# Improved Hybrid Consensus Scheme with Privacy-preserving Property

*Abstract*—**Proof-of-work-based consensus, adopted in Bitcoin, has already drawn much attention from cryptocurrency and block chain community. Despite its nice decentralization property, it has significant limitation in terms of efficiency since transactions can not be confirmed within seconds. In 2016, hybrid consensus was proposed to partially deal with this issue by introducing committee responsible for validating transactions. However, there still exists some issues with respect to this hybrid consensus such as selfish mining, fairness to the election of committee member, incentives for the consensus scheme, and so on.**

**To improve the hybrid consensus further, we first present a substitution for proof-of-work, named as fair-proof-of-work (FPoW), to solve the issues related to selfish mining and fair committee election. We also demonstrate the incentives for our improved hybrid consensus. Then, based on this consensus, we build privacy-preserving constructions to make the consensus more applicable and powerful. It is expected that this novel consensus scheme could be adopted in block chains which require decentralization, high efficiency, as well as privacy-preserving.**

*Index Terms*—**Block chain, Proof-of-work, Consensus, Privacy-preserving, Hybrid Consensus.**

## I. INTRODUCTION

Decentralized ledger-based currency systems (such as *Bitcoin*[Nak08]) provided a novel means of constructing payment system without a trusted central bank [BMC+15].

Blockchain requires miners to solve a hash problem, that is to find a nonce $\mathsf{nc}$, so that $H(\mathsf{rec}, \mathsf{nc}) \in \mathsf{target}$, where $\mathsf{rec}$ is all transaction records in this block, and $H(\cdot)$ is a collision-resistant hash function.

Removal of central bank leads to cost of transaction privacy and efficiency. Traditional block chain is facing two tough problems. Firstly, it has no guarantee on privacy of identity and transaction amount, although pseudo-identity might be utilized to achieve anonymity, some previous works have shown that such anonymity is not dependable. Secondly, traditional block chain grows at unsatisfactory speed.

Works have been done in attempt to achieve real anonymity. *Zerocoin* and *ZeroCash* proposed in [MGGR13] and [BCG+14], provided us a novel vision of transaction anonymity by implementing zero-knowledge proof. Also, new mechanisms were proposed to achieve better efficiency by replacing *Proof-of-work* by *Proof-of-stake* (see [BGM16]) or PBFT (see *Hybrid Consensus*).

Hybrid consensus proposed in [PS16b] gave an outline of new utilization of *proof-of-work* by taking Nakamoto Chain or FruitChain (see [PS16a]), which were named snailchain, as generator of a rotating committee, and all transactions are validated by the committee through a Permissioned Byzantine Fault-tolerance(PBFT) [CL99] protocol. Committee members of each round are miners of csize blocks on-chain, that

is to say, committee members will wait till generation of csize new blocks to perform a switchover. Soundness of this construction is guaranteed as long as over $3/4$ (or $2/3$ for Fruit Chain) computing power is at hands of honest nodes. In *Hybrid Consensus*, by term *round*, it meant time interval for creation of csize blocks in snailchain, which was also the duration of service of current committee. Rounds were denoted with consequent natural numbers starting from 1. Validated transaction log of round $R$ is denoted as $\mathsf{rec}_R$, and $\mathsf{rec}_R[l]$ means $l$-th transaction in daily log ($l$ is called a sequence number). By notation $\mathsf{CM}_R = [\mathsf{ID}_1, \mathsf{ID}_2, \dots, \mathsf{ID}_{\mathsf{csize}}]$, we denote set of committee members for round $R$.

Here are **Some drawbacks of original *Hybrid Consensus***:

- **Privacy.** In original work of *Hybrid Consensus*, transactions were stored in the clear. Hence, privacy was not guaranteed.
- **Motivation for honesty.** In *Hybrid Consensus*, honesty of committee members were guaranteed from block reward and transaction fee. However, it merely guaranteed honesty and hard-working of nodes as committee candidates (i.e. miners), not as committee members.
- **Existence of forking.** In *Hybrid Consensus*, forking of underlying snailchain existed, wasting great amount of time and energy, leading to security hazards such as possibility of *Selfish Mining* (that is the reason why it required $3/4$ overall honesty rate instead of $2/3$).

In this paper, we will propose a new cryptocurrency mechanism. It is an improvement of *Hybrid Consensus*, providing us privacy-preserving property, as well as satisfactory efficiency and stronger security guarantees. Demand on honest rate of total computing power is $2/3$ in our scheme.

### A. Our contribution

In our construction, we use modified *Hybrid Consensus* as underlying protocol. We can achieve following properties.

- **Transaction Privacy.** In this construction, all transactions are accessible only to members of rotating committee, with techniques in Section IV.
- **Permissionless model with excellent Efficiency.** This is a permissionless model, where nodes can join and leave dynamically. In traditional constructions, a permissionless model means terrible performance in efficiency of transaction validation. However, with rotating committee elected from underlying snailchain (see *Hybrid Consensus*), we can validate transactions through BFT network among committee members. In such way, satisfactory efficiency can be achieved. Inherited from *Hybrid Consensus*, our confirmation time is bounded only by actual delay, instead of theoretical upper-bound of delay.

- **Forking-free.** In traditional blockchain, forking has to be implemented in order to tackle with ambiguous. However, forking is waste of time and energy. Users have to wait for generation of sufficiently many new blocks to conform a transaction. Power consumed by miners who followed "wrong" block has to be wasted in vain. For this reason, fairness is lost. Also, existence of forking leads to existence of selfish mining. In *Hybrid Consensus*, forking still exists in underlying snailchain. While in our construction from Section III, forking can be prevented.
- **Security.** Compared with related works (including *Hybrid Consensus*), our construction is endowed with following security properties.

