On the Exact Round Complexity of Secure Three-Party Computation

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Abstract. We settle the exact round complexity of three-party computation (3PC) in honest-majority setting, for a range of security notions such as selective abort, unanimous abort, fairness and guaranteed output delivery. Selective abort security, the weakest in the lot, allows the corrupt parties to selectively deprive some of the honest parties of the output. In the mildly stronger version of unanimous abort, either all or none of the honest parties receive the output. Fairness implies that the corrupted parties receive their output only if all honest parties receive output and lastly, the strongest notion of guaranteed output delivery implies that the corrupted parties cannot prevent honest parties from receiving their output. It is a folklore that the implication holds from the guaranteed output delivery to fairness to unanimous abort to selective abort. We focus on two network settings– pairwise-private channels without and with a broadcast channel.

In the minimal setting of pairwise-private channels, 3PC with selective abort is known to be feasible in just two rounds, while guaranteed output delivery is infeasible to achieve irrespective of the number of rounds. Settling the quest for exact round complexity of 3PC in this setting, we show that three rounds are necessary and sufficient for unanimous abort and fairness. Extending our study to the setting with an additional broadcast channel, we show that while unanimous abort is achievable in just two rounds, three rounds are necessary and sufficient for fairness and guaranteed output delivery. Our lower bound results extend for any number of parties in honest majority setting and imply tightness of several known constructions.

The fundamental concept of garbled circuits underlies all our upper bounds. Concretely, our constructions involve transmitting and evaluating only constant number of garbled circuits. Assumption-wise, our constructions rely on injective (one-to-one) one-way functions.

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1 Introduction

In secure multi-party computation (MPC) [GMW87, CDG87, Yao82], n parties wish to jointly perform a computation on their private inputs in a secure way, so that no adversary $\mathcal{A}$ actively corrupting a coalition of t parties can learn more information than their outputs (*privacy*), nor can they affect the outputs of the computation other than by choosing their own inputs (*correctness*). MPC has been a subject of extensive research and has traditionally been divided into two classes: MPC with dishonest majority [GMW87, DO10, BDOZ11, DPSZ12, GGHR14, BHP17, ACJ17] and MPC with honest majority [BGW88, CCD88, RB89, BMR90, Bea91, DN07, BFO12, BH06, CDD+99]. While the special case of MPC with dishonest majority, namely the two-party computation (2PC) has been at the focus of numerous works [Yao82, IPS08, Lin13, SS13, HKK+14, AMPR14, RR16, MR17], the same is not quite true for the special case of MPC protocols with honest majority.

The three-party computation (3PC) and MPC with small number of parties maintaining an honest majority make a fascinating area of research due to myriad reasons as highlighted below. First, they present useful use-cases in practice, as it seems that the most likely scenarios for secure MPC in practice would involve a small number of parties. In fact, the first large scale implementation of secure MPC, namely the Danish sugar beet auction [BCD+09] was designed for the three-party setting. Several other applications solved via 3PC include statistical data analysis [BTW12], email-filtering [LADM14], financial data analysis [BTW12] and distributed credential encryption service [MRZ15]. The practical efficiency of 3PC has thus got considerable emphasis in the past and some of them have evolved to technologies [Gei07, BLW08, LDDA12, LADM14, CMF+14, FLNW17, AFL+16]. Second, in practical deployments of secure computation between multiple servers that may involve long-term sensitive information, three or more servers are preferred as opposed to two. This enables recovery from faults in case one of the servers malfunctions. Third and importantly, practical applications usually demand strong security goals such as fairness (corrupted parties receive their output only if all honest parties receive output) and guaranteed output delivery (corrupted parties cannot prevent honest parties from receiving their output) which are feasible *only* in honest majority setting [Cle86]. Fourth and interestingly, there are evidences galore that having to handle a single corrupt party can be leveraged conveniently and taken advantage of to circumvent known lower bounds and impossibility results. A lower bound of three rounds has been proven in [GIKR02] for *fair* MPC with $t \geq 2$ and arbitrary number of parties, even in the presence of broadcast channels. [IKKP15] circumvents the lower bound by presenting a *two-round* 4PC protocol tolerating a *single* corrupt party that provides guaranteed output delivery without even requiring a broadcast channel. Verifiable secret sharing (VSS) which serves as an important tool in constructing MPC protocols are known to be impossible with $t \geq 2$ with one round in the sharing phase irrespective of the computational power of the adversary [GIKR01, PCRR09, BKP11]. Interestingly enough, a perfect VSS with $(n = 5, t = 1)$ [GIKR01], statistical VSS with $(n = 4, t = 1)$ [PCRR09, IKKP15] and cryptographic VSS with $(n = 4, t = 1)$ [BKP11] are shown to be achievable with one round in the sharing phase.

The world of MPC for small population in honest majority setting witnesses a few more interesting phenomena. Assumption-wise, MPC with 3, 4 and 5 parties can be built from just One-way functions (OWF) or injective one-way functions/permutations [IKKP15, MRZ15, CGMV17], shunning public-key primitives such as Oblivious Transfer (OT) entirely, which is the primary building block in the 2-party setting. Last but not the least, the known constructions for small population in the honest majority setting perform arguably better than the constructions with two parties while offering the same level of security. For instance, 3PC with honest majority [IKKP15, MRZ15] allows to circumvent certain inherent challenges in malicious 2PC such as enforcing correctness of garbling which incurs additional communication. The exact round complexity is yet another measure that sets apart the protocols with three parties over the ones with two parties. For instance, 3PC protocol is achievable just in two rounds with the minimal network setting of pairwise-private channels [IKKP15]. The 2PC (and MPC with dishonest majority) protocols achieving the same level of security (with abort) necessarily require 4 rounds [KO04] and have to resort to a common reference string (CRS) to shoot for the best possible round complexity of 2 [HLP11].

With the impressive list of motivations that are interesting from both the theoretical and practical viewpoint, we explore 3PC in the honest majority setting tolerating a malicious adversary. In this work, we set our focus on the exact round complexity of 3PC. To set the stage for our contributions, we start with a set of relevant works below.

Related Works. Since round complexity is considered an important measure of efficiency of MPC protocols, there is a rich body of work studying the round complexity of secure 2PC and MPC protocols under various adversarial settings and computational models. We highlight some of them below. Firstly, it is known that two rounds of interaction are essential for realizing an MPC protocol irrespective of the setting. This is because in a 1-round protocol, a corrupted party could repeatedly evaluate the "residual function" with the inputs of the honest parties fixed on many different inputs of its own (referred as "residual function" attack) [HLP11]. In the plain model, any actively secure 2PC is known to require 5 rounds in non-simultaneous message model [KO04] (under black-box simulation). The bound can be improved to 4 even in the dishonest majority setting [GMPP16] in simultaneous message model and tight upper bounds are presented in [BHP17, ACJ17, HHPV17]. With a common reference string (CRS), the lower bound can be further improved to 2 rounds [HLP11]. Tight upper bounds are shown in [GGHR14] under indistinguishability obfuscation (assumption weakened to witness encryption by [GLS15]), and in [MW16] under a variant of Fully Homomorphic Encryption (FHE) and Non-interactive Zero-knowledge (NIZK).

In the honest majority setting which is shown to be necessary [Cle86] and sufficient [BGW88, CCD88, CL14] for the feasibility of protocols with fairness and guaranteed output delivery, the study on round complexity has seen the following interesting results. Three is shown to be the lower bound for fair protocols in the broadcast-only model (no private channels), surprisingly even with access to a CRS [GLS15]. Several matching upper bounds can be found in [GLS15, ACGJ18, BJMS18] relying on tools such as Zaps, multi-key FHE, dense crypto-systems. The protocol of [GLS15] can be collapsed to two rounds given access to PKI where the infrastructure carries the public keys corresponding to the multi-key FHE they use.

In the plain model, three rounds are shown to be necessary for MPC with fairness and $t \geq 2$, even in the presence of a broadcast channel and arbitrary number of parties [GIKR02]. In an interesting work, [IKKP15] circumvents the above result by considering 4PC with *one* corruption. The protocol provides guaranteed output delivery, yet does not use a broadcast channel. In the same setting (plain model and no broadcast), [IKKP15] presents a 2-round 3PC protocol tolerating single corruption; whose communication and computation efficiency was improved by the 3-round protocol of [MRZ15]. Both these protocols achieve a weaker notion of security known as security with selective abort. Selective abort security [IKP10] (referred as 'security with abort and no fairness' in [GL02]) allows the corrupt parties to selectively deprive some of the honest parties of the output. In the mildly stronger version of unanimous abort (referred as 'security with unanimous abort and no fairness' in [GL02]), either all or none of the honest parties receive the output. An easy observation concludes that the 3PC of [MRZ15] achieves unanimous abort, when its third round message is broadcasted, albeit for functions giving the same output to all. The works relevant to honest majority setting are listed below.

