# Conditional Blind Signatures

Alexandros Zacharakis, Panagiotis Grontas, and Aris Pagourtzis

School of Electrical and Computer Engineering
National Technical University of Athens, 15780 Athens, Greece
azach@corelab.ntua.gr, pgrontas@corelab.ntua.gr, pagour@cs.ntua.gr

**Abstract.** We propose a novel cryptographic primitive that we call *conditional blind signatures*. Our primitive allows a user to request blind signatures on messages of her choice. The signer has a secret Boolean input which determines if the supplied signature is valid or not. The user should not be able to distinguish between valid and invalid signatures. A designated verifier, however, can tell which signatures verify correctly, and is in fact the only entity who can learn the secret input associated with the signed message after the unblinding process. We instantiate our primitive as an extension of the Okamoto-Schnorr blind signature scheme. We analyze and prove the security properties of the new scheme and explore potential applications.

**Keywords:** digital signatures, blind signatures, designated verifier signatures

## 1 Introduction

Digital signatures, proposed in [1], are one of the most successful public key cryptographic primitives. A *user* $\mathcal{U}$ submits a message to a *signer* $\mathcal{S}$, who applies a function of his secret signing key $sk$ and generates a signature that can be verified by everybody that possesses the corresponding public verification key $vk$. They allow message integrity, authenticity and non repudiation in a publicly verifiable manner. A digital signature scheme is secure if no probabilistic adversary $\mathcal{A}$, running in polynomial time (PPT), can output a forgery of a signature, a valid signature, that is, without the possession of the signing key. Instantiations of digital signatures schemes base their security on well know cryptographic problems such as the RSA problem [2] the Discrete Log problem ([3],[4]) with its many variations and many more. Moreover, in [5] a method is given to construct digital signatures from interactive proofs of knowledge.

Digital signatures, are also quite versatile, as attested by the plethora of variations that have been proposed in the literature. Indeed many useful schemes can arise, if one fiddles with the basic setting of a digital signature scheme. For instance, blind signatures [6], hide the message to be signed from the signer, thus allowing the user to maintain her privacy while keeping the signature publicly

This paper has been presented at the 7th International Conference on Algebraic Informatics, June 25-28, 2017, Kalamata, Greece (CAI-2017).

verifiable. The security of blind signatures has been studied in [7], [8] and [9]. The relevant security properties are *blindness* or *unlinkability*, which models the fact that the signer cannot have access to the message, and resistance to *one more forgery*, which states that the user cannot herself create more signatures than the signer provided. Blind signatures have many important applications such as electronic cash [6] and electronic voting [10].

Another variation of digital signatures is group signatures [11]. They aim to provide *signer anonymity within a group*. This means that the signature is validated as coming from the group as a whole, without giving evidence as to which member of the group actually signed. Of course in the case of a dispute, the *traceability* property allows the group manager to specify which group member actually signed.

A less studied primitive related to signatures via zero knowledge proofs, are designated verifier proofs, proposed in [12]. They sacrifice the public verifiability of a proof and propose a scheme where its validity can only be verified by an entity that has a specific piece of knowledge (e.g. a private key). This entity is designated by the prover. Such a scheme might seem of no particular use, but this is not the case, since in [12], the authors propose a very interesting application in the context of receipt free and coercion resistant electronic voting. In particular, they propose that a voting authority uses designated verifier proofs, in order to convince a voter that her ballot was correctly counted. However this proof is only verifiable by the voter herself and not by any third party. As a result it cannot be presented voluntarily or involuntarily to a bidder or a coercer.

*Motivation.* In this paper we aim to create a primitive that can be used as a building block for protocols that require strong guarantees for coercion resistance and privacy. Such a primitive can be used, for example, in auction and payment systems, but the primary application we have in mind is remote electronic voting, where the lack of a controlled environment for vote casting, leaves the voters vulnerable to coercion attacks. The most well known way to defeat such attacks was proposed in the JCJ framework [13]. Its main idea, stems from the fact that the coercer has no incentive to carry out his attack if he cannot tell whether it has succeeded or not. This can be achieved, if we allow the voter to cast multiple ballots, by attaching a different but indistinguishable anonymous credential to each vote. One of these credentials is *valid* and it is used to cast the vote when the coercer is not present - JCJ assumes that each voter has a moment of privacy. Only the votes cast with valid credentials are included in the election tally. In order to filter out the invalid credentials the authors of [13] propose a quadratic number of checks in the number of votes cast. Such a complexity is not practical for real large scale elections.

A more practical solution, would involve a signer that uses voter identification information to efficiently retrieve the valid credential and check it against the one that is provided during the voting process. If the credentials are different, then the voter is under coercion and the vote should not be counted. This bit of information has to be conveyed to the counter in a manner undetectable by the coercer. Of course the signer should not have any access to the contents of the

vote, in order to keep it anonymous. As a result the signatures have to be blind, as well. A well known voting scheme built on blind signatures was given in [10], but it is not coercion resistant. What is needed, is a primitive that can integrate the coercion resistance property of [13] and the increased privacy guarantees of [10].