  **Tolerated corruption.** In this work, we require roughly $2/3$ overall honesty to achieve $2/3$-chain quality, so as to assure $2/3$ BFT committee members are honest. (*Hybrid Consensus* utilizing traditional blockchain as underlying snailchain required $3/4$ overall honesty)

  **Looser assumption against mildly agile corruption.** In *Hybrid Consensus*, adversary is allowed to perform mildly agile corruption, i.e., they can choose nodes to corrupt according to the configuration of environment. $\tau$-agility, which means an adversary has to wait for $\tau$ time to corrupt a honest node, is defined to describe assumption on adversary's capability. In our work, assumption on $\tau$ can be much looser than that of *Hybrid Consensus*.

  **Preventing selfish mining.** In traditional block chain, with existence of transaction pool, selfish mining may happen (see [ES14]). However, in our forking-free construction, selfish mining has no reason to exist.
- **Fairness in competition.** Without existence of forking, selfish mining is prevented. Also, with our FPoW, better fairness to committee candidates can be guaranteed in face of network delay.

## II. NOTATIONS AND PRELIMINARIES

Notations are shown in TABLE I. In following contents we show some preliminaries.

### A. Proof-of-work

*Proof-of-work*, has been introduced to bitcoin system in order to make sure any newly generated block is mined by an honest node with probability equal to fraction of total honest computing power.

In detail: suppose $H$ is a cryptographic hash function, $\boldsymbol{B}_i$ is $i$-th block on chain, and target is a target range to adjust difficulty of puzzle. Miners of block prove their computing power by trying to solve hash-puzzles $H(\boldsymbol{B}_{i-1}, \mathsf{nc}_i) \in \mathsf{target}$ after block (say, $\boldsymbol{B}_{i-1}$) on chain, so that (s)he could propose $\boldsymbol{B}_i$ with $\mathsf{nc}_i$ (along with reward transaction to reward itself) appended onto it.

### B. Blockchain

*Block Chain* (for short, *blockchain*), was digital currency system firstly proposed by *Satoshi Nakamoto* in [Nak08]. In bitcoin system, network links transactions by time-stamping

technique, and hashing them into a chain of hash-based proof-of-work. In such way, history that cannot be tampered without redoing the proof-of-work is formed.

### C. Hybrid consensus

*Hybrid Consensus*, proposed in [PS16b], was a brand-new cryptographic scheme utilizing Nakamoto blockchain or *FruitChain* (see [PS16a]) as underlying snailchain, so as to dynamically maintaining a rotating committee. All transactions are verified by a BFT network among committee members.

### D. Permissioned model

Permissionless models, where nodes are allowed to join and leave dynamically, often lacks effciency in practice. On the other hand, Permissioned models, where nodes are pre-determined, can achieve satisfactory efficiency. In *Hybrid Consensus*, a rotating committee is elected from a permissionless environment, so as to perform permissioned BFT among committee members. With this technique, efficiency of transaction validation is guaranteed in a permissionless environment.

### E. Mildly agile corruption

In fully adaptive corruption model, we assume that adversary can perform any corruption without any cost of time. This assumption is too strong in practice, since adversary has to spend long time locating a node, when adversary in fact only know its pseudo-identity.

$\tau$-**agile corruption** is assumption that adversary has to spend time $\tau$ to corrupt a node. That is to say, time interval between adversary's deciding to corrupt a node and node's getting finally corrupted should be at least $\tau$.

## III. IMPROVED HYBRID CONSENSUS WITH FAIR-PROOF-OF-WORK

In *Hybrid Consensus*, for each round, transactions are validated through PBFT network among committee members. However, committee election in *Hybrid Consensus* is based on traditional PoW, hence is not forking-free (which we will unfold in following contents). To improve this, we propose our *Fair-Proof-of-Work* (for short, FPoW).

### A. Demand for forking-free construction

When multiple nodes mine a block at almost same time, how to make sure all nodes concede to "miner of next block"? In traditional block chain, forking of chain helps to solve this issue. Forking is necessary in classical cryptocurrency systems, because forking also helps tackle with misbehaviour of miners. But it is not the case in *Hybrid Consensus*, since validation of transactions has nothing to do with non-committee miners.

From our perspective, demand for forking-free construction arises mainly for three reasons. Firstly, forking is waste of energy, as much computation power would be wasted in vain merely in order to tackle with ambiguity. Secondly, forking is waste of time and long confirmation time means

