# Blockcipher-based Authenticated Encryption: How Small Can We Go? * 

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#### Abstract

This paper presents a lightweight blockcipher based authenticated encryption mode mainly focusing on minimizing the implementation size, i.e., hardware gates or working memory on software. The mode is called COFB, for COmbined FeedBack. COFB uses an $n$-bit blockcipher as the underlying primitive, and relies on the use of a nonce for security. In addition to the state required for executing the underlying blockcipher, COFB needs only $n / 2$ bits state as a mask. Till date, for all existing constructions in which masks have been applied, at least $n$ bit masks have been used. Thus, we have shown the possibility of reducing the size of a mask without degrading the security level much. Moreover, it requires one blockcipher call to process one input block. We show COFB is provably secure up to $O\left(2^{n / 2} / n\right)$ queries which is almost up to the standard birthday bound. We first present an idealized mode iCOFB along with the details of its provable security analysis. Next, we extend the construction to the practical mode COFB. We instantiate COFB with two 128-bit blockciphers, AES-128 and GIFT-128, and present their implementation results on FPGAs. When instantiated with AES-128, COFB achieves only a few more than 1000 Look-Up-Tables (LUTs) while maintaining almost the same level of provable security as standard AES-based AE, such as GCM. When instantiated with GIFT-128, COFB performs much better in hardware area. It consumes less than 1000 LUTs while maintaining the same security level. Both these figures show competitive implementation results compared to other authenticated encryption constructions.


Keywords: COFB, AES, GIFT, authenticated encryption, blockcipher.

## 1 Introduction

Authenticated encryption (AE) is a symmetric-key cryptographic primitive for providing both confidentiality and authenticity. Due to the recent rise in communication networks operated on small devices, the era of the so-called Internet of Things, AE is expected to play a key role in securing these networks.

[^0]In this paper, we study blockcipher modes for AE with primary focus on the hardware implementation size. Here, we consider the overhead in size, thus the state memory size beyond the underlying blockcipher itself (including the key schedule) is the criteria we want to minimize, which is particularly relevant for hardware implementation. We observe this direction has not received much attention until the launch of CAESAR competition (see below), while it would be relevant for future communication devices requiring ultra low-power operations.

Generic Approaches. One generic approach for reducing the implementation size of blockcipher modes is to use lightweight blockciphers. It covers a broad area of use cases, where standard AES is not suitable due to the implementation constraints, and one of the major criteria is area minimization. One of the most popular lightweight blockciphers is PRESENT [19] proposed in 2007. Since then, many have been proposed in the last decade, such as KATAN [22], LED [33], PICCOLLO [58], PRINCE [21] and TWINE [59]. SIMON and SPECK [15] are proposed by NSA in 2014. More recent designs are SKINNY (which is a tweakable blockcipher [16]) and GIFT [13, 14].

The other approach is to use standard AES implemented in a tiny, serialized core [47], where the latter is shown to be effective for various schemes including popular CCM [5] or OCB [40] modes, as shown in [20] and [12]. Still, this requires much larger number of clock cycles for each AES encryption than the standard round-based implementation, and hence is not desirable when speed or energy is also a criteria in addition to size.

AE Modes with Small Memory. CAESAR [3] is a competition for AE started in 2012. It attracted 57 AE schemes, and there are new schemes that were designed to minimize the implementation size while designed as a blockcipher mode (i.e. it uses a blockcipher as a black box). Among them, JAMBU [62] is considered to be one of the most relevant mode to our purpose, which can be implemented with $(1.5 n+k)$-bit state memory, using $n$-bit blockcipher with $k$-bit key. However, the provable security result is not published for this scheme ${ }^{\ddagger}$, and the security claim about the confidentiality in the nonce misuse scenario was shown to be flawed [50]. We also point out that the rate of JAMBU is $1 / 2$, i.e., it makes two blockcipher calls to process one input block. CLOC and SILC [36, 37] have provable security results and were designed to minimize the implementation size, however, they have $(2 n+k)$-bit state memory and the rate is also $1 / 2$.

NIST Lightweight Cryptography Project. Recently, the growing importance of lightweight applications have also been addressed by NIST's lightweight cryptography project [44], which recognizes the apparent lack of suitable AE standards to be used for lightweight applications. They highlighted the requirements under the backdrop of several arising applications like sensor networks, health care, distributed control systems and several others, where highly resource constrained devices communicate among themselves.

[^1]We next summarize our contributions.
A New Type of Feedback Function. To reduce the state memory, it is natural to use feedback from the blocks involved in each blockcipher call, at the cost of losing parallelizability. There are existing feedback modes (such as ciphertextfeedback of CBC encryption), however, we found that none of them is enough to fulfill our needs. We first formalize the feedback function as a linear function to take blockcipher output $(Y)$ and plaintext block $(M)$ to produce the corresponding ciphertext block $(C)$ and the chain value as the next input to blockcipher $(X)$. This formalization covers all previous popular feedback functions. Then, we propose a new type of feedback function, called combined feedback, where $X$ is a linear function (not a simple XOR) of $M$ and $Y$. We show that if the above linear function satisfies certain conditions we could build a provably-secure, small-state AE. We first present a mode of tweakable random function which has additional input called tweak in addition to $n$-bit block input, to demonstrate the effectiveness of combined feedback and intuition for provable security. The proposed scheme (iCOFB for idealized COmbined FeedBack) has a quite high provable security, comparable to $\Theta C B 3$ presented in the proof of OCB3 [40], and has small memory ( $n$-bit block memory plus those needed for the primitive). In addition it needs one primitive call to process $n$-bit message block.

Blockcipher AE mode with Combined Feedback Function. Starting from iCOFB, we take a further step to propose a blockcipher mode using combined feedback. The main obstacle is the instantiation of tweakable random function (or, equivalently tweakable blockcipher [43]) using a blockcipher. We could use existing tweakable blockcipher mode for this purpose, e.g. XEX [51] by Rogaway, and thanks to the standard birthday type security of XEX, the resulting blockcipher mode would also have standard birthday type security. However, the implementation of XEX or similar ones needs $n$-bit memory used as input mask to blockcipher, in addition to the main $n$-bit state block, implying $(2 n+k)$-bit state memory. Therefore, instead of relying on the existing tweakable blockcipher modes, we instantiate the tweakable random function using only $n / 2$-bit mask and provide a dedicated security proof for our final proposal (mode), which we call COFB. We show COFB achieves almost birthday bound security, roughly up to $O\left(2^{n / 2} / n\right)$ queries, based on the standard PRP assumption on the blockcipher.

COFB needs $n / 2$-bit register for mask in addition to the registers requires for holding round keys and the internal $n$-bit state for the blockcipher computation. Hence the state size of COFB is $1.5 n+k$ bits. The rate of COFB is 1 , i.e, it makes one blockcipher call to process one input block, meaning it is as fast as encryption-only modes. On the downside, COFB is completely serial both for encryption and decryption, which is inherent to the use of combined feedback. However, we argue that this is a reasonable trade-off, as tiny devices are our primal target platform for COFB. See Table 1 for comparison of COFB with others. The description and the security analysis of COFB in Sect. 4 and 5 have been described at the proceedings version of our paper in CHES 2017 [23].

Table 1. Comparison of AE modes, using an $n$-bit blockcipher with $k$-bit keys. An inverse-free mode is a mode that does not need the blockcipher inverse (decryption) function for both encryption and decryption. For JAMBU, the authenticity bound was briefly presented in [62].

| Scheme | State Size | Rate | Parallel | Inverse-Free | Sec. Proof | Ref |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| COFB | $1.5 n+k$ | 1 | No | Yes | Yes | This work |
| JAMBU | $1.5 n+k$ | $1 / 2$ | No | Yes | Partial | $[62]$ |
| CLOC/ SILC | $2 n+k$ | $1 / 2$ | No | Yes | Yes | $[36,37]$ |
| iFEED | $3 n+k$ | 1 | Only for Enc | Yes | Flawed $[57]$ | $[64]$ |
| OCB | $\geq 3 n+k$ | 1 | Yes | No | Yes | $[40,51,52]$ |

Instantiations and Hardware Implementations. We instantiate and implement COFB with the 128-bit version of the blockcipher AES known as AES-128. We also implement COFB with the 128 -bit version of the blockcipher GIFT (described as GIFT-128 in $[13,14]$ ) to get an idea of the lightweight property of the COFB mode by checking how small (hardware area) it can go with a lightweight blockcipher. For the sake of completeness we compare our implementation figures with various schemes (not limited to blockcipher modes) listed in the hardware benchmark framework called ATHENa [1]. The implementation details of COFB[AES] have already been described in [23,24]. COFB[AES] shows the impressive performance figures of COFB both for size and speed compared to other AES-based AE modes. Moreover, if we implement COFB with GIFT, then it achieves much smaller area than COFB[AES] and is quite competitive to even ad-hoc designs (see Sect. 6). The implementation details of COFB[GIFT] are also described in Sect. 6, which is a new contribution compared to [23]. We have to warn that this is a rough comparison ignoring differences in several implementation factors (see Sect. 6). Nevertheless, we think this comparison implies a good performance of COFB among others even using the standard AES-128, and implies COFB with a lightweight blockcipher to hit the limit of blockcipher-based AE's speed and size.

## 2 Preliminaries

Notation. We fix a positive integer $n$ which is the block size in bits of the underlying blockcipher $E_{K}$. Typically, we consider $n=128$ and AES-128 [7] is the underlying blockcipher, where $K$ is the 128 -bit AES key. The empty string is denoted by $\lambda$. For any $X \in\{0,1\}^{*}$, where $\{0,1\}^{*}$ is the set of all finite bit strings (including $\lambda$ ), we denote the number of bits of $X$ by $|X|$. Note that $|\lambda|=0$. For two bit strings $X$ and $Y, X \| Y$ denotes the concatenation of $X$ and $Y$. A bit string $X$ is called a complete (or incomplete) block if $|X|=n$ (or $|X|<n$ respectively). We write the set of all complete (or incomplete) blocks as $\mathcal{B}$ (or $\mathcal{B}^{<}$respectively). Let $\mathcal{B} \leq=\mathcal{B}^{<} \cup \mathcal{B}$ denote the set of all blocks. For $B \in \mathcal{B} \leq$, we
define $\bar{B}$ as follows:

$$
\bar{B}= \begin{cases}0^{n} & \text { if } B=\lambda \\ B \| 10^{n-1-|B|} & \text { if } B \neq \lambda \text { and }|B|<n \\ B & \text { if }|B|=n\end{cases}
$$

Given non-empty $Z \in\{0,1\}^{*}$, we define the parsing of $Z$ into $n$-bit blocks as

$$
\begin{equation*}
(Z[1], Z[2], \ldots, Z[z]) \stackrel{n}{r}_{\leftarrow} Z \tag{1}
\end{equation*}
$$

where $z=\lceil|Z| / n\rceil,|Z[i]|=n$ for all $i<z$ and $1 \leq|Z[z]| \leq n$ such that $Z=$ $(Z[1]\|Z[2]\| \cdots \| Z[z])$. If $Z=\lambda$, we let $z=1$ and $Z[1]=\lambda$. We write $\|Z\|=z$ (number of blocks present in $Z$ ). We similarly write $(Z[1], Z[2], \ldots, Z[z]){ }_{\leftarrow}^{m} Z$ to denote the parsing of the bit string $Z$ into $m$-bit strings $Z[1], Z[2], \ldots, Z[z-1]$ and $1 \leq|Z[z]| \leq m$. Given any sequence $Z=(Z[1], \ldots, Z[s])$ and $1 \leq a \leq b \leq s$, we represent the sub sequence $(Z[a], \ldots, Z[b])$ by $Z[a . . b]$. For integers $a \leq b$, we write $[a . . b]$ for the set $\{a, a+1, \ldots, b\}$. For two bit strings $X$ and $Y$ with $|X| \geq|Y|$, we define the extended xor-operation as

$$
\begin{aligned}
X \oplus Y & =X[1 . .|Y|] \oplus Y \text { and } \\
X \bar{\oplus} Y & =X \oplus\left(Y \| 0^{|X|-|Y|}\right),
\end{aligned}
$$

where $(X[1], X[2], \ldots, X[x]) \stackrel{1}{\leftarrow} X$ and thus $X[1 . .|Y|]$ denotes the first $|Y|$ bits of $X$. When $|X|=|Y|$, both operations reduce to the standard $X \oplus Y$.

Let $\gamma=(\gamma[1], \ldots, \gamma[s])$ be a tuple of equal-length strings. We define $\operatorname{mcoll}(\gamma)=$ $r$ if there exist distinct $i_{1}, \ldots, i_{r} \in[1 . . s]$ such that $\gamma\left[i_{1}\right]=\cdots=\gamma\left[i_{r}\right]$ and $r$ is the maximum of such integer. We say that $\left\{i_{1}, \ldots, i_{r}\right\}$ is an $r$-multi-collision set for $\gamma$.