3PC has been studied in different settings as well. High-throughput MPC with non-constant round complexity are studied in [FLNW17, AFL+16]. [CKMZ14] studies 3PC with dishonest majority. Recently, [CGMV17] presents a practically efficient 5-party MPC protocol in honest majority setting, going beyond 3-party case, relying on distributed garbling technique based on [BMR90].

Ref.	Setting	Round	Network Setting / Assumption	Security	Comments
[AJL+12]	$t < n/2$	≥ 5	private channel, Broadcast / CRS, FHE, NIZK	fairness	upper bound
[GLS15]	$t < n/2$	3	broadcast-only / CRS, FHE	guaranteed output delivery	upper bound
[GLS15]	$t < n/2$	2	broadcast-only / CRS, PKI, FHE	guaranteed output delivery	upper bound
[BJMS18]	$t < n/2$	3	broadcast-only / Zaps, FHE, Dense Crypto-systems	guaranteed output delivery	upper bound
[ACGJ18]	$t < n/2$	3	broadcast-only / Zaps, public-key encryption	guaranteed output delivery	upper bound
[IKP10]	$n = 5, t = 1$	2	private channel / OWF	guaranteed output delivery	upper bound
[IKKP15]	$n = 3, t = 1$	2	private channel / OWF	selective abort	upper bound
[IKKP15]	$n = 4, t = 1$	2	private channel / (injective) OWF	guaranteed output delivery	upper bound
[MRZ15]	$n = 3, t = 1$	3	private channel, Broadcast / PRG	unanimous abort	upper bound
[GLS15]	$t < n/2$	3	broadcast-only / CRS	fairness	lower bound
[GIKR02]	$n; t > 1$	3	private channel, Broadcast	fairness	lower bound

1.1 Our Results

In this paper, we set our focus on the exact round complexity of 3PC protocols with one active corruption achieving a range of security notions, namely selective abort, unanimous abort, fairness and guaranteed output delivery in a setting with pair-wise private channels and without or with a broadcast channel (and no additional setup). In the minimal setting of pair-wise private channels, it is known that 3PC with selective abort is feasible in just two rounds [IKKP15], while guaranteed output delivery is infeasible to achieve irrespective of the number of rounds [CHOR16]. No bound on round complexity is known for unanimous abort or fairness. In the setting with a broadcast channel, the result of [MRZ15] implies 3-round 3PC with unanimous abort. Neither the round optimality of the [MRZ15] construction, nor any bound on round complexity is known for protocols with fairness and guaranteed output delivery.

This work settles all the above questions via two lower bound results and three upper bounds. Both our lower-bounds extend for general n and t with strict honest majority i.e. $n/3 \leq t < n/2$. They imply tightness of several known constructions of [IKKP15] and complement the lower bound of [GIKR02] which holds for only $t > 1$. Our upper bounds are from injective (one-to-one) one-way functions. The fundamental concept of garbled circuits (GC) contributes as their key basis, following several prior works in this domain [CKMZ14, IKKP15, MRZ15]. The techniques in our upper bounds do not seem to extend for $t > 1$, leaving open designing round-optimal protocols for the general case with various security notions. We now elaborate on the results below:

Without Broadcast Channel. In this paper, we show that three rounds are necessary to achieve 3PC with unanimous abort and fairness, in the absence of a broadcast channel. The sufficiency is proved via a 3-round fair protocol (which also achieves unanimous abort security). Our lower bound result immediately implies tightness of the 3PC protocol of [IKKP15] achieving selective abort in two rounds, in terms of security achieved. This completely settles the questions on exact round complexity of 3PC in the minimal setting of pair-wise private channels. Our 3-round fair protocol uses a sub-protocol that is reminiscent of Conditional Disclosure of Secrets (CDS) [GIKM00], with an additional property of authenticity that allows a recipient to detect the correct secret. Our implementation suggests a realisation of authenticated CDS from privacy-free GCs.

With Broadcast Channel. With access to a broadcast channel, we show that it takes just two rounds to get 3PC with unanimous abort, implying non-optimality of the 3-round construction of [MRZ15]. On the other hand, we show that three rounds are necessary to construct a 3PC protocol with fairness and guaranteed output delivery. The sufficiency for fairness already follows from our 3-round fair protocol without broadcast. The sufficiency for guaranteed output delivery is shown via yet another construction in the presence of broadcast. The lower bound result restricted for $t = 1$ complements the lower bound of [GIKR02] making three rounds necessary for MPC with fairness in the honest majority setting for all the values of t. The lower bound further implies that for two-round fair (or guaranteed output delivery) protocols with one corruption, the number of parties needs to be at least four, making the 4PC protocol of [IKKP15] an optimal one. Notably, our result does not contradict with the two-round protocol of [GLS15] that assumes PKI (where the infrastructure contains the public keys of a 'special' FHE), CRS and also broadcast channel.

The table below captures the complete picture of the round complexity of 3PC. The necessity of two rounds for any type of security follows from [HLP11] via the 'residual attack'. Notably, broadcast facility only impacts the round complexity of unanimous abort and guaranteed output delivery, leaving the round complexity of selective abort and fairness unperturbed.

Security	Without Broadcast	References Necessity/Sufficiency	With Broadcast	References Necessity/Sufficiency
Selective Abort	2	[HLP11] / [IKKP15]	2	[HLP11] / [IKKP15]
Unanimous Abort	3	**This paper / This paper**	2	[HLP11] / **This paper**
Fairness	3	**This paper / This paper**	3	**This paper / This paper**
Guaranteed output delivery	Impossible	[CHOR16]	3	**This paper / This paper**

1.2 Techniques

Lower Bounds. We present two lower bounds– **(a)** three rounds are necessary for achieving fairness in the presence of pair-wise channels and a broadcast channel; **(b)** three rounds are necessary for achieving unanimous abort in the presence of just pair-wise channels. The lower bounds are shown by taking a special 3-party function and by devising a sequence hybrid executions under different adversarial strategies, allowing to conclude any 3PC protocol computing the considered function cannot be simultaneously private and fair or secure with unanimous abort.

Upper Bounds. We present three upper bounds– **(a)** 3-round fair protocol; **(b)** 2-round protocol with unanimous abort and **(c)** 3-round protocol with guaranteed output delivery. The former in the presence of just pairwise channels, the latter two with an additional broadcast channel. The known generic transformations such as, unanimous abort to (identifiable) fairness [IKP+16] or identifiable fairness to guaranteed output delivery [CL14], does not help in any of our constructions. For instance, any 3-round fair protocol without broadcast cannot take the former route as it is not round-preserving and unanimous abort in two rounds necessarily requires broadcast as shown in this work. A 3-round protocol with guaranteed output delivery cannot be constructed combining both the transformations due to inflation in round complexity.

Building on the protocol of [MRZ15], the basic building block of our protocols needs two of the parties to enact the role of the garbler and the remaining party to carry out the responsibility of circuit evaluation. Constrained with just two or three rounds, our protocols are built from the parallel composition of three sub-protocols, each one with different party enacting the role of the evaluator (much like [IKKP15]). Each sub-protocol consumes two rounds. Based on the security needed, the sub-protocols deliver distinct flavours of security with 'identifiable abort'. For the fair and unanimous abort, the identifiability is in the form of conflict that is local (privately known) and public/global (known to all) respectively, while for the protocol with guaranteed output delivery, it is local identification of the corrupt. Achieving such identifiability in just two rounds (sometime without broadcast) is challenging in themselves. Pulling up the security guarantee of these subprotocols via entwining three executions to obtain the final goals of fairness, unanimous abort and guaranteed output delivery constitute yet another novelty of this work. Maintaining the input consistency across the three executions pose another challenge that are tackled via mix of novel techniques (that consume no additional cost in terms of communication) and existing tricks such as 'proof-of-cheating' or 'cheat-recovery' mechanism [Lin13, CKMZ14]. The issue of input consistency does not appear in the construction of [MRZ15] at all, as it does not deal with parallel composition. On the other hand, the generic input consistency technique adopted in [IKKP15] can only (at the best) detect a conflict locally and cannot be extended to support the stronger form of identifiability that we need.