| Notation | Description |
|---|---|
| $\kappa, \lambda$ | security parameters for encryption and consensus |
| ID | in this work, by term identity, we mean a pseudo-identity named by node itself (collision can be prevented) |
| $R$ | round number |
| $\ell$ | sequence number |
| $\mathsf{CM}_R$ | committee members for round $R$ |
| csize | size of rotating committee $\mathsf{csize} := \Theta(\lambda)$ |
| ek | key for symmetric encryption |
| $(\mathsf{pk}, \mathsf{sk})$ | public and private key for digital signature scheme |
| $(pk, sk)$ | public and private key for hybrid encryption scheme |
| $(\mathsf{PK}, \mathsf{SK})$ | public and private key for public key encryption scheme |
| $(k, \psi)$ | session key $k$ and its encapsulation $\psi$ in hybrid encryption scheme |
| $in = (R, l)$ | $in$ is index of one transaction, $R$ is its round number and $l$ is its sequence number of that round |
| $\mathsf{round}(in)$ | $\mathsf{round}((R,l)) = R$ |
| $\mathsf{seq}(in)$ | $\mathsf{seq}((R,l)) = l$ |
| $\langle \mathsf{tx}, \mathsf{tx}' \rangle$ | pair of payment transaction and corresponding charge transaction |
| $\langle \mathsf{TX}, \mathsf{TX}' \rangle$ | pair of payment transaction and corresponding charge transaction after encryption |
| txc, TXC | transaction command txc and its encryption TXC |
| txp, TXP | transaction proof txp and its encryption TXP |
| val(tx) | transaction amount for tx |
| $\mathsf{val}_{\mathsf{ek}}(\mathsf{TX})$ | transaction amount for TX, which requires symmetric key ek to be disclosed |
| $(pk_{com}, sk_{com})$ | public and private key for committee in hybrid encryption scheme |

TABLE I
NOTATIONS FOR CONSTRUCTION

that committee members might be more vulnerable under target corruption. Thirdly, forking-free helps to prevent *selfish-mining* (since selfish mining exists only when forking is possible, see [ES14]).

### B. Fair-proof-of-work

In original construction of *Hybrid Consensus*, where committee members are selected from traditional *Proof-of-work*, controversial problems arises. Compared with original *Hybrid Consensus*, FPoW mainly has following advantages:

- **Forking-free.** One issue of *Hybrid Consensus* is that, forking still happens in competition for block mining. However, we hope to prevent forking. To avoid forking, we can revise the rule and stipulate that blocks should be appended with broadcast time and the one that is broadcast first should be the next block. Certainly this is impractical, since it is hard for all nodes to share the same clock, and nodes have no reason to behave honestly when appending broadcast time.

  In our construction, we need neither guarantee on time synchronization nor honesty of nodes, to achieve a fair competition for "*first* miner of next block" without existence of forking.
- **Fairness in competition.** Our construction guarantees on better fairness on candidates suffering network delay. In **Appendix C**, we will give a formal proof of why our newly proposed construction excels ordinary PoW considering the existence of network delaying.
- **Robust against target corruption.** Secondly, committee switchover in *Hybrid Consensus* happens every creation

of csize blocks, and due to the existence of forking, nodes have to wait for creation of at least $\lambda$ blocks to finally become a committee member. This is too long an interval, as well as great exposure to target corruption.
- **Preventing selfish mining.** In our forking-free construction, selfish mining has no reason to work. In *Hybrid Consensus*, $3/4$ overall honesty has to be guaranteed (if underlying snailchain is *Nakamoto Chain*) to achieve $2/3$-chain quality, due to existence of selfish mining. However, we have no such concern in this work.

Our proposed *Fair-Proof-of-Work* (FPoW) is novel version of *Proof-of-Work*. We denote it as FPoW in order to distinguish it from traditional PoW. In traditional construction, for each candidate, probability of mining a nonce for each block is roughly proportional to its computing power, similarly, in our construction, we lower down difficulty of mining puzzle to make expected number of nonce found proportional to its computing power.

We can view list of committee members as a queue, each day one lucky miner enters and one leaves. In our construction, difficulty of nonce-puzzle is smaller than that of PoW. In each round, each candidate $u$ finds some nonce solutions, say, $\mathsf{nc}_{u,1}, \mathsf{nc}_{u,2}, \ldots, \mathsf{nc}_{u,P_u}$. Before end of this round, candidate submits all solutions it found to the committee, then committee members arrange all received solutions in an array $L$ in a certain order, and decides one random number $1 \leq r \leq |L| = \sum_u P_u$. Finally, committee announces one new committee member of next round, who is the miner corresponding to the $r$-th item of $L$. We will give a more detailed description in following contents.

## C. Committee from FPoW

In our construction, one node enters committee and one leaves for each round. Hence our round interval is much shorter than that of *Hybrid Consensus*(which is called "Day" in *Hybrid Consensus*). Now we present introduction to FPoW.

*1) On side of candidates:* In round $R$, one candidate, say, Tom, collects all transactions of round $R-1$ (signed by at least $2/3$ committee members) and arrange them according to sequence order into $\mathsf{rec}_{R-1}$, then finds as much as possible nonces $\mathsf{nc}_{tom,1}, \mathsf{nc}_{tom,2}, \ldots, \mathsf{nc}_{tom,P_{tom}}$ such that

$$H(\boldsymbol{B}_{R-1}||\mathsf{ID}_{tom}||\mathsf{nc}_{tom,i}) \in \mathsf{target} \quad (1 \le i \le P_{tom})$$

where block content $\boldsymbol{B}_{R-1} := \{\mathsf{rec}_{R-1}, H(\boldsymbol{B}_{R-2}), \mathsf{CM}_{R-1}\}$ is block of previous round. Note that differently from traditional Bitcoin block chain, $\mathsf{rec}_{R-1}$ here includes users' transactions handled by previous round's committee, reward transactions for previous round's committee. $\mathsf{CM}_{R-1}$ is identity list of previous round's committee members.

Tom arranges all nonces found into $L_{tom}$:

$$L_{tom} = \begin{bmatrix} \mathsf{nc}_{tom,1} & \mathsf{ID}_{tom} \\ \mathsf{nc}_{tom,2} & \mathsf{ID}_{tom} \\ \vdots & \vdots \\ \mathsf{nc}_{tom,P_{tom}} & \mathsf{ID}_{tom} \end{bmatrix}$$

and submit all items in $L_{tom}$ to the rotating committee before the end of round $R$.

