

# Determining the Nonexistent Terms of Non-linear Multivariate Polynomials: How to Break Grain-128 More Efficiently

Ximing Fu<sup>1</sup>, Xiaoyun Wang<sup>1,2,3,4\*</sup>, Jiazhe Chen<sup>5</sup> and Marc Stevens<sup>6</sup>

<sup>1</sup> Department of Computer Science and Technology, Tsinghua University

<sup>2</sup> Institute for Advanced Study, Tsinghua University

`xiaoyunwang@mail.tsinghua.edu.cn`

<sup>3</sup> School of Mathematics, Shandong University

<sup>4</sup> Key Laboratory of Cryptologic Technology and Information Security, Ministry of Education, Shandong University

<sup>5</sup> China Information Technology Security Evaluation Center

<sup>6</sup> Centrum Wiskunde & Informatica, Amsterdam, The Netherlands

**Abstract.** In this paper, we propose a reduction technique that can be used to determine the density of IV terms of a complex multivariable boolean polynomial. Using this technique, we revisit the dynamic cube attack on Grain-128. Based on choosing one more nullified state bit and one more dynamic bit, we are able to obtain the IV terms of degree 43 with various of complicated reduction techniques for polynomials, so that the nonexistent IV terms can be determined. As a result, we improve the time complexity of the best previous attack on Grain-128 by a factor of  $2^{16}$ . Moreover, our attack applies to all keys.

**Keywords:** Stream ciphers, Grain-128, Polynomial reduction, Dynamic cube attack

## 1 Introduction

Most cryptanalytic problems of symmetric ciphers can be reduced to the problem of solving large non-linear multivariate polynomial systems. This problem is NP-complete [6]. Solving the system using linearization or relinearization methods directly will result in space and time complexities that are beyond the power of current computers. As a result, algebraic attacks such as cube attack [3], cube tester [5,1] and dynamic cube attack [4] were proposed in order to reduce the complexities.

Stream cipher Grain-128 [7] is a refined version of Grain scheme, one of the finalists of eSTREAM Project. The output bit is a high degree boolean function in initial vector (IV) bits and key bits. Since the proposal of Grain-128, a number of cryptanalytic results have been presented in the literatures. Fischer et. al.

---

\* Corresponding author.

applied the statistical analysis to recover the key of reduced Grain-128 up to 180 out of 256 iterations with a complexity slightly better than brute force [5]. They pointed out that Grain-128 may be immune to this attack due to the very high degree of the polynomial of the output bit. Knellwolf et al. proposed the conditional differential cryptanalysis on NLFSR-based stream ciphers including Grain scheme ciphers [8]. Conditional differential cryptanalysis exploited the message modification technique introduced in [10,9], which controlled the diffusion by controlling the plaintexts. It was applied to recover 3 bits' key of the Grain-128 reduced to 213 rounds with a probability up to 0.59 and distinguish Grain-128 reduced to 215 rounds from random primitives [8]. Another message controlling method is to nullify some state bits which may be more important than the others for reducing the degree or enhancing the sparsity. After nullifying some state bits, the representation of the output bit with IV bits can be simplified; and the output bit becomes nonrandom. This technique is called dynamic cube attack which combines the conditional differential and cube tester. With dynamic cube attack, Dinur and Shamir [4] proposed two key-recovery attacks on reduced-round Grain-128 for arbitrary keys and an attack on the full Grain-128 that holds for  $2^{-10}$  of the key space. Furthermore, Dinur et al. [2] improved the dynamic cube attack on full Grain-128, and tested the main component with a dedicated hardware. Their experimental results showed that for about 7.5% of the keys, the proposed attack beat the exhaustive search by a factor of  $2^{38}$ .

In this paper, we further improve the attack in [2] by obtaining the non-existent IV terms. The time complexity of our attack is about  $2^{74}$  cipher executions, which is applicable to all keys of Grain128.

**Our Contributions.** The contributions of this paper are five fold. Firstly, we exploit the nullification technique introduced in [2] and improve the nullification by a better choice of nullified state bits and dynamic IV bits. Secondly, we propose IV controlling techniques to reduce the IV terms of high degrees. Thirdly, we give several reduction techniques for boolean functions in order to reduce the number of terms we need to process. The primary techniques are those to remove the repeated terms and covered terms. Fourthly, with the aforementioned techniques, we mathematically compute the IV terms of degree 43. The major difference between our attack and the previous dynamic cube attacks is that we focus on obtaining the IV terms mathematically instead of choosing the suitable (sub)cubes with testing technique. Finally, we present an attack with a complexity  $2^{16}$  faster than the best result before. In this way, our method not only improves the previous attacks but also can naturally be applied to all keys with the same complexity. So the successful ratio of our attack is 100%.

Due to the difference, we also simplified the attack procedure of the dynamic cube attack. Briefly, our attack can be decomposed to the following phases:

1. Determine the dynamic variables, state/IV bits to be nullified, as well as the key bits to be guessed. Calculate the IV terms of degree 43 afterwards. The cube can be chosen freely from those disappeared IV terms. This is the preprocessing phase of the attack. However, most of the dedicated work in this paper contributes to this phase.

In this phase, we exploit various of techniques to obtain the exact IV terms. We exploit two algorithms to remove the repeated and covered terms, which help reduce the polynomial dramatically. We also use IV representation to calculate the IV terms of degree 43.

2. Guess the key bits, and get the output bits with IVs chosen according to the principle in the previous phase. The summation of the output bits over the cube can be used as a distinguisher since for the correct key the summation will always be 0, while for wrong keys 0 and 1 are supposed to occur with the same probability. This is the only on-line phase of our attack.

The detail of our attack will be demonstrated in the rest of the paper. In Section 2, the outline of Grain-128 and basic concepts of cube attacks, cube tester and dynamic cube attack will be introduced. In Section 3, we present a couple of methods to control the IV terms and reduction techniques in order to reduce the number of terms to be processed. The preprocessing phase is introduced in Section 3 as well. Section 4 will give the on-line phase of the attack. Finally, Section 5 summarizes the paper.

## 2 Preliminaries

In this section, we will first briefly introduce Grain-128. Then the techniques related to our work will be introduced, including cube attack/tester, dynamic cube attack and nullification technique.

### 2.1 Outline of Grain-128

The state of Grain-128 is represented by a 128-bit LFSR and a 128-bit NLFSR. The feedback function of the LFSR and NLFSR are defined as

$$\begin{aligned}
 s_{i+128} &= s_i + s_{i+7} + s_{i+38} + s_{i+70} + s_{i+81} + s_{i+96}, \\
 b_{i+128} &= s_i + b_i + b_{i+26} + b_{i+56} + b_{i+91} + b_{i+96} + b_{i+3}b_{i+67} + b_{i+11}b_{i+13} + \\
 &\quad b_{i+17}b_{i+18} + b_{i+27}b_{i+59} + b_{i+40}b_{i+48} + b_{i+61}b_{i+65} + b_{i+68}b_{i+84}.
 \end{aligned}$$

The output function is

$$\begin{aligned}
 z_i &= b_{i+2} + b_{i+15} + b_{i+36} + b_{i+45} + b_{i+64} + b_{i+73} + b_{i+89} + s_{i+93} + b_{i+12}s_{i+8} + \\
 &\quad s_{i+13}s_{i+20} + b_{i+95}s_{i+42} + s_{i+60}s_{i+79} + b_{i+12}b_{i+95}s_{i+95}.
 \end{aligned}$$

During the initialization step, the 128-bit key is loaded into the NLFSR and 96 bits of IV are loaded into the LFSR, with the other IV bits setting to 1. The state runs 256 rounds with the output feeding back, and the first output bit is  $z_{257}$ . For the detail of Grain-128, we refer to [7].

The output bit  $z_{257}$  is a boolean function over IV bits and key bits. We define in this paper the degree of the polynomial in terms of the IV variables, since the key bits are constant though they are unknown in our attack.

Next, we will give some more definitions used throughout the paper. Assuming that there are  $n$  IV bits, i.e.,  $v_1, v_2, \dots, v_n$  and  $m$  key bits, i.e.,  $k_1, k_2, \dots, k_m$ , the output bit  $x$  can be illustrated as

$$x = \sum_{I,J} t_I g_I(k), \quad (1)$$

where  $t_I$  is the multiplication of all the IV bits whose indices are in  $I$ , i.e., it can be represented by  $t_I = \prod_{i \in I} v_i$ ,  $I$  and  $J$  are subsets of  $\{1, 2, \dots, n\}$  and  $\{1, 2, \dots, m\}$  respectively.  $g_I(k)$  is defined as the coefficient function, which is a function over key bits:

$$g_I(k) = \prod_{l_1 \in J_1} k_{l_1} + \prod_{l_2 \in J_2} k_{l_2} + \dots + \prod_{l_p \in J_p} k_{l_p}, \quad (2)$$

where  $J_1, J_2, \dots, J_p$  are subsets of  $\{1, 2, \dots, m\}$ . In the rest of the paper, we will call it coefficient for short, when there is no ambiguity. Each  $t_I \prod_{l_j \in J_j} k_{l_j}$  in (1) is a term of  $x$ , and  $t_I$  is called an IV term. Note that any term has a unique IV term. But an IV term may occur in a large number of terms because  $g_I(k)$  is probably very complicated.

## 2.2 Cube Attack and Cube Tester

Cube attack [3] exploits the IV terms whose coefficient is linear over key bits, and then retrieves the keys once enough independent linear functions are obtained, by solving linear equations. A methodology is proposed [3,1] to test if a coefficient is linear and which key bits are involved.