*Our contribution.* Our approach is based on the observation that a combination of a simple group signature scheme with a designated verifier proof can be used to convey a piece of secret information from a signer to a specified verifier. For instance if we imagine the group members, as possible responses to the message to be signed, a designated group signature is equivalent to sending a particular response to the verifier. As a result, we propose a new primitive, called *conditional blind signatures*, that implements this functionality. We define its security properties and provide an instantiation that is based on the well known Okamoto-Schnorr blind signatures [4]. We use our definitions and the particular instantiation to provide proofs for the security properties. Despite the fact that the motivation behind our primitive is specific, we think that it can stand on its own and be used in many applications apart from electronic voting.

*Related work.* On a conceptual level our scheme resembles Chaum's and van Antwerpen's Designated Confirmer Signatures (DCS) [14]. Indeed, DCS were also proposed as a combination of digital signatures and zero knowledge proofs, to solve a problem of undeniable signatures [15]. In undeniable signatures, if the signer becomes unavailable during the verification process, the signature cannot be validated. To fix this, DCS adds a third party to the protocol, a designated confirmer, that can also verify (confirm) the signature. In addition he can produce normal digital signatures. Designated confirmer signatures have been studied extensively since their introduction and many variations have been proposed [16].

Our proposal, resembles DCS in the basic usage scenario, since in our case, as well, the verifier can be a third party which is 'designated' during the signature creation process. However, in our scheme the signer cannot himself verify the signature. Moreover, the problem we wish to solve is the secure passing of a single bit of information to the verifier through the signature and not the unavailability of the signer. Finally, in our usage scenario the messages are blinded, so that the signer cannot have access to their contents. To the best of our knowledge a similar primitive has not been proposed in the literature so far.

## 2 Preliminaries

In this section we briefly review the necessary concepts for the construction and security analysis of our proposal.

### 2.1 Security Assumptions

In section 5 we prove that our scheme is secure by showing that breaking it would imply that the Computational Diffie Hellman (CDH) and the Decisional

Diffie Hellman (DDH) assumptions are false. Informally, the CDH assumption states that given a group $\mathbb{G}$ and a generator $g$, two group elements $g^a, g^b$ the group element $g^{ab}$ cannot be efficiently computed. The DDH assumption states that the triples of group elements $(g^a, g^b, g^{ab})$ and $(g^a, g^b, g^c)$ where $a, b, c$ are randomly selected from $\{1, \cdots, |\mathbb{G}|\}$ cannot be efficiently distinguished.

More formally [17], let $G$ be a group family and $g$ a generator of a particular member $\mathbb{G}$ of $G$. We denote the security parameter with $\lambda$.

**Definition 1.** Computational Diffie Hellman *Assumption.*
*A* CDH *algorithm $A$ is a probabilistic polynomial time algorithm satisfying:*

$$\Pr[A(g, g^a, g^b) = g^{ab}] > \frac{1}{\lambda^k}$$

*for some fixed $k \in \mathbb{Z}$, where the probability is taken over the selection of $\mathbb{G}$, $a, b$ and the random bits of $A$. The group family satisfies the* CDH *assumption if there is no* CDH *algorithm for it.*

**Definition 2.** Decisional Diffie Hellman *Assumption.*
*A* DDH *algorithm $A$ is a probabilistic polynomial time algorithm satisfying:*

$$|\Pr[A(g, g^a, g^b, g^{ab}) = 1] - \Pr[A(g, g^a, g^b, g^c) = 1]| > \frac{1}{\lambda^k}$$

*for some fixed $k \in \mathbb{Z}$, where the probability is taken over the selection of $\mathbb{G}$, $a, b, c$ and the random bits of $A$. The group family satisfies the* DDH *assumption if there is no* DDH *algorithm for it.*

There are many groups where the Decisional Diffie Hellman Assumption is believed to hold [17].

### 2.2 Okamoto-Schnorr Blind Signatures

In section 4 we provide an instantiation of our scheme built on the Okamoto-Schnorr blind signatures. For completeness, we repeat their definition here from [4].

The public parameters of the protocol are a group $\mathbb{G}$ with prime order $q$, two generators $g_1, g_2$ and a hash function $\mathcal{H}$. The signer $\mathcal{S}$ selects the private signing key $s_1, s_2 \in \mathbb{Z}_q$ and computes the public verification key $v = g_1^{-s_1} g_2^{-s_2}$. The user $\mathcal{U}$ wants to sign the message $m$. The protocol is executed in the following phases:

1. In the commitment phase, $\mathcal{S}$ randomly selects $r_1, r_2 \in \mathbb{Z}_q$ and commits to the value $x = g_1^{r_1} g_2^{r_2}$.
2. In the blinding phase, $\mathcal{U}$ selects the blinding factors $u_1, u_2, d \in \mathbb{Z}_q$ and computes the following values:
   - $x^* = g_1^{u_1} g_2^{u_2} v^d x$
   - $e^* = \mathcal{H}(m, x^*)$
   - $e = e^* - d \bmod q$

Finally she sends the value of $e$ to $\mathcal{S}$.

