PIR with compressed queries and amortized query processing

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Abstract

Private information retrieval (PIR) is a key building block in many privacy-preserving systems. Unfortunately, existing constructions remain very expensive. This paper introduces two complementary techniques that make the computational variant of PIR (CPIR) more efficient in practice. The first technique targets a recent class of CPU-efficient CPIR protocols where the PIR query sent by the client is a vector containing a number of ciphertexts proportional to the size of the server's database. We propose a new method to compresses this vector into a single ciphertext, thereby reducing query sizes by up to 256×.

The second technique is a new data encoding called a *probabilistic batch code* (PBC). We use PBCs to build a multi-query PIR scheme that allows the server to amortize the computational cost of processing a batch of requests. The protocol achieves up to 50× speedup over processing queries one at a time, and is significantly more efficient than related encodings. We apply our techniques to the Pung unobservable communication system which relies on a custom multi-query CPIR protocol for its privacy guarantees. Replacing Pung's protocol with our schemes, we find that we can simultaneously reduce network costs by 36× and increase throughput by 3×.

1 Introduction

A key cryptographic building block in recent privacy-preserving systems is *private information retrieval* (PIR) [32]. Examples include anonymous and unobservable communication [11, 58, 63], privacy-preserving media streaming [8, 48], ad delivery [46], friend discovery [18], and subscriptions [30].

PIR allows a client to download an element (e.g., movie, Web page, friend record) from a database held by an untrusted server (e.g., streaming service, social network) without revealing to the server *which* element was downloaded. PIR is very powerful, but it is also very expensive—and unfortunately this expense is fundamental: PIR schemes force the database server to perform some computation on every element in the database to answer a single client query [32]. After all, if the server were to omit an element when answering a query it would learn that the omitted element is of no interest to the client.

We are interested in the computational variant of PIR (CPIR) [57]. This variant is desirable because it relies only on cryptographic hardness assumptions; the information-theoretic variant of PIR requires multiple non-colluding servers, an assumption that conflicts with many deployment scenarios. Unfortunately, the costs of even the most recent CPIR constructions [8, 56, 60] are so significant that all CPIR-backed system of which we are aware must settle with supporting small databases with fewer than 100,000 entries [8, 11, 46, 48].

In this paper we discuss two orthogonal but complementary techniques that make CPIR more efficient in practice. The first is the introduction of SealPIR, a new CPIR library that extends the most computationally efficient CPIR protocol, XPIR [8], with a new query compression procedure that reduces network costs (§3). Specifically, a query in XPIR (and its base protocol [72]), consists of a vector of n ciphertexts, where n is the number of elements in the server's database. Stern [72] showed that it is possible to reduce the number of entries in the vector to $d\sqrt[d]{n}$ ciphertexts for any positive integer d, therefore making network costs *sublinear* in the size of the database. The downside of Stern's approach is that it comes at an exponential increase in the size of the response (§3.4). As we show in our evaluation, values of $d > 3$ in XPIR lead to responses that are so large that they outweigh any reduction in query size (§7.1).

SealPIR adopts a fundamentally different approach. Instead of creating a query vector, SealPIR has the client send a single ciphertext containing an encoding of the index of the desired element. The server then executes a new *oblivious expansion procedure* that extracts the corresponding n-ciphertext vector from the single ciphertext, without leaking any information about the client's index, and without increasing the size of the response (§3.3). The server can then proceed with the XPIR protocol on the extracted vector as before.

In terms of concrete savings over XPIR, SealPIR results in queries that are 256× smaller and are 28× less expensive for the client to construct. However, SealPIR introduces between 8% and 31% CPU overhead to the server (over XPIR) to obliviously expand queries. We think this is an excellent trade-off since XPIR's protocol is embarrassingly parallel and one can regain the lost throughput by employing additional servers. Also, PIR with low network overheads will make it usable in applications where clients are likely to be devices with limited bandwidth (e.g., private variants of mobile messaging apps). Furthermore, many ISPs set strict data limits on wireless and wired plans [7].

Our second contribution is a new technique to amortize the server's CPU cost when processing multiple queries from the same client. Our technique is a relaxation of *batch codes* [51], a data encoding that can in principle be used for this purpose. In practice, most batch code constructions target a different domain—providing load balancing and availability guarantees to distributed storage [67, 69] and network switches [77]; using these constructions to amortize the processing of a batch of PIR queries is not worthwhile since they introduce absurd network costs while yielding only modest CPU speedups (§4.1).

Our encoding, called *probabilistic batch codes (PBC)*, addresses this issue at the expense of introducing a small probability of failure (§4.2). In the context of multi-query PIR, failure simply means that a client only gets some (not all) of her queries answered in a single interaction. While the implications of failure depend on the application, we argue that in many cases this is not an issue in practice (§5). Furthermore, the failure prob-

ability of our constructions is very low—about 1 in a trillion multi-queries would be affected.

The key idea behind our PBC construction is a simple new technique called *hashing in the head* (§4.3). This technique flips the way that hashing (e.g., multi-choice [64], cuckoo [66]) is typically used in distributed systems to achieve load balancing: *instead of executing the hashing algorithm during data placement, it is executed during data retrieval*. Like batch codes, our PBC constructions amortize CPU costs when processing a batch of queries. Unlike batch codes, they are more network-efficient: they introduce orders of magnitude less network overheads than existing batch codes (§7.3). Furthermore, PBCs are general and can be used to amortize computation on *any* PIR scheme (even the information-theoretic variants).

We demonstrate the concrete benefits of our techniques through an extensive evaluation of several deployments that include well-provisioned clients, bandwidth-limited mobile clients, and geo-distributed clients on databases of up to four million entries. We also integrate SealPIR and PBCs into a recent unobservable communication system called Pung [11] that uses CPIR for its privacy guarantees.

In summary, the contributions of this work are:

- SealPIR, a new CPIR library that reduces network costs through a novel oblivious query expansion procedure (§3).
- The introduction of PBC, a new probabilistic data encoding suitable to building multi-query PIR protocols that amortize computational costs (§4.2).
- The design of a PBC construction from a simple technique that we call *hashing in the head* (§4.3).
- The implementation and evaluation of SealPIR and PBC on a variety of settings (§7), including porting these techniques to the Pung communication system (§7.4).

Despite the above contributions, there remains a large performance gap between current CPIR implementations and widespread adoption. Nevertheless, we hope that the content of this work can help usher a way forward.

2 Background and related work

We begin by giving some background on PIR and existing multi-query proposals that relate to our work.

2.1 Private information retrieval (PIR)

Chor et al. [32] introduced private information retrieval (PIR) to address the following problem: how can a client retrieve an element from a remote database managed by an untrusted server (or set of servers) such that the server does not learn *which* element was retrieved by the client? And whether this can be done more efficiently than the trivial solution where the client simply downloads the entire database from the server and locally selects the desired element? This effort served as the catalyst for two lines of work that remain highly active: *information theoretic* PIR (IT-PIR) and *computational* PIR (CPIR).[1]

In IT-PIR schemes [15, 32, 35, 36, 45] the database is replicated across several non-colluding servers. The client issues a carefully-crafted query to each server and combines the responses from all the servers locally. IT-PIR schemes have two benefits. First, the servers' computation is relatively inexpensive (an XOR for each entry in the database). Second, the privacy guarantees are information-theoretic, meaning they hold against a computationally-unbounded adversary and avoid cryptographic hardness assumptions. However, basing systems on IT-PIR poses significant deployment challenges since it is difficult to enforce the non-collusion assumption in practice.

On the other hand, CPIR protocols [8, 23, 26, 39, 44, 56, 57, 59, 60, 78] can be used with a database controlled by a single administrative domain (e.g., a company), under cryptographic hardness assumptions. The drawback is that they are more expensive than IT-PIR protocols as they require the database operator to perform costly cryptographic operations on each database element. Fortunately, there is a long line of work to improve the resource overheads of CPIR (see [8, 56] for the state-of-the-art), and recent work [8] proposes a construction that achieves, for the first time, plausible (although still high) computational costs. Unfortunately, this construction has high network costs that scale unfavorably with the size of the database (e.g., $O(\sqrt{n})$). Figure 1 depicts a simplified version of this protocol (more specifically it depicts Stern's protocol [72]).

Regardless of which flavor of PIR a system implements, the costs remain undeniably high. As a result, it is hard for systems to support large databases or handle many requests. While addressing the former issue (i.e., supporting large databases) remains elusive—although Section 3 makes progress towards this goal—supporting many concurrent queries is the focus of many existing proposals. We discuss them next.

2.2 Existing multi-query PIR schemes

Given PIR's high costs, it is desirable to amortize the servicing of many requests. Such scenarios include privacy-preserving variants of databases that serve many users concurrently (e.g., Netflix, Spotify), or databases that process a batch of request from the same user (e.g., Gmail, Slack, bulletin boards). The most general approach to achieve this goal is to use *batch codes* [51]. In a batch code, the database is encoded such that the server (or servers) can respond to *any* k requests (from the same user) more cheaply (computationally) than the baseline solution of running k parallel instances of PIR. The trade-off is that batch codes require more network resources (than the baseline solution). Furthermore, in concrete terms, this network overhead is onerous; we discuss this further in Section 4.1.

Other existing proposals tailor the amortization to particular PIR protocols or particular applications, as we discuss next.

Amortization for particular PIR protocols. Beimel et al. [16] describe two query amortization techniques. The first is based on the observation that queries in many PIR schemes consist of a vector of entries, and answering these queries is equivalent to computing a matrix-vector product (where the product could be over ciphertexts instead of plaintexts, or it could be an XOR operation). By aggregating multiple queries—even from different users—the server's work can be expressed as a product of two matrices. As a result, leveraging sub-cubic matrix multiplication algorithms (e.g., [33, 73]) provides amor-

[1] These two lines of work are sometimes known as multi-database PIR (for IT-PIR) and single-database PIR (for CPIR).

tization over multiple matrix-vector multiplication instances. This approach is further studied by Lueks and Goldberg [62] in the context of Goldberg's IT-PIR scheme [45].

