ON THE SECURITY OF THE WOTS-PRF SIGNATURE SCHEME

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Abstract. We identify a flaw in the security proof and a flaw in the concrete security analysis of the WOTS-PRF variant of the Winternitz one-time signature scheme, and discuss the implications to its concrete security.

1. Introduction

The Winternitz one-time signature (WOTS) scheme (see [22, 8]) is an optimization of a one-time signature scheme first described by Lamport [20]; the latter is now called the Lamport-Diffie one-time signature scheme. The WOTS scheme is widely believed to be resistant to attacks by large-scale quantum computers, and therefore is a prime candidate for inclusion in emerging standards for post-quantum cryptography.

Several variants of WOTS have been proposed and studied in the literature. The original WOTS scheme used a one-way function and was analyzed by Dods et al. [6]. The Leighton and Micali scheme WOTS-LM is described in an IETF Internet-Draft [21], and has been analyzed in the random oracle model [17] and the quantum random oracle model [7]. Buchmann et al. [4] (see also [3, 11]) proposed a variant, called WOTS-PRF, that uses a pseudorandom function (PRF) instead of a hash function. Another hash-based WOTS variant, called WOTS$^+$, was proposed by Hülsing [12] and has been included in an IETF Internet-Draft [14]. In [16], a modification of WOTS$^+$ specifically designed to resist multi-target attacks was studied.

The practicality of a one-time signature scheme is enhanced by using a Merkle tree [22] to simultaneously authenticate many public keys for the one-time signature scheme. Merkle tree-based signature schemes that use a WOTS variant as the underlying one-time signature scheme include the eXtended Merkle Signature Scheme (XMSS) [5], XMSS$^+$ [13], XMSS^{MT} [15], and XMSS-T [16].

The most attractive feature of WOTS-PRF is that it has a reductionist security proof with minimal assumptions [4], namely the existence of a secure PRF whose existence in turn is guaranteed by the existence of one-way functions [9, 10]. This is unlike, say, WOTS-LM whose only known security proof assumes that the underlying hash function is a purely random function [17], or WOTS$^+$ whose security proof assumes the existence of a one-way function that is also second-preimage resistant and 'undetectable' [12].

In this paper, we show that the security proof for WOTS-PRF in [4] is flawed. Furthermore, we show that even if the flaw can be repaired, the concrete security analysis in [4] is incorrect since it underestimates the possible number of "key collisions" for the PRF by using an unconstructible reductionist argument to relate this number to PRF security. We

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show that this underestimation leads to a drastic overestimation of the concrete security of WOTS-PRF and the Merkle signature schemes that employ it including XMSS and XMSS$^+$.

The remainder of the paper is organized as follows. The WOTS-PRF signature scheme is described in §2. In §3 we identify a flaw in the reductionist security proof. The flaw in the concrete security analysis and its implications are presented in §4. We make some concluding remarks in §5.

2. The WOTS-PRF signature scheme

The WOTS-PRF signature scheme [4] has the following ingredients:

(1) A security parameter $n \in \mathbb{N}$.
(2) The bitlength m of messages.
(3) A Winternitz parameter $w \in \mathbb{N}$, which for simplicity we will take to be a power of two: $w = 2^e$.
(4) A pseudorandom function $f : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$. For $(k, x) \in \{0,1\}^n \times \{0,1\}^n$, we will denote $f(k, x)$ by $f_k(x)$. The *iterates* of f are defined as follows. For $(k, x) \in \{0,1\}^n \times \{0,1\}^n$,
$$f_k^0(x) = k \quad \text{and} \quad f_k^i(x) = f_{f_k^{i-1}(x)}(x) \text{ for } i \geq 1.$$
Thus, $f_k^1(x) = f_k(x)$, $f_k^2(x) = f_{f_k(x)}(x)$, and so on.
(5) A checksum C on messages defined as follows: set
$$\ell_1 = \left\lceil \frac{m}{e} \right\rceil, \quad \ell_2 = \left\lfloor \frac{\log_2(\ell_1(w-1))}{e} \right\rfloor + 1, \quad \ell = \ell_1 + \ell_2.$$
Define $C : \{0,1\}^m \to \{0,1\}^{e\ell_2}$ as follows. Let $M \in \{0,1\}^m$. Obtain M^0 by prepending M with 0's until the bitlength of M^0 is $e\ell_1$, and then write $M^0 = M_1 \| M_2 \| \cdots \| M_{\ell_1}$ where each M_i has bitlength e. Interpret each M_i as a non-negative integer and compute $c(M) = \sum_{i=1}^{\ell_1}(w - 1 - M_i)$. The checksum $C(M)$ is obtained by converting $c(M)$ to a binary string and then prepending 0's as necessary to obtain a binary string of bitlength exactly $e\ell_2$.