**Remark:** In practice, it is not good idea to submit nonces to all committee members and assume they will all receive same number of nonces during whole interval of round $R$. Consensus on nonce acceptance should be reached through another PBFT network. However, in following contents, we merely assume that a safe submission protocol exists, to simplify representation.

*2) On side of current committee member:* For simplicity, we order all committee members in $1, 2, \ldots, \mathsf{csize}$.

Each honest committee member receives $L_u$ from all candidates, putting all $L_u$ into $L$, sorts all items in the same order, to get

$$L = \begin{bmatrix} \mathsf{nc}_{A,1} & \mathsf{ID}_A \\ \mathsf{nc}_{A,2} & \mathsf{ID}_A \\ \mathsf{nc}_{B,1} & \mathsf{ID}_B \\ \vdots & \vdots \\ \mathsf{nc}_{tom,1} & \mathsf{ID}_{tom} \\ \mathsf{nc}_{tom,2} & \mathsf{ID}_{tom} \\ \vdots & \vdots \\ \mathsf{nc}_{tom,P_{tom}} & \mathsf{ID}_{tom} \\ \vdots & \vdots \end{bmatrix}_{|L| \times 2}$$

Before beginning of next round, committee members in $\mathsf{CM}_R = [\mathsf{ID}_1, \mathsf{ID}_2, \ldots, \mathsf{ID}_{\mathsf{csize}}]$ produce a random number $1 \le r \le |L| = \sum_u P_u$ by the procedures in Figure 1. That is, all members firstly find random number $r_j$ (for $j$-th one) and broadcast $H(r_j)$. After that, all members broadcast $r_j$. In such way, adversary can control nothing about generated random number, as long as any one committee member is honest.

Details are shown in Fig.1. By term "Accuse", we mean to vote denial during final voting process.

Finally, committee members declare ("Enter", $\mathsf{ID}'$) along with their signatures, where $\mathsf{ID}'$ is identity of miner of $r$-th nonce. This lucky candidate is enrolled into committee if this declaration is signed by at least $2/3$ current committee members. After that, this round ends and next round begins.

## IV. PRIVACY-PRESERVING CONSTRUCTIONS BUILT ON OUR IMPROVED HYBRID CONSENSUS

In this section, we present one additional construction for stronger security guarantee. In previous constructions, although transactions have been properly encrypted so that only participants and committee members of current round could take view of them. However, in fact, some committee members might be malicious or garrulous. In this case, it would be great danger to unveil all transaction contents to committee members.

To tackle with such problem, we can blur out identities of transactions senders and receipts with certain techniques. This technique can be implemented to DSA signature scheme using homomorphism property of its key pairs. In this construction, we consider all signatures follow DSA signature scheme. (Certainly, this technique can also be implemented to other signatures schemes satisfying homomorphism property of key pairs)

For any key pair $(\mathsf{pk}, \mathsf{sk})$ of DSA, and any random number $r$, $(\mathsf{pk}^r, \mathsf{sk} \cdot r)$ should also be valid key pair of DSA. We will use this homomorphism property to construct our scheme.

We consider all transaction commands are arranged in form

$$\begin{cases} \mathsf{tx} = (\mathsf{pk}_1^{r_1}, \mathsf{pk}_2^{r_2}, m), \\ \mathsf{tx'} = (\mathsf{pk}_1^{r_1}, \mathsf{pk}_1^{r_3}, m_0 - m), \\ \mathsf{tx}_0 = (\mathsf{pk}_3^{r_0}, \mathsf{pk}_1^{r'_0}, m_0), \\ in_0 = (R_{\mathsf{tx}_0}, \ell_{\mathsf{tx}_0}), \\ \sigma_{\mathsf{tx}} = \Pi_{\mathsf{DSA}}.\mathsf{sig}(\mathsf{sk} \cdot r'_0, \ \mathsf{tx}||\mathsf{tx'}||in_0), \end{cases}$$

where $r_0, r'_0$ are random numbers utilized in income transaction, $r_1, r_2, r_3$ are random numbers picked at random, $\mathsf{tx}_0$ is the income transaction, $\mathsf{tx}$ is payment transaction and $\mathsf{tx'}$ is charge transaction. $m_0$ is amount of coins included in income transaction, $m$ is amount of payment. $in_0$ is address of income transaction, which should be appended to transaction command for committee to check up.

To facilitate description, in this construction, we assume that all transactions will include only one income transaction, in fact this method can be easily generalized to fit in case of multi-income scenario. For a user with public key $\mathsf{pk}_1$ to spend transaction $\mathsf{tx}_0 = (\mathsf{pk}_3^{r_0}, \mathsf{pk}_1^{r'_0}, m_0)$ with a newly constructed transaction in order to pay $\mathsf{pk}_2$. $\mathsf{pk}_1$ chooses random numbers $r_1, r_2$. Then the payment transaction will be $\mathsf{tx} = (\mathsf{pk}_1^{r_1}, \mathsf{pk}_2^{r_2}, m)$, along with a charge transaction $\mathsf{tx'} = (\mathsf{pk}_1^{r_1}, \mathsf{pk}_1^{r_3}, m_0 - m)$, where $r_3$ is another random number to make $\mathsf{tx}, \mathsf{tx'}$ look similar, so that charge transaction can be indistinguishable from payment transactions to eyes of the publicity. Within transaction command, address $in_0$ of income transaction must be included, and signature $\sigma_{\mathsf{tx}}$