Even if the coefficient is nonlinear but only involves a small number of key bits (where the degree on key bits is low), the coefficient can also be obtained by the technique called cube tester [1]. This technique is constitute of two steps: First testing which key bits are involved in the coefficient, and then determining the specific expression (if it is linear, then the first step is enough).

## 2.3 Dynamic Cube Attack

In [4], Dinur and Shamir proposed the dynamic cube attack to recover the secret key by exploiting distinguishers obtained from cube testers, with application to Grain-128.

In dynamic cube attack, some dynamic bits in the IV that are determined by key bits are chosen in order to nullify some state bits that will greatly simplify the output function. Then one expects to acquire certain nonrandom property which can be exploited by cube tester. The nonrandom property can be used as a distinguisher for key recovery.

As mentioned in Section 1, the attack in [2] is an improvement of that in [4]. The dynamic cube attacks introduced and exploited in [4,2] are based on the

nullification technique. The major difference between the two work is the different choices of nullified bits<sup>7</sup>. As a consequence, we will detail the nullification technique in the next subsection.

## 2.4 Nullification Technique

For Grain-128, the first output bit  $z_{257}$  can be represented by state bits as

$$z_{257} = b_{269}b_{352}s_{352} + b_{352}s_{299} + s_{317}s_{336} + s_{270}s_{277} + b_{269}s_{265} + s_{350} + b_{346} + b_{330} + b_{321} + b_{302} + b_{293} + b_{272} + b_{259}.$$

The most significant term of its ANF (algebraic normal form) is  $b_{269}b_{352}s_{352}$ . In fact, the terms resulted from  $b_{269}b_{352}s_{352}$  are much more than those from the other terms. Similarly, the most significant terms of the ANF of  $b_{269}$ ,  $b_{352}$  and  $s_{352}$  are  $b_{153}b_{236}s_{236}$ ,  $b_{236}b_{319}s_{319}$  and  $b_{236}b_{319}s_{319}$  respectively. The common factor is  $b_{236}$ , so  $b_{269}$ ,  $b_{352}$ ,  $s_{352}$  and  $z_{257}$  can be simplified if nullifying  $b_{236}$ . But  $b_{236}$  is too complicated, i.e.,

$$b_{236} = b_{120}b_{203}s_{203} + b_{203}s_{150} + b_{176}b_{192} + s_{168}s_{187} + b_{169}b_{173} + b_{148}b_{156} + b_{135}b_{167} + b_{125}b_{126} + b_{119}b_{121} + b_{111}b_{175} + s_{121}s_{128} + b_{120}s_{116} + b_{204} + s_{201} + b_{199} + b_{197} + b_{181} + b_{172} + b_{164} + b_{153} + b_{144} + b_{134} + b_{123} + b_{110} + b_{108} + 1.$$

Nullifying  $b_{236}$  needs too many guessed key bits, so the scheme in [4] retrieved a subset of  $2^{-10}$  of all possible keys by fixing 10 key bits to be zero.

Another approach is to simplify  $b_{236}$  by nullifying  $b_{203}$ , which is adopted by [2].  $b_{203}$  is still too complicated to be nullified directly, i.e.,

$$b_{203} = b_{87}b_{170}s_{170} + b_{170}s_{117} + b_{143}b_{159} + s_{135}s_{154} + b_{136}b_{140} + b_{115}b_{123} + b_{102}b_{134} + b_{92}b_{93} + b_{86}b_{88} + b_{78}b_{142} + s_{88}s_{95} + b_{87}s_{83} + b_{171} + s_{168} + b_{166} + b_{164} + b_{148} + b_{139} + b_{131} + b_{120} + b_{111} + b_{101} + b_{90} + b_{77} + b_{75} + s_{75}.$$

In order to nullify  $b_{203}$ , one should first nullify  $b_{170}$ ,  $b_{159}$ ,  $b_{138}$ ,  $s_{135}$ ,  $b_{136}$ ,  $b_{134}$ ,  $b_{133}$ , and  $b_{131}$ . All of these bits but  $b_{170}$  can be nullified directly by choosing IV bits. We know that

$$b_{170} = b_{54}b_{137}s_{137} + b_{137}s_{84} + b_{110}b_{126} + s_{102}s_{121} + b_{103}b_{107} + b_{82}b_{90} + b_{69}b_{101} + b_{59}b_{60} + b_{53}b_{55} + b_{45}b_{109} + s_{55}s_{62} + b_{54}s_{50} + b_{138} + s_{135} + b_{133} + b_{131} + b_{115} + b_{106} + b_{98} + b_{87} + b_{78} + b_{68} + b_{57} + b_{44} + b_{42} + s_{42}.$$

In order to nullify  $b_{170}$ ,  $b_{137}$  can be nullified first by setting  $s_9$  to be a dynamic value.

In fact,  $b_{170}$  can be nullified directly as well since it is not too complicated. However, nullification of  $b_{137}$  can not only nullify  $b_{170}$  but also simplify  $b_{253}$  which

<sup>7</sup> The authors also experimentally verified the main component of the attack by a dedicated hardware in [2]

**Table 1.** Nullification Scheme in [2]

|               |   |
|---------------|---|
| nullification | $b_{131}, b_{133}, b_{134}, s_{135}, b_{136}, b_{137}, b_{138}, b_{145}, b_{153}, b_{159}, b_{170}, b_{176}, b_{203}$ |
| dynamic bits  | $s_3, s_5, s_6, s_{77}, s_8, s_9, s_{10}, s_{17}, s_{25}, s_{31}, s_{42}, s_{83}, s_1$                                |

contributes a lot to the degree and IV terms. The term  $b_{143}b_{159}$  can be nullified by nullifying either  $b_{143}$  or  $b_{159}$ . The authors chose to nullify  $b_{159}$  because it can help reduce  $b_{275}$  whose ANF significant term is  $b_{159}b_{242}s_{242}$ .  $b_{145}$ ,  $b_{153}$  and  $b_{176}$  are also nullified in order to simplify  $s_{261}$ ,  $b_{269}$  and  $b_{292}$ . The nullified state bits and dynamic IV bits of [2] are shown in Table 1.

### 3 Obtaining the IV terms of Boolean Polynomials

In this paper, the main purpose is to obtain all the IV terms of degree 43. In order to achieve this goal, we first propose new nullification (Section 3.1) and IV choosing (Section 3.2) techniques. Then term reduction techniques (Section 3.3) are also presented to remove the terms that will not contribute to the IV terms of degree 43 in the polynomial. With these techniques, we are able to obtain the exact IV terms of degree 43 in the output bit in Section 3.4.

#### 3.1 Nullification of State Bits

Motivated by the nullification technique in [4,2], we choose to nullify some state bits. Following, we will introduce how to obtain the nullification scheme.

**Step 1:** We obtain the exact representations of  $b_i$  and  $s_i$  ( $0 \leq i \leq 222$ ) and hence obtain the exact degree. Then we estimate degrees of the other state bits  $b_i$  and  $s_i$  for  $i \in [223, 320]$ .

**Step 2:** We substitute the state terms and preserve the high degree state terms, and then calculate the frequency of occurrence of each state bit involved.

**Step 3:** Nullify the high frequency state bits and set them to 0, then the corresponding terms will disappear. As a result, the nullification of these state bits decreases the degree and terms of high degree dramatically.

**Step 4:** Then we substitute again and repeat the procedure above.

From the process of choosing nullified state bits, after nullifying one state bits in **Step 3**, **Step 1** is repeated again because the degree of state bits may change due to the change of dynamic bits. Then we obtain the nullification scheme of state bits, which are shown in Table 2. Finally we can determine the dynamic IV bits and the corresponding key bits that have to be guessed during the on-line attack. The dynamic IV bits are shown in Table 2 as well. The more details of the nullification are shown in Table 8 in Appendix A.

### 3.2 IV Choosing Techniques

After the nullifications shown in Table 2, there are 82 IV bits left, which probably leads to a very high degree of the output bit polynomial. Thus in this subsection we will show how to choose some IV bits to reduce the degree and make some IV terms disappear as well.

**IV Nullification Technique** For Grain-128, the output is generated by iteration of IV and key bits, some IV or key bits tend to (dis)appear simultaneously in high degree terms. For example, for state bits in the first 32 initialization rounds, the only degree 2 terms are  $s_{i+13}s_{i+20}$  and  $s_{i+60}s_{i+79}$ . Choosing to nullify  $s_{i+13}$  may result in disappearance of  $s_{i+20}$ .

Furthermore, the degrees of some state bits are decreased to 1 by nullifying some IV bits of the terms. However, some state bits may have higher degrees due to the higher-degree dynamic bits. As a result, the high frequency IV bits in the high frequency state bits and terms are chosen to be nullified, in order to make as many dynamic bits disappear as possible.

The other nullified IV bits can be chosen as follows.

**Step 1:** We substitute the state terms with previous state bits and preserve the high degree terms, of which sum of the degrees of state bits involved in state terms is high.

**Step 2:** Repeat **Step 1** until we can obtain the IV terms using IV representation, which will be introduced in Section 3.3.

**Step 3:** Calculate the frequency of IV bits in high degree IV terms and set the high frequency IV bit(s) with zeros.

**Step 4:** Update the degree of state bits and repeat *Step 1-Step 3*.

In addition, the IV nullification is an iterative process since nullifying any IV bit may result in changing the degree of many state bits.

In summary, carefully choosing the nullified IV bits will result in the disappearance of many IV terms.

Hence, totally 36 IV bits are nullified (set to zeros) and degrees of all state bits can be updated. There are remaining 46 IV bits and we consider the nonexistent IV terms in the 46-bit set.