We next present the WOTS-PRF signature scheme.

Key generation. Each user A does the following:

(1) Select $x \in_R \{0,1\}^n$.
(2) Select $sk_1, sk_2, \ldots, sk_\ell \in_R \{0,1\}^n$.
(3) Compute $pk_i = f_{sk_i}^{w-1}(x)$ for $i = 1, 2, \ldots, \ell$; $(sk_i, f_{sk_i}^1(x), f_{sk_i}^2(x), \ldots, f_{sk_i}^{w-1}(x))$ is called the i-th Winternitz hash chain.
(4) A's public signature verification key is $pk = (pk_0, pk_1, \ldots, pk_\ell)$ where $pk_0 = x$. A's secret signature generation key is $sk = (sk_1, sk_2, \ldots, sk_\ell)$.

Signature generation. To sign a message $M \in \{0,1\}^m$, A does the following:

(1) Compute the checksum $C = C(M)$, and let $B = M^0 \| C = b_1 \| b_2 \| \cdots \| b_\ell$ where each b_i has bitlength e.
(2) Compute $\sigma_i = f_{sk_i}^{b_i}(x)$ for $i = 1, 2, \ldots, \ell$.
(3) A's signature on M is $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_\ell)$.

Signature verification. To verify A's signed message (M, σ), the verifier does the following:

(1) Compute the checksum $C = C(M)$, and let $B = M^0 \| C = b_1 \| b_2 \| \cdots \| b_\ell$ where each b_i has bitlength e.
(2) Compute $pk_i' = f_{\sigma_i}^{w-1-b_i}(pk_0)$ for $i = 1, 2, \ldots, \ell$.
(3) Accept the signature if and only if $pk_i' = pk_i$ for all $i = 1, 2, \ldots, \ell$.

3. The WOTS-PRF security proof

This section presents the WOTS-PRF reductionist security proof from [4] and the flaw we observed in the analysis of its success probability. We begin with the definitions of a secure one-time signature scheme, a secure pseudorandom function, and the maximum and minimum number of key collisions.

Definition 1. A one-time signature scheme $\mathcal{S}$ is said to be (t, ϵ)-secure if all adversaries $\mathcal{A}_{\mathcal{S}}$ whose running times are bounded by t have success probability less than ϵ in the following game: $\mathcal{A}_{\mathcal{S}}$ is given a public key pk for $\mathcal{S}$ and can query a signing oracle (with respect to pk) for the signature σ of one message m of its choosing; $\mathcal{A}_{\mathcal{S}}$'s challenge is to generate a valid signed message (m^*, σ^*) with $m^* \neq m$. The security level of $\mathcal{S}$ is $\log_2(t/\epsilon)$ bits.