Below, we present the common issues faced and approach taken in all our protocols before turning towards the challenges and way-outs specific to our constructions. Two of the major efficiency bottlenecks of 2PC from garbled circuits, namely the need of multiple garbled circuits due to cut-and-choose approach and Oblivious Transfer (OT) for enabling the evaluator to receive its input in encoded form are bypassed in the 3PC

scenario through two simple tricks [IKKP15, MRZ15]. First, the garblers use common randomness to construct the same garbled circuit individually. A simple comparison of the GCs received from the two garblers allows to conclude the correctness of the GC. Since at most one party can be corrupt, if the received GCs match, then its correctness can be concluded. Second, the evaluator shares its input additively among the garblers at the onset of the protocol, reducing the problem to a secure computation of a function on the garblers' inputs alone. Specifically, assuming P_3 as the evaluator, the computation now takes inputs from P_1 and P_2 as (x_1, x_{31}) and (x_2, x_{32}) respectively to compute $C(x_1, x_2, x_{31}, x_{32}) = f(x_1, x_2, x_{31} \oplus x_{32})$. Since the garblers possess all the inputs needed for the computation, OT is no longer needed to transfer the evaluator's input in encoded form to P_3.

Next, to force the garblers to input encoding and decoding information (the keys) that are consistent with the GCs, the following technique is adopted. Notice that the issue of input consistency where a corrupt party may use different inputs as an evaluator and as a garbler in different instances of the sub-protocols is distinct and remains to be tackled separately. Together with the GC, each garbler also generates the commitment to the encoding and decoding information using the common shared randomness and communicates to the evaluator. Again a simple check on whether the set of commitments are same for both the garblers allows to conclude their correctness. Now it is infeasible for the garblers to decommit the encoded input corresponding to their own input and the evaluator's share to something that are inconsistent to the GC without being caught. Following a common trick to hide the inputs of the garblers, the commitments on the encoding information corresponding to every bit of the garblers' input are sent in permuted order that is privy to the garblers. The commitment on the decoding information is relevant only for the fair protocol where the decoding information is withheld to force a corrupt evaluator to be fair. Namely, in the third round of the final protocol, the evaluator is given access to the decoding information only when it helps the honest parties to compute the output. This step needs us to rely on the obliviousness of our garbling scheme, apart from privacy. The commitment on the decoding information and its verification by crosschecking across the garblers are needed to prevent a corrupt party to lie later. Now we turn to the challenges specific to the constructions.

Achieving fairness in 3 rounds. The sub-protocol for our fair construction only achieves a weak form of identifiability, a local conflict to be specific, in the absence of broadcast. Namely, the evaluator either computes the encoded output ('happy' state) or it just gets to know that the garblers are in conflict ('confused' state) in the worst case. The latter happens when it receives conflicting copies of GCs or commitments to the encoding/decoding information. In the composed protocol, a corrupt party can easily breach fairness by keeping one honest evaluator happy and the other confused in the end of round 2 and *selectively* enable the happy party to compute the output by releasing the decoding information in the third round (which was withheld until Round 2). Noting that the absence of a broadcast channel ensues conflict and confusion, we handle this using a neat trick of 'certification mechanism' that tries to enforce honest behaviour from a sender who is supposed to send a common information to its fellow participants.

A party is rewarded with a 'certificate' for enacting an honest sender and emulating a broadcast by sending the same information to the other two parties, for the common

information such as GCs and commitments. This protocol internally mimics a CDS protocol [GIKM00] for equality predicate, with an additional property of 'authenticity', a departure from the traditional CDS. An authenticated CDS allows the receiver to detect correct receipt of the secret/certificate (similar to authenticated encryption where the receiver knows if the received message is the desired one). As demonstrated below, the certificate allows to identify the culprit behind the confusion on one hand, and to securely transmit the decoding information from a confused honest party to the happy honest party in the third round, on the other. The certificate, being a proof of correct behaviour, when comes from an honest party, say P_i, the other honest party who sees conflict in the information distributed by P_i communicated over point-to-point channel, can readily identify the corrupt party responsible for creating the conflict in Round 3. This aids the latter party to compute the output using the encoded output of the former honest party. The certificate further enables the latter party to release the decoding information in Round 3 in encrypted form so that the other honest party holding a certificate can decrypt it. The release of encryption is done only for the parties whose distributed information are seen in conflict, so that a corrupt party either receives its certificate or the encryption but *not* both. Consequently, it is forced to assist at least one honest party in getting the certificate and be happy to compute the output, as only a happy party releases the decoding information on clear. In a nutshell, the certification mechanism ensures that when one honest party is happy, then no matter how the corrupt party behaves in the third round, both the honest parties will compute the output in the third round. When no honest party is happy, then none can get the output. Lastly, the corrupt party must keep one honest party happy, for it to get the output.

Yet again, we use garbled circuits to implement the above where a party willing to receive a certificate acts as an evaluator for a garbled circuit implementing 'equality' check of the inputs. The other two parties act as the garblers with their inputs as the common information dealt by the evaluator. With no concern of input privacy, the circuit can be garbled in a privacy-free way [JKO13, FNO15]. The certificate that is the key for output 1 is accessible to the evaluator only when it emulates a broadcast by dealing identical copies of the common information to both the other parties. Notably, [IW14] suggests application of garbling to realise CDS.

Achieving unanimous abort in 2 rounds. Moving on to our construction with unanimous abort, the foremost challenge comes from the fact that it must be resilient to any corrupt Round 2 private communication. Because there is no time to report this misbehaviour to the other honest party who may have got the output and have been treated with honest behaviour all along. Notably, in our sub-protocols, the private communication from both garblers in second round inevitably carries the encoded share of the evaluator's input (as the share themselves arrives at the garblers' end in Round 1). This is a soft spot for a corrupt garbler to selectively misbehave and cause selective abort. While the problem of transferring encoded input shares of the evaluator without relying on second round private communication seems unresolvable on the surface, our take on the problem uses a clever 'two-part release mechanism'. The first set of encoding information for random inputs picked by the garblers themselves is released in the first round privately and any misbehaviour is brought to notice in the second round. The second set of encoding information for the offsets of the random values and the actual shares of the evaluator's

input is released in the second round via broadcast without hampering security, while allowing public detection. Thus the sub-protocol achieves global/public conflict and helps the final construction to exit with $\perp$ unanimously when any of the sub-protocol detects a conflict.

Achieving guaranteed output delivery in 3 rounds. For achieving this stronger notion, the sub-protocol here needs a stronger kind of identifiability, identifying the corrupt locally to be specific, to facilitate all parties to get output within an additional round no matter what. To this effect, our sub-protocol is enhanced so that the evaluator either successfully computes the output or identifies the corrupt party. We emphasise that the goals of the sub-protocols for unanimous abort and guaranteed output delivery, namely global conflict vs. local identification, are orthogonal and do not imply each other. The additional challenge faced in composing the executions to achieve guaranteed output delivery lies in determining the appropriate 'committed' input of the corrupt party based on which round and execution of sub-protocol it chooses to strike.

Tackling input consistency. We take a uniform approach for all our protocols. We note that a party takes three different roles across the three composed execution: an evaluator, a garbler who initiate the GC generation by picking the randomness, a co-garbler who verifies the sanity of the GC. In each instance, it gets a chance to give inputs. We take care of input consistency in two parts. First, we tie the inputs that a party can feed as an evaluator and as a garbler who initiates a GC construction via a mechanism that needs no additional communication at all. This is done by setting the permutation strings (used to permute the commitments of encoding information of the garblers) to the shares of these parties' input in a certain way. The same trick fails to work in two rounds for the case when a party acts as a garbler and a co-garbler in two different executions. We tackle this by superimposing two mirrored copies of the sub-protocol where the garblers exchange their roles. Namely, in the final sub-protocol, each garbler initiates an independent copy of garbled circuit and passes on the randomness used to the fellow garbler for verification. The previous trick is used to tie the inputs that a party feeds as an evaluator and as a garbler for the GC initiated by it (inter-execution consistency). The input consistency of a garbler for the two garbled circuits (one initiated by him and the other by the co-garbler) is taken care using 'proof-of-cheating' mechanism [Lin13] where the evaluator can unlock the clear input of both the other parties using conflicting output wire keys (intra-execution consistency). While this works for our protocols with unanimous abort and guaranteed output delivery, the fair protocol faces additional challenges. First, based on whether a party releases a clear or encoded input, a corrupt garbler feeding two different inputs can conclude whether f leads to the same output for both his inputs, breaching privacy. This is tackled by creating the ciphertexts using conflicting input keys. Second, inspite of the above change, a corrupt garbler can launch 'selective failure attack' [MF06, KS06] and breach privacy of his honest co-garbler. We tackle this using 'XOR-tree approach' [LP07] where every input bit is broken into s shares and security is guaranteed except with probability $2^{-(s-1)}$ per input bit. We do not go for the refined version of this technique, known as probe-resistant matrix, [LP07, SS13] for simplicity.