Authenticated Encryption and Security Definitions. An authenticated encryption (AE) is an integrated scheme that provides both privacy of a plaintext $M \in\{0,1\}^{*}$ and authenticity of $M$ as well as associated data $A \in\{0,1\}^{*}$. Taking a nonce $N$ (which is a value never repeats at encryption) together with associated data $A$ and plaintext $M$, the encryption function of AE, $\mathcal{E}_{K}$, produces a tagged-ciphertext $(C, T)$ where $|C|=|M|$ and $|T|=t$. Typically, $t$ is fixed and we assume $n=t$ throughout the paper. The corresponding decryption function, $\mathcal{D}_{K}$, takes $(N, A, C, T)$ and returns a decrypted plaintext $M$ when the verification on $(N, A, C, T)$ is successful, otherwise returns the atomic error symbol denoted by $\perp$.

Privacy. Given an adversary $\mathcal{A}$, we define the $P R F$-advantage of $\mathcal{A}$ against $\mathcal{E}$ as $\operatorname{Adv}_{\mathcal{E}}^{\text {prf }}(\mathcal{A})=\left|\operatorname{Pr}\left[\mathcal{A}^{\mathcal{E}_{K}}=1\right]-\operatorname{Pr}\left[\mathcal{A}^{\mathbb{S}}=1\right]\right|$, where $\$$ returns a random string of the same length as the output length of $\mathcal{E}_{K}$, by assuming that the output length of $\mathcal{E}_{K}$ is uniquely determined by the query. The PRF-advantage of $\mathcal{E}$ is defined as

$$
\mathbf{A d v}_{\mathcal{E}}^{\mathrm{prf}}(q, \sigma, t)=\max _{\mathcal{A}} \mathbf{A d}_{\mathcal{E}}^{\mathrm{prf}}(\mathcal{A})
$$

where the maximum is taken over all adversaries running in time $t$ and making $q$ queries with the total number of blocks in all the queries being at most $\sigma$. If $\mathcal{E}_{K}$ is an encryption function of AE , we call it the privacy advantage and write as $\operatorname{Adv}_{\mathcal{E}}^{\text {priv }}(q, \sigma, t)$, as the maximum of all nonce-respecting adversaries (that is, the adversary can arbitrarily choose nonces provided all nonce values in the encryption queries are distinct).

Authenticity. We say that an adversary $\mathcal{A}$ forges an AE scheme $(\mathcal{E}, \mathcal{D})$ if $\mathcal{A}$ is able to compute a tuple $(N, A, C, T)$ satisfying $\mathcal{D}_{K}(N, A, C, T) \neq \perp$, without querying $(N, A, M)$ for some $M$ to $\mathcal{E}_{K}$ and receiving $(C, T)$, i.e. $(N, A, C, T)$ is a non-trivial forgery.

In general, a forger is nonce-respecting with respect to encryption queries, but can make $q_{f}$ forging attempts without restriction on $N$ in the decryption queries, that is, $N$ can be repeated in the decryption queries and an encryption query and a decryption query can use the same $N$. The forging advantage for an adversary $\mathcal{A}$ is written as $\operatorname{Adv}_{\mathcal{E}}^{\text {auth }}(\mathcal{A})=\operatorname{Pr}\left[\mathcal{A}^{\mathcal{E}_{K}, \mathcal{D}_{K}}\right.$ forges $]$, and we write

$$
\mathbf{A d v}_{\mathcal{E}}^{\text {auth }}\left(\left(q, q_{f}\right),\left(\sigma, \sigma_{f}\right), t\right)=\max _{\mathcal{A}} \mathbf{A d v}_{\mathcal{E}}^{\text {auth }}(\mathcal{A})
$$

to denote the maximum forging advantage for all adversaries running in time $t$, making $q$ encryption and $q_{f}$ decryption queries with total number of queried blocks being at most $\sigma$ and $\sigma_{f}$, respectively.

Unified Security Notion for AE. The privacy and authenticity advantages can be unified into a single security notion as introduced in [31,53]. Let $\mathcal{A}$ be an adversary that only makes non-repeating queries to $\mathcal{D}_{K}$. Then, we define the AE-advantage of $\mathcal{A}$ against $\mathcal{E}$ as

$$
\operatorname{Adv}_{\mathcal{E}}^{\mathrm{AE}}(\mathcal{A})=\left|\operatorname{Pr}\left[\mathcal{A}^{\mathcal{E}_{K}, \mathcal{D}_{K}}=1\right]-\operatorname{Pr}\left[\mathcal{A}^{\Phi, \perp}=1\right]\right|,
$$

where $\perp$-oracle always returns $\perp$ and $\$$-oracle is as the privacy advantage. We similarly define $\operatorname{Adv}_{\mathcal{E}}^{\mathrm{AE}}\left(\left(q, q_{f}\right),\left(\sigma, \sigma_{f}\right), t\right)=\max _{\mathcal{A}} \mathbf{A d v}_{\mathcal{E}}^{\mathrm{AE}}(\mathcal{A})$, where the maximum is taken over all adversaries running in time $t$, making $q$ encryption and $q_{f}$ decryption queries with the total number of blocks being at most $\sigma$ and $\sigma_{f}$, respectively.

Blockcipher Security. We use a blockcipher $E$ as the underlying primitive, and we assume the security of $E$ as a PRP (pseudorandom permutation). The PRPadvantage of a blockcipher $E$ is defined as $\operatorname{Adv}_{E}^{\operatorname{prp}}(\mathcal{A})=\mid \operatorname{Pr}\left[\mathcal{A}^{E_{K}}=1\right]-\operatorname{Pr}\left[\mathcal{A}^{\mathrm{P}}=\right.$ $1] \mid$, where $P$ is a random permutation uniformly distributed over all permutations over $\{0,1\}^{n}$. We write

$$
\mathbf{A d v}_{E}^{\operatorname{prp}}(q, t)=\max _{\mathcal{A}} \mathbf{A} \mathbf{d v}_{E}^{\operatorname{prp}}(\mathcal{A})
$$

where the maximum is taken over all adversaries running in time $t$ and making $q$ queries. Here, $\sigma$ does not appear as each query has a fixed length.


Fig. 3.1. Different types of feedback modes. We introduce the last feedback mode (called the combined feedback mode) in our construction.

## 3 Idealized Combined Feedback Mode

In this section, we introduce our idealized combined feedback mode. Let $E_{K}$ be the underlying primitive, a blockcipher, with key $K$. Depending on how the next input block of $E_{K}$ is determined from the previous output of $E_{K}$, a plaintext block, or a ciphertext block, we can categorize different types of feedback modes. Some of the feedback modes are illustrated in Fig. 3.1. The first three modes are known as the message feedback mode, ciphertext feedback mode, and output feedback mode, respectively. The examples using the first three modes can be found in the basic encryption schemes [4] or AE schemes [5, 36, 37, 64]. The fourth mode, which uses additional (linear) operation $G: \mathcal{B} \rightarrow \mathcal{B}$, is new. We call it combined feedback. In the combined feedback mode, the next input block $X[i]$ of the underlying primitive $E_{K}$ depends on at least two of the following three values: (i) previous output $E_{K}(X[i-1])$, (ii) plaintext $M[i]$, and (iii) ciphertext $C[i]$. With an appropriate choice of $G$, this feedback mode turns out to be useful for building small and efficient AE schemes. We provide a unified presentation of all types of feedback functions below.

Definition 1 (Feedback Function). A function $\rho: \mathcal{B} \times \mathcal{B} \rightarrow \mathcal{B} \times \mathcal{B}$ is called a feedback function (for an encryption) if there exists a function $\rho^{\prime}: \mathcal{B} \times \mathcal{B} \rightarrow \mathcal{B} \times \mathcal{B}$ (used for decryption) such that

$$
\begin{equation*}
\forall Y, M \in \mathcal{B}, \quad \rho(Y, M)=(X, C) \Rightarrow \rho^{\prime}(Y, C)=(X, M) . \tag{2}
\end{equation*}
$$

$\rho$ is called a plaintext or output feedback if $X$ depends only on $M$ or $Y$, respectively (e.g., the first and third mode in Fig. 3.1). Similarly, it is called ciphertext feedback if $X$ depends only on $C$ in the function $\rho^{\prime}$ (e.g., the second mode in Fig. 3.1). All other feedback functions are called combined feedback.

The condition stated in Eq. (2) is sufficient for inverting the feedback computation from the ciphertext. Given the previous output block $Y=E_{K}(X[i-1])$ and


Fig. 3.2. iCOFB: It is based on a tweakable random function $\mathrm{R}_{N, A,(a, b)}$ and a feedback function $\rho$. The diagram shows how the tag and ciphertext computed for a three complete blocks message.
a ciphertext block $C=C[i-1]$, we are able to compute $(X, M)=(X[i], M[i])$ by using $\rho^{\prime}(Y, C)$.

In particular, when $G$ is not the zero function nor the identity function, the combined feedback mode using this $G$ is not reduced to the remaining three modes. It can be described as $\rho(Y, M)=(X, C)=(G(Y) \oplus M, Y \oplus M)$.

## 3.1 iCOFB Construction

The idealized version of our construction is described in Fig. 3.3 and illustrated in Fig. 3.2. Here we idealize in many ways from a real implementable AE construction. This is a simple warm up for the sake of simplicity and to understand the basic structure of our main construction. In the following construction, we assume that the last message block is a complete block. In other words, all messages are elements of $\mathcal{B}^{+} \stackrel{\text { def }}{=} \cup_{i \geq 1} \mathcal{B}^{i}$. We denote the set of all non negative integers as $\mathbb{Z}_{\geq 0}$. We also consider a tweakable random function R which takes tweak $(N, A, i, j) \in \mathcal{N} \times\{0,1\}^{*} \times \mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0}$ where $N$ is called a nonce chosen from a nonce space $\mathcal{N}, A$ is associated data, and the pair of non negative integers $(i, j)$ is called a position-tweak.

### 3.2 The feedback function $\rho$

The function $\rho: \mathcal{B} \times \mathcal{B} \rightarrow \mathcal{B} \times \mathcal{B}$ in the encryption algorithm is called a feedback function. The function $\rho$ should be chosen in a way such that there exists a function $\rho^{\prime}$ (as used in the decryption algorithm) for which decryption algorithm correctly decrypts. In other words, we need an appropriate condition on $\rho$ for the correctness of the encryption algorithm. A necessary and sufficient condition for $\rho: \mathcal{B} \times \mathcal{B} \rightarrow \mathcal{B} \times \mathcal{B}$ is the following: there exists a function $\rho^{\prime}: \mathcal{B} \times \mathcal{B} \rightarrow \mathcal{B} \times \mathcal{B}$ such that Eq. (2) holds.

It is easy to see that for such a function $\rho$, the decryption algorithm correctly decrypts a ciphertext. If we closely look into the correctness property, what we need that given $(Y, C)$, the value of $M$ should be uniquely computable. Once $M$

```
Algorithm iCOFB-E \((N, A, M)\)
    \((M[1], M[2], \ldots, M[m]) \stackrel{n}{\leftarrow} M\)
    \(t[0] \leftarrow(0,0)\)
    \(Y[0] \leftarrow \mathrm{R}_{N, A, t[0]}\left(0^{n}\right)\)
    for \(i=1\) to \(m\)
    if \(i<m\) then \(t[i] \leftarrow(i, 0)\)
    else \(t[m] \leftarrow(m, 1)\)
        \((X[i], C[i]) \leftarrow \rho(Y[i-1], M[i])\)
        \(Y[i] \leftarrow \mathrm{R}_{N, A, t[i]}(X[i])\)
    \(C \leftarrow(C[1], \ldots, C[m])\)
    \(T \leftarrow Y[m]\)
    return \((C, T)\)
```


## Algorithm iCOFB-D $(N, A, C, T)$

1. $(C[1], C[2], \ldots, C[c]) \stackrel{n}{\leftarrow} C$
$t[0] \leftarrow(0,0)$
$Y[0] \leftarrow \mathrm{R}_{N, A, t[0]}\left(0^{n}\right)$
for $i=1$ to $c$
if $i<c$ then $t[i] \leftarrow(i, 0)$
else $t[c] \leftarrow(c, 1)$
$(X[i], M[i]) \leftarrow \rho^{\prime}(Y[i-1], C[i])$
$Y[i] \leftarrow \mathrm{R}_{N, A, t[i]}(X[i])$
$M \leftarrow(M[1], \ldots, M[c])$
2. if $T=Y[c]$ then return $M$
3. else return $\perp$

Fig. 3.3. Encryption and decryption algorithms of iCOFB AE-mode. Here $M \in \mathcal{B}^{m}$, $C \in \mathcal{B}^{c}$ for some $m, c \geq 1$ and $\rho, \rho^{\prime}: \mathcal{B}^{2} \rightarrow \mathcal{B}^{2}$. The choices of these functions are described in Sect. 3.2.
is computed, $X$ can be computed by applying $\rho$ again. In this paper, we require very lightweight function, e.g. linear function, on the choice of $\rho$. If $\rho$ is a linear function then we can express $\rho$ by a $2 n \times 2 n$ binary matrix

$$
\left(\begin{array}{ll}
E_{1,1} & E_{1,2} \\
E_{2,1} & E_{2,2}
\end{array}\right)
$$

where $E_{i, j}$ 's are $n \times n$ binary matrices and the line 7 in the encryption algorithm of Fig. 3.3 becomes

$$
\begin{aligned}
& X[i]=E_{1,1} \cdot Y[i-1]+E_{1,2} \cdot M[i], \\
& C[i]=E_{2,1} \cdot Y[i-1]+E_{2,2} \cdot M[i] .
\end{aligned}
$$

We have the following lemma.
Lemma 1. If $\rho$ is a linear function satisfying Eq. (2), then $E_{2,2}$ must be invertible.