3. In the signing phase $\mathcal{S}$ computes the values $y_1 = r_1 + es_1 \bmod q$ and $y_2 = r_2 + es_2 \bmod q$. The blind signature is $(x, e, y_1, y_2)$.
4. In the unblinding phase $\mathcal{U}$ computes the values $y_1^* = y_1 + u_1 \bmod q$ and $y_2^* = y_2 + u_2 \bmod q$. The plain signature is $(x^*, e^*, y_1^*, y_2^*)$.
5. To verify the signature the following two relations are checked:
   - $e^* =_? \mathcal{H}(m, x^*)$
   - $x^* =_? g_1^{y_1^*} g_2^{y_2^*} v^{e^*}$

## 3 Definitions for Conditional Blind Signatures

Our new primitive can be abstractly viewed as a protocol implementing the following functionality $f$:

$$\text{signature} = f(b, sk, pk, c)$$

where:

- $b$ is the secret information to be conveyed from the signer to the verifier. We restrict the secret information to a single bit.
- $sk$ is the signing key.
- $pk$ is the corresponding public key.
- $c$ is the blinded message to be signed.

The participants of the protocol are:

- The user $\mathcal{U}$ is the entity that requests blind signatures on messages of her choice.
- The signer $\mathcal{S}$ is the entity that creates the signatures on the message provided by the user. The signer wants to use the signature to convey the secret information to the verifier.
- The verifier $\mathcal{V}$ is the entity that checks the validity of the signatures and learns the secret information of $\mathcal{S}$. The verifier can be the signer himself at a future time.

The adversary $\mathcal{A}$ may be any entity apart from the designated verifier. This means that, apart from external attackers, both the signer and the user may have incentive to attack our scheme. For instance, the signer signs a blinded message, so he might want to learn its contents. The user, or an agent acting on her behalf, on the other hand, might want to retrieve the signer's secret information.

The desired security properties of our scheme extend the security properties of digital and blind signatures:

- The signatures given must be statistically blind.
- The scheme must be secure against one more forgery.
- No PPT adversary can check the validity of the produced signatures nor can he extract the secret information, but with probability negligible to a security parameter.

Concretely, our primitive can be defined as follows:

**Definition 3.** *A conditional blind signature scheme is a triple* (Gen, Sign, Vrfy) *with the following properties:*

- Gen *is an algorithm that takes as input the security parameter* $1^\lambda$ *and outputs two pair of keys* $(sk_{\mathcal{S}}, pk_{\mathcal{S}})$ *for signing and* $(sk_{\mathcal{V}}, pk_{\mathcal{V}})$ *for verification, the message space* $\mathbb{M}$ *and the signature space* $\mathbb{S}$*. These sets are described by a set of parameters (e.g. group generators) collectively denoted as* params*. We also refer to the set of public keys as* $pk = (pk_{\mathcal{S}}, pk_{\mathcal{V}})$*.*
- $\mathsf{Sign}(\text{params}, pk) = \langle \mathcal{S}(sk_{\mathcal{S}}, b), \mathcal{U}(m) \rangle$ *is a protocol executed between the signer and the user. The public input to the signing protocol consists of the parameters and the public keys. The secret input of the signer is the signing key* $sk_{\mathcal{S}}$ *and the secret information bit* $b$*, while the secret input of the user is the message* m *to be signed. The protocol outputs a signature* sig *of* m *to* $\mathcal{U}$*.*
- Vrfy *is an algorithm which on input* $(sk_{\mathcal{V}}, m, sig)$ *outputs* 1 *if and only if* $sig$ *is the output of the execution of the protocol* Sign *on message* m *and the secret information bit of* $\mathcal{S}$ *is* $b = 1$*, except with negligible probability.*

Conditional Blind Signatures inherit the Blindness and the security against One More Forgery properties from the conventional blind signatures schemes (with minor modifications). For the sake of completeness we repeat the definitions of [9].

We first formally define the blindness property using the following game, which states that a malicious signer cannot tell which of the two messages $m_0, m_1$ was signed first except with negligible probability:

**Algorithm 1:** $\text{BlindExp}_{\mathcal{A},\Pi}$

**Input :** security parameter $\lambda$
**Output:** $b \in \{0, 1\}$

$(pk, m_0, m_1, st_{find}) \leftarrow \mathcal{A}(\mathbf{find}, 1^\lambda)$
$c \leftarrow_R \{0, 1\}$
$st_{issue} \leftarrow \mathcal{A}^{\langle \cdot, U(pk, m_b) \rangle, \langle \cdot, U(pk, m_{1-b}) \rangle}(\mathbf{issue}, st_{find})$
**if** $\sigma_0 = \perp \vee \sigma_1 = \perp$ **then**
  $(\sigma_0, \sigma_1) = (\perp, \perp)$
**else**
  $c^* \leftarrow \mathcal{A}(\mathbf{guess}, \sigma_0, \sigma_1, st_{issue})$
**end**
**if** $c = c^*$ **then**
  return 1
**else**
  return 0
**end**

**Definition 4.** *A blind signature scheme $\Pi$ is statistically blind if for every (unbounded) $\mathcal{A}$:* $\Pr[\text{BlindExp}_{\mathcal{A},\Pi}(\lambda) = 1] \leq \frac{1}{2} + negl(\lambda)$.