| Generating random number $1 \le r \le |L|$ on round $R$ |
|---|
| (for member of identity $\mathsf{ID}_i$ , $1 \le i \le \mathsf{csize}$) |
| $\boxed{\mathsf{CM}_R = [\mathsf{ID}_1, \mathsf{ID}_2, \ldots, \mathsf{ID}_{\mathsf{csize}}];}$ |
| Choose randomly $r_i \leftarrow \{0,1\}^{\kappa}$; |
| Broadcast commitment $\mathsf{commit}_i := H(r_i)$ (and its signature); |
| Receive $\mathsf{commit}_j := H(r_j)$ broadcast by other members $j \neq i$; |
| Broadcast $r_i$ (and its signature) through PBFT towards all other members; |
| Receive all $r_j$ ($j \neq i$) from other members; |
| For $j \neq i$: if $H(r_j) \neq \mathsf{commit}_j$, then {set $r_j \leftarrow 0$; Accuse $j$;} |
| $r \leftarrow 1 + \lceil \left( \mathsf{PRF}(\bigoplus_{j=1}^{\mathsf{csize}} r_j, R) \right) \cdot \frac{|L|}{2^{\kappa}} \rceil$ |

Fig. 1. Generating random number for committee election

signed with pseudo-identity belonging to receipt of income transaction should be appended to prove its rights to spend income transaction.

And then, it can send transaction command to the committee in the same way as that of our original construction, after KEM/DEM encapsulation with public key of the committee.

After that, $r_3$ must be sent to $\mathsf{pk}_2$ so that $\mathsf{pk}_2$ can spend revenues included in tx afterwards. After receiving $r_3$ from $\mathsf{pk}_1$, and receiving tx broadcast by committee after consensus process, $\mathsf{pk}_2$ should check whether pseudo-identity of receipt in tx is $pk_2^{r_3}$ or not.

## V. CONCLUSION AND DISCUSSION

We have proposed a privacy-preserving bitcoin protocol from rotating committee elected from FPoW, where FPoW is *fair-proof-of-work* firstly proposed in this paper. We've provided detailed description of this protocol and security analysis.

### A. How to achieve better privacy-preserving property?

For convenience, we denote transactions in lowercase tx as transaction before encryption, and transaction encrypted with AES in uppercase TX. We use ek to denote a symmetric key for encryption. Additionally, since indexes of all income transactions of current transaction have to be made public (so that committee of different rounds could check whether double-spending happens), TX should include all income indexes.

**Remark:** Similarly to that of bitcoin, we assume all transactions may have multiple incomes, but only two outcomes: one for payment, one for charge (sent to itself or another account held by itself).

We use notation $\mathsf{val}(\mathsf{tx})$ to denote quantity of currency in transaction tx, $\mathsf{val}_{\mathsf{ek}}(\mathsf{TX})$ for transaction TX encrypted with ek.

For example, for Alice to transmit $m$ coins to Bob, she may not use all coins from income transactions, so she may need to return charge of amount $m'$ to herself. More formally, she makes such transaction pair:

$$\langle \mathsf{tx}, \mathsf{tx}' \rangle = \langle (\mathsf{ID}_{Alice}, \mathsf{ID}_{Bob}, \mathsf{m}), (\mathsf{ID}_{Alice}, \mathsf{ID}'_{Alice}, \mathsf{m}') \rangle$$

so that once this transaction goes through consensus, *Bob* later on will be able to spend the $m$ coins in later transaction tx' when he needs, he can include round and sequence number this transaction as source reference of his later transaction. Of course, it must guarantee that

$$\sum_{i=1}^{t} [\mathsf{val}_{\mathsf{ek}_{in_i}}(\mathsf{TX}_{in_i})] = m + m'$$

We note

$$\mathsf{TX} = \mathsf{SE.Enc}(\mathsf{ek}, \mathsf{tx})$$
$$\mathsf{TX}' = \mathsf{SE.Enc}(\mathsf{ek}, \mathsf{tx}')$$

Within an index of income source $in = (R, l)$, $R$ is the round and $l$ is one sequence number of that round.

*Transaction command:* In our construction, transactions command includes transaction $\langle \mathsf{TX}, \mathsf{TX}' \rangle$, symmetric key ek to get $\mathsf{tx} \leftarrow \mathsf{SE.Dnc}(\mathsf{ek}, \mathsf{TX})$, as well as symmetric keys $(\mathsf{ek}_{in_1}, \mathsf{ek}_{in_2}, \ldots, \mathsf{ek}_{in_t})$ to open up all previous transactions which serve as income of the current transaction.

For convenience, we denote each transaction command as

$\mathsf{txc} = (\langle \mathsf{TX}, \mathsf{TX}' \rangle, \mathsf{ek}, t, in_1, in_2, \ldots, in_t, \ \mathsf{ek}_{in_1}, \mathsf{ek}_{in_2}, \ldots, \mathsf{ek}_{in_t})$
$(k', \psi) \leftarrow \mathsf{KEM.Enc}(pk_{com})$
$\mathsf{TXC} = (\psi, \mathsf{DEM.Enc}(k', \mathsf{txc}))$

In *Hybrid Consensus*, encryption on transactions is not introduced, hence transactions are open to all publicity. In this section, we propose framework of a privacy-preserving consensus, in which all transactions are sent to committee with KEM/DEM encapsulation, with public key negotiated by committee members. In this way, all transactions can be blurred out to publicity.