Proof. If not, then there exist $M \neq M^{\prime}$ with $E_{2,2} \cdot M=E_{2,2} \cdot M^{\prime}$. Then, for any $Y, \rho(Y, M)=(X, C)$ and $\rho\left(Y, M^{\prime}\right)=\left(X^{\prime}, C\right)$. However, $\rho^{\prime}(Y, C)$ cannot be both $(X, M)$ and $\left(X^{\prime}, M^{\prime}\right)$.

Let $\rho$ be a linear feedback function satisfying Eq. (2) (equivalently $E_{2,2}$ is invertible with the above matrix representation). Then, $\rho^{\prime}$ can be chosen to be a linear function defined as follows:

$$
\begin{aligned}
\left(E_{1,1}+E_{1,2} E_{2,2}^{-1} E_{2,1}\right) \cdot Y[i-1]+E_{1,2} \cdot C[i] & =X[i] \\
E_{2,2}^{-1} E_{2,1} \cdot Y[i-1]+E_{2,2}^{-1} \cdot C[i] & =M[i] .
\end{aligned}
$$

We also express the above system of linear equations as

$$
\left(\begin{array}{ll}
D_{1,1} & D_{1,2} \\
D_{2,1} & D_{2,2}
\end{array}\right) \cdot\binom{Y[i-1]}{C[i]}=\binom{X[i]}{M[i]}
$$

where $D_{i, j}$ 's are $n \times n$ matrix determined from the above linear equations. In particular, $D_{1,1}=\left(E_{1,1}+E_{1,2} E_{2,2}^{-1} E_{2,1}\right), D_{1,2}=E_{1,2}, D_{2,1}=E_{2,2}^{-1} E_{2,1}$ and $D_{2,2}=E_{2,2}^{-1}$. Throughout the paper we assume that $E_{2,2}$ is invertible.

Let $\boldsymbol{I}$ and $\boldsymbol{O}$ denote the identity matrix and zero matrix, respectively, of size $n$. We have seen that in all types of feedback modes, we define the ciphertext block $C$ as $M \oplus Y$. They differ how the next input block $X$ is defined. Let $\rho_{\mathrm{OFB}}$, $\rho_{\text {CFB }}, \rho_{\text {PFB }}$ denote the feedback functions for output, ciphertext and plaintext feedback mode respectively. Then, we have

$$
\rho_{\mathrm{OFB}}=\left(\begin{array}{cc}
\boldsymbol{I} & \boldsymbol{O} \\
\boldsymbol{I} & \boldsymbol{I}
\end{array}\right), \quad \rho_{\mathrm{CFB}}=\left(\begin{array}{cc}
\boldsymbol{I} & \boldsymbol{I} \\
\boldsymbol{I} & \boldsymbol{I}
\end{array}\right), \quad \rho_{\mathrm{PFB}}=\left(\begin{array}{cc}
\boldsymbol{O} & \boldsymbol{I} \\
\boldsymbol{I} & \boldsymbol{I}
\end{array}\right) .
$$

In this paper, we consider combined feedback function. By combined feedback, we mean that the following four matrices $E_{1,1}, E_{1,2}, D_{1,1}$ and $D_{1,2}$ are nonzero. Note that these matrices represent the effect of output vector and plaintext or ciphertext block to the next input block. In this paper we fix our choice of $\rho$ (and $\rho^{\prime}$ ) as

$$
\rho=\left(\begin{array}{cc}
\boldsymbol{G} & \boldsymbol{I} \\
\boldsymbol{I} & \boldsymbol{I}
\end{array}\right), \quad \rho^{\prime}=\left(\begin{array}{cc}
\boldsymbol{I}+\boldsymbol{G} & \boldsymbol{I} \\
\boldsymbol{I} & \boldsymbol{I}
\end{array}\right)
$$

where $\boldsymbol{G}$ is an invertible matrix such that $\boldsymbol{I}+\boldsymbol{G}$ is also invertible (see Fig. 3.1). We will specify one choice of $\boldsymbol{G}$ later.

### 3.3 Security Analysis of the Idealized Construction

In this section we provide the security analysis of the idealized construction. Now we prove that under a very minimal assumption on $\rho$, the idealized version has perfect privacy and authenticity with negligible advantage. We say that a linear feedback function $\rho$ is valid (which is true for our choice of the feedback function) if
( $\mathbf{P 1}$ ) $E_{2,1}$ is invertible, (A1) $D_{1,2}$ is invertible and (A2) $D_{1,1}$ is invertible,
where ( $\mathbf{P 1}$ ) is needed for the privacy notion and (A1) and (A2) are needed for the authenticity notion. Here, A1 implies that for any two $C \neq C^{\prime}$ and for any $Y, D_{1,1} \cdot Y+D_{1,2} \cdot C \neq D_{1,1} \cdot Y+D_{1,2} \cdot C^{\prime}$. Note that we assume that $E_{2,2}^{-1}$ is invertible for correctness. Thus, A2 means that the $2 n \times 2 n$ feedback matrix for $\rho$ is also invertible. Another important implication of A2 is the following:

$$
\operatorname{Pr}\left[Y \stackrel{\S}{\leftarrow} \mathcal{B}: D_{1,1} \cdot Y+D_{1,2} \cdot C=X\right]=2^{-n}, \forall(C, X) \in \mathcal{B}^{2} .
$$

We have the following theorem.
Theorem 1. If $\rho$ is valid then for adversary $\mathcal{A}$ making q encryption queries and $q_{f}$ forging attempts having at most $\ell_{f}$ many blocks, we have

$$
\operatorname{Adv}_{\mathrm{iCOFB}}^{\text {priv }}(\mathcal{A})=0, \quad \operatorname{Adv}_{\mathrm{iCOFB}}^{\text {auth }}(\mathcal{A}) \leq \frac{q_{f}\left(\ell_{f}+1\right)}{2^{n}}
$$

Proof. We consider an adversary $\mathcal{A}$ which make $q$ nonce-respecting encryption queries $\left(A_{i}, N_{i}, M_{i}\right)$ and receives $\left(C_{i}, T_{i}\right), 1 \leq i \leq q$, and makes $q_{f}$ decryption queries $\left(N_{i}^{*}, A_{i}^{*}, C_{i}^{*}, T_{i}^{*}\right), 1 \leq i \leq q_{f}$. The intermediate variable $Z$ appeared in the both encryption and decryption algorithms are represented by $Z_{i}[j]$ for the $j$-th computation of the $i$-th query, where $Z$ can be $A, M, C, X, Y$ and $t$ (recall that $t$ is a position-tweak). Note that $T_{i}, T_{i}^{*}$ and $N_{i}$ 's are single blocks.

Perfect Privacy. We prove the perfect privacy under the assumption that $E_{2,1}$ is invertible (i.e. P1). To show perfect privacy, it would be sufficient to show that $C_{1}, \ldots, C_{q}$ are uniformly and independently distributed and this would be true provided $Y_{1}, \ldots, Y_{q}$ are uniformly and independently distributed (due to P1 which says that keeping all other fixed, influence from $Y_{i}[j]$ to $C_{i}[j]$ is bijective). Note that $Y_{i}[j]=\mathrm{R}_{N_{i}, A_{i}, t_{i}[j]}\left(X_{i}[j]\right)$. We know that a tweakable random function returns a random string if the input concatenated with the tweak is fresh. So it is sufficient to show that for all $i, j,\left(N_{i}, A_{i}, t_{i}[j], X_{i}[j]\right)$ is fresh. But this is easy to see as $\mathcal{A}$ is a nonce-respecting adversary and for any $i$, the values of $t_{i}[j]$ 's are distinct and hence $\left(N_{i}, t_{i}[j]\right)$ 's are distinct for all $(i, j)$.

Authenticity Advantage. We prove it in different cases of forging attempt.
$\operatorname{Case}\left(N^{*}, A^{*}\right)=\left(N_{i}, A_{i}\right)$.
W.o.l.g. we assume that $i=1$. Let $p$ be the length of the largest common prefix of $\left(\left(C_{1}[1], t_{1}[1]\right), \ldots,\left(C_{1}\left[m_{1}\right], t_{1}\left[m_{1}\right]\right)\right)$ and $\left(\left(C^{*}[1], t^{*}[1]\right), \ldots,\left(C^{*}\left[m^{*}\right], t^{*}\left[m^{*}\right]\right)\right)$. From the definition of tweak $t[\cdot]$, it is easy to see that $p<\min \left\{m_{1}, m^{*}\right\}$. So, we have

$$
Y_{1}[p]=Y^{*}[p], \quad\left(C_{1}[p+1], t_{1}[p+1]\right) \neq\left(C^{*}[p+1], t^{*}[p+1]\right)
$$

Claim. $\left(N^{*}, A^{*}, t^{*}[p+1], X^{*}[p+1]\right)$ is fresh among all tweaked inputs.
For the time being let us assume that this claim is true. So $Y^{*}[p+1]$ is uniformly distributed given the values obtained so far. By A2 condition, the probability of the next input also remains fresh with probability at least $\left(1-2^{-n}\right)$. We can continue this until the last tweaked input and so the last tweaked input remains fresh with probability at least $1-\frac{m^{*}}{2^{n}}$. So the forging probability is at most $\frac{\left(m^{*}+1\right)}{2^{n}}$ for a single attempt.
Case $\left(N^{*}, A^{*}\right) \neq\left(N_{i}, A_{i}\right)$ for all $i$.
In this case the first tweaked input $\left(N^{*}, A^{*}, t^{*}[0], 0^{n}\right)$ is fresh. We can similarly apply the previous argument to claim that the last tweaked input remains fresh with probability at least $1-\frac{m^{*}}{2^{n}}$. So in this case also, the forging probability is at most $\frac{\left(m^{*}+1\right)}{2^{n}}$.

In the case of $q_{f}$ forging attempts, the success probability is at most $\frac{q_{f}\left(\ell_{f}+1\right)}{2^{n}}$ (from definition, $m^{*} \leq \ell_{f}$ ). This completes the proof, and it remains to show the proof of the claim.

Proof (of Claim). We prove this in two sub-cases. We first note that for all $i \neq$ $p+1, t_{1}[i] \neq t^{*}[p+1]$ and so it would be sufficient to show that $\left(t_{1}[p+1], X_{1}[p+\right.$ 1]) $\neq\left(t^{*}[p+1], X^{*}[p+1]\right)$. If $C_{1}[p+1]=C^{*}[p+1]$ then $X_{1}[p+1]=X^{*}[p+1]$ but $t_{1}[p+1] \neq t^{*}[p+1]$. Similarly, when $C_{1}[p+1] \neq C^{*}[p+1]$, by A1 condition, the next tweaked inputs are distinct.

Remark 1. We would like to note the one of key argument in the proof. It says that whenever we obtain a fresh tweaked input, with high probability the last tweaked input remains fresh. So it would be sufficient to identify a position in which the tweaked input for the forging attempt is fresh with high probability. In the above proof for the idealized version, the position is $(p+1)$ and for this position, tweaked input is fresh with probability one. For our main construction, the freshness occurs with high probability instead of probability one. However, the position will be determined in exactly the same way as we did here.

Now we see that P1 and A1 are also necessary. For example, if P1 is not satisfied then we find a nonzero block $d$ such that, $d^{t r} \cdot E_{2,1}=0^{n}$ where $d^{t r}$ denotes the transposition of the vector. Then, for any $Y, d^{t r} \cdot E_{2,2} \cdot M=d^{t r} \cdot C$ where $C=E_{2,1} \cdot Y+E_{2,2} \cdot M$. This observation can be used as a privacy distinguisher.

Similarly if A1 is not satisfied then $D_{1,2}$ is not invertible. So there exists a nonzero $d$ such that $D_{1,2} \cdot d=0^{n}$. Thus, $D_{1,1} \cdot Y+D_{1,2} \cdot C^{*}=D_{1,1} \cdot Y+D_{1,2} \cdot C$ where $C^{*}=C+d$. This observation can be extended to an authenticity attack.