On the assumption needed. While the garbled circuits can be built just from OWF, the necessity of injective OWF comes from the use of commitments that need bind-

ing property for any (including adversarially-picked) public parameter. Our protocols, having 2-3 rounds, seem unable to spare rounds for generating and communicating the public parameters by a party who is different from the one opening the commitments.
On concrete efficiency. Though the focus is on the round complexity, the concrete efficiency of our protocols is comparable to Yao [Yao82] and require transmission and evaluation of few GCs (upto 9) (in some cases we only need privacy-free GCs which permit more efficient constructions than their private counterparts [JKO13, FNO15]). The broadcast communication of the optimized variants of our protocols is independent of the GC size via applying hash function. We would like to draw attention towards the new tricks such as the ones used for input consistency, getting certificate of good behaviour via garbled circuits, which may be of both theoretical and practical interest. We believe the detailed take on our protocols will help to lift them or their derivatives to practice in future.

1.3 Roadmap

We present a high-level overview of the primitives used in Section 2. The security definition and the functionalities appear in Appendix B. We present our 3-round fair protocol, 2-round protocol with unanimous abort and 3-round protocol with guaranteed output delivery in Section 3, 4 and 5 respectively. The respective security proofs appear in Appendices D, E and F and the common optimizations in Appendix C. Our lower bound results appear in Section 6. We define authenticated CDS in Appendix G and show its realisation from one of the sub-protocol used in our 3-round fair protocol.

2 Preliminaries

2.1 Model

We consider a set of $n = 3$ parties $\mathcal{P} = \{P_1, P_2, P_3\}$, connected by pair-wise secure and authentic channels. Each party is modelled as a probabilistic polynomial time Turing (PPT) machine. We assume that there exists a PPT adversary $\mathcal{A}$, who can actively corrupt at most $t = 1$ out of the $n = 3$ parties and make them behave in any arbitrary manner during the execution of a protocol. We assume the adversary to be static, who decides the set of t parties to be corrupted at the onset of a protocol execution. For our 2-round protocol achieving unanimous abort and 3-round protocol achieving guaranteed output delivery, a broadcast channel is assumed to exist.

We denote the cryptographic security parameter by κ. A negligible function in κ is denoted by $\mathtt{negl}(\kappa)$. A function $\mathtt{negl}(\cdot)$ is *negligible* if for every polynomial $p(\cdot)$ there exists a value N such that for all $m > N$ it holds that $\mathtt{negl}(m) < \frac{1}{p(m)}$. We denote by $[x]$, the set of elements $\{1, \ldots, x\}$ and by $[x, y]$ for $y > x$, the set of elements $\{x, x+1, \ldots, y\}$. For any $x \in_R \{0,1\}^m$, x^i denotes the bit of x at index i for $i \in [m]$. Let S be an infinite set and $X = \{X_s\}_{s \in S}, Y = \{Y_s\}_{s \in S}$ be distribution ensembles. We say X and Y are computationally indistinguishable, if for any PPT distinguisher $\mathcal{D}$ and all sufficiently large $s \in S$, we have $|\Pr[\mathcal{D}(X_s) = 1] - \Pr[\mathcal{D}(Y_s) = 1]| < 1/p(|s|)$ for every polynomial $p(\cdot)$.

2.2 Primitives

Garbling Schemes. The term 'garbled circuit' (GC) was coined by Beaver [BMR90], but it had largely only been a technique used in secure protocols until they were formalized as a primitive by Bellare et al. [BHR12]. 'Garbling Schemes' as they were termed, were assigned well-defined notions of security, namely *correctness*, *privacy*, *obliviousness*, and *authenticity*. A garbling scheme $\mathcal{G}$ is characterised by a tuple of PPT algorithms $\mathcal{G} = (\mathsf{Gb}, \mathsf{En}, \mathsf{Ev}, \mathsf{De})$ described below.

- $\mathsf{Gb}\,(1^\kappa, C)$ is invoked on a circuit C in order to produce a 'garbled circuit' $\mathbf{C}$, 'input encoding information' e, and 'output decoding information' d.
- $\mathsf{En}\,(x, e)$ encodes a clear input x with encoding information e in order to produce a garbled/encoded input $\mathbf{X}$.
- $\mathsf{Ev}\,(\mathbf{C}, \mathbf{X})$ evaluates $\mathbf{C}$ on $\mathbf{X}$ to produce a garbled/encoded output $\mathbf{Y}$.
- $\mathsf{De}\,(\mathbf{Y}, d)$ translates $\mathbf{Y}$ into a clear output y as per decoding information d.

We give an informal intuition of the notion captured by each of the security properties, namely *correctness*, *privacy*, *obliviousness*, and *authenticity*. Correctness enforces that a correctly garbled circuit, when evaluated, outputs the correct output of the underlying circuit. Privacy aims to protect the privacy of encoded inputs. Authenticity enforces that the evaluator can only learn the output label that corresponds to the value of the function. Obliviousness captures the notion that when the decoding information is withheld, the garbled circuit evaluation leaks no information about *any* underlying clear values; be they of the input, intermediate, or output wires of the circuit. The formal definitions are deferred to Appendix A.1.

We are interested in a class of garbling schemes referred to as *projective* in [BHR12]. When garbling a circuit $C : \{0,1\}^n \mapsto \{0,1\}^m$, a projective garbling scheme produces encoding information of the form $e = \left(e_i^0, e_i^1\right)_{i\in[n]}$, and the encoded input $\mathbf{X}$ for $x = (x_i)_{i\in[n]}$ can be interpreted as $\mathbf{X} = \mathsf{En}(x, e) = (e_i^{x_i})_{i\in[n]}$.

Our 3-round fair protocol relies on garbling schemes that are simultaneously correct, private and oblivious. One of its subroutine uses a garbling scheme that is only authentic. Such schemes are referred as *privacy-free* [JKO13, FNO15]. Our protocols with unanimous abort and guaranteed output delivery need a correct, private and authentic garbling scheme that need not provide obliviousness. Both these protocols as well as the privacy-free garbling used in the fair protocol further need an additional decoding mechanism denoted as *soft decoding* algorithm sDe [MRZ15] that can decode garbled outputs without the decoding information d. The soft-decoding algorithm must comply with correctness: $\mathsf{sDe}(\mathsf{Ev}(\mathbf{C}, \mathsf{En}(e, x))) = C(x)$ for all $(\mathbf{C}, e, d)$. While both sDe and De can decode garbled outputs, the authenticity needs to hold only with respect to De. In practice, soft decoding in typical garbling schemes can be achieved by simply appending the truth value to each output wire label.

Non-interactive Commitment Schemes. A non-interactive commitment scheme (NICOM) consists of two algorithms $(\mathsf{Com}, \mathsf{Open})$ defined as follows. Given a security parameter κ, a common parameter pp, message x and random coins r, PPT algorithm Com outputs commitment c and corresponding opening information o. Given κ, pp, a

commitment and corresponding opening information (c, o), PPT algorithm Open outputs the message x. The algorithms should satisfy correctness, binding (i.e. it must be hard for an adversary to come up with two different openings of any c and *any* pp) and hiding (a commitment must not leak information about the underlying message) properties. We need this kind of strong binding as the same party who generates the pp and commitment is required to open later. Two such instantiations of NICOM based on symmetric key primitives (specifically, injective one-way functions) and the formal definitions of the properties are given in Appendix A.2.

We also need a NICOM scheme that admits equivocation property. An equivocal non-interactive commitment (eNICOM) is a NICOM that allows equivocation of a certain commitment to any given message with the help of a trapdoor. The formal definitions and instantiations appear in Appendix A.3.

Symmetric-Key Encryption (SKE) with Special Correctness. Our fair protocol uses a SKE $\pi = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ which satisfies CPA security and a special correctness property [JW16, LP09]– if the encryption and decryption keys are different, then decryption fails with high probability. The definition and an instantiation appear in Appendix A.4.

3 3-round 3PC with Fairness

This section presents a tight upper bound for 3PC achieving fairness in the setting with just pair-wise private channels. Our result from Section 6.2 rules out the possibility of achieving fairness in 2 rounds in the same setting. Our result from Section 6.1 further shows tightness of 3 rounds even in the presence of a broadcast channel.

Building on the intuition given in the introduction, we proceed towards more detailed discussion of our protocol. Our fair protocol is built from parallel composition of three copies of each of the following two sub-protocols: (a) fair_i where P_i acts as the evaluator and the other two as garblers for computing the desired function f. This sub-protocol ensures that honest P_i either computes its encoded output or identifies just a conflict in the worst case. The decoding information is committed to P_i, yet not opened. It is released in Round 3 of the final composed protocol under subtle conditions as elaborated below. (b) cert_i where P_i acts as the evaluator and the other two as garblers for computing an equality checking circuit on the common information distributed by P_i in the first round of the final protocol. Notably, though the inputs come solely from the garblers, they are originated from the evaluator and so the circuit can be garbled in a privacy-free fashion. This sub-protocol ensures either honest P_i gets its certificate, the key for output 1 (meaning the equality check passes through), or identifies a conflict in the worst case. The second round of cert_i is essentially an 'authenticated' CDS for equality predicate tolerating one active corruption as discussed in Appendix G. Three *global* variables are maintained by each party P_i to keep tab on the conflicts and the corrupt. Namely, $\mathcal{C}_i$ to keep the identity of the corrupt, flag_j and flag_k (for distinct $i, j, k \in [3]$) as indicators of detection of conflict with respect to information distributed by P_j and P_k respectively. The sub-protocols fair_i and cert_i assure that if neither the two flags nor $\mathcal{C}_i$ is set, then P_i must be able to evaluate the GC successfully and get its certificate respectively.