The unforgeability property is captured using the notion of *One More Forgery* [8], which states that, if $l$ is an integer, polynomial in the security parameter $\lambda$, an attacker produces $l+1$ valid signatures, after fewer than $l$ successful interactions with the signer. The *Strong* One More Forgery [8] is a variation of the above case, where $l$ is *polylogarithmically* bound to the security parameter. More formally:

**Algorithm 2:** $\text{OneMoreForge}_{\mathcal{A},\Pi}$

**Input :** security parameter $\lambda$
**Output:** $b \in \{0, 1\}$

$(sk, pk) \leftarrow \text{Gen}(1^{\lambda})$
$((m_1, \sigma_1), \ldots, (m_{l+1}, \sigma_{l+1})) \leftarrow \mathcal{A}^{\langle S(sk_{\mathcal{S}}),\cdot\rangle}(pk)$
`/* k: number of successful interactions for the Sign protocol */`
**if** $(\forall i, j \text{ with } i \neq j \Rightarrow m_i \neq m_j) \wedge (\forall i \ \text{Vrfy}(vk, m_i, \sigma_i) = 1) \wedge k \leq l$ **then**
  return 1
**else**
  return 0
**end**

**Definition 5.** *A blind signature scheme $\Pi$ is one more unforgeable if for every* PPT $\mathcal{A}$ *there is a negligible function of $\lambda$ where:* $\Pr[\text{OneMoreForge}_{\mathcal{A},\Pi}(\lambda) = 1] \leq negl(\lambda)$.

The above games can be easily extended to accommodate for the secret information bit specified in the proposed primitive. In particular, since in BlindExp the secret bit $b$ can be used to distinguish between two messages, we restrict the adversary in issuing the two signatures with the same secret bit. In OneMoreForge, invalid signatures might assist the aspiring forger, so we allow him to get signatures with a $b$ of his choice.

Additionally, for our primitive we define an extra property which is called *Conditional Verifiability*, which is defined in the game CondVerExp presented in algorithm 3.

In the particular game, the adversary $\mathcal{A}$ adaptively gets valid and invalid signatures of his choice and creates a challenge message. He then gets a valid or invalid signature on this message based on a coin flip and afterwards he tries to determine the value of the coin flip. He may continue to get signatures of his choice. The scheme is secure with respect to conditional verifiability if there is no PPT adversary who can succeed in guessing the result of the coin flip with non negligible advantage. More formally:

**Definition 6.** *A conditional blind signature scheme $\Pi$ has the Conditional Verifiability property if for each* PPT *adversary $\mathcal{A}$ there is a negligible function* negl

**Algorithm 3:** $\text{CondVerExp}_{\mathcal{A},\Pi}$

**Input :** security parameter $\lambda$
**Output:** $x \in \{0,1\}$

$c \leftarrow_R \{0,1\}$
$(sk, pk, \text{params}) \leftarrow \text{Gen}(1^\lambda)$
$\{(m_i, sig_i) \leftarrow \text{Sign}\langle S(\text{params}, sk_{\mathcal{S}}, b_i), \mathcal{A}(\text{params}, pk, \{m_j, sig_j\}_{j=1}^{i-1}, b_i)\rangle\}_{i=1}^{l_1}$
$m_c \leftarrow \mathcal{A}(pk, \text{params}, \{(m_i, sig_i)\}_{i=1}^{l_1}, \textbf{Challenge})$
$(\epsilon, sig_c) \leftarrow \text{Sign}\langle S(\text{params}, sk, b), \mathcal{A}(\text{params}, pk, m_c)\rangle$
`/* ϵ is the empty string */`
$\{(m_i, sig_i) \leftarrow \text{Sign}\langle S(\text{params}, sk_{\mathcal{S}}, b_i), \mathcal{A}(\text{params}, pk, \{m_j, sig_j\}_{j=1}^{i-1}, b_i)\rangle\}_{i=l_1+1}^{l_2}$
$c' = \mathcal{A}(\{m_i, sig_i\}_{i=1}^{l_1+l_2}, m_c, sig_c, \textbf{Guess})$
**if** $c = c'$ **then**
  return 1
**else**
  return 0
**end**

*with regard to the security parameter* $\lambda$ *such that* $\Pr[\text{CondVerExp}_{\mathcal{A},\Pi} = 1] \leq \frac{1}{2} + \text{negl}(\lambda)$.