In our construction, one transaction tx will be encrypted with a symmetric encryption key to form TX, this symmetric key should be generated by the payer, and acknowledged to payee and committee members of current round. After that, TX is send to committee members through KEM/DEM encapsulation.

*1) Committee side:* Before starting each round, committee members of current round determine key pair $(pk_{com}, sk_{com})$ from negotiation, description of this negotiation process will be similar to generation of random number (we suppose ElGamal-based scheme is adopted). More details of this negotiation are shown in Appendix A.

During this round, upon receiving a transaction command TXC, each committee member verifies amount of TXC and identity of sender (it should map receipt identity of income transaction) then checks whether double-spending of income transaction exists. After that, committee members reach consensus through PBFT within this committee.

To make judgement on one transaction command, each honest committee member gets $k' \leftarrow$ KEM.Decap $(sk_{com}, \psi)$, and then txc $\leftarrow$ DEM.Dec$(k', \mathsf{TXC})$, after that, gets $b \in \{0,1\} \leftarrow$ judge(txc), where judge(txc) is to take validation procedure of committee as a black box, including checking of transaction identities, double-spending detection, and amount verification.

$$
\begin{array}{|l|}
\hline
\text{txc} = (\langle \mathsf{TX}, \mathsf{TX}' \rangle, \text{ek}, t, \text{ek}_{in_1}, \text{ek}_{in_2}, \ldots, \text{ek}_{in_t}) \\
(k', \psi) \leftarrow \text{KEM.Enc}(pk_{com}) \\
\mathsf{TXC} = (\psi, \text{DEM.Enc}(k', \text{txc})) \\
\hline
\text{txp} = (\text{tx}, \text{ek}) \\
(k'_2, \psi_2) \leftarrow \text{KEM.Enc}(pk_{com}) \\
\mathsf{TXP} = (\psi_2, \text{DEM.Enc}(k'_2, \text{txp})) \\
\hline
\end{array}
$$

Fig. 3. Transaction command to committee and transaction proof to receipt

*2) User side:* For Alice to pay Bob, it makes a transaction command txc and encrypts it with session key generated from committee public key $pk_{com}$ in order to get TXC, then delivers it to the committee through a gossip network. At the same time, in order to prove to Bob, she makes transaction proof txp and sends $\mathsf{TXP} = (\psi_2, \mathsf{DEM.Enc}(k'_2, \text{txp}))$ to Bob, where $pk_{bob}$ is public key of Bob.

After transaction goes through consensus of committee, Alice finds the index of this encrypted transaction pair $\langle \mathsf{TX}, \mathsf{TX}' \rangle$, which are denoted as $ix_1 = (R, l)$, $ix_2 = (R, l+1)$ (where $R$ is the round of transaction, $l$ and $l + 1$ are adjacent sequence number). Then Alice sends $ix_1$ to Bob.

For Bob to check whether Alice has payed or not, it waits till receiving $ix_1$ from Alice, then he finds TX on chain according to the index.

Finally, he checks tx = SE.Dec(ek, TX) and finish confirmation.

## APPENDIX A

*Key Negotiation for committee members*

In our construction, committee members need to negotiate for two key pairs: $(\mathsf{PK}_{com}, \mathsf{SK}_{com})$ for public key encryption scheme, and $(pk_{com}, sk_{com})$ for hybrid encryption (KEM/DEM) scheme. The main difficulty of key generation is to generate random numbers, and such randomness should not be controlled by any dishonest member in committee. Similarly to the case in committee election, we give an outline of how to generate a random number. In fact, the only difference of this procedure (from that in committee election) is that all broadcasting content should be encrypted with public key of all committee members, to prevent eavesdropping.

We suppose $R$-round committee consists of $\mathsf{CM}_R = [\mathsf{ID}_1, \mathsf{ID}_2, \ldots, \mathsf{ID}_{\mathsf{csize}}]$, public key and secret key (in public key encryption scheme) for $\mathsf{ID}_i$ is $\mathsf{PK}_i$ and corresponding secret key is $\mathsf{SK}_i$.

Similar to that of random number generation, term "Accuse" means to show denial during final voting process.

## APPENDIX B

*Why FPoW excels ordinary PoW given existence of delaying?*

For any $0 < c < 1$, any natural number $N$: $c \cdot \sum_{i=0}^{\infty}(1 - c)^{(iN)} - \frac{1}{N} = o(\frac{1}{N})$.

*Proof.*

$$
c \cdot \sum_{i=0}^{\infty}(1-c)^{(iN)} = c \cdot \lim_{k \to \infty} \frac{1-(1-c)^{Nk}}{1-(1-c)^N} = \frac{c}{1-(1-c)^N}
$$
$$
= \frac{c}{1 - \left(1^N - \binom{N}{1}1^{N-1}c + o(c)\right)}
$$
$$
= \frac{c}{Nc - o(c)}
$$

And then we get

$$
c \cdot \sum_{i=0}^{\infty}(1-c)^{(iN)} - \frac{1}{N} = \frac{c}{Nc - o(c)} - \frac{c}{Nc}
$$
$$
= \frac{c \cdot o(c)}{(Nc - o(c)) \cdot Nc} = o(\frac{1}{N})
$$
$\square$

For any integers $\Delta > \delta > \delta' > 0$, any $0 < c < 1$, there exists sufficiently large $N$, s.t.

$$
\sum_{i=\delta}^{\infty}(1-c)^{i-\delta} \cdot c \cdot (1-c)^{(N-1)(i-\delta')} < \frac{\Delta - \delta}{\Delta - \delta'} \cdot \frac{1}{N}
$$

