,3}, \widetilde{\mathsf{c}}'_{t,2}, \widetilde{\mathsf{c}}'_{t,3})$ (which may be $\bot$).

By assumption on Hyb_3, by 2-extractability of NMCom and 3-extractability of the extractable commitment, when RWI uses witness $(\bot, \mathsf{t}_V, r_t)$ in the main threads, we have that $\mathcal{A}_{\mathsf{RWI}}$ outputs the honest party trapdoor t with probability $\frac{q}{\mathsf{poly}(\lambda)}$ for some polynomial $\mathsf{poly}(\cdot)$. On the other hand, by Claim 4 for Hyb_2, $\mathcal{A}_{\mathsf{RWI}}$ outputs the honest party trapdoor t with probability $\mathsf{negl}(\lambda)$ when RWI uses witness $(w, \bot, \bot)$ in the main threads. This contradicts the bounded rewinding security of WI according to Definition 6. □

Claim 7. $\mathsf{Hyb}_2 \approx_c \mathsf{Hyb}_3$.

Proof. The main difference between Hyb_2 and Hyb_3 is that in Hyb_2, the simulator computes the bounded rewinding WI argument using the adversary's trapdoor value in the left session. Suppose there exists an adversary V^* that can distinguish the two hybrids with noticeable probability, we can use V^* to design an adversary $\mathcal{A}_{\mathsf{WI}}$ (identical to $\mathcal{A}_{\mathsf{RWI}}$ in Claim 6 above except that it does not perform the last step) that breaks the bounded rewinding security of the WI. □

Claim 8. *Assuming 2-extractability of the* NMCom *according to Definition 4, invariant T holds in* Hyb_4.

Proof. The only difference between Hyb_3 and Hyb_4 is that in Hyb_4, in the main thread of the left session, the simulator uses instance $x' \xleftarrow{\$} \mathcal{X}'$ instead of $x \xleftarrow{\$} \mathcal{X}$. Assume for the sake of contradiction that there exists an MIM adversary that causes event E_t to occur with probability q in Hyb_4, where $q = \frac{1}{\mathsf{poly}(\cdot)}$ for some polynomial $\mathsf{poly}(\cdot)$. We will use $\mathcal{A}$ to design an adversary $\mathcal{A}_x$ that breaks the indistinguishability between distributions $\mathcal{X}$ and $\mathcal{X}'$.

$\mathcal{A}_{\mathsf{RWI}}$ interacts with the MIM as challenger, behaving as follows:

- Obtain input x (that is sampled from one out of $\mathcal{X}$ or $\mathcal{X}'$).

- Generate lookahead threads according to Hyb_3, and obtain trapdoor t_V.

- Just as in Hyb_3, if $\mathsf{t}_V = \bot$, abort, else generate rounds 2 and 3 for the main thread according to Hyb_3 using t_V for instance x.

- Next, rewind to the end of round 1 and again generate rounds 2 and 3 (with fresh uniform randomness) according to Hyb_2 using t_V, and using instance x. Record the MIM's non-malleable commitment $(\widetilde{\mathsf{c}}_{t,1}, \widetilde{\mathsf{c}}'_{t,2}, \widetilde{\mathsf{c}}'_{t,3})$.

- Output $\mathsf{NMExtract}(\widetilde{\mathsf{c}}_{t,1}, \widetilde{\mathsf{c}}_{t,2}, \widetilde{\mathsf{c}}_{t,3}, \widetilde{\mathsf{c}}'_{t,2}, \widetilde{\mathsf{c}}'_{t,3})$ (which may be $\bot$).

By assumption on Hyb_4, by 2-extractability of NMCom, when RWI uses witness $(\bot, \mathsf{t}_V, r_t)$ in the main threads, we have that $\mathcal{A}_x$ outputs the honest party trapdoor t with probability $\frac{q}{\mathsf{poly}(\lambda)}$ for some polynomial $\mathsf{poly}(\cdot)$. On the other hand, by Claim 6 for Hyb_3, $\mathcal{A}_x$ outputs the honest party trapdoor t with probability $\mathsf{negl}(\lambda)$. This contradicts the indistinguishability of distributions $\mathcal{X}$ and $\mathcal{X}'$. □

Claim 9. $\mathsf{Hyb}_3 \approx_c \mathsf{Hyb}_4$.

Proof. The only difference between these hybrids is whether the simulator obtains $x \leftarrow \mathcal{X}$ or $x' \leftarrow \mathcal{X}'$. By indistinguishability of the distributions $\mathcal{X}$ and $\mathcal{X}'$, the two hybrids are indistinguishable. □

This proves that the real and ideal distributions remain indistinguishable. Moreover, by Claim 8 and by soundness of WI, it holds that the MIM can only output accepting proofs for $x \in L$ in the right session. This completes the proof of simulation soundness.

6 Three Round List Coin Tossing

We first define the notion of list coin tossing.

Definition 12. *An n-party protocol π in the simultaneous message setting is a* secure list coin tossing protocol *if for every PPT adversary $\mathcal{A}$ corrupting at most $n-1$ parties, there exists an expected PPT simulator $\mathcal{S}$ such that the output of the experiments $\mathsf{REAL}_{\mathcal{A}}$ and $\mathsf{IDEAL}_{\mathcal{S}}$, defined below, are computationally indistinguishable. In the real world, we denote by $(\mathsf{c}, \mathsf{view}_{\mathcal{A}})$ the pair, where $\mathsf{view}_{\mathcal{A}}$ is the view of the adversary $\mathcal{A}$ interacting with honest parties in the protocol π, and $\mathsf{c} \in \{0,1\}^{\ell} \cup \{\perp\}$ is the output of the honest parties in this protocol execution. Similarly, we use the pair $(\tilde{\mathsf{c}}, \mathsf{view}_{\mathcal{S}})$ in the ideal world, defined in Figure 1 below.*

We use ℓ to denote the length of each (honest) party's output after the running the protocol (assuming the parity did not output the abort symbol $\perp$).

$\mathsf{REAL}_{\mathcal{A}}(1^{\lambda}, 1^{\ell})$	$\mathsf{IDEAL}_{\mathcal{S}}(1^{\lambda}, 1^{\ell})$
$(\mathsf{c}, \mathsf{view}_{\mathcal{A}}) \leftarrow \mathsf{REAL}_{\mathcal{A}}(1^{\lambda}, 1^{\ell})$	$1^k \leftarrow \mathcal{S}(1^{\lambda}, 1^{\ell})$
	$(\mathsf{c}_1, \ldots, \mathsf{c}_k) \leftarrow \{0,1\}^{\ell \cdot k}$
	$(\tilde{\mathsf{c}}, \mathsf{view}_{\mathcal{S}}) \leftarrow \mathcal{S}(\mathsf{c}_1, \ldots, \mathsf{c}_k, 1^{\lambda}, 1^{\ell})$
Output $(\mathsf{c}, \mathsf{view}_{\mathcal{A}})$	If $\tilde{\mathsf{c}} \in \{\mathsf{c}_1, \ldots, \mathsf{c}_k, \perp\}$, then output $(\tilde{\mathsf{c}}, \mathsf{view}_{\mathcal{S}})$
	Else, output fail.

Table 1: List Coin Tossing

Definition 13. *We say that an n-party protocol π is a secure list coin tossing protocol in the simultaneous message setting with* black-box simulation *if there exists a (universal) expected PPT oracle machine $\mathcal{S}$ such that for every PPT adversary $\mathcal{A}$ corrupting at most $n-1$ parties, the simulator $\mathcal{S}^{\mathcal{A}}$ satisfies that*

$$\mathsf{REAL}_{\mathcal{A}}(1^{\lambda}, 1^{\ell}) \approx \mathsf{IDEAL}_{\mathcal{S}^{\mathcal{A}}}(1^{\lambda}, 1^{\ell})$$

where $\mathsf{REAL}_{\mathcal{A}}(1^{\lambda}, 1^{\ell})$ and $\mathsf{IDEAL}_{\mathcal{S}^{\mathcal{A}}}(1^{\lambda}, 1^{\ell})$ are defined as in Definition 12.

Theorem 9. *There exists a 3-round secure multi-party list coin tossing protocol π in the simultaneous message setting with* black-box simulation*, assuming the existence of a simulation extractable promise ZK scheme for NP.*

This, together with Theorem 8, implies the following corollary.

Corollary 10. *There exists a 3-round secure multi-party list coin tossing protocol π in the simultaneous message setting with* black-box simulation*, assuming the existence of injective one-way functions.*

As mentioned earlier, by applying the transformation of [GMPP16][17] to the protocol from Corollary 10 for the 2-party case, we can obtain a 4-round 2-party list coin-tossing protocol in the *unidirectional-message* model. This result overcomes the barrier of five rounds for standard 2-party coin-tossing established by [KO04].

Corollary 11 (Informal)**.** *Assuming the existence of injective one-way functions, there exists a 4-round 2-party secure list coin-tossing protocol in the unidirectional-message model (with black-box simulation).*

The rest of this section is devoted to proving Theorem 9. We provide the construction of our list coin-tossing protocol in Section 6.1, and provide the security proof in Section 6.2.

6.1 Our List Coin Tossing Protocol

Consider n parties $\mathsf{P}_1, \ldots, \mathsf{P}_n$ who wish to evaluate the List Coin Tossing functionality. In this section, we construct a protocol for computing the List Coin Tossing functionality using any simulation extractable promise ZK argument system (see Section 5).

We first list some notation and building blocks that we use use in our protocol.

Building Blocks

- Let Com be any non-interactive commitment scheme. We know that such a commitment scheme can be built assuming injective one way functions.
- $\pi^{\mathsf{SE-PZK}} = (P_{\mathsf{ZK}}, V_{\mathsf{ZK}}, \mathsf{Valid}, \mathcal{E})$ any 3-round simulation-extractable delayed-input distributional promise ZK argument system (such as the one constructed in Section 5).

  For all $i \in [3]$, we use $P_{\mathsf{ZK}i}$ to denote the algorithm used by the prover P to compute the i^{th} round messages, and we use $V_{\mathsf{ZK}i}$ to denote the algorithm used by the verifier V to compute the i^{th} round messages. Further, let $V_{\mathsf{ZK}4}$ denote the algorithm used by the verifier V to compute the output bit 0 or 1.

NP Languages. In our construction, we use proofs for the NP language L characterized by the following relation R.
Statement : $\mathsf{x} = (\mathsf{c}, \mathsf{r})$
Witness : $\mathsf{w} = \mathsf{s}$
$R(\mathsf{x}, \mathsf{w}) = 1$ if and only if $\mathsf{c} = \mathsf{Com}(\mathsf{r}; \mathsf{s})$.

Notation :

- We assume broadcast channels.
- λ denotes the security parameter.
- In the superscript, we use $i \to j$ to denote that the message was sent by party P_i to party P_j, where party P_i is taking the role of the prover in the underlying promise-ZK protocol and P_j is taking the role of the verifier. We use $j \leftarrow i$ to denote that the message was sent by party P_i to party P_j, where party P_i is taking the role of the verifier in the underlying promise-ZK protocol and P_j is taking the role of the prover. (Recall that all messages are broadcast).

[17] The work of Garg et al. [GMPP16] establishes an impossibility result for 3-round multi-party coin-tossing by transforming any 3-round two-party coin-tossing protocol in the simultaneous-message model into a 4-round two-party coin-tossing protocol in the unidirectional-message model, and then invoking the impossibility of [KO04].

- The round number of the sub-protocol $\pi^{\mathsf{SE-PZK}}$ being used is written in the subscript.

Our 3-round list coin tossing protocol π^{CT} is described in Figure 4.

1. **Round 1:** $\forall j \in [n]$ with $j \neq i$, P_i does the following:
   - Choose random strings $r^{i\to j}, r^{j\leftarrow i} \leftarrow \{0,1\}^\lambda$ and compute $\mathsf{prove}_1^{i\to j} = P_{\mathsf{ZK}1}(1^\lambda, r^{i\to j})$ and $\mathsf{ver}_1^{j\leftarrow i} \leftarrow V_{\mathsf{ZK}1}(1^\lambda, r^{j\leftarrow i})$.
   - Broadcast $\left\{\mathsf{prove}_1^{i\to j}, \mathsf{ver}_1^{j\leftarrow i}\right\}_{j\neq i}$.

   For every $i \neq j$, let $\mathsf{view}_1^{i\rightleftarrows j} \triangleq (\mathsf{prove}_1^{i\to j}, \mathsf{ver}_1^{i\leftarrow j})$.
2. **Round 2:** P_i does the following for each $j \in [n]$ with $j \neq i$:
   - Compute $\mathsf{prove}_2^{i\to j} = P_{\mathsf{ZK}2}(\mathsf{view}_1^{i\rightleftarrows j}; r^{i\to j})$ and $\mathsf{ver}_2^{j\leftarrow i} = V_{\mathsf{ZK}2}(\mathsf{view}_1^{i\rightleftarrows j}; r^{j\leftarrow i})$.
   - Choose a random string $\mathsf{r}_i \leftarrow \{0,1\}^\ell$ and a random string $\mathsf{s}_i \leftarrow \{0,1\}^\lambda$, and compute $\mathsf{c}_i = \mathsf{Com}(\mathsf{r}_i; \mathsf{s}_i)$.
   - Broadcast $\left(\mathsf{c}_i, \left\{\mathsf{prove}_2^{i\to j}, \mathsf{ver}_2^{j\leftarrow i}\right\}_{j\neq i}\right)$.

   For every $i \neq j$, let $\mathsf{view}_2^{i\rightleftarrows j} \triangleq (\mathsf{prove}_2^{i\to j}, \mathsf{ver}_2^{i\leftarrow j})$.
3. **Round 3:** P_i does the following:
   - For each $j \in [n]$ with $j \neq i$, do:
     - Generate $\mathsf{prove}_3^{i\to j} = P_{\mathsf{ZK}3}(\mathsf{view}_1^{i\rightleftarrows j}, \mathsf{view}_2^{i\rightleftarrows j}, (\mathsf{c}_i, \mathsf{r}_i, \mathsf{s}_i); r^{i\to j})$, for the statement $(c_i, r_i) \in L$ using the witness s_i.
     - Generate $\mathsf{ver}_3^{j\leftarrow i} \leftarrow V_{\mathsf{ZK}3}(\mathsf{view}_1^{i\rightleftarrows j}, \mathsf{view}_2^{i\rightleftarrows j}; r^{j\leftarrow i})$.
   - Broadcast $\left(\mathsf{r}_i, \left\{\mathsf{prove}_3^{i\to j}, \mathsf{ver}_3^{j\leftarrow i}\right\}_{j\neq i}\right)$.

   For every $i \neq j$, let $\mathsf{view}_3^{i\rightleftarrows j} \triangleq (\mathsf{prove}_3^{i\to j}, \mathsf{ver}_3^{i\leftarrow j})$.
4. **Output Computation:** Each party checks if there exists $i \neq j$ such that

$$\mathsf{Valid}\left(\mathsf{view}_1^{i\rightleftarrows j}, \mathsf{view}_2^{i\rightleftarrows j}, \mathsf{view}_3^{i\rightleftarrows j}, (\mathsf{c}_i, \mathsf{r}_i)\right) = 0.$$

   If so, output $\perp$, and otherwise, output $\mathsf{r} = \bigoplus_{i\in[n]} \mathsf{r}_i$.

Figure 4: 3-round secure protocol π^{CT} for list coin tossing.

6.2 Security Proof

In this section, we prove Theorem 9.

Proof. Fix any adversary $\mathcal{A}$ who corrupts at most $n-1$ parties in the protocol π^{CT}. Assume without loss of generality that $\mathcal{A}$ corrupts exactly $n-1$ parties. This is without loss of generality

since in the case of input-less functionalities allowing the adversary to corrupt more parties strictly makes the result stronger. Assume without loss of generality that party P_1 is the only honest party. Moreover, assume without loss of generality that $\mathcal{A}$ is deterministic. This is without loss of generality since we can think of the coins of $\mathcal{A}$ as being fixed and hardwired. Let $\mathcal{E}$ denote the simulator (and extractor) of the simulation extractable promise ZK system as defined in Section 5 (see Definition 10). We construct a simulator $\mathcal{S}$ for the ideal world, that has oracle access to $\mathcal{A}$, as follows:

Description of the simulator $\mathcal{S}^{\mathcal{A}}(1^\lambda, 1^\ell)$.

1. **First Round Simulation:** Simulate all the messages sent in the first round, by simulating the messages of honest party honestly, and using the oracle access to $\mathcal{A}$ to simulate the messages sent by the corrupted parties. Denote all these messages by $\mathsf{view}_{\mathcal{A},1}$, and denote the secret state of the honest party by st. (This state will be needed to continue with the simulation.)

2. **Check if Abort:** Continue to run the protocol π^{CT} to completion, while continuing to simulate the honest party using the secret state st, and using oracle access to $\mathcal{A}$ to simulate the messages sent by corrupted parties. If at the end of this execution the honest parties output $\perp$, then $\mathcal{S}$ outputs $(\perp, \mathsf{view}_{\mathcal{A}})$ where $\mathsf{view}_{\mathcal{A}}$ consists of all the messages exchanged in the simulated protocol. Otherwise, continue.

3. **Validity Approximation:** Estimate the probability that all the zero-knowledge proofs are valid, conditioned on the first round messages being $\mathsf{view}_{\mathcal{A},1}$. This is done by creating a set of look-ahead threads that run only the second and third rounds of π^{CT}, until receiving non-aborting transcripts in λ threads. Formally, this is done as follows.

   (a) Set counters $i = 0$ and $T = 0$.

   (b) Set $T = T + 1$.

   (c) Use st to simulate the protocol π^{CT} to completion, while conditioning on the messages of the first round being $\mathsf{view}_{\mathcal{A},1}$.

   (d) If the honest parties output $\perp$ in this execution, then set $i = i + 1$.

   (e) If $i = \lambda$ then output $p \triangleq \frac{\lambda}{T}$. Otherwise, go back to Item (b).

4. **Learning the Adversary's Randomness:** Denote by $\mathsf{view}^{\text{partial}}_{\mathcal{A},1}$ only the messages in $\mathsf{view}_{\mathcal{A},1}$, sent between parties P_1 and P_j (recall that we assume that P_1 is the only honest party). $\mathcal{S}^{\mathcal{A}}(1^\lambda, 1^\ell, \mathsf{view}_{\mathcal{A},1})$ does the following:

   Generate a valid look ahead thread using the extractor $\mathcal{E} = (\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3)$ from Definition 10, as follows.

   (a) Run $\mathcal{E}_1^{\mathcal{A}}(\mathsf{view}^{\text{partial}}_{\mathcal{A},1}, \mathsf{st}, p) = \widetilde{\mathsf{st}}_1$.

   (b) Choose at random $\mathsf{r}_1 \leftarrow \{0,1\}^\ell$ and $\mathsf{s} \leftarrow \{0,1\}^\lambda$, and let $\mathsf{c}_1 = \mathsf{Com}(\mathsf{r}_1; \mathsf{s})$.

   (c) Run $\mathcal{E}_2^{\mathcal{A}}(\mathsf{c}_1, \mathsf{view}^{\text{partial}}_{\mathcal{A},1}, \widetilde{\mathsf{st}}_1) = (\mathsf{view}^{\text{partial}}_{\mathcal{A},2}, \widetilde{\mathsf{st}}_2)$, where the oracle is w.r.t. $\mathcal{A} = \mathcal{A}(\mathsf{view}_{\mathcal{A},1})$.

   (d) Let $\mathsf{view}_{\mathcal{A},2}$ denote the unique extension of $\mathsf{view}^{\text{partial}}_{\mathcal{A},2}$ that is consistent with $(\mathsf{view}_{\mathcal{A},1}, \mathsf{view}^{\text{partial}}_{\mathcal{A},2})$.

   Note that the only messages in $\mathsf{view}_{\mathcal{A},2}$ that are not in $\mathsf{view}^{\text{partial}}_{\mathcal{A},2}$ are messages sent between malicious players, and since we assume (without loss of generality) that $\mathcal{A}$ is deterministic, $\mathsf{view}_{\mathcal{A},2}$ is a deterministic function of $(\mathsf{view}_{\mathcal{A},1}, \mathsf{view}^{\text{partial}}_{\mathcal{A},2})$.

(e) Run $\mathcal{E}_3^{\mathcal{A}}(\mathsf{r}_1, \mathsf{view}_{\mathcal{A},1}^{\text{partial}}, \mathsf{view}_{\mathcal{A},2}^{\text{partial}}, \widetilde{\mathsf{st}}_2)$, where the oracle is w.r.t. $\mathcal{A} = \mathcal{A}(\mathsf{view}_{\mathcal{A},1}, \mathsf{view}_{\mathcal{A},2})$.

(f) Let $\mathsf{view}_{\mathcal{A},3}$ denote the unique extension of $\mathsf{view}_{\mathcal{A},3}^{\text{partial}}$ that is consistent with $(\mathsf{view}_{\mathcal{A},1}, \mathsf{view}_{\mathcal{A},2}, \mathsf{view}_{\mathcal{A},3}^{\text{partial}})$.

(g) If $\mathsf{Valid}(\mathsf{view}_{\mathcal{A},1}, \mathsf{view}_{\mathcal{A},2}, \mathsf{view}_{\mathcal{A},3}) = 0$, then goto Step (4b). Otherwise, $\mathsf{view}_{\mathcal{A},2}$ includes commitments $\{\mathsf{c}_j\}_{j\in[n]}$, and $\mathsf{view}_{\mathcal{A},3}$ includes the (supposedly) committed values $\{\mathsf{r}_j\}_{j\in[n]}$.

(h) Output $\{\mathsf{r}_j\}_{j\in[n]}$.

5. **Second Validity Approximation:** Estimate the probability that the continuation of $(\mathsf{view}_{\mathcal{A},1}, \mathsf{view}_{\mathcal{A},2})$ by $\mathcal{E}_3$ is valid. This is done by creating a set of look-ahead threads that run only the third round of π^{CT}, until it receives non-aborting transcripts in λ threads. Formally,

   (a) Set counters $i = 0$ and $T = 0$.

   (b) Set $T = T + 1$.

   (c) Run $\mathcal{E}_3^{\mathcal{A}}(\mathsf{r}_1, \mathsf{view}_{\mathcal{A},1}^{\text{partial}}, \mathsf{view}_{\mathcal{A},2}^{\text{partial}}, \widetilde{\mathsf{st}}_2) = \mathsf{view}_{\mathcal{A},3}$, where the oracle is w.r.t. $\mathcal{A} = \mathcal{A}(\mathsf{view}_{\mathcal{A},1}, \mathsf{view}_{\mathcal{A},2})$.

   (d) $\mathsf{Valid}(\mathsf{view}_{\mathcal{A},1}, \mathsf{view}_{\mathcal{A},2}, \mathsf{view}_{\mathcal{A},3}) = 1$ then set $i = i + 1$.

   (e) If $i = \lambda$ then output $q \triangleq \frac{\lambda}{T}$. Otherwise, go back to Item (b).

6. **Query to Ideal Functionality:** Set $k = \lceil \frac{\lambda}{q} \rceil$, and send 1^k to the ideal functionality, and receive k random strings $\{\mathsf{ans}_1, \ldots, \mathsf{ans}_k\}$, each of length ℓ.

7. **Forcing the Output:** $\mathcal{S}^{\mathcal{A}}(1^\lambda, 1^\ell, \mathsf{view}_{\mathcal{A},1}, \mathsf{view}_{\mathcal{A},2}, \widetilde{\mathsf{st}})$ does the following:

   (a) Set counter $i = 1$.

   (b) If $i > k$ then output $\perp$ and abort.

   (c) Otherwise, set $\mathsf{r}_1^* = \mathsf{ans}_i \bigoplus \left(\bigoplus_{j=2}^n \mathsf{r}_j\right)$.

   (d) Run $\mathcal{E}_3^{\mathcal{A}}(\mathsf{r}_1^*, \mathsf{view}_{\mathcal{A},1}, \mathsf{view}_{\mathcal{A},2}, \widetilde{\mathsf{st}}, p) = (\mathsf{view}_{\mathcal{A},3}, \{\widetilde{w}_j\})$

   (e) If $\mathsf{Valid}(\mathsf{view}_{\mathcal{A},1}, \mathsf{view}_{\mathcal{A},2}, \mathsf{view}_{\mathcal{A},3}) = 0$ then set $i = i + 1$ and goto Item (b).

   (f) Otherwise, output $(\mathsf{ans}_i, \mathsf{view}_{\mathcal{A},1}, \mathsf{view}_{\mathcal{A},2}, \mathsf{view}_{\mathcal{A},3})$.

We next prove that the simulator is an expected PPT machine.

Claim 10. *Simulator $\mathcal{S}$ runs in expected polynomial time in λ.*

Proof. We analyze the runtime of each step of the simulation separately. Clearly, steps 1 and 2 takes only $\mathsf{poly}(\lambda)$ time. Denote by ϵ the probability with which the adversary $\mathcal{A}$ does not abort. Namely, ϵ is the probability that $\mathcal{S}^{\mathcal{A}}$ proceeds to Step 3, and the expected runtime of Step 3 (given that it is executed) is $\frac{\mathsf{poly}(\lambda)}{\epsilon}$. Thus, on average, this contributes another $\mathsf{poly}(\lambda)$ to the simulator's runtime.