**Definition 7.** *A conditional blind signature scheme* $\Pi$ *is secure if it has the properties* Statistical Blindness*,* One More Forgery *and* Conditional Verifiability*.*

We must note here that the user cannot validate the signature she receives, since she does not have access to the secret bit $b$. Although this seems counter intuitive with respect to traditional signatures, in our setting it is the exact property we want to capture.

## 4 An instantiation based on Okamoto-Schnorr Blind Signatures

In this section we propose an instantiation of our scheme based on the Okamoto-Schnorr Blind Signatures [4]. The intuition behind our construction is that we replace the elements $(y_1, y_2)$ of the standard blind signature of [4] with a 'lifted' signature $(k^{y_1}, y_2)$ where $k$ is some element of the underlying group with logarithm known only to the verifier.

Firstly we define the key generation algorithm for the 3 participating entities, namely the user, the signer and the verifier presented in algorithm 4.

For the signing protocol we assume a hash function $\mathcal{H} : \{0,1\}^* \rightarrow \mathbb{Z}_q$, which is modelled as a random oracle. The algorithm begins as in the Okamoto-Schnorr blind signatures [4]. The signer commits to a random value. The user selects the blinding factors and blinds the commitment and the hash to be signed. Our variation actually begins when the signer is ready to sign the blinded values. We consider 2 cases:

**Algorithm 4:** Key Generation Algorithm

**Input :** security parameter $\lambda$
**Output:** $(sk_S, vk_S), (sk_V, vk_V), params$

/* We select a group $\mathbb{G}$ with prime order $q$ with $q > 2^\lambda$ where the DDH problem is hard */
$(q, \mathbb{G}) \leftarrow \text{GroupGen}(1^\lambda)$
/* Select the appropriate generators */
$(g_1, g_2) \leftarrow_R \mathbb{G}$
params $\leftarrow (q, \mathbb{G}, g_1, g_2)$
/* Select the secret $sk_S$ and public signing keys $vk_S$ for $\mathcal{S}$ */
$s_1, s_2 \leftarrow_R \mathbb{Z}_q$
$v \leftarrow g_1^{-s_1} g_2^{-s_2}$
$(sk_S, vk_S) \leftarrow ((s_1, s_2), v)$
/* Select secret $sk_V$ and public verification keys $vk_V$ for $\mathcal{V}$ */
$s \leftarrow_R \mathbb{Z}_q$
$k \leftarrow g_1^s$
$(sk_V, pk_V) \leftarrow (s, k)$

- If the hidden bit of $\mathcal{S}$ is 1, instead of generating the standard Okamoto-Schnorr tuple, the signer raises the public key of the verifier to the first part of the signature.
- If the hidden bit of $\mathcal{S}$ is 0, then the signature is invalidated merely by selecting a random element from the underlying group.

In both cases, the second part of the tuple is calculated in the standard way. The details are given in Figure 1. Note that the unblinding of the first part of the signature, occurs on the exponent.

For the verification algorithm, the verifier checks the verification equation using the hash of the message and the commitment. If the secret signer bit is 1, then the signature will be valid, otherwise the verification equation will not hold. Thus the verifier will learn the secret bit of the signer. Details are presented in algorithm 5.

Note that in this specific instantiation of conditional blind signatures, the verifier can issue valid signatures by choosing a random $bsig_2$ and calculating the corresponding $bsig_1$ by the verification equation. So, in the specific scheme, it is natural to assume that the signer and the verifier are the same entity, but we intentionally distinguish them to comply with the broader definition of conditional blind signatures.

In the case of $b = 1$ the signatures are valid and can be verified by the designated verifier. The correctness property follows easily from the verification equation:

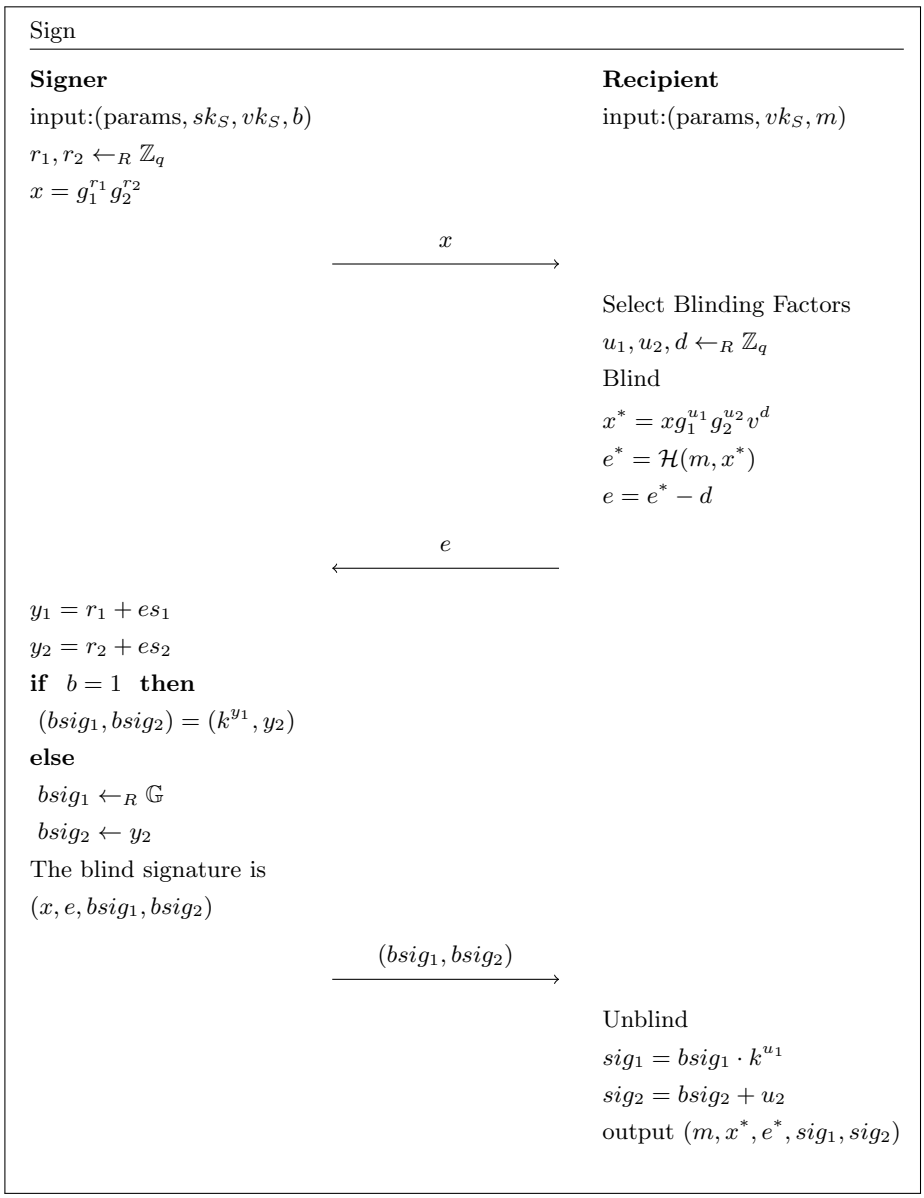


**Fig. 1.** Signing algorithm for Conditional Blind Signatures

**Algorithm 5:** Signature Verification

**Input :** $sk_e, pk_e, \text{params}, \mathcal{H}, m, sig = (x^*, e^*, sig_1, sig_2)$
**Output:** $b \in \{0, 1\}$

**if** $m \neq m'$ **then**
  return 0
**end**
$e^* \leftarrow H(m, x^*)$
${y_1}' = sig_1$
${y_2}' = sig_2$
**if** $x^{*s} = {y_1}' {g_2}^{{y_2}' \cdot s} v^{e^* \cdot s}$ **then**
  return 1
**else**
  return 0
**end**

$$
\begin{aligned}
x^{*s} = {y_1}' {g_2}^{{y_2}' \cdot s} v^{e^* \cdot s} &\Leftrightarrow (x g_1^{u_1} g_2^{u_2} v^d)^s = k^{y_1 + u_1} {g_2}^{(y_2 + u_2) \cdot s} v^{(e+d) \cdot s} \\
&\Leftrightarrow x^s k^{u_1} g_2^{s \cdot u_2} v^{s \cdot d} = k^{y_1} k^{u_1} {g_2}^{s y_2} g_2^{s u_2} v^{se} v^{sd} \\
&\Leftrightarrow x^s = k^{y_1} {g_2}^{s y_2} v^{se} \\
&\Leftrightarrow x^s = {g_1}^{s y_1} {g_2}^{s y_2} v^{se} \\
&\Leftrightarrow x = {g_1}^{y_1} {g_2}^{y_2} v^e \\
&\Leftrightarrow g_1^{r_1} g_2^{r_2} = g_1^{r_1 + e s_1} g_2^{r_2 + e s_2} (g_1^{-s_1} g_2^{-s_2})^e \\
&\Leftrightarrow g_1^{r_1} g_2^{r_2} = g_1^{r_1} g_1^{e s_1} g_2^{r_2} g_2^{e s_2} {g_1}^{-e s_1} {g_2}^{-e s_2} \\
&\Leftrightarrow g_1^{r_1} g_2^{r_2} = g_1^{r_1} g_2^{r_2}
\end{aligned}
$$

## 5 Security Analysis

### 5.1 Blindness

For the blindness property we can apply the arguments of the original Okamoto-Schnorr scheme [4]. More specifically, the commitment is blinded in exactly the same way in both schemes and the second parts of the signatures are identical in both cases. In addition, the message hash is hidden using the value $d$ exactly as in [4]. The first part of the signature is 'lifted' in our case, but the mapping from $y_1$ to $k^{y_1}$ is one to one and onto. As a result in the blindness game in section 3, the probability that an unbounded adversary succeeds in linking two protocol executions to the corresponding messages and signature pairs is exactly 1/2.