**Exploit the Low-frequency IV Bits** Since some IV bits will disappear in high degree terms after setting some other IV bits to zero, IV terms with these IV bits such as  $s_{80}$  will be sparse in high degree terms. We call these common IV bits that lead to sparse terms *low-frequency bits*.

In order to find out the *low-frequency bits*, we execute the following steps.

**Step 1:** We substitute the state terms with previous state bits and preserve the high degree terms.

**Step 2:** Repeat **Step 1** until we can obtain the IV terms using IV representation.

**Step 3:** Calculate the frequency of IV bits in high degree IV terms and choose the IV bit(s) of low frequency.

**Table 2.** Our Nullification Scheme

|               |  |
|---------------|--|
| nullification | $b_{131}, b_{133}, b_{134}, s_{135}, b_{136}, b_{137}, b_{138}, b_{143}, b_{145}, b_{153}, b_{159}, b_{170}, b_{176}, b_{203}$ |
| dynamic bits  | $s_3, s_5, s_6, s_{77}, s_8, s_9, s_{10}, s_{15}, s_{17}, s_{25}, s_{31}, s_{42}, s_{83}, s_1$                                 |

**Step 4:** Repeat *Step 1-Step 3*. Note that the state terms that do not contain the *low-frequency bits* can be discarded directly.

We find out at least 7 of them, which are listed in Table 5 along with the corresponding state bits in the first 32 rounds.

### 3.3 Reduction Techniques for Polynomial Terms

We presented the methods to reduce the boolean function of the output bit in Section 3.1 and Section 3.2. However, the specific IV terms that appear are under consideration, so that distinguishers can be deduced. Thus we propose in this subsection the manners to obtain the IV terms of boolean functions of Grain-128.

We first introduce an important property for Grain-128 as follows.

*Property 1.* Let  $b_{i+128} = b'_{i+128} + z_i$  and  $s_{i+128} = s'_{i+128} + z_i$ , then  $b_{i+128}s_{i+128} = b'_{i+128}s'_{i+128} + z_i(b'_{i+128} + s'_{i+128}) + z_i$ , where  $z_i$  is the feedback bit,  $b'_{i+128}$  and  $s'_{i+128}$  are the state bits before the feedback of  $z_i$ .

Property 1 holds because there is a collision, i.e.,  $z_i \cdot z_i = z_i$

**Degree estimation of state bits** After the nullification and IV choosing scheme introduced in Section 3.1 and Section 3.2, we first estimate the degree of some state bits. Degrees of some state bits are shown in Table 3, which can be obtained directly on PC by obtaining their boolean function in IV bits and key bits. In fact, the exact degree of state bits before round 150 can be obtained, while the others can be estimated by substitution.

In addition to the degrees of the state bits, another observation of the degree reduction of the multiplication of two state bits can be also obtained, that is we can obtain  $\deg(b_i) + \deg(s_i) - \deg(b_i s_i)$ , which is shown in Table 4. This degree reduction is resulted from Property 1. Hence, some state terms involving  $b_i s_i$  that are in Table 4 can be discarded by increasing the degree threshold by the degree reduction. So Table 4 can help to discard state terms in advance.

**Remove the Repeated Terms** We know that it is much easier to illustrate the output with the state bits than using the key and IV bits directly. Moreover, the state bits can be expressed by earlier ones that are simpler functions of the IV and key bits. As a consequence, we can iteratively express the output bits.



**Table 3.** Degree of partial state bits

|             |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|-------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $i$         | 128 | 129 | 130 | 131 | 132 | 133 | 134 | 135 | 136 | 137 | 138 | 139 | 140 | 141 | 142 | 143 | 144 | 145 | 146 |
| $\deg(b_i)$ | 1   | 2   | 1   | 0   | 1   | 0   | 0   | 1   | 0   | 0   | 0   | 1   | 2   | 2   | 2   | 0   | 1   | 0   | 1   |
| $\deg(s_i)$ | 1   | 2   | 2   | 1   | 1   | 1   | 1   | 0   | 1   | 1   | 1   | 1   | 2   | 1   | 2   | 0   | 1   | 1   | 1   |
| $i$         | 147 | 148 | 149 | 150 | 151 | 152 | 153 | 154 | 155 | 156 | 157 | 158 | 159 | 160 | 161 | 162 | 163 | 164 | 165 |
| $\deg(b_i)$ | 1   | 1   | 1   | 0   | 2   | 1   | 0   | 2   | 2   | 1   | 1   | 1   | 0   | 2   | 2   | 3   | 3   | 2   | 2   |
| $\deg(s_i)$ | 1   | 1   | 1   | 1   | 2   | 1   | 1   | 2   | 2   | 1   | 1   | 1   | 0   | 2   | 2   | 3   | 3   | 2   | 2   |
| $i$         | 166 | 167 | 168 | 169 | 170 | 171 | 172 | 173 | 174 | 175 | 176 | 177 | 178 | 179 | 180 | 181 | 182 | 183 | 184 |
| $\deg(b_i)$ | 2   | 2   | 2   | 1   | 0   | 1   | 2   | 3   | 3   | 3   | 0   | 2   | 2   | 2   | 2   | 2   | 2   | 2   | 3   |
| $\deg(s_i)$ | 1   | 2   | 2   | 1   | 1   | 1   | 2   | 3   | 3   | 3   | 2   | 2   | 2   | 2   | 2   | 2   | 2   | 2   | 3   |
| $i$         | 185 | 186 | 187 | 188 | 189 | 190 | 191 | 192 | 193 | 194 | 195 | 196 | 197 | 198 | 199 | 200 | 201 | 202 | 203 |
| $\deg(b_i)$ | 2   | 2   | 2   | 3   | 2   | 2   | 2   | 2   | 3   | 3   | 5   | 4   | 3   | 3   | 3   | 3   | 3   | 3   | 0   |
| $\deg(s_i)$ | 2   | 2   | 2   | 3   | 2   | 2   | 2   | 2   | 3   | 3   | 5   | 4   | 3   | 3   | 3   | 3   | 3   | 3   | 2   |
| $i$         | 204 | 205 | 206 | 207 | 208 | 209 | 210 | 211 | 212 | 213 | 214 | 215 | 216 | 217 | 218 | 219 | 220 | 221 | 222 |
| $\deg(b_i)$ | 3   | 3   | 5   | 4   | 4   | 3   | 4   | 4   | 4   | 3   | 3   | 4   | 3   | 5   | 4   | 4   | 4   | 5   | 5   |
| $\deg(s_i)$ | 3   | 3   | 5   | 4   | 4   | 3   | 4   | 4   | 4   | 3   | 3   | 4   | 3   | 5   | 4   | 4   | 4   | 5   | 4   |

**Table 4.**  $\deg(b_i) + \deg(s_i) - \deg(b_i s_i)$

|   |     |     |     |     |     |     |     |     |     |     |     |     |     |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $i$                                     | 129 | 140 | 142 | 151 | 154 | 155 | 160 | 161 | 162 | 163 | 164 | 165 | 167 |
| $\deg(b_i) + \deg(s_i) - \deg(b_i s_i)$ | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 2   | 1   | 1   | 1   |
| $i$                                     | 168 | 172 | 173 | 174 | 175 | 180 | 181 | 182 | 183 | 184 | 188 | 193 | 194 |
| $\deg(b_i) + \deg(s_i) - \deg(b_i s_i)$ | 1   | 1   | 1   | 2   | 2   | 1   | 1   | 1   | 1   | 1   | 1   | 2   | 1   |
| $i$                                     | 195 | 196 | 197 | 198 | 199 | 206 |     |     |     |     |     |     |     |
| $\deg(b_i) + \deg(s_i) - \deg(b_i s_i)$ | 2   | 3   | 1   | 1   | 1   | 1   |     |     |     |     |     |     |     |

During this procedure, we will try to reduce the complexity of the polynomial by removing the so called **repeated terms** (RT).

Two terms are repeated terms if they are the same by ignoring  $s_i$  ( $i \in [96, 127]$ ) (which are all 1s). It is obvious that the boolean addition of two RT is zero.

The collision which is shown in Property 1 can result in a number of repeated terms when substituting the terms of  $z_i$ . Actually we find that the repeated term are mostly caused from collisions. Property 1 is trivial, however, it helps a lot when we remove the RT with computer program but not manually before our RT removing algorithm, which is shown in Algorithm 1.

Suppose that there are  $n$  terms, the complexity of Algorithm 1 is  $O(n)$ .

**Identifying the IV Terms of Degree 43** Removing the RT is not enough since the number of terms still increases dramatically, which pushes us to reduce the terms furthermore.

**Table 5.** Low frequency bits

| low-frequency bits | corresponding state bits                      |
|--------------------|---|
| $s_0$              | $b_{128}, s_{128}, b_{160}, s_{160}$          |
| $s_2$              | $b_{130}, s_{130}$                            |
| $s_{37}$           | $b_{157}, s_{157}, s_{158}$                   |
| $s_{43}$           | $s_{133}$                                     |
| $s_{60}$           | $b_{146}, s_{146}, s_{150}$                   |
| $s_{80}$           | $b_{129}, s_{129}, s_{138}, b_{148}, s_{148}$ |
| $s_{90}$           | $s_{137}, s_{148}, s_{158}$                   |

---

**Algorithm 1** Repeated-term Removing Algorithm

---

**Input:** The vector  $\mathbf{T}$  with  $n$  terms, i.e.,  $T_1, T_2, \dots, T_n$ .**Output:** Updated  $\mathbf{T}$  with  $m$  terms, where  $m \leq n$ .