The second proposal described by Beimel et al. [16] is to perform preprocessing over the database in certain IT-PIR schemes to reduce the cost of future queries. This works well, so recent works [19, 25] employ an analogous preprocessing approach in CPIR schemes. However, making the database accessible by more than a single client under existing CPIR preprocessing schemes requires prohibitively expensive cryptography (a virtual black-box obfuscation primitive [14] instantiated from *indistinguishability obfuscation* [42]).

Groth et al. [47] extend Gentry and Ramzan's [44] CPIR scheme to retrieve k elements at lower amortized network cost by having the client compute k discrete logarithms (with tractable but expensive parameters) on the server's answer. This results in very low network costs, but Gentry and Ramzan's scheme is computationally expensive (on the order of hours to process a single PIR query, based on our estimates); Groth et al.'s extension compounds this issue. With a similar goal but in the context of amortizing CPU rather than network resources, Henry et al. [49, 50] and RAID-PIR [35] extend specific IT-PIR protocols. While these techniques result in good amortization, they are only applicable to particular IT-PIR schemes.

Amortization for particular apps. Popcorn [48] pipelines the processing of queries in IT-PIR in order to amortize disk I/O which is a bottleneck for databases with very large files. Pung [11] hybridizes an existing batch code due to Ishai et al. [51] with a probabilistic protocol that exploits the setting of online communication where users can coordinate a priori (e.g., chat, e-mail). This enables Pung to amortize CPU costs with less network expense than traditional batch codes.

In contrast with the above, our multi-query scheme is agnostic to the particular PIR protocol or application being used. Compared to batch codes [51], our technique has weaker properties (sufficient for most applications) but is significantly more efficient. Compared to Pung's technique, our approach is general, more efficient, and application-independent (§4.2).

3 SealPIR: An efficient CPIR library

Our starting point for SealPIR is XPIR [8], a recent CPIR construction that introduces several optimizations to Stern's CPIR scheme [72]. We give a rough sketch of these protocols in Figure 1. The key idea in XPIR is to perform the encryption and homomorphic operations using a lattice-based cryptosystem (the authors use the BV cryptosystem [21]), and preprocess the database in a way that greatly reduces the cost of the operations in Lines 11 and 12 in Figure 1. To our knowledge, this makes XPIR the only CPIR implementation that is usable in practice.

A major drawback of XPIR is network costs. In particular, the query sent by the client is large: in the basic scheme, it contains one ciphertext (encrypting 0 or 1) for each entry in an n-element database. Furthermore, lattice-based cryptosystems have a high *expansion factor*, F, which is the size ratio between a ciphertext and the largest plaintext that can be encrypted; for recommended security parameters, $F \geq 6.4$ [10, 27].

```
1:  function SETUP(DB)
2:      Represent DB in an amenable format (see [8, §3.2])
3:
4:  function QUERY(pk, idx, n)
5:      for i = 0 to n − 1 do
6:          c_i ← Enc(pk, i == idx? 1 : 0)
7:      return q ← {c_0, ..., c_{n−1}}
8:
9:  function ANSWER(q = {c_0, ..., c_{n−1}}, DB)
10:     for i = 0 to n − 1 do
11:         a_i ← DB_i · c_i          // plaintext-ciphertext multiplication
12:     return a ← Σ_{i=0}^{n−1} a_i  // homomorphic addition
13:
14: function EXTRACT(sk, a)
15:     return Dec(sk, a)
```

FIGURE 1—CPIR protocol from Stern [72] and XPIR [8] on a database DB of n elements. This protocol requires an *additively homomorphic* cryptosystem with algorithms (KeyGen, Enc, Dec), where (pk, sk) is the public and secret key pair generated using KeyGen. We omit the details of all optimizations. The client runs the QUERY and EXTRACT procedures, and the server runs the SETUP and ANSWER procedures. Each element in DB is assumed to fit inside a single ciphertext. Otherwise, each element can be split into ℓ smaller chunks, and Lines 11 and 12 can be performed on each chunk individually; in this case ANSWER would return ℓ ciphertexts instead of one.

To improve network costs, Stern [72] describes a way to represent the query using $d\sqrt[d]{n}$ ciphertexts (instead of n) for any positive integer d. Unfortunately, this increases the response size exponentially from 1 to F^{d-1} ciphertexts (Section 3.4 explains this). If the goal is to minimize network costs, a value of $d = 2$ or 3 is optimal in XPIR for the databases that we evaluate (§7.1). As a result, even with this technique the query vector is made up of hundreds or thousands of ciphertexts.

3.1 Compressing queries

At a high level, our goal is to realize the following picture: the client sends one ciphertext containing an encryption of its desired index i to the server, and the server inexpensively evaluates a function EXPAND that outputs n ciphertexts containing an encryption of 0 or 1 (where the i^{th} ciphertext encrypts 1 and others encrypt 0). The server can then use these n ciphertexts as a query and execute the protocol as before (Figure 1, Line 9).

A straw man approach to construct EXPAND is to create a Boolean circuit that computes the following function: "if the index encrypted by the client is i return 1, else return 0". The server can then evaluate this circuit on the client's ciphertext using a fully homomorphic encryption (FHE) scheme (e.g., BV [21], BGV [20], FV [40]) passing in values of $i \in [0, n-1]$ to obtain the n ciphertexts. Unfortunately, this approach is very expensive. First, FHE supports addition and multiplication operations, but not Boolean operations (AND, XOR, etc.), which are needed for comparisons. As a result, the client has to express its index as a bit string and encrypt each bit individually, resulting in a query of $\log(n)$ ciphertexts. Second, to operate on these encrypted bits, the server has to emulate Boolean operations

operation	CPU cost (ms)	noise growth
addition	0.002	additive
plaintext multiplication	0.141	multiplicative*
multiplication	1.514	multiplicative
substitution	0.279	additive

FIGURE 2—Cost of operations in SEAL [4]. The parameters used are given in Section 7. Every operation increases the *noise* in a ciphertext. Once the noise passes a threshold, the ciphertext cannot be decrypted. For a given computation, parameters must be chosen to accommodate the expected noise. *While plaintext multiplication yields a multiplicative increase in the noise, the factor is always 1 (i.e., no noise growth) in EXPAND because it is based on the number of non-zero coefficients in the plaintext [27, §6.2].

using addition and multiplication,[2] resulting in a $\log(n)$-depth circuit. Finally, this circuit must be evaluated n times, one for each possible value of i.

Instead, we propose a new algorithm to implement EXPAND. It relies on FHE, but perhaps surprisingly, it does not require encrypting each bit of the index individually, emulating Boolean gates, or performing any homomorphic multiplications. This last point is the most critical for performance, since homomorphic multiplications are very expensive and require using larger security parameters (Figure 2). We note that the underlying cryptosystem used by XPIR (BV [21]) is an FHE scheme, so we could implement EXPAND using that. However, we choose to implement all of SealPIR using the SEAL homomorphic library [4] which implements the Fan-Vercauteren (FV) [40] cryptosystem instead. We make this choice for pragmatic reasons: EXPAND requires the implementation of a new homomorphic group operation, and the SEAL library is a mature code base that already implements many of the necessary building blocks. Below we give some background on FV.

Fan-Vercauteren FHE cryptosystem (FV). In FV, plaintexts are polynomials of degree at most N with integer coefficients modulo t. The polynomials are from the quotient ring $R_t = \mathbb{Z}_t[x]/(x^N+1)$, where N is a power of 2, and t is the *plaintext modulus* that determines how much data can be packed into a single FV plaintext. In Section 6 we discuss how regular binary data, for example a PDF file, is encoded in an FV plaintext, and what these polynomials actually look like in code.

Ciphertexts in FV consist of two polynomials, each in $R_q = \mathbb{Z}_q[x]/(x^N+1)$. Here q is the *coefficient modulus* that affects how much *noise* a ciphertext can contain, and the security of the cryptosystem. When a ciphertext is created it contains noise that increases as operations are performed on the ciphertext. Once the noise passes a threshold the ciphertext cannot be decrypted. The noise growth of operations depends heavily on t, so t should be kept small. However, lower t means that more FV plaintexts are needed to represent the binary data (PDF, movie, etc.). Larger q supports more noise, but results in lower security. The expansion factor is $F = 2\log(q)/\log(t)$. We discuss concrete parameters in Section 7.

[2]For example, when both operands a and b are single bits, AND ($a \wedge b$) is the same as multiplication ($a \cdot b$); NAND ($\neg(a \wedge b)$), which is a universal gate that can be used to represent all other Boolean gates, is $1 + (-1 \cdot (a \cdot b))$.

In addition to the standard operations of a cryptosystem (key generation, encryption, decryption), FV also supports homomorphic addition, multiplication, and relinearization (which is performed after multiplications to keep the number of polynomials in the ciphertext at two); for our purposes we care about the following operations.

- **Addition**: Given ciphertexts c_1 and c_2, which encrypt FV plaintexts $p_1(x), p_2(x) \in R_t$, the operation $c_1 + c_2$ results in a ciphertext that encrypts their sum, $p_1(x) + p_2(x)$.
- **Plaintext multiplication**: Given a ciphertext c that encrypts $p_1(x) \in R_t$, and given a plaintext $p_2(x) \in R_t$, the operation $p_2(x) \cdot c$ results in a ciphertext that encrypts $p_1(x) \cdot p_2(x)$.
- **Substitution**: Given a ciphertext c that encrypts plaintext $p(x) \in R_t$ and an odd integer a, the operation $\mathsf{Sub}(c, a)$ returns an encryption of $p(x^a)$. For instance if c encrypts $p(x) = 7 + x^2 + 2x^3$, then $\mathsf{Sub}(c, 3)$ returns an encryption of $p(x^3) = 7 + (x^3)^2 + 2(x^3)^3 = 7 + x^6 + 2x^9$.

Our implementation of the substitution group operation is based on the plaintext slot permutation technique discussed by Gentry et al. [43, §4.2]. Fortunately, substitution requires only a subset of the operations needed by the arbitrary permutations that Gentry et al. consider, so we can implement it very efficiently, as shown in the last row of Figure 2.

3.2 Encoding the index.

A client that wishes to retrieve the i^{th} element from the server's database using SealPIR generates an FV plaintext that encodes this index. The client does so by representing $i \in [0, n-1]$ as the monomial $x^i \in R_t$. The client then encrypts this plaintext to obtain $query = Enc(x^i)$, which is then sent to the server. The case where the database is so large that the index cannot be represented by a single FV plaintext is discussed in Section 3.5.