### 5.2 Strong One More Forgery

Our system is also secure against the strong version of the one more forgery assumption [8]. We note here that an adversary can create invalid signatures

by randomly choosing $y_2 \in \mathbb{Z}_q$ and a random element of $\mathbb{G}$. As a result, in the security proof, an interaction with the signer for an invalid signature does not provide any advantage, so we may assume that the adversary only interacts with the signer to obtain valid signatures.

The following theorem demonstrates that the system is secure under the (strong) one more forgery definition.

**Theorem 1.** *Suppose there exists a* PPT *adversary* $\mathcal{A}$ *that wins the* OneMoreForge *experiment, for* $l$ *polylogarithmic in the security parameter* $\lambda$*, with non negligible probability. Then there exists a* PPT *algorithm* $\mathcal{B}$ *that solves* Computational Diffie Hellman *problem with non negligible probability.*

*Proof.* Let $\mathcal{A}$ be such an adversary. If $\mathcal{A}$ receives two signatures $(m, x, e, k^{y_1}, y_2)$, $(m, x, \bar{e}, k^{\bar{y_1}}, \bar{y_2})$ for the same message with the same initial commitment and $y_2 - s_2 e \neq \bar{y_2} - s_2 \bar{e}$ then we can efficiently solve the CDH problem, that is calculate $g^{ab}$ by using $g, g^a, g^b$.

In order to obtain these signatures we apply a Replay Attack as in ([18], [8]). More specifically we run the algorithm with a random oracle $\mathcal{H}_1$ and then we repeat the same process with a random oracle $\mathcal{H}_2$ such that $\mathcal{H}_2$ yields the same answers to the first $i-1$ questions. We expect that with non negligible probability, we will achieve the collision in the $i$-th query.

This follows because there is an one to one and onto correspondence between $y_1$ and $k^{y_1}$ and so the probabilistic analysis presented in ([18],[8]) also holds for our scheme.

In more detail the reduction is as follows:

- Our input is $g, g^a, g^b$ and we want to compute $g^{ab}$
- We set $g = g_1, g_2 = g^a, k = g_1^s = g^s = g^b$. We select $s_1, s_2$ and compute the public key $v$
- We supply the public values $g_1, g_2, k, v$ to $\mathcal{A}$.
- We execute the forgery game with $\mathcal{A}$ and random oracle $\mathcal{H}_1$.
- We repeat the attack substituting $\mathcal{H}_1$ with $\mathcal{H}_2$.
- If we receive the required signatures since they are valid: $x^s = k^{y_1} {g_2}^{y_2 s} v^{es}$ and $x^s = k^{\bar{y_1}} {g_2}^{\bar{y_2} s} v^{\bar{e}s}$
- This means that:
$$k^{y_1} = x^s {g_2}^{-y_2 s} v^{-es}$$
$$k^{\bar{y_1}} = x^s {g_2}^{-\bar{y_2} s} v^{-\bar{e}s}$$
- In turn we have:
$$k^{y_1} k^{-\bar{y_1}} = {g_2}^{(-y_2 + \bar{y_2})s} v^{(-e+\bar{e})s}$$
- We know:
  - $t = k^{y_1} k^{-\bar{y_1}}$
  - $y = (-y_2 + \bar{y_2})$
  - $c = (-e + \bar{e})$
  - the values $s_1, s_2$ from the secret key that was generated by $\mathcal{A}$

Now we can calculate $g^{ab}$ from the above known values and $g_1, g_2, k$:

$$\begin{aligned} t = g_2^{ys} v^{cs} &\Rightarrow t = (g^a)^{ys}(g^{-s_1} g_2^{-s_2})^{cs} \\ &\Rightarrow t = g^{ays}(g^{-s_1} g^{-s_2 a})^{cs} \\ &\Rightarrow t = g^{ays} g^{-s_1 cs} g^{-s_2 acs} \\ &\Rightarrow t g^{s_1 cs} = g^{as(y - s_2 c)} \\ &\Rightarrow g^{as} = (t \cdot (g^s)^{s_1 c})^{(y - s_2 c)^{-1}} \\ &\Rightarrow g^{ab} = (t \cdot (g^b)^{s_1 c})^{(y - s_2 c)^{-1}} \end{aligned}$$

By using the same techniques as in ([18], [8]) it follows that the probability that this attack succeeds is non-negligible. □

### 5.3 Conditional Verifiability

Finally, we show that the system is conditionally verifiable by a reduction from the DDH problem:

**Theorem 2.** *Suppose there exist a* PPT *adversary* $\mathcal{A}$ *that wins the* CondVerExp *with non negligible probability. Then there exists a* PPT *algorithm* $\mathcal{B}$ *that solves* Decisional Diffie Hellman *problem with non negligible probability.*

*Proof.* We will construct $\mathcal{B}$.