Definition 2. A function $f : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ is said to be a (t, ϵ)-secure PRF if all adversaries $\mathcal{A}_f$ whose running times are bounded by t have advantage less than ϵ in the following game: $\mathcal{A}_f$ is given blackbox access to an oracle $O(\cdot)$ that with equal probability is either $f_k(\cdot)$ for hidden key $k \in_R \{0,1\}^n$ or else a random function $R : \{0,1\}^n \to \{0,1\}^n$; $\mathcal{A}_f$'s challenge is to determine which it is. ($\mathcal{A}_f$'s advantage is the absolute value of the differences in probabilities that $\mathcal{A}_f$ declares that $O(\cdot)$ is $f_k(\cdot)$ in the case where $O(\cdot)$ is $f_k(\cdot)$ and the case where $O(\cdot)$ is $R(\cdot)$.) The security level of f is $\log_2(t/\epsilon)$ bits.

Definition 3. Consider the function $f : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$. For each pair $(k, x) \in \{0,1\}^n \times \{0,1\}^n$, let

$$N_{k,x} = \#\{k' \in \{0,1\}^n \; : \; f_{k'}(x) = f_k(x)\},$$

and

$$T_x = \max_k \{N_{k,x}\} \quad \text{and} \quad S_x = \min_k \{N_{k,x}\}.$$

Then the maximum number κ and minimum number κ' of key collisions are

$$\kappa = \max_x \{T_x\} \quad \text{and} \quad \kappa' = \min_x \{S_x\}.$$

Observe that $N_{k,x} \geq 1$, and so $1 \leq \kappa' \leq \kappa$. We note that the definition of κ' in [4] is incorrect, as are the definitions of κ and κ' in [3]. Our definitions of κ and κ' are equivalent to those given in [11].

In [4], the following notion of a key one-way (KOW) function is introduced.

Definition 4. A function $f : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ is said to be (t, ϵ)-KOW if all adversaries $\mathcal{A}_{KOW}$ whose running times are bounded by t have advantage less than ϵ in the following game: $\mathcal{A}_{KOW}$ is given (x, y), where $x, k \in_R \{0,1\}^n$ and $y = f_k(x)$; $\mathcal{A}_{\text{KOW}}$'s challenge is to find some $k' \in \{0,1\}^n$ with $f_{k'}(x) = y$.

Proposition 2.7 in [4] shows that a (t,ϵ)-secure PRF is a $(t-2, \epsilon/(1/\kappa - 1/2^n))$-KOW. The following is the main security claim in [4]. We include a summary of the proof from [4].

Theorem 1 (Theorem 2.8 in [4])**.** *Let $f : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ be a (t', ϵ')-secure PRF. Then WOTS-PRF is a (t,ϵ)-secure one-time signature scheme with*

$$t = t' - t_{\mathrm{Kg}} - t_{\mathrm{Vf}} - 2, \tag{1}$$

$$\epsilon \leq \epsilon' \ell^2 w^2 \kappa^{w-1} \frac{1}{1/\kappa - 1/2^n}, \tag{2}$$

where t_{Kg} and t_{Vf} denotes the running times of the WOTS-PRF key generation and verification algorithms, respectively.

Summary of proof from [4]. Suppose that $\mathcal{A}_{\mathrm{WOTS}}$ is a forger that runs in time t and produces a WOTS-PRF forgery with probability at least ϵ. We construct an adversary $\mathcal{A}_{\mathrm{KOW}}$ that uses $\mathcal{A}_{\mathrm{WOTS}}$ to solve the KOW challenge.

The adversary $\mathcal{A}_{\mathrm{KOW}}$ is given a KOW challenge (x, y). It begins by generating a WOTS-PRF key pair as specified in §2 with one exception. It selects random indices $\alpha \in_R [1, \ell]$ and $\beta \in_R [1, w-1]$. Instead of selecting the secret key component sk_α and computing $pk_\alpha = f^{w-1}_{sk_\alpha}(x)$, $\mathcal{A}_{\mathrm{KOW}}$ sets $pk_\alpha = f^{w-1-\beta}_y(x)$; i.e., it inserts y at position β in the Winternitz hash chain used to compute pk_α.