We next argue that the expected runtime of Step 4 (given that it is executed) is $\mathsf{poly}(\lambda)/\epsilon$. Note that this step consists of running algorithm $\mathcal{E}_{\mathcal{A}}^1$ *once*, and then repeatedly running $\mathcal{E}_2^{\mathcal{A}}$ and $\mathcal{E}_3^{\mathcal{A}}$. Recall that the runtime of $\mathcal{E}_1^{\mathcal{A}}$ is $\frac{\mathsf{poly}(\lambda)}{p}$, whereas the runtime of $\mathcal{E}_2^{\mathcal{A}}$ and $\mathcal{E}_3^{\mathcal{A}}$ is $\mathsf{poly}(\lambda)$. Note that $\mathbb{E}[p] = \epsilon$. Thus, it suffices to prove that on expectation, the number of times that $\mathcal{E}_2^{\mathcal{A}}$ and $\mathcal{E}_3^{\mathcal{A}}$ are run (in Step 4) is $O(1/\epsilon)$. This follows from the fact that conditioned on $(\mathsf{view}_{\mathcal{A},1}, \mathsf{st})$ generated in Steps 1 and 2, the probability that $\mathcal{E}$ generates a valid transcript is $\Omega(\epsilon)$. This is the case since otherwise, it would contradict the zero-knowledge property of Definition 10.

We next analyze the expected runtime of Step 5 (given that it is executed). For each message $(\mathsf{view}_{\mathcal{A},2}, \widetilde{\mathsf{st}}_2)$ there is a different probability that the continuation will be valid. If for a given message $(\mathsf{view}_{\mathcal{A},2}, \widetilde{\mathsf{st}}_2)$ this probability is q, then the expected runtime of Step 5 is $\mathsf{poly}(\lambda)/q$. Since $\mathbb{E}[q] = \mathbb{E}[p] = \epsilon$, we conclude that the expected runtime of Step 5 is $\mathsf{poly}(\lambda)/\epsilon$, as desired.

The observation that $\mathbb{E}[q] = \epsilon$, immediately implies that Step 6 runs in expected $\frac{\lambda}{\epsilon}$ time. It thus remains to notice that Step 7 takes time $k \cdot \mathsf{poly}(\lambda)$, which together with the observation above, implies that the expected runtime is $\frac{\mathsf{poly}(\lambda)}{\epsilon}$, as desired.

We conclude that altogether, the expected runtime of Steps 3-7 is $\frac{\mathsf{poly}(\lambda)}{\epsilon}$. This, together with the fact that these steps are invoked with probability ϵ, implies that the overall expected runtime of $\mathcal{S}$ is $\epsilon \cdot \frac{\mathsf{poly}(\lambda)}{\epsilon} = \mathsf{poly}(\lambda)$, as desired.

□

We also note that by Definition 10, on completion of Step 4, the simulator correctly learns the adversary's randomness r. Definition 10 also guarantees that the adversary uses the same randomness when the simulator forces the output in Step 7, implying correctness of forced output.

Claim 11. *The ensembles* $\{\mathsf{REAL}_{\mathcal{A}}(1^\lambda, 1^\ell)\}_{\lambda\in\mathbb{N}}$ *and* $\{\mathsf{IDEAL}_{\mathcal{S}^{\mathcal{A}}}(1^\lambda, 1^\ell)\}_{\lambda\in\mathbb{N}}$ *are computationally indistinguishable.*

Proof. Suppose for contradiction that there exists a PPT distinguisher $\mathcal{D}$ and there exists a constant $c \in \mathbb{N}$ such that for infinitely many security parameters λ,

$$\left|\Pr[\mathcal{D}(\mathsf{REAL}_{\mathcal{A}}(1^\lambda, 1^\ell)) = 1] - \Pr[\mathcal{D}(\mathsf{IDEAL}_{\mathcal{S}^{\mathcal{A}}}(1^\lambda, 1^\ell)) = 1]\right| \geq \lambda^{-c}.$$

In what follows we get a contradiction, by considering a series of hybrids where the first hybrid Hyb_0 corresponds to the ideal world and the last hybrid Hyb_4 corresponds to the real world. We reach a contradiction by proving that

$$|\Pr[\mathcal{D}(\mathsf{Hyb}_4) = 1] - \Pr[\mathcal{D}(\mathsf{Hyb}_0) = 1]| < \lambda^{-c}. \tag{1}$$

- Hyb_0 **- Ideal World:** This hybrid is identical to the ideal world simulation $S^{\mathcal{A}}(1^\lambda, 1^\ell)$
- Hyb_1**:** This hybrid is similar to Hyb_0 except that in Step 4b, we choose $c_1 = \mathsf{Com}(0; s)$ (rather than choosing it with a random r_1), and yet Step 4e is executed w.r.t. a random $\mathsf{r}_1 \leftarrow \{0,1\}^\ell$.
- Hyb_2**:** This hybrid executes Hyb_1 until the end of Step 4, and outputs the view of $\mathcal{A}$ in this simulation.
- Hyb_3**:** This hybrid is similar to Hyb_2, except that rather than choosing p as in Step 3, it sets $p \triangleq \mathsf{pExtract}^{\mathcal{A}}_{2c}$, with respect to the distribution of instances (c, r), generated by sampling at random $r \leftarrow \{0,1\}^\ell$ and random $s \leftarrow \{0,1\}^\lambda$ and setting $c = \mathsf{Com}(r; s)$. If $p = 0$ then simply output $\perp$. Otherwise, execute Step 4 with this new value of p.
- Hyb_4 **- Real World:** In this hybrid, the simulator plays the role of the honest party (honestly) and emulates the malicious parties as instructed by $\mathcal{A}$. It outputs the view of $\mathcal{A}$.

We prove Equation 1, via the following sequence of claims.

Claim 12.

$$\mathsf{Hyb}_0 \approx \mathsf{Hyb}_1.$$

This claim follows immediately from the hiding property of the commitment scheme Com.

Claim 13.

$$\mathsf{Hyb}_1 \equiv \mathsf{Hyb}_2.$$

This claim follows from the observation that Hyb_2 aborts in Step 7 only with negligible probability, and these hybrids are identical conditioned on the event that Hyb_2 does not abort in Step 7.

Claim 14. *For every PPT distinguisher $\mathcal{D}$,*

$$|\Pr[\mathcal{D}(\mathsf{Hyb}_2) = 1] - \Pr[\mathcal{D}(\mathsf{Hyb}_3) = 1]| \leq 2\lambda^{-2c} + \mathsf{negl}(\lambda).$$

This claim follows from the observation that if $\mathsf{pExtract}_{2c}^{\mathcal{A}} = 0$ then the probability that these hybrids abort immediately after Step 2 is at least $1 - 2\lambda^{-2c}$. Moreover, if $\mathsf{pExtract}_{2c}^{\mathcal{A}} > 0$, then by Definition 10, together with the fact that Step 3 (the validity approximation step) is a good approximation for the validity.

Claim 15.

$$\mathsf{Hyb}_3 \approx \mathsf{Hyb}_4.$$

This claim follows immediately from Definition 10. □

□

7 Four Round Malicious Secure MPC

7.1 Building Block: Extractable Commitments with Additional Properties

In this section, we describe a three round extractable commitment protocol $\mathsf{ECom} = (S, R)$. While several constructions of three round extractable commitment schemes are known in the literature (see, e.g., [PRS02, Ros04]), the commitment scheme we describe here achieves some stronger security properties that we describe below:

- *Bounded-Rewinding Security:* The commitment scheme satisfies a "bounded-rewinding security" property, which roughly means that the value committed by a sender in an execution of the commitment protocol remains hidden even if a malicious receiver can rewind the sender back to the start of the second round of the protocol an a priori bounded B number of times. In our application, we set $B = 4$; however, our construction also supports larger values of B.
- *Reusability:* The commitment scheme also satisfies a "reusability" property, which roughly means that the values committed by a sender in polynomially many executions of the commitment protocol remain hidden even if all of the executions share the same first two messages.

For technical reasons, we don't define or prove B-rewinding security property and reusability property for our extractable commitment protocol. Instead, this is done inline in the application - the four round MPC protocol in the next subsection.

Construction. Let Com denote a non-interactive perfectly binding commitment scheme based on injective one-way functions. Let N and B be positive integers such that $N - B - 1 \geq \frac{N}{2} + 1$. For $B = 4$, it suffices to set $N = 12$.

The three round extractable commitment protocol ECom is described in Figure 5.

Sender S has input x.

Commitment Phase:

1. **Round 1:**
   S does the following:
   - Pick N random degree B polynomials $\mathsf{p}_1, \ldots, \mathsf{p}_N$ over $\mathbb{Z}_q$, where q is a prime larger than 2^λ.
   - Compute $\mathsf{ECom}_{1,\ell}^{S\to R} \leftarrow \mathsf{Com}(\mathsf{p}_\ell; r_\ell)$ using a random string r_ℓ, for every $\ell \in [N]$.
   - Send $\mathsf{ECom}_1^{S\to R} = (\mathsf{ECom}_{1,1}^{S\to R}, \ldots, \mathsf{ECom}_{1,N}^{S\to R})$ to R.
2. **Round 2:**
   R does the following:
   - Pick random values $\mathsf{z}_\ell \xleftarrow{\$} \mathbb{Z}_q$ for every $\ell \in [N]$.
   - Send $\mathsf{ECom}_2^{R\to S} = (\mathsf{z}_1, \ldots, \mathsf{z}_N)$ to S.
3. **Round 3:**
   S does the following:
   - Sample a PRF key k and a random string s.
   - Compute $\mathsf{ECom}_{3,\ell}^{S\to R} \leftarrow (k \oplus \mathsf{p}_\ell(0), \mathsf{p}_\ell(\mathsf{z}_\ell))$ for all $\ell \in [N]$.
   - Set $\mathsf{ECom}_{3,N+1}^{S\to R} = s$ and compute $\mathsf{ECom}_{3,N+2}^{S\to R} \leftarrow \mathsf{PRF}(k, s) \oplus x$.
   - Send $\mathsf{ECom}_3^{S\to R} = (\mathsf{ECom}_{3,1}^{S\to R}, \ldots, \mathsf{ECom}_{3,N+2}^{S\to R})$ to R.

Decommitment Phase:

1. S outputs $\mathsf{p}_1, \ldots, \mathsf{p}_N$ together with the randomness $r_1, \ldots, r_N$ used in the first round commitments.
2. R first verifies the following:
   - For each $\ell \in [N]$, $\mathsf{ECom}_{1,\ell}^{S\to R} = \mathsf{Com}(\mathsf{p}_\ell; r_\ell)$.
   - Parse $\mathsf{ECom}_{3,\ell}^{S\to R} = (\alpha_\ell, \beta_\ell)$. Verify that $\beta_\ell = \mathsf{p}_\ell(\mathsf{z}_\ell)$.
   - For each $\ell \in [N]$, compute $k_\ell = \mathsf{p}_\ell(0) \oplus \alpha_\ell$. Verify that all the k_ℓ values are equal.

   If any of the above verifications fail, R outputs $\perp$. Otherwise, R computes $x \leftarrow \mathsf{PRF}(k, \mathsf{ECom}_{3,N+1}) \oplus \mathsf{ECom}_{3,N+2}$.

Figure 5: Extractable Commitment Scheme ECom.

Well-Formedness of ECom Transcripts. We now define a "well-formedness" property of an execution transcript of ECom. Roughly, we say that a transcript $(\mathsf{ECom}_1^{S\to R}, \mathsf{ECom}_2^{R\to S}, \mathsf{ECom}_3^{S\to R})$ is well-formed w.r.t. an input x and randomness r if:

- $N-1$ out of the N tuples $\mathsf{ECom}_{3,\ell}^{S\to R} = (\alpha_\ell, \beta_\ell)$ (where $\ell \in [N]$) are "honestly" computed using randomness $r = \left(k, \{\mathsf{p}_i\}_{i=1}^N, \{r_i\}_{i=1}^N\right)$ in the sense that: each α_ℓ is a one-time pad of k w.r.t. the key $\mathsf{p}_\ell(0)$ where p_ℓ is a polynomial committed (using randomness r_ℓ) in the first round message $\mathsf{ECom}_1^{S\to R}$, and each β_ℓ is a correct evaluation of the polynomial p_ℓ over the "challenge" value z_ℓ contained in $\mathsf{ECom}_2^{R\to S}$.

- $\mathsf{ECom}^{S\to R}_{3,N+2}$ is a one-time pad of x w.r.t. the key $\mathsf{PRF}(k, \mathsf{ECom}_{3,N+1})$.

We now proceed to formally define the well-formedness property. For any set T, let $T[i]$ denote the i^{th} element of T.

Definition 14 (Well-Formed Transcripts)**.** *An execution transcript* $(\mathsf{ECom}^{S\to R}_1, \mathsf{ECom}^{R\to S}_2, \mathsf{ECom}^{S\to R}_3)$ *of* ECom *is said to be* well-formed *with respect to an input* x *and randomness* $r = \left(k, \{\mathsf{p}_i\}_{i=1}^N, \{r_i\}_{i=1}^N\right)$ *if there exists an index set* $\mathcal{I}$ *of size* $N-1$ *such that the following holds:*

- *For every* $j \in |\mathcal{I}|$, $\mathsf{ECom}^{S\to R}_{1,\mathcal{I}[j]} = \mathsf{Com}(\mathsf{p}_{\mathcal{I}[j]}; r_{\mathcal{I}[j]})$ *(AND)*
- *For every* $j \in |\mathcal{I}|$, $\mathsf{ECom}^{S\to R}_{3,\mathcal{I}[j]} = (k \oplus \mathsf{p}_{\mathcal{I}[j]}(0), \mathsf{p}_{\mathcal{I}[j]}(\mathsf{z}_{\mathcal{I}[j]}))$, *where* $\mathsf{ECom}^{R\to S}_2 = (\mathsf{z}_1, \ldots, \mathsf{z}_N)$ *(AND)*
- $x = \mathsf{PRF}(k, \mathsf{ECom}_{3,N+1}) \oplus \mathsf{ECom}_{3,N+2}$.

We remark that the above well-formedness property is "weak" in the sense that we only require $N-1$ out of the N tuples $\mathsf{ECom}^{S\to R}_{3,\ell} = (\alpha_\ell, \beta_\ell)$ to be honestly generated (instead of requiring that all N tuples are honestly generated). This relaxation is crucial to establishing the B-rewinding-security property for ECom. In fact, a slightly "trimmed" version of our construction where we do not use a PRF in the third round already suffices for achieving B-rewinding-security property. We use a PRF in the third round to achieve reusability property, in a similar manner to [JKKR17].

We now define an "admissibility" property for any input to the extractor.

Definition 15 (Admissible Inputs)**.** *An input set* $(\mathsf{ECom}_1, \{\mathsf{ECom}^i_2, \mathsf{ECom}^i_3\}_{i=1}^{B+1})$ *is said to be* admissible *if for every* $i, j \in [B+1]$ *s.t.* $i \neq j$ *and every* $\ell \in [N]$, *we have that* $\mathsf{z}^i_\ell \neq \mathsf{z}^j_\ell$, *where* $\mathsf{ECom}^t_2 = (\mathsf{z}^t_1, \ldots, \mathsf{z}^t_N)$.

Extractor $\mathsf{Ext}_{\mathsf{ECom}}$**.** The extractor algorithm $\mathsf{Ext}_{\mathsf{ECom}}$ is described in Figure 6.[18]

Lemma 1. *There exists a PPT extractor algorithm* $\mathsf{Ext}_{\mathsf{ECom}}$ *such that, given a set of* $(B+1)$ *"well-formed" and "admissible" execution transcripts of* ECom *where each transcript consists of the same first round sender message, the extractor successfully extracts the value committed in each transcript, except with negligible probability.*

Proof. We now analyze the extraction algorithm. Recall that for every $i \in [B+1]$, the transcript $(\mathsf{ECom}_1, \mathsf{ECom}^i_2, \mathsf{ECom}^i_3)$ is well-formed w.r.t. some value x_i. By the definition of well-formedness, we have that for every i, there exists at most one $j \in [N]$ such that $\mathsf{ECom}^i_{3,j}$ was not computed correctly and consistently with the other $\mathsf{ECom}^i_{3,j'}$. This means that overall, across all $i \in [B+1]$ execution transcripts, there exists at most $(B+1)$ values of $\mathsf{ECom}^i_{3,j}$ that were not computed correctly. This implies that for at least $(N-B-1)$ values of j, the values $\mathsf{ECom}^i_{3,j}$ were computed correctly in all $B+1$ transcripts. This means that for every $i \in [B+1]$, $(N-B-1)$ out of N values $\{k^i_1, \ldots, k^i_N\}$ computed by the extractor are the same. Then, since $N - B - 1 \geq \frac{N}{2} + 1$, we have that the extractor computes the correct values k^i and x_i for every $i \in [B]$. □

[18] An admissible input set consisting of $(B+1)$ "well-formed" execution transcripts of ECom that share the same first round sender message can be obtained from a malicious sender via an expected PPT rewinding procedure. The expected PPT simulator in our application performs the necessary rewindings to obtain such transcripts and then feeds them to the extractor $\mathsf{Ext}_{\mathsf{ECom}}$.

Input: An admissible set $(\mathsf{ECom}_1, \{\mathsf{ECom}_2^i, \mathsf{ECom}_3^i\}_{i=1}^{B+1})$ where $\forall i$, $(\mathsf{ECom}_1, \mathsf{ECom}_2^i, \mathsf{ECom}_3^i)$ is well-formed w.r.t. some value x_i.

1. For every $i \in [B+1]$, parse $\mathsf{ECom}_2^i = (\mathsf{z}_1^i, \ldots, \mathsf{z}_N^i)$ and $\mathsf{ECom}_3^i = (\mathsf{ECom}_{3,1}^i, \ldots, \mathsf{ECom}_{3,N+2}^i)$.
2. For each $\ell \in [N]$:
   - Parse $\mathsf{ECom}_{3,\ell}^i = (\alpha_\ell^i, \beta_\ell^i)$.
   - Using polynomial interpolation, compute a degree B polynomial p_ℓ over $\mathbb{Z}_q$ such that on point z_ℓ^i, $\mathsf{p}_\ell(\mathsf{z}_\ell^i) = \beta_\ell^i$.
   - Compute $k_\ell^i = (\alpha_\ell^i \oplus \mathsf{p}_\ell(0))$.
3. For every $i \in [B]$, let k^i be the value that equals a majority of the values in the set $\{k_1^i, \ldots, k_N^i\}$, and compute $x_i = \mathsf{PRF}(k^i, \mathsf{ECom}_{3,N+1}^i) \oplus \mathsf{ECom}_{3,N+2}^i$. If no such k^i value exists, set $x_i = \bot$.
4. Output $(x_1, \ldots, x_B)$.

Figure 6: Strategy of algorithm $\mathsf{Ext}_{\mathsf{ECom}}$.

7.2 The MPC Protocol

In this section, we describe our four round MPC protocol that achieves security against any malicious PPT adversary that corrupts a dishonest majority of parties. We obtain our result by "compiling" any three-round semi-malicious MPC protocol in which the first message of the protocol is public-coin (i.e., an honest party simply samples a random string and sends it as the first message) into a four round maliciously secure protocol.

Formally, we prove the following theorem.

Theorem 12. *Assuming polynomially secure:*

- *DDH or Quadratic Residuosity or N^{th} Residuosity, AND*
- *a three round semi-malicious MPC protocol π^{SM} for any functionality f secure against a dishonest majority and the first round is public coin,*

the protocol π presented below is a four round MPC protocol for f secure against a dishonest majority.

[BHP17, GS18, BL18][19] construct such three round semi-malicious protocols. Instantiating the above theorem with the constructions of [GS18, BL18], we get the following corollary.

Corollary 13. *Assuming polynomially secure:*

- *DDH or Quadratic Residuosity or N^{th} Residuosity,*

the protocol π presented below is a four round MPC protocol for any functionality f.

Below, we first list the ingredients used in our construction, followed by some relevant notation. We formally describe our MPC protocol in Section 7.3.

[19]Note that [GS18, BL18] construct two round semi-malicious MPC protocols and hence they are trivially three round with the first round being public coin.

Ingredients. We list all the cryptographic ingredients that are used in our MPC protocol. As mentioned in the introduction, our construction uses a Promise ZK argument system in a non-black-box manner, and therefore we list all of its ingredients separately.

- NCom is a non-interactive commitment scheme based on injective one way functions.
- $\mathsf{WZK} = (\mathsf{WZK}_1, \mathsf{WZK}_2, \mathsf{WZK}_3, \mathsf{WZK}_4)$ is a three-message delayed-input distributional weak zero-knowledge argument system, where the algorithm WZK_4 is used to compute the decision of the verifier. Such a scheme was constructed by Jain et al.[JKKR17] based on DDH or Quadratic Residuosity or N^{th} Residuosity assumption.
- $\mathsf{ECom} = (\mathsf{ECom}_1, \mathsf{ECom}_2, \mathsf{ECom}_3, \mathsf{Ext}_{\mathsf{ECom}})$ is the three-message delayed-input extractable commitment scheme described in Section 7.1 based on injective one way functions. We set the rewinding parameter B associated with ECom to be 4. Here $\mathsf{Ext}_{\mathsf{ECom}}$ is the extractor algorithm associated with ECom s.t. given an admissible input set consisting of 5 well-formed execution transcripts of the ECom protocol that share the same first round message, $\mathsf{Ext}_{\mathsf{ECom}}$ outputs all of the committed values with overwhelming probability.
- $\mathsf{TDGen} = (\mathsf{TDGen}_1, \mathsf{TDGen}_2, \mathsf{TDGen}_3, \mathsf{TDOut}, \mathsf{TDValid}, \mathsf{TDExt})$ is a three-message trapdoor generation protocol as defined in Section 4 based on one way functions. The first three algorithms are used to generate the messages of the protocol while TDOut computes the receiver's output. Algorithm $\mathsf{TDValid}$ determines whether an input trapdoor value is valid w.r.t. the first message of a protocol transcript. Algorithm TDExt computes a valid trapdoor given three protocol transcripts that share the same first round message.
- $\mathsf{WI} = (\mathsf{WI}_1, \mathsf{WI}_2, \mathsf{WI}_3, \mathsf{WI}_4)$ is a three-message delayed-input witness-indistinguishable argument system, where WI_4 is used to compute the decision of the verifier. Such schemes are known from injective one-way functions [LS90].
- $\mathsf{RWI} = (\mathsf{RWI}_1, \mathsf{RWI}_2, \mathsf{RWI}_3, \mathsf{RWI}_4)$ is a three round delayed-input witness-indistinguishable argument with non-adaptive B-rewinding security (for $B = 6$) as well as reusability security (see Definition 19 and 18 in Appendix C). Informally, by non-adaptive rewinding security, we mean that the verifier challenges are non-adaptive, that is - the second round message of the verifier in all the threads (main and rewound) are committed in advance before the challenger responds to any of them in the third round. This means that the verifier generates each second round query independent of the third round response to the previous query. This rewinding security property suffices for our MPC construction.

  We describe how to construct the protocol RWI in Appendix C based on DDH or Quadratic Residuosity or N^{th} Residuosity assumption. Briefly, we consider protocol 6 of [JKKR17] that is non-adaptive unbounded rewinding and reusable secure and instead instantiate the ZAP in that protocol with our bounded rewinding secure WI from Appendix A to achieve only non-adaptive bounded rewinding and reusability security. We elaborate more on it in Appendix C.
- $\mathsf{NMCom} = (\mathsf{NMCom}_1, \mathsf{NMCom}_2, \mathsf{NMCom}_3)$ is the three-message special non-malleable commitment scheme of Goyal et al. [GPR16] satisfying Definition 4. It is based on injective one way functions. Let $\mathsf{Ext}_{\mathsf{NMCom}}$ denote the PPT extractor associated with the 2-extraction property satisfied by NMCom.

- π^{SM} is a three-round semi-malicious MPC protocol with the first round being public coin. Let $(\pi_1^{\mathsf{SM}}, \pi_2^{\mathsf{SM}}, \pi_3^{\mathsf{SM}})$ denote the algorithms used by any party to compute the messages in each of the three rounds and OUT denotes the algorithm to compute the final output. Also, let Trans_i denote all the messages sent in an execution of π^{SM} up to the completion of round i. Let $\mathcal{S} = (\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3)$ denote the straight line simulator for this protocol, where $\mathcal{S}_i$ is the simulator's algorithm to compute the i^{th} round messages.

Notation. Let $\mathsf{P}_1, \ldots, \mathsf{P}_n$ denote the n parties, and let λ denote the security parameter. For simplicity, without loss of generality, we assume that n is at most λ. We consider communication in the broadcast model where all of the protocol messages are sent over a broadcast channel. Below, we describe some additional notation:

- We augment our notation with $i \to j$ in the superscript to denote that a message is being sent by party P_i with party P_j as the intended recipient.
- The round number of any sub-protocol (such as the non-malleable commitment, bounded rewinding secure WI arguments etc.) is written in the subscript.
- We use two instantiations of the extractable commitment scheme ECom in our construction. We use a and b in the subscript to differentiate between the two.

NP Languages. In our construction, we use proofs for the following NP languages:

- Language L_1 is characterized by the following relation R_1:
  Statement : $\mathsf{st} = \mathsf{nc}$
  Witness : $\mathsf{w} = \mathsf{r}_{\mathsf{nc}}$
  $R_1(\mathsf{st}, \mathsf{w}) = 1$ if and only if $\mathsf{nc} = \mathsf{NCom}(1; \mathsf{r}_{\mathsf{nc}})$

  In our protocol, we use this language for delayed-input distributional WZK proofs. When restricting our attention to such proofs between parties P_i and P_j, where P_i is the prover and P_j is the verifier, we denote the corresponding language by $L_1^{i \to j}$.