*Proof.* We denote $d = \delta - \delta'$, hence $\delta' = \delta - d$;

$$
\sum_{i=\delta}^{\infty}(1-c)^{i-\delta} \cdot c \cdot (1-c)^{(N-1)(i-\delta')}
$$
$$
= \sum_{i=\delta}^{\infty}(1-c)^{i-\delta} \cdot c \cdot (1-c)^{(N-1)(i-\delta+d)}
$$
$$
= (1-c)^{(N-1)d} \cdot c \cdot \sum_{i=\delta}^{\infty}(1-c)^{N(i-\delta)}
$$
$$
= (1-c)^{(N-1)d} \cdot c \cdot \sum_{i=0}^{\infty}(1-c)^{(iN)}
$$

we use previous lemma and get:

$$
(1-c)^{(N-1)d} \cdot c \cdot \sum_{i=0}^{\infty}(1-c)^{(iN)}
$$
$$
= (1-c)^{(N-1)d} \cdot (\frac{1}{N} + o(\frac{1}{N}))
$$
$$
\approx \frac{1}{N}(1-c)^{(N-1)d}
$$

Meanwhile,

$$
\frac{\Delta - \delta}{\Delta - \delta'} \cdot \frac{1}{N} = \frac{\Delta - \delta}{\Delta - \delta + d} \cdot \frac{1}{N}
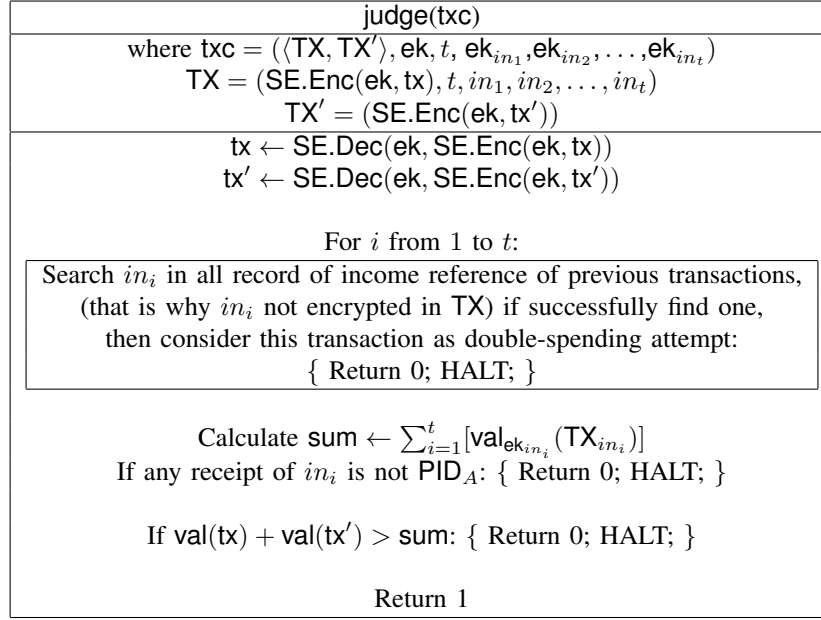$$

| judge(txc) |
|---|
| where $\text{txc} = (\langle \text{TX}, \text{TX}' \rangle, \text{ek}, t, \text{ek}_{in_1}, \text{ek}_{in_2}, \ldots, \text{ek}_{in_t})$ <br> $\text{TX} = (\text{SE.Enc}(\text{ek}, \text{tx}), t, in_1, in_2, \ldots, in_t)$ <br> $\text{TX}' = (\text{SE.Enc}(\text{ek}, \text{tx}'))$ |
| $\text{tx} \leftarrow \text{SE.Dec}(\text{ek}, \text{SE.Enc}(\text{ek}, \text{tx}))$ <br> $\text{tx}' \leftarrow \text{SE.Dec}(\text{ek}, \text{SE.Enc}(\text{ek}, \text{tx}'))$ |
| For $i$ from 1 to $t$: <br> Search $in_i$ in all record of income reference of previous transactions, <br> (that is why $in_i$ not encrypted in TX) if successfully find one, <br> then consider this transaction as double-spending attempt: <br> { Return 0; HALT; } |
| Calculate $\text{sum} \leftarrow \sum_{i=1}^{t}[\text{val}_{\text{ek}_{in_i}}(\text{TX}_{in_i})]$ <br> If any receipt of $in_i$ is not $\text{PID}_A$: { Return 0; HALT; } <br><br> If $\text{val}(\text{tx}) + \text{val}(\text{tx}') > \text{sum}$: { Return 0; HALT; } |
| Return 1 |