Next, $\mathcal{A}_{\mathrm{KOW}}$ invokes $\mathcal{A}_{\mathrm{WOTS}}$ with public key pk and answers its signing oracle query as follows. If $b_\alpha < \beta$, then $\mathcal{A}_{\mathrm{KOW}}$ terminates the experiment since it doesn't know the first β entries of the α'th Winternitz hash chain. Otherwise, if $b_\alpha \geq \beta$, then $\mathcal{A}_{\mathrm{WOTS}}$ produces the required signature as specified in §2. If $\mathcal{A}_{\mathrm{WOTS}}$ produces a valid forger (M', σ') within its allotted time, and if $b'_\alpha < \beta$, then $\mathcal{A}_{\mathrm{WOTS}}$ computes $k' = f^{\beta-1-b'_\alpha}_{\sigma'_\alpha}(x)$ and outputs k' if $f_{k'}(x) = y$; otherwise $\mathcal{A}_{\mathrm{WOTS}}$ terminates with failure. See Figure 1.

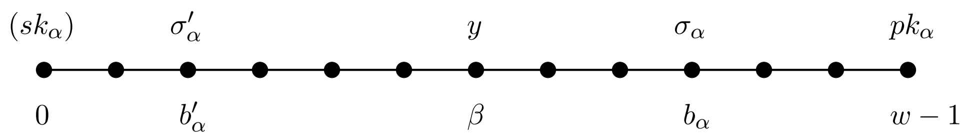

FIGURE 1. The α'th Winternitz hash chain in $\mathcal{A}_{\mathrm{KOW}}$'s experiment.

$\mathcal{A}_{\mathrm{KOW}}$'s success probability ϵ_{KOW} is assessed as follows. The probability that $b_\alpha \geq \beta$ is at least $(\ell w)^{-1}$. The probability that $\mathcal{A}_{\mathrm{WOTS}}$ succeeds is at least ϵ subject to the condition that pk is a valid public key, i.e., there exists $sk_\alpha \in \{0,1\}^n$ such that $f^{\beta}_{sk_\alpha}(x) = y$. This happens with probability at least $1/\kappa^\beta$ according to Definition 3. The probability that $b'_\alpha < \beta$ is at least $(\ell w)^{-1}$. The probability that $y = f_{k'}(x)$ holds where $k' = f^{\beta-1-b'_\alpha}_{\sigma'_\alpha}(x)$ is at least $1/\kappa^{w-1-\beta}$. This is because there exists at most κ^{w-1} keys mapping x to pk_α after $w-1$ iterations of f and only κ^β of these keys maps x to y after β iterations.

In summary we have $\epsilon_{\mathrm{KOW}} \geq \epsilon/(\ell^2 w^2 \kappa^\beta \kappa^{w-1-\beta})$ and $t_{\mathrm{KOW}} = t + t_{\mathrm{Kg}} + t_{\mathrm{Vf}}$. This yields a PRF forger $\mathcal{A}_{\mathrm{PRF}}$ with $\epsilon_{\mathrm{PRF}} \geq \epsilon(1/\kappa - 1/2^n)/(\ell^2 w^2 \kappa^{w-1})$ and $t_{\mathrm{PRF}} = t + t_{\mathrm{Kg}} + t_{\mathrm{Vf}} + 2$. □

We observe a flaw in the proof of Theorem 1, which pertains to the probability analysis of the reduction. To aid in our explanations, we introduce the notion of a *keychain*.

Definition 5. Let $f : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ be a PRF, and fix $x \in \{0,1\}^n$. For any $\gamma \in \mathbb{N}$ and $y \in \{0,1\}^n$, a *γ-keychain to y* is an ordered tuple $(k_1, k_2, \ldots, k_\gamma)$ of n-bit keys such that $k_{i+1} = f_{k_i}(x)$ for $i = 1, 2, \ldots, \gamma - 1$ and $k_\gamma = y$.