## 4 COFB: a Small-State, Rate-1, Inverse-Free AE Mode

In this section, we present our proposal, COFB, which has rate- 1 (i.e. needs one blockcipher call for one input block), and is inverse-free, i.e., it does not need a blockcipher inverse (decryption). In addition to these features, this mode has a quite small state size, namely $1.5 n+k$ bits, in case the underlying blockcipher has an $n$-bit block and $k$-bit keys. We first specify the basic building blocks and parameters used in our construction.

### 4.1 Specification

Key and Blockcipher. The underlying cryptographic primitive is an $n$-bit blockcipher, $E_{K}$. We assume that $n$ is a multiple of 4 . The key of the scheme is the key of the blockcipher, i.e. $K$.

Masking Function. We define the masking function mask : $\{0,1\}^{n / 2} \times \mathbb{N}^{2} \rightarrow$ $\{0,1\}^{n / 2}$ as follows:

$$
\begin{equation*}
\operatorname{mask}(\Delta, a, b)=\alpha^{a} \cdot(1+\alpha)^{b} \cdot \Delta \tag{3}
\end{equation*}
$$

We may write mask $\Delta(a, b)$ to mean mask $(\Delta, a, b)$. Here, $\cdot$ denotes the multiplication over $\operatorname{GF}\left(2^{n / 2}\right)$, and $\alpha$ denotes the primitive element of the field. For the



Fig. 4.1. Encryption of COFB for 3-block associated data and plaintext.
primitive polynomial defining the field, we choose the lexicographically first one, that is, $p(x)=\mathrm{x}^{64}+\mathrm{x}^{4}+\mathrm{x}^{3}+\mathrm{x}+1$ following [6,35]. Rogaway [51] showed that for all $(a, b) \in\left\{0, \ldots, 2^{51}\right\} \times\left\{0, \ldots, 2^{10}\right\}$, the values of $\alpha^{a} \cdot(1+\alpha)^{b}$ are distinct. If we follow the notations of [51], the right hand side of Eq. (3) could be written as $2^{a} 3^{b} \Delta$. For other values of $n$, we need to identify the primitive element $\alpha$ of the primitive polynomial and an integer $L$ such that $\alpha^{a} \cdot(1+\alpha)^{b}$ are distinct for all $(a, b) \in\{0, \ldots, L\} \times\{0, \ldots, 4\}$. Then the total allowed size of a message and associated data would be at most $n L$ bits. We need this condition to prove the security claim. In particular, we have the following properties of the masking function.

Lemma 2. For any $(a, b) \neq\left(a^{\prime}, b^{\prime}\right)$ chosen from the set $\{0, \ldots, L\} \times\{0, \ldots, 4\}$ (as described above), $c \in\{0,1\}^{n / 2}$ and a random $n / 2$ bit string $\Delta$, we have
$\operatorname{Pr}\left[\operatorname{mask}_{\Delta}(a, b) \oplus \operatorname{mask}_{\Delta}\left(a^{\prime}, b^{\prime}\right)=c\right]=\frac{1}{2^{n / 2}}$, and $\operatorname{Pr}\left[\operatorname{mask}_{\Delta}(a, b)=c\right]=\frac{1}{2^{n / 2}}$.
Proof of the first equation trivially follows from the fact that $\alpha^{a} \cdot(1+\alpha)^{b}$ are distinct for all $(a, b) \in\{0, \ldots, L\} \times\{0, \ldots, 4\}$.

Similar masking functions are frequently used in other modes, such as $[9$, $45,51]$, however, the masks are full $n$ bits. The use of $n$-bit masking function
usually allows to redefine the AE scheme as a mode of XE or XEX tweakable blockcipher [51], which significantly reduces the proof complexity. In our case, to reduce the state size, we decided to use the $n / 2$-bit masking function, and as a result the proof is ad-hoc and does not rely on XE or XEX.

Feedback Function. Let $Y \in\{0,1\}^{n}$ and $(Y[1], Y[2], Y[3], Y[4]) \stackrel{n / 4}{\longleftarrow} Y$, where $Y[i] \in\{0,1\}^{n / 4}$. We define $G: \mathcal{B} \rightarrow \mathcal{B}$ as $G(Y)=(Y[2], Y[3], Y[4], Y[4] \oplus Y[1])$. We also view $G$ as the $n \times n$ non-singular matrix, so we write $G(Y)$ and $G \cdot Y$ interchangeably. For $M \in \mathcal{B} \leq$ and $Y \in \mathcal{B}$, we define $\rho_{1}(Y, M)=G \cdot Y \oplus \bar{M}$. The feedback function $\rho$ and its corresponding $\rho^{\prime}$ are defined as

$$
\begin{aligned}
\rho(Y, M) & =\left(\rho_{1}(Y, M), Y \underline{\oplus} M\right) \\
\rho^{\prime}(Y, C) & =\left(\rho_{1}(Y, Y \underline{\oplus} C), Y \underline{\oplus} C\right) .
\end{aligned}
$$

Note that when $(X, M)=\rho^{\prime}(Y, C)$ then $X=(G \oplus I) \cdot Y \oplus C$. Our choice of $G$ ensures that $I \oplus G$ is also invertible matrix. So when $Y$ is chosen randomly for both computations of $X$ (through $\rho$ and $\rho^{\prime}$ ), $X$ also behaves randomly. We need this property when we bound probability of bad events later.

Tweak Value for The Last Block. Given $B \in\{0,1\}^{*}$, we define $\delta_{B} \in\{1,2\}$ as follows:

$$
\delta_{B}= \begin{cases}1 & \text { if } B \neq \lambda \text { and } n \text { divides }|B|  \tag{4}\\ 2 & \text { otherwise }\end{cases}
$$

This will be used to differentiate the cases that the last block of $B$ is $n$ bits or shorter, for $B$ being associated data or plaintext or ciphertext. We also define a formatting function Fmt for a pair of bit strings $(A, Z)$, where $A$ is associated data and $Z$ could be either a plaintext or a ciphertext. Let $(A[1], \ldots, A[a]) \stackrel{n}{\leftarrow} A$ and $(Z[1], \ldots, Z[z]) \stackrel{n}{\leftarrow} Z$. We define $\mathrm{t}[i]$ as follows:

$$
\mathrm{t}[i]= \begin{cases}(i, 0) & \text { if } i<a \\ \left(a-1, \delta_{A}\right) & \text { if } i=a \\ \left(i-1, \delta_{A}\right) & \text { if } a<i<a+z \\ \left(a+z-2, \delta_{A}+\delta_{Z}\right) & \text { if } i=a+z\end{cases}
$$

Now, the formatting function $\operatorname{Fmt}(A, Z)$ returns the following sequence:

$$
((A[1], \mathrm{t}[1]), \ldots,(\overline{A[a]}, \mathrm{t}[a]),(Z[1], \mathrm{t}[a+1]), \ldots,(\overline{Z[z]}, \mathrm{t}[a+z]))
$$

where the first coordinate of each pair specifies the input block to be processed, and the second coordinate specifies the exponents of $\alpha$ and $1+\alpha$ to determine the constant over $\operatorname{GF}\left(2^{n / 2}\right)$. Let $\mathbb{Z}_{>0}$ be the set of non-negative integers and $\mathcal{X}$ be some non-empty set. We say that a function $f: \mathcal{X} \rightarrow\left(\mathcal{B} \times \mathbb{Z}_{\geq 0} \times \mathbb{Z}_{\geq 0}\right)^{+}$is prefix-free if for all $X \neq X^{\prime}, f(X)=(Y[1], \ldots, Y[\ell])$ is not a prefix of $f\left(X^{\prime}\right)=$ $\left(Y^{\prime}[1], \ldots, Y^{\prime}\left[\ell^{\prime}\right]\right)$ (in other words, $\left.(Y[1], \ldots, Y[\ell]) \neq\left(Y^{\prime}[1], \ldots, Y^{\prime}[\ell]\right)\right)$. Here, for a set $\mathcal{S}, \mathcal{S}^{+}$means $\mathcal{S} \cup \mathcal{S}^{2} \cup \cdots$, and we have the following lemma.

```
Algorithm Mask-Gen \((K, N)\)
    . \(Y[0] \leftarrow E_{K}\left(0^{n / 2} \| N\right)\)
    \(\left(Y^{1}[0], \ldots, Y^{4}[0]\right) \stackrel{n / 4}{\longleftarrow} Y[0]\)
    \(\Delta \leftarrow Y^{2}[0] \| Y^{3}[0]\)
    return \((\Delta, Y[0])\)
Algorithm COFB- \(\mathcal{E}_{K}(N, A, M)\)
    \((\Delta, Y[0]) \leftarrow \operatorname{Mask}-\operatorname{Gen}(K, N)\)
    \((A[1], \ldots, A[a]) \leftarrow^{n} A\)
    \((M[1], \ldots, M[m]) \stackrel{n}{\leftarrow} M\)
    for \(i=1\) to \(a-1\)
        \(\Delta \leftarrow 2 \Delta\)
        \(X[i] \leftarrow(A[i] \oplus G \cdot Y[i-1]) \bar{\oplus} \Delta\)
        \(Y[i] \leftarrow E_{K}(X[i])\)
if \(|A[a]|=n\) then \(\Delta \leftarrow 3 \Delta\)
else \(\Delta \leftarrow 3^{2} \Delta\)
\(X[a] \leftarrow(\overline{A[a]} \oplus G \cdot Y[a-1]) \bar{\oplus} \Delta\)
\(Y[a] \leftarrow E_{K}(X[a])\)
for \(i=1\) to \(m-1\)
    \(X[i+a] \leftarrow(M[i] \oplus G \cdot Y[i+a-1]) \oplus \Delta\)
    \(Y[i+a] \leftarrow E_{K}(X[i+a])\)
    \(C[i] \leftarrow Y[i+a-1] \oplus M[i]\)
    if \(i<m-1\) then \(\Delta \leftarrow 2 \Delta\)
if \(|M[m]|=n\) then \(\Delta \leftarrow 3 \Delta\)
else \(\Delta \leftarrow 3^{2} \Delta\)
\(X[a+m] \leftarrow(\overline{M[m]} \oplus G \cdot Y[a+m-1]) \bar{\oplus} \Delta\)
\(C[m] \leftarrow Y[a+m-1] \oplus M[m]\)
\(T \leftarrow E_{K}(X[a+m])\)
return \((C, T)\)
```

Fig. 4.2. The encryption and decryption algorithms of COFB.

Lemma 3. The function $\operatorname{Fmt}(\cdot)$ is prefix-free.
The proof is more or less straightforward and hence we skip it.
We present the specifications of COFB in Fig. 4.2, where $\alpha$ and $(1+\alpha)$ in Eq. (3) are written as 2 and 3. See also Fig. 4.1. The encryption and decryption algorithms are denoted by COFB- $\mathcal{E}_{K}$ and COFB- $\mathcal{D}_{K}$. We remark that the nonce length is $n / 2$ bits, which is enough for the security up to the birthday bound. The nonce is processed as $E_{K}\left(0^{n / 2} \| N\right)$ to yield the first internal chaining value. The encryption algorithm takes non-empty $A$ and non-empty $M$, and outputs $C$ and $T$ such that $|C|=|M|$ and $|T|=n$. The decryption algorithm takes ( $N, A, C, T$ ) with $|A|,|C| \neq 0$ and outputs $M$ or $\perp$. Note that some of building blocks described above are not presented in Fig. 4.2, since they are introduced for the proof. An equivalent presentation using them is presented in Fig. 5.1.

## 5 Security of COFB

We present the security analysis of COFB in Theorem 2. Before going to the proof, as mentioned earlier, we would like to mention that we use the function

Fmt and Lemma 3 in the proof to make it easy to understand. We would also like to mention that, we instantiate iCOFB with COFB by choosing

$$
\mathrm{R}_{N, A,(i, j)}(X)= \begin{cases}f(N, A) & \text { if } i=0, j=0 \\ E_{K}\left(X \oplus \operatorname{mask}_{\Delta}\left(a+i-1, \delta_{A}\right)\right) & \text { if } i<m, j=0 \\ E_{K}\left(X \oplus \operatorname{mask}_{\Delta}\left(a+m-2, \delta_{A}+\delta_{M}\right)\right) & \text { if } i=m, j=1\end{cases}
$$

where $f(N, A)$ is the function that simulates the associated data phase and outputs $Y[a]$ (Line $1-11$, Fig. 4.2, $K$ is implicit and chosen uniformly from the key space and $X=0^{n}$ in this case). $\Delta$ (computed using $E_{K}$ and $N$ ), $a, m, \delta_{A}$ and $\delta_{M}$ are described as in the previous section, and we instantiate $\rho$ by the feedback function described in the previous section. However, the security proof of COFB does not follow from that of iCOFB, since as a tweakable PRF, the security of R is only guaranteed up to $n / 4$ bits, and thus we cannot rely on the hybrid argument to show the security of COFB. We next proceed with our proof for our instantiation.