- $\mathcal{B}$ gets as input $g, g^a, g^s, g^c$. She tries to find whether $c = as$ or $c$ is uniformly distributed in $\mathbb{G}$.
- $\mathcal{B}$ sets $g_1 = g$, $g_2 = g^a$ and $k = g_1^s$. She randomly chooses $s_1, s_2$ and sets $v = g_1^{-s_1} g_2^{-s_2}$. She gives $g_1, g_2, k, v$ to $\mathcal{A}$.
- Using the secret key $(s_1, s_2)$ $\mathcal{B}$ can answer $\mathcal{A}$'s valid signature requests.
- When $\mathcal{B}$ gets a challenge request from $\mathcal{A}$ she does the following:
  - She randomly chooses $r_1, r_2$ and sends $x = g_1^{r_1} g_2^{r_2}$ to $\mathcal{A}$.
  - $\mathcal{A}$ responds with $e$.
  - $\mathcal{B}$ chooses a random $y_2$ and sets

$$k^{y_1} = (g^s)^{r_1}(g^c)^{r_2}(g^c)^{-y_2}(g^s)^{s_1 e}(g^c)^{s_2 e}$$

  - $\mathcal{B}$ sends the signature pair $(bsig_1, bsig_2) \leftarrow (k^{y_1}, y_2)$
- As before $\mathcal{B}$ responds to $\mathcal{A}$'s signing requests using the secret key $(s_1, s_2)$.
- $\mathcal{B}$ outputs 1 (the input is a DDH tuple) iff $\mathcal{A}$ outputs 1 (valid signature).

The validity of the signature is fully defined by the message $(bsig_1, bsig_2)$ sent by the signer. The signature is valid iff $k^{y_1} = x^s {g_2}^{-y_2 s} v^{-es}$. Now we have:

$$\begin{aligned} k^{y_1} = x^s {g_2}^{-y_2 s} v^{-es} &\Leftrightarrow (g^s)^{r_1}(g^c)^{r_2}(g^c)^{-y_2}(g^s)^{s_1 e}(g^c)^{s_2 e} = x^s {g_2}^{-y_2 s} v^{-es} \\ &\Leftrightarrow g^{sr_1} g^{cr_2} g^{-cy_2} g^{ss_1 e} g^{cs_2 e} = g^{sr_1} g_2^{sr_2} g_2^{-sy_2} g^{ss_1 e} g_2^{ss_2 e} \\ &\Leftrightarrow (g^c)^{(r_2 - y_2 + s_2 e)} = (g^{as})^{(r_2 - y_2 + s_2 e)} \end{aligned}$$

This means that if $r_2 - y_2 + s_2 e \neq 0$ then the signature is valid iff $g^c = g^{as}$ which means that the input is a DDH tuple. Since $y_2$ is chosen randomly $r_2 - y_2 + s_2 e = 0$ only with negligible probability which yields the result. □

The theorems above prove that the system is secure. We must note however that security for one more forgery depends on the fact that the number of valid signatures is poly logarithmic to the security parameter, which is not strong enough. We leave it as future work to strengthen our scheme to attacks that require a polynomial number of signatures.

## 6 Conclusion, Applications and Future Work

In this paper, we proposed a new digital signature primitive called Conditional Blind Signatures. We defined and proved its security properties based on a particular instantiation.

Our scheme is general and as such, it can be used in every case where the act of signing a message, must convey some additional piece of information to a designated verifier. As we noted in the introduction, one major such application can be found in the case of coercion resistant electronic voting, where a voter must defeat a strong adversary that wishes to dictate the vote and threatens with countermeasures if the voter does not comply. The possibility of coercion is the most important obstacle to the large scale application of internet voting.

Our primitive can be used to design an efficient protocol that utilizes the JCJ coercion resistance framework [13], where the voter can cast many ballots authenticated using indistinguishable anonymous credentials. Before the election one such credential is registered as authentic with the registration authority. A vote should be counted only if it is accompanied by this specific credential. As a result, when the voter is under coercion she can provide a different one, in effect cancelling this ballot. When she gets her moment of privacy, as required by the JCJ framework, she can cast a vote with the registered credential. Of course the coercer should not be able to distinguish the two cases. This does not apply to the tallier, who must be able to tell which votes should be counted and which should not.

A protocol that utilizes our primitive to implement the above scenario involves a registration authority that compares the credentials supplied during voting with the one that is registered before. The secret bit of the signer is the result of this comparison and indicates whether the credential is valid or not. The comparison can be easily carried out by the registrar, since he has access to the voter identity. By applying our primitive, he can convey this bit of information, about the validity of the vote to the tallier, who will act as the designated verifier. Thus, she will learn if the votes are under coercion or not and proceed to count them in the former case. We leave it as future work to fully design a complete electronic voting protocol based on our primitive and reason about its properties.

Future work will also aim to design protocols that utilize the security properties of our primitive in other scenarios such as electronic auctions. Finally, we

plan to investigate different instantiations of the primitive and improve it. One thing that must be done in this regard, is to efficiently allow the signer to extend the secret information to more than 1 bits. The most important weakness that we must overcome, however, is the fact that our scheme is proved secure against forgeries, only if the adversary is restricted to a polylogarithmic number of issued signatures. We must design protocols that are secure against stronger adversaries that can request a polynomial number of signatures in their forgery attempt.

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