3.3 Expanding queries obliviously

To explain how the server expands $query = Enc(x^i)$ into a vector of n ciphertexts where the i^{th} ciphertext is $Enc(1)$ and all other are $Enc(0)$, we first give a description for $n = 2$.

As we discuss in the previous section, the server receives $query = Enc(x^i)$, with $i \in \{0, 1\}$ in this case (since $n = 2$) as the client's desired index. The server first expands *query* into two ciphertexts $c_0 = query$ and $c_1 = query \cdot x^{-1}$:

$$c_0 = \begin{cases} Enc(1) & \text{if } i = 0 \\ Enc(x) & \text{if } i = 1 \end{cases}$$

$$c_1 = \begin{cases} Enc(x^i \cdot x^{-1}) = Enc(x^{-1}) & \text{if } i = 0 \\ Enc(x^i \cdot x^{-1}) = Enc(1) & \text{if } i = 1 \end{cases}$$

The server computes $c'_j = c_j + \mathsf{Sub}(c_j, N+1)$ for $j \in \{0, 1\}$. Since operations in R_t are defined modulo $x^N + 1$,[3] a substitution with $N+1$ transforms the plaintext encrypted by c_0 and c_1 from $p(x)$ to $p(-x)$. Specifically, we have:

$$c'_0 = \begin{cases} Enc(1) + Enc(1) = Enc(2) & \text{if } i = 0 \\ Enc(x) + Enc(-x) = Enc(0) & \text{if } i = 1 \end{cases}$$

[3]For example, $x^N + 1 \equiv 0 \pmod{x^N+1}$ and $x^{N+1} \equiv -x \pmod{x^N+1}$.

1: **function** EXPAND($query = Enc(x^i)$)
2: find smallest $m = 2^\ell$ such that $m \geq n$
3: $ciphertexts \leftarrow [query]$
4: // each outer loop iteration doubles the number of ciphertexts,
5: // and only one ciphertext ever encrypts a non-zero polynomial
6: **for** $j = 0$ to $\ell - 1$ **do**
7: **for** $k = 0$ to $2^j - 1$ **do**
8: $c_0 \leftarrow ciphertexts[k]$
9: $c_1 \leftarrow c_0 \cdot x^{-2^j}$
10: $c'_k \leftarrow c_0 + \mathsf{Sub}(c_0, N/2^j + 1)$
11: $c'_{k+2^j} \leftarrow c_1 + \mathsf{Sub}(c_1, N/2^j + 1)$
12: $ciphtertexts \leftarrow [c'_0, \ldots, c'_{2^{j+1}-1}]$
13: // ciphertext at position j encrypts m and all others encrypt 0
14: $inverse \leftarrow m^{-1} \pmod t$
15: **for** $j = 0$ to $n - 1$ **do**
16: $o_j \leftarrow ciphertexts[j] \cdot inverse$
17: **return** $output \leftarrow [o_0, \ldots, o_{n-1}]$

FIGURE 3—Procedure that expands a single ciphertext *query* that encodes an index i into a vector of n ciphertexts, where the i^{th} entry is an encryption of 1, and all other entries are encryptions of 0. We introduce a new group operation Sub (see text for details). Plaintexts are in the polynomial quotient ring $\mathbb{Z}_t[x]/(X^N + 1)$. $N \geq n$ is a power of 2, and n is the number of elements in the server's database.

$$c'_1 = \begin{cases} Enc(x^{-1}) + Enc(-x^{-1}) = Enc(0) & \text{if } i = 0 \\ Enc(1) + Enc(1) = Enc(2) & \text{if } i = 1 \end{cases}$$

Finally, assuming t is odd, we can compute the multiplicative inverse of 2 in $\mathbb{Z}_t$, say α, encode it as the monomial $\alpha \in R_t$, and compute $o_j = \alpha \cdot c'_j$. It is the case that o_0 and o_1 contain the desired output of EXPAND: o_i encrypts 1, and o_{1-i} encrypts 0.

We can generalize this approach to any power of 2 as long as $n \leq N$. In cases where n is not a power of 2, we can run the algorithm for the next power of 2, and take the first n output ciphertexts as the client's query. Figure 3 gives the generalized algorithm, and Figure 4 depicts an example for a database of 4 elements. We prove the correctness of EXPAND in Appendix A.1, and bound its noise growth in Appendix A.2.

3.4 Reducing the cost of expansion

One issue with EXPAND is that despite each operation being inexpensive (Figure 2), $O(n)$ operations are needed to extract the n-entry query vector. This is undesirable, since EXPAND could end up being more expensive to the server than computing the answer to a query (see Figure 1, Line 9). We show how to reduce this cost by having the client send multiple ciphertexts.

Stern [72] proposes the following modification to the protocol in Figure 1. Instead of structuring the database *DB* as an n-entry vector (where each entry is an element), the server structures the database as a $\sqrt{n} \times \sqrt{n}$ matrix M: each cell in M is a different element in *DB*. The client sends 2 query vectors, v_{row} and v_{col}, each of size $\sqrt{n}$. The vector v_{row} has the encryption of 1 at position r, while v_{col} has the encryption of 1 at position c (where $M[r, c]$ is the client's desired element). The server, upon receiving v_{row} and v_{col}, computes the following matrix-vector product: $A_c = M \cdot v_{col}$, where each multiplication is between a plaintext and ciphertexts, and additions are on ciphertexts.

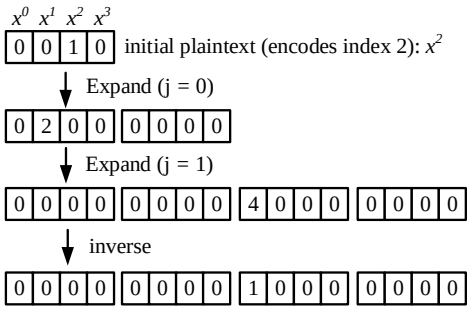


FIGURE 4—Example of EXPAND's effect on the extracted vector's plaintext on each iteration of its outer loop. This example assumes a database with 4 elements, and a query retrieving the third item. Each plaintext is a polynomial represented as an array of coefficients. Note that the server only sees the corresponding ciphertexts (not depicted).

Observe that A_c is a vector containing the encryption of all entries in column c of M.

The server then performs a similar step using A_c and v_{row}. There is, however, one technical challenge: each entry in A_c is a ciphertext, so it is too big to fit inside another ciphertext (recall that the largest plaintext that can fit in a ciphertext has size $|ciphertext|/F$). To address this, the server splits elements in A_c into F chunks, so A_c can be thought of as a $\sqrt{n}$ by F matrix. The server can now repeat the process as before on the transpose of this matrix: it computes $A_c^T \cdot v_{row}$, to yield a vector of F ciphertexts, which it sends to the client. The client then decrypts all F ciphertexts and combines the result to obtain $Enc(M[r, c])$. The client can then decrypt $Enc(M[r, c])$ to obtain $M[r, c]$—the desired element in *DB*. This scheme generalizes by structuring the database as a d-dimensional hypercube and having the client send d query vectors of size $\sqrt[d]{n}$. The server then returns F^{d-1} ciphertexts as the response.

We use the above scheme to reduce the computational cost of EXPAND (in contrast, Stern and XPIR use the above technique to reduce network costs by reducing the size of the query vector). Instead of encoding one index, the client encodes d indices (on different ciphertexts), one for each dimension of the database. The server then calls EXPAND on each of the d ciphertexts, and extracts a $\sqrt[d]{n}$-entry vector from each. The server uses the above scheme with the extracted d vectors, which results in the CPU costs of EXPAND being $O(d\sqrt[d]{(n)})$. Of course, this approach has the downside that the PIR response gets larger because of the cryptosystem's expansion factor (F). Specifically, the network cost is d ciphertexts to encode the indices, and F^{d-1} ciphertexts to encode the response. The good news is that for small values of d (2 or 3), this results in major computational savings while still reducing network costs by orders of magnitude over XPIR.

3.5 Indexing large databases

As we discuss in Section 3.3, the size of the query vector that EXPAND can generate is bounded by N, which bounds the degree of the polynomials used in FV. Based on recommended security parameters [10, 27], N is typically 2048 or 4096 (larger N improves security but reduces performance). So how can one index into databases with more than N elements?

We propose two solutions. First, the client sends multiple

ciphertexts and the server expands them and concatenates the results. For instance, if N is 2048, the database has 4096 elements, and the client wishes to get the element at index 2050, the client sends 2 ciphertexts: the first encrypts 0 and the second encrypts x^2. The server expands both ciphertexts into 2048-entry vectors and concatenates them to get a 4096-entry vector where the entry at index 2050 encrypts 1, and all others encrypt 0.

A more efficient solution is to represent the database as a d-dimensional hypercube as we discuss in Section 3.4. This allows the client to send d ciphertexts to index a database of size N^d. For $d = 2$ and $N = 4096$, two ciphertexts are sufficient to index 16.7 million entries. One can also use a combination of these solutions. For example, given a database with 2^{30} entries, SealPIR would use $d = 2$ (so the database is a $2^{15} \times 2^{15}$ matrix), and will represent the index for each dimension using $2^{15}/4096 = 8$ ciphertexts. The server expands these 8 ciphertexts and concatenates them to obtain a vector of 2^{15} entries. In total, this approach requires the client to send 16 ciphertexts as the query (8 per dimension), and receive $F \approx 7$ ciphertexts as the response ($d = 3$ would lead to 3 ciphertexts as the query, but F^2 ciphertexts as the response).

4 Amortizing computational costs in PIR

Answering a PIR query requires computation that is linear in the size of the database, so a promising way to save computational resources is for the server to amortize costs by processing a batch of queries. *Batch codes* [51] are a data encoding that, among other applications, can be used to achieve this goal. In particular, the server can use a batch code to encode its database in a way that it can answer a batch of queries more cheaply (computationally) than answering each query individually. Unfortunately, despite a large body of work on batch codes, we find that most constructions do not focus on PIR amortization. Instead, they target load balancing in distributed storage systems [67, 69] and network switches [77], which have different requirements. Using batch codes to amortize PIR query processing would incur prohibitive network costs.