- Language L_2 is characterized by the following relation R_2:
  Statement : $\mathsf{st} = \left(\{\mathsf{ecom}_{a,i}\}_{i=1}^3, \{\mathsf{ecom}_{b,i}\}_{i=1}^3, \mathsf{msg}_2, \mathsf{Trans}_1, \{\mathsf{nmcom}_i\}_{i=1}^3, \mathsf{td}_1, \mathsf{nc}\right)$
  Witness : $\mathsf{w} = (\mathsf{inp}, \mathsf{r}, \mathsf{r}_{a,\mathsf{ecom}}, \mathsf{r}_{b,\mathsf{ecom}}, \mathsf{t}, \mathsf{r}_{\mathsf{nmcom}}, \mathsf{r}_{\mathsf{nc}})$
  $R_2(\mathsf{st}, \mathsf{w}) = 1$ if and only if :

  1. Either $(\mathsf{ecom}_{a,1}, \mathsf{ecom}_{a,2}, \mathsf{ecom}_{a,3})$ or $(\mathsf{ecom}_{b,1}, \mathsf{ecom}_{b,2}, \mathsf{ecom}_{b,3})$ is a well-formed transcript of ECom w.r.t. input $(\mathsf{inp}, \mathsf{r})$ and randomness $\mathsf{r}_{a,\mathsf{ecom}}$ (see Definition 14), and msg_2 is an honestly computed second round message in protocol π^{SM} w.r.t. input inp and randomness r and round 1 protocol transcript Trans_1 (OR)
  2. $(\mathsf{nmcom}_1, \mathsf{nmcom}_2, \mathsf{nmcom}_3)$ is a transcript of a non-malleable commitment to a value t that is a valid trapdoor w.r.t. td_1. (OR)
  3. nc is a commitment to 0.

  Formally, $R_2(\mathsf{st}, \mathsf{w}) = 1$ if and only if :

  - $\mathsf{ecom}_{a,1} = \mathsf{ECom}_1(\mathsf{r}_{a,\mathsf{ecom}})$ AND
  - $\mathsf{ecom}_{a,3} = \mathsf{ECom}_3(\mathsf{inp}, \mathsf{r}, \mathsf{ecom}_{a,1}, \mathsf{ecom}_{a,2}; \mathsf{r}_{a,\mathsf{ecom}})$ AND
  - $\mathsf{msg}_2 = \pi_2^{\mathsf{SM}}(\mathsf{inp}, \mathsf{Trans}; \mathsf{r})$ AND

– $(\mathsf{ecom}_{a,1}, \mathsf{ecom}_{a,2}, \mathsf{ecom}_{a,3})$ is well-formed w.r.t. input $(\mathsf{inp}, \mathsf{r})$ and randomness $\mathsf{r}_{a,\mathsf{ecom}}$

(OR)

– $\mathsf{ecom}_{b,1} = \mathsf{ECom}_1(\mathsf{r}_{b,\mathsf{ecom}})$ AND
– $\mathsf{ecom}_{b,3} = \mathsf{ECom}_3(\mathsf{inp}, \mathsf{r}, \mathsf{ecom}_{b,1}, \mathsf{ecom}_{b,2}; \mathsf{r}_{b,\mathsf{ecom}})$ AND
– $\mathsf{msg}_2 = \pi_2^{\mathsf{SM}}(\mathsf{inp}, \mathsf{Trans}_1; \mathsf{r})$ AND
– $(\mathsf{ecom}_{b,1}, \mathsf{ecom}_{b,2}, \mathsf{ecom}_{b,3})$ is well-formed w.r.t. committed value $(\mathsf{inp}, \mathsf{r})$ and randomness $\mathsf{r}_{b,\mathsf{ecom}}$.

(OR)

– $\mathsf{TDValid}(\mathsf{td}_1, \mathsf{t}) = 1$ AND
– $\mathsf{nmcom}_1 = \mathsf{NMCom}_1(\mathsf{r}_{\mathsf{nmcom}})$ AND
– $\mathsf{nmcom}_3 = \mathsf{NMCom}_3(\mathsf{t}, \mathsf{nmcom}_1, \mathsf{nmcom}_2; \mathsf{r}_{\mathsf{nmcom}})$.

(OR)

– $\mathsf{nc} = \mathsf{NCom}(0; \mathsf{r}_{\mathsf{nc}})$.

In our protocol, we use language L_2 for bounded-rewinding secure delayed-input RWI proofs. When restricting our attention to such proofs between parties P_i and P_j, where P_i is the prover and P_j is the verifier, we denote the corresponding language by $L_2^{i \to j}$.

- Language L_3 is characterized by the following relation R_3:
  Statement : $\mathsf{st} = \left(\{\mathsf{ecom}_{a,i}\}_{i=1}^3, \{\mathsf{ecom}_{b,i}\}_{i=1}^3, \mathsf{msg}_3, \mathsf{Trans}_2, \{\mathsf{nmcom}_i\}_{i=1}^3, \mathsf{td}_1\right)$
  Witness : $\mathsf{w} = (\mathsf{inp}, \mathsf{r}, \mathsf{r}_{a,\mathsf{ecom}}, \mathsf{r}_{b,\mathsf{ecom}}, \mathsf{t}, \mathsf{r}_{\mathsf{nmcom}})$
  $R_3(\mathsf{st}, \mathsf{w}) = 1$ if and only if :

  1. Either $(\mathsf{ecom}_{a,1}, \mathsf{ecom}_{a,2}, \mathsf{ecom}_{a,3})$ or $(\mathsf{ecom}_{b,1}, \mathsf{ecom}_{b,2}, \mathsf{ecom}_{b,3})$ is a well-formed transcript of ECom w.r.t. input $(\mathsf{inp}, \mathsf{r})$ and randomness $\mathsf{r}_{a,\mathsf{ecom}}$ (see Definition 14), and msg_3 is an honestly computed third round message in protocol π^{SM} w.r.t. input inp and randomness r and round 2 protocol transcript Trans_2 (OR)
  2. $(\mathsf{nmcom}_1, \mathsf{nmcom}_2, \mathsf{nmcom}_3)$ is a transcript of a non-malleable commitment to a value t that is a valid trapdoor w.r.t. td_1.

  The formal description of R_3 is similar to R_2; we skip the details to avoid repetition.

  In our protocol, we use language L_3 for delayed-input WI proofs between parties. When restricting our attention to such proofs between parties P_i and P_j, where P_i is the prover and P_j is the verifier, we denote the corresponding language by $L_3^{i \to j}$.

7.3 The Protocol

We now proceed to formally describe our four round protocol π between n parties $\mathsf{P}_1, \ldots, \mathsf{P}_n$. The input of party P_i is denoted as x_i.

Round 1: P_i does the following:

- Compute $\mathsf{msg}_{1,i} \leftarrow \pi_1^{\mathsf{SM}}(\mathsf{r}_i)$. Recall that π^{SM} is public-coin in the first round; thus, $\mathsf{msg}_{1,i}$ is a random string.
- Compute $\mathsf{td}_{1,i} \leftarrow \mathsf{TDGen}_1(\mathsf{r}_{\mathsf{td},i})$ using a random string $\mathsf{r}_{\mathsf{td},i}$.
- Compute the first message of each of the three round delayed-input subprotocols. That is, for each $j \in [n]$ with $j \neq i$, P_i does the following:
  - Compute $\mathsf{ecom}_{a,1}^{i\to j} \leftarrow \mathsf{ECom}_1(\mathsf{r}_{a,\mathsf{ecom}}^{i\to j})$, $\mathsf{ecom}_{b,1}^{i\to j} \leftarrow \mathsf{ECom}_1(\mathsf{r}_{b,\mathsf{ecom}}^{i\to j})$, and $\mathsf{nmcom}_1^{i\to j} \leftarrow \mathsf{NMCom}_1(\mathsf{r}_{\mathsf{nmcom}}^{i\to j})$ using random strings $\mathsf{r}_{a,\mathsf{ecom}}^{i\to j}$, $\mathsf{r}_{b,\mathsf{ecom}}^{i\to j}$ and $\mathsf{r}_{\mathsf{nmcom}}^{i\to j}$, respectively.
  - Compute $\mathsf{wi}_1^{i\to j} \leftarrow \mathsf{WI}_1(1^\lambda)$, $\mathsf{rwi}_1^{i\to j} \leftarrow \mathsf{RWI}_1(1^\lambda)$ and $\mathsf{wzk}_1^{i\to j} \leftarrow \mathsf{WZK}_1(1^\lambda)$.
- Broadcast $(\mathsf{msg}_{1,i}, \mathsf{ecom}_{a,1}^{i\to j}, \mathsf{ecom}_{b,1}^{i\to j}, \mathsf{nmcom}_1^{i\to j}, \mathsf{td}_{1,i}, \mathsf{wi}_1^{i\to j}, \mathsf{rwi}_1^{i\to j}, \mathsf{wzk}_1^{i\to j})$.

Note that round 1 does not require the use of private input x_i.

Round 2: P_i now generates the second message of each of the three round delayed-input subprotocols. In more detail, for each $j \in [n]$ with $j \neq i$, P_i does the following:

- Compute $\mathsf{ecom}_{a,2}^{i\to j} \leftarrow \mathsf{ECom}_2(\mathsf{ecom}_{a,1}^{j\to i})$, $\mathsf{ecom}_{b,2}^{i\to j} \leftarrow \mathsf{ECom}_2(\mathsf{ecom}_{b,1}^{j\to i})$, and $\mathsf{nmcom}_2^{i\to j} \leftarrow \mathsf{NMCom}_2(\mathsf{nmcom}_1^{j\to i})$
- Compute $\mathsf{td}_2^{i\to j} \leftarrow \mathsf{TDGen}_2(\mathsf{td}_{1,j})$.
- Compute $\mathsf{wi}_2^{i\to j} \leftarrow \mathsf{WI}_2(\mathsf{wi}_1^{j\to i}), \mathsf{rwi}_2^{i\to j} \leftarrow \mathsf{RWI}_2(\mathsf{rwi}_1^{j\to i})$ and $\mathsf{wzk}_2^{i\to j} \leftarrow \mathsf{WZK}_2(\mathsf{wzk}_1^{j\to i})$.
- Broadcast $(\mathsf{ecom}_{a,2}^{i\to j}, \mathsf{ecom}_{b,2}^{i\to j}, \mathsf{nmcom}_2^{i\to j}, \mathsf{td}_2^{i\to j}, \mathsf{wi}_2^{i\to j}, \mathsf{rwi}_2^{i\to j}, \mathsf{wzk}_2^{i\to j})$.

Similar to round 1, round 2 also does not require the use of private input x_i.

Round 3: P_i does the following:

- Compute $\mathsf{msg}_{2,i} \leftarrow \pi_2^{\mathsf{SM}}(\mathsf{x}_i, \mathsf{Trans}_1; \mathsf{r}_i)$, where Trans_1 denotes the round 1 transcript of π^{SM}. (This is the first step where P_i uses its private input x_i.)
- Let $\mathsf{td}_{2,i} = (\mathsf{td}_2^{1\to i}||\ldots||\mathsf{td}_2^{n\to i})$ with $\mathsf{td}_2^{i\to i} = \perp$.[20] Compute the third message of the trapdoor generation protocol, i.e., $\mathsf{td}_{3,i} \leftarrow \mathsf{TDGen}_3(\mathsf{td}_{1,i}, \mathsf{td}_{2,i}; \mathsf{r}_{\mathsf{td},i})$ using randomness $\mathsf{r}_{\mathsf{td},i}$.
- Compute $\mathsf{nc}_i \leftarrow \mathsf{NCom}(1; \mathsf{r}_{\mathsf{nc},i})$.
- For each $j \in [n]$ with $j \neq i$, compute the following:
  - The third messages of the two extractable commitment schemes corresponding to inputs $(\mathsf{x}_i, \mathsf{r}_i)$ and $\perp$, respectively, and third message of the non-malleable commitment corresponding to input $\perp$. That is,

$$\mathsf{ecom}_{a,3}^{i\to j} \leftarrow \mathsf{ECom}_3((\mathsf{x}_i, \mathsf{r}_i), \mathsf{ecom}_{a,1}^{i\to j}, \mathsf{ecom}_{a,2}^{j\to i}; \mathsf{r}_{a,\mathsf{ecom}}^{i\to j}),$$

[20] From Section 4, note that this concatenation still defines valid second round message of the trapdoor generation protocol because that just requires a string to be signed. We do this to establish a single global trapdoor for each party instead of pairwise trapdoors between every two parties.

$$\mathsf{ecom}_{b,3}^{i\to j} \leftarrow \mathsf{ECom}_3(\bot, \mathsf{ecom}_{b,1}^{i\to j}, \mathsf{ecom}_{b,2}^{j\to i}; \mathsf{r}_{b,\mathsf{ecom}}^{i\to j}),$$

$$\mathsf{nmcom}_3^{i\to j} \leftarrow \mathsf{NMCom}_3(\bot, \mathsf{nmcom}_1^{i\to j}, \mathsf{nmcom}_2^{j\to i}; \mathsf{r}_{\mathsf{nmcom}}^{i\to j}),$$

using random strings $\mathsf{r}_{a,\mathsf{ecom}}^{i\to j}$, $\mathsf{r}_{b,\mathsf{ecom}}^{i\to j}$, and $\mathsf{r}_{\mathsf{nmcom}}^{i\to j}$, respectively.

- The third message of RWI, i.e., $\mathsf{rwi}_3^{i\to j} \leftarrow \mathsf{RWI}_3(\mathsf{rwi}_1^{i\to j}, \mathsf{rwi}_2^{j\to i}, \mathsf{st}_2^{i\to j}, \mathsf{w}_2^{i\to j})$ for the statement

$$\mathsf{st}_2^{i\to j} = \left(\{\mathsf{ecom}_{a,\ell}^{i\to j}\}_{\ell=1}^3, \{\mathsf{ecom}_{b,\ell}^{i\to j}\}_{\ell=1}^3, \mathsf{msg}_{2,i}, \mathsf{Trans}_1, \{\mathsf{nmcom}_\ell^{i\to j}\}_{\ell=1}^3, \mathsf{td}_{1,j}, \mathsf{nc}_i\right) \in L_2^{i\to j}$$

using witness $\mathsf{w}_2^{i\to j} = (\mathsf{x}_i, \mathsf{r}_i, \mathsf{r}_{a,\mathsf{ecom}}^{i\to j}, \bot, \bot, \bot, \bot)$.

- The third message of the weak zero-knowledge protocol proving that nc_i is a commitment to 1. That is, $\mathsf{wzk}_3^{i\to j} \leftarrow \mathsf{WZK}_3(\mathsf{wzk}_1^{i\to j}, \mathsf{wzk}_2^{j\to i}, \mathsf{st}_1^{i\to j}, \mathsf{w}_1^{i\to j})$ for the statement $\mathsf{st}_1^{i\to j} = \mathsf{nc}_1^{i\to j} \in L_1^{i\to j}$ using witness $\mathsf{w}_1^{i\to j} = \mathsf{r}_{\mathsf{nc},i}$.

- Broadcast $(\mathsf{msg}_{2,i}, \mathsf{nc}_i, \mathsf{ecom}_{a,3}^{i\to j}, \mathsf{ecom}_{b,3}^{i\to j}, \mathsf{nmcom}_3^{i\to j}, \mathsf{td}_{3,i}, \mathsf{rwi}_3^{i\to j}, \mathsf{wzk}_3^{i\to j})$.

Round 4: P_i does the following:

- Abort if for any $j \in [n]$ s.t. $j \neq i$:
  - $\mathsf{TDOut}(\mathsf{td}_{1,j}, \mathsf{td}_{2,j}, \mathsf{td}_{3,j}) \neq 1$ where $\mathsf{td}_{2,j}$ is computed as in round 3. (OR)
  - $\mathsf{WZK}_4(\mathsf{wzk}_1^{j\to i}, \mathsf{wzk}_2^{i\to j}, \mathsf{wzk}_3^{j\to i}, \mathsf{st}_1^{j\to i}) \neq 1$ where $\mathsf{st}_1^{j\to i}$ is as defined above. (OR)
  - $\mathsf{RWI}_4(\mathsf{rwi}_1^{j\to i}, \mathsf{rwi}_2^{i\to j}, \mathsf{rwi}_3^{j\to i}, \mathsf{st}_2^{j\to i}) \neq 1$ where $\mathsf{st}_2^{j\to i}$ is as defined above.
- Compute $\mathsf{msg}_{3,i} \leftarrow \pi_3^{\mathsf{SM}}(\mathsf{x}_i, \mathsf{Trans}_2; \mathsf{r}_i)$, where Trans_2 denotes the round 2 transcript of π^{SM}.
- For each $j \in [n]$ with $j \neq i$, compute $\mathsf{wi}_3^{i\to j} \leftarrow \mathsf{WI}_3(\mathsf{wi}_1^{i\to j}, \mathsf{wi}_2^{j\to i}, \mathsf{st}_3^{i\to j}, \mathsf{w}_3^{i\to j})$ for the statement

$$\mathsf{st}_3^{i\to j} = (\{\mathsf{ecom}_{a,\ell}^{i\to j}\}_{\ell=1}^3, \{\mathsf{ecom}_{b,\ell}^{i\to j}\}_{\ell=1}^3, \mathsf{msg}_{3,i}, \mathsf{Trans}_2, \{\mathsf{nmcom}_\ell^{i\to j}\}_{\ell=1}^3, \mathsf{td}_{1,j}) \in L_3^{i\to j}$$

using witness $\mathsf{w}_3^{i\to j} = (\mathsf{x}_i, \mathsf{r}_i, \mathsf{r}_{a,\mathsf{ecom}}^{i\to j}, \bot, \bot, \bot)$.

- Broadcast $(\mathsf{msg}_{3,i}, \mathsf{wi}_3^{i\to j})$.

Output Computation: P_i does the following:

- Abort if for any $j \in [n]$ s.t. $j \neq i$:
  - $\mathsf{WI}_4(\mathsf{wi}_1^{j\to i}, \mathsf{wi}_2^{i\to j}, \mathsf{wi}_3^{j\to i}, \mathsf{st}_3^{j\to i}) \neq 1$ where $\mathsf{st}_3^{j\to i}$ is as defined above.
- Compute output $\mathsf{y}_i \leftarrow \mathsf{OUT}(\mathsf{x}_i, \mathsf{Trans}_3; \mathsf{r}_i)$, where Trans_3 denotes the round 3 transcript of π^{SM}.

This completes the description of protocol π.

7.4 Security Proof

In this section, we formally prove Theorem 12. Consider a malicious non-uniform PPT adversary $\mathcal{A}$ who corrupts $t < n$ parties. Let p be a polynomial such that $p(\lambda)$ denotes the total length of the input and randomness of each party P_i in protocol π^{SM}, i.e., $|(\mathsf{x}_i, \mathsf{r}_i)| = p(\lambda)$. We start by constructing an expected PPT black-box simulator Sim that simulates the view of $\mathcal{A}$.

7.4.1 Description of Simulator

Let us first recall the basic strategy followed by any black-box simulator who rewinds the adversary in order to simulate its view. Such a simulator creates a "main thread" of execution and a set of "look-ahead" threads. The main thread is the execution thread that is output at the end of the simulation, while the look-ahead threads facilitate the extraction of the adversary's inputs and some additional trapdoor information that is used to simulate the adversary's view on the main thread.

We construct an expected PPT black-box simulator Sim for protocol π. At a high-level, Sim works in the following two modes:

- **Case 1: Adversary does not abort in third round.** Suppose that the adversary does not cause all the honest parties to abort after the third round of π. In this case, Sim rewinds the adversary in the second and third round of π to create look-ahead threads. Thus, each look-ahead thread created by Sim shares the first round with the main thread, but contains different messages in the second and third rounds. Further, each look-ahead thread is terminated at the end of the third round. An important property of Sim is that it follows the honest party's strategy in all of the look ahead threads using input 0 for each honest party. Sim uses the adversary's messages in the third round of these look-ahead threads to extract the adversary's inputs as well as trapdoor information and then use them to simulate the main thread.
- **Case 2: Adversary aborts in third round.** In this case, Sim simply follows the honest party's strategy using input 0 on the main thread. It does not create any look-ahead threads.

To prevent a cluttered description, we overload notation when referring to the same object in the main thread and the look-ahead threads. However, it will be clear from context which thread's object is being referred to.

Simulator Sim. The simulator's description now follows:

Step 1 - Check Abort:

1. **Round 1:**
   For each honest party P_i, Sim follows the honest party algorithm in the first round. In more detail:
   - Compute $\mathsf{msg}_{1,i} \leftarrow \mathcal{S}_1(\mathsf{r}_i)$. Recall that similar to the honest party algorithm, this step simply consists of sampling a random string.
   - Compute $\mathsf{td}_{1,i} \leftarrow \mathsf{TDGen}_1(\mathsf{r}_{\mathsf{td},i})$ using a random string $\mathsf{r}_{\mathsf{td},i}$.
   - For each $j \in [n]$ with $j \neq i$, Sim does the following:
     - Compute $\mathsf{ecom}_{a,1}^{i\to j} \leftarrow \mathsf{ECom}_1(\mathsf{r}_{a,\mathsf{ecom}}^{i\to j})$, $\mathsf{ecom}_{b,1}^{i\to j} \leftarrow \mathsf{ECom}_1(\mathsf{r}_{b,\mathsf{ecom}}^{i\to j})$, and $\mathsf{nmcom}_1^{i\to j} \leftarrow \mathsf{NMCom}_1(\mathsf{r}_{\mathsf{nmcom}}^{i\to j})$ using random strings $\mathsf{r}_{a,\mathsf{ecom}}^{i\to j}$, $\mathsf{r}_{b,\mathsf{ecom}}^{i\to j}$ and $\mathsf{r}_{\mathsf{nmcom}}^{i\to j}$, respectively.
     - Generate $(\mathsf{wi}_1^{i\to j}) \leftarrow \mathsf{WI}_1(1^\lambda)$, $(\mathsf{rwi}_1^{i\to j}) \leftarrow \mathsf{RWI}_1(1^\lambda)$ and $(\mathsf{wzk}_1^{i\to j}) \leftarrow \mathsf{WZK}_1(1^\lambda)$.
   - Send $(\mathsf{msg}_{1,i}, \mathsf{ecom}_{a,1}^{i\to j}, \mathsf{ecom}_{b,1}^{i\to j}, \mathsf{nmcom}_1^{i\to j}, \mathsf{td}_{1,i}, \mathsf{wi}_1^{i\to j}, \mathsf{rwi}_1^{i\to j}, \mathsf{wzk}_1^{i\to j})$ to $\mathcal{A}$.
2. **Round 2:**
   The simulator continues to follow the honest party algorithm in the second round. That is, for each honest party P_i and for each $j \in [n]$ with $j \neq i$, Sim does the following:

- Compute $\mathsf{ecom}_{a,2}^{i\to j} \leftarrow \mathsf{ECom}_2(\mathsf{ecom}_{a,1}^{j\to i})$, $\mathsf{ecom}_{b,2}^{i\to j} \leftarrow \mathsf{ECom}_2(\mathsf{ecom}_{b,1}^{j\to i})$ and $\mathsf{nmcom}_2^{i\to j} \leftarrow \mathsf{NMCom}_2(\mathsf{nmcom}_1^{j\to i})$.
- Compute $\mathsf{td}_2^{i\to j} \leftarrow \mathsf{TDGen}_2(\mathsf{td}_{1,i})$.
- Compute $\mathsf{wi}_2^{i\to j} \leftarrow \mathsf{WI}_2(\mathsf{wi}_1^{j\to i}), \mathsf{rwi}_2^{i\to j} \leftarrow \mathsf{RWI}_2(\mathsf{rwi}_1^{j\to i})$ and $\mathsf{wzk}_2^{i\to j} \leftarrow \mathsf{WZK}_2(\mathsf{wzk}_1^{j\to i})$.
- Send $(\mathsf{ecom}_{a,2}^{i\to j}, \mathsf{ecom}_{b,2}^{i\to j}, \mathsf{nmcom}_2^{i\to j}, \mathsf{td}_2^{i\to j}, \mathsf{wi}_2^{i\to j}, \mathsf{rwi}_2^{i\to j}, \mathsf{wzk}_2^{i\to j})$ to $\mathcal{A}$.