Fig. 2. Committee validation as black box

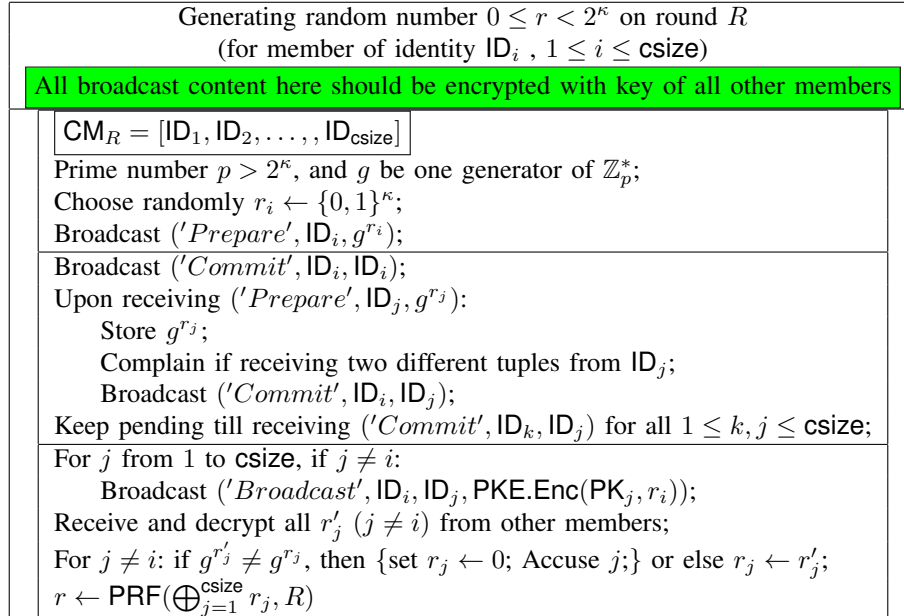| Generating random number $0 \le r < 2^\kappa$ on round $R$ <br> (for member of identity $\text{ID}_i$ , $1 \le i \le \text{csize}$) |
|---|
| **All broadcast content here should be encrypted with key of all other members** |
| $\boxed{\text{CM}_R = [\text{ID}_1, \text{ID}_2, \ldots, , \text{ID}_{\text{csize}}]}$ <br> Prime number $p > 2^\kappa$, and $g$ be one generator of $\mathbb{Z}_p^*$; <br> Choose randomly $r_i \leftarrow \{0,1\}^\kappa$; <br> Broadcast $('Prepare', \text{ID}_i, g^{r_i})$; |
| Broadcast $('Commit', \text{ID}_i, \text{ID}_i)$; <br> Upon receiving $('Prepare', \text{ID}_j, g^{r_j})$: <br>     Store $g^{r_j}$; <br>     Complain if receiving two different tuples from $\text{ID}_j$; <br>     Broadcast $('Commit', \text{ID}_i, \text{ID}_j)$; <br> Keep pending till receiving $('Commit', \text{ID}_k, \text{ID}_j)$ for all $1 \le k, j \le \text{csize}$; |
| For $j$ from 1 to $\text{csize}$, if $j \ne i$: <br>     Broadcast $('Broadcast', \text{ID}_i, \text{ID}_j, \text{PKE.Enc}(\text{PK}_j, r_i))$; <br> Receive and decrypt all $r'_j$ $(j \ne i)$ from other members; <br> For $j \ne i$: if $g^{r'_j} \ne g^{r_j}$, then {set $r_j \leftarrow 0$; Accuse $j$;} or else $r_j \leftarrow r'_j$; <br> $r \leftarrow \text{PRF}(\bigoplus_{j=1}^{\text{csize}} r_j, R)$ |

Fig. 4. Generating random number for key generation

Since for sufficiently large $N$:

$$(1-c)^{(N-1)d} \ll \frac{\Delta - \delta}{\Delta - \delta + d}$$

And this lemma has been proved.

$$\sum_{i=\delta}^{\infty}(1-c)^{i-\delta} \cdot c \cdot (1-c)^{(N-1)(i-\delta')} < \frac{\Delta - \delta}{\Delta - \delta'} \cdot \frac{1}{N}$$

$\square$

Given the lemma above, we now illustrate how an inequality proves FPoW excels PoW in sense of stability when network delaying exists.

In following discussion, for simplicity, we consider such case: we have $N$ candidates sharing the same computing power, i.e., their expectation of timing of find one nonce solution in FPoW is $T_s$. We assume one of them suffers from certain network delaying and has to begin puzzle-solving at time $\delta$, and all other nodes start puzzle-solving at time $\delta' < \delta$, and we denote $\Delta$ as ending of current round.

Then, in FPoW, the probability that the node suffering network delaying (say, Tom) would become new committee of next round is:

$$\gamma_1 = \frac{\mathbb{E}[\text{sol}_\delta]}{(N-1) \cdot \mathbb{E}[\text{sol}_{\delta'}] + \mathbb{E}[\text{sol}_\delta]} = \frac{\frac{\Delta - \delta}{T_s}}{(N-1) \cdot \frac{\Delta - \delta'}{T_s} + \frac{\Delta - \delta}{T_s}}$$

$$= \frac{\Delta - \delta}{N(\Delta - \delta')} + o(\frac{1}{N})$$

where $\mathsf{sol}_\delta$ denotes number of nonce solutions to be found if starting puzzle-solving at time $\delta$.

In the case that we want our election forking-free, with traditional PoW, we have to stipulate that first block mined should be the on-chain block. In this case, we take a glance at the probability of Tom entering committee next round in ordinary PoW scheme:

$$\gamma_2 = \sum_{i=\delta}^{\infty} (1-c)^{i-\delta} \cdot c \cdot (1-c)^{(N-1)(i-\delta')}$$

where $c$ is the probability that one (since we assume they share same computing power) find a nonce within one unit of time.

When $\delta = \delta'$, from **Lemma B**, we get $\gamma_1 - \gamma_2 = o(\frac{1}{N})$, which fits our scenario since all them share the same probability of entering committee next round is no delaying exists (or suffering exactly same delaying).

When $\delta' < \delta < \Delta$, then from **Lemma B**, FPoW excels ordinary PoW in the sense of stability since it makes the damage of delaying less $\gamma_2 < \gamma_1$.

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