Our key observation is that certain guarantees of batch codes are not necessary for many PIR-backed systems. Relaxing those guarantees leads to constructions that are not only asymptotically better, but also concretely efficient—without compromising the functionality of our target system. Below we give a description of batch codes, highlight the sources of overhead, and then introduce our construction.

4.1 Batch codes and their cost

A (n, m, k, b)-batch code $\mathcal{B}$ takes as input a collection DB consisting of n elements and produces a set of m codewords C. These codewords are then distributed among b buckets. Formally, $\mathcal{B} : DB \rightarrow (C_0, \ldots, C_{m-1})$, where $|C_i|$ is the number of codewords in bucket i, and the sum of codewords across all buckets is $m = \Sigma_{i=0}^{b-1}|C_i| \geq n$. The goal of these codes is two-fold. First, they ensure that any k elements from DB can be retrieved from the b buckets by fetching at most one codeword from each bucket. Second, they keep the number of total codewords (i.e., m) lower than $k \cdot n$.

Example. We now give an example of a $(4, 6, 2, 3)$-batch code, specifically the *subcube batch code* [51]. Let $DB = \{x_1, x_2, x_3, x_4\}$. For the encoding, DB is split in half to produce 2 buckets, and the XOR of the entries in these buckets produces elements that are placed in a third bucket: $\mathcal{B}(DB) = \{x_1, x_2\}, \{x_3, x_4\}, \{x_1 \oplus x_3, x_2 \oplus x_4\}$. Observe that one can obtain any 2 elements in DB by querying each bucket at most once. For example, to obtain x_1 and x_2, one would get x_1 from the first bucket, x_4 from the second bucket, and $x_2 \oplus x_4$ from the third bucket. One would then compute $x_2 = x_4 \oplus (x_2 \oplus x_4)$.

This encoding is helpful for PIR because a client wishing to retrieve 2 elements from DB can instead issue one query to each bucket, which forces the server to compute over 3 "databases" of 2 elements each. This results in 25% fewer computation than answering two queries to the original database DB (which requires the server to compute over 4 elements twice).

Costs of PIR with batch codes. Figure 5 depicts the relationship between the number of codewords (m) and the number of buckets b, as a function of the database size (n) and the batch size (k) for several constructions. In multi-query PIR, the client issues one query to each of the b buckets, and therefore receives b responses (§5). To answer these b queries, the server computes over all m codewords exactly once; lower values of m lead to less computation, and lower values of b lead to lower network costs. Since $m < k \cdot n$, the total computation done by the server is lower than answering each of the k queries individually without a batch code. The drawback is that existing batch codes produce many buckets (cubic or worse in k). As a result, they *introduce* significant network overhead over not using a batch code at all. This makes batch codes unappealing in practice.

4.2 Probabilistic batch codes (PBC)

Batch codes have exciting properties, but existing constructions offer an unattractive trade-off: they reduce computation but introduce significant network overhead. While we cannot construct codes that save computation *and* avoid network overhead, we can make this trade-off more appealing.

A *probabilistic batch code* (PBC) differs from a traditional batch code in that it fails to be *complete* with probability p. That is, a collection encoded with a PBC may have no way to recover a specific set of k elements by retrieving exactly one codeword from each bucket. The probability of encountering one such set is p. In the example of Section 4.1, this would mean that under a PBC, a client may be unable to retrieve both x_1 and x_2 by querying each bucket at most once (whereas a traditional batch code guarantees that this is always possible). In practice, this is not really an issue: our construction has parameters that can result in roughly 1 in a trillion queries failing, which we think is a sufficiently rare occurrence. Furthermore, as we discuss in Section 5, this is an easy failure case to address in multi-query PIR since a client learns whether or not it can get all of the elements *before* issuing any queries.

Definition 1 (PBC). A (n, m, k, b, p)-PBC is given by three polynomial-time algorithms (Encode, GenSchedule, Decode):

- $(C_0, \ldots, C_{b-1}) \leftarrow \mathsf{Encode}(DB)$: Given an n-element collection DB, output a b-tuple of buckets, where $b \geq k$, each

batch code	codewords (m)	buckets (b)	probability of failure (p)
subcube ($\ell \geq 2$) [51, §3.2]	$n \cdot ((\ell+1)/\ell)^{\log_2(k)}$	$(\ell+1)^{\log_2(k)}$	0
combinatorial ($\binom{r}{k-1} \leq n/(k-1)$) [67, §2.2]	$kn - (k-1) \cdot \binom{r}{k-1}$	r	0
Balbuena graphs [69, §IV.A]	$2(k^3 - k \cdot \lceil n/(k^3-k) \rceil)$	$2(k^3-k)$	0
Pung hybrid* [11, §4.4]	$4.5n$	$9k$	$\approx 2^{-20}$
3-way cuckoo hashing in the head (**this work**, §4.5)	$3n$	$1.5k$	$\approx 2^{-40}$

FIGURE 5—Cost of existing batch codes and the probabilistic batch code (PBC) construction given in Section 4.5. n indicates the number of elements in the database DB. k gives the number of elements that can be retrieved from DB by querying each bucket in $\beta(DB)$ at most once, where β is the batch code. Building a multi-query PIR scheme from any of the above constructions leads to computational costs to the server proportional to $O(m)$, and network communication proportional to $O(b)$. We list batch codes that have explicit constructions and can amortize CPU costs for multi-query PIR. Other batch codes have been proposed (e.g., [61, 70, 71, 76]) but they either have no known constructions, or they seek additional properties (e.g., tolerate data erasures, optimize for the case where $n = b$, support multisets) that introduce structure or costs that makes them a poor fit for multi-query PIR. *The scheme in Pung is neither a batch code nor a PBC since it relies on clients replicating the data to buckets (rather than the server). It is, however, straightforward to port Pung's allocation logic via hashing in the head (§4.4) to construct a PBC.

bucket contains zero or more codewords and the total number of codewords across all buckets is $m = \Sigma_{i=0}^{b-1} |C_i| \geq n$.

- $\{\sigma, \perp\} \leftarrow \mathsf{GenSchedule}(I)$: Given a set of k indices I corresponding to the positions of elements in DB, output a *schedule* $\sigma : I \rightarrow \{0, \ldots, b-1\}^+$. The schedule σ gives, for each position $i \in I$, the index of one or more buckets from which to retrieve a codeword that can be used to reconstruct element $DB[i]$. GenSchedule outputs $\perp$ if it cannot produce a schedule where each index $\in I$ is associated with at least one bucket, and where no buckets is used more than once. This failure event occurs with probability p.
- *element* $\leftarrow \mathsf{Decode}(W)$: Given a set of codewords $W \subseteq C$, outputs the corresponding element $\in DB$.

We are now ready to discuss an efficient PBC construction. Our key idea is as follows. Observe that batch codes are designed to spread out elements in a clever way such that retrieval requests are well-balanced among the buckets. Relatedly, many data structures and networking applications use different variants of hashing—consistent [53], asymmetric [75], weighted [74], multi-choice [13, 64], cuckoo [12, 66], and others [22, 38]—to achieve the same goal. While there is no obvious way to use these hashing schemes to implement multi-query PIR directly, we can do it indirectly: we first build a PBC from a simple technique that we call *hashing in the head*,[4] and then use the PBC to implement multi-query PIR (§5). We detail this process in the next few sections.

4.3 Randomized load balancing

A common use case for (non-cryptographic) hash functions is to build efficient data structures such as hash tables. In a hash table, the insert procedure consists of computing one or more hash functions on the key of the item being inserted. Each application of a hash function returns an index into an array of buckets in the hash table. The item is then placed into one of these buckets following an allocation algorithm. For example, in multi-choice hashing [13, 64], the item is placed in the bucket least full among several candidate buckets. In Cuckoo hashing [66], items

[4]The phrase "in the head" was introduced by Ishai et al. [52] to describe the action of an entity who simulates the execution of a protocol (multiparty computation in their case). We borrow this phrase.

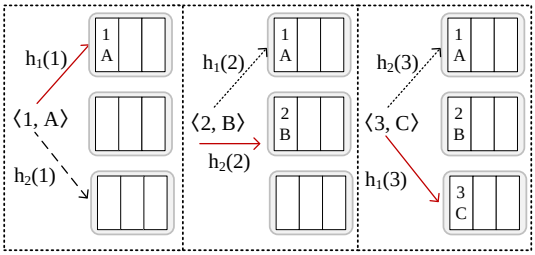


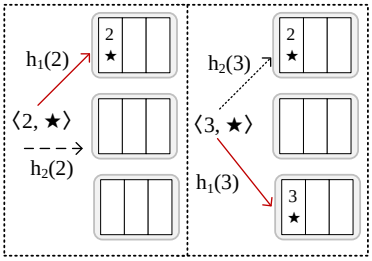
FIGURE 6—Logic for two-choice hashing [13] when allocating three key-value tuples to buckets: $\langle 1, A\rangle$, $\langle 2, B\rangle$, $\langle 3, C\rangle$. Tuples are inserted into the bucket least full. Arrows represent the choices for each tuple based on different hashes of the tuple's key (here we depict an optimistic scenario). The red solid arrow indicates the chosen mapping.

are moved around following the Cuckoo hashing algorithm (we explain this algorithm in Section 4.5).

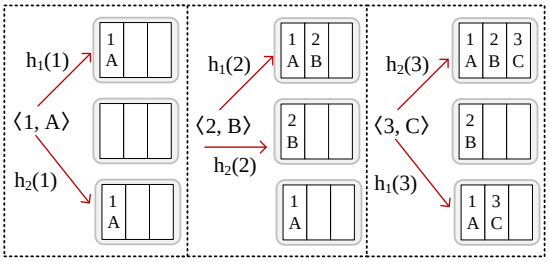
An ideal allocation results in items being assigned to buckets such that all buckets have roughly the same number of items (since this lowers the cost of lookup). In practice, *collisions* are frequent and many items might map to the same bucket. To look up an item by its key, one computes the different hash functions on the key to obtain the list of buckets in which the item could have been placed; one then scans each of those buckets for the desired item. An example of the insertion process for multi-choice hashing is given in Figure 6.