- 1: Initialize an empty Hash Set  $\mathbf{H}$ .
  - 2: **for**  $i \leftarrow 1 : n$  **do**
  - 3:   Compute the Hash value of  $T_i$ , i.e.,  $H(T_i)$
  - 4:   **if**  $H.contains(T_i)$  is **true** **then**
  - 5:      $H.delete(T_i)$
  - 6:   **else**
  - 7:      $H.insert(T_i)$
  - 8:   **end if**
  - 9: **end for**
- 

**IV representation.** Replacing the terms with their corresponding IV terms is called *IV representation*. Due to the neglect of the key information, using IV representation will result in repeated IV terms.

**Repeated IV terms.** When two IV terms are the same, then one of them can be removed directly, which is shown in Algorithm 2. Supposing there are  $n$  IV terms, the complexity of Algorithm 2 is  $O(n)$ .

---

**Algorithm 2** Repeated-IV term Removing Algorithm

---

**Input:** The vector  $\mathbf{T}$  with  $n$  IV terms, i.e.,  $T_1, T_2, \dots, T_n$ .**Output:** Updated  $\mathbf{T}$  with  $m$  IV terms, where  $m \leq n$ .

- 1: Initialize an empty Hash set  $\mathbf{H}$ .
  - 2: **for**  $i \leftarrow 1 : n$  **do**
  - 3:   Compute the Hash value of  $T_i$ , i.e.,  $H(T_i)$ .
  - 4:   **if**  $H.contains(T_i)$  is **false** **then**
  - 5:      $H.insert(T_i)$ .
  - 6:   **end if**
  - 7: **end for**
- 

**Covered IV terms.** In addition to repeated IV terms, we give the definition of covered IV terms: If  $I \subseteq J$ , then IV term  $s_1 = \prod_{i \in I} v_i$  is said to be covered

by IV term  $s_2 = \prod_{j \in J}$ , where  $I$  and  $J$  are subsets of  $\{0, 1, \dots, 95\}$  respectively. So repeated IV terms are special cases of covered IV terms.

Covered IV terms will show up frequently using IV representation, so we develop an algorithm to remove the covered IV terms, which is shown in Algorithm 3.

---

**Algorithm 3** Covered-(IV)term Removing Algorithm

---

**Input:** The vector  $\mathbf{T}$  of  $n$  terms, i.e.,  $T_1, T_2, \dots, T_n$ .

**Output:** Updated  $\mathbf{T}$  with  $m$  terms, where  $m \leq n$ .

```

1: for  $i \leftarrow 1 : n - 1$  do
2:   if  $T_i \neq 0$  then
3:     for  $j \leftarrow i + 1 : n$  do
4:       if  $T_j \neq 0$  then
5:         if ( $T_i$  is covered by  $T_j$ ) then
6:            $T_i \leftarrow 0$ 
7:            $i \leftarrow i + 1$ 
8:         else if ( $T_j$  is covered by  $T_i$ ) then
9:            $T_j \leftarrow 0$ 
10:           $j \leftarrow j + 1$ 
11:        end if
12:      end if
13:    end for
14:  end if
15: end for
16:  $m \leftarrow 1$ 
17: for  $i \leftarrow 1 : n$  do
18:   if  $T_i \neq 0$  then
19:      $T_m \leftarrow T_i$ 
20:      $m \leftarrow m + 1$ 
21:   end if
22: end for

```

---

Here we give an example to illustrate the use of IV representation, repeated IV term removing and covered IV term removing. Assume that  $x_1 = v_0(k_1 + k_0k_2) + v_0v_1k_2$ ,  $x_2 = v_1k_0 + v_1v_2k_1$ , then the IV representations of  $x_1$  and  $x_2$  are  $\hat{x}_1 = v_0 + v_0v_1$  and  $\hat{x}_2 = v_1 + v_1v_2$  respectively. After removing the repeated IV terms of  $\hat{x}_1\hat{x}_2 = v_0v_1 + v_0v_1v_2 + v_0v_1 + v_0v_1v_2$  using Algorithm 2, the resultant IV terms are  $v_0v_1$  and  $v_0v_1v_2$ , so that the existent IV terms can be determined. When only the highest degree is under consideration, then Algorithm 3 can be used. After removing the covered IV terms,  $\hat{x}_1 = v_0v_1$ ,  $\hat{x}_2 = v_1v_2$ . Then  $\hat{x}_1\hat{x}_2 = v_0v_1v_1v_2 = v_0v_1v_2$ . So the highest degree of  $x_1x_2$  is 3, within the IV term  $v_0v_1v_2$ . Using the IV representation and Algorithm 3, only 1 IV multiplication is needed for this concrete example, while 4 IV multiplication and 6 key multiplication are needed in the trivial way.

Since the coefficient function on key bits is usually quite complicated, the proposed method reduces the computing complexity dramatically, which enables

us to obtain the IV terms of degree 43. From the observation of Grain-128, we believe there are nonexistent IV terms of some degrees. For example, there are nonexistent IV terms of degree 42 and 44. We choose to determine nonexistent IV terms of degree 43 for two reasons: 1) there are enough nonexistent IV terms that can be used as distinguishers for degree 43 with consideration of the density while there may be no enough distinguishers of degree more than 43; 2) the computing complexity of obtaining the nonexistent IV terms of degree 43 is within our computing ability while it is much more complicated to determine the nonexistent IV terms of degree lower than 43.

The computational complexity of Algorithm 3 is  $O(n^2)$  if there are  $n$  IV terms. This is the worst-case complexity; while for Grain-128 the complexity will be much lower due to the fact that an higher degree term may cover a larger number of lower degree ones. Normally, processing 30 million IV terms by Algorithm 3 only needs about several minutes on a single core, which is quite efficient.

It is obvious that Algorithm 2 will not lose any information of IV terms. Now we have to make sure that the above method does not lose any information about the degree, i.e., the degree of the original polynomial will be bounded by the deduced degree.

*Property 2.* The degree of the multiplication of two state bits is strictly bounded by the estimated one deduced by *IV representation* and Algorithm 3.

*Proof.* This proof is composed of two steps. First, we need to prove that IV representation of the state bits with Algorithm 3 will guarantee the keeping of the IV terms with the highest degree. Then, we need to prove that applying Algorithm 3 on the multiplication of two IV-represented state bits will guarantee the keeping of the IV terms with the highest degree. It is obvious right for the latter one, we just need to prove the former one.

Consider state terms  $A = a_1 + a_2 + \dots + a_m$  and  $B = b_1 + b_2 + \dots + b_n$ , where  $a_i$  and  $b_j$  are IV terms. Assuming that  $a_{i_1}$  is covered by  $a_{i_2}$ , then  $a_{i_1}B$  will be covered by  $a_{i_2}B$ . Hence,  $a_{i_1}$  can be removed from  $A$ , which is done by Algorithm 3. The removal of IV terms in  $B$  is similar.  $\square$

This property can be easily extended to the multiplication of multiple state bits, which holds by iteratively using Property 2.

Actually, Algorithm 3 can be used to remove the so called **covered terms** (CT), where CT is defined as follows:

Let state terms

$$t_1 = \prod_{i_1 \in I_1} b_{i_1} \prod_{i_2 \in I_2} b_{i_2} \prod_{i_3 \in I_3} s_{i_3}$$

and

$$t_2 = \prod_{j_1 \in J_1} b_{j_1} \prod_{j_2 \in J_2} b_{j_2} \prod_{j_3 \in J_3} s_{j_3},$$

where  $I_1, J_1 \subseteq [0, 127]$ ,  $I_2, J_2 \subseteq [128, 352]$ ,  $I_3, J_3 \subseteq [0, 95] \cup [128, 352]$ . Term  $t_1$  is covered by  $t_2$  if  $I_2 \subseteq J_2$  and  $I_3 \subseteq J_3$ .

If the degree of a state term can be bounded by a bound using *IV representation*, then IV terms produced by the covered state terms will also be covered and hence will be removed using Algorithm 3. So covered state terms can be removed first. As a result, the state terms are partitioned into two sets, of which degrees of state terms in the first one are bounded by our bound while degrees of state terms in the other may be higher. Of course, most state terms will be in the first set. Algorithm 3 can be applied to the first set and most terms would be dropped off.

When there are billions of terms, removing the covered terms by executing Algorithm 3 on all terms with a single core is difficult as well, so we remove the covered terms in parallel. Here we propose a partition scheme that guarantees all covered terms can be removed. A term covers another if only the degree of the first is no less than that of the second. So the terms can be partitioned into different sets according to their degrees. First we apply Algorithm 3 to the highest degree terms, and remove the second highest degree terms that are covered by the highest ones. Then we operate Algorithm 3 on second highest degree terms, and so on, until all covered terms are removed. Normally, operating Algorithm 3 on 10 million state terms needs just several minutes in a single core when analyzing Grain-128. This is because a state term may cover a large number of state terms. Therefore, these terms will be set to 0 after a scan of all the state terms.

For state terms in the second set, we need to substitute the terms and remove the repeated terms using Algorithm 1 until the IV terms can be obtained using *IV representation*. Combined with Algorithm 2, all IV terms of degree 43 can be obtained.

### 3.4 Preprocessing Phase of the Proposed Attack

Now we are ready to describe the preprocessing phase of our attack, using the techniques proposed in the previous three subsections.