Once $\{\mathsf{fair}_i, \mathsf{cert}_i\}_{i\in[3]}$ complete by the end of round 2 of the final protocol fair, any honest party will be in one of the three states: (a) no corruption and no conflict detected ($(\mathcal{C}_i = \emptyset) \wedge (\mathsf{flag}_j = 0) \wedge (\mathsf{flag}_k = 0)$); (b) corruption detected ($\mathcal{C}_i \neq \emptyset$); (c) conflict detected $(\mathsf{flag}_j = 1) \vee (\mathsf{flag}_k = 1)$. An honest party, guaranteed to have computed its encoded output and certificate *only* in the first state, releases these as well as the decoding information for both the other parties unconditionally in the third round. In the other two states, an honest party conditionally releases only the decoding information. This step is extremely crucial for maintaining fairness. Specifically, a party that belongs to the second state, releases the decoding information only to the party identified to be honest. A party that belongs to the third state, releases the decoding information in encrypted form *only* to the party whose distributed information are not agreed upon, so that the encryption can be unlocked only via a valid certificate. A corrupt party will either have its certificate or the encrypted decoding information, but *not* both. The former when it distributes its common information correctly and the latter when it does not. The only way a corrupt party can get its decoding information is by keeping one honest party in the first state, in which case both the honest parties will be able to compute the output as follows. The honest party in state one, say P_i, either gets it decoding information on clear or in encrypted form. The former when the other honest party, P_j is in the first or second state and the latter when P_j is in the third state. P_i retrieves the decoding information no matter what, as it also holds the certificate to open the encryption. An honest party P_j in the second state, on identifying P_i as honest, takes the encoded output of P_i and uses its own decoding information to compute the output. The case for an honest party P_j in the third state is the most interesting. Since honest P_i belongs to the first state, a corrupt party must have distributed its common information correctly as otherwise P_i will find a conflict and would be in third state. Therefore, P_j in the third state must have found P_i's information on disagreement due the corrupt party's misbehaviour. Now, P_i's certificate that proves his correct behaviour, allows P_j to identify the corrupt, enter into the second state and compute the output by taking the encoded output of honest P_i. In the following, we describe execution fair_i assuming input consistency, followed by cert_i. Entwining the six executions, tackling the input consistency and the final presentation of protocol fair appear in the end.

3.1 Protocol fair_i

At a high level, fair_i works as follows. In the first round, the evaluator shares its input additively between the two garblers making the garblers the sole input contributors to the computation. In parallel, each garbler initiates construction of a GC and commitments on the encoding and decoding information. While the GC and the commitments are given to the evaluator P_i, the co-garbler, acting as a verifier, additionally receives the source of the used randomness for GC and openings of commitments. Upon verification, the co-garbler either approves or rejects the GC and commitments. In the former case, it also releases its own encoded input and encoded input for the share of P_i via opening the commitments to encoding information in second round. In the latter case, P_i sets the flag corresponding to the generator of the GC to true. Failure to open a verified commitment readily exposes the corrupt to the evaluator. If all goes well, P_i evaluates both circuits and obtains encoded outputs. The correctness of the evaluated GC

follows from the fact that it is either constructed or scrutinised by a honest garbler. The decoding information remains hidden (yet committed) with P_i and the obliviousness of GC ensures that P_i cannot compute the output until it receives the correct opening.

To avoid issues of adaptivity, the GCs are not sent on clear in the first round to P_i who may choose its input based on the GCs. Rather, a garbler sends a commitment to its GC to P_i and it is opened only by the co-garbler after successful scrutiny. The correctness of evaluated GC still carries over as a corrupt garbler cannot open to a different circuit than the one committed by an honest garbler by virtue of the binding property of the commitment scheme. We use an eNICOM for committing the GCs and decoding information as equivocation is needed to tackle a technicality in the security proof. The simulator of our final protocol needs to send the commitments on GC, encoding and decoding information without having access to the input of an evaluator P_i (and thus also the output), while acting on behalf of the honest garblers in fair_i. The eNICOM cannot be used for the encoding information, as they are opened by the ones who generate the commitments and eNICOM does not provide binding in such a case. Instead, the GCs and the decoding information are equivocated based on the input of the evaluator and the output.

Protocol fair_i appears in Figure 1 where P_i returns encoded outputs $\mathbf{Y}_i = (\mathbf{Y}_i^j, \mathbf{Y}_i^k)$ (initially set to $\perp$) for the circuits created by P_j, P_k, the commitments to the respective decoding information $C_j^{\mathsf{dec}}, C_k^{\mathsf{dec}}$ and the flags $\mathsf{flag}_j, \mathsf{flag}_k$ (initially set to false) to be used in the final protocol. The garblers output their respective corrupt set, flag for the fellow garbler and opening for the decoding information corresponding to its co-garbler's GC and *not* its own. This is to ensure that it cannot break the binding of eNICOM which may not necessarily hold for adversarially-picked public parameter.

Lemma 1. *During* fair_i, $P_\beta \notin \mathcal{C}_\alpha$ *holds for honest* P_α, P_β.

Proof. An honest P_α would include P_β in $\mathcal{C}_\alpha$ only if one of the following hold: (a) Both are garblers and P_β sends commitments to garbled circuit, encoding and decoding information inconsistent with the randomness and openings shared privately with P_α (b) P_α is an evaluator and P_β is a garbler and either (i) P_β's opening of a committed garbled circuit fails or (ii) P_β's opening of a committed encoded input fails. It is straightforward to verify that the cases will never occur for honest (P_α, P_β). □

Lemma 2. *If honest* P_i *has* $\mathcal{C}_i = \emptyset$ *and* $\mathsf{flag}_j = \mathsf{flag}_k = 0$, *then* $\mathbf{Y}_i = (\mathbf{Y}_i^j, \mathbf{Y}_i^k) \neq \perp$.

Proof. According to fair_i, P_i fails to compute $\mathbf{Y}_i$ when it identifies the corrupt or finds a mismatch in the common information $\mathcal{D}_j$ or $\mathcal{D}_k$ or receives a `nOK` signal from one of its garblers. The first condition implies $\mathcal{C}_i \neq \emptyset$. The second condition implies, P_i would have set either flag_j or flag_k to true. For the third condition, if P_j sends `nOK` then P_i would set $\mathsf{flag}_k = 1$. Lastly, if P_k sends `nOK`, then P_i sets $\mathsf{flag}_j = 1$. Clearly when $\mathcal{C}_i = \emptyset \wedge \mathsf{flag}_j = 0 \wedge \mathsf{flag}_k = 0$, P_i evaluates both $\mathbf{C}_j, \mathbf{C}_k$ and obtains $\mathbf{Y}_i = (\mathbf{Y}_i^j, \mathbf{Y}_i^k) \neq \perp$. □

3.2 Protocol cert_i

When a party P_i in fair_i is left in a confused state and has no clue about the corrupt, it is in dilemma on whether or whose encoded output should be used to compute output and

Protocol $\mathsf{fair}_i()$

Inputs: Party P_α has x_α for $\alpha \in [3]$.
Common Inputs: The circuit $C(x_1, x_2, x_3, x_4)$ that computes $f(x_1, x_2, x_3 \oplus x_4)$.
Output: A garbler P_l ($l \in \{j,k\}$) outputs corrupt set $\mathcal{C}_l, \mathsf{flag}_{\{j,k\}\setminus l}$ and O_i^{dec}. P_i outputs ($\mathcal{C}_i$, $\mathbf{Y}_i = (\mathbf{Y}_i^j, \mathbf{Y}_i^k), C_j^{\mathsf{dec}}, C_k^{\mathsf{dec}}, \mathsf{flag}_j, \mathsf{flag}_k$) where $\mathbf{Y}_i$ denote a pair of encoded outputs or $\perp$.
Primitives: A garbling scheme $\mathcal{G} = (\mathsf{Gb}, \mathsf{En}, \mathsf{Ev}, \mathsf{De})$ that is correct, private and oblivious, a NICOM $(\mathsf{Com}, \mathsf{Open})$, an eNICOM $(\mathsf{eGen}, \mathsf{eCom}, \mathsf{eOpen}, \mathsf{Equiv})$ and a PRG G.