The flaw is in the claim that the probability that $y = f_{k'}(x)$ holds is at least $1/\kappa^{w-1-\beta}$. Consider the tree of all w-keychains to pk_α; see Figure 2. By definition of κ, there exist at

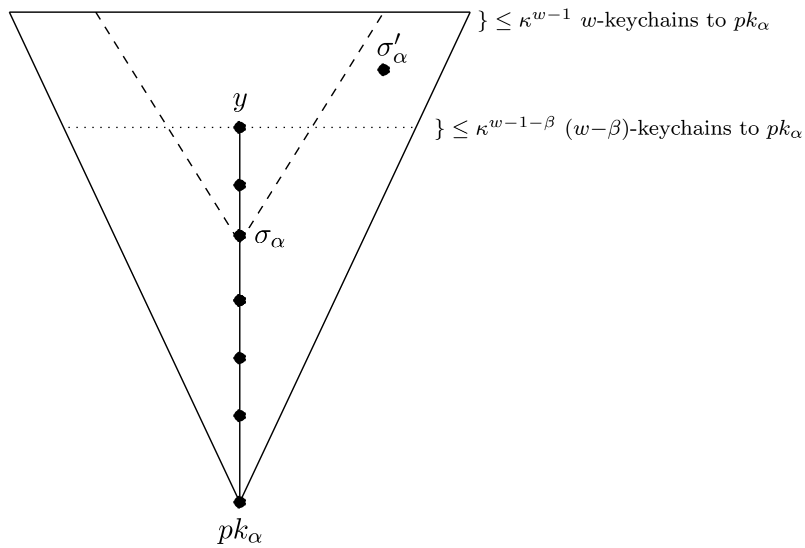

FIGURE 2. The tree of w-keychains to pk_α.

most $\kappa^{w-1-\beta}$ $(w-\beta)$-keychains to pk_α. Note that y is the first coordinate of one of these keychains. Now, since $b'_\alpha < \beta$, the $(w - b'_\alpha)$-keychain to pk_α beginning at σ'_α must connect with one of the $(w-\beta)$-keychains to pk_α. If the connecting keychain is selected uniformly at random, then the probability that the connecting keychain begins with y (and thus $y = f_{k'}(x)$) is indeed at least $1/\kappa^{w-1-\beta}$. However, there is no justification for assuming that $\mathcal{A}_{\mathrm{WOTS}}$ selects a connecting chain uniformly at random. Indeed, since $\mathcal{A}_{\mathrm{WOTS}}$ knows σ_α, it is conceivable that it always selects σ'_α so that the $(w - b'_\alpha)$-keychain beginning at σ'_α does not pass through σ_α, and thus never connects with y; in this event, the probability that $y = f_{k'}(x)$ holds is zero.

4. Concrete security of WOTS-PRF

In [4], the following relationship between the security level of the PRF f and the maximum number of key collisions κ for f is proven.

Lemma 2. *Let $f : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$ be a (t, ϵ)-secure PRF with security level $b = \log_2(t/\epsilon)$. Then $\kappa \leq 2^{n-b} + 1$.*

Proof, paraphrased from [4]. Suppose that $\kappa > 2^{n-b}+1$ and let $(x, y) \in \{0,1\}^n \times \{0,1\}^n$ be a pair for which there exist κ keys k for which $f_k(x) = y$. We construct a PRF-adversary $\mathcal{A}_f$ as follows. $\mathcal{A}_f$ queries its oracle $O(\cdot)$ with x. If $O(x) = y$ then $\mathcal{A}_f$ declares that