1. We deduce the key bits to be guessed (the number is 40), as well as the corresponding dynamic IV bits, to nullify the state bits shown in Table 2. The nullified IV bits and low-frequency bits shown in Table 6 are also chosen.
2. Iteratively express the output bit and discard the terms whose degrees are likely to be below certain threshold. Algorithm 1 is then used to remove the repeated terms. Note that there is no information lost in Algorithm 1. Not all state terms can be preserved in each substitution because there are large quantities of state terms. We just preserve high degree state terms, which depends on the storage and computing ability. Repeat this step until the IV multiplication can be executed, i.e., we express the state terms until all the state bits involved in the state terms are within the range of  $[0, 159]$ .
3. Use IV representation, combined with Algorithm 2 and Algorithm 3 to obtain the existent IV terms of degree 43. Whether Algorithm 2 or Algorithm 3 is

**Table 6.** Nullified and low-frequency IV bits.

|                    |  |
|--------------------|--|
| nullified bits     | $s_{14}, s_{16}, s_{20}, s_{22}, s_{23}, s_{24}, s_{28}, s_{30}, s_{32}, s_{33}, s_{35}, s_{36}, s_{38}, s_{41}, s_{44}, s_{50}, s_{51}, s_{53}, s_{55}, s_{56}, s_{61}, s_{64}, s_{67}, s_{68}, s_{69}, s_{70}, s_{71}, s_{75}, s_{76}, s_{79}, s_{81}, s_{82}, s_{84}, s_{85}, s_{86}, s_{94}$ |
| low-frequency bits | $s_0, s_2, s_{37}, s_{43}, s_{60}, s_{80}, s_{90}$   |

used depends on the real data. In fact, we use Algorithm 3 heuristically first to obtain the covered IV terms. If the resulted IV terms are of degree no more than 43, then finish; otherwise, Algorithm 2 is used to obtain the IV terms. This step may be repeated over the high degree state terms that deduce high degree IV terms.

4. We process the other state terms that have not been preserved. In fact, few of them can deduce IV terms of degree more than 43. We deduce the IV terms by heuristic similar to that in **Step 3** over state terms: we use Algorithm 3 first, if there are IV terms of degree more than 43, then we replace it with Algorithm 2. Note that most remaining state terms can not deduce IV terms of degree more than 43.

In Step 2, we will remove as many repeated terms as possible within our computing and storage ability. After Step 2, a large number of state terms are discarded. However, most of the terms are of degrees lower than our bound, which is actually proved by using Algorithm 3.

The preprocessing process is quite complicated, so we use a computer cluster with 740 nodes (8880 cores in total) to do most of the time consuming work. A dynamic number of cores are used (which are between 600 and 4000), depending on the specific program to be paralleled. Finally, we can determine all the IV terms that may appear, which are shown in Appendix B. The IV terms in Appendix B occur with probability 1/2 while the others do not occur with probability 1 in  $z_{257}$ .

### 3.5 Example

In this section, we use an example to illustrate the process of obtaining the nonexistent IV terms, with the same nullification and IV choosing schemes in previous sections. In order to exploit the nullification and IV choosing schemes, we choose to obtain the nonexistent IV terms of  $z_{191}$ , where the result is very easy to verify in a single core.

We express  $z_{191}$  as the following formula

$$z_{191} = b_{193} + b_{206} + b_{227} + b_{236} + b_{255} + b_{264} + b_{280} + s_{284} + b_{203}s_{199} + s_{204}s_{211} + b_{286}s_{233} + s_{251}s_{270} + b_{203}b_{286}s_{286}.$$

Due to the nullification,  $b_{203}$  is nullified,

$$z_{191} = b_{193} + b_{206} + b_{227} + b_{236} + b_{255} + b_{264} + b_{280} + s_{284} + s_{204}s_{211} + b_{286}s_{233} + s_{251}s_{270}.$$

**Table 7.** Nonexistent IV terms of degree 11 in  $z_{191}$

|            |            |            |            |            |            |            |            |
|------------|------------|------------|------------|------------|------------|------------|------------|
| 2008100800 | 2000300800 | 2000500800 | 2000900800 | 2000110800 | 2000120800 | 2000140800 | 4000500800 |
| 2000180800 | 2000101800 | 2000102800 | 2000104800 | 2000108800 | 2000109900 | 2000100A00 | 4000900800 |
| 2000100C00 | 2000100810 | 2000100820 | 2000100840 | 2000100880 | 2000100801 | 2000100802 | 4000110800 |
| 2000100804 | C000100800 | 4100100800 | 4200100800 | 4400100800 | 4800100800 | 4010100800 | 4000120800 |
| 4020100800 | 4040100800 | 4080100800 | 4001100800 | 4002100800 | 4004100800 | 4008100800 | 4000180800 |

If we consider the nonexistent IV terms of degree more than 11, then the other state terms except  $b_{264} + b_{280} + s_{284} + b_{286}s_{233} + s_{251}s_{270}$  can be discarded directly because their degrees are less than 11, under the degree estimation in Table 3. After that, we substitute again and preserve the state terms that are of degree more than 11. Then we discard those state terms that can not deduce the low-frequency bits which are shown in Table 5. The number of state terms drops dramatically. We substitute again and preserve the state terms that are of degree more than 11 and that can deduce the low-frequency bits. After all the state bits are within the range of  $[b_0, b_{159}]$  and  $[s_0, s_{159}]$ , we use IV representation, combined with Algorithm 3 and 2, to obtain the nonexistent IV terms. Here, we use Algorithm 3 to conduct the IV terms for convenience and find out the existent IV terms of degree more than 11 are very sparse. The highest degree is 15 and there are only 70 existent IV terms of degree 15. Furthermore, there is a large number of nonexistent IV terms of degree 11. For example, we choose the 40 nonexistent IV terms as distinguishers in Table 7. Each hexadecimal number in this table indicates a multiplication of 43 IV bits. Let  $H = H_0H_1H_2H_3H_4H_5H_6H_7H_8H_9$ , where  $H_i$  is a hexadecimal number with the range of  $[0, 15]$ . As there are 39 bits, so  $H_9$  is within the range of  $[0, 7]$ . Define  $h_{ij}$  as the  $j$ -th lowest bit of  $H_i$ . Let  $S$  be the vector whose elements are 4, 7, 11, 12, 13, 18, 19, 21, 26, 27, 29, 34, 39, 40, 45, 46, 47, 48, 49, 52, 54, 57, 58, 59, 62, 63, 65, 66, 72, 73, 74, 78, 87, 88, 89, 91, 92, 93 and 95 sequently, then the cube defined by  $H$  is  $v_0v_2v_3v_7v_{43}v_{60}v_{80}v_{90} \prod_{i \in [0,9]} v_{S_{i*4+j}}^{h_{ij}}$ .

## 4 On-line Phase of the Attack

In this section, we will introduce the on-line phase of our attack on Grain-128 and analyze its complexity. Actually, the on-line phase of our attack is much simpler than the preprocessing phase.

Since we determine that totally 2581 IV terms of degree 43 are possible to be existent, which means there are  $C_{46-7}^{43-7} - 2581 = 6558$  nonexistent IV terms. The density of IV terms is about 28%. We choose cubes from these nonexistent IV terms. It is known that summing over each of the cubes, the output will be always zero, which results in distinguishers for key recovery. Since for the correct key guess the summation is always 0, while for the wrong ones the summation could be 0 or 1 with random probability. On average, 40 cubes are needed to get the correct key.

Thus, in this phase, we first guess the 40 key bits and choose the dynamic bits to nullify partial state bits. We sum over the output bits with the first

chosen cube. If the result is 1, then we conclude that the guess is wrong. About half of the keys will be discarded in this way. For the remaining keys, we repeat the procedure with the second cube. And so on. Then after all the 40 cubes are used, it is supposed that only the correct key will be kept. Because there are more than 6000 nonexistent IV terms, there are enough such distinguishers.

**Time Complexity:** Sum over the first cube needs  $2^{40} \cdot 2^{43} = 2^{83}$  bit operations. After the first sum, about half wrong key guesses are dropped off, that is  $(2^{40} - 1)/2$  wrong key guesses and 1 right key remain. So there are  $(2^{40} - 1)/2 + 1$  guesses for the second sum and the time complexity for the second sum is  $((2^{40} - 1)/2 + 1) \cdot 2^{43}$ . Generally, there are  $(2^{40} - 1)/2^{n-1} + 1$  key guesses for the  $n$ -th sum over a nonexistent IV term. So the time complexity for the  $n$ -th sum is  $2^{43}((2^{40} - 1)/2^{n-1} + 1)$ . Totally, the time complexity is

$$\sum_{n=1}^{40} 2^{43}((2^{40} - 1)/2^{n-1} + 1) \approx 2^{84}.$$

According to the estimation in [2], one encryption needs at least 1000 bit operations, which is equivalent to  $2^{10}$  Grain-128 encryptions. So the attack needs about  $2^{74}$  cipher executions.

**Data Complexity:** The data complexity is  $2^{14} \cdot 43 \cdot 2^{43} \approx 2^{62.4}$ .

After recovering the 40 key bits, there are various of methods to recover the remaining key bits. For example,  $b_{236}$  can be easily nullified with 23 key guesses. Nonexistent IV terms of lower dimensions can be obtained using the techniques in Section 3. For example, nonexistent IV terms of dimension 42 can be chosen as distinguishers. Then the complexity to recover these bits is about  $23 \cdot 2^{23} \cdot 2^{42} \approx 2^{72}$  bit operations. Then the other key bits can be recovered by guessing with a complexity of  $2^{65}$ . As a result, the complexity of our attack is dominated by the recovery of the first 40 bits.

## 5 Conclusion

In this paper, we improved the attack on full-round Grain-128. Our attack is based on the knowledge that a lot of IV terms will disappear, after nullifying some state bits and IV bits. In addition, we find out the low-frequency IV bits and exploit them in the high degree terms. We also propose a method to cancel the terms with lower degree, and exploit the IV representation to obtain the IV terms with much lower complexity. Then the nonexistent IV terms are used as distinguishers so that we improved the attack in [2] by a factor of  $2^{16}$ . Our attack is not based on any key information, so we can attack Grain-128 with any arbitrary selected keys. Although the nonexistent IV terms can be tested by cube tester technique on super computers, our method can also work for higher dimensions, in which case the computing complexities for cube tester are beyond our ability.