Round 1:

- P_i randomly secret shares his input x_i as $x_i = x_{ij} \oplus x_{ik}$ and sends x_{ij} to P_j and x_{ik} to P_k.
- P_l for $l \in \{j,k\}$ samples $s_l \in_R \{0,1\}^\kappa$, epp_l and pp_l for G, eNICOM and NICOM resp. and:
  - compute garbled circuit $(\mathbf{C}_l, e_l, d_l) \leftarrow \mathsf{Gb}(1^\kappa, C)$ using randomness from $\mathsf{G}(s_l)$. Assume $\{e_{l\alpha}^0, e_{l\alpha}^1\}_{\alpha\in[\ell]}, \{e_{l(\ell+\alpha)}^0, e_{l(\ell+\alpha)}^1\}_{\alpha\in[\ell]}, \{e_{l(2\ell+\alpha)}^0, e_{l(2\ell+\alpha)}^1\}_{\alpha\in[2\ell]}$ denote the encoding information for the input of P_j, P_k and the secret shares of P_i respectively.
  - compute commitments for GC and decoding information. $(c_l, o_l) \leftarrow \mathsf{eCom}(\mathsf{epp}_l, \mathbf{C}_l)$ and $(c_l^{\mathsf{dec}}, o_l^{\mathsf{dec}}) \leftarrow \mathsf{eCom}(\mathsf{epp}_l, d_l)$.
  - sample permutation strings $p_{lj}, p_{lk} \in_R \{0,1\}^\ell$ for the inputs of P_j and P_k. Compute commitments to encoding information as: for $b \in \{0,1\}$, $(c_{l\alpha}^b, o_{l\alpha}^b) \leftarrow \mathsf{Com}(\mathsf{pp}_l, e_{l\alpha}^{p_{lj}^\alpha \oplus b})$, $(c_{l(\ell+\alpha)}^b, o_{l(\ell+\alpha)}^b) \leftarrow \mathsf{Com}(\mathsf{pp}_l, e_{l(\ell+\alpha)}^{p_{lk}^\alpha \oplus b})$ when $\alpha \in [\ell]$, $(c_{l(2\ell+\alpha)}^b, o_{l(2\ell+\alpha)}^b) \leftarrow \mathsf{Com}(\mathsf{pp}_l, e_{l(2\ell+\alpha)}^b)$ when $\alpha \in [2\ell]$.
  - send $\mathcal{D}_l = (\mathsf{epp}_l, \mathsf{pp}_l, c_l, \{c_{l\alpha}^b, \}_{\alpha\in[4\ell], b\in\{0,1\}}, c_l^{\mathsf{dec}})$ to both the other parties and send $\{s_l, p_{lj}, p_{lk}, o_l, \{o_{l\alpha}^b, \}_{\alpha\in[4\ell],b\in\{0,1\}}, o_l^{\mathsf{dec}}\}$ only to co-garbler $P_{\{j,k\}\setminus l}$.
- P_j sets $\mathcal{C}_j = P_k$ if $\mathcal{D}_k$ and $\{s_k, p_{kj}, p_{kk}, o_k, \{o_{k\alpha}^b, \}_{\alpha\in[4\ell],b\in\{0,1\}}, o_k^{\mathsf{dec}}\}$ are inconsistent. Else, set $O_i^{\mathsf{dec}} = o_k^{\mathsf{dec}}$. P_k performs similar steps for the values received from P_j.

Round 2:

- P_i sends $\mathcal{D}_j$ to P_k and $\mathcal{D}_k$ to P_j. P_j sets $\mathsf{flag}_k = 1$ if $\mathcal{D}_k$ received from P_i and P_k does not match. Similar step is executed by P_k.
- P_j computes the indicator strings $m_{jj} = p_{jj} \oplus x_j, m_{kj} = p_{kj} \oplus x_j$ for its inputs. If $P_k \notin \mathcal{C}_j$, then send $\Big(\mathtt{OK}, \mathcal{D}_k, (o_k, \{o_{k\alpha}^{m_{kj}^\alpha}, o_{k(2\ell+\alpha)}^{x_{ij}^\alpha}\}_{\alpha\in[\ell]}, m_{kj}), (\{o_{j\alpha}^{m_{jj}^\alpha}, o_{j(2\ell+\alpha)}^{x_{ij}^\alpha}\}_{\alpha\in[\ell]}, m_{jj})\Big)$ to P_i. Else, send $\mathtt{nOK}$ to P_i. P_k performs similar steps.
- (Local Computation) P_i sets $\mathbf{Y}_i^j = \perp$ and $\mathsf{flag}_j = 1$ when **(a)** P_k sent $\mathtt{nOK}$ or **(b)** $\mathcal{D}_j$ sent by P_j and P_k do not match. Otherwise, P_i sets $C_j^{\mathsf{dec}} = c_j^{\mathsf{dec}} \in \mathcal{D}_j$ and does:
  - open $\mathbf{C}_j \leftarrow \mathsf{eOpen}(\mathsf{epp}_j, c_j, o_j)$ with o_j received from P_k. Set $\mathcal{C}_i = P_k$ if $\mathbf{C}_j = \perp$.
  - open $\mathbf{X}_j^\alpha = \mathsf{Open}(\mathsf{pp}_j, c_{j\alpha}^{m_{jj}^\alpha}, o_{j\alpha}^{m_{jj}^\alpha})$, $\mathbf{X}_{ij}^\alpha = \mathsf{Open}(\mathsf{pp}_j, c_{j(2\ell+\alpha)}^{x_{ij}^\alpha}, o_{j(2\ell+\alpha)}^{x_{ij}^\alpha})$, for $\alpha \in [\ell]$, for the opening received from P_j and the commitments taken from $\mathcal{D}_j$. Include P_j in $\mathcal{C}_i$ if any of the opened input labels above is opened to $\perp$.
  - open $\mathbf{X}_k^\alpha = \mathsf{Open}(\mathsf{pp}_j, c_{j(\ell+\alpha)}^{m_{jk}^\alpha}, o_{j(\ell+\alpha)}^{m_{jk}^\alpha})$ and $\mathbf{X}_{ik}^\alpha = \mathsf{Open}(\mathsf{pp}_j, c_{j(3\ell+\alpha)}^{x_{ik}^\alpha}, o_{j(3\ell+\alpha)}^{x_{ik}^\alpha})$ for $\alpha \in [\ell]$, for the opening received from P_k and the commitments taken from $\mathcal{D}_j$. Include P_k in $\mathcal{C}_i$ if any of the opened input labels above is opened to $\perp$.
  - If $\mathcal{C}_i = \emptyset$, set $\mathbf{X} = \mathbf{X}_j|\mathbf{X}_k|\mathbf{X}_{ij}|\mathbf{X}_{ik}$, run $\mathbf{Y}_i^j \leftarrow \mathsf{Ev}(\mathbf{C}_j, \mathbf{X})$. Else set $\mathbf{Y}_i^j = \perp$

  Similar steps for $\mathbf{C}_k$ will be executed to compute $\mathbf{Y}_i^k$, populate $\mathcal{C}_i$ and update flag_k.

Fig. 1: Protocol fair_i

who should it release the decoding information (that it holds as a garbler) to in the final protocol. Protocol cert_i, in a nutshell, is introduced to help a confused party to identify the corrupt and take the honest party's encoded output for output computation, on one hand, and to selectively deliver the decoding information only to the other honest party, on the other. Protocol cert_i implements evaluation of an equality checking function that takes inputs from the two garblers and outputs 1 when the test passes and outputs the inputs themselves otherwise. In the final protocol, the inputs are the common information (GCs and commitments) distributed by P_i across all executions of fair_j. The certificate is the output key corresponding to output 1. Since input privacy is not a concern here, the circuit is enough to be garbled in privacy-free way and authenticity of garbling will ensure a corrupt P_i does not get the certificate. cert_i follows the footstep of fair_i with the following simplifications: (a) Input consistency need not be taken care across the executions implying that it is enough one garbler alone initiates a GC and the other garbler simply extends its support for verification. To divide the load fairly, we assign garbler P_j where $i = (j+1) \mod 3$ to act as the generator of GC in cert_i. (b) The decoding information need not be committed or withheld. We use soft decoding that allows immediate decoding.

Similar to fair_i, at the end of the protocol, either P_i gets its certificate (either the key for 1 or the inputs themselves), or sets its flags (when GC and commitment do not match) or sets its corrupt set (when opening of encoded inputs fail). P_i outputs its certificate, the flag for the GC generator and corrupt set, to be used in the final protocol. The garblers output the key for 1, flag for its fellow garbler and the corrupt set. Notice that, when cert_i is composed in the bigger protocol, P_i will be in a position to identify the corrupt when the equality fails and the certificate is the inputs fed by the garblers. The protocol appears in Figure 2.