## Theorem 2 (Main Theorem).

$$
\begin{gathered}
\mathbf{A d v}_{\mathrm{COFB}}^{\mathrm{AE}}\left(\left(q, q_{f}\right),\left(\sigma, \sigma_{f}\right), t\right) \leq \mathbf{A d v}_{\mathrm{AES}}^{\mathrm{prp}}\left(q^{\prime}, t^{\prime}\right)+\frac{0.5\left(q^{\prime}\right)^{2}}{2^{n}}+\frac{4 \sigma+0.5 n q_{f}}{2^{n / 2}} \\
+\frac{q_{f}+\left(q+\sigma+\sigma_{f}\right) \cdot \sigma_{f}}{2^{n}},
\end{gathered}
$$

where $q^{\prime}=q+q_{f}+\sigma+\sigma_{f}$, which corresponds to the total number of blockcipher calls through the game, and $t^{\prime}=t+O\left(q^{\prime}\right)$.

Proof. Without loss of generality, we can assume $q^{\prime} \leq 2^{\frac{n}{2}-1}$, since otherwise the bound obviously holds as the right hand side becomes more than one. The first transition we make is to use an $n$-bit (uniform) random permutation P instead of $E_{K}$, and then to use an $n$-bit (uniform) random function R instead of P . This twostep transition requires the first two terms of our bound, from the standard PRPPRF switching lemma and from the computation to the information security reduction (e.g., see [17]). Then what we need is a bound for COFB using R, denoted by COFB-R. That is, we prove

$$
\begin{equation*}
\mathbf{A d v}_{\mathrm{COFB}-\mathrm{R}}^{\mathrm{AE}}\left(\left(q, q_{f}\right),\left(\sigma, \sigma_{f}\right), \infty\right) \leq \frac{4 \sigma+0.5 n q_{f}}{2^{n / 2}}+\frac{q_{f}+\left(q+\sigma+\sigma_{f}\right) \cdot \sigma_{f}}{2^{n}} \tag{5}
\end{equation*}
$$

For $i=1, \ldots, q$, we write $\left(N_{i}, A_{i}, M_{i}\right)$ and $\left(C_{i}, T_{i}\right)$ to denote the $i$-th encryption query and response. Here, $A_{i}=\left(A_{i}[1], \ldots, A_{i}\left[a_{i}\right]\right), M_{i}=\left(M_{i}[1], \ldots, M_{i}\left[m_{i}\right]\right)$, and $C_{i}=\left(C_{i}[1], \ldots, C_{i}\left[m_{i}\right]\right)$. Let $\ell_{i}=a_{i}+m_{i}$, which denotes the total input block length for the $i$-th encryption query. We write $X_{i}[j]$ (resp. $Y_{i}[j]$ ) for $i=1, \ldots, q$ and $j=0, \ldots, \ell_{i}$ to denote the $j$-th input (resp. output) of the internal R invoked at the $i$-th encryption query, where the order of invocation follows the specification shown in Fig. 4.2. We remark that $X_{i}[0]=0^{n / 2} \| N_{i}$ and $Y_{i}\left[\ell_{i}\right]=T_{i}$ for all $i=1, \ldots, q$. Similarly, we write $\Delta_{i}$ to denote $Y_{i}^{2}[0] \| Y_{i}^{3}[0]$ where $Y_{i}^{1}[0]\|\cdots\| Y_{i}^{4}[0] \stackrel{n / 4}{{ }^{\frac{1}{4}}} Y_{i}[0]$.

We introduce the following relaxations in the game, which only gain the advantage. First, after completing all queries and forging attempts (i.e. decryption queries), let the adversary learn all the $Y$-values for all encryption queries only. We remark that any $X$-values computed at the message processing phase (not the AD processing phase) of the $i$-th encryption query are immediately determined by the $i$-th query-response tuple, $\left(N_{i}, A_{i}, M_{i}, C_{i}, T_{i}\right)$ and $Y_{i}$ values from the property of feedback function, and $\Delta$-values (it is a part of $Y[0]$ ).

In case of the ideal oracle, all these variables corresponding to $Y$ will be chosen uniformly and independently, where at the plaintext encryption phase $Y_{i}[j]$ is randomly chosen and used to determine $C_{i}[j]$ as $C_{i}[j]=Y_{i}[j-1] \oplus M_{i}[j]$, and at AD processing phase it is a dummy and has no influence to the response $\left(C_{i}, T_{i}\right)$. For decryption queries, the ideal oracle always returns $\perp$ (here we assume that the adversary makes only fresh queries).

Coefficients-H Technique. We outline the Coefficients-H technique developed by Patarin, which serves as a convenient tool for bounding the advantage (see $[49,60]$ ). We will use this technique (without giving a proof) to prove our main theorem. Consider two oracles $\mathcal{O}_{0}=(\$, \perp)$ (the ideal oracle for the relaxed game) and $\mathcal{O}_{1}$ (real, i.e. our construction in the same relaxed game). Let $\mathcal{V}$ denote the set of all possible views an adversary can obtain. For any view $\tau \in \mathcal{V}$, we will denote the probability to realize the view as $\mathrm{ip}_{\text {real }}(\tau)$ (or $\mathrm{ip}_{\text {ideal }}(\tau)$ ) when it is interacting with the real (or ideal respectively) oracle. We call these interpolation probabilities. Without loss of generality, we assume that the adversary is deterministic and fixed. Then, the probability space for the interpolation probabilities is uniquely determined by the underlying oracle. As we deal with stateless oracles, these probabilities are independent of the order of query responses in the view. Suppose we have a set of views, $\mathcal{V}_{\text {good }} \subseteq \mathcal{V}$, which we call good views, and the following conditions hold:

1. In the game involving the ideal oracle $\mathcal{O}_{0}$ (and the fixed adversary), the probability of getting a view in $\mathcal{V}_{\text {good }}$ is at least $1-\epsilon_{1}$.
2. For any view $\tau \in \mathcal{V}_{\text {good }}$, we have $\mathrm{ip}_{\text {real }}(\tau) \geq\left(1-\epsilon_{2}\right) \cdot \mathrm{ip}_{\text {ideal }}(\tau)$.

Then we have $\left|\operatorname{Pr}\left[\mathcal{A}^{\mathcal{O}_{0}}=1\right]-\operatorname{Pr}\left[\mathcal{A}^{\mathcal{O}_{1}}=1\right]\right| \leq \epsilon_{1}+\epsilon_{2}$. The proof can be found at (say) [60]. Now we proceed with the proof of Theorem 2 by defining certain $\mathcal{V}_{\text {good }}$ for our games, and evaluating the bounds, $\epsilon_{1}$ and $\epsilon_{2}$.

Views. In our case, a view $\tau$ is defined by the following tuple:

$$
\tau=\left(\left(N_{i}, A_{i}, M_{i}, Y_{i}\right)_{i \in\{1, \ldots, q\}},\left(N_{i^{\prime}}^{*}, A_{i^{\prime}}^{*}, C_{i^{\prime}}^{*}, T_{i^{\prime}}^{*}, Z_{i^{\prime}}^{*}\right)_{i^{\prime} \in\left\{1, \ldots, q_{f}\right\}}\right)
$$

where $Z_{i^{\prime}}^{*}$ denotes the output of the decryption oracle $\mathcal{D}$ (it is always $\perp$ when we interact with the ideal oracle) for the $i^{\prime}$-th decryption query ( $N_{i^{\prime}}^{*}, A_{i^{\prime}}^{*}, C_{i^{\prime}}^{*}, T_{i^{\prime}}^{*}$ ). Note that $Y_{i}$ denotes $\left(Y_{i}[0], \ldots, Y_{i}\left[\ell_{i}\right]\right)=Y_{i}\left[0 . . \ell_{i}\right]$, where $\ell_{i}=a_{i}+m_{i}$, and $a_{i}$ (resp. $m_{i}$ ) denotes the block length of $A_{i}\left(\right.$ resp. $\left.M_{i}\right)$. Here we implicitly use the fact that given a complete block $M_{i}[j]$, the mapping from $Y_{i}[j]$ to $C_{i}[j]$ is bijective
and hence keeping those $Y_{i}[j]$ values instead of $C_{i}[j]$ is sufficient. Similarly we define $c_{i^{\prime}}^{*}$ and $a_{i^{\prime}}^{*}$, and write $\ell_{i^{\prime}}^{*}=a_{i^{\prime}}^{*}+c_{i^{\prime}}^{*}$.

Let $\left(L_{i}[j], R_{i}[j]\right) \stackrel{n / 2}{\leftrightarrows} X_{i}[j]$ for all $i \in[1 . . q]$ and $j \in\left[1 . . \ell_{i}\right]$. For any $i$, let $p_{i}$ denote the length of the longest common prefix of $\operatorname{Fmt}\left(A_{i}^{*}, C_{i}^{*}\right)$ and $\operatorname{Fmt}\left(A_{j}, C_{j}\right)$ where $N_{j}=N_{i}^{*}$. If there is no such $j$, we define $p_{i}=-1$. Since Fmt is prefix-free, it holds that $p_{i}<\min \left\{\ell_{i}^{*}, \ell_{j}\right\}$. We observe that $p_{i}$ is unique for all $i=1, \ldots, q_{f}$, as there is at most one encryption query that uses the same nonce as $N_{i}^{*}$.

Bad Views. Now we define a bad view. The complement of the set of bad views is defined to be the set of good views. A view is called bad if one of the following events occurs:

B1: $L_{i}[j]=0^{n / 2}$ for some $i \in[1 . . q]$ and $j>0$.
B2: $X_{i}[j]=X_{i^{\prime}}\left[j^{\prime}\right]$ for some $(i, j) \neq\left(i^{\prime}, j^{\prime}\right)$ where $j, j^{\prime}>0$.
B3: $\operatorname{mcoll}(R)>n / 2$, where $R$ is the tuple of all $R_{i}[j]$ values. Recall that $\left(L_{i}[j], R_{i}[j]\right) \stackrel{n / 2}{\rightleftarrows} X_{i}[j]$.
B4: $X_{i}^{*}\left[p_{i}+1\right]=X_{i_{1}}\left[j_{1}\right]$ for some $i, i_{1}, j_{1}$ with $p_{i}$ as defined above. Note that when $p_{i} \geq 0, X_{i}^{*}\left[p_{i}+1\right]$ is determined from the values of $Y$.
B5: For some $Z_{i}^{*} \neq \perp$. This clearly cannot happen for the ideal oracle case.
We add some intuitions on these events. When $\mathbf{B 1}$ does not hold, then $X_{i}[j] \neq$ $X_{i^{\prime}}[0]$ for all $i, i^{\prime}$, and $j>0$. Hence $\Delta_{i}$ will be completely random. When B2 does not hold, then all the inputs for the random function are distinct for encryption queries, which makes the responses from encryption oracle completely random in the "real" game. When B3 does not hold, then at the right half of $X_{i}[j]$ we see at most $n / 2$ multi-collisions. A successful forgery is to choose one of the $n / 2$ multicollision blocks and forge the left part so that the entire block collides. Forging the left part has $2^{-n / 2}$ probability due to randomness of masking. Finally, when $\mathbf{B 4}$ does not hold, then the $\left(p_{i}+1\right)$-st input for the $i$-th forging attempt will be fresh with a high probability and so all the subsequent inputs will remain fresh with a high probability.

A view is called good if none of the above events hold. Let $\mathcal{V}_{\text {good }}$ be the set of all such good views. The following lemma bounds the probability of not realizing a good view while interacting with a random function (this will complete the first condition of the Coefficients-H technique).