In this paper, we have various of strategies in choosing IV bits such as choosing the low-frequency IV bits, so that the IV terms of degree 43 are very sparse.



We believe that attacker can enhance the sparsity with lower degree, which is much more complicated, where more nullified IV bits and low-frequency IV bits should be exploited. So finding the lowest degree for sparse IV terms is an open problem for further research. In addition, we exploit the methods to the cryptanalysis of Trivium and find the nullification technique does not work, where nullification one state bits may result in the increased degrees of many state bits and hence the boolean polynomial of the output bit is more complicated. So how to use bit controlling technique to reduce the polynomials of Trivium will be next research point.

## References

1. Aumasson, J., Dinur, I., Meier, W., Shamir, A.: Cube testers and key recovery attacks on reduced-round MD6 and Trivium. In: Fast Software Encryption, 16th International Workshop, FSE 2009. pp. 1–22. Springer (2009)
2. Dinur, I., Güneysu, T., Paar, C., Shamir, A., Zimmermann, R.: An experimentally verified attack on full Grain-128 using dedicated reconfigurable hardware. In: Advances in Cryptology-ASIACRYPT2011. pp. 327–343. Springer (2011)
3. Dinur, I., Shamir, A.: Cube attacks on tweakable black box polynomials. In: Joux, A. (ed.) Advances in Cryptology-EUROCRYPT2009. LNCS, vol. 5479, pp. 278–299. Springer, Heidelberg (2009)
4. Dinur, I., Shamir, A.: Breaking Grain-128 with dynamic cube attacks. In: Fast Software Encryption - 18th International Workshop, FSE 2011. pp. 167–187 (2011)
5. Fischer, S., Khazaei, S., Meier, W.: Chosen IV statistical analysis for key recovery attacks on stream ciphers. In: Progress in Cryptology - AFRICACRYPT 2008. pp. 236–245. Springer (2008)
6. Garey, M.R., Johnson, D.S.: Computers and Intractability: A Guide to the Theory of NP-Completeness. W. H. Freeman (1979)
7. Hell, M., Johansson, T., Maximov, A., Meier, W.: A stream cipher proposal: Grain-128. In: IEEE International Symposium on Information Theory (ISIT 2006) (2006)
8. Knellwolf, S., Meier, W., Naya-Plasencia, M.: Conditional differential cryptanalysis of NLFSR-based cryptosystems. In: Advances in Cryptology-ASIACRYPT2010. pp. 130–145. Springer (2010)
9. Wang, X., Yin, Y.L., Yu, H.: Finding collisions in the full SHA-1. In: Advances in Cryptology - CRYPTO 2005. pp. 17–36. Springer (2005)
10. Wang, X., Yu, H.: How to break MD5 and other hash functions. In: Advances in Cryptology-EUROCRYPT 2005. pp. 19–35. Springer (2005)

## A The details of nullifications

The nullification detail is shown in Table 8. The first column is the state bits to be nullified and the second column is the corresponding equation. The third column is the subkey bits guessed for nullifications. For example, in order to nullify  $b_{131}$ , we just need to set  $s_3$  to  $b_{15}b_{98} + b_{98}s_{45} + b_{71}b_{87} + s_{63}s_{82} + b_{64}b_{68} + b_{43}b_{51} + b_{30}b_{62} + b_{20}b_{21} + b_{14}b_{16} + b_6b_{70} + s_{16}s_{23} + \underline{b_{15}}s_{11} + b_{99} + b_{94} + b_{92} + b_{76} + b_{67} + b_{59} + b_{48} + b_{39} + b_{29} + b_{18} + b_5 + b_3 + 1$ , where  $b_{98}$  and  $b_{15}$  underlined are the key bits guessed. Besides these two bits, one expression on key bits indicated

by \*, that is  $b_{15}b_{98} + b_{71}b_{87} + b_{64}b_{68} + b_{43}b_{51} + b_{30}b_{62} + b_{20}b_{21} + b_{14}b_{16} + b_6b_{70} + b_{99} + b_{94} + b_{92} + b_{76} + b_{67} + b_{59} + b_{48} + b_{39} + b_{29} + b_{18} + b_5 + b_3$  should be guessed. Hence, three bits need to be guessed to nullify  $b_{131}$ . Totally, 40 bits should be guessed for nullifying the state bits in Table 2.

## **B All IV terms of degree 43 in $z_{257}$**

The resulted IV terms that may appear are shown in Table 9, 10, 11, 12, 13 and 14. The expression follows that in Section 3.5.