Lemma 3. *During* cert_i*,* $P_\beta \notin \mathcal{C}_\alpha$ *holds for honest* P_α, P_β*.*

Proof. An honest P_α would include P_β in $\mathcal{C}_\alpha$ only if one of the following holds: (a) P_β sends inconsistent $(s_\beta, \mathcal{W}_\beta)$ to P_α. (b) P_β's opening of committed encoded input or garbled circuit fails. It is straightforward to verify that the cases will never occur for honest (P_β, P_α). □

Lemma 4. *If an honest* P_i *has* $\mathcal{C}_i = \emptyset$ *and* $\mathsf{flag}_j = \mathsf{flag}_k = 0$*, then,* $\mathbf{cert}_i \neq \perp$*.*

Proof. The proof follows easily from the steps of the protocol. □

3.3 Protocol fair

Building on the intuition laid out before, we only discuss input consistency that is taken care in two steps: Inter-input consistency (across executions) and intra-input consistency (within an execution). In the former, P_i's input as an evaluator in fair_i is tied with its input committed as garblers for its own garbled circuits in fair_j and fair_k. In the latter, the consistency of P_i's input for both garbled circuits in fair_j (and similarly in fair_k) is tackled. We discuss them one by one.

We tackle the former in a simple yet clever way without incurring any additional overhead. We explain the technique for enforcing P_1's input consistency on input x_1 as

$\mathsf{cert}_i()$

Common Inputs: The circuit $C(\gamma_j, \gamma_k)$ that outputs 1 if $(\gamma_j = \gamma_k)$ and $(0, \gamma_j, \gamma_k)$ otherwise. For distinct $i, j, k \in [3]$, P_i is assumed to be the evaluator and (P_j, P_k) as the garblers. We assume $i = (j+1) \mod 3, k = (j+2) \mod 3$.

Primitives: A correct, authentic, privacy-free garbling scheme $\mathcal{G} = (\mathsf{Gb}, \mathsf{En}, \mathsf{Ev}, \mathsf{De})$ that has the property of *soft decoding*, a PRG G, a NICOM $(\mathsf{Com}, \mathsf{Open})$

Output: A garbler P_l for $l \in \{j, k\}$ outputs corrupt set $\mathcal{C}_l$ and $\mathbf{key}_i$. P_i outputs $(\mathbf{cert}_i, \mathcal{C}_i, \mathsf{flag}_j, \mathsf{flag}_k)$. Garbler P_k additionally outputs flag_j.

Round 1: P_j does the following:

- Choose a seed $s_i \in_R \{0,1\}^\kappa$ for G and construct a garbled circuit $(\mathbf{C}_i, e_i, d_i) \leftarrow \mathsf{Gb}(1^\kappa, C)$. Generate commitment on garbled circuit $\mathbf{C}_i$ as $(\mathsf{c}_i, \mathsf{o}_i) \leftarrow \mathsf{Com}(\mathbf{C}_i)$ and on the encoding information e_i as $(c_i, o_i) \leftarrow \mathsf{Com}(e_i)$ using randomness from $\mathsf{G}(s_i)$. Let $\mathcal{W}_i = \{\mathsf{c}_i, c_i\}$. Send $(s_i, \mathcal{W}_i)$ to P_k and $\mathcal{W}_i$ to P_i.
- (Local Computation by P_k) P_k adds P_j to $\mathcal{C}_k$ if $(s_i, \mathcal{W}_i)$ are inconsistent and is not as per what an honest P_j should do. P_j and P_k output $\mathbf{key}_i$ equals to the key for output 1 of $\mathbf{C}_i$.

Round 2:

- P_i sends $\mathcal{W}_i$ to P_k. P_k sets $\mathsf{flag}_j = 1$ if $\mathcal{W}_i$ received from P_i and P_j is not identical.
- P_j opens its encoded input $\mathbf{X}_j$ (corresponding to γ_j) to P_i by sending the opening of the corresponding commitment in c_i.
- If $P_j \in \mathcal{C}_k$, P_k sends $\mathtt{nOK}$ to P_i. Else P_k sends $\mathcal{W}_i$, opening for garbled circuit o_i and its encoded input $\mathbf{X}_k$ (for γ_k) to P_i.
- (Local Computation by P_i) If P_i does not receive identical $\mathcal{W}_i$ from P_j and P_k or receives $\mathtt{nOK}$ from P_k, P_i sets $\mathbf{cert}_i = \perp$ and $\mathsf{flag}_j = 1$. Else, P_i uses the opening information sent by P_j, P_k to retrieve $\mathbf{X}_j, \mathbf{X}_k$. P_i adds P_l $(l \in \{j, k\})$ to $\mathcal{C}_i$ and sets $\mathbf{cert}_i = \perp$ if any of the openings sent by P_l result in $\perp$. Else, P_i runs $\mathbf{Y} \leftarrow \mathsf{Ev}(\mathbf{C}_i, \mathbf{X}_j, \mathbf{X}_k)$. If $\mathsf{sDe}(\mathbf{Y}) = 1$, then set $\mathbf{cert}_i = \mathbf{Y}$, else set $\mathbf{cert}_i = (\gamma'_j, \gamma'_k)$ where these two are decoded from $\mathbf{Y}$.

Fig. 2: Protocol cert_i

an evaluator during fair_1 and as a garbler during $\mathsf{fair}_2, \mathsf{fair}_3$ with respect to his GC $\mathbf{C}_1$. Since the protocol is symmetric in terms of the roles of the parties, similar tricks are adopted for P_2 and P_3. Let in the first round of fair_1, P_1 shares its input x_1 by handing x_{12} and x_{13} to P_2 and P_3 respectively. Now corresponding to $\mathbf{C}_1$ during fair_2, P_1 and P_3 who act as the garblers use x_{13} as the permutation vector p_{11} that defines the order of the commitments of the bits of x_1. Now input consistency of P_1's input is guaranteed if m_{11} transferred by P_1 in fair_2 is same as x_{12}, P_1's share for P_2 in fair_1. For an honest P_1, the above will be true since $m_{11} = p_{11} \oplus x_1 = x_{13} \oplus x_1 = x_{12}$. If the check fails, then P_2 identifies P_1 as corrupt. This simple check forces P_1 to use the same input in both fair_1 and fair_2 (corresponding to $\mathbf{C}_1$). A similar trick is used to ensure input consistency of the input of P_1 across fair_1 and fair_3 (corresponding to $\mathbf{C}_1$) where P_1 and P_2 who act as the garblers use x_{12} as the permutation vector p_{11} for the commitments of the bits of x_1. The evaluator P_3 in fair_3 checks if m_{11} transferred by P_1 in fair_3 is same as x_{13} that P_3 receives from P_1 in fair_1. While the above technique enforces the

consistency with respect to P_1's GC, unfortunately, the same technique cannot be used to enforce P_1's input consistency with respect to $\mathbf{C}_2$ in fair_3 (or fair_2) since p_{21} cannot be set to x_{12} which is available to P_2 only at the end of first round. While, P_2 needs to prepare and broadcast the commitments to the encoding information in jumbled order as per permutation string p_{21} in the first round itself. We handle it differently as below.

The consistency of P_i's input for both garbled circuits in fair_j (and similarly in fair_k) is tackled via 'cheat-recovery mechanism' [Lin13]. We explain with respect to P_1's input in fair_3. P_2 prepares a ciphertext (cheat recovery box) with the input keys of P_1 corresponding to the mismatched input bit in the two garbled circuits, $\mathbf{C}_1$ and $\mathbf{C}_2$ in fair_3. This ciphertext encrypts the the input shares of garblers that P_3 misses, namely, x_{12} and x_{21}. This would allow P_3 to compute the function on clear inputs directly. To ensure that the recovered missing shares are as distributed in fair_1 and fair_2, the shares are not simply distributed but are committed via NICOM by the input owners and the openings are encrypted by the holders. Since there is no way for an evaluator to detect any mismatch in the inputs to and outputs from the two GCs as they are in encoded form, we use encryption scheme with special correctness (Definition 6) to enable the evaluator to identify the relevant decryptions. Crucially, we depart from the usual way of creating the cheat recovery boxes using conflicting encoded outputs. Based on whether the clear or encoded output comes out of honest P_3 in round 3, corrupt garbler P_1 feeding two different inputs to $\mathbf{C}_1$ and $\mathbf{C}_2$ can conclude whether its two different inputs lead to the same output or not, breaching privacy. Note that the decoding information cannot be given via this cheat recovery box that uses conflicting encoded outputs as key, as that would result in circularity.