3. **Round 3:**
   For each honest party P_i, Sim does the following:

   - Compute $\mathsf{msg}_{2,i}$ by executing the honest party algorithm with private input set to 0 and randomness r_ia.
   - Compute $\mathsf{nc}_i \leftarrow \mathsf{Com}(1; \mathsf{r}_{\mathsf{nc},i})$.
   - Let $\mathsf{td}_{2,i} = (\mathsf{td}_2^{1\to i}||\ldots||\mathsf{td}_2^{n\to i})$ with $\mathsf{td}_2^{i\to i} = \perp$. Compute the third message of the trapdoor generation protocol, i.e., $\mathsf{td}_{3,i} \leftarrow \mathsf{TDGen}_3(\mathsf{td}_{1,i}, \mathsf{td}_{2,i}; \mathsf{r}_{\mathsf{td},i})$ using randomness $\mathsf{r}_{\mathsf{td},i}$.
   - For each $j \in [n]$ with $j \neq i$, compute:
     - $\mathsf{ecom}_{a,3}^{i\to j} \leftarrow \mathsf{ECom}_3(0, \mathsf{r}_i, \mathsf{ecom}_{a,1}^{i\to j}, \mathsf{ecom}_{a,2}^{j\to i}; \mathsf{r}_{a,\mathsf{ecom}}^{i\to j})$, $\mathsf{ecom}_{b,3}^{i\to j} \leftarrow \mathsf{ECom}_3(\perp, \mathsf{ecom}_{b,1}^{i\to j}, \mathsf{ecom}_{b,2}^{j\to i}; \mathsf{r}_{b,\mathsf{ecom}}^{i\to j})$, and $\mathsf{nmcom}_3^{i\to j} \leftarrow \mathsf{NMCom}_3(\perp, \mathsf{nmcom}_1^{i\to j}, \mathsf{nmcom}_2^{j\to i}; \mathsf{r}_{\mathsf{nmcom}}^{i\to j})$ using random strings $\mathsf{r}_{a,\mathsf{ecom}}^{i\to j}$, $\mathsf{r}_{b,\mathsf{ecom}}^{i\to j}$ and $\mathsf{r}_{\mathsf{nmcom}}^{i\to j}$, respectively.
     - $\mathsf{rwi}_3^{i\to j} \leftarrow \mathsf{RWI}_3(\mathsf{rwi}_1^{i\to j}, \mathsf{rwi}_2^{j\to i}, \mathsf{st}_2^{i\to j}, \mathsf{w}_2^{i\to j})$ for the statement

       $$\mathsf{st}_2^{i\to j} = \left(\{\mathsf{ecom}_{a,\ell}^{i\to j}\}_{\ell=1}^3, \{\mathsf{ecom}_{b,\ell}^{i\to j}\}_{\ell=1}^3, \mathsf{msg}_{2,i}, \mathsf{Trans}_1, \{\mathsf{nmcom}_\ell^{i\to j}\}_{\ell=1}^3, \mathsf{td}_{1,j}, \mathsf{nc}_i\right) \in L_2^{i\to j}$$

       using witness $\mathsf{w}_2^{i\to j} = (0, \mathsf{r}_i, \mathsf{r}_{a,\mathsf{ecom}}^{i\to j}, \perp, \perp, \perp, \perp)$.
     - $\mathsf{wzk}_3^{i\to j} \leftarrow \mathsf{WZK}_3(\mathsf{wzk}_1^{i\to j}, \mathsf{wzk}_2^{j\to i}, \mathsf{st}_1^{i\to j}, \mathsf{w}_1^{i\to j})$ for the statement $\mathsf{st}_1^{i\to j} = \mathsf{nc}_1^{i\to j} \in L_1^{i\to j}$ using witness $\mathsf{w}_1^{i\to j} = (\mathsf{r}_{\mathsf{nc},i})$.
   - Send $(\mathsf{msg}_{2,i}, \mathsf{nc}_i, \mathsf{ecom}_{a,3}^{i\to j}, \mathsf{ecom}_{b,3}^{i\to j}, \mathsf{nmcom}_3^{i\to j}, \mathsf{td}_{3,i}, \mathsf{rwi}_3^{i\to j}, \mathsf{wzk}_3^{i\to j})$ to $\mathcal{A}$.

4. **Check Abort Condition:**
   Simulator now checks whether $\mathcal{A}$ aborted in the third round. That is, Sim sets $\mathsf{flag} = \mathtt{abort}$ if: for every honest party P_i, there exists a malicious party P_j such that,

   - $\mathsf{TDOut}(\mathsf{td}_{1,j}, \mathsf{td}_{2,j}, \mathsf{td}_{3,j}) \neq 1$ where $\mathsf{td}_{2,j}$ is computed as in round 3. (OR)
   - $\mathsf{WZK}_4(\mathsf{wzk}_1^{j\to i}, \mathsf{wzk}_2^{i\to j}, \mathsf{wzk}_3^{j\to i}, \mathsf{st}_1^{j\to i}) \neq 1$ where $\mathsf{st}_1^{j\to i} = \mathsf{nc}_1^{j\to i}$. (OR)
   - $\mathsf{RWI}_4(\mathsf{rwi}_1^{i\to j}, \mathsf{rwi}_2^{i\to j}, \mathsf{rwi}_3^{j\to i}, \mathsf{st}_2^{j\to i}) \neq 1$ where $\mathsf{st}_2^{j\to i}$ is as defined above.

If $\mathsf{flag} = \mathtt{abort}$, then Sim outputs the partial view generated so far and stops. Otherwise, we will say that the "Check Abort" step succeeded. In this case, Sim proceeds to the next step.

Step 2 - Rewinding:

- Sim now rewinds $\mathcal{A}$ to the end of round 1 and freezes the main thread at this point. Then, Sim creates a set of T (defined later) look-ahead threads, where on each thread, only rounds 2 and 3 of the protocol are executed in the following manner:

5. **Round 2:**
   In every look-ahead thread, for each honest party P_i and for each $j \in [n]$ with $j \neq i$, Sim executes the same strategy as in round 2 of step 1, using fresh randomness.
6. **Round 3:**
   In every look-ahead thread, for each honest party P_i and for each $j \in [n]$ with $j \neq i$, Sim executes the same strategy as in round 3 of step 1, using fresh randomness.

- For each thread above, define it to be BAD if the "Check Abort" step fails (otherwise, we call it GOOD). That is, a thread is called BAD if for every honest party P_i, there exists a malicious party P_j such that:
  - $\mathsf{TDOut}(\mathsf{td}_{1,j}, \mathsf{td}_{2,j}, \mathsf{td}_{3,j}) \neq 1$ where $\mathsf{td}_{2,j}$ is computed as in round 3. (OR)
  - $\mathsf{WZK}_4(\mathsf{wzk}_1^{j\to i}, \mathsf{wzk}_2^{i\to j}, \mathsf{wzk}_3^{j\to i}, \mathsf{st}_1^{j\to i}) \neq 1$ where $\mathsf{st}_1^{j\to i} = \mathsf{nc}_1^{j\to i}$. (OR)
  - $\mathsf{RWI}_4(\mathsf{rwi}_1^{i\to j}, \mathsf{rwi}_2^{i\to j}, \mathsf{rwi}_3^{j\to i}, \mathsf{st}_2^{j\to i}) \neq 1$ where $\mathsf{st}_2^{j\to i}$ is as defined earlier.
- Alternately, a thread is defined to be "GOOD with respect to P_{i^*}" if, for all malicious parties P_j:
  - $\mathsf{TDOut}(\mathsf{td}_{1,j}, \mathsf{td}_{2,j}, \mathsf{td}_{3,j}) = 1$ where $\mathsf{td}_{2,j}$ is computed as in round 3. (AND)
  - $\mathsf{WZK}_4(\mathsf{wzk}_1^{j\to i^*}, \mathsf{wzk}_2^{i^*\to j}, \mathsf{wzk}_3^{j\to i^*}, \mathsf{st}_1^{j\to i^*}) = 1$ where $\mathsf{st}_1^{j\to i^*} = \mathsf{nc}_1^{j\to i^*}$. (AND)
  - $\mathsf{RWI}_4(\mathsf{rwi}_1^{i^*\to j}, \mathsf{rwi}_2^{i^*\to j}, \mathsf{rwi}_3^{j\to i^*}, \mathsf{st}_2^{j\to i^*}) = 1$ where $\mathsf{st}_2^{j\to i^*}$ is as defined earlier.
- The number of threads T created is such that at least $(12 \cdot \lambda)$ GOOD threads exist. That is, Sim keeps running till it obtains $(12 \cdot \lambda)$ GOOD threads.

Step 3 - Input and Trapdoor Extraction:
Sim does the following:

- Select 5 threads that are "GOOD with respect to P_{i^*}" for some honest party P_{i^*}. That is, in each GOOD thread, we know that $\exists$ an honest party P_i such that for all malicious parties P_j, the adversary does not cause P_i to abort. Since $(12 \cdot \lambda) > (5 \cdot n)$, (without loss of generality, assuming the number of parties $n = \lambda$), there must exist one honest party P_i^* corresponding to a set of 5 GOOD threads.
- **Trapdoor Extraction:** For every corrupted party P_j, extract a trapdoor t_j by running the trapdoor extractor TDExt on input the transcripts of the trapdoor generation protocol with P_j playing the role of sender, a.k.a. trapdoor generator from any 3 GOOD threads. That is, compute $\mathsf{t}_j \leftarrow \mathsf{TDExt}\Big(\mathsf{td}_{1,j}, \{\mathsf{td}_{2,j}, \mathsf{td}_{3,j}\}_{\ell=1}^{3}\Big)$, where $(\mathsf{td}_{1,j}, \mathsf{td}_{2,j}, \mathsf{td}_{3,j})$ denotes the transcript of the trapdoor generation protocol with P_j as the sender on the ℓ^{th} GOOD thread.
- **Input Extraction:** For every corrupted party P_j, extract an input and randomness pair $(\mathsf{x}_{a,j}, \mathsf{r}_{a,j})$ by running the extractor $\mathsf{Ext}_{\mathsf{ECom}}$ on input the transcripts of the first extractable commitment protocol between P_j and P_{i^*} from the 5 GOOD threads picked above. That is, compute:
$$(\mathsf{x}_{a,j}, \mathsf{r}_{a,j}) \leftarrow \mathsf{Ext}_{\mathsf{ECom}}\Big(\mathsf{ecom}_{a,1}^{j\to i^*}, \{\mathsf{ecom}_{a,2,\ell}^{i^*\to j}, \mathsf{ecom}_{a,3,\ell}^{j\to i^*}\}_{\ell=1}^{5}\Big),$$
where $(\mathsf{ecom}_{a,1}^{j\to i^*}, \mathsf{ecom}_{a,2,\ell}^{i^*\to j}, \mathsf{ecom}_{a,3,\ell}^{j\to i^*})$ denotes the transcript of the first extractable commitment protocol between P_j and P_{i^*} on the ℓ^{th} GOOD thread. Similarly, compute $(\mathsf{x}_{b,j}, \mathsf{r}_{b,j})$

by running the extractor $\mathsf{Ext}_{\mathsf{ECom}}$ on input the transcripts of the second extractable commitment protocol between P_j and P_{i^*} from those 5 GOOD threads. Check which of $(\mathsf{x}_{a,j}, \mathsf{r}_{a,j})$ and $(\mathsf{x}_{b,j}, \mathsf{r}_{b,j})$ are consistent with the (partial) transcript of π^{SM} and set that pair to be $(\mathsf{x}_j, \mathsf{r}_j)$. Let R denote the set of all $\{\mathsf{x}_j, \mathsf{r}_j\}$ pairs extracted.

- Output "Special Abort" if any of the above two steps fail.

Step 4 - Query to Ideal Functionality:

- Sim queries the ideal functionality with the set of values $\{\mathsf{x}_j\}$ where x_j is the input of adversarial party P_j that was extracted in the previous step.
- Sim receives output y from the ideal functionality.

Step 5 - Abort Probability Estimation:
Set $\epsilon' = \frac{12\lambda}{T}$ as the probability with which the adversary doesn't abort (Recall that T is the number of threads created in Step 2).

Step 6 - Resampling the Main Thread:
First, Sim sets a counter value to 0. Sim now continues the main thread from the end of round 1. (Note that this step also involves rewinding of its own as elaborated below.)

7. **Round 2:**
   Run exactly as before - i.e. as done in Step 1.

8. **Round 3:**
   Now, there are two key differences from the threads generated in previous steps:

   - The first is that in the non-malleable commitment execution between honest party i and adversarial party j, Sim commits to the trapdoor $\mathsf{t}^{j\to i}$ (extracted above).
   - The second is that the Simulator invokes the semi-malicious simulator of the protocol to generate messages on behalf of honest parties, instead of committing to 0s.

   In more detail, for each honest P_i, Sim does the following:

   - Compute $\mathsf{msg}_{2,i} \leftarrow \mathcal{S}_2(\mathsf{Trans}_1; \mathsf{r}_i)$, where Trans_1 is the round 1 transcript of π^{SM}.
   - Let $\mathsf{td}_{2,i} = (\mathsf{td}_2^{1\to i} || \ldots || \mathsf{td}_2^{n\to i})$ with $\mathsf{td}_2^{i\to i} = 0$. Compute the third message of the trapdoor generation protocol, i.e., $\mathsf{td}_{3,i} \leftarrow \mathsf{TDGen}_3(\mathsf{td}_{1,i}, \mathsf{td}_{2,i}; \mathsf{r}_{\mathsf{td},i})$ using randomness $\mathsf{r}_{\mathsf{td},i}$.
   - Compute $\mathsf{nc}_i \leftarrow \mathsf{Com}(1; \mathsf{r}_{\mathsf{nc},i})$.
   - For each $j \in [n]$ with $j \neq i$, compute:
     - Compute $\mathsf{ecom}_{a,3}^{i\to j} \leftarrow \mathsf{ECom}_3(0, \mathsf{r}_i, \mathsf{ecom}_{a,1}^{i\to j}, \mathsf{ecom}_{a,2}^{j\to i}; \mathsf{r}_{a,\mathsf{ecom}}^{i\to j})$, $\mathsf{ecom}_{b,3}^{i\to j} \leftarrow \mathsf{ECom}_3(\perp, \mathsf{ecom}_{b,1}^{i\to j}, \mathsf{ecom}_{b,2}^{j\to i}; \mathsf{r}_{b,\mathsf{ecom}}^{i\to j})$, and $\mathsf{nmcom}_3^{i\to j} \leftarrow \mathsf{NMCom}_3(\mathsf{t}_j, \mathsf{nmcom}_1^{i\to j}, \mathsf{nmcom}_2^{j\to i}; \mathsf{r}_{\mathsf{nmcom}}^{i\to j})$ using random strings $\mathsf{r}_{a,\mathsf{ecom}}^{i\to j}$, $\mathsf{r}_{b,\mathsf{ecom}}^{i\to j}$ and $\mathsf{r}_{\mathsf{nmcom}}^{i\to j}$, respectively.
     - $\mathsf{wzk}_3^{i\to j} \leftarrow \mathsf{WZK}_3(\mathsf{wzk}_1^{i\to j}, \mathsf{wzk}_2^{j\to i}, \mathsf{st}_1^{i\to j}, \mathsf{w}_1^{i\to j})$ for the statement $\mathsf{st}_1^{i\to j} = \mathsf{nc}_1^{i\to j} \in L_1^{i\to j}$ using witness $\mathsf{w}_1^{i\to j} = (\mathsf{r}_{\mathsf{nc},i})$.
     - $\mathsf{rwi}_3^{i\to j} \leftarrow \mathsf{RWI}_3(\mathsf{rwi}_1^{i\to j}, \mathsf{rwi}_2^{j\to i}, \mathsf{st}_2^{i\to j}, \mathsf{w}_2^{i\to j})$ for the statement $\mathsf{st}_2^{i\to j}$ as defined earlier, using witness $\mathsf{w}_2^{i\to j} = (\perp, \perp, \perp, \perp, \mathsf{t}_j, \mathsf{r}_{\mathsf{nmcom}}^{i\to j}, \perp)$.

- Send $(\mathsf{msg}_{2,i}, \mathsf{nc}_i, \mathsf{ecom}_{a,3}^{i\to j}, \mathsf{ecom}_{b,3}^{i\to j}, \mathsf{nmcom}_3^{i\to j}, \mathsf{td}_{3,i}, \mathsf{rwi}_3^{i\to j})$ to $\mathcal{A}$.

9. **Abort Condition:**
   For each honest party P_i, Sim does the following:

   - For each $j \in [n]$ with $j \neq i$, increase the counter value by 1 if
     - $\mathsf{TDOut}(\mathsf{td}_{1,j}, \mathsf{td}_{2,j}, \mathsf{td}_{3,j}) \neq 1$ where $\mathsf{td}_{2,j}$ is computed as in round 3. (OR)
     - $\mathsf{WZK}_4(\mathsf{wzk}_1^{j\to i}, \mathsf{wzk}_2^{i\to j}, \mathsf{wzk}_3^{j\to i}, \mathsf{st}_1^{j\to i}) \neq 1$ where $\mathsf{st}_1^{j\to i} = \mathsf{nc}_1^{j\to i}$. (OR)
     - $\mathsf{RWI}_4(\mathsf{rwi}_1^{i\to j}, \mathsf{rwi}_2^{i\to j}, \mathsf{rwi}_3^{j\to i}, \mathsf{st}_2^{j\to i}) \neq 1$ where $\mathsf{st}_2^{j\to i}$ is as defined earlier.
   - If Sim's running time equals 2^λ, Abort.
   - If the counter value was not increased, continue to Step 7.
   - Else, if counter value is less than $\frac{\lambda^2}{\epsilon^r}$, rewind back to the beginning of round 2 in Step 6 and resample the main thread. Abort otherwise.

Step 7 - Finishing the Main Thread:

10. **Round 4:**
    In the main thread, for each honest party P_i, Sim does the following:

    - Compute $\mathsf{msg}_{3,i} \leftarrow \mathcal{S}_3(\mathsf{y}, \mathsf{R}, \mathsf{Trans}_2, i)$, where Trans_2 denotes the round 2 transcript of protocol π^{SM}.
    - For each $j \in [n]$ with $j \neq i$,
      Compute $\mathsf{wi}_3^{i\to j} \leftarrow \mathsf{WI}_3(\mathsf{wi}_1^{i\to j}, \mathsf{wi}_2^{j\to i}, \mathsf{st}_3^{i\to j}, \mathsf{w}_3^{i\to j})$ for the statement

      $$\mathsf{st}_3^{i\to j} = (\{\mathsf{ecom}_{a,\ell}^{i\to j}\}_{\ell=1}^3, \{\mathsf{ecom}_{b,\ell}^{i\to j}\}_{\ell=1}^3, \mathsf{msg}_{3,i}, \mathsf{Trans}_2, \{\mathsf{nmcom}_\ell^{i\to j}\}_{\ell=1}^3, \mathsf{td}_{1,j}) \in L_3^{i\to j}$$

      using witness $\mathsf{w}_3^{i\to j} = (\bot, \bot, \bot, \bot, \mathsf{t}_j, \mathsf{r}_{\mathsf{nmcom}}^{i\to j})$.
    - Send $(\mathsf{msg}_{3,i}, \mathsf{wi}_3^{i\to j})$ to $\mathcal{A}$.

11. **Output Computation:**
    Let Trans_3 denote the transcript of protocol π^{SM} after round 3. In the main thread, for each honest party P_i and every $j \in [n]$ with $j \neq i$, Sim does the following:

    - Abort if $\mathsf{WI}_4(\mathsf{wi}_1^{j\to i}, \mathsf{wi}_2^{i\to j}, \mathsf{wi}_3^{j\to i}, \mathsf{st}_3^{j\to i}) \neq 1$ where $\mathsf{st}_3^{j\to i}$ is as defined above. Also abort if any adversarial party aborted in the 4th round.

    In case of no abort, instruct the ideal functionality to deliver output to the honest parties.

Running Time. We now prove that the simulator is an expected PPT machine.

Claim 16. *Simulator* Sim *runs in expected time that is polynomial in* λ.

Proof. We analyze the running time of each step performed by Sim:

- It is easy to see that Step 1 ("Check Abort") takes only $\mathsf{poly}(\lambda)$ time for some polynomial.
- Let ϵ be the probability with which "Check Abort" step succeeds. That is, Sim proceeds to Step 2 only with probability ϵ. Now, since the probability of the adversary not aborting is ϵ, the expected number of threads to be run by the simulator to get one non-aborting transcript is $\frac{1}{\epsilon}$. Therefore, the expected total number of threads created in Step 2 is $\frac{12\cdot\lambda}{\epsilon}$ and each thread takes only $\mathsf{poly}(\lambda)$ time.

- In Step 3, Sim simply runs the extractors TDExt and $\mathsf{Ext}_{\mathsf{ECom}}$ that are both PPT machines.
- Steps 4 and 5 are trivially polynomial time.
- As shown in [GK96, Lin17], the probability that the estimate ϵ' computed in Step 5 is not within a factor of 2 of ϵ is at most 2^{λ}. An exact computation of how to achieve this exact bound using Chernoff bounds can be found in [HL10], Section 6.5.3. (which also explains why we chose to run Step 2 till we get $12 \cdot \lambda$ non-aborting transcripts). Therefore, the number of threads created in step 6 is at most $\frac{\lambda^2}{\epsilon}$ (ignoring the constant factor). Note that Step 6 might still take time 2^{λ} but this happens only when the estimate of ϵ' is incorrect: that is, when ϵ' is not within a constant factor of ϵ and this happens only with probability $2^{-\lambda}$.
- Finally, it is easy to see that Step 7 runs in $\mathsf{poly}(\lambda)$.

Therefore, we can bound the overall running time by :

$$\mathsf{T}_{\mathsf{Sim}} = \mathsf{poly}(\lambda) + \mathsf{poly}(\lambda) \cdot \epsilon \left(\frac{12 \cdot \lambda}{\epsilon} + \left(1 - \frac{1}{2^{\lambda}}\right) \cdot \frac{\lambda^2}{\epsilon} + \left(\frac{1}{2^{\lambda}}\right) \cdot 2^{\lambda} \right)$$

$$\leq \mathsf{poly}(\lambda).$$

This concludes the analysis. □

7.4.2 Hybrids

We now show that the above simulation strategy is successful against all malicious PPT adversaries. That is, the view of the adversary along with the output of the honest parties is computationally indistinguishable in the real and ideal worlds. We will show this via a series of computationally indistinguishable hybrids where the first hybrid $\mathsf{Hyb}_{\mathsf{Real}}$ corresponds to the real world and the last hybrid $\mathsf{Hyb}_{\mathsf{Ideal}}$ corresponds to the ideal world.

First, assume by contradiction that there exists an adversary $\mathcal{A}$ that can distinguish the real and ideal worlds with some non-negligible probability μ. We will use this value μ in the hybrids.

- $\mathsf{Hyb}_{\mathsf{Real}}$ **- Real World:** In this hybrid, consider a simulator $\mathsf{Sim}_{\mathsf{Hyb}}$ that plays the role of the honest parties. This corresponds to the real world experiment.
- Hyb_0 **- Determining Abort in 3rd Round and Extraction:**

  In this hybrid, $\mathsf{Sim}_{\mathsf{Hyb}}$ does the following:

  1. First runs the "Check Abort" step (Step 1 in the description of Sim) to check if the adversary aborts. That is, $\mathsf{Sim}_{\mathsf{Hyb}}$ executes the first 3 rounds of the protocol using the honest parties' strategy. If the adversary does cause an abort, then $\mathsf{Sim}_{\mathsf{Hyb}}$ simply outputs the view of the adversary and stops. *In this case, the rest of the hybrids are skipped.*
  2. Otherwise, if the adversary doesn't cause an abort (i.e., the "Check Abort" step succeeded), then, $\mathsf{Sim}_{\mathsf{Hyb}}$ rewinds back to the end of round 1 of the protocol and freezes the main thread. Then, $\mathsf{Sim}_{\mathsf{Hyb}}$ creates a set of $\frac{5 \cdot n \cdot \lambda}{\mu}$ look-ahead threads in a similar manner as described in Step 2 of Sim. In all the threads, $\mathsf{Sim}_{\mathsf{Hyb}}$ plays the role of the honest parties exactly as in $\mathsf{Hyb}_{\mathsf{Real}}$.

3. $\mathsf{Sim}_{\mathsf{Hyb}}$ also runs the "Input and Trapdoor Extraction" phase and "Query to Ideal Functionality" phase described in steps 3 and 4 of the description of Sim using the first 5 look-ahead threads that are "GOOD with respect to P_{i^*}" for some honest party P_{i^*}. That is, it extracts the adversary's input, randomness and a set of valid trapdoors and also, queries the ideal functionality using the adversary's inputs to receive the function output.
4. $\mathsf{Sim}_{\mathsf{Hyb}}$ outputs "Special Abort" if the above step fails.
5. Then, $\mathsf{Sim}_{\mathsf{Hyb}}$ continues executing the main thread from the end of round 1 of the protocol and plays the role of the honest parties exactly as in $\mathsf{Hyb}_{\mathsf{Real}}$. If the adversary causes an abort, $\mathsf{Sim}_{\mathsf{Hyb}}$ goes back to the end of round 1 and resamples the main thread exactly as in the last sentence (playing the role of the honest parties). This process is repeated at most $\frac{\lambda}{\mu}$ times.

Observe that $\mathsf{Sim}_{\mathsf{Hyb}}$ runs in polynomial time because, by assumption, μ was non-negligible. (The same argument will hold in the subsequent hybrids as well.)

The remaining hybrids are only considered if the "Check Abort" step succeeded.

- Hyb_1 **- Using input 0 in the Aborting Step:** In this hybrid, $\mathsf{Sim}_{\mathsf{Hyb}}$ runs the "Check Abort" step using the honest parties' strategy *but using input* 0. If the adversary does cause an abort, then $\mathsf{Sim}_{\mathsf{Hyb}}$ simply outputs the view of the adversary and stops.

  Otherwise, if the adversary doesn't cause an abort (i.e., the "Check Abort" step succeeded), then, $\mathsf{Sim}_{\mathsf{Hyb}}$ proceeds as in Hyb_0.

  Once again, the remaining hybrids are only considered if the "Check Abort" step succeeded.

- Hyb_2 **- Using input 0 in the look-ahead threads:** In this hybrid, in each look-ahead thread, $\mathsf{Sim}_{\mathsf{Hyb}}$ runs the honest parties' strategy *but using input* 0.