$O(\cdot)$ is $f_k(\cdot)$; otherwise it declares that $O(\cdot)$ is $R(\cdot)$. Clearly $\mathcal{A}_f$'s runs in time $t' = 1$. Furthermore,

$$\Pr[\mathcal{A}_f \text{ declares that } O(\cdot) \text{ is } f_k(\cdot) \mid O(\cdot) \text{ is indeed } f_k(\cdot)] = \frac{\kappa}{2^n} > 2^{-b} + 2^{-n}$$

and

$$\Pr[\mathcal{A}_f \text{ declares that } O(\cdot) \text{ is } f_k(\cdot) \mid O(\cdot) \text{ is indeed } R(\cdot)] = 2^{-n}.$$

Hence $\mathcal{A}_f$'s advantage is $\epsilon' > 2^{-b}$, which contradicts the assumed PRF security level of b for f. □

Since the only way for the adversary of a good PRF f to gain an advantage is to guess the hidden key, the authors of [4] conclude that f can be expected to have security level $b = n$, whence $\kappa \leq 2$. However, we will argue that $\kappa = 2$ is a severe underestimation of the maximum number of key collisions for f. The problem with the proof of Lemma 2 is that the adversary $\mathcal{A}_f$ described is *non-constructive* since no efficient method for determining the pair (x, y) for f may be known. On the other hand, the security level b of the PRF f is usually assessed by considering all known *constructible* algorithms for the PRF security game in Definition 2. Thus, $\mathcal{A}_f$'s advantage $\epsilon' > 2^{-b}$ in the proof does not contradict the assumed security level of f.

We show in §4.1 that κ can be expected to be considerably larger than 2 even for 'good' PRFs. The implications of the underestimation of κ to the concrete security guarantees for WOTS-PRF are explored in §4.2.

Remark 1. As argued in [18, 19] (see also [2]), the security level of a PRF f against attacks that might be unconstructible is expected to be significantly lower that when only constructible attacks are considered. In particular, if f is a good PRF with security level n against constructible attacks, then f can be expected to have security level no more than $n/2$ against unconstructible attacks. Furthermore, determining the exact security level of f against unconstructible attacks is expected to be a very challenging undertaking. The significance of the difference in the constructible and unconstructible security levels of f to the concrete security guarantees of Bellare's security proof [1] for the HMAC authentication scheme is discussed in [18, 19].

Remark 2. A one-time signature scheme $\mathcal{S}$ is said to be (t, ϵ)-strongly secure if, in addition to satisfying Definition 1, it is required that the signed message (m^*, σ^*) produced by the adversary $\mathcal{A}_{\mathcal{S}}$ satisfies $(m^*, \sigma^*) \neq (m, \sigma)$. Theorem 3.5 of [4] proves that WOTS-PRF is strongly secure assuming that the underlying PRF f is second-key resistant (SKR) or key-collision resistant (KCR). Furthermore, it is assumed that the minimum number of key collisions κ' for f (see Definition 3) satisfies $\kappa' \geq 2$. However, since

$$\kappa' = \min_{(k,x)} \{N_{k,x}\},$$

it is highly unlikely that $\kappa' \neq 1$ for PRFs f used in practice. Indeed, one would expect with overwhelming probability that $N_{k,x} = 1$ for at least one pair (k, x) for a function f selected uniformly at random from the space of all functions from $\{0,1\}^n \times \{0,1\}^n$ to $\{0,1\}^n$. Thus, the claim that WOTS-PRF is strongly secure if $\kappa' \geq 2$ is vacuous for common constructions of PRFs.

4.1. **Balls and bins.** Consider an experiment wherein N balls are thrown, independently and uniformly at random, into N bins. Of interest is the expected maximum number of balls in any bin. This study is analogous to the determination of the expected value of T_x for a fixed $x \in \{0,1\}^n$ (cf. Definition 3) for a uniform random function $f : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$. Here, the balls are the keys $k \in \{0,1\}^n$ (so $N = 2^n$), the bins are the elements of the codomain $\{0,1\}^n$, and ball k is placed in bin $f_k(x)$. Then the expected maximum number M of balls in a bin is equal to the expected value of T_x, which in turn is at most the expected value of κ.