Abstract problem: balls and bins. In the above example, hashing is used to solve an instance of the classic n balls and b bins problem, which arises during insertion. The items to be inserted into a hash table are the n balls, and the buckets in the hash table are the b bins; using w hash functions to hash a key to w candidate buckets approximates an independent and uniform random assignment of a ball to w bins. The number of collisions in a bucket is the load of a bin, and the highest load across all bins is the *max load*. In the worst case, the max load is n/w (all balls map to the same w candidate buckets), but there are useful bounds that hold with high probability.

Interestingly, if we examine other scenarios abstracted by the balls and bins problem, a pattern becomes clear: the allocation algorithm is always executed during data placement. In the hash table example, the allocation algorithm determines where to insert an element. In the context of a transport protocol [54],



(a) consumer's simulation



(b) producer's allocation

FIGURE 7—Example of two-choice hashing in the head. (a) shows the consumer's simulation when inserting two tuples $\langle 2, \star \rangle$, $\langle 3, \star \rangle$. The $\star$ indicates that the value is not known, so an arbitrary value is used. (b) shows a modification to two-choice hashing where the producer stores the tuple in all possible choices. This ensures that the final allocation is always compatible with the consumer's simulation.

the allocation algorithm dictates on which path to send a packet. In the context of a job scheduler [65], the allocation algorithm selects the server on which to run a task. The result is that the load balancing effect is achieved at the time of "data placement". However, to build a PBC, we must do it at the time of "data retrieval". Hashing in the head achieves this.

4.4 Hashing in the head

We start by introducing two principals: the *producer* and the *consumer*. The producer holds a collection of n items where each item is a key-value tuple. It is in charge of data placement: taking each of the n elements and placing them into one of b buckets based on their keys (e.g., insert procedure in a hash table). The consumer holds a set of k keys ($k \leq n$), and is in charge of data retrieval: it fetches items by their key from the buckets that were populated by the producer (e.g., lookup procedure in a hash table). The goal is for the consumer to get all k items by probing each bucket as few times as possible. That is, the consumer has an instance of a k balls and b bins problem, and the goal is to reduce its max load.

Note that the consumer is not inserting elements into buckets (that is the job of the producer). Instead, the consumer is placing "retrieval requests" into the buckets. The challenge is that any clever allocation chosen by the consumer must be *compatible* with the actions of the producer (who populates the buckets). That is, if the consumer, after running its allocation algorithm (e.g., multi-choice hashing) decides to retrieve items x_1, x_2, and x_3, from buckets 2, 3, and 7, it better be the case that the producer previously placed those elements in those exact buckets. We describe how we guarantee compatibility below.

Protocol. The consumer starts by imagining in its head that it is a producer with a collection of k elements. In particular, the consumer converts its k keys into k key-value tuples by assigning a dummy value to each key (since it does not know actual values). In this simulation, the consumer follows a specific allocation algorithm (e.g., 2-choice hashing, cuckoo hashing) and populates the b buckets accordingly. The result is an allocation that balances the load of the k elements among the b buckets (as we discuss in Section 4.3). The consumer then ends its simulation and uses the resulting allocation to fetch the k elements from the buckets that were populated by the real producer.

Guaranteeing that the consumer's allocation is compatible with the producer's actions is challenging. One reason is that the consumer's simulation is acting on k items whereas the real producer is acting on n items. If the allocation algorithm being used (by the consumer and the producer) is randomized or depends on prior choices (this is the cases with most multi-choice hashing schemes), the allocations will be different. Indeed, observe that if a producer generates the allocation in Figure 6 it would not be compatible with the consumer's simulation in Figure 7a despite both entities using the same algorithm (since the producers places the item under key "2" in the middle bucket, but the consumer's simulation maps it to the top bucket).

To guarantee compatibility we employ a simple solution: the producer follows the same allocation algorithm as the consumer's simulation (e.g., 2-choice hashing) on its n elements but stores the elements in all candidate buckets. That is, whenever the algorithm chooses one among w candidate buckets to store an element, the producer stores the element in all w buckets. This ensures that regardless of which k elements are part of the consumer's simulation or which non-deterministic choices the algorithm makes, the allocations are always compatible (Figure 7b). Of course this means that the producer is replicating elements, which defeats the point of load balancing. However, PBCs only need load balancing during data retrieval.

4.5 A PBC from cuckoo hashing in the head

We give a construction that uses cuckoo hashing [66] to allocate balls to bins. However, the same method can be used with other algorithms (e.g., multi-choice Greedy [13], LocalSearch [55]) to obtain different parameters. We give a brief summary of Cuckoo hashing's allocation algorithm below.

Cuckoo hashing algorithm. Given n balls, b buckets, and w independent hash functions $h_0, \ldots, h_{w-1}$, compute w candidate buckets for each ball by applying the w hash functions: $h_i(b) \bmod b$. For each ball x, place x in any empty candidate bucket. If none of the w candidate buckets are empty, select one of the candidate buckets at random, remove the ball currently in that bucket (x_{old}), place x in the bucket, and re-insert x_{old} as before. If re-inserting x_{old} causes another ball to be removed, this process continues recursively for a maximum number of rounds. If this maximum number is reached, the algorithm aborts.

Construction. Let H be an instance (producer, consumer) of hashing in the head where the allocation algorithm is Cuckoo hashing with w hash functions and b bins (we discuss concrete values for w and b later in this section). We construct a

(n, m, k, b, p)-PBC as follows.

Encode(DB). Given a collection DB of n elements, follow H's producer algorithm to allocate the n elements to the b buckets. This results in $m = wn$ total elements distributed across the b buckets (each bucket may contain a different number of elements). Return the buckets.

GenSchedule(I). Given a set of indices I, follow H's consumer algorithm to allocate the k indices to the b buckets. Return the mapping of indices to buckets. If more than one index maps to the same bucket (i.e., if there are collisions), return $\perp$ instead.

Decode(W). Since Encode performs only replication, all codewords are elements in DB and require no decoding. Furthermore, σ, which is returned by GenSchedule, has only one entry for each index. As a result, W contains only one codeword. Decode returns that codeword.

Concrete parameters. Analyzing the exact failure probability of Cuckoo hashing remains an open problem (see [37] for the most recent progress). However, several works [28, 68] have estimated this probability empirically for different parameter configurations. Following the analysis in [28, §4.2], we choose $w = 3$ and $b = 1.5k$. In this setting, the failure probability is estimated to be $p \approx 2^{-40}$ for $k > 200$ (for smaller k it is closer to 2^{-20}). This means that, assuming the mapping from indices to buckets is pseudorandom, the probability that GenSchedule(I) returns $\perp$ for a set of indices I is p. Figure 5 compares this result with existing batch code constructions and the scheme proposed in Pung [11, §4.4].

5 Multi-query PIR from PBCs

We give the pseudocode for a PBC-based multi-query CPIR scheme in Figure 8. At a high level, the server encodes its database by calling the PBC's Encode procedure. This produces a set of buckets, each of which can be treated as an independent database on which clients can perform PIR. A client who wishes to retrieve elements at indices $I = \{i_0, \ldots, i_{k-1}\}$ can then locally call GenSchedule(I) to obtain a schedule σ. This schedule states, for each index, the bucket from which to retrieve an element using PIR. Because of the semantics of GenSchedule it is guaranteed that no bucket is queried more than once (or $\sigma = \perp$). As a result, the client can run one instance of PIR on each bucket. However, a challenge is determining *which* index to retrieve from each bucket: by assumption (of PIR) the client knows the index in DB, but this has no relation to the index of that same element in each bucket. To address this, we introduce an oracle $\mathcal{O}$ that provides this information (we discuss it below). If the client has nothing to retrieve from a given bucket, the client simply queries a random index for that bucket.

Constructing the oracle $\mathcal{O}$. There are several ways that the client can construct $\mathcal{O}$. The simplest solution is to obtain the mapping from each bucket to the index of elements in DB (for example, items 3, 4, 5 in DB are in bucket 0, 3, 7, 5 in DB are in bucket 1, etc.). While this might sound unreasonable, observe that PIR has an implicit assumption that the client knows the index in DB of the desired element. The client could use the same technique to obtain the corresponding w indices in $\mathcal{B}(DB)$. For example, in the Pung communication system [11], clients obtain this mapping in a succinct Bloom filter [17].

```
1:  function SETUP(DB)
2:      (C_0, ..., C_{b-1}) ← Encode(DB)
3:      for j = 0 to b − 1 do
4:          SETUP(C_j)                                  // See Fig. 1, Line 1
5:
6:  function MULTIQUERY(pk, I, M = {|C_0|, ..., |C_{b-1}|})
7:      σ ← GenSchedule(I)
8:      if σ ≠ ⊥ then
9:          // get an element for each bucket
10:         // pick a random index if the bucket is not used in σ
11:         for j = 0 to b − 1 do
12:             idx_j ← index for bucket j (based on σ and O)
13:             q_j ← QUERY(pk, idx_j, |C_j|)           // see Fig. 1, Line 4
14:         return q ← (q_0, ..., q_{b-1})
15:     else
16:         Deal with failure (see §5)
17:
18: function MULTIANSWER(q, (C_0, ..., C_{b-1}))
19:     for j = 0 to b − 1 do
20:         a_j ← ANSWER(q_j, C_j)                      // see Fig. 1, Line 9
21:     return a ← (a_0, ..., a_{b-1})
22:
23: function MULTIEXTRACT(sk, a, I, σ)
24:     // extract the codewords from the provided PIR answers into cw
25:     for j = 0 to b − 1 do
26:         cw_j ← EXTRACT(sk, a_j)                     // see Fig. 1, Line 14
27:     // select codewords from cw that are relevant to each index in I
28:     for i = 0 in k − 1 do
29:         W ← codewords from cw (based on σ[I_i])
30:         e_i ← Decode(W)
31:     return (e_0, ..., e_{k-1})
```

FIGURE 8—Multi-query CPIR protocol based on a CPIR protocol and a PBC (Encode, GenSchedule, Decode). I is the set of k desired indices and M is the set of bucket lengths. As in Figure 1, this protocol requires an additively homomorphic cryptosystem with algorithms (KeyGen, Enc, Dec), where (pk, sk) are generated from KeyGen.