- Hyb_3 **- Changing NMCom on main thread:** In the main thread, for each honest party P_i and every malicious party P_j, $\mathsf{Sim}_{\mathsf{Hyb}}$ does the following: in round 3, compute $\mathsf{nmcom}_3^{i\to j} \leftarrow \mathsf{NMCom}_3(\mathsf{t}_j, \mathsf{nmcom}_1^{i\to j}, \mathsf{nmcom}_2^{j\to i}; \mathsf{r}_{\mathsf{nmcom}}^{i\to j})$ where t_j is a valid trapdoor extracted as in the previous hybrid. That is, the simulator now commits to a trapdoor t_j in the non-malleable commitment between P_i (committer) and P_j (receiver).

  Note: The above change happens in each of the "resamples" to generate the main thread till $\mathsf{Sim}_{\mathsf{Hyb}}$ receives a non-aborting transcript after round 3 or the number of resamples reaches $\frac{\lambda}{\mu}$.

- Hyb_4 **- Switching RWI proofs on main thread:** In the main thread, for each honest party P_i and every malicious party P_j, $\mathsf{Sim}_{\mathsf{Hyb}}$ does the following: in round 3, compute $\mathsf{rwi}_2^{i\to j} \leftarrow \mathsf{RWI}_2(\mathsf{rwi}_1^{j\to i}, \mathsf{st}_2^{i\to j}, \mathsf{w}_2^{i\to j})$ for the statement

$$\mathsf{st}_2^{i\to j} = \left(\{\mathsf{ecom}_{a,\ell}^{i\to j}\}_{\ell=1}^3, \{\mathsf{ecom}_{b,\ell}^{i\to j}\}_{\ell=1}^3, \mathsf{msg}_{2,i}, \mathsf{Trans}_1, \{\mathsf{nmcom}_\ell^{i\to j}\}_{\ell=1}^3, \mathsf{td}_{1,j}, \mathsf{nc}_i\right) \in L_2^{i\to j}$$

  using witness $\mathsf{w}_2^{i\to j} = (\bot, \bot, \bot, \bot, \mathsf{t}_j, \mathsf{r}_{\mathsf{nmcom}}^{i\to j}, \bot)$.
  That is, in the main thread, the simulator now uses the "trapdoor" witness (which establishes that it committed to a valid trapdoor in the non-malleable commitment) to compute the last message of each RWI proof where an honest party plays the role of the prover.

- Hyb_5 **- Switching WI proofs on main thread:** In the main thread, for each honest party P_i and every malicious party P_j, $\mathsf{Sim}_{\mathsf{Hyb}}$ does the following: in round 4, compute $\mathsf{wi}_3^{i\to j} \leftarrow \mathsf{WI}_3(\mathsf{wi}_1^{i\to j}, \mathsf{wi}_2^{j\to i}, \mathsf{st}_3^{i\to j}, \mathsf{w}_3^{i\to j})$ for the statement

$$\mathsf{st}_3^{i\to j} = (\{\mathsf{ecom}_{a,\ell}^{i\to j}\}_{\ell=1}^3, \{\mathsf{ecom}_{b,\ell}^{i\to j}\}_{\ell=1}^3, \mathsf{msg}_{3,i}, \mathsf{Trans}_2, \{\mathsf{nmcom}_{\ell}^{i\to j}\}_{\ell=1}^3, \mathsf{td}_{1,j}) \in L_3^{i\to j}$$

  using witness $\mathsf{w}_3^{i\to j} = (\bot, \bot, \bot, \bot, \mathsf{t}_j, \mathsf{r}_{\mathsf{nmcom}}^{i\to j})$.
  That is, in the main thread, the simulator now uses the "trapdoor" witness (which establishes that it committed to a valid trapdoor in the non-malleable commitment) to compute the last message of each WI proof where an honest party plays the role of the prover.

- Hyb_6 **- Changing EComs on main thread:** In the main thread, for each honest party P_i and every malicious party P_j, $\mathsf{Sim}_{\mathsf{Hyb}}$ does the following: in round 3, compute $\mathsf{ecom}_{a,3}^{i\to j} = \mathsf{ECom}_3(0, \mathsf{r}_i, \mathsf{ecom}_{a,1}^{i\to j}, \mathsf{ecom}_{a,2}^{j\to i}; \mathsf{r}_{a,\mathsf{ecom}}^{i\to j})$.
  That is, in the main thread, the simulator now commits to input 0 in the extractable commitment scheme.

- Hyb_7 **- Simulate π^{SM} on main thread:** In the main thread, for each honest party P_i, $\mathsf{Sim}_{\mathsf{Hyb}}$ does the following:

  - in round 1, compute $\mathsf{msg}_{1,i} \leftarrow \mathcal{S}_1(\mathsf{r}_i)$. The description of $\mathsf{Sim}_{\mathsf{Hyb}}$ now corresponds to the ideal world simulator Sim.
  - In round 3, compute $\mathsf{msg}_{2,i} \leftarrow \mathcal{S}_2(\mathsf{Trans}_1; \mathsf{r}_i)$ where Trans_1 denotes the transcript of protocol π^{SM} after round 1.
  - in round 4, compute $\mathsf{msg}_{3,i} \leftarrow \mathcal{S}_3(\mathsf{y}, \mathsf{R}, \mathsf{Trans}_2, i)$ where y is the output from the ideal functionality, R denotes the set of all $\{\mathsf{x}_j, \mathsf{r}_j\}$ extracted and Trans_2 denotes the transcript of protocol π^{SM} after round 2.

- $\mathsf{Hyb}_{\mathsf{Ideal}}$ **- Run the actual probability estimation:** In this hybrid, the number of look-ahead threads is increased from $\frac{5\cdot n\cdot\lambda}{\mu}$ to as many as needed to estimate the probability of the adversary not aborting - ϵ'.

  Additionally, at this point, $\mathsf{Sim}_{\mathsf{Hyb}}$ doesn't resample the main thread $\frac{\lambda}{\mu}$ times. Instead, $\mathsf{Sim}_{\mathsf{Hyb}}$ resamples the main thread for $\mathsf{min}(2^\lambda, \frac{\lambda^2}{\epsilon'})$ times as in the ideal world. This hybrid corresponds exactly to the ideal world.

7.4.3 Indistinguishability of hybrids

Throughout the sequence of hybrids, starting with $\mathsf{Hyb}_{\mathsf{Real}}$, we will maintain the following invariant which will be useful to argue the proof of indistinguishability.

Definition 16 (Invariant)**.** *Consider any malicious party P_j and any honest party P_i. $\mathsf{td}_{1,i}$ denotes the first message of the trapdoor generation protocol with P_i as the trapdoor generator. The tuple $(\mathsf{nmcom}_1^{j\to i}, \mathsf{nmcom}_2^{i\to j}, \mathsf{nmcom}_3^{j\to i})$ denote the messages of the non-malleable commitment with P_j as the committer.*
Consider the following event E which occurs if $\exists(j, i, \mathsf{t}_i, \mathsf{r}_{\mathsf{nmcom}}^{j\to i})$ such that:

- $\mathsf{nmcom}_1^{j\to i} = \mathsf{NMCom}_1(\mathsf{r}_{\mathsf{nmcom}}^{j\to i})$ *(AND)*

- $\mathsf{nmcom}_3^{j\to i} = \mathsf{NMCom}_3(\mathsf{t}_i, \mathsf{nmcom}_1^{j\to i}, \mathsf{nmcom}_2^{i\to j}; \mathsf{r}_{\mathsf{nmcom}}^{j\to i})$. *(AND)*
- $\mathsf{TDValid}(\mathsf{td}_{1,i}, \mathsf{t}_i) = 1$.

That is, the event E *occurs if any corrupted party* P_j *commits to a valid trapdoor* t_i *(corresponding to the trapdoor generation protocol where* P_i *was the trapdoor generator) in the non-malleable commitment protocol with* P_i*.*

The invariant is : $\mathsf{Pr}[$ *Event* E *occurs* $] \leq \mathsf{negl}(\lambda)$.

Claim 17. *Assuming the "1-rewinding security" of the trapdoor generation protocol* TDGen *and the existence of an extractor* $\mathsf{Ext}_{\mathsf{NMCom}}$ *for the non-malleable commitment scheme* NMCom*, the invariant holds in* $\mathsf{Hyb}_{\mathsf{Real}}$.

Proof. We will prove this by contradiction. Assume that the invariant doesn't hold in $\mathsf{Hyb}_{\mathsf{Real}}$. That is, there exists an adversary $\mathcal{A}$ such that for some honest party P_i^* and malicious party P_j^*, $\mathcal{A}$ causes event E to occur with non-negligible probability. We will use this adversary to design an adversary $\mathcal{A}_{\mathsf{TDGen}}$ that breaks the "1-rewinding security" of the trapdoor generation protocol TDGen as defined in Section 4 with non-negligible probability.

$\mathcal{A}_{\mathsf{TDGen}}$ interacts with a challenger $\mathcal{C}_{\mathsf{TDGen}}$ and receives a first round message td_1 corresponding to the protocol TDGen. $\mathcal{A}_{\mathsf{TDGen}}$ performs the role of $\mathsf{Sim}_{\mathsf{Hyb}}$ in its interaction with $\mathcal{A}$ exactly as done in $\mathsf{Hyb}_{\mathsf{Real}}$. $\mathsf{Sim}_{\mathsf{Hyb}}$ picks an honest party P_i uniformly at random and a malicious party P_j uniformly at random. Now, in round 1 of protocol π, $\mathcal{A}_{\mathsf{TDGen}}$ sets $\mathsf{td}_{1,i}$ as td_1 received from $\mathcal{C}_{\mathsf{TDGen}}$. On receiving all the values $\mathsf{td}_2^{1\to i} \ldots, \mathsf{td}_2^{n\to i}$ including the value $\mathsf{td}_2^{j\to i}$ from $\mathcal{A}$ in round 2, $\mathcal{A}_{\mathsf{TDGen}}$ sets $\mathsf{td}_{2,i} = (\mathsf{td}_2^{1\to i} || \ldots || \mathsf{td}_2^{n\to i})$ and forwards this message to $\mathcal{C}_{\mathsf{TDGen}}$ as its second round message for the protocol TDGen. $\mathcal{A}_{\mathsf{TDGen}}$ receives td_3 from $\mathcal{C}_{\mathsf{TDGen}}$ which is set as $\mathsf{td}_{3,i}$ in its interaction with $\mathcal{A}$. $\mathcal{A}_{\mathsf{TDGen}}$ continues with the rest of protocol π exactly as in Hyb_0 up to round 3. At this point, $\mathcal{A}_{\mathsf{TDGen}}$ rewinds the adversary $\mathcal{A}$ back to the beginning of round 2. To be consistent with our earlier terminology, this can be interpreted as follows: $\mathcal{A}_{\mathsf{TDGen}}$ creates a look-ahead thread that runs only rounds 2 and 3 of protocol π.[21] As in the main thread, $\mathcal{A}_{\mathsf{TDGen}}$ forwards the message $\mathsf{td}_{2,i}$ from $\mathcal{A}$ in round 2 to $\mathcal{C}_{\mathsf{TDGen}}$ and receives td_3 from $\mathcal{C}_{\mathsf{TDGen}}$ which is set as $\mathsf{td}_{3,i}$ in its interaction with $\mathcal{A}$. $\mathcal{A}_{\mathsf{TDGen}}$ continues with the rest of protocol π exactly as in $\mathsf{Hyb}_{\mathsf{Real}}$.

Now, $\mathcal{A}_{\mathsf{TDGen}}$ runs the extractor $\mathsf{Ext}_{\mathsf{NMCom}}$ of the non-malleable commitment scheme using the messages in both the threads that correspond to the non-malleable commitment from malicious party P_j to honest party P_i. Let the output of $\mathsf{Ext}_{\mathsf{NMCom}}$ be t^*. $\mathcal{A}_{\mathsf{TDGen}}$ outputs t^* as a valid trapdoor to $\mathcal{C}_{\mathsf{TDGen}}$.

Since the invariant doesn't hold by our assumption, the adversary P_j^*, using the non-malleable commitment, commits to a valid trapdoor t_i^* for the trapdoor generation messages of the honest party P_i^* with non-negligible probability ϵ. With probability $\frac{1}{n^2}$, where n is the total number of parties, this corresponds to honest party P_i and malicious party P_j picked randomly by $\mathcal{A}_{\mathsf{TDGen}}$. Therefore, with non-negligible probability $\frac{\epsilon}{n^2}$, the adversary P_j, using the non-malleable commitment, commits to a valid trapdoor t_i^* for the trapdoor generation messages of the honest party P_i. Now, by the 2-extractability property, given the messages of the non-malleable commitment in 2 threads, the extractor $\mathsf{Ext}_{\mathsf{NMCom}}$ is successful with some non-negligible probability ϵ'. Therefore, with non-negligible probability $\frac{\epsilon\cdot\epsilon'}{n^2}$, $\mathcal{A}_{\mathsf{TDGen}}$ outputs t^* as a valid trapdoor to $\mathcal{C}_{\mathsf{TDGen}}$ which breaks the 1-rewinding security of the trapdoor generation protocol TDGen. Thus, it must be the case that the invariant holds in $\mathsf{Hyb}_{\mathsf{Real}}$. □

[21] Note that this look-ahead thread exists only in the proof of the invariant and not in the description of $\mathsf{Hyb}_{\mathsf{Real}}$.

Claim 18. *The invariant holds in* Hyb_0.

Proof. Since there is no difference in the main thread in the first 3 rounds between $\mathsf{Hyb}_{\mathsf{Real}}$ and Hyb_0 , the invariant continues to hold true. □

Claim 19. Hyb_0 *is indistinguishable from* $\mathsf{Hyb}_{\mathsf{Real}}$ *except with probability atmost* $\frac{\mu}{4} + \mathsf{negl}(\lambda)$.

Proof. We argue the proof via the following two cases.
Case 1: Pr[not abort] $\geq \frac{\mu}{4}$**:**
Suppose the adversary doesn't cause an abort with probability greater than $\frac{\mu}{4}$. First, lets analyze the probability that $\mathsf{Sim}_{\mathsf{Hyb}}$ outputs "Special Abort" in Hyb_0 in the extraction phase. By Chernoff's bound, in Hyb_0, except with negligible probability, in the set of $\frac{5 \cdot n \cdot \lambda}{\mu}$ threads, there will be at least 5 "GOOD threads with respect to P_{i^*}" for some honest party P_{i^*}. Now, we show that the algorithms $\mathsf{Ext}_{\mathsf{ECom}}$ and TDExt successfully extract except with negligible probability.

By the definition of ECom, algorithm $\mathsf{Ext}_{\mathsf{ECom}}$ is successful except with negligible probability if given as input $(\mathsf{ecom}_1, \{\mathsf{ecom}_2^k, \mathsf{ecom}_3^k\}_{k=1}^5)$ such that $(\mathsf{ecom}_1, \mathsf{ecom}_2^k, \mathsf{ecom}_3^k)$ constitute "well-formed" and "admissible" extractable commitment messages for all k. "Admissibility" follows trivially since $\mathsf{Sim}_{\mathsf{Hyb}}$ picks random challenges z for the extractable commitment. Now, notice that since the invariant holds in Hyb_0, from the soundness of the schemes $\mathsf{WI}, \mathsf{RWI}$ and WZK, in each "Good thread with respect to P_{i^*}" for some honest party P_{i^*}, for every malicious party P_j, either the first or second statements must hold true for the proofs given in round 3. That is, for every malicious party P_j and honest party P_i, either the values $(\mathsf{ecom}_{a,1}^{j \to i}, \mathsf{ecom}_{a,2}^{i \to j}, \mathsf{ecom}_{a,3}^{j \to i})$ or the values $(\mathsf{ecom}_{b,1}^{j \to i}, \mathsf{ecom}_{b,2}^{i \to j}, \mathsf{ecom}_{b,3}^{j \to i})$ form a "well-formed" tuple of the scheme ECom as defined in Section 7.1. Hence, $\mathsf{Ext}_{\mathsf{ECom}}$ is successful except with negligible probability.

By the definition of the scheme TDGen, the algorithm TDExt is successful except with negligible probability if given as input $(\mathsf{td}_1, \{\mathsf{td}_2^i, \mathsf{td}_3^i\}_{i=1}^3)$ where td_1 is the first message of the protocol TDGen and $\mathsf{td}_2^i, \mathsf{td}_3^i$ denote the second and round messages of the i^{th} execution of protocol TDGen using the same first round message. Since we have 5 "GOOD" threads, we can extract every malicious party's trapdoor except with negligible probability.

Finally, again by Chernoff bound, in the set of $\frac{\lambda}{\mu}$ resampled main threads, there will be at least one completed execution. Thus, in this case, the adversary's view in $\mathsf{Hyb}_{\mathsf{Real}}$ is computationally indistinguishable to that in Hyb_0.
Case 2: Pr[not abort] $< \frac{\mu}{4}$**:**
Suppose the adversary doesn't cause an abort at the end of round 3 with probability less than $\frac{\mu}{4}$. Then, in both hybrids, $\mathsf{Sim}_{\mathsf{Hyb}}$ aborts at the end of the "Check Abort" step except with probability $\frac{\mu}{4}$. Thus, in this case, the adversary's view in $\mathsf{Hyb}_{\mathsf{Real}}$ is indistinguishable to that in Hyb_0 except with probability atmost $\frac{\mu}{4} + \mathsf{negl}(\lambda)$. □

Claim 20. *The invariant holds in* Hyb_1.

Proof. Since there is no difference in the main thread between Hyb_0 and Hyb_1, the invariant continues to hold true. □

Claim 21. *Assuming the hiding property of* Com *and* NCom*, the weak ZK property of* WZK*, the witness indistinguishability with reusability property of* RWI*, the security of the protocol* π^{SM}*,* Hyb_0 *is indistinguishable from* Hyb_1.

Proof. The only difference between the two hybrids is in the scenario when the "Check Abort" step doesn't succeed. In that case, in Hyb_0, $\mathsf{Sim}_{\mathsf{Hyb}}$ uses the honest parties' inputs to run the protocol while in Hyb_1, $\mathsf{Sim}_{\mathsf{Hyb}}$ runs the protocol using input 0 for every honest party. We show

in Appendix B via a sequence of sub-hybrids that these two hybrids are indistinguishable. Here, note that we can't just directly use a three round strong WI from [JKKR17] (instead of the RWI protocol) to prove this claim because that protocol is strictly delayed-input and in our case, we crucially do require part of the statement to be decided before the third round. Hence, we resort to the sequence of sub-hybrids in Appendix B to emulate a delayed-input strong WI. □

We now get back to proving security of the main simulation.

Claim 22. *The invariant holds in* Hyb_2*.*

Proof. There is no difference in the main thread between Hyb_1 and Hyb_2 and so the invariant continues to hold true. □

Claim 23. *Assuming the hiding property of* Com *and* NCom*, the weak ZK property of* WZK*, the witness indistinguishability with reusability property of* RWI*, the security of the protocol* π^{SM}*,* Hyb_1 *is indistinguishable from* Hyb_2*.*

Proof. The difference between the two hybrids is in the way the look-ahead threads are generated. Therefore, the only difference in the distinguisher's view is if $\mathsf{Sim}_{\mathsf{Hyb}}$ outputs "Special Abort" in the extraction phase of one hybrid and not the other.

We argue that this doesn't happen except with negligible probability. First, observe that in order to perform extraction successfully, all we need is 5 "GOOD" look-ahead threads. Therefore, it suffices to argue that the probability with which a given thread is "GOOD" does not change by more than a negligible amount between the two hybrids. As a result, in order to ensure that the probability of extraction remains the same in both the hybrids, it is enough to prove that each look-ahead thread, in isolation, is indistinguishable in both the hybrids.

Now, in order to prove this, first observe that for every look-ahead thread, the difference between Hyb_1 and Hyb_2 is exactly the same as the difference on the main thread between Hyb_0 and Hyb_1 when the "Check Abort" step doesn't succeed. That is, in Hyb_1, $\mathsf{Sim}_{\mathsf{Hyb}}$ runs the look-ahead thread using honest parties' inputs while in Hyb_2, $\mathsf{Sim}_{\mathsf{Hyb}}$ runs the look-ahead thread using input 0. So, by using the same set of arguments as in Appendix B, we can show that, in isolation, every look-ahead thread in Hyb_1 is indistinguishable from the corresponding look-ahead thread in Hyb_2.

Therefore, since every look-ahead thread in isolation is indistinguishable in both the hybrids, for each look-ahead thread, the probability with which the thread is "GOOD with respect to P_{i^*}" for some honest party P_{i^*} remains the same. Hence, the probability with which $\mathsf{Sim}_{\mathsf{Hyb}}$ outputs "Special Abort" in the extraction phase is same in both hybrids and this completes the proof.

□

Claim 24. *Assuming* NMCom *is a secure non-malleable commitment scheme, the invariant holds in* Hyb_3*.*

Proof. First, observe that if the number of "GOOD" look-ahead threads is less than 5 with respect to all honest parties, then $\mathsf{Sim}_{\mathsf{Hyb}}$ outputs "Special Abort" in both hybrids and they are identical. Therefore, suppose we have 5 "GOOD" look-ahead threads in both hybrids with respect to some honest party P_{i^*}.

We know that the invariant holds in Hyb_2. The only difference between Hyb_2 and Hyb_3 is that in Hyb_3, the simulator now computes the non-malleable commitment in the main thread using the adversary's trapdoor value. Assume for the sake of contradiction that the invariant doesn't hold in Hyb_3. That is, there exists an adversary $\mathcal{A}$ such that for some honest party P_i^* and malicious party P_j^*, $\mathcal{A}$ causes event E to occur in the main thread with non-negligible probability. We will use $\mathcal{A}$ to design an adversary $\mathcal{A}_{\mathsf{NMCom}}$ that breaks the security of the non-malleable commitment scheme.

$\mathcal{A}_{\mathsf{NMCom}}$ interacts with a challenger $\mathcal{C}_{\mathsf{NMCom}}$. $\mathcal{A}_{\mathsf{NMCom}}$ performs the role of $\mathsf{Sim}_{\mathsf{Hyb}}$ in its interaction with $\mathcal{A}$ exactly as done in Hyb_2. $\mathcal{A}_{\mathsf{NMCom}}$ picks an honest party P_i and a malicious party P_j uniformly at random. $\mathcal{A}_{\mathsf{NMCom}}$ interacts with a challenger $\mathcal{C}_{\mathsf{NMCom}}$ and receives a first round message nmcom_1^L on the left side which is set as $\mathsf{nmcom}_1^{i\to j}$ in its interaction with $\mathcal{A}$ in round 1 of protocol π. On receiving $\mathsf{nmcom}_1^{j\to i}$, $\mathcal{A}_{\mathsf{NMCom}}$ forwards this to $\mathcal{C}_{\mathsf{NMCom}}$ as its first round message on the right side.

$\mathcal{A}_{\mathsf{NMCom}}$ creates a set of 5 look-ahead threads, in each of which, it runs rounds 2 and 3 of the protocol alone. In each look-ahead thread, $\mathcal{A}_{\mathsf{NMCom}}$ computes $\mathsf{nmcom}_3^{i\to j}$ as a commitment to $\perp$. Recall from the definition of the scheme NMCom that $\mathcal{A}_{\mathsf{NMCom}}$ can do this even without knowing the randomness used to generate $\mathsf{nmcom}_1^{i\to j}$. As in the proof of Claim 25, recall that using these 5 threads (that are all "GOOD with respect to some honest party H" with noticeable probability), $\mathcal{A}_{\mathsf{NMCom}}$ can successfully run the input extraction phase.

Then, on the main thread, $\mathcal{A}_{\mathsf{NMCom}}$ receives a value nmcom_2^R from $\mathcal{C}_{\mathsf{NMCom}}$ as the second round message on the right side which it sets as the value $\mathsf{nmcom}_2^{i\to j}$ in round 2 of protocol π on the main thread. Then, on receiving $\mathsf{nmcom}_2^{j\to i}$ in the main thread, $\mathcal{A}_{\mathsf{NMCom}}$ sends this to $\mathcal{C}_{\mathsf{NMCom}}$ as its second round message on the left side along with the pair of values $(\perp, \mathsf{t}_j)$ where t_j is generated in the input extraction phase. (Recall that NMCom is a delayed-input scheme.)

$\mathcal{A}_{\mathsf{NMCom}}$ receives a third round message nmcom_3^L which is either a commitment to $\perp$ or t_j. This is sent to $\mathcal{A}$ as the value $\mathsf{nmcom}_3^{i\to j}$ in the main thread and the rest of protocol π is performed exactly as in Hyb_1. On receiving the value $\mathsf{nmcom}_3^{i\to j}$ from $\mathcal{A}$ in the main thread, $\mathcal{A}_{\mathsf{NMCom}}$ sends it to $\mathcal{C}_{\mathsf{NMCom}}$ as its third round message in the right thread.

Observe that the first case corresponds to Hyb_2 while the second case corresponds to Hyb_3. By our assumption, the invariant doesn't hold in Hyb_3 so there exists honest party P_i^* and malicious party P_j^* such that event E doesn't hold. That is, the adversary P_j^*, using the non-malleable commitment, commits to a valid trapdoor t_i^* for the trapdoor generation messages of the honest party P_i^* with non-negligible probability ϵ. With probability $\frac{1}{n^2}$ where n is the total number of parties, this corresponds to honest party P_i and malicious party P_j picked randomly by $\mathcal{A}_{\mathsf{NMCom}}$. Therefore, with non-negligible probability $\frac{\epsilon}{n^2}$, the adversary P_j, using the non-malleable commitment, commits to a valid trapdoor t_i^* for the trapdoor generation messages of the honest party P_i in Hyb_3. Recall that this is same as the messages sent by $\mathcal{A}_{\mathsf{NMCom}}$ to $\mathcal{C}_{\mathsf{NMCom}}$ as its commitment messages on the right side. In Hyb_2, since the invariant holds, the adversary did not commit to a valid trapdoor from any malicious party P_j to honest party P_i.