## Lemma 4.

$$
\operatorname{Pr}_{\text {ideal }}\left[\tau \notin \mathcal{V}_{\text {good }}\right] \leq \frac{4 \sigma+0.5 n q_{f}}{2^{n / 2}}
$$

Proof (of Lemma 4). Throughout the proof, we assume all probability notations are defined over the ideal game. We bound all the bad events individually and then by using the union bound, we will obtain the final bound. We first develop some more notation. Let $\left(Y_{i}^{1}[j], Y_{i}^{2}[j], Y_{i}^{3}[j], Y_{i}^{4}[j]\right) \stackrel{n / 4}{\longleftarrow} Y_{i}[j]$. Similarly, we denote $\left(M_{i}^{1}[j], M_{i}^{2}[j]\right) \stackrel{n / 2}{\leftrightarrows} M_{i}[j]$.
(1) $\operatorname{Pr}[\mathbf{B} 1] \leq \sigma / 2^{n / 2}$ : We fix a pair of integers $(i, j)$ for some $i \in[1 . . q]$ and $j \in\left[1 . . \ell_{i}\right]$. Now, $L_{i}[j]$ can be expressed as

$$
\left(Y_{i}^{2}[j-1] \| Y_{i}^{3}[j-1]\right) \oplus\left(\alpha^{a} \cdot(1+\alpha)^{b} \cdot \Delta_{i}\right) \oplus M_{i}^{1}[j]
$$

for some $a$ and $b$. Note that when $j>1, \Delta_{i}$ and $Y_{i}[j-1]$ are independently and uniformly distributed, and hence for those $j$, we have $\operatorname{Pr}\left[L_{i}[j]=0^{n / 2}\right]=$ $2^{-n / 2}$ (apply Lemma 2 after conditioning $Y_{i}[j-1]$ ). Now when $j=1$, we have the following three possible choice: (i) $L_{i}[1]=(1+\alpha) \cdot \Delta_{i} \oplus$ Cons if $a_{i} \geq 2$, (ii) $L_{i}[1]=\alpha \cdot \Delta_{i} \oplus$ Cons if $a_{i}=1$ and the associated data block is full, and (iii) $L_{i}[1]=\alpha^{2} \cdot \Delta_{i} \oplus$ Cons if $a_{i}=1$ and the associated data block is not full, for some constant Cons. In all cases by applying Lemma 2, $\operatorname{Pr}[\mathbf{B} 1] \leq \sigma / 2^{n / 2}$.
(2) $\operatorname{Pr}[\mathbf{B 2}] \leq \sigma / 2^{n / 2}$ : For any $(i, j) \neq\left(i^{\prime}, j^{\prime}\right)$ with $j, j^{\prime} \geq 1$, the equality event $X_{i}[j]=X_{i^{\prime}}\left[j^{\prime}\right]$ has a probability at most $2^{-n}$ since this event is a non-trivial linear equation on $Y_{i}[j-1]$ and $Y_{i^{\prime}}\left[j^{\prime}-1\right]$ and they are independent to each other. Note that $\sigma^{2} / 2^{n} \leq \sigma / 2^{n / 2}$ as we are estimating probabilities.
(3) $\operatorname{Pr}[\mathbf{B} 3] \leq 2 \sigma / 2^{n / 2}$ : The event $\mathbf{B 3}$ is a multi-collision event for randomly chosen $\sigma$ many $n / 2$-bit strings as $Y$ values are mapped in a regular manner (see the feedback function) to $R$ values. From the union bound, we have
$\operatorname{Pr}[\mathbf{B} 3] \leq\binom{\sigma}{n / 2} \frac{1}{2^{(n / 2) \cdot((n / 2)-1)}} \leq \frac{\sigma^{n / 2}}{2^{(n / 2) \cdot((n / 2)-1)}} \leq\left(\frac{\sigma}{2^{(n / 2)-1}}\right)^{n / 2} \leq \frac{2 \sigma}{2^{n / 2}}$, where the last inequality follows from the assumption $\left(\sigma \leq 2^{(n / 2)-1}\right)$.
(4) $\operatorname{Pr}\left[\mathbf{B} 4 \wedge \mathbf{B} 1^{c} \wedge \mathbf{B 3}^{c}\right] \leq 0.5 n q_{f} / 2^{n / 2}$ : We fix some $i$ and want to bound the probability $\operatorname{Pr}\left[X_{i}^{*}\left[p_{i}+1\right]=X_{i_{1}}\left[j_{1}\right] \wedge \mathbf{B 1}^{c} \wedge \mathbf{B 3}{ }^{c}\right]$ for some $i_{1}, j_{1}$. If $p_{i}=-1$ (i.e., $N_{i}^{*}$ does not appear in encryption queries), then $N_{i}^{*}$ is fresh as left $n / 2$ bits of all $X_{i}[j]$ is non-zero for all $j>0$ (since we also consider B1 does not hold). So the probability is zero. Now we consider $p_{i} \geq 0$. The event B3 ${ }^{c}$ implies that at most $n / 2$ possible values of $\left(i_{1}, j_{1}\right)$ are possible for which $X_{i}^{*}\left[p_{i}+1\right]=X_{i_{1}}\left[j_{1}\right]$ can hold. Fix any such $\left(i_{1}, j_{1}\right)$. Now it is sufficient to bound the probability for equality for the left $n / 2$ bits. We first consider the case where $j_{1}=p_{i}+1$. Now from the definition of $p_{i}$, $\left(C_{i}^{*}\left[p_{i}+1\right], \mathrm{t}_{i}^{*}\left[p_{i}+1\right]\right) \neq\left(C_{i_{1}}\left[p_{i}+1\right], \mathrm{t}_{i_{1}}\left[p_{i}+1\right]\right)$. If $\mathrm{t}_{i}\left[p_{i}+1\right]=\mathrm{t}_{i_{1}}\left[p_{i}+1\right]$ then the bad event cannot hold with probability one. Otherwise, we obtain a non-trivial linear equation in $\Delta_{i_{1}}$ and apply Lemma 2 , and we also use the fact that $G+I$ is non singular. A similar argument holds for the other choices of $j_{1}$. Therefore, the probability for the atomic case is at most $2^{-n / 2}$, and because we have at most $q_{f} \cdot n / 2$ chances, $\operatorname{Pr}\left[\mathbf{B} 4 \wedge \mathbf{B} 1^{c} \wedge \mathbf{B} 3^{c}\right]$ is at most $(n / 2) \cdot q_{f} \cdot 1 / 2^{n / 2}$.
Summarizing, we have

$$
\begin{aligned}
\operatorname{Pr}_{\text {ideal }}\left[\tau \notin \mathcal{V}_{\text {good }}\right] & \leq \operatorname{Pr}[\mathbf{B} 1]+\operatorname{Pr}[\mathbf{B 2}]+\operatorname{Pr}[\mathbf{B} 3]+\operatorname{Pr}\left[\mathbf{B} 4 \wedge \mathbf{B 1}^{c} \wedge \mathbf{B 3}^{c}\right] \\
& \leq \frac{\sigma}{2^{n / 2}}+\frac{\sigma}{2^{n / 2}}+\frac{2 \sigma}{2^{n / 2}}+\frac{0.5 n q_{f}}{2^{n / 2}}=\frac{4 \sigma+0.5 n q_{f}}{2^{n / 2}}
\end{aligned}
$$

which concludes the proof.

Lower Bound of $\mathrm{ip}_{\text {real }}(\tau)$. We consider the ratio of $\mathrm{ip}_{\text {real }}(\tau)$ and $\mathrm{ip}_{\text {ideal }}(\tau)$. In this paragraph we assume that all the probability space, except for $\mathrm{ip}_{\text {ideal }}(*)$, is defined over the real game. We fix a good view

$$
\tau=\left(\left(N_{i}, A_{i}, M_{i}, Y_{i}\right)_{i \in\{1, \ldots, q\}},\left(N_{i^{\prime}}^{*}, A_{i^{\prime}}^{*}, C_{i^{\prime}}^{*}, T_{i^{\prime}}^{*}, Z_{i^{\prime}}^{*}\right)_{i^{\prime} \in\left\{1, \ldots, q_{f}\right\}}\right),
$$

where $Z_{i^{\prime}}^{*}=\perp$. We separate $\tau$ into

$$
\tau_{e}=\left(N_{i}, A_{i}, M_{i}, Y_{i}\right)_{i \in\{1, \ldots, q\}} \text { and } \tau_{d}=\left(N_{i^{\prime}}^{*}, A_{i^{\prime}}^{*}, C_{i^{\prime}}^{*}, T_{i^{\prime}}^{*}, Z_{i^{\prime}}^{*}\right)_{i^{\prime} \in\left\{1, \ldots, q_{f}\right\}}
$$

and we first see that for a good view $\tau$, ip $_{\text {ideal }}(\tau)$ equals to $1 / 2^{n(q+\sigma)}$.
Now we consider the real case. Since B1 and B2 do not hold with $\tau$, all inputs of the random function inside $\tau_{e}$ are distinct, which implies that the released $Y$-values are independent and uniformly random. The variables in $\tau_{e}$ are uniquely determined given these $Y$-values, and there are exactly $q+\sigma$ distinct input-output of R. Therefore, $\operatorname{Pr}\left[\tau_{e}\right]$ is exactly $2^{-n(q+\sigma)}$.

We next evaluate

$$
\begin{equation*}
\mathrm{ip}_{\text {real }}(\tau)=\operatorname{Pr}\left[\tau_{e}, \tau_{d}\right]=\operatorname{Pr}\left[\tau_{e}\right] \cdot \operatorname{Pr}\left[\tau_{d} \mid \tau_{e}\right]=\frac{1}{2^{n(q+\sigma)}} \cdot \operatorname{Pr}\left[\tau_{d} \mid \tau_{e}\right] . \tag{6}
\end{equation*}
$$

We observe that $\operatorname{Pr}\left[\tau_{d} \mid \tau_{e}\right]$ equals to $\operatorname{Pr}\left[\perp_{\text {all }} \mid \tau_{e}\right]$, where $\perp_{\text {all }}$ denotes the event that $Z_{i}^{*}=\perp$ for all $i=1, \ldots, q_{f}$, as other variables in $\tau_{d}$ are determined by $\tau_{e}$.

Let $\eta$ denote the event that, for all $i=1, \ldots, q_{f}, X_{i}^{*}[j]$ for $p_{i}<j \leq \ell_{i}^{*}$ is not colliding to $X$-values in $\tau_{e}$ and $X_{i}^{*}\left[j^{\prime}\right]$ for all $j^{\prime} \neq j$. For $j=p_{i}+1$, the above condition is fulfilled by $\mathbf{B 4}$, and thus $Y_{i}^{*}\left[p_{i}+1\right]$ is uniformly random, and hence $X_{i}^{*}\left[p_{i}+2\right]$ is also uniformly random, due to the property of feedback function (here, observe that the mask addition between the chain of $Y_{i}^{*}[j]$ to $X_{i}^{*}[j+1]$ does not reduce the randomness).

Now we have $\operatorname{Pr}\left[\perp_{\text {all }} \mid \tau_{e}\right]=1-\operatorname{Pr}\left[\left(\perp_{\text {all }}\right)^{c} \mid \tau_{e}\right]$, and we also have $\operatorname{Pr}\left[\left(\perp_{\text {all }}\right)^{c} \mid \tau_{e}\right]=$ $\operatorname{Pr}\left[\left(\perp_{\text {all }}\right)^{c}, \eta \mid \tau_{e}\right]+\operatorname{Pr}\left[\left(\perp_{\text {all }}\right)^{c}, \eta^{c} \mid \tau_{e}\right]$. Here, $\operatorname{Pr}\left[\left(\perp_{\text {all }}\right)^{c}, \eta \mid \tau_{e}\right]$ is the probability that at least one $T_{i}^{*}$ for some $i=1, \ldots, q_{f}$ is correct as a guess of $Y_{i}^{*}\left[\ell_{i}^{*}\right]$. Here $Y_{i}^{*}\left[\ell_{i}^{*}\right]$ is completely random from $\eta$, hence using the union bound we have

$$
\operatorname{Pr}\left[\left(\perp_{\mathrm{all}}\right)^{c}, \eta \mid \tau_{e}\right] \leq \frac{q_{f}}{2^{n}}
$$

For $\operatorname{Pr}\left[\left(\perp_{\text {all }}\right)^{c}, \eta^{c} \mid \tau_{e}\right]$ which is at most $\operatorname{Pr}\left[\eta^{c} \mid \tau_{e}\right]$, the above observation suggests that this can be evaluated by counting the number of possible bad pairs (i.e. a pair that a collision inside the pair violates $\eta$ ) among the all $X$-values in $\tau_{e}$ and all $X^{*}$-values in $\tau_{d}$, as in the same manner to the collision analysis of e.g., CBC-MAC using R. For each $i$-th decryption query, the number of bad pairs is at most $\left(q+\sigma+\ell_{i}^{*}\right) \cdot \ell_{i}^{*} \leq\left(q+\sigma+\sigma_{f}\right) \cdot \ell_{i}^{*}$. Therefore, the total number of bad pairs is $\sum_{1 \leq i \leq q_{f}}\left(q+\sigma+\sigma_{f}\right) \cdot \ell_{i}^{*} \leq\left(q+\sigma+\sigma_{f}\right) \cdot \sigma_{f}$, and we have

$$
\operatorname{Pr}\left[\left(\perp_{\mathrm{all}}\right)^{c}, \eta^{c} \mid \tau_{e}\right] \leq \frac{\left(q+\sigma+\sigma_{f}\right) \cdot \sigma_{f}}{2^{n}}
$$

```
Module Mask-Gen( \(K, N\) )
    \(Y[0] \leftarrow E_{K}\left(0^{n / 2} \| N\right)\)
    \(\left(Y^{1}[0], \ldots, Y^{4}[0]\right) \stackrel{n / 4}{\longleftarrow} Y[0]\)
    \(\Delta \leftarrow Y^{2}[0] \| Y^{3}[0]\)
    return \((\Delta, Y[0])\)
Algorithm COFB- \(\mathcal{E}_{K}(N, A, M)\)
    \((\Delta, Y[0]) \leftarrow \operatorname{Mask}-\operatorname{Gen}(K, N)\)
    \((A[1], \ldots, A[a]) \leftarrow^{n} A\)
    \((M[1], \ldots, M[m]) \stackrel{n}{\leftarrow} M\)
    \(\ell \leftarrow a+m\)
    \(((B[1], \mathrm{t}[1]), \ldots,(B[\ell], \mathrm{t}[\ell])) \leftarrow \operatorname{Fmt}(A, M)\)
    for \(i=1\) to \(\ell\)
    \(X[i] \leftarrow(B[i] \oplus G \cdot Y[i-1]) \oplus \operatorname{mask}_{\Delta}(\mathrm{t}[i])\)
    \(Y[i] \leftarrow E_{K}(X[i])\)
    if \(i>a\) then
        \(C[i-a] \leftarrow Y[i-1] \oplus M[i-a]\)
    \(T \leftarrow Y[\ell]\)
    return \((C, T)\)
```