Table 8. Nullification equations

| Nullified bits | Equations for nullification   | Subkey bits guessed   |
|----------------|---|---|
| $b_{203}$      | $s_1 = b_{115}b_{123} + b_{92}b_{93} + b_{86}b_{88} + s_{88}s_{95} + b_{87}s_{83} + b_{111}b_{127} + b_{104}b_{108} + b_{83}b_{91} + b_{48}$<br>$+ b_{60}b_{61} + b_{54}b_{56} + b_{46}b_{110} + s_{56}s_{63} + b_{107} + b_{99} + b_{88} + b_{79} + b_{58} + b_{45} + b_{43} + s_{14}s_{21}$<br>$+ s_{43} + b_{52}s_{48} + b_{113} + b_{104} + b_{85} + b_{55} + b_{42} + s_{78} + s_{47} + s_{40} + b_{106}b_{122} + b_{99}b_{103} + b_5$<br>$+ b_{78}b_{86} + b_{65}b_{97} + b_{55}b_{56} + b_{49}b_{51} + b_{41}b_{105} + b_{50}s_{46} + b_{102} + b_{83} + b_{64} + b_{53} + b_{96}s_{43}$<br>$+ b_{40} + b_{104}b_{120} + b_{97}b_{101} + b_{76}b_{84} + b_{63}b_{95} + b_{53}b_{54} + b_{47}b_{49} + b_{39}b_{103} + b_{120} + s_3b_{13}b_{96}$<br>$+ b_{101} + b_{90} + b_{125} + b_{109} + b_{100} + b_{92} + b_{81} + b_{72} + b_{62} + b_{51} + b_{36} + b_{77} + b_{75} + s_{82}$<br>$+ s_{75} + b_{32}b_{115} + b_{115}s_{62} + b_{88}b_{104} + s_{80} + b_{81}b_{85} + b_{60}b_{68} + b_{47}b_{79} + b_{37}b_{38} + s_{39} + s_8$<br>$+ b_{31}b_{33} + b_{23}b_{87} + b_{111} + b_{109} + b_{93} + b_{84} + b_{76} + b_{65} + b_{56} + b_{46} + b_{70}b_{102} + b_{16} + b_3$<br>$+ b_{35} + b_{22} + b_{20} + b_{100}s_{47} + s_{18}s_{25} + b_{17}s_{13} + b_{78} + b_{50} + b_{41} + b_{13}s_9 + s_{94} + b_{90} + b_{65}$<br>$+ b_{20} + b_7 + s_{101} + s_{86} + s_{43} + s_{12} + s_5b_{15}b_{98} + b_{98}s_{45} + b_{15}s_{11} + b_{92} + b_{67} + b_{46} + b_{37}$<br>$+ b_{18} + b_{39} + s_{73} + s_{10} + 1$ | $b_{87}, b_{52}, b_{115}, b_{96},$<br>$b_{13}, *$               |
| $b_{176}$      | $s_{83} = s_{48} + b_{48} + b_{74} + b_{104} + s_{11} + b_{11} + b_{37} + b_{67} + b_{14}b_{78} + b_{22}b_{24} + b_{28}b_{29} + b_{38}b_{70}$<br>$+ b_{51}b_{59} + b_{72}b_{76} + b_{79}b_{95} + b_{13} + b_{26} + b_{47} + b_{56} + b_{75} + b_{100} + s_{19}b_{23} + b_{23}b_{106} + b_{16}$<br>$+ b_{42} + b_{72} + b_{19}b_{83} + b_{27}b_{29} + b_{33}b_{34} + b_{43}b_{75} + b_{56}b_{64} + b_{77}b_{81} + b_{84}b_{100} + b_{18} + b_{31}$<br>$+ b_{52} + b_{61} + b_{80} + b_{89} + b_{105} + b_{28}b_{111} + s_{58}b_{111} + b_{51}b_{115} + b_{59}b_{61} + b_{65}b_{66} + b_{75}b_{107}$<br>$+ b_{88}b_{96} + b_{109}b_{113} + s_4b_{116} + b_4b_{116} + b_{30}b_{116} + b_{60}b_{116} + b_{95}b_{116} + b_{100}b_{116} + b_7b_{71}b_{116}$<br>$+ b_{15}b_{17}b_{116} + b_{21}b_{22}b_{116} + b_{31}b_{63}b_{116} + b_{44}b_{52}b_{116} + b_{65}b_{69}b_{116} + b_{72}b_{88}b_{116} + b_6b_{116}$<br>$+ b_{19}b_{116} + b_{40}b_{116} + b_{49}b_{116} + b_{68}b_{116} + b_{77}b_{116} + b_{93} + b_{93}b_{116} + s_{12}b_{16}b_{116} + b_{16}b_{99}b_{116}$<br>$+ b_{116} + s_{46}b_{99}b_{116} + b_{50} + b_{63} + b_{121} + s_{13} + b_{15} + b_{28} + b_{49} + b_{58} + b_{77} + b_{86} + s_{21}b_{25}$<br>$+ b_{25}b_{108} + s_{73}s_{92}$  | $b_{23}, b_{111}, b_{16}b_{116},$<br>$b_{25}, b_{99}b_{116}, *$ |
| $b_{170}$      | $s_{42} = b_{110}b_{126} + b_{103}b_{107} + b_{82}b_{90} + b_{69}b_{101} + b_{59}b_{60} + b_{53}b_{55} + b_{45}b_{109} + s_{55}s_{62} + b_{54}s_{50}$<br>$+ b_{115} + b_{106} + b_{98} + b_{87} + b_{78} + b_{68} + b_{57} + b_{44} + b_{42} + 1$   | *   |
| $b_{159}$      | $s_{31} = b_{43}b_{126} + b_{126}s_{73} + b_{99}b_{115} + s_{91} + b_{92}b_{96} + b_{71}b_{79} + b_{58}b_{90} + b_{48}b_{49} + b_{42}b_{44}$<br>$+ b_{34}b_{98} + s_{44}s_{51} + b_{43}s_{39} + b_{127} + b_{122} + b_{120} + b_{104} + b_{95} + b_{87} + b_{76} + b_{67} + b_{57} + b_{46}$<br>$+ b_{33} + b_{31} + 1$   | $b_{43}, b_{126}, *$  |
| $b_{153}$      | $s_{25} = b_{37}b_{120} + b_{120}s_{67} + b_{93}b_{109} + s_{85} + b_{86}b_{90} + b_{65}b_{73} + b_{52}b_{84} + b_{42}b_{43} + b_{36}b_{38} + b_{121}$<br>$+ b_{28}b_{92} + s_{38}s_{45} + b_{37}s_{33} + b_{116} + b_{114} + b_{98} + b_{89} + b_{81} + b_{70} + b_{61} + b_{51} + b_{40} + b_{27}$<br>$+ b_{25} + 1$  | *   |
| $b_{145}$      | $s_{17} = b_{29}b_{112} + b_{112}s_{59} + b_{85}b_{101} + s_{77} + b_{78}b_{82} + b_{57}b_{65} + b_{44}b_{76} + b_{34}b_{35}$<br>$+ b_{28}b_{30} + b_{20}b_{84} + s_{30}s_{37} + b_{29}s_{25} + b_{113} + b_{108} + b_{106} + b_{90}$<br>$+ b_{81} + b_{73} + b_{62} + b_{53} + b_{43} + b_{32} + b_{19} + b_{17} + 1$  | $b_{29}, b_{112}, *$  |
| $b_{143}$      | $s_{15} = b_{27}b_{110} + b_{110}s_{57} + b_{83}b_{99} + s_{75}s_{94} + b_{76}b_{80} + b_{55}b_{63} + b_{42}b_{74} + b_{32}b_{33}$<br>$+ b_{26}b_{28} + b_{18}b_{82} + s_{28}s_{35} + b_{27}s_{23} + b_{111} + b_{106} + b_{104} + b_{88} + b_{79}$<br>$+ b_{71} + b_{60} + b_{51} + b_{41} + b_{30} + b_{17} + b_{15} + 1$   | $b_{27}, b_{110}, *$  |
| $b_{138}$      | $s_{10} = b_{22}b_{105} + b_{105}s_{52} + b_{78}b_{94} + s_{70}s_{89} + b_{71}b_{75} + b_{50}b_{58} + b_{37}b_{69} + b_{27}b_{28}$<br>$+ b_{21}b_{23} + b_{13}b_{77} + s_{23}s_{30} + b_{22}s_{18} + b_{106} + b_{101} + b_{99} + b_{83} + b_{74}$<br>$+ b_{66} + b_{55} + b_{46} + b_{36} + b_{25} + b_{12} + b_{10} + 1$  | $b_{22}, b_{105}, *$  |
| $b_{137}$      | $s_9 = b_{21}b_{104} + b_{104}s_{51} + b_{77}b_{93} + s_{69}s_{88} + b_{70}b_{74} + b_{49}b_{57} + b_{36}b_{68} + b_{26}b_{27}$<br>$+ b_{20}b_{22} + b_{12}b_{76} + s_{22}s_{29} + b_{21}s_{17} + b_{105} + b_{100} + b_{98} + b_{82} + b_{73}$<br>$+ b_{65} + b_{54} + b_{45} + b_{35} + b_{24} + b_{11} + b_9 + 1$  | $b_{21}, *$   |
| $b_{136}$      | $s_8 = b_{20}b_{103} + b_{103}s_{50} + b_{76}b_{92} + s_{68}s_{87} + b_{69}b_{73} + b_{48}b_{56} + b_{35}b_{67} + b_{25}b_{26}$<br>$+ b_{19}b_{21} + b_{11}b_{75} + s_{21}s_{28} + b_{20}s_{16} + b_{104} + b_{99} + b_{97} + b_{81} + b_{72} + b_{64}$<br>$+ b_{53} + b_{44} + b_{34} + b_{23} + b_{10} + b_8 + 1$   | *   |
| $s_{135}$      | $s_{77} = b_{19}b_{102} + b_{102}s_{49} + s_{67}s_{86} + s_{20}s_{27} + b_{19}s_{15} + b_{96} + s_{88}$<br>$+ b_{80} + s_7 + b_{71} + b_{52} + s_{45} + b_{43} + b_{22} + s_{14} + b_9$   | $b_{19}, b_{102}, *$  |
| $b_{134}$      | $s_6 = b_{18}b_{101} + b_{101}s_{48} + b_{74}b_{90} + s_{66}s_{85} + b_{67}b_{71} + b_{46}b_{54} + b_{33}b_{65} + b_{23}b_{24}$<br>$+ b_{17}b_{19} + b_9b_{73} + s_{19}s_{26} + b_{18}s_{14} + b_{102} + b_{97} + b_{95} + b_{79} + b_{70} + b_{62}$<br>$+ b_{51} + b_{42} + b_{32} + b_{21} + b_8 + b_6 + 1$   | $b_{101}, *$  |
| $b_{133}$      | $s_5 = b_{17}b_{100} + b_{100}s_{47} + b_{73}b_{89} + s_{65}s_{84} + b_{66}b_{70} + b_{45}b_{53} + b_{32}b_{64} + b_{22}b_{23}$<br>$+ b_{16}b_{18} + b_8b_{72} + s_{18}s_{25} + b_{17}s_{13} + b_{101} + b_{96} + b_{94} + b_{78} + b_{69} + b_{61}$<br>$+ b_{50} + b_{41} + b_{31} + b_{20} + b_7 + b_5 + 1$   | $b_{17}, b_{100}, *$  |
| $b_{131}$      | $s_3 = b_{15}b_{98} + b_{98}s_{45} + b_{71}b_{87} + s_{63}s_{82} + b_{64}b_{68} + b_{43}b_{51} + b_{30}b_{62} + b_{20}b_{21}$<br>$+ b_{14}b_{16} + b_6b_{70} + s_{16}s_{23} + b_{15}s_{11} + b_{99} + b_{94} + b_{92} + b_{76} + b_{67}$<br>$+ b_{59} + b_{48} + b_{39} + b_{29} + b_{18} + b_5 + b_3 + 1$  | $b_{15}, b_{98}, *$   |

<sup>1</sup> Each \* in this table indicates a different expression of partial key bits.