Therefore, when the value committed to by the honest party in the left execution changes, the value committed to by the adversary in the right execution has also changed except with negligible probability. This breaks the security of the scheme NMCom which is a contradiction. Thus, the invariant must hold in Hyb_2 as well.

Remark: To simplify the exposition, we will implicitly use this argument in every subsequent proof that the two hybrids are identical if the number of "GOOD" look-ahead threads is less than 5 with respect to all honest parties. Therefore, we argue that there exists some honest party H, such that if we create just 5 look-ahead threads they are all "GOOD with respect to H" with noticeable probability and hence we can extract the adversary's input and trapdoor with noticeable probability. We will skip describing the specific party H in the rest of the exposition. For a similar reason, we will consider only one execution of the main thread in the reductions and not all the resampling. That is, without resampling, just one execution should be "GOOD" with noticeable probability and that suffices. □

Claim 25. *Assuming the hiding property of the non-malleable commitment scheme* NMCom*,* Hyb_2 *is computationally indistinguishable from* Hyb_3*.*

Proof. The only difference between Hyb_2 and Hyb_3 is that in Hyb_3, the simulator now computes the non-malleable commitment using the adversary's trapdoor value. Suppose there exists an adversary $\mathcal{A}$ that can distinguish between the two hybrids. We can use $\mathcal{A}$ to design an adversary $\mathcal{A}_{\mathsf{Hid}}$ that breaks the hiding property of the non-malleable commitment scheme. The rest of the proof is similar to the proof of Claim 24 above.

□

Claim 26. *Assuming the bounded rewinding security of the protocol* RWI *and the existence of an extractor* $\mathsf{Ext}_{\mathsf{NMCom}}$ *for the non-malleable commitment scheme* NMCom*, the invariant holds in* Hyb_4*.*

Proof. We know that the invariant holds in Hyb_3. The only difference between Hyb_3 and Hyb_4 is that in Hyb_4, in the main thread, the simulator now computes the rewinding secure WI using a witness for the non-malleable commitment. Assume for the sake of contradiction that the invariant doesn't hold in Hyb_4. That is, there exists an adversary $\mathcal{A}$ such that for some honest party P_i^* and malicious party P_j^*, $\mathcal{A}$ causes event E to occur in the main thread with non-negligible probability.

We will use $\mathcal{A}$ to design an adversary $\mathcal{A}_{\mathsf{RWI}}$ that breaks the bounded security of the protocol RWI.

$\mathcal{A}_{\mathsf{RWI}}$ interacts with a challenger $\mathcal{C}_{\mathsf{RWI}}$. $\mathcal{A}_{\mathsf{RWI}}$ performs the role of $\mathsf{Sim}_{\mathsf{Hyb}}$ in its interaction with $\mathcal{A}$ exactly as done in Hyb_2. $\mathcal{A}_{\mathsf{RWI}}$ picks an honest party P_i and a malicious party P_j uniformly at random. Initially, it receives a message rwi_1 from $\mathcal{C}_{\mathsf{RWI}}$ which it sets as the value $\mathsf{rwi}_1^{i\to j}$ in its interaction with $\mathcal{A}$ in round 1. Then, after receiving the round 2 message $\mathsf{rwi}_2^{j\to i}$ from $\mathcal{A}$, $\mathcal{A}_{\mathsf{RWI}}$ creates a set of 5 look-ahead threads that run only rounds 2 and 3 of the protocol. For each thread, on receiving $\mathsf{rwi}_2^{j\to i}$ in round 2, $\mathcal{A}_{\mathsf{RWI}}$ forwards this to $\mathcal{C}_{\mathsf{RWI}}$ as its second round message for that look-ahead thread. For each thread, $\mathcal{A}_{\mathsf{RWI}}$ also sends the statement

$$\mathsf{st}_2^{i\to j} = \left(\{\mathsf{ecom}_{a,\ell}^{i\to j}\}_{\ell=1}^{3}, \{\mathsf{ecom}_{b,\ell}^{i\to j}\}_{\ell=1}^{3}, \mathsf{msg}_{2,i}, \mathsf{Trans}_1, \{\mathsf{nmcom}_{\ell}^{i\to j}\}_{\ell=1}^{3}, \mathsf{td}_{1,j}, \mathsf{nc}_i\right) \in L_2^{i\to j}$$

where the other values are generated as in Hyb_3.

In the main thread, $\mathcal{A}_{\mathsf{RWI}}$ also sends the pair of witnesses $(\mathsf{x}_i, \mathsf{r}_i, \mathsf{r}_{\mathsf{ecom}}^{i\to j}, \bot, \bot, \bot, \bot)$ and $(\bot, \bot, \bot, \bot, \mathsf{t}_j, \mathsf{r}_{\mathsf{nmcom}}^{i\to j}, \bot)$ where t_j is generated in the input extraction phase. For each look-ahead thread, $\mathcal{A}_{\mathsf{RWI}}$ sends the witness $(\mathsf{x}_i, \mathsf{r}_i, \mathsf{r}_{\mathsf{ecom}}^{i\to j}, \bot, \bot, \bot, \bot)$. For each thread, $\mathcal{A}_{\mathsf{RWI}}$ receives a message rwi_3 which is set as $\mathsf{rwi}_3^{i\to j}$ in its interaction with $\mathcal{A}$ in round 3 of protocol π. The rest of protocol π is performed exactly as in Hyb_3. As in the proof of Claim 25, recall that using these 5 threads (that are all "GOOD with respect to some honest party H" with noticeable probability), $\mathcal{A}_{\mathsf{RWI}}$ can successfully run the input extraction phase.

Observe that the first case corresponds to Hyb_3 while the second case corresponds to Hyb_4.

Now, let's see how $\mathcal{A}_{\mathsf{RWI}}$ breaks the security of the protocol RWI. Recall that RWI is secure even in the presence of 6 total threads. $\mathcal{A}_{\mathsf{RWI}}$ runs the extractor $\mathsf{Ext}_{\mathsf{NMCom}}$ of the non-malleable commitment scheme using the messages in all the threads that correspond to the non-malleable commitment from malicious party P_j to honest party P_i. Let the output of $\mathsf{Ext}_{\mathsf{NMCom}}$ be t^*. If $\mathsf{TDValid}(\mathsf{td}_{1,i}, \mathsf{t}^*) = 1$, then $\mathcal{A}_{\mathsf{RWI}}$ outputs case 2 indicating that the WI was constructed using the second witness on the main thread. Else, it outputs case 1.

Let's analyze why this works. We know that the invariant doesn't hold in Hyb_4 so there exists honest party P_i^* and malicious party P_j^* such that event E doesn't hold. That is, the adversary P_j^*, using the non-malleable commitment, commits to a valid trapdoor t_i^* for the trapdoor generation

messages of the honest party P_i^* with non-negligible probability ϵ. With probability $\frac{1}{n^2}$ where n is the total number of parties, this corresponds to honest party P_i and malicious party P_j picked randomly by $\mathcal{A}_{\mathsf{RWI}}$. Therefore, with non-negligible probability $\frac{\epsilon}{n^2}$, the adversary P_j, using the non-malleable commitment, commits to a valid trapdoor t_i^* for the trapdoor generation messages of the honest party P_i in Hyb_3. Therefore, since the invariant holds in Hyb_3 with non-negligible probability, by the 2-extractability property of the non-malleable commitment, when the extractor $\mathsf{Ext}_{\mathsf{NMCom}}$ outputs a valid trapdoor t^*, it must be the case that we are in Hyb_4 with non-negligible probability. That is, when $\mathsf{Ext}_{\mathsf{NMCom}}$ outputs a valid trapdoor, it corresponds to $\mathcal{A}_{\mathsf{RWI}}$ receiving a proof using the second (alternate) witness and otherwise, it corresponds to $\mathcal{A}_{\mathsf{RWI}}$ receiving a proof using the first witness. Thus, $\mathcal{A}_{\mathsf{RWI}}$ breaks the bounded rewinding security of the scheme RWI which is a contradiction. Hence, the invariant holds in Hyb_4 as well.

□

Claim 27. *Assuming the bounded rewinding security of the protocol* RWI*,* Hyb_3 *is indistinguishable from* Hyb_4*.*

Proof. The only difference between Hyb_3 and Hyb_4 is that in Hyb_4, in the main thread, the simulator now computes the rewinding secure WI using a witness for the alternate statement. Suppose there exists an adversary $\mathcal{A}$ that can distinguish between the two hybrids, we can use $\mathcal{A}$ to design an adversary $\mathcal{A}_{\mathsf{RWI}}$ that breaks the bounded rewinding security of the protocol RWI. The rest of the proof is very similar to the proof of Claim 26 above.

□

Claim 28. *Assuming* WI *is a secure delayed input witness indistinguishable argument, the invariant holds in* Hyb_5*.*

Proof. Notice that the only difference between Hyb_4 and Hyb_5 is in the witness used in the WI proof of the main thread that is completed in round 4 of the protocol. However, since WI is a delayed-input scheme, there is no difference in the first 2 rounds of the WI protocol. In other words, there is no difference between Hyb_4 and Hyb_5 in the messages sent in the first 3 rounds of the protocol by $\mathsf{Sim}_{\mathsf{Hyb}}$. Since the invariant depends only on the first 3 rounds, and since the invariant holds in Hyb_4, it continues to hold in Hyb_5. □

Claim 29. *Assuming* WI *is a secure delayed input witness indistinguishable argument,* Hyb_4 *is indistinguishable from* Hyb_5*.*

Proof. This is very similar to (and in fact, simpler than) the proof of Claim 27 with the main difference being that there is no rewinding here in the reduction. That is, the reduction uses the external challenger's messages only once - to generate the proof only in the final round of the main thread and not in each look-ahead thread. Therefore, we don't need any rewinding security for the scheme WI. □

Claim 30. *Assuming the bounded rewinding security of the* RWI *, the hiding property of the commitment scheme* Com *and the pseudorandom function* PRF *used in the construction of* ECom *and the existence of an extractor* $\mathsf{Ext}_{\mathsf{NMCom}}$ *for the non-malleable commitment scheme* NMCom*, the invariant holds in* Hyb_6*.*

Proof. We know that the invariant holds in Hyb_5. The only difference between Hyb_5 and Hyb_6 is that in Hyb_6, the simulator now computes the extractable commitment in the main thread for every honest party P_i using input 0 whereas in Hyb_5 it was computed using input $(\mathsf{x}_i, \mathsf{r}_i)$. We drop the subscript "a" throughout the description of this proof.

First, throughout this proof, we drop the subscript "a" since it is clear from context that we are referring only to the first invocation of the extractable commitment. The second invocation is not used in the proof at all.

Let's briefly recall the construction of the scheme ECom from Section 7.1 in the context of protocol π. Consider a party P_i as committer interacting with a party P_j as receiver using input message m. In round 1, P_i computes $\mathsf{ecom}_1^{i\to j} = \{\mathsf{ecom}_{1,1}^{i\to j}, \ldots, \mathsf{ecom}_{1,N}^{i\to j}\}$ where $\mathsf{ecom}_{1,\ell}^{i\to j} = \mathsf{Com}(\mathsf{p}_\ell^{i\to j})$ where each $\mathsf{p}_\ell^{i\to j}$ is a random polynomial of degree 4 (defined in Section 7.1). In round 2, P_j generates $\mathsf{ecom}_2^{j\to i} = (\mathsf{z}_1^{j\to i}, \ldots, \mathsf{z}_N^{j\to i})$ where each $\mathsf{z}_\ell^{j\to i}$ is a random value. Then, in round 3, P_i outputs $\mathsf{ecom}_3^{i\to j} = \{\mathsf{ecom}_{3,1}^{i\to j}, \ldots, \mathsf{ecom}_{3,N+2}^{i\to j}\}$ where $\mathsf{ecom}_{3,\ell}^{i\to j} = (k^{i\to j} \oplus \mathsf{p}_\ell^{i\to j}(0), \mathsf{p}_\ell^{i\to j}(\mathsf{z}_\ell^{j\to i}))$ for each $\ell \in [N]$ and $\mathsf{ecom}_{3,N+1} = s^{i\to j}$ and $\mathsf{ecom}_{3,N+2} = \mathsf{PRF}(k^{i\to j}, s^{i\to j}) \oplus m$.

We will design a set of intermediate hybrids $\mathsf{Sub.Hyb}_1$ to $\mathsf{Sub.Hyb}_5$ where $\mathsf{Sub.Hyb}_1$ denotes Hyb_5 and $\mathsf{Sub.Hyb}_5$ denotes Hyb_6. We will then show that the invariant holds in every intermediate hybrid to complete the proof.

- $\mathsf{Sub.Hyb}_1$**:** This is same as H_5.
- $\mathsf{Sub.Hyb}_2$**:** In this hybrid, for every honest party P_i and malicious party P_j, only on the main thread, compute $\mathsf{ecom}_{3,\ell}^{i\to j} = (0 \oplus \mathsf{p}_\ell^{i\to j}(0), \mathsf{p}_\ell^{i\to j}(\mathsf{z}_\ell^{j\to i}))$ for all $\ell \in [N]$. Observe that all the look-ahead threads continue committing to a random value $\mathsf{r}^{i\to j}$.
- $\mathsf{Sub.Hyb}_3$**:** In this hybrid, for every honest party P_i and malicious party P_j, in the main thread, compute $\mathsf{ecom}_{3,N+2}^{i\to j}$ uniformly at random.
- $\mathsf{Sub.Hyb}_4$**:** In this hybrid, for every honest party P_i and malicious party P_j, in the main thread, compute $\mathsf{ecom}_{3,N+2}^{i\to j} = \mathsf{PRF}(k^{i\to j}, \mathsf{ecom}_{3,N+1}^{i\to j}) \oplus 0$.
- $\mathsf{Sub.Hyb}_5$**:** In this hybrid, for every honest party P_i and malicious party P_j, in the main thread, compute $\mathsf{ecom}_{3,\ell}^{i\to j} = (k^{i\to j} \oplus \mathsf{p}_\ell^{i\to j}(0), \mathsf{p}_\ell^{i\to j}(\mathsf{z}_\ell^{j\to i}))$ for all $\ell \in [N]$. This is same as H_6.

Sub-Claim 1. *Assuming the bounded rewinding security of the* RWI*, the hiding property of the underlying commitment scheme* Com *used in the construction of* ECom *and the existence of an extractor* $\mathsf{Ext}_{\mathsf{NMCom}}$ *for the non-malleable commitment scheme* NMCom*, the invariant holds in* $\mathsf{Sub.Hyb}_2$*.*

Proof. The only difference between the two hybrids is that in $\mathsf{Sub.Hyb}_1$, for all $\ell \in [N]$, in $\mathsf{ecom}_{3,\ell}^{i\to j}$ we use $k^{i\to j}$ while in $\mathsf{Sub.Hyb}_2$, we use 0. We know that the invariant holds in $\mathsf{Sub.Hyb}_1$.

We will design a set of intermediate hybrids $\mathsf{Sub.Hyb}_{1,0}$ to $\mathsf{Sub.Hyb}_{1,N}$ where $\mathsf{Sub.Hyb}_{1,0}$ denotes $\mathsf{Sub.Hyb}_1$ and $\mathsf{Sub.Hyb}_{1,N}$ denotes $\mathsf{Sub.Hyb}_2$. We will then show that the invariant holds in every intermediate hybrid to complete the proof. **For** $\ell = 1 \ldots N$**,** $\mathsf{Sub.Hyb}_{1,\ell}$**:** For every honest party P_i and malicious party P_j, do: (to ease the exposition, we will skip the superscript $i \to j$).

- Pick a new degree 4 polynomial q_ℓ such that $(k \oplus \mathsf{p}_\ell(0)) = (0 \oplus \mathsf{q}_\ell(0))$.
- In round 1, compute $\mathsf{ecom}_{1,\ell} = \mathsf{Com}(\mathsf{q}_\ell)$.
- in the main thread, compute $\mathsf{ecom}_{3,\ell}$ as $(0 \oplus \mathsf{q}_\ell(0), \mathsf{q}_\ell(\mathsf{z}_\ell))$.
- In every look-ahead thread, compute $\mathsf{ecom}_{3,\ell}$ as before, i.e using input k but using polynomial q_ℓ.

That is, in $\mathsf{Sub.Hyb}_{1,\ell}$, we switch the ℓ^{th} index of the extractable commitment scheme from using input (k) to using input 0.

Recall that the goal was to show that the invariant holds in every intermediate hybrid. Assume for the sake of contradiction that there exists ℓ such that the invariant doesn't hold in $\mathsf{Sub.Hyb}_{1,\ell}$ but holds in $\mathsf{Sub.Hyb}_{1,0}, \ldots, \mathsf{Sub.Hyb}_{1,\ell-1}$. That is, there exists an adversary $\mathcal{A}$ such that for some honest party P_i^* and malicious party P_j^*, $\mathcal{A}$ causes event E to occur with non-negligible probability in $\mathsf{Sub.Hyb}_{1,\ell}$. We will use $\mathcal{A}$ to arrive at a contradiction.

We will again prove this using a series of intermediate hybrids. We know that the invariant holds in $\mathsf{Sub.Hyb}_{1,\ell-1}$. Consider a set of hybrids $\mathsf{H}_1, \ldots, \mathsf{H}_5$ as follows where H_1 corresponds to $\mathsf{Sub.Hyb}_{1,\ell-1}$ and H_5 corresponds to $\mathsf{Sub.Hyb}_{1,\ell}$.

- H_2**:** In round 3 of every look-ahead thread, for every honest party P_i and malicious party P_j, the algorithm RWI_3 proves that the commitment $(\mathsf{ecom}_1, \mathsf{ecom}_2, \mathsf{ecom}_3)$ is "well-formed" using all indices from $1, \ldots, \lambda$ except index ℓ.
- H_3**:** In round 1, for every honest party P_i and malicious party P_j, compute $\mathsf{ecom}_{1,\ell} = \mathsf{Com}(0)$.
- H_4**:** For every honest party P_i and malicious party P_j:
  - In the main thread, compute $\mathsf{ecom}_{3,\ell}$ as $(0 \oplus \mathsf{q}_\ell(0), \mathsf{q}_\ell(\mathsf{z}_\ell))$.
  - In every look-ahead thread, compute $\mathsf{ecom}_{3,\ell}$ as before but using polynomial q_ℓ.
- H_5**:** In round 1, for every honest party P_i and malicious party P_j, compute $\mathsf{ecom}_{1,\ell} = \mathsf{Com}(\mathsf{q}_\ell)$.

Note: as before, we drop the superscript $i \to j$ to ease the exposition. We will now show that the invariant holds in $\mathsf{H}_2, \ldots, \mathsf{H}_5$ and this completes the proof.

Lemma 2. *Assuming the bounded rewinding security of the protocol* RWI *and the existence of an extractor* $\mathsf{Ext}_{\mathsf{NMCom}}$ *for the non-malleable commitment scheme* NMCom*, the invariant holds in* H_2*.*

Proof. We know that the invariant holds in H_1. Suppose it doesn't hold in H_2. That is, there exists an adversary $\mathcal{A}$ such that for some honest party P_i^* and malicious party P_j^*, $\mathcal{A}$ causes event E to occur with non-negligible probability. The only difference between the two hybrids is in the witness used to compute the RWI in round 3 of protocol π for every look-ahead thread. We will use $\mathcal{A}$ to design an adversary $\mathcal{A}_{\mathsf{RWI}}$ that breaks the bounded rewinding security of the scheme RWI.

$\mathcal{A}_{\mathsf{RWI}}$ interacts with a challenger $\mathcal{C}_{\mathsf{RWI}}$. $\mathcal{A}_{\mathsf{RWI}}$ performs the role of $\mathsf{Sim}_{\mathsf{Hyb}}$ in its interaction with $\mathcal{A}$ exactly as done in H_1. $\mathcal{A}_{\mathsf{RWI}}$ picks an honest party P_i, a malicious party P_j. Initially, it receives a message rwi_1 from $\mathcal{C}_{\mathsf{RWI}}$ which it sets as the value $\mathsf{rwi}_1^{i\to j}$ in its interaction with $\mathcal{A}$ in round 1. Then, $\mathcal{A}_{\mathsf{RWI}}$ creates a set of 5 look-ahead threads. On receiving $\mathsf{rwi}_2^{j\to i}$ in round 2 for each look-ahead thread, $\mathcal{A}_{\mathsf{RWI}}$ forwards this to $\mathcal{C}_{\mathsf{RWI}}$ as its second round message for that look-ahead thread. $\mathcal{A}_{\mathsf{RWI}}$ also sends the statement

$$\mathsf{st}_2^{i\to j} = \left(\{\mathsf{ecom}_{a,\ell}^{i\to j}\}_{\ell=1}^3, \{\mathsf{ecom}_{b,\ell}^{i\to j}\}_{\ell=1}^3, \mathsf{msg}_{2,i}, \mathsf{Trans}_1, \{\mathsf{nmcom}_\ell^{i\to j}\}_{\ell=1}^3, \mathsf{td}_{1,j}, \mathsf{nc}_i\right) \in L_2^{i\to j}$$

where the other values are generated as in H_1 along with the pair of witnesses used in H_1 and H_2 respectively. Corresponding to each look-ahead thread, $\mathcal{A}_{\mathsf{RWI}}$ receives a message rwi_3 which is set as $\mathsf{rwi}_3^{i\to j}$ in its interaction with $\mathcal{A}$ in round 3 of protocol π. The rest of protocol π is performed exactly as in H_1. As in the proof of Claim 25, recall that using these 5 threads (that are all "GOOD with respect to some honest party H" with noticeable probability), $\mathcal{A}_{\mathsf{RWI}}$ can successfully run the input extraction phase.

Observe that the first case corresponds to H_1 while the second case corresponds to H_2.

Now, let's see how $\mathcal{A}_{\mathsf{RWI}}$ breaks the bounded rewinding security of the scheme RWI. $\mathcal{A}_{\mathsf{RWI}}$ runs the extractor $\mathsf{Ext}_{\mathsf{NMCom}}$ of the non-malleable commitment scheme using the messages in all the threads that correspond to the non-malleable commitment from malicious party P_j to honest party P_i. Let the output of $\mathsf{Ext}_{\mathsf{NMCom}}$ be t^*. If $\mathsf{TDValid}(\mathsf{td}_{1,i}, \mathsf{td}_{2,i}, \mathsf{td}_{3,i}, \mathsf{t}^*) = 1$, then $\mathcal{A}_{\mathsf{RWI}}$ outputs case 2 indicating that the WI was constructed using the witness as in H_2. Else, it outputs case 1.

Let's analyze why this works. We know that the invariant doesn't hold in H_2 so there exists honest party P_i^* and malicious party P_j^* such that event E doesn't hold. That is, the adversary P_j^*, using the non-malleable commitment, commits to a valid trapdoor t_i^* for the trapdoor generation messages of the honest party P_i^* with non-negligible probability ϵ. With probability $\frac{1}{n^2}$ where n is the total number of parties, this corresponds to honest party P_i and malicious party P_j picked randomly by $\mathcal{A}_{\mathsf{RWI}}$. Therefore, with non-negligible probability $\frac{\epsilon}{n^2}$, the adversary P_j, using the non-malleable commitment, commits to a valid trapdoor t_i^* for the trapdoor generation messages of the honest party P_i in H_2. Therefore, since the invariant holds in H_1 with non-negligible probability, when the extractor $\mathsf{Ext}_{\mathsf{NMCom}}$ outputs a valid trapdoor t^*, it must be the case that we are in H_2 with non-negligible probability. Thus, $\mathcal{A}_{\mathsf{RWI}}$ breaks the bounded rewinding security of the scheme RWI which is a contradiction. Hence, the invariant holds in H_2 as well. □

Lemma 3. *Assuming the hiding of the commitment scheme* Com*, the invariant holds in* H_3*.*

Proof. We know that the invariant holds in H_2. The only difference between H_2 and H_3 is that in H_3, the simulator now computes the commitment $\mathsf{ecom}_{1,\ell}$ in round 1 as $\mathsf{ecom}_{1,\ell} = \mathsf{Com}(0)$ for every honest party P_i and malicious party P_j. Assume for the sake of contradiction that the invariant doesn't hold in H_3. That is, there exists an adversary $\mathcal{A}$ such that for some honest party P_i^* and malicious party P_j^*, $\mathcal{A}$ causes event E to occur with non-negligible probability. We will use $\mathcal{A}$ to design an adversary $\mathcal{A}_{\mathsf{Com}}$ that breaks the hiding of the commitment scheme.

$\mathcal{A}_{\mathsf{Com}}$ interacts with a challenger $\mathcal{C}_{\mathsf{Com}}$. $\mathcal{A}_{\mathsf{Com}}$ performs the role of $\mathsf{Sim}_{\mathsf{Hyb}}$ in its interaction with $\mathcal{A}$ exactly as done in Hyb_1. $\mathcal{A}_{\mathsf{NMCom}}$ picks an honest party P_i and a malicious party P_j uniformly at random. $\mathcal{A}_{\mathsf{Com}}$ creates a set of 5 look-ahead threads. $\mathcal{A}_{\mathsf{Com}}$ interacts with a challenger $\mathcal{C}_{\mathsf{Com}}$ and sends two strings: (p_ℓ) and (0). $\mathcal{A}_{\mathsf{Com}}$ receives a message nmcom which is sent to $\mathcal{A}$ as the value $\mathsf{ecom}_{1,\ell}$ round 1 and the rest of protocol π is performed exactly as in H_2. As in the proof of Claim 25, recall that using these 5 threads (that are all "GOOD with respect to some honest party H" with noticeable probability), $\mathcal{A}_{\mathsf{Com}}$ can successfully run the input extraction phase.