Algorithm COFB-D $\mathcal{D}_{K}(N, A, C, T)$

```
Algorithm COFB-D \(\mathcal{D}_{K}(N, A, C, T)\)
\((\Delta, Y[0]) \leftarrow \operatorname{Mask}-\operatorname{Gen}(K, N)\)
\((\Delta, Y[0]) \leftarrow \operatorname{Mask}-\operatorname{Gen}(K, N)\)
    \((A[1], \ldots, A[a]) \stackrel{n}{\leftarrow} A\)
    \((A[1], \ldots, A[a]) \stackrel{n}{\leftarrow} A\)
    \((C[1], \ldots, C[c]) \stackrel{n}{n}_{\leftarrow} C\)
```

    \((C[1], \ldots, C[c]) \stackrel{n}{n}_{\leftarrow} C\)
    ```
```

    \(\ell \leftarrow a+c\)
    ```
    \(\ell \leftarrow a+c\)
    \(((B[1], \mathrm{t}[1]), \ldots,(B[\ell], \mathrm{t}[\ell])) \leftarrow \operatorname{Fmt}(A, C)\)
    \(((B[1], \mathrm{t}[1]), \ldots,(B[\ell], \mathrm{t}[\ell])) \leftarrow \operatorname{Fmt}(A, C)\)
    for \(i=1\) to \(\ell\)
    for \(i=1\) to \(\ell\)
        if \(i \leq a\) then
        if \(i \leq a\) then
            \(X[i] \leftarrow(B[i] \oplus G \cdot Y[i-1]) \bar{\oplus}\) mask \(_{\Delta}(\mathrm{t}[i])\)
            \(X[i] \leftarrow(B[i] \oplus G \cdot Y[i-1]) \bar{\oplus}\) mask \(_{\Delta}(\mathrm{t}[i])\)
        else \(X[i] \leftarrow(B[i] \oplus Y[i-1] \oplus G \cdot Y[i-1])\)
        else \(X[i] \leftarrow(B[i] \oplus Y[i-1] \oplus G \cdot Y[i-1])\)
                                \(\bar{\oplus}\) mask \(_{\Delta}(\mathrm{t}[i])\)
                                \(\bar{\oplus}\) mask \(_{\Delta}(\mathrm{t}[i])\)
        \(Y[i] \leftarrow E_{K}(X[i])\)
        \(Y[i] \leftarrow E_{K}(X[i])\)
        or \(i=1\) to \(c\)
        or \(i=1\) to \(c\)
        \(M[i] \leftarrow Y[i+a-1] \oplus C[i]\)
        \(M[i] \leftarrow Y[i+a-1] \oplus C[i]\)
        \(M \leftarrow(M[1], \ldots, M[c])\)
        \(M \leftarrow(M[1], \ldots, M[c])\)
    14. \(T^{\prime} \leftarrow Y[\ell]\)
    14. \(T^{\prime} \leftarrow Y[\ell]\)
    15. if \(T^{\prime}=T\) then return \(M\)
    15. if \(T^{\prime}=T\) then return \(M\)
    16. else return \(\perp\)
```

    16. else return \(\perp\)
    ```

Fig. 5.1. A presentation of COFB using Fmt function. This is equivalent to Fig. 4.2.

Combining all, we have
\[
\begin{aligned}
\mathrm{ip}_{\text {real }}(\tau) & =\frac{1}{2^{n(q+\sigma)}} \cdot \operatorname{Pr}\left[\tau_{d} \mid \tau_{e}\right]=\mathrm{ip}_{\text {ideal }}(\tau) \cdot \operatorname{Pr}\left[\perp_{\text {all }} \mid \tau_{e}\right] \\
& \geq \mathrm{ip}_{\text {ideal }}(\tau) \cdot\left(1-\left(\operatorname{Pr}\left[\left(\perp_{\text {all }}\right)^{c}, \eta \mid \tau_{e}\right]+\operatorname{Pr}\left[\left(\perp_{\text {all }}\right)^{c}, \eta^{c} \mid \tau_{e}\right]\right)\right) \\
& \geq \operatorname{ip}_{\text {ideal }}(\tau) \cdot\left(1-\frac{q_{f}+\left(q+\sigma+\sigma_{f}\right) \cdot \sigma_{f}}{2^{n}}\right) .
\end{aligned}
\]

\section*{6 Hardware Implementation of COFB}

\subsection*{6.1 Overview}

COFB primarily aims to achieve a lightweight implementation on small hardware devices. For such devices, the hardware resource for implementing memory is often the dominant factor of the size of entire implementation, and the scalability by parallelizing the internal components is not needed. In this respect, COFB's small state size and completely serial operation is quite desirable.

For implementation aspects, COFB is simple, as it consists of a blockcipher and several basic operations (bitwise XOR, the feedback function, and the constant multiplications over \(\operatorname{GF}\left(2^{n / 2}\right)\) ). Combined with the small state size, this implies that the implementation size of COFB is largely dominated by the underlying blockcipher. In this section we provide hardware implementation details of COFB using two blockciphers, AES and GIFT. Here, GIFT is a family of

Table 2. Clock cycles per message byte for \(\operatorname{COFB}[A E S]\).
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline & \multicolumn{9}{|c|}{ Message length (Bytes) } \\
\cline { 2 - 12 } & 16 & 32 & 64 & 128 & 256 & 512 & 1024 & 2048 & 4096 & 16384 & 32768 \\
\hline cpb & 2.93 & 2.22 & 1.86 & 1.68 & 1.59 & 1.54 & 1.52 & 1.51 & 1.50 & 1.50 & 1.50 \\
\hline
\end{tabular}

Table 3. Clock cycles per message byte for COFB[GIFT].
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline & \multicolumn{9}{|c|}{ Message length (Bytes) } \\
\cline { 2 - 11 } & 16 & 32 & 64 & 128 & 256 & 512 & 1024 & 2048 & 4096 & 16384 & 32768 \\
\hline cpb & 5.441 & 5.283 & 5.204 & 5.164 & 5.145 & 5.135 & 5.130 & 5.127 & 5.126 & 5.125 & 5.125 \\
\hline
\end{tabular}
lightweight blockcipher proposed by Banik et al. [13]. It employs a structure similar to PRESENT [19] while improves efficiency by carefully choosing S-box and the bit permutation. It has 64 -bit and 128 -bit block versions, both have 128 -bit key. We write GIFT-128 or simply write GIFT to denote the 128 -bit-block version. We write COFB[AES] and GIFT-128 to denote COFB using AES-128 and COFB[GIFT] respectively.

We provide the number of clock cycles needed to process input bytes, as a conventional way to estimate the speed. Here, COFB[AES] taking \(a\)-block AD (associated data) and an \(m\)-block message needs \(12(a+m)+23\) cycles. Table 2 shows the number of average cycles per input message bytes, which we call cycles per byte (cpb), assuming AD has the same length as message and the underlying blockcipher has 128 -bit block. That is, the table shows \((12 \cdot 2 m+23) / 16 m\).

Similarly, COFB[GIFT] needs \(41 \cdot(a+m)+81\) cycles for \(a\)-block AD and an \(m\)-block message. Table 3 shows the number of average cycles per input message bytes, which we call cycles per byte (cpb), assuming AD has the same length as message and the underlying blockcipher has 128 -bit block. That is, the table shows \((41 \cdot 2 m+81) / 16 m\).

\subsection*{6.2 Hardware Architecture}

We describe the implementation details of both COFB[AES] and COFB[GIFT]. These are basic round based implementations without any pipelining, and employ module architecture. We primary focus on the encryption-only circuit, however, the combined encryption and decryption circuit should have very small amount of overhead thanks to the inverse-freeness (i.e. no blockcipher decryption routine is needed) and simplicity of the mode. Due to the similarity between the associated data and the message processing phase, the same hardware modules are used in both phases. A single bit switch is used to distinguish between the two types of input data. The main architecture consists of the modules described below. We remark that, there is also a Finite State Machine (FSM) which controls the flow by sending signal to these modules. The FSM has a rather simple structure, and is described below. Then, the overall hardware architecture is
described in Fig. 6.2. We would like to mention that both the versions can be described with the same hardware architecture as they have exactly the same interface. Hence, we often use \(B C\) instead of the underlying blockcipher, where \(B C \in\{\) AES-128, GIFT-128\}. We also assume that \(B C\) comprises of \(r\) rounds.
1. State Registers. The state registers are used to store the intermediate states after each iteration. We use a 128 -bit State register to store the 128bit \(B C\) block state, a 64 -bit \(\Delta\) register to store the 64 -bit mask applied to each \(B C\) input, and a 128 -bit Key register to store the 128 -bit key. The round key of \(B C\) is stored in the additional 128-bit register (Round Key), however, this is included in the \(B C\) module.
2. BC Round. \(B C\) round function module runs one \(B C\) round computation and produces a 128 -bit output, using two 128 -bit inputs, one from the State and the other from (internal) Round Key registers. The latter register is initialized by loading the master key, stored in the Key register, each time the \(B C\) function is invoked. The output of \(B C\) module is stored into the State register, which is the input for the next round. The entire operation is serial, while the internal round computation and the round key generation run in parallel, and needs \(r+1\) cycles to perform full \(B C\) encryption.
3. Feedback Function \(\rho\). The \(\rho\) module is to compute the linear feedback function \(\rho\) on the 128-bit data block and the 128-bit intermediate state value (output from the \(B C\) computation). The output is a 128 -bit ciphertext and a 128-bit intermediate state (to be masked and stored to the State register).
4. Mask Update. uMask module updates the mask stored in \(\Delta\) register. uMask receives the current mask value and updates it by multiplying with \(\alpha\) or \((1+\alpha)\) or \((1+\alpha)^{2}\) based on the signals generated by the FSM, where signals are to indicate the end of the message and the completeness of the final block process.
5. FSM. The control of the complete design can be described by a finite state machine (FSM). We provide a separate and simple view of FSM in Fig. 6.1. The FSM consists of 9 states and starts with the Reset_St. This state is idle and followed by a Load_St, which initializes the \(B C\) state by loading nonce (before the first \(B C\) invocation). After the initialization, FSM enters into the \(B C\) invocation phase to encrypt the nonce. This phase consists of BC_Reset_St to reset BC parameters, BC_Start_St for key whitening, \(B C \_\)Round_St to run one \(B C\) round and \(B C_{-}\)Done_St to indicate the end of the \(B C\) invocation. Depending on whether the current blockcipher call is final or not, the FSM either releases the tag or it enters to the Compute_ \(\rho_{-} A d d_{-} M a s k_{-} S t\), which computes the \(\rho\) function, updates mask and partially masks the blockcipher input. The FSM sends two additional bits \(E O M\) to denote the end of data block and isComplete to denote the last data block is complete or not. Next it enters the BC_Reset_St for the next blockcipher invocation. After the last \(B C\) invocation it enters the Release_Tag_St. Finally, the FSM enters the end state. We use a 4 -bit register to keep track of the states. It is to be noted that, in addition to the
state transition, FSM also sends the corresponding relevant signals to the top modules.


Fig. 6.1. FSM for \(\mathrm{COFB}[B C]\) Hardware Implementation

Basic Implementation. We describe a basic flow of our implementation, which generally follows the pseudocode of Fig. 4.2. Prior to the initialization, State register is loaded with \(0^{64} \| N\). Once State register is initialized, the initialization process starts by encrypting the nonce \(\left(0^{64} \| N\right)\) with \(B C\). Then, 64 bits of the encrypted nonce is chopped by the "chop" function as in Fig. 6.2, and this chopped value is stored into the \(\Delta\) register (this is initialization of \(\Delta\) ). After the initialization, 128 -bit associated data blocks are fetched and sent to the \(\rho\) module along with the previous \(B C\) output to produce a 128 bit intermediate state. This state is partially masked with 64 -bit \(\Delta\) for every BC call. After all the associated data blocks are processed, the message blocks are processed in the same manner, except that the \(\rho\) function produces 128 -bit ciphertext blocks in addition to the intermediate state values. Finally, after the message processing is over, the tag is generated using an additional \(B C\) call.