Theorem 3 ([23])**.** *Consider an experiment wherein N balls are randomly assigned to N bins. Let M be the random variable that counts the maximum number of balls in a bin. Then*

$$E[M] = \frac{\ln N}{\ln \ln N}(1 + o(1)) \text{ with probability } 1 - o(1).$$

Moreover,

$$\Pr[\text{ there is at least one bin with } \geq \alpha \frac{\ln N}{\ln \ln N} \text{ balls }] = \begin{cases} 1 - o(1), & \text{if } 0 < \alpha < 1, \\ o(1), & \text{otherwise.} \end{cases}$$

Clearly the value $\ln N / \ln \ln N$ can be made arbitrarily large. Hence, for any $t \in \mathbb{N}$ one can produce values $0 < \alpha < 1$ and $N \in \mathbb{N}$ such that $\alpha \ln N / \ln \ln N \geq t$. Thus, even though the PRF f is not uniformly random, this gives strong evidence that $\kappa \leq 2$ is in general false.

4.2. **Concrete security assurances of WOTS-PRF and XMSS.** The tightness gap in the security reduction of Theorem 1 is

$$\ell^2 w^2 \kappa^{w-1} \frac{1}{1/\kappa - 1/2^n} \approx \ell^2 w^2 \kappa^w,$$

which is sensitive to the value to κ. For example, suppose that the PRF f is instantiated using AES with 128-bit keys, whereby it is reasonable to assume that it has a security level of 128 bits. The authors of [4], take $\kappa = 2$, $m = 128$, $w = 16$ and conclude that Theorem 1 guarantees a security level of at least 91 bits for WOTS-PRF. However, since one expects that

$$\kappa \geq \frac{\ln(2^{128})}{\ln(\ln(2^{128}))} \approx 20,$$

Theorem 1 can guarantee a security level of at most 39 bits for WOTS-PRF, which is insufficient in practice.

As a second example, consider XMSS when instantiated with WOTS-PRF. The security proof in [11] yields an XMSS security level of

$$b > n - h - 3 - \max\{h + 1, w \log_2(\kappa) + \log_2(\ell w)\}, \tag{3}$$

where h is the height of the XMSS tree. Taking $n = m = 256$, $w = 64$, $\kappa = 2$ and $h = 16$, Table 7.1 concludes that XMSS has a security level of at least 161 bits. However, since one expects that

$$\kappa \geq \frac{\ln(2^{256})}{\ln(\ln(2^{256}))} \approx 34.3,$$

the security bound (3) can at best guarantee that $b > -100$, which is vacuous.

Similar conclusions can be drawn about the concrete security levels given for XMSS in [5] and XMSS$^+$ in [13].

5. Concluding remarks

We emphasize that our observations on the WOTS-PRF security proof have no bearing on the security proofs for other variants of WOTS such as WOTS-LM and WOTS$^+$. Furthermore, our remarks in §4.2 on the concrete security bounds for XMSS and XMSS$^+$ only apply when these signature schemes are instantiated with WOTS-PRF. In particular, they are not applicable to XMSS as described in the IETF Internet-Draft [14] where WOTS$^+$ is the underlying one-time signature scheme.

An open problem is to devise a (tight) reductionist security proof for WOTS-PRF (or a variant of it) under the sole assumption that f is a secure PRF.

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References

[1] M. Bellare, "New proofs for NMAC and HMAC: Security without collision resistance", *Advances in Cryptology — CRYPTO 2006*, LNCS 4117 (2006), 602–619.

[2] D. Bernstein and T. Lange, "Non-uniform cracks in the concrete: the power of free computation", *Advances in Cryptology — ASIACRYPT 2013*, LNCS 8270 (2013), 321–340.

[3] J. Buchmann, E. Dahmen, S. Ereth, A. Hülsing and M. Rückert, "On the security of the Winternitz one-time signature scheme", *Progress in Cryptology — AFRICACRYPT 2011*, LNCS 6737 (2011), 363–378.