Another option is for the client to fetch elements using PIR not by index but by some label using PIR-by-keywords [31]. Examples of labels include the name or UUID of a movie, the index in the original DB, etc. One last option is for the clients to construct $\mathcal{O}$ directly. This requires the server to share with clients its source of randomness (e.g., a PRF seed). Clients can then simulate the server's encoding procedure on a database of n dummy elements (replicating each element into w candidate buckets), which yields $\mathcal{O}$. Furthermore, this process is incremental for many hashing schemes: if a client has $\mathcal{O}$ for an n-element database, it can construct $\mathcal{O}$ for a database with $n+1$ elements by simulating the insertion of the last element.

Dealing with failures in the schedule. If the PBC being used has $p > 0$, then it is possible that for a client's choice of indices, $\sigma = \perp$. In this case, the client is unable to fetch all k elements that it wishes to retrieve privately. However, notice that the client learns of this fact *before* issuing any PIR query (see Figure 8, Line 8). As a result, the client has a few options. First, the client can adjust its set of indices (i.e., choose different

elements to retrieve). This is possible in applications where the client needs to retrieve more than a batch of k items. Second, the client can retrieve a subset of the elements. In a messaging application, this would mean that the client would not retrieve all unread messages. In many cases, this is acceptable since messages are not ephemeral so the client can try again at a later time (presumably with a new set of indices). Lastly, the client can fail silently. Which of these strategies is taken by a client depends on the application.

6 Implementation

We build SealPIR by implementing XPIR's protocol [8] on top of version 2.3.0-4 of the SEAL homomorphic encryption library [4]. This required around 2,000 lines of C++ and Rust. The most difficult component to implement was EXPAND (Figure 3), which required the introduction of the substitution homomorphic operation (§3.1). We implement this group operation in SEAL by porting the Galois group actions algorithm from Gentry et al. [43, §4.2]. This required 400 lines of C++, and our changes are now part of the latest version of SEAL.

SealPIR exposes the API described in Figure 1 to applications. One difference with XPIR is that the substitution operation used in EXPAND requires a special cryptographic key (Galois key) to be generated by the client and be sent to the server. However, a client can reuse this key across any number of requests and the key is relatively small (2.9 MB).

Encoding elements into FV plaintexts. In SealPIR, an FV plaintext is represented as an array of 64-bit integers, where each integer is mod t. Each element in the array represents a coefficient of the corresponding polynomial. We encode an element $e \in DB$ into an FV plaintexts $p(x)$ by storing $\log(t)$ bits of e into each coefficient of $p(x)$. If elements are small, we store many elements into a single FV plaintext (for example, the first element is stored in the first 20 coefficients, the second element in the next 20 coefficients, etc.).

Optimizations. We implement an optimization for EXPAND. In FV, an encryption of $2^\ell \pmod{2^y}$, for $y \geq \ell$, is equivalent to an encryption of $1 \pmod{2^{y-\ell}}$. Observe that in Lines 14–16 of Figure 3, EXPAND multiplies the n ciphertexts by the inverse of m where $m = 2^\ell$. Instead, we change the plaintext modulus of the n ciphertexts from $t = 2^y$ to $t' = 2^{y-\ell}$, which allows us to avoid the plaintext multiplications and the inversion, and reduces the noise growth of EXPAND. The result is $n-1$ ciphertexts encoding 0, and one ciphertext encoding 1, as we expect. This also allows us to use any value of t and not just an odd integer (since we avoid inverting m). The one (minor) drawback is that the server must represent the database using FV plaintexts defined with the plaintext modulus t' (rather than t). As a result, we must pack fewer database elements into a single FV plaintext.

Implementing PBCs. We have also implemented *mPIR*, a multi-query PIR library based on PBCs. mPIR implements 5 different PBC constructions: each is a different instance of hashing in the head (§4.4) with a different allocation algorithms (e.g., two-choice hashing, Cuckoo hashing, the Hybrid allocation scheme in Pung [11]). This library works transparently on top of both XPIR and SealPIR, and is written in 1,700 lines of Rust. It uses SHA-256 with varying counters to implement the different hash functions.

7 Evaluation

Our evaluation answers four questions:

1. What are the concrete resource costs of SealPIR, and how do they compare to XPIR?
2. What is the throughput and latency achieved by SealPIR under different deployment scenarios?
3. What are the concrete benefits provided by PBCs, and how do they compare to existing batch codes?
4. What is the impact of using SealPIR and mPIR on a representative system?

Experimental setup. We run our experiments using Microsoft's Azure instances in three data centers: West US, South India, and West Europe. We run the PIR servers on H16 instances (16-core 3.6 GHz Intel Xeon E5-2667 and 112 GB RAM), and clients on F16s instances (16-core, 2.4 GHz Intel Xeon E5-2673 and 32 GB RAM), all running Ubuntu 16.04. We compile all our code with Rust's nightly version 1.25. For XPIR, we use the publicly available source code [9]. We report all network costs measured at the application layer. We run each experiment 10 times and report averages from those 10 trials. Standard deviations are less than 10% of the reported means.

Parameters. We choose FHE's security parameters following XPIR's latest estimates [5], which are based on the analysis by Albrecht et al. [10]. We set the degree of ciphertexts' polynomials to 2048, and the size of the coefficients to 60 bits (N and q in the terminology of Section 3). More specifically, SEAL needs values of $q \equiv 1 \pmod{2^{18}}$, whereas XPIR needs values of $q \equiv 1 \pmod{2^{14}}$ [6]. $q = 2^{60} - 2^{18} + 1$ works for both.

Each database element is 288 bytes. We choose this size since the Pung communication system uses 288-byte messages (§7.4). Unless otherwise stated, for SealPIR we use a plaintext modulus value of $t = 2^{23}$. A larger value of t leads to lower network and computational costs, but might cause noise to grow too much, which can make the result to be not decryptable (we lower t in some experiments to ensure that we can always decrypt the result). For XPIR, we use $\alpha = 16$, meaning that we pack α elements into a single logical element, thereby reducing the number of elements in the database by a factor of α. For 288-byte elements and our security parameters, setting $\alpha = 16$ has roughly the same effect as setting $t = 2^{23}$ in SealPIR (although our optimization to EXPAND, which we discuss in Section 6, means that SealPIR packs fewer elements together).

7.1 Cost and performance of SealPIR

To evaluate SealPIR, we run a series of microbenchmarks to measure: (i) the time to generate, expand, and answer a query; (ii) the time to extract the response; and (iii) the time to preprocess the database. We study several database sizes and repeat the same experiment for XPIR using two different depth parameters d (§3.4). Figure 9 tabulates our results.

CPU costs. We find that the computational costs of the client are lower under SealPIR than under XPIR. This is because the client in SealPIR generates d ciphertexts as a query rather

	XPIR ($d = 2$)			XPIR ($d = 3$)			SealPIR ($d = 2$)		
database size (n)	65,536	262,144	1,048,576	65,536	262,144	1,048,576	65,536	262,144	1,048,576
client CPU costs (ms)									
QUERY	18.43	44.16	90.39	7.51	11.26	20.08	3.11	3.20	3.20
EXTRACT	0.86	0.86	0.86	6.03	6.61	6.91	1.91	1.91	1.91
server CPU costs (sec)									
SETUP	0.61	2.43	7.53	0.18	2.14	7.24	0.32	1.18	5.46
EXPAND	N/A	N/A	N/A	N/A	N/A	N/A	0.068	0.13	0.28
ANSWER	0.26	0.71	2.50	0.25	1.17	3.22	0.15	0.51	1.99
network costs (KB)									
query	4,096	8,192	16,384	1,248	2,464	3,872	64	64	64
answer	512	512	512	3,424	3,872	3,872	256	256	256

FIGURE 9—Microbenchmarks of CPU and network costs for XPIR and SealPIR under varying database sizes (n). Elements are of size 288 bytes.

than $d \cdot \sqrt[d]{n}$ ciphertexts as in XPIR (§3.4). Furthermore, XPIR with $d = 3$ produces larger answers (i.e., they contain more ciphertexts) which require more time to decrypt.

However, SealPIR's EXPAND procedure introduces CPU overheads to the server. Specifically, the overhead over computing an answer using an expanded query vector (as in XPIR) is between 14% and 45% depending on the database size. While this is high, we think this is an excellent trade-off given the significant network savings (which we discuss below). Also, for answering a query (after it has been expanded), SealPIR is always faster than XPIR. Furthermore, the total CPU cost of answering a query including the expansion of the query is competitive with XPIR's cost of answering a query.

We note that larger values of d lead to more computation for the server for two reasons. First, structuring the database as a d-dimensional hyperrectangle often requires padding the database with dummy plaintexts to fit all dimensions. Second, as we discuss in Section 3.4, the ciphertext expansion factor effectively increases the size of the elements by a factor of F after processing each dimension, necessitating more computation.

Network costs. For network costs, SealPIR enjoys a significant reduction owing to its query encoding and EXPAND procedure (§3.3). For larger databases, the query size reductions over XPIR are 256× when $d = 2$, and 60× when $d = 3$.

7.2 SealPIR's response time and throughput

While microbenchmarks are useful for understanding how SealPIR compares to XPIR, another important axis is understanding how these costs affect response time and throughput.

7.2.1 Response times

To measure response time, we run experiments where we deploy a PIR server in Azure's US West data center, and place a PIR client under four deployment scenarios. We then measure the time to retrieve a 288-byte element using SealPIR, XPIR, and `scp` (i.e., secure copy command line tool). We use `scp` to emulate a naive version of PIR in which a client downloads the entire database.