Observe that the first case corresponds to H_2 while the second case corresponds to H_3.

Now, let's analyze why $\mathcal{A}_{\mathsf{Com}}$ breaks the hiding of the scheme Com. We know that the invariant doesn't hold in H_3 so there exists honest party P_i^* and malicious party P_j^* such that event E doesn't hold. That is, the adversary P_j^*, using the non-malleable commitment, commits to a valid trapdoor t_i^* for the trapdoor generation messages of the honest party P_i^* with non-negligible probability ϵ. With probability $\frac{1}{n^2}$ where n is the total number of parties, this corresponds to honest party P_i and malicious party P_j picked randomly by $\mathcal{A}_{\mathsf{Com}}$. Therefore, with non-negligible probability $\frac{\epsilon}{n^2}$, the adversary P_j, using the non-malleable commitment, commits to a valid trapdoor t_i^* for the trapdoor generation messages of the honest party P_i in H_3. Therefore, since the invariant holds in H_2 with non-negligible probability, when the extractor $\mathsf{Ext}_{\mathsf{NMCom}}$ outputs a valid trapdoor t^*, it must be the case that we are in H_3 with non-negligible probability. Thus, $\mathcal{A}_{\mathsf{Com}}$ can use this to break the hiding of the commitment scheme Com which is a contradiction. Hence, the invariant holds in H_3 as well. □

Lemma 4. *Invariant holds in* H_4*.*

Proof. We know that the invariant holds in H_3. The difference between H_3 and H_4 is only a statistical change and hence the invariant continues to hold in H_4 as well. □

Lemma 5. *Invariant holds in* H_5.

Proof. This is same as the proof of Lemma 3. □

This completes the proof of Sub-Claim 1. □

Sub-Claim 2. *Assuming the security of the pseudorandom function* PRF *and the existence of an extractor* $\mathsf{Ext}_{\mathsf{NMCom}}$ *for the non-malleable commitment scheme* NMCom*, the invariant holds in* $\mathsf{Sub.Hyb}_3$.

Proof. The only difference between the two hybrids is that, in $\mathsf{Sub.Hyb}_3$, on the main thread, the value $\mathsf{ecom}^{i\to j}_{3,N+2}$ is computed uniformly at random while in $\mathsf{Sub.Hyb}_2$, it was computed as $\mathsf{PRF}(k^{i\to j}, \mathsf{ecom}^{i\to j}_{3,N+1}) \oplus \mathsf{x}_i$. Therefore, if there exists an adversary $\mathcal{A}$ for which the invariant holds in $\mathsf{Sub.Hyb}_2$ but not in $\mathsf{Sub.Hyb}_3$, we can use the extractor $\mathsf{Ext}_{\mathsf{NMCom}}$ to extract the value inside the adversary's non-malleable commitment and check whether the invariant holds or not. Thus, we can build a reduction that breaks the security of the PRF. □

Sub-Claim 3. *Assuming the security of the pseudorandom function* PRF *and the existence of an extractor* $\mathsf{Ext}_{\mathsf{NMCom}}$ *for the non-malleable commitment scheme* NMCom*, the invariant holds in* $\mathsf{Sub.Hyb}_4$.

Proof. This is identical to the previous proof. □

Sub-Claim 4. *Assuming the bounded rewinding security of the scheme* RWI*, the hiding property of the underlying commitment scheme* Com *used in the construction of* ECom *and the existence of an extractor* $\mathsf{Ext}_{\mathsf{NMCom}}$ *for the non-malleable commitment scheme* NMCom*, the invariant holds in* $\mathsf{Sub.Hyb}_5$.

Proof. This is identical to the proof of Sub-Claim 1. □

This completes the proof of Claim 30. □

Claim 31. *Assuming the bounded rewinding security of the scheme* RWI *and the hiding property of the underlying commitment scheme* Com *used in the construction of* ECom*,* Hyb_5 *is indistinguishable from* Hyb_6.

Proof. This is similar to the proof of Claim 30 except that we no longer have to run the extractor $\mathsf{Ext}_{\mathsf{NMCom}}$ of the non-malleable commitment scheme.

□

Claim 32. *Assuming the security of protocol* π^{SM} *and the existence of an extractor* $\mathsf{Ext}_{\mathsf{NMCom}}$ *for the non-malleable commitment scheme* NMCom*, the invariant holds in* Hyb_7.

Proof. We know that the invariant holds in Hyb_6. The only difference between Hyb_6 and Hyb_7 is that in Hyb_7, in the main thread, the simulator now computes the messages of protocol π^{SM} using the simulated algorithms $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$. Assume for the sake of contradiction that the invariant doesn't hold in Hyb_7. That is, there exists an adversary $\mathcal{A}$ such that for some honest party P^*_i and malicious party P^*_j, $\mathcal{A}$ causes event E to occur in the main thread with non-negligible probability. We will use $\mathcal{A}$ to design an adversary $\mathcal{A}_{\pi^{\mathsf{SM}}}$ that breaks the security of the protocol π^{SM}.

$\mathcal{A}_{\pi^{\mathsf{SM}}}$ performs the role of $\mathsf{Sim}_{\mathsf{Hyb}}$ in its interaction with $\mathcal{A}$ exactly as done in Hyb_6. $\mathcal{A}_{\pi^{\mathsf{SM}}}$ also interacts with a challenger $\mathcal{C}_{\pi^{\mathsf{SM}}}$ and corrupts the same parties as done by $\mathcal{A}$. For every honest party P_i, $\mathcal{A}_{\pi^{\mathsf{SM}}}$ receives a first round message $\mathsf{msg}_{1,i}$ which is sent to $\mathcal{A}$ in round 1 of protocol π on the main thread. On receiving $\mathsf{msg}_{1,j}$ for every malicious party P_j in round 1 of the main thread from $\mathcal{A}$, $\mathcal{A}_{\pi^{\mathsf{SM}}}$ forwards this to $\mathcal{C}_{\pi^{\mathsf{SM}}}$ as the first round messages of the malicious parties.

$\mathcal{A}_{\pi^{\mathsf{SM}}}$ then creates a set of 5 look-ahead threads, in each of which, it runs rounds 2 and 3 of the protocol alone. In each look-ahead thread, $\mathcal{A}_{\pi^{\mathsf{SM}}}$ computes msg_2^i using input 0. Recall from the properties of the scheme π^{SM} that $\mathcal{A}_{\pi^{\mathsf{SM}}}$ can do since msg_1^i is public coin. As in the proof of Claim 25, recall that using these 5 threads (that are all "GOOD with respect to some honest party H" with noticeable probability), $\mathcal{A}_{\pi^{\mathsf{SM}}}$ can successfully run the input extraction phase.

Then, on the main thread, as before, the messages $\mathsf{msg}_{2,i}$ and $\mathsf{msg}_{2,j}$ corresponding to every honest party P_i and malicious party P_j are sent across between $\mathcal{C}_{\pi^{\mathsf{SM}}}$ and $\mathcal{A}$ via $\mathcal{A}_{\pi^{\mathsf{SM}}}$ in round 3 of protocol π on the main thread. $\mathcal{A}_{\pi^{\mathsf{SM}}}$ also sends the values $(\mathsf{y}, \{\mathsf{x}_j, \mathsf{r}_j\})$ (obtained in the input extraction phase) to $\mathcal{C}_{\pi^{\mathsf{SM}}}$. Then, for every honest party P_i $\mathcal{A}_{\pi^{\mathsf{SM}}}$ receives a third round message $\mathsf{msg}_{3,i}$ which is sent to $\mathcal{A}$ in round 4 of protocol π. On receiving $\mathsf{msg}_{3,j}$ for every malicious party P_j in round 3 from $\mathcal{A}$, $\mathcal{A}_{\pi^{\mathsf{SM}}}$ forwards this to $\mathcal{C}_{\pi^{\mathsf{SM}}}$ as the third round messages of the malicious parties. The rest of protocol π is performed exactly as in Hyb_6. Observe that when $\mathcal{C}_{\pi^{\mathsf{SM}}}$ computes the messages of protocol π^{SM} honestly, $\mathcal{A}$'s view corresponds to Hyb_6 and then $\mathcal{C}_{\pi^{\mathsf{SM}}}$ computes simulated messages, $\mathcal{A}$'s view corresponds to Hyb_7.

Now, let's see how $\mathcal{A}_{\pi^{\mathsf{SM}}}$ breaks the security of protocol π^{SM}. For every honest party P_i and malicious party P_j, $\mathcal{A}_{\pi^{\mathsf{SM}}}$ runs the extractor $\mathsf{Ext}_{\mathsf{NMCom}}$ of the non-malleable commitment scheme using the messages in all the "Good" threads that correspond to the non-malleable commitment from P_j to P_i. Let the output of $\mathsf{Ext}_{\mathsf{NMCom}}$ be t_i^*. If for some pair of parties, $\mathsf{TDValid}(\mathsf{td}_{1,i}, \mathsf{t}_i^*) = 1$, then $\mathcal{A}_{\pi^{\mathsf{SM}}}$ outputs case 2 indicating that the messages sent by the challenger were simulated. Else, it outputs case 1 indicating that the messages were generated honestly.

Let's analyze why this works. We know that the invariant doesn't hold in Hyb_7 so there exists honest party P_i^* and malicious party P_j^* such that event E doesn't hold. That is, the adversary P_j^*, using the non-malleable commitment, commits to a valid trapdoor t_i^* for the trapdoor generation messages of the honest party P_i^* with non-negligible probability ϵ. Therefore, since the invariant holds in Hyb_6 with non-negligible probability, when the extractor $\mathsf{Ext}_{\mathsf{NMCom}}$ outputs a valid trapdoor t_i^* corresponding to this pair of parties, it must be the case that we are in Hyb_6 with non-negligible probability. That is, when $\mathsf{Ext}_{\mathsf{NMCom}}$ outputs a valid trapdoor, it corresponds to $\mathcal{A}_{\pi^{\mathsf{SM}}}$ receiving simulated messages and otherwise, it corresponds to $\mathcal{A}_{\pi^{\mathsf{SM}}}$ receiving honestly generated messages. Thus, $\mathcal{A}_{\pi^{\mathsf{SM}}}$ breaks the security of the protocol π^{SM} which is a contradiction. Hence, the invariant holds in Hyb_7 as well.

Remark: Note that the adversary could potentially cause some honest parties to abort at the end of round 3 and some to continue the protocol. In the event that the adversary causes all the honest parties to abort, there is no difference between the two hybrids. In the case when the adversary causes at least one honest party to not abort, □

Claim 33. *Assuming the security of protocol π^{SM}, Hyb_6 is indistinguishable from Hyb_7.*

Proof. The only difference between Hyb_6 and Hyb_7 is that in Hyb_7, in the main thread, the simulator now computes the messages of protocol π^{SM} using the simulated algorithms $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$. If there exists an adversary $\mathcal{A}$ that can distinguish between the two hybrids, we can use $\mathcal{A}$ to design an adversary $\mathcal{A}_{\pi^{\mathsf{SM}}}$ that breaks the security of the protocol π^{SM}. The rest of the proof is similar to the proof of Claim 32 above.

□

Claim 34. *The invariant holds in* Hyb_8.

Proof. There is no difference in the main thread between Hyb_7 and Hyb_8 and so the invariant continues to remain true. □

Claim 35. Hyb_7 *is computationally indistinguishable from* $\mathsf{Hyb}_{\mathsf{Ideal}}$ *except with probability atmost* $\frac{\mu}{4} + \mathsf{negl}(\lambda)$.

Proof. We argue the proof via the following two cases.
Case 1: Pr[not abort] $\geq \frac{\mu}{4}$:
Suppose the adversary doesn't abort with probability greater than $\frac{\mu}{4}$, then, by Chernoff's bound, in Hyb_7, except with negligible probability, in the set of $\frac{5 \cdot n \cdot \lambda}{\mu}$ threads, there will be at least 5 "GOOD threads with respect to P_{i^*} for some honest party P_{i^*}. Also, in $\mathsf{Hyb}_{\mathsf{Ideal}}$ (the final ideal world simulator), $\mathsf{Sim}_{\mathsf{Hyb}}$ will run an expected polynomial number of threads to get at least 12λ (which is greater than $5 \cdot n$) "GOOD" threads. Thus, the input and trapdoor extraction will be successful in both hybrids except with negligible probability.

Therefore, the only difference between Hyb_7 and $\mathsf{Hyb}_{\mathsf{Ideal}}$ is that in Hyb_7, after the input extraction, $\mathsf{Sim}_{\mathsf{Hyb}}$ resamples the main thread $\frac{\lambda}{\mu}$ times while in $\mathsf{Hyb}_{\mathsf{Ideal}}$, $\mathsf{Sim}_{\mathsf{Hyb}}$ first estimates the probability of not aborting to be ϵ' and then resamples the main thread $\mathsf{min}(2^\lambda, \frac{\lambda^2}{\epsilon'})$ times. The rest of the proof follows in a very similar manner to the proof of claim 5.8 in [Lin17]. That is, we show that if the "Check Abort" step succeeds, the simulator fails in $\mathsf{Hyb}_{\mathsf{Ideal}}$ only with negligible probability using the claim in [Lin17]. Also, by a Markov argument, we know that in Hyb_7, if the "Check Abort" step succeeds, the simulation successfully forces the output and hence, this completes the proof.
Case 2: Pr[not abort] $< \frac{\mu}{4}$:
Suppose the adversary doesn't cause an abort at the end of round 3 with probability less than $\frac{\mu}{4}$. Then, in both hybrids, $\mathsf{Sim}_{\mathsf{Hyb}}$ aborts at the end of the "Check Abort" step except with probability $\frac{\mu}{4}$. Thus, in this case, the adversary's view in Hyb_7 is indistinguishable to that in $\mathsf{Hyb}_{\mathsf{Ideal}}$ except with probability atmost $\frac{\mu}{4} + \mathsf{negl}(\lambda)$. □

We now calculate the probability that the adversary can distinguish between $\mathsf{Hyb}_{\mathsf{Real}}$ and $\mathsf{Hyb}_{\mathsf{Ideal}}$.

Observe that except in 2 cases, every pair of consecutive hybrids is computationally indistinguishable. In each of those 2 cases (the indistinguishability between Hyb_7 and $\mathsf{Hyb}_{\mathsf{Ideal}}$, Hyb_0 and $\mathsf{Hyb}_{\mathsf{Real}}$), the pair of hybrids are indistinguishable except with probability atmost $\frac{\mu}{4} + \mathsf{negl}(\lambda)$. Thus, $\mathsf{Hyb}_{\mathsf{Real}}$ and $\mathsf{Hyb}_{\mathsf{Ideal}}$ are indistinguishable except with probability atmost $\frac{\mu}{2} + \mathsf{negl}(\lambda)$. This contradicts our assumption that there exists an adversary $\mathcal{A}$ that can distinguish the real and ideal worlds with some non-negligible probability μ. This completes the proof.

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A WI with Bounded Rewinding Security

In this section, we sketch the proof of Theorem 7.

More generally, for any constant $B > 0$, we construct a 3 round delayed-input witness indistinguishable argument with B-rewinding security according to Definition 6 assuming the existence of injective one way functions.

Overview of the Protocol and Proof. At a very high level, the protocol consists of combining the 3-round delayed-input WI protocol in [LS90] with the bounded rewinding secure 3-round "MPC in the head" based 3-round protocol of [IKOS07]. The 3-round delayed-input WI protocol in [LS90] for proving that a Graph G contains Hamiltonian cycle H is as follows. The prover first commits to a random hamiltonian matrix H' in the first round. Upon receiving verifier challenge, if this challenge is 0, the prover decommits the entire matrix H' (containing 0's representing non-edges and 1's representing edges), and the verifier checks if it is Hamiltonian. If this challenge is 1, the prover uses input G, H to compute a random permutation π that maps H to H'. It outputs π, and additionally decommits all values in H' that correspond to non-edges in $\pi(G)$. The verifier checks that all opened values in H' are 0.

The main idea to obtain bounded rewinding security, is to offload the process of verifying the commitment to the Hamiltonian H' (corresponding to $b = 0$), to an "MPC-in-the-Head" protocol [IKOS07] with bounded rewinding security. Opening this up, the commitment $c = \mathsf{Com}(H')$ will be accompanied by commitments to views of $n = (6B + 1)$ virtual players in a *robust* MPC protocol, with privacy against any subset of $2B$ collusions. The MPC protocol will output 1 to all virtual players iff H' is indeed a commitment to a hamiltonian. The verifier challenge corresponding to $b = 0$, will be replaced by $\binom{n}{2}$ possible challenges revealing the views of different subsets of 2 players, and the verifier will check these views for correctness and consistency. By $2B$-privacy of MPC, even given the views of $2B$ players, a malicious verifier will learn no information about H', giving us the desired bounded-rewinding security. Soundness will follow by [LS90] because the soundness of MPC-in-the-head will guarantee that H' is Hamiltonian.

To prove B-rewinding security, we note that each query of the malicious verifier V^* can be interpreted as consisting of two parts - the first is a bit b that is either 0 or 1 and if $b = 0$, the

second part consists of a random subset of 2 parties whose views must be opened. The simulator runs by guessing the set of queries made across all the B-rewinds in advance, and appropriately running the [LS90] and [IKOS07] simulators based on this guess. We now describe the construction for completeness.

MPC-in-the-Head. We make black-box use of a 3-round zero knowledge protocol (non delayed-input) with constant soundness error and bounded rewinding security.

Definition 17 (3-Round Constant Soundness ZK with Bounded Rewinding Security)**.** *Fix a positive integer B. A delayed-input 3-round interactive argument (as defined in Definition 1) for an NP language L, with an NP relation R_L is said to have B-*Rewinding Security *if there exists a simulator* Sim *such that for every non-uniform PPT interactive Turing Machine V^*, it holds that $\{\mathsf{REAL}^{V^*}(1^\lambda)\}_\lambda$ and $\{\mathsf{IDEAL}^{V^*}(1^\lambda)\}_\lambda$ are computationally indistinguishable, where the random variable $\mathsf{REAL}^{V^*}(1^\lambda)$ is defined via the following experiment. In what follows we denote by $P_{\mathsf{ZK}1}$ the prover's algorithm in the first round, and similarly we denote by $P_{\mathsf{ZK}3}$ his algorithm in the third round.*

Experiment $\mathsf{REAL}^{V^*}(1^\lambda)$ *is defined as follows:*

1. *Run $P_{\mathsf{ZK}1}(1^\lambda, x, w; r)$ and obtain output rwi_1 to be sent to the verifier.*
2. *Run the verifier $V^*(1^\lambda, \mathsf{rwi}_1)$ and interpret its output as message rwi_2.*
3. *Run $P_{\mathsf{ZK}3}(1^\lambda, \mathsf{rwi}_2, x, w; r)$, where $P_{\mathsf{ZK}3}$ is the (honest) prover's algorithm for generating the third message of the WI protocol, and send its output rwi_3 to V^*.*
4. *Set a counter $i = 0$.*
5. *If $i < B$, then set $i = i + 1$, and V^* (given all the information so far) generates another message rwi_2^i, and receives the (honest) prover's message $P_{\mathsf{ZK}3}(\mathsf{rwi}_2^i, x, w; r)$. Repeat this step until $i = B$.*
6. *The output of the experiment is the view of V^*.*

Experiment $\mathsf{IDEAL}^{V^*}(1^\lambda)$ *is the output of the experiment* $\mathsf{Sim}^{V^*}(1^\lambda, x; r)$.

Imported Theorem 2. *[IKOS07] Assume the existence of injective one-way functions. Then, there exists a 3-round constant-soundness zero-knowledge protocol for proving NP statements, that is simulatable under B-bounded rewinding according to Definition 17.*

The work of [IKOS07] constructs such a protocol by having the prover emulate a $t = 2B$-private MPC "in his head".

Construction. We construct the protocol RWI with constant soundness error for the Graph Hamiltonicity problem, in Figure 7. Soundness of this protocol can be amplified via parallel repetition while preserving the WI property: and this can be used to prove general NP statements via a Karp reduction. The protocol RWI consists of 4 algorithms $(\mathsf{RWI}_1, \mathsf{RWI}_2, \mathsf{RWI}_3, \mathsf{RWI}_4)$ where the first 3 denote the algorithms used by the prover and verifier to send their messages and the last is the final verification algorithm.

We use the protocol from Imported Theorem 2. We denote its algorithms by $\mathsf{Head.ZK} = (\mathsf{Head.ZK}_1, \mathsf{Head.ZK}_2, \mathsf{Head.ZK}_3, \mathsf{Head.ZK}_4)$, where the first 3 denote the algorithms used by the prover and verifier to generate their messages, and the last is the final verification algorithm.

Inputs: Prover P obtains input (in the third round) an efficiently sampleable graph G (the statement) and a corresponding hamiltonian cycle H (the witness).

1. **Round 1: Prover message:**
   - Pick a random cyclic graph H', uniform randomness r and compute $\mathsf{nmcom} = \mathsf{Com}(H'; \mathsf{r})$ to the adjacency matrix of H' using a non-interactive commitment Com.
   - Compute $\mathsf{hzk}_1^{P \to V} \leftarrow \mathsf{Head.ZK}_1(1^\lambda)$ and send $(\mathsf{nmcom}, \mathsf{prove}_1^{P \to V})$ to V.
2. **Round 2: Verifier message:**
   - Pick a random bit b.
   - If $b = 0$, compute $\mathsf{hzk}_2^{V \to P} \leftarrow \mathsf{Head.ZK}_2(\mathsf{hzk}_1^{P \to V})$ and send $(0, \mathsf{hzk}_2^{V \to P})$ to P.
   - If $b = 1$, send $(1, \perp)$ to P.
3. **Round 3: Prover message:**
   - On input graph G with hamiltonian cycle H, do the following:
   - If $b = 0$, compute and send $\mathsf{hzk}_3^{P \to V} \leftarrow \mathsf{Head.ZK}_3(\mathsf{hzk}_1^{P \to V}, \mathsf{hzk}_2^{V \to P})$ to prove using witness $w = (H', \mathsf{r})$ that H' is a cyclic graph and $\mathsf{nmcom} = \mathsf{Com}(H'; \mathsf{r})$.
   - If $b = 1$, compute and send a permutation π that maps H' onto H. Also, decommit to all edges in the adjacency graph of H' that correspond to non-edges in $\pi^{-1}(G)$.
4. **Verifier Output:**
   - If $b = 0$, compute the output of the algorithm $\mathsf{Head.ZK}_4$.
   - If $b = 1$, check that all opened values are 0 and map (via π) to all non-edges of G.

Figure 7: 3 round Bounded Rewinding Secure WI

B MPC - Proof of Aborting Case

Recall that the only difference between the two hybrids is if the adversary causes $\mathsf{Sim}_{\mathsf{Hyb}}$ to abort at the end of round 3 - that is, the "Check Abort" step doesn't succeed. In that case, in Hyb_0, $\mathsf{Sim}_{\mathsf{Hyb}}$ uses the honest parties' inputs to run the protocol while in Hyb_1, $\mathsf{Sim}_{\mathsf{Hyb}}$ runs the protocol using input 0 for every honest party. We will now show that these two hybrids are indistinguishable via a sequence of intermediate hybrids H_0 to H_{12}. Here, H_0 will correspond to Hyb_0 and H_{12} will correspond to Hyb_1.

- H_0 : This is same as Hyb_0.
- H_1 **- Simulate Weak ZK:** In this hybrid, $\mathsf{Sim}_{\mathsf{Hyb}}$ creates two sets of look-ahead threads that run only round 3 of the protocol. The first set is identical to the main thread in Hyb_0. The second set is identical to the main thread in Hyb_1 (same as H_{12}) - that is, it computes an extractable commitment using input 0 and the messages for the underlying semi-malicious MPC protocol π^{SM} using input 0.

  These look-ahead threads are used to simulate the Weak ZK argument WZK. Using all these look-ahead threads, $\mathsf{Sim}_{\mathsf{Hyb}}$ runs the algorithm $\mathsf{Sim}_{\mathsf{WZK}}$ of the weak ZK protocol to simulate the weak ZK argument given by every honest party in round 3 of the main thread. Note that in the look-ahead threads, the weak ZK is computed honestly.

Recall from [JKKR17] that $\mathsf{Sim}_{\mathsf{WZK}}$ is a distinguisher-dependent simulator that uses the distinguisher's response on all these look-ahead threads to simulate the weak ZK. Note that these look-ahead threads that are created here are local to this proof and have nothing to do with the look-ahead threads that are created by the actual simulator Sim for the overall protocol. In particular, these local look-ahead threads are not created by Sim in the overall proof. As a result, though $\mathsf{Sim}_{\mathsf{WZK}}$ depends on the distinguisher, we use it only in the underlying reductions here and not in the overall simulation and hence our overall simulator Sim is not distinguisher dependent.

- H_2 **- Change Second ECom:** In the main thread, for each honest party P_i, $\mathsf{Sim}_{\mathsf{Hyb}}$ does the following: in round 3, compute $\mathsf{ecom}_{b,3}^{i\to j} = \mathsf{ECom}_3(\mathsf{x}_i, \mathsf{r}_i, \mathsf{ecom}_{b,1}^{i\to j}, \mathsf{ecom}_{b,2}^{j\to i}; \mathsf{r}_{b,\mathsf{ecom}}^{i\to j})$. That is, in the main thread, the simulator now commits to the input in the second extractable commitment scheme too.