Table 9. IV terms of degree 43 in  $z_{257}$ -part 1

|            |            |            |            |            |            |
|------------|------------|------------|------------|------------|------------|
| FFFFFDFB6  | FBFFFFFB6  | FFFFFDBB7  | FFFFFDDFA7 | FFDFDFDFB7 | FBFFFFFB7  |
| FBFFFFFA7  | FF9FFFFB7  | BFFFFDFB7  | DFDFDFDFB7 | BFBFFFFB7  | FDBFFFFB7  |
| FFFFFD7B7  | FBFFFF7B7  | FFFFBDFB7  | FFFFDDDFB7 | FBBBFFFFB7 | FFBFDFFB7  |
| FBFFFFDFB7 | FFFFFDFB3  | F7FFFFDFB7 | FEFFFFDFB7 | FBFFFFFB3  | F7BFFFFB7  |
| EBFFFFFB7  | EFFFFDFB7  | EBFFFFFB7  | FFFFFD97   | FFFDDFB7   | FEFFFFDFB7 |
| FFFFFDEB7  | 7FFFFDFB7  | FBFFFF97   | FBFDFFFB7  | FAFFFFFB7  | FBFFFFEB7  |
| 7BFFFFFB7  | BEFFFFFB7  | 9FFFFFB7   | 9EFFFFFB7  | DFFFFFFB7  | BFDFFFFB7  |
| BFEDFFFF7  | 9FFDFFFF7  | DFDFFFFB7  | DFEDFFFF7  | FEFFFFFB6  | FEFFFFCF7  |
| FFFFF6F6   | FFFBFFFB6  | FFFFDFFB6  | FFFFFEB2   | F7FFFFFB6  | FEFFFFFB6  |
| FFFFFEEF6  | FFFFFEE6   | FFDFFFFB6  | FBFFFFFB6  | FFFFFED6   | FDFFFFFB6  |
| FFFFFDEF6  | FFFFDFFB6  | FFFF7FB6   | FFFFFDF3   | FFFFFDF3   | FFFFFCE7   |
| FFDFFFCF7  | F7FFFFCF7  | FEFFFFCF7  | FFFFF4F7   | FFFBFFCF7  | FFFDFFCF7  |
| FFFFDFCF7  | FBFFFFCF7  | FFEDFFFB7  | FFFFF7F7   | FFFDFFDF7  | FFEFFDF3   |
| 7EFFFFDF7  | FEFFFF9F7  | EFDFDFDF7  | F7EFFFFDF7 | FEFFFF5F7  | FFFBFFDF7  |
| FEFFFFDF7  | FFEDFFDF7  | FAFFFFDF7  | FEFFFFDE7  | FFCFFDF7   | FFEFFDF7   |
| 7FFFFDF3   | FFFFF9F3   | EFFFFDF3   | FFFFF5F3   | FFFBFFDF3  | FFDFFDF3   |
| FFFFDFDF3  | FEFFFFDF3  | FFFFFDE3   | FFDFDFDF3  | FBBBFFDF3  | FFFDFFDF3  |
| 7FFFFDE7   | 7FFDFDF7   | 77FFFFDF7  | 7EFFFFDF7  | FFFFF9E7   | FFDFFF9F7  |
| F7FFFF9F7  | FEFFFF9F7  | EFFFFDE7   | EFFDFDF7   | E7FFFFDF7  | EEFFFFDF7  |
| FFFFF5F7   | FFDBFFDF7  | FFDFDFDF7  | 7FFFF5F7   | FFFFF1F7   | EFFFF5F7   |
| 7FFBFFDF7  | FFFFBFF9F7 | EFFBFFDF7  | 7FFDFDF7   | 7FFDFDF7   | 7FBFFDF7   |
| FFFFDF9F7  | FFFFDF9F7  | FBBBFF9F7  | EFFDFDF7   | EFFDFDF7   | EBBFFDF7   |
| F7FFFF5F7  | FEFFFF5F7  | F7BFFDF7   | FEFFBFFDF7 | FEFFDFDE7  | FDFFDFDE7  |
| FDDDFDF7   | FBDBFFDF7  | F6FFDFDF7  | F7DFDFDF7  | FEEDFFDF7  | F7BFFDF7   |
| FBBFFDF7   | F7FFFFDE7  | FEFFFFDE7  | F7DFFDF7   | FEDEFFDF7  | F7BFFDF7   |
| FEFFDFDF7  | F7FFDFDF7  | FFFFDF5F7  | FFFFF5E7   | FFDFF5F7   | FBFF5F7    |
| FFFF9FDF7  | FFFBFFDE7  | FFDFBFFDF7 | FBBBFFDF7  | FEFDFFDF7  | FFDDFFDF7  |
| FFBDFDF7   | FFFFDFDE7  | FFDFDFDF7  | FFFFDDDF7  | FFFFDFDE7  | FFDFDFDF7  |
| FFBFFDFE7  | F9FFFFDF7  | FFBFFDF7   | FEFFFFEF7  | FFEDFFEF7  | FFDFFFEF7  |
| DFEFFFEF7  | DEFFFFEF7  | DFDFFFFEF7 | FBFFDFFEF7 | FBFFFFEF7  | FBFFFEF7   |
| FBDFDFEF7  | FBFFFFFAF7 | EBFFFFEF7  | FBFFFF6F7  | FBFFFFEF3  | FBFFFEF7   |
| BDDFDFEF7  | FBFFDFDF7  | FBBFFFEF7  | F3FFFFEF7  | FAFFFFEF7  | FBFBFFEF7  |
| DBFFFFEF7  | F9FFFFEF7  | FBFFDFDF7  | FBFFFFED7  | FBFF7FEF7  | BFFDFFEF7  |
| BFFFFFEF7  | BFFFFFEF7  | BFFDFDFEF7 | BFFFFFAF7  | AFFFFFEF7  | BFFFF6F7   |
| BFFFFFEF3  | BFFFFFEF7  | BFDFDFEF7  | BFFDFDFEF7 | BFBFFFEF7  | B7FFFFEF7  |
| BEFFFFEF7  | BFFBFFEF7  | 9FFFFFEF7  | BDFFFFEF7  | BFFFFDFEF7 | BFFFFED7   |
| BFFF7FEF7  | FEFFDFEF7  | FFFFDFEF7  | FFDDFFEF7  | DFFFDFEF7  | FFFFFE76   |
| FFFF7F76   | 7EFFFFEF7  | 7FFFFFEF7  | 7FFDFFEF7  | FEFFFAF7   | FFFFFAF7   |
| FFDFFAF7   | 7FFFFFAF7  | EFFFFFEF7  | EFFFFFEF7  | EFFDFFEF7  | 6FFFFEF7   |
| EFFFFFAF7  | FEFFFF6F7  | FFFFFE6F7  | FFDFFF6F7  | FFFBFFFEF7 | FFFFBEEF7  |
| FFDFBFFEF7 | FFDFBFFEF3 | FFFFDFEF3  | FEFFFFEF3  | FFFFFEF3   | FFFFBEE3   |
| FFDFFFEF3  | FEFFFFEF7  | FFDFDFEF7  | FFCFFFEF7  | FFDFDFEF7  | FFFFBEE7   |
| FFDFFFEF7  | FFFFDFEF7  | FFDFFFEF7  | FFDFFFEF7  | FFDFDFEF7  | FFDDFFEF7  |
| FFDFDFEF7  | FFFFF6F3   | FFFFF6E7   | FFDFFF6F7  | FFFFDF6F7  | 7FFFF6F7   |
| FFFFF2F7   | EFFFF6F7   | 7FFFFF3    | 7FFFFFE7   | 7DFFFFEF7  | 7FFFDFF7   |
| FFFFF2F3   | FFFFF2AF7  | FFDFFFAF7  | FFFFDFAF7  | EFFFFFEF3  | EFFFFFE7   |
| FDFFFFEF7  | EFFFDFF7   | FFFFFDF6   | FAFFFFEF7  | FBFFFFEF7  | FFBDFFEF7  |
| F6FFFFEF7  | F7EFFFF7   | FEFFFFEF7  | F7FFFFEF7  | FEFFFFEF7  | F7DFFFEF7  |
| FFDFFFEF7  | F7BFFFF7   | EBFFFFEF7  | 7BFFFFEF7  | 77FFFFEF7  | 7EFFFFEF7  |
| FBFFFFAF7  | F7FFFFAF7  | FEFFFFAF7  | EBFFFFEF7  | E7FFFFEF7  | EEFFFFEF7  |
| FBBFF6F7   | F7FFFF6F7  | FEFFFF6F7  | FBBFFFEF3  | FBBFFFE7   | FF9FFFEF7  |
| FBFFDFEF7  | F7FFFFEF3  | FEFFFFEF3  | F7FFFFEF7  | FEFFFFEF7  | F7DFFFEF7  |
| FEDFFFEF7  | F7FFDFEF7  | FEFFDFEF7  | 7FFBFFEF7  | FFFFBFAF7  | EFFBFFEF7  |
| 5FFFFFEF7  | DEFFFFAF7  | CFFFFFEF7  | FFFFB6F7   | DEFFFF6F7  | FFFFBFEF3  |
| FFFFBFE7   | FFDBFFEF7  | FFFFBDFEF7 | DFFBFFEF7  | DEFFFFEF3  | DFFBFFEF7  |
| DFDFFFEF7  | DEFFDFEF7  | FBBBFFEF7  | F7FBFFEF7  | FEFFBFEF7  | DFBFFFEF7  |
| D7FFFFEF7  | DEFFFFEF7  | FEFFBFFEF7 | FDFFBFEF7  | FDFFBFEF7  | FEFFBDEF7  |
| FFFFFCE7   | FFDFDFEF7  | FEFFBFFEF7 | FFFFBFE7   | FFDFBFE7   | FFEFFFE7   |
| FFFF7EF7   | FFDF7FE7   | DDFFFFEF7  | DDFFBFE7   | DDFFBFE7   | DDFF7FE7   |
| 7DFFFFEF7  | FDFFFFAF7  | EDFFFFEF7  | PDFFFF6F7  | FDFFBFEF7  | FDFFFFEF3  |
| FDFFFFEE7  | FDFFFFEF7  | FDFFDFEF7  | FDFFFFCF7  | FFFFDFCF7  | FFFFDFCD7  |
| FFFF7FCF7  | FDBFFFEF7  | F5FFFFEF7  | FCFFFFFEF7 | FEFFFEF6   | FFEDFFFE6  |
| FFDFFEF6   | DEFFFFF6   | DFFFFFEF6  | DDFFFFFE6  | 7FFDFFEF7  | 7FFFFDF7   |
| 7FFFFED7   | FFFDFFAF7  | FFFFFDFAF7 | FFFFFAD7   | EFFDFFEF7  | EFFFDDEF7  |
| EFFFFED7   | 7FFFF7FEF7 | FFFF7FAF7  | EFFF7FEF7  | FFFDFF6F7  | FFFFDF6F7  |
| FFFFF6D7   | FFFF9FF7   | FFFFBDFEF7 | FFFFBFFED7 | FFFDFFEF3  | FFFFDFFE7  |
| FFDFDFEF7  | FFFDDFEF7  | FFFFDFEF3  | FFFFFDE3   | FFFFFDEE7  | FFDFDFEF7  |
| FFFFDDEF7  | FFFFFEC7   | FFDFFFED7  | FFFFDFED7  | FFFF7F6F7  | FFFFB7FEF7 |
| FFFF7FEF3  | FFFF7FE7   | FFDF7FEF7  | FFFF5FEF7  | FBBBDFEF7  | FFBDFFEF7  |
| FBBFFED7   | F7FFDFEF7  | FEFFDFEF7  | F7FFFFEF7  | FEFFDFEF7  | F7FFFFDF7  |
| FEFFFE7    | FBBF7FEF7  | F7FF7FEF7  | FEFF7FEF7  | FDFFDFEF7  | DFFFDFEF7  |
| FDFFFE7    | FFFDDFEF7  | FFFFDFED7  | FFFFDFED7  | FFFF7FEF7  | FDFF7FEF7  |