Deployment scenarios. We consider a variety of deployment scenarios as detailed below to measure response times.

intra-DC: the client and the server are both in the US West data center. The bandwidth between the two VMs is approximately 3.4 Gbps (measured using the `iperf` measurement tool). This scenario is optimistic since it makes little sense to use PIR inside two VMs in the same data center controlled by the same party. Nevertheless, it gives an idea of the performance that PIR schemes could achieve if network bandwidth were plentiful.

inter-DC: the client is placed in the South India data center. The bandwidth between the two VMs is approximately 800 Mbps. This scenario represents clients who deploy their applications in a data center (or well-provisioned proxy) that they trust, and access content from an untrusted data center.

home network: the client is placed in the South India data center. However, we use the `tc` traffic control utility to configure the Linux kernel packet scheduler in both VMs to maintain a 20 Mbps bandwidth. We choose this number as it is slightly over the mean download speed in the U.S. (18.7 Mbps) according to Akamai's latest connectivity report [1, §4]. This scenario is optimistic to XPIR since it ignores the asymmetry present in home networks where the uplink bandwidth is typically much lower (meanwhile in XPIR, the queries are large). Nevertheless our aim is to give a rough estimate of a common PIR use case in which a client accesses an element from their home machine.

mobile carrier: the client is placed in the South India data center. We use the `tc` utility to configure VMs to maintain a 10 Mbps bandwidth. We choose this number as it approximates the average data speed achieved by users across all U.S. carriers according to OpenSignal's 2017 State of Mobile Networks report [2] and Akamai [1, §8]. As with the home network, this scenario is optimistic (for XPIR) as it ignores the discrepancy between download and upload speeds. It aims to represent the use of PIR from a mobile device, which is a common deployment for applications such as private communication (§7.4).

Results. Figure 10 depicts the results. At very high speeds (intra-DC), naive PIR (`scp`) is currently the best option, which is not surprising given the computational costs introduced by PIR. However, for all other network speeds, XPIR and SealPIR significantly outperform downloading the entire database. As network bandwidth decreases (e.g., home, mobile), SealPIR's lower network consumption and competitive computational costs yield up to a 42% reduction in response time.

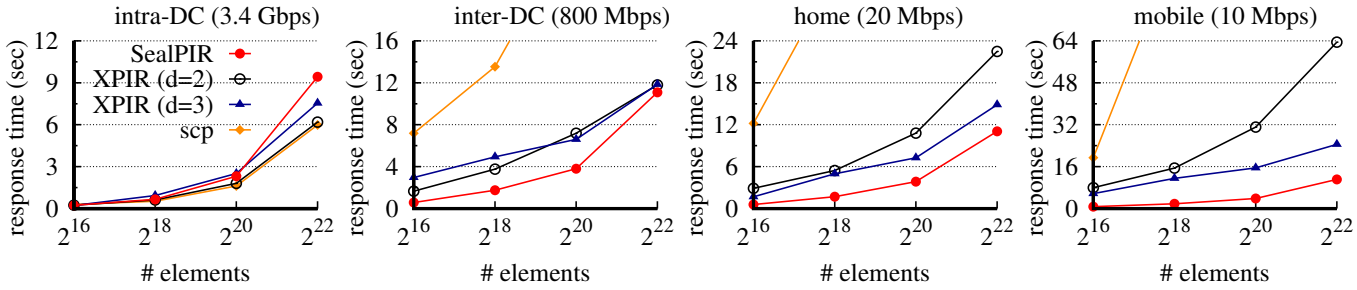


FIGURE 10—Mean response time experienced by a client under different deployments (see text for description of different network conditions) with different PIR schemes. When the network bandwidth is plentiful (e.g., intra-DC), downloading the entire database from the server (`scp`) achieves the best response time. However, when the network bandwidth is limited (e.g., home, mobile), SealPIR outperforms both XPIR and `scp`.

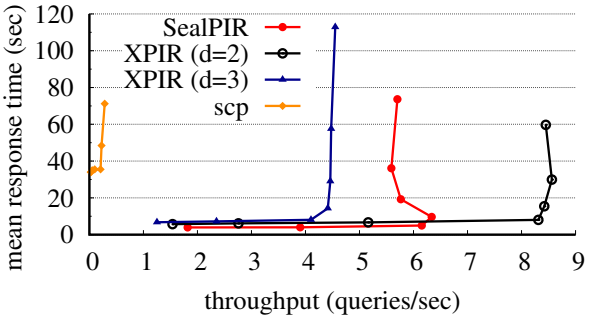


FIGURE 11—Comparing throughput vs. mean response time under SealPIR and XPIR (with $d = 2$ and $d = 3$) when using a database with 2^{20} elements where each element is 288 bytes long. We find that XPIR with $d = 2$ saturates at 8 requests/second whereas SealPIR saturates at 6 requests/second (a 25% reduction in throughput). When XPIR uses $d = 3$, SealPIR achieves about 50% higher throughput.

7.2.2 *Throughput*

To measure throughput, we deploy the PIR server in Azure's US West data center, but access it with an increasing number of concurrent PIR clients deployed across the South India and EU West data centers. The server's bandwidth is 800 Mbps.

We then measure the number of requests serviced per minute at the server, and the request completion times at the clients. Figure 11 depicts the results of running from 4 to 256 clients each requesting one 288-byte element from a database with 2^{20} entries. In our experiments, we ensure that the bottleneck is the server's CPU or WAN network connection, and not the clients or some link between specific data centers.

We find that SealPIR achieves a 50% higher throughput than XPIR with $d = 3$, but a 25% lower throughput than XPIR with $d = 2$. Most of the difference can be attributed to EXPAND, but we believe that with further engineering we can close this gap (since SealPIR is computationally more efficient than XPIR according to microbenchmarks). Compared to naive PIR via `scp`, SealPIR and XPIR achieve over 20× higher throughput since the primary bottleneck in naive PIR is network bandwidth and not CPU (which are bottlenecks for both SealPIR and XPIR).

7.3 Benefits of PBCs

To understand how PBCs can improve throughput and what type of network overhead they add, we repeat the microbenchmark experiments of Section 7.1, but this time we use mPIR. To put the benefits and costs in context, we also evaluate the multi-query PIR scheme found in Pung [11]. Pung's protocol, like PBCs, is probabilistic and significantly improves over existing batch codes in terms of costs.

Figure 12 tabulates the results. We find that mPIR does a better job than Pung's scheme at amortizing CPU costs across all batch sizes. This is a direct effect of the Cuckoo PBC producing fewer total codewords (see Figure 5), since computational costs are proportional to the number of elements after encoding (m). At $k = 256$ and 288-byte elements, mPIR achieves a 2.7× reduction in CPU cost for the server when answering queries over Pung's scheme. Over the naive approach of processing queries independently, the per-request CPU cost of mPIR is 50.3× lower. Repeating the experiment in Figure 11 we find that mPIR (with SealPIR as the underlying PIR scheme) and a batch of $k = 256$ achieves a throughput of 197 queries/sec.

The difference in network costs is more pronounced. This owes to Pung's scheme building on the subcube batch code of Ishai et al. [51] which creates a large number of buckets (see Figure 5); to preserve privacy, clients must issue a PIR query to each bucket. In terms of concrete savings, mPIR is 7× more network efficient (upload and download) than Pung's scheme. Considering that mPIR also has a lower failure probability (around 2^{-40}, compared to Pung's 2^{-20}), this suggests that mPIR is an attractive replacement to Pung's multi-query protocol, offering improvements on all axes.

7.4 Case study: Pung with SealPIR and mPIR

To get a sense of the end-to-end benefits that SealPIR and mPIR provide to actual applications, we modify the available implementation of the Pung unobservable communication system [3]. Pung is a messaging service that allows users to exchange messages in rounds without leaking any metadata (who they are talking to, how often, or when). We choose Pung because it uses XPIR to achieve its privacy guarantees, and because it also relies on multi-query PIR to allow clients to receive multiple messages simultaneously. Consequently, we can switch Pung's PIR engine from XPIR to SealPIR, and we can replace Pung's custom multi-query PIR scheme with mPIR.

Experiment. In our experiment, we have clients send and retrieve k messages in a closed-loop, meaning that we advance rounds as soon as all clients have sent and retrieved the mes-

	single-query	Pung's multi-retrieval			mPIR (Cuckoo hashing)		
batch size (k)	1	16	64	256	16	64	256
client CPU costs (ms)							
MultiQuery	3.19	28.77	28.56	28.2	4.85	4.33	4.17
MultiExtract	2.58	19.56	15.97	15.97	2.87	2.96	2.97
server CPU costs (sec)							
MultiSetup	7.41	2.18	0.68	0.30	1.67	0.42	0.12
MultiAnswer	3.62	1.31	0.47	0.19	0.65	0.21	0.072
network costs (KB)							
query	64	577	577	577	84	84	83
answer	384	2,885	2,308	2,308	420	420	333

FIGURE 12—Per-request (amortized) CPU and network costs of two multi-query PIR schemes on a database consisting of 2^{20} elements, with varying batch sizes. The schemes are Pung's multi-retrieval protocol and mPIR, which is based on PBCs (Cuckoo variant). The second column gives the cost of retrieving a single element (no amortization). The underlying PIR library is SealPIR with $t = 2^{20}$ and elements are 288 bytes.

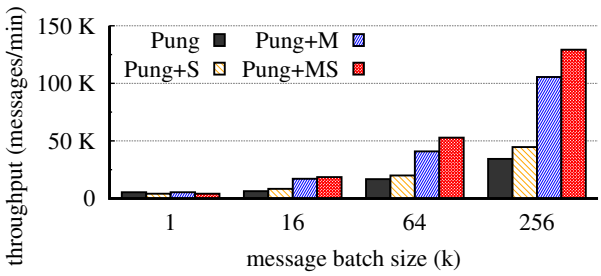


FIGURE 13—Throughput of Pung on a deployment of 4 servers with 256K users, each sending and retrieving k 288 byte messages per round. The label "Pung" indicates the implementation as given in [3], with updated security and XPIR parameters (§7). "Pung+S" corresponds to a version of Pung that uses SealPIR with $t = 2^{20}$; "Pung+M" corresponds to a version of Pung that uses mPIR; and "Pung+MS" corresponds to a version of Pung that uses both mPIR and SealPIR.

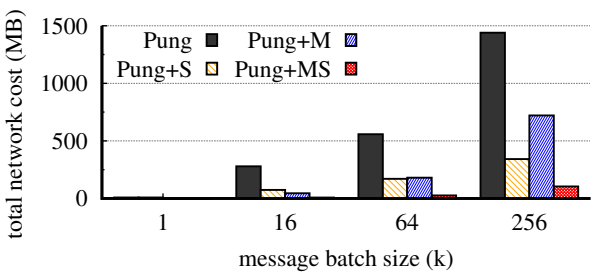


FIGURE 14—Per-user total network cost (upload and download) of a Pung deployment with 256K users. Each user sends and retrieves k 288-byte messages. See Figure 13 for an explanation of the legend.

sages, rather than waiting for a timeout. To experiment with many clients we employ the same simulation technique used in Pung: we have 64 real clients accessing each server, and simulate additional clients by pre-populating the servers' databases with random messages.