  Further, $\mathsf{Sim}_{\mathsf{Hyb}}$ also changes the first set of look-ahead threads. In the first set of look-ahead threads, the second extractable commitment is computed as in the main thread: that is, $\mathsf{ecom}_{b,3}^{i\to j} = \mathsf{ECom}_3(\mathsf{x}_i, \mathsf{r}_i, \mathsf{ecom}_{b,1}^{i\to j}, \mathsf{ecom}_{b,2}^{j\to i}; \mathsf{r}_{b,\mathsf{ecom}}^{i\to j})$. In the second set of look-aheads, the second extractable commitment continues to be computed using input 0: that is, $\mathsf{ecom}_{b,3}^{i\to j} = \mathsf{ECom}_3(0, \mathsf{r}_i, \mathsf{ecom}_{b,1}^{i\to j}, \mathsf{ecom}_{b,2}^{j\to i}; \mathsf{r}_{b,\mathsf{ecom}}^{i\to j})$ where r_i is sampled fresh in each look-ahead thread.

- H_3 **- Switch RWI:** In every thread (main and look-aheads), in round 3, $\mathsf{Sim}_{\mathsf{Hyb}}$ computes the RWI from each honest party P_i to malicious party P_j using the witness for the second statement: that is, using the second extractable commitment.

- H_4 **- Switch Commitment:** In the main thread, in round 3, $\mathsf{Sim}_{\mathsf{Hyb}}$ computes $\mathsf{nc}_i \leftarrow \mathsf{NCom}(0; \mathsf{r}_{\mathsf{nc},i})$. Note that in the look ahead threads, nc_i continues to be computed as a commitment to 1 so that the weak ZK arguments in the look-ahead threads can be honestly generated.

- H_5 **- Switch RWI:** In the main thread, in round 3, $\mathsf{Sim}_{\mathsf{Hyb}}$ computes the RWI from each honest party P_i to malicious party P_j using the witness for the fourth statement: that is, using the commitment nc_i to input 0.

- H_6 **- Change First ECom:** In the main thread, for each honest party P_i, $\mathsf{Sim}_{\mathsf{Hyb}}$ does the following: in round 3, compute $\mathsf{ecom}_{a,3}^{i\to j} = \mathsf{ECom}_3(0, \mathsf{r}_i, \mathsf{ecom}_{a,1}^{i\to j}, \mathsf{ecom}_{a,2}^{j\to i}; \mathsf{r}_{a,\mathsf{ecom}}^{i\to j})$. That is, in the main thread, the simulator now commits to input 0 in the first extractable commitment scheme.

- H_7 **- Change π^{SM}:** In the main thread, for every honest party P_i, $\mathsf{Sim}_{\mathsf{Hyb}}$ compute $\mathsf{msg}_{2,i} \leftarrow \mathcal{S}_2(\mathsf{Trans}_2; \mathsf{r}_i)$. Note that this is same as computing $\mathsf{msg}_{2,i} \leftarrow \pi_2^{\mathsf{SM}}(0, \mathsf{Trans}_1; \mathsf{r}_i)$.

- H_8 **- Switch RWI:** In every thread (main and look-aheads), in round 3, $\mathsf{Sim}_{\mathsf{Hyb}}$ computes the RWI from each honest party P_i to malicious party P_j using the witness for the first statement: that is, using the first extractable commitment.

- H_9 **- Change Second ECom:** In the main thread and every look-ahead thread in the first set, for each honest party P_i, $\mathsf{Sim}_{\mathsf{Hyb}}$ does the following: in round 3, compute $\mathsf{ecom}_{b,3}^{i\to j} = \mathsf{ECom}_3(\bot, \mathsf{ecom}_{b,1}^{i\to j}, \mathsf{ecom}_{b,2}^{j\to i}; \mathsf{r}_{b,\mathsf{ecom}}^{i\to j})$. That is, the simulator now commits to $\bot$ in the second extractable commitment scheme.

- H_{11} **- Switch Commitment:** In the main thread, in round 3, $\mathsf{Sim}_{\mathsf{Hyb}}$ computes $\mathsf{nc}_i \leftarrow \mathsf{NCom}(1; \mathsf{r}_{\mathsf{nc},i})$.

- H_{12} **- Stop Simulating Weak ZK:** In this hybrid, $\mathsf{Sim}_{\mathsf{Hyb}}$ stops the two sets of look-ahead threads and computes the Weak ZK argument honestly. This hybrid exactly corresponds to Hyb_1.

We now show that each pair of successive hybrids in the above list is computationally indistinguishable and that completes the proof of Claim 21.

Sub-Claim 5. *Assuming the Weak Zero Knowledge property of the argument system* WZK*,* H_0 *is computationally indistinguishable from* H_1*.*

Proof. The only difference between the two hybrids is that in H_0, the Weak ZK argument is computed honestly while in H_1, the Weak ZK argument on the main thread is computed using the simulator $\mathsf{Sim}_{\mathsf{WZK}}$ from [JKKR17]. The proof of indistinguishability of the two hybrids directly follows from the security of the Weak ZK argument system constructed in [JKKR17].

Here, note that for the second set of look-ahead threads, we can generate the extractable commitment using input 0 even though the first 2 rounds are already fixed because the scheme is delayed input and allows sampling a fresh value s for the term $\mathsf{ecom}_{b,3,N+1}$ in each third round message. Informally, recall that this is the reusability property mentioned in Section 7.1. That is, in round 3 of each look-ahead thread in the second set, corresponding to the commitment from every honest party P_i to malicious party P_j, $\mathsf{Sim}_{\mathsf{Hyb}}$ computes $\mathsf{ecom}_{b,3}^{i\to j}$ as $(\mathsf{ecom}_{b,3,1}^{i\to j}, \ldots, \mathsf{ecom}_{b,3,N+2}^{i\to j})$ where $\mathsf{ecom}_{b,3,1}^{i\to j}, \ldots, \mathsf{ecom}_{b,3,N}^{i\to j}$ are the same in all the threads, $\mathsf{ecom}_{b,3,N+1} = s^{i\to j}$ is sampled afresh in each thread and $\mathsf{ecom}_{b,3,N+2} = \mathsf{PRF}(k^{i\to j}, s^{i\to j}) \oplus 0$. Also, note that the second round message of protocol π^{SM} can also be generated using input 0 in the second set of look-ahead threads because the first round message doesn't depend on the input at all. □

Sub-Claim 6. *Assuming the security of the non-interactive commitment* Com *and the pseudo-random function* PRF *used inside the scheme* ECom*,* H_1 *is computationally indistinguishable from* H_2*.*

Proof. Let's briefly recall the construction of the scheme ECom from Section 7.1 in the context of protocol π. Consider a party P_i as committer interacting with a party P_j as receiver using input message m. In round 1, P_i computes $\mathsf{ecom}_1^{i\to j} = \{\mathsf{ecom}_{1,1}^{i\to j}, \ldots, \mathsf{ecom}_{1,N}^{i\to j}\}$ where $\mathsf{ecom}_{1,\ell}^{i\to j} = \mathsf{Com}(\mathsf{p}_\ell^{i\to j})$ where each $\mathsf{p}_\ell^{i\to j}$ is a random polynomial of degree 4 (defined in Section 7.1). In round 2, P_j generates $\mathsf{ecom}_2^{j\to i} = (\mathsf{z}_1^{j\to i}, \ldots, \mathsf{z}_N^{j\to i})$ where each $\mathsf{z}_\ell^{j\to i}$ is a random value. Then, in round 3, P_i outputs $\mathsf{ecom}_3^{i\to j} = \{\mathsf{ecom}_{3,1}^{i\to j}, \ldots, \mathsf{ecom}_{3,N+2}^{i\to j}\}$ where $\mathsf{ecom}_{3,\ell}^{i\to j} = (k^{i\to j} \oplus \mathsf{p}_\ell^{i\to j}(0), \mathsf{p}_\ell^{i\to j}(\mathsf{z}_\ell^{j\to i}))$ for each $l \in [N]$ and $\mathsf{ecom}_{3,N+1} = s^{i\to j}$ and $\mathsf{ecom}_{3,N+2} = \mathsf{PRF}(k^{i\to j}, s^{i\to j}) \oplus m$.

We will now prove this subclaim via a series of intermediate sub-hybrids $\mathsf{Sub.Hyb}_1$ to $\mathsf{Sub.Hyb}_5$ where $\mathsf{Sub.Hyb}_1$ corresponds to H_1 and $\mathsf{Sub.Hyb}_5$ corresponds to H_2. Throughout, we skip the subscript "b" to ease the exposition.

- $\mathsf{Sub.Hyb}_1$**:** This is same as H_1.

- $\mathsf{Sub.Hyb}_2$**:** In the main thread and each look ahead thread in the first set, for every honest party P_i and malicious party P_j, in round 1, compute $\mathsf{ecom}_{1,\ell}^{i\to j} = \mathsf{Com}(0)$ for all $\ell \in [N]$. This is indistinguishable from the previous hybrid by the hiding property of the scheme Com.

- $\mathsf{Sub.Hyb}_3$**:** In the main thread and each look ahead thread in the first set, for every honest party P_i and malicious party P_j, pick $k_1^{i\to j}$ uniformly at random. Then, for each $\ell \in [N]$, pick a new degree B polynomial $\mathsf{q}_\ell^{i\to j}$ such that $(k^{i\to j} \oplus \mathsf{p}_\ell^{i\to j}(0)) = (k_1^{i\to j} \oplus \mathsf{q}_\ell^{i\to j}(0))$. Compute $\mathsf{ecom}_{3,\ell}$ as $(k_1^{i\to j} \oplus \mathsf{q}_\ell^{i\to j}(0), \mathsf{q}_\ell^{i\to j}(\mathsf{z}_\ell^{i\to j}))$.
  This hybrid is statistically indistinguishable from the previous hybrid.

- $\mathsf{Sub.Hyb}_4$**:** In the main thread and each look ahead thread in the first set, for every honest party P_i and malicious party P_j, in round 1, compute $\mathsf{ecom}_{1,\ell}^{i\to j} = \mathsf{Com}(\mathsf{q}_\ell^{i\to j})$ for all $\ell \in [N]$.
  This is indistinguishable from the previous hybrid by the hiding property of the scheme Com.

- $\mathsf{Sub.Hyb}_5$**:** In the main thread and each look ahead thread in the first set, for every honest party P_i and malicious party P_j, compute $\mathsf{ecom}_{3,N+2}^{i\to j} = \mathsf{PRF}(k_1^{i\to j}, \mathsf{ecom}_{3,N+1}^{i\to j}) \oplus \mathsf{x}_i$. This is same as H_2.
  This is indistinguishable from the previous sub-hybrid by the security of the pseudorandom function PRF.

□

Sub-Claim 7. *Assuming the reusability security of the scheme* RWI*,* H_2 *is computationally indistinguishable from* H_3*.*

Proof. This follows from the reusability property of RWI in the same manner as in [JKKR17]. (In particular, we make L invocations of the reusability security, where L is the total number of threads.) □

Sub-Claim 8. *Assuming the hiding property of the non-interactive commitment scheme* NCom*,* H_3 *is computationally indistinguishable from* H_4*.*

Proof. The only difference between the two hybrids is that in H_3, for every honest party P_i, nc_i is computed as a commitment of 1 in the main thread while in H_4 it is computed as a commitment of 0. Note that in both hybrids, there is no change on any of the look-ahead threads. Thus, if there exists an adversary that can distinguish between the two hybrids, there exists a reduction that can break the hiding property of the commitment scheme. □

Sub-Claim 9. *Assuming the reusability security of the scheme* RWI*,* H_4 *is computationally indistinguishable from* H_5*.*

Proof. The proof is similar to the proof of Sub-Claim 7. The only difference being that here, the witness is changed only in the main thread and not in all the threads. □

Sub-Claim 10. *Assuming the hiding property of the commitment scheme* Com *and the security of the pseudorandom function* PRF *used inside the scheme* ECom*,* H_5 *is computationally indistinguishable from* H_6*.*

Proof. The proof is similar to the proof of Sub-Claim 6.

□

Sub-Claim 11. *Assuming the security of the semi-malicious MPC protocol* π^{SM}*,* H_6 *is computationally indistinguishable from* H_7*.*

Proof. The only difference between H_6 and H_7 is that in H_7, in the main thread, the simulator now computes the second message of protocol π^{SM} using the simulated algorithms $\mathcal{S}_2$. Assume for the sake of contradiction that there exists an adversary $\mathcal{A}$ that can distinguish between the two hybrids. We will use $\mathcal{A}$ to design an adversary $\mathcal{A}_{\pi^{\mathsf{SM}}}$ that breaks the security of the protocol π^{SM}.

$\mathcal{A}_{\pi^{\mathsf{SM}}}$ performs the role of $\mathsf{Sim}_{\mathsf{Hyb}}$ in its interaction with $\mathcal{A}$ exactly as done in H_6. $\mathcal{A}_{\pi^{\mathsf{SM}}}$ also interacts with a challenger $\mathcal{C}_{\pi^{\mathsf{SM}}}$ and corrupts the same parties as done by $\mathcal{A}$. For every honest party P_i, $\mathcal{A}_{\pi^{\mathsf{SM}}}$ receives a first round message $\mathsf{msg}_{1,i}$ which is sent to $\mathcal{A}$ in round 1 of protocol π on the main thread. On receiving $\mathsf{msg}_{1,j}$ for every malicious party P_j in round 1 of the main thread from $\mathcal{A}$, $\mathcal{A}_{\pi^{\mathsf{SM}}}$ forwards this to $\mathcal{C}_{\pi^{\mathsf{SM}}}$ as the first round messages of the malicious parties. Similarly, the messages $\mathsf{msg}_{2,i}$ and $\mathsf{msg}_{2,j}$ corresponding to every honest party P_i and malicious party P_j are sent across between $\mathcal{C}_{\pi^{\mathsf{SM}}}$ and $\mathcal{A}$ via $\mathcal{A}_{\pi^{\mathsf{SM}}}$ in round 3 of protocol π on the main thread. The rest of protocol π is performed exactly as in H_6. In particular, $\mathcal{A}_{\pi^{\mathsf{SM}}}$ generates the messages of protocol π^{SM} in the look-ahead threads on its own even though the first message of the protocol was received externally from $\mathcal{C}_{\pi^{\mathsf{SM}}}$. Recall that this follows from a property of the protocol π^{SM} that the first round message is independent of the input and doesn't have any secret information used to generate the second round messages.

Observe that when $\mathcal{C}_{\pi^{\mathsf{SM}}}$ computes the messages of protocol π^{SM} honestly, $\mathcal{A}$'s view corresponds to H_6 and when $\mathcal{C}_{\pi^{\mathsf{SM}}}$ computes simulated messages, $\mathcal{A}$'s view corresponds to H_7. Therefore, if $\mathcal{A}$ can distinguish between these two hybrids, $\mathcal{A}_{\pi^{\mathsf{SM}}}$ will use the same distinguishing guess to break the security of protocol π^{SM}.

□

Sub-Claim 12. *Assuming the reusability security of the scheme* RWI*,* H_7 *is computationally indistinguishable from* H_8*.*

Proof. The proof is same as the proof of Sub-Claim 7 discussed above. □

Sub-Claim 13. *Assuming the hiding property of the commitment scheme* Com *and the security of the pseudorandom function* PRF *used inside the scheme* ECom*,* H_8 *is computationally indistinguishable from* H_9*.*

Proof. The proof is same as the proof of Sub-Claim 6 discussed above. □

Sub-Claim 14. *Assuming the hiding property of the non-interactive commitment scheme* NCom*,* H_9 *is computationally indistinguishable from* H_{10}*.*

Proof. The proof is same as the proof of Sub-Claim 8 discussed above. □

Sub-Claim 15. *Assuming the Weak Zero Knowledge property of the argument system* WZK*,* H_{10} *is computationally indistinguishable from* H_{11}*.*

Proof. The proof is same as the proof of Sub-Claim 5 discussed above. □

C Non-adaptive Bounded Rewinding WI with Reusability Security

In this section, we first define 3 round delayed input WI with reusability security verbatim from Definition 10 in [JKKR17]. We then define 3 round delayed input WI with non-adaptive rewinding security which is a modification of Definition 6. In the next subsection, we show how to construct a 3 round WI protocol that is non-adaptive bounded rewinding secure and reusable secure.

Definition 18 (3-Round Delayed-Input WI with Reusability Security)**.** *A delayed-input 3-round interactive argument (as defined in Definition 1) for an NP language L, with an NP relation R_L is said to be WI with* Resuable Security *if for every non-uniform PPT interactive Turing Machine V^*, every $k = \mathsf{poly}(\lambda)$ and every sequence $(x^1, w^1), (x^2, w^2), ...(x^{k-1}, w^{k-1}), (x^k, w_0^k, w_1^k)$ it holds that $\{\mathsf{REAL}_0^{V^*}(1^\lambda)\}_\lambda$ and $\{\mathsf{REAL}_1^{V^*}(1^\lambda)\}_\lambda$ are computationally indistinguishable, where for $b \in \{0,1\}$ the random variable $\mathsf{REAL}_b^{V^*}(1^\lambda)$ is defined via the following experiment. In what follows we denote by $P_{\mathsf{ZK}1}$ the prover's algorithm in the first round, and similarly we denote by $P_{\mathsf{ZK}3}$ his algorithm in the third round.*

Experiment $\mathsf{REAL}_b^{V^*}(1^\lambda)$**:**

1. *Run $P_{\mathsf{ZK}1}(1^\lambda)$ and denote its output by (rwi_1, σ), where σ is its secret state, and rwi_1 is the message to be sent to the verifier.*
2. *Run the verifier $V^*(1^\lambda, \mathsf{rwi}_1)$, who outputs a message rwi_2.*
3. *Next, P receives inputs $((x^1, w^1), (x^2, w^2), ...(x^{k-1}, w^{k-1}), (x^k, w_0^k, w_1^k))$ and V receives $(x^1, x^2...x^k)$.*
4. *Run $P_{\mathsf{ZK}3}(\sigma, \mathsf{rwi}_2, x^1, w^1), \ldots, P_{\mathsf{ZK}3}(\sigma, \mathsf{rwi}_2, x^{k-1}, w^{k-1}), P_{\mathsf{ZK}3}(\sigma, \mathsf{rwi}_2, x^k, w_b^k)$, where $P_{\mathsf{ZK}3}$ is the (honest) prover's algorithm for generating the third message of the WI protocol, and send its messages $(\mathsf{rwi}_3^1, \ldots, \mathsf{rwi}_3^k)$ to V^*.*
5. *The output of the experiment is the output of V^*.*

Definition 19 (3-Round Delayed-Input WI with Non-Adaptive Bounded Rewinding Security)**.** *Fix a positive integer B. A delayed-input 3-round interactive argument (as defined in Definition 1) for an NP language L, with an NP relation R_L is said to be WI with* Non-Adaptive B-Rewinding Security *if for every non-uniform PPT interactive Turing Machine V^*, it holds that $\{\mathsf{REAL}_0^{V^*}(1^\lambda)\}_\lambda$ and $\{\mathsf{REAL}_1^{V^*}(1^\lambda)\}_\lambda$ are computationally indistinguishable, where for $b \in \{0,1\}$ the random variable $\mathsf{REAL}_b^{V^*}(1^\lambda)$ is defined via the following experiment. In what follows we denote by $P_{\mathsf{ZK}1}$ the prover's algorithm in the first round, and similarly we denote by $P_{\mathsf{ZK}3}$ his algorithm in the third round.*

Experiment $\mathsf{REAL}_b^{V^*}(1^\lambda)$**:**

1. *Run $P_{\mathsf{ZK}1}(1^\lambda)$ and denote its output by (rwi_1, σ), where σ is its secret state, and rwi_1 is the message to be sent to the verifier.*
2. *Run the verifier $V^*(1^\lambda, \mathsf{rwi}_1)$, who outputs $\{(x^i, w_0^i, w_1^i)\}_{i\in[B]}$ and a set of messages $\{\mathsf{rwi}_2^i\}_{i\in[B]}$.*
3. *For each $i \in [B]$, run $P_{\mathsf{ZK}3}(\sigma, \mathsf{rwi}_2^i, x^i, w_b^i)$, where $P_{\mathsf{ZK}3}$ is the (honest) prover's algorithm for generating the third message of the WI protocol, and send its message rwi_3 to V^*.*
4. *The output of the experiment is the output of V^*.*

C.1 Construction

In this section, we show how that protocol 6 in [JKKR17] is a 3 round non-adaptive bounded rewinding and reusable secure WI argument if the underlying WI in that construction is instead instantiated using our 3 round delayed-input bounded rewinding WI argument from Appendix A. Formally, we show the following theorem:

Theorem 14. *For any constant $B > 0$, assuming polynomially secure:*

- *DDH or Quadratic Residuosity or N^{th} Residuosity,*

the protocol π presented below is a 3 round delayed-input non-adaptive B-rewinding and reusable secure witness indistinguishable argument.

Our protocol π, which is a modified version of protocol 6 from [JKKR17] is described in Figure 8. Let $\mathsf{BR.WI}$ denote the 3 round delayed-input bounded rewinding WI argument from Appendix A. let OT denote a two round oblivious transfer protocol with security against malicious receivers and semi-honest senders. Such oblivious transfer protocols can be instantiated based on DDH/QR/Nth Residuosity [NP01, HK12]. We also assume the existence of dense cryptosystems which are known based on DDH/QR/Nth Residuosity.

Prover Input: Instance $x \in L$, witness w such that $R_L(x, w) = 1$.
Verifier Input: Instance x, language L.

- **Prover Message:** Pick $r_1, r_2, r_1', r_2' \in \{0,1\}^*$, send $c_1 = \mathsf{com}(r_1; r_1'), c_2 = \mathsf{com}(r_2; r_2')$ using non-interactive statistically binding commitment com. Also, send $\mathsf{br.wi}_1$ as the first message of the WI argument.

- **Verifier Message:** Pick challenge $e \xleftarrow{\$} \{0,1\}^\lambda$ for the Σ-protocol, and for $i \in [\lambda]$, send $\mathsf{OT}_{1,i}(e_i)$ in parallel. Each e_i is encrypted with a fresh OT instance. Additionally send $\widetilde{r}_1, \widetilde{r}_2 \xleftarrow{\$} \{0,1\}^*$, and send $\mathsf{br.wi}_2$ as the second message of the WI argument.

- **Prover Message:** Send r_1, r_2 with $\mathsf{br.wi}_3$ as the third message of the WI argument proving that $\exists r_1'$ such that $c_1 = \mathsf{com}(r_1; r_1')$ OR $\exists r_2'$ such that $c_2 = \mathsf{com}(r_2; r_2')$. Set $\mathsf{pk}_1 = r_1 \oplus \widetilde{r}_1, \mathsf{pk}_2 = r_2 \oplus \widetilde{r}_2$ as public keys for a dense cryptosystem.

  Define $\mathsf{commit}(M; R) = \mathsf{enc}_{\mathsf{pk}_1}(M; s_1), \mathsf{enc}_{\mathsf{pk}_2}(M; s_2)$ and $R = s_1 || s_2$, which is decommitted by revealing R. For $i \in [\lambda]$, and send $\mathsf{commit}(h_i), \mathsf{OT}_{2,i}(z_i^0, z_i^1)$ in parallel using the scheme commit. The decommitment information in z_i^0, z_i^1 corresponding to any commitment, only consists of the randomness R used to generate the commitment using commit.

- **Verifier Output:** The verifier V recovers z_i as the output of OT_i for $i \in [\lambda]$, and outputs accept if and only if $(\mathsf{br.wi}_1, \mathsf{br.wi}_2, \mathsf{br.wi}_3)$ is an accepting transcript and $(a_i, e_i, z_i)_{i \in [\lambda]}$ is an accepting transcript of the underlying Σ-protocol, according to the commitment scheme commit.

Figure 8: 3 round delayed-input non-adaptive bounded rewinding WI with reusability security

C.2 Overview of Security

The delayed-input non-adaptive bounded rewinding WI with reusability security of the protocol in Figure 8 follows by a straightforward adaptation of their proof to rely on bounded-rewinding secure WI instead of ZAPs.

Soundness and Reusable Security. Note that the proof of soundness and reusable security of this protocol([JKKR17]) only relies on soundness and witness indistinguishability of the underlying building block $\mathsf{br} \cdot \mathsf{wi}$. As such, since our bounded-rewinding secure WI satisfies both soundness and (standard) witness indistinguishability, these properties follow exactly by the proofs in [JKKR17].

That is, in the proof of soundness, instead of relying on the soundness of any generic WI as is used in their construction, one can rely on the soundness of the WI we instantiate with here. Similarly, the reusable witness indistinguishability follows ([JKKR17]) by using the reusability of two-message OT, and noting that the underlying WI component is used only to prove the same statement in every invocation.

Non-adaptive Bounded Rewinding Security. The proof of non-adaptive bounded rewinding security of the primitive follows the proof in [JKKR17]. They note that all primitives satisfy non-adaptive rewinding security and allow the same sequence to go through as the non-rewinding setting. We note that their proof can be modified only so that in the hybrid argument that relies on resettable witness indistinguishability of the WI (or ZAP) protocol in [JKKR17], we instead rely on bounded-rewinding security of the WI protocol. As a result, in this hybrid, we reduce non-adaptive bounded-rewinding security of the protocol in Figure 8 to bounded-rewinding security of the underlying WI, by only creating a bounded number of rewinding or look-ahead threads.