[4] J. Buchmann, E. Dahmen, S. Ereth, A. Hülsing and M. Rückert, "On the security of the Winternitz one-time signature scheme", *International Journal of Applied Cryptography*, 3 (2013), 84–96.

[5] J. Buchmann, E. Dahmen and A. Hülsing, "XMSS – a practical forward secure signature scheme based on minimal security assumptions", *Post-Quantum Cryptography — PQCrypto 2011*, LNCS 7071 (2011), 117–129.

[6] C Dods, N. Smart and M. Stam, "Hash based digital signature schemes", *Cryptography and Coding*, LNCS 3796 (2005), 96–115.

[7] E. Eaton, "Leighton-Micali hash-based signatures in the quantum random-oracle model", *Selected Areas in Cryptography — SAC 2017*, to appear. Available at eprint.iacr.org/2017/607.

[8] S. Even, O. Goldreich and S. Micali, "On-line/off-line digital signatures", *Journal of Cryptology*, 9 (1996), 35–67.

[9] O. Goldreich, S. Goldwasser and S. Micali, "How to construct random functions", *Journal of the ACM*, 33 (1986), 792–807.

[10] J. Håstad, R. Impagliazzo, L. Levin and M. Luby, "A pseudorandom generator from any one-way function", *SIAM Journal on Computing*, 28 (1999), 1364–1396.

[11] A. Hülsing, "Practical forward secure signatures using minimal security assumptions", Ph.D. thesis, Technical University of Darmstadt, 2013.

[12] A. Hülsing, "W-OTS$^+$ — Shorter signatures for hash-based signature schemes", *Progress in Cryptology — AFRICACRYPT 2013*, LNCS 7918 (2013), 173–188.

[13] A. Hülsing, C. Busold and J. Buchmann, "Forward secure signatures on smart cards", *Selected Areas in Cryptography — SAC 2012*, LNCS 7707 (2013), 66–80.

[14] A. Hülsing, D. Butin, S. Gazdag, J. Rijneveld and A. Mohaisen, "XMSS: Extended hash-based signatures", Internet Draft, July 24, 2017; available at https://tools.ietf.org/html/draft-irtf-cfrg-xmss-hash-based-signatures-10.

[15] A. Hülsing, L. Rausch and J. Buchmann, "Optimal parameters for XMSS^{MT}", *Availability, Reliability, and Security in Information Systems and HCI — CD-ARES 2013*, LNCS 8128 (2013), 194–208.

[16] A. Hülsing, J. Rijneveld and F. Song, "Mitigating multi-target attacks in hash-based signatures", *Public-Key Cryptography — PKC 2016*, LNCS 9614 (2016), 387–416.

[17] J. Katz, "Analysis of a proposed hash-based signature scheme", *Security Standardisation Research — SSR 2016*, LNCS 10074 (2016), 261–273.

[18] N. Koblitz and A. Menezes, "Another look at HMAC", *Journal of Mathematical Cryptology*, 7 (2013), 225–251.

[19] N. Koblitz and A. Menezes, "Another look at non-uniformity", *Groups Complexity Cryptology*, 5 (2013), 117–140.

[20] L. Lamport, "Constructing digital signatures from a one way function", Technical Report CSL-98, SRI International, 1979.

[21] D. McGrew, M. Curcio and S. Fluhrer, "Hash-based signatures", Internet Draft, June 20, 2017; available at https://tools.ietf.org/html/draft-mcgrew-hash-sigs-07.

[22] R. Merkle, "A digital signature based on a conventional encryption function", *Advances in Cryptology — CRYPTO '87*, LNCS 293 (1988), 369–378.

[23] M. Raab and A. Steger, ""Balls into bins" – a simple and tight analysis", *Randomization and Approximation Techniques in Computer Science – RANDOM 1998*, LNCS 1518 (1998), 159–170.

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