Figure 13 shows the throughput in messages per minute that Pung achieves with mPIR and SealPIR ("Pung+MS"). Pung+MS yields better performance than the existing Pung code base for all batch sizes greater than 1. There are three reasons for this. First, Pung's multi-retrieval scheme produces 50% more codewords than mPIR, and therefore has to process over more elements. Second, Pung's multi-retrieval scheme produces 7× more buckets than mPIR. This forces Pung to run XPIR on many small databases that contain an average of 500 to 8,000 elements (depending on the batch size), which exacerbates XPIR's fixed costs (it is more efficient to run one instance of XPIR on a database of 100,000 elements than two instances of XPIR on databases of 50,000 elements).

Last, even though SealPIR incurs additional CPU costs than XPIR ($d = 2$) on large databases as we show in Section 7.1 (this is also why Pung has higher throughput than Pung-MS when the batch size is 1), SealPIR is actually faster when the database is small (see the columns with 65,536 elements in Figure 9). Ultimately, thanks to these factors we find that if clients send $k = 64$ messages per round, Pung+MS processes 3.1× more messages per minute than Pung.

When it comes to network costs, the benefits of SealPIR and mPIR are considerable. Figure 14 depicts the total network cost incurred by a single client for one round of the Pung protocol. We find that the compressed queries and fewer buckets result in savings of over 36×. In particular, the per-client communication costs are cut down to 7 MB per round for a batch size of 16 (versus 279 MB in the original Pung implementation).

8 Discussion

SealPIR significantly reduces the network cost of XPIR, while introducing modest computational overheads. However, there are several opportunities to reduce CPU costs further. Observe that in EXPAND and Stern's protocol, when the database dimension (d) is greater than 1 (see Section 3.4) the computation consists of several matrix-vector products. We can therefore implement the optimization described by Beimel et al. [16] where multiple queries (from potentially different users) are aggregated to form a matrix; the server can then use a subcubic matrix multiplication algorithm to compute the result (§2.2).

Another area of potential improvement is in the design of PBCs. As we show in our evaluation, PBCs built from hashing in the head reduce costs over existing methods, but so far we have only studied allocation strategies that are typically used for *online* load balancing (i.e., balls arrive one at a time). We could also consider strategies that optimize for the *offline* setting in which all balls are available at the same time (which is the case

in PBCs). In this setting, the allocation process can be phrased in terms of orienting the edges of undirected graphs in order to obtain directed graphs with minimum in-degree [24]. Optimal solutions for this problem can be computed in polynomial time [29], and linear time approximations also exist [24, 34, 41].

Appendix

A Query expansion

A.1 Correctness of query expansion

Below we prove that EXPAND (Figure 3) correctly expands one ciphertext into a vector of n ciphertexts with the desired contents. The following theorem makes this formal.

Theorem 1. Let N be a power of 2, $N \geq n$, and $query = Enc(x^i)$ be the client's encoding of index i. The n output ciphertexts $o_0, \ldots, o_{n-1}$ of EXPAND($query$) satisfy, for all $0 \leq k \leq n-1$:

$$o_k = \begin{cases} Enc(1) & \text{if } i = k \\ Enc(0) & \text{otherwise} \end{cases}$$

Proof. It suffices to prove the case for $n = 2^\ell$. For $j = \{0, 1, \ldots, \ell - 1\}$, we claim that after the j^{th} iteration of the outer loop, we have $ciphertexts = [c'_0, \ldots, c'_{2^{j+1}-1}]$ such that

$$ciphertexts[k] = \begin{cases} Enc(2^{j+1}x^{i-k}) & \text{if } i \equiv k \pmod{2^{j+1}} \\ Enc(0) & \text{otherwise} \end{cases}$$

We prove the claim by induction on j. The base case $j = 0$ is explained in the main text of Section 3.3. Suppose the claim is true for some $j \geq 0$. Then in the next iteration, we compute an array $ciphertexts'$.

For the first half of the array, i.e., $0 \leq k < 2^{j+1}$, we have $ciphertexts'[k] = ciphertexts[k] + \mathsf{Sub}(ciphertexts[k], N/2^{j+1} + 1)$. If $i \not\equiv k \pmod{2^{j+1}}$, then $ciphertexts'[k]$ is an encryption of 0; otherwise, there is an integer r such that $i - k = 2^{j+1} \cdot r$, and $\mathsf{Sub}(ciphertexts[k], N/2^{j+1} + 1) = Enc(2^{j+1}x^{(N/2^{j+1}+1)(2^{j+1}r)}) = Enc(2^{j+1}(-1)^r x^{i-k})$. Hence, if r is odd, then $ciphertexts'[k]$ is an encryption of 0; otherwise, $ciphertexts'[k]$ is an encryption of $2^{j+2}x^{i-k}$. So the claim follows because r is even if and only if $i \equiv k \pmod{2^{j+2}}$.

We now prove the claim for the second half of the array $ciphertexts'$. The only interesting case is $i \equiv k - 2^{j+1} \pmod{2^{j+1}}$. In this case, it is easy to see that $ciphertexts'[k]$ is again $Enc(2^{j+1}(-1)^{(i-k)/2^{j+1}}x^{i-k})$. So the same argument applies.

Finally, using the above claim we can show that after the outer loop in EXPAND, we have an array of 2^ℓ ciphertexts such that:

$$ciphertexts[k] = \begin{cases} Enc(2^\ell x^{i-k}) & \text{if } i \equiv k \pmod{2^\ell} \\ Enc(0) & \text{otherwise} \end{cases}$$

However, note that $i < n = 2^\ell$, so $i \equiv k \pmod{2^\ell}$ implies $i = k$. Hence $ciphertexts[k]$ is either an encryption of 0 or an encryption of 2^ℓ. To obtain an encryption of 0 or 1, we multiply $ciphertexts[k]$ by the inverse of 2^ℓ modulo t in the last step (Figure 3, Line 14). □

A.2 Noise growth of query expansion

One advantage of our query expansion technique over the straw man FHE solution given in Section 3.1 (besides the one mentioned in that section) is that our approach has *much* smaller noise growth. We bound the noise growth of EXPAND (Figure 3) in the theorem below. Before stating the theorem, we give some background on noise. See the SEAL manual [27] for a more detailed explanation. We have that the noise of the addition of two ciphertexts is the sum of their individual noises. Plain multiplication by a monomial x^j (for some j) with coefficient 1 does not change the noise, and plain multiplication by a constant α multiplies the noise by α. Substitution adds a constant additive term B_{sub} to the noise, which depends on the FV parameters.

Theorem 2. Let v_{out} be the output noise of EXPAND, and v_{in} be the input noise. Let t denote the plaintext modulus in EXPAND, and let $k = \lceil \log(n) \rceil$. We have that

$$v_{out} \leq t \cdot (2^k(v_{in} + 2B_{sub}))$$

Proof. Let v_i be the noise after the i^{th} iteration in EXPAND (setting $v_0 = v_{in}$). Then $v_i = 2(v_{i-1} + B_{sub})$. Carrying out the sum, we get

$$v_k = 2^k v_0 + 2(2^k - 1)B_{sub} < 2^k(v_0 + 2B_{sub})$$

Since inverse $\leq t$, the final plain multiplication results in $v_{out} \leq t v_k$. This completes the proof. □

B Cost of PBC variants

We have implemented five PBCs with different allocation algorithms using hashing in the head. Our goal is to show that all of them admit efficient encoding and decoding procedures. For the purpose of building a multi-query PIR scheme, we wish to select a PBC variant that reduces the number of codewords (m) and buckets (b). Our hypothesis is that PBCs that produce low values of m and b result in more expensive encoding and schedule generation procedures.

To test this hypothesis we create a collection with 131K elements, each of which is 1 KB, and encode the collection with the different PBCs for a batch size of $k = 64$. We then measure the time to encode, decode, and generate a schedule. We also experiment with other element and collections sizes and find that while the absolute costs vary, they are still small (considering Encode is a one-time operation), and the relative costs are consistent.

Figure 15 lists the CPU time taken by various operations for all the variants we have implemented. Our hypothesis holds to an extent: all the variants that are based on replication (for the producer) and hashing (for the consumer) follow our prediction. The source of costs for schedule generation corresponds to the time taken to find a solution to a k balls, b bins, and w choices problem. The different allocation strategies approximate the optimal solution, and among them, Cuckoo hashing yields the best approximation by recursively relocating elements when there are collisions (§4.5). Encoding performance, on the other hand, is based on the number and the cost of the memory copies, since encoding is a simple repetition code.

PBC scheme	Encode	GenSchedule	Decode
k-way replication	22.5 ms	5.8 μs	0.1 μs
sharding	52.1 ms	112.8 μs	0.3 μs
2-choice hashing	103.6 ms	212.9 μs	0.2 μs
Pung Hybrid	101.8 ms	42.3 μs	1.2 μs
Cuckoo hashing	154.1 ms	319.2 μs	0.15 μs

FIGURE 15—Cost of operations for five PBCs implemented as part of mPIR. The collection size (n) is 524,288 and the batch size (k) is 64. Each element in the collection is 288 bytes. k-way replication simply replicates the n balls into k-bins during the producer's allocation, and picks a different bin for the k balls during the consumer's simulation. Sharding maps balls to a single bin during the producer's allocation, and the consumer uses a hash function during simulation (this variant has a high failure rate which we improve by replicating buckets).

Our hypothesis does not hold for the PBC variant that corresponds to a port of Pung's Hybrid multi-retrieval protocol. The reason is that this variant is partially based on the subcube batch code of Ishai et al. [51], for which the final position of each input element is statically determined and does not require computing a hash function (unlike our hashing variants). This allows computing a schedule by consulting a lookup table.

Finally, as mentioned above, our goal with this experiment was to confirm that all PBCs have reasonably efficient encoding, decoding, and schedule generation procedures. As such, our evaluation (§7) focuses only on the Cuckoo variant since it yields the most efficient parameters, and the second lowest failure probability (k-way replication never fails, but has a very high value of m).

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