Combined Encryption and Decryption. As mentioned earlier, we here focus on the encryption-only circuit. However, due to the similarity between the encryption and the decryption modes, the combined hardware for encryption and decryption can be built with a small increase in the area, with the same throughput. This can be done by adding a control flow to a binary signal for mode selection.


Fig. 6.2. Hardware Circuit Diagram
Table 4. FPGA implementation results of COFB[AES]
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline Platform & \begin{tabular}{c} 
Slice \\
Registers
\end{tabular} & LUTs & Slices & \begin{tabular}{c} 
Frequency \\
(MHz)
\end{tabular} & \begin{tabular}{c} 
Throughput \\
(Gbps)
\end{tabular} & Mbps/LUT & Mbps/Slice \\
\hline Virtex 6 & 594 & 1051 & 449 & 267.20 & 2.85 & 2.71 & 6.35 \\
\hline Virtex 7 & 593 & 1440 & 564 & 274.84 & 2.93 & 2.03 & 5.19 \\
\hline
\end{tabular}

\subsection*{6.3 Implementation Results}

We have implemented both COFB[AES] and COFB[GIFT] on Xilinx Virtex 6 and Virtex 7, using VHDL and Xilinx ISE 13.4. Table 4 presents the implementation results of COFB on Virtex 7 with the target device \(x c 7 v x 330 t\) and Virtex 6 with the target device \(x c 6 v 1 \times 760\). We employ RTL approach and a basic iterative type architecture (128-bit round based implementation). The areas are listed in the number of Slice Registers, Slice LUTs and Occupied Slices. We also report frequency ( MHz ), Throughput (Gbps), and throughput-area efficiency. Table 4 presents the mapped hardware results of COFB[AES]. In this paper, we have slightly optimized the implementation in \([23,24]\) to get a better estimate of the number of slice registers.

For AES-128, we use the implementation available from Athena [1] maintained by George Mason University. This implementation stores all the round subkeys in a single register to make the AES implementation faster and parallelizable. However, the main motivation of COFB is to reduce hardware footprint. Hence, we change the above implementation to a sequential one such that it processes only one AES round in a single clock cycle. This in turn eliminates the need to store all the round subkeys in a single register and reduces the hardware area consumed by the AES module.

For GIFT-128, we use our own implementation in FPGA. The implementation is round based without any pipelining. The architecture uses three registers State, RK and Round to hold the blockcipher state, current round key and the round counter respectively. The architecture is divided into four modules \(S N\),

Table 5. FPGA implementation results of COFB[GIFT]
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline Platform & \begin{tabular}{c} 
Slice \\
Registers
\end{tabular} & LUTs & Slices & \begin{tabular}{c} 
Frequency \\
(MHz)
\end{tabular} & \begin{tabular}{c} 
Throughput \\
(Gbps)
\end{tabular} & Mbps/LUT & Mbps/Slice \\
\hline Virtex 6 & 342 & 771 & 355 & 612.91 & 1.91 & 2.48 & 5.51 \\
\hline Virtex 7 & 342 & 771 & 316 & 712.99 & 2.23 & 2.89 & 6.62 \\
\hline
\end{tabular}
\(B P, A R K\) and \(A R C, U K E Y\). operations. \(S N\) module applies a 4-bit sbox to each of the 4 -bit nibbles of the state. \(B P\) applies the bit permutation on the state. \(A R K\) performs the round key addition on the state and \(A R C\) applies round constant addition on the state. \(U K E Y\) updates the round key and stores it in \(R K\). The architecture also uses another module EXT to extract a part of the round key to be added to the state. The hardware implementation results in slice registers, slice LUTs and Slices are presented in Table 5.

\subsection*{6.4 Hardware Flexibility of the COFB Design}

COFB is itself very lightweight and it uses a few operations other than the blockcipher computations. Below in Table 6 and 7 , we present the hardware area occupied by the blockcipher and the other modules for both COFB[AES] and COFB[GIFT] on Vertex 6 . We observe that COFB[AES] consumes low hardware footprint and the majority of the hardware footprint is used by AES, whereas in COFB[GIFT] the implementation size is much smaller as the underlying blockcipher GIFT is much lighter than AES. This depicts that implementation area optimized blockcipher will be the most efficient one.

Table 6. COFB[AES]: Area utilization by modules in Virtex 6
\begin{tabular}{|c|c|c|}
\hline Modules & Slices & LUTs \\
\hline Total & 449 & 1051 \\
\hline AES & 311 & 657 \\
\hline Others & 138 & 394 \\
\hline
\end{tabular}

Table 7. COFB[GIFT]: Area utilization by modules in Virtex 6
\begin{tabular}{|c|c|c|}
\hline Modules & Slices & LUTs \\
\hline Total & 355 & 771 \\
\hline GIFT & 155 & 346 \\
\hline Others & 200 & 425 \\
\hline
\end{tabular}

\subsection*{6.5 Benchmarking with ATHENa Database}

We compare our implementation of COFB with the results published in ATHENa Database [2], taking Virtex 6 and Virtex 7 as our target platforms. We first warn that this is a rough comparison. Here, we ignore the overhead to support the GMU API and the fact that ours is encryption-only while the others are (to the best of our knowledge) supporting both encryption and decryption, and the difference in the achieved security level, both quantitative and qualitative. We acknowledge that supporting GMU API will require some additional overhead to the current figures of COFB. Nevertheless, we think the current figures of COFB suggest that small hardware implementations (small for AES-128 and even smaller with GIFT-128) are possible compared with other blockcipher AE modes shown in the table, using the same AES-128 and GIFT-128, even if we add a circuit for supporting GMU API and decryption.

We also remark that it is basically hard to compare COFB using AES-128 or GIFT-128 with other non-block-cipher-based AE schemes in the right way, because of the difference in the primitives and the types of security guarantee. For example, ACORN is built from scratch and does not have any provable security result, and is subjected to several cryptanalyses [27, 55, 54, 42]. Joltik and JAMBU-SIMON employ lightweight (tweakable) blockciphers allowing smaller implementation than AES, and Sponge AE schemes (ASCON, Ketje, NORX, and PRIMATES-HANUMAN) use a keyless permutation of a large block size to avoid key scheduling circuit and have the provable security relying on the random permutation model. In Table 8 and 9 , we provide the comparison table both Vertex 6 and Vertex 7 platforms.

\section*{7 Conclusion}

This paper presents COFB, a blockcipher mode for AE focusing on minimizing the state size. When instantiated with an \(n\)-bit blockcipher, COFB operates at rate-1, and requires state size of \(1.5 n\) bits, and is provable secure up to \(O\left(2^{n / 2} / n\right)\) queries based on the standard PRP assumption on the blockcipher. In fact this is the first scheme fulfilling these features at once. A key idea of COFB is a new type of feedback function combining both plaintext and ciphertext blocks. We first present an idealized version of COFB, named iCOFB along with its provable security analysis. We instantiate COFB with the AES-128 blockcipher. We also present two hardware implementation results for COFB with AES-128 and GIFT-128 blockcipher respectively. These two implementations demonstrate the effectiveness of our approach.

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Table 8. Comparison on Virtex 6 [2]. In the "Primitive" column, SC denotes Stream cipher, (T)BC denotes (Tweakable) blockcipher, and BC-RF denotes the blockcipher's round function.
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline Scheme & Primitive & LUT & Slices & T'put (Gbps) & Mbps / LUT & Mbps / Slice \\
\hline ACORN [61] & SC & 455 & 135 & 3.112 & 6.840 & 23.052 \\
\hline AEGIS [63] & BC-RF & 7592 & 2028 & 70.927 & 9.342 & 34.974 \\
\hline AES-COPA [10] & BC & 7754 & 2358 & 2.500 & 0.322 & 1.060 \\
\hline AES-GCM [29] & BC & 3175 & 1053 & 3.239 & 1.020 & 3.076 \\
\hline AES-OTR [46] & BC & 5102 & 1385 & 2.741 & 0.537 & 1.979 \\
\hline AEZ [34] & BC-RF & 4597 & 1246 & 8.585 & 0.747 & 2.756 \\
\hline ASCON [28] & Sponge & 1271 & 413 & 3.172 & 2.496 & 7.680 \\
\hline CLOC [37] & BC & 3145 & 891 & 2.996 & 0.488 & 1.724 \\
\hline DEOXYS [39] & TBC & 3143 & 951 & 2.793 & 0.889 & 2.937 \\
\hline ELmD [26] & BC & 4302 & 1584 & 3.168 & 0.736 & 2.091 \\
\hline JAMBU-AES [62] & BC & 1836 & 652 & 1.999 & 1.089 & 3.067 \\
\hline JAMBU-SIMON [62] & BC (non-AES) & 1222 & 453 & 0.363 & 0.297 & 0.801 \\
\hline Joltik [38] & TBC & 1292 & 442 & 0.853 & 0.660 & 0.826 \\
\hline Ketje [18] & Sponge & 1270 & 456 & 7.345 & 5.783 & 16.107 \\
\hline Minalpher [56] & BC (non-AES) & 2879 & 1104 & 1.831 & 0.636 & 1.659 \\
\hline NORX [11] & Sponge & 2964 & 1016 & 11.029 & 3.721 & 10.855 \\
\hline PRIMATES-HANUMAN [8] & Sponge & 1012 & 390 & 0.964 & 0.953 & 2.472 \\
\hline OCB [41] & BC & 4249 & 1348 & 3.122 & 0.735 & 2.316 \\
\hline SCREAM [32] & TBC & 2052 & 834 & 1.039 & 0.506 & 1.246 \\
\hline SILC [37] & BC & 3066 & 921 & 4.040 & 1.318 & 4.387 \\
\hline Tiaoxin [48] & BC-RF & 7123 & 2101 & 52.838 & 7.418 & 25.149 \\
\hline TriviA-ck [25] & SC & 2118 & 687 & 15.374 & 7.259 & 22.378 \\
\hline COFB[AES] & BC & 1051 & 449 & 2.850 & 2.710 & 6.350 \\
\hline COFB[GIFT] & BC & 771 & 355 & 1.91 & 2.48 & 5.51 \\
\hline
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Table 9. Comparison on Virtex 7 [2].
\begin{tabular}{|c|c|c|c|c|c|}
\hline Scheme & LUT & Slices & T'put (Gbps) & Mbps / LUT & Mbps / Slice \\
\hline ACORN & 499 & 155 & 3.437 & 6.888 & 22.174 \\
\hline AEGIS & 7504 & 1983 & 94.208 & 12.554 & 47.508 \\
\hline AES-COPA & 7795 & 2221 & 2.770 & 0.355 & 1.247 \\
\hline AES-GCM & 3478 & 949 & 3.837 & 1.103 & 4.043 \\
\hline AES-OTR & 4263 & 1204 & 3.187 & 0.748 & 2.647 \\
\hline AEZ & 4686 & 1645 & 8.421 & 0.719 & 2.047 \\
\hline ASCON & 1373 & 401 & 3.852 & 2.806 & 9.606 \\
\hline CLOC & 3552 & 1087 & 3.252 & 0.478 & 1.561 \\
\hline DEOXYS & 3234 & 954 & 1.472 & 0.455 & 2.981 \\
\hline ELmD & 4490 & 1306 & 4.025 & 0.896 & 3.082 \\
\hline JAMBU-AES & 1595 & 457 & 1.824 & 1.144 & 3.991 \\
\hline JAMBU-SIMON & 1200 & 419 & 0.368 & 0.307 & 0.878 \\
\hline Joltik & 1261 & 390 & 0.402 & 0.319 & 1.031 \\
\hline Ketje & 1125 & 351 & 8.718 & 7.749 & 24.838 \\
\hline Minalpher & 2941 & 802 & 2.447 & 0.832 & 3.051 \\
\hline NORX & 2881 & 857 & 10.328 & 3.585 & 12.051 \\
\hline PRIMATES-HANUMAN & 1148 & 370 & 1.072 & 0.934 & 2.897 \\
\hline OCB & 4269 & 1228 & 3.608 & 0.845 & 2.889 \\
\hline SCREAM & 2315 & 696 & 1.100 & 0.475 & 1.580 \\
\hline SILC & 3040 & 910 & 4.365 & 1.436 & 4.796 \\
\hline Tiaoxin & 7556 & 1985 & 75.776 & 10.029 & 38.174 \\
\hline TriviA-ck & 2221 & 684 & 14.852 & 6.687 & 21.713 \\
\hline COFB[AES] & 1440 & 564 & 2.93 & 2.03 & 5.19 \\
\hline COFB[GIFT] & 771 & 316 & 2.23 & 2.89 & 6.62 \\
\hline & & & & \\
\hline
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[^0]:    * A preliminary version of this paper was presented at CHES 2017 [23].

[^1]:    $\ddagger$ The authenticity result was briefly presented in the latest specification [62].