Despite using the above fix, the mechanism as discussed above is susceptible to 'selective failure attack', an attack well-known in the 2-party domain. While in the latter domain, the attack is launched to breach the privacy of the evaluator's input based on whether it aborts or not. Here, a corrupt garbler can prepare the ciphertexts in an incorrect way and can breach privacy of its honest co-garbler based on whether clear or encoded output comes out of the evaluator. We elaborate the attack in fair_3 considering a corrupt P_1 and single bit inputs. P_1 is supposed to prepare two ciphertexts corresponding to P_2's input bit using the following key combinations– (a) key for 0 in $\mathbf{C}_1$ and 1 in $\mathbf{C}_2$ and (b) vice-versa. Corrupt P_1 may replace one of the ciphertexts using key based on encoded input 0 of P_2 in both the GCs. In case P_2 indeed has input 0 (that he would use consistently across the 2 GCs during fair_3), then P_3 would be able to decrypt the ciphertext and would send clear output in Round 3. P_1 can readily conclude that P_2's input is 0. This attack is taken care via the usual technique of breaking each input bit to s number of xor-shares, referred as 'XOR-tree approach' [LP07] (probe-resistance matrix [LP07, SS13] can also be used; we avoid it for simplicity). The security is achieved except with probability $2^{-(s-1)}$.

Given that input consistency is enforced, at the end of round 2, apart from the three states– (a) no corruption and no conflict detected (b) corrupt identified (c) conflict detected, a party can be in yet another state. Namely, no corruption and no conflict detected and the party is able to open a ciphertext and compute f on clear. A corrupt party cannot be in this state since the honest parties would use consistent inputs and therefore the corrupt would not get access to conflicting encoded inputs that constitute the key

of the ciphertexts. If any honest party is in this state, our protocol results in all parties outputting this output. In Round 3, this party can send the computed output along with the opening of the shares he recovered via the ciphertexts as 'proof' to convince the honest party of the validity of the output. The protocol fair appears in Figure 4 and the schematic diagram is given in Appendix D.1.

We now prove the correctness of fair.

Lemma 5. *During* fair, $P_j \notin \mathcal{C}_i$ *holds for honest* P_i, P_j.

Proof. An honest P_i will not include P_j in its corrupt set in the sub-protocols $\{\mathsf{fair}_\alpha, \mathsf{cert}_\alpha\}_{\alpha\in[3]}$ following Lemma 1, Lemma 3. Now we prove the statement individually investigating the three rounds of fair.

In Round 1 of fair, P_i includes P_j as corrupt only if (a) P_i, P_j are garblers and P_j sets $p_{jj} \neq x_{ji}$ or (b) P_j sends $\mathsf{pp}_j, c_{ji}, o_{ji}, x_{ji}$ to P_i such that $\mathsf{Open}(\mathsf{pp}_j, c_{ji}, o_{ji}) \neq x_{ji}$. None of them will be true for an honest P_j. In Round 2 of fair, P_i includes P_j as corrupt only if (a) P_j is a garbler and P_i is an evaluator and $m_{jj} \neq x_{ji}$ or (b) P_i obtains $\mathbf{cert}_i = (\gamma'_j, \gamma'_k)$ and detects P_j's input γ'_j in cert_i to be different from the information sent by him. The former will not be true for an honest P_j. The latter also cannot hold for honest P_j by correctness of the privacy-free garbling used. In the last round of fair, P_i will identify P_j as corrupt, if it has $\mathsf{flag}_k = 1$ and yet receives $\mathbf{cert}_k$ which is same as $\mathbf{key}_k$ from P_k. A corrupt P_k receives $\mathbf{key}_k$ only by handing out correct and consistent common information to P_i and P_j until the end of Round 1. Namely, the following must be true for P_k to obtain $\mathbf{key}_k$ (except for the case when it breaks the authenticity of the GC): (i) γ_i and γ_j for cert_k must be same and (ii) P_k must not be in the corrupt set of any honest party at the end of Round 1. In this case, flag_k cannot be 1. □

Lemma 6. *No corrupt party can be in* $\mathtt{st}_1$ *by the end of Round 1, except with negligible probability.*

Proof. For a corrupt P_k, its honest garblers P_i and P_j creates the ciphertexts cts using keys with opposite meaning for their respective inputs from their garbled circuits. Since honest P_i and P_j use the same input for both the circuits, P_k will not have a key to open any of the ciphertexts. The openings (o_{ij}, o_{ji}) are therefore protected due to the security of the encryption scheme. Subsequently, P_k cannot compute y. □

Definition 1. *A party* P_i *is said to be 'committed' to a unique input* x_i*, if* P_j *holds* $(c_{ij}, c_{ik}, o_{ij}, x_{ij})$ *and* P_k *holds* $(c_{ij}, c_{ik}, o_{ik}, x_{ik})$ *such that:* **(a)** $x_i = x_{ij} \oplus x_{ik}$ *and* **(b)** c_{ij} *opens to* x_{ij} *via* o_{ij} *and likewise,* c_{ik} *opens to* x_{ik} *via* o_{ik}.

We next prove that a corrupt party must have committed its input if some honest party is in $\mathtt{st}_1$ or $\mathtt{st}_2$. To prove correctness, the next few lemmas then show that an honest party computes its output based on its own output or encoded output if it is in $\mathtt{st}_1$ or $\mathtt{st}_2$ or relies on the output or encoded output of the other *honest* party. In all cases, the output will correspond to the committed input of the corrupt party.

Lemma 7. *If an honest party is in* $\{\mathtt{st}_1, \mathtt{st}_2\}$*, then corrupt party must have committed a unique input.*

Protocol $\mathsf{fair}()$

Inputs: Party P_i has x_i for $i \in [3]$.
Output: $y = f(x_1, x_2, x_3)$ or $\perp$ where the inputs and the function output belong to $\{0,1\}^\ell$.
Subprotocols: fair_i for $i \in [3]$ (Figure 1), cert_i for $i \in [3]$ (Figure 2), SKE $(\mathsf{Enc}, \mathsf{Dec})$ with 'special correctness' (Definition 6).

Round 1: For $i \in [3]$ and for distinct indices $j, k \in [3] \setminus \{i\}$

- Each P_i computes an encoding of length ℓs corresponding to its input x_i. For each bit b of x_i, the encoding $b_1, \ldots b_s$ is such that $b = \oplus_{\alpha=1}^{s} b_\alpha$. Reusing the notation, we refer to this encoding as P_i's input x_i and its length by ℓ.
- **Round 1** of cert_i is run.
- **Round 1** of fair_i are run with the following amendments: **(1)** The circuit in fair_i is changed as follows: each input wire is replaced by a gate whose input consists of s new input wires and whose output is the exclusive-or of these wires. **(2)** P_j and P_k work with the permutation strings p_{jj} and p_{kk} respectively as x_{jk} and x_{kj}.
- P_i samples pp_i, generates $(c_{ij}, o_{ij}) \leftarrow \mathsf{Com}(\mathsf{pp}_i, x_{ij})$, $(c_{ik}, o_{ik}) \leftarrow \mathsf{Com}(\mathsf{pp}_i, x_{ik})$ and sends $\{\mathsf{pp}_i, c_{ij}, c_{ik}\}$ to P_j, P_k. Additionally, P_i sends o_{ij}, o_{ik} to P_j, P_k respectively.
- (Local Computation by P_i) P_i adds P_ℓ in $\mathcal{C}_i$ if $\mathsf{Open}(c_{li}, o_{li}) \neq x_{li}$. P_j adds P_k in $\mathcal{C}_j$ if: **(a)** p_{kk} not taken as x_{kj} or **(b)** the check in fair_i or cert_i fails. P_k adds P_j in $\mathcal{C}_k$ if: **(a)** p_{jj} not taken as x_{jk} or **(b)** the check in fair_i or cert_i fails.

Round 2: For $i \in [3]$ and for distinct indices $j, k \in [3] \setminus \{i\}$:

- If $P_i \notin \mathcal{C}_j$, P_j sends $(\mathsf{pp}_i, c_{ij}, c_{ik})$ to P_k. If $P_i \notin \mathcal{C}_k$, P_k sends $(\mathsf{pp}_i, c_{ij}, c_{ik})$ to P_j. They set $\mathsf{flag}_i = 1$ in case of mismatch or no communication.
- If $P_i \notin \mathcal{C}_j$, P_j participates in cert_i as a garbler with input γ_j as $\{\mathcal{D}_i^j, \mathcal{D}_i^k, \mathcal{W}_k, \mathsf{pp}_i, c_{ij}, c_{ik}\}$ where $\mathcal{D}_i^j, \mathcal{D}_i^k, \mathcal{W}_k$ and $(\mathsf{pp}_i, c_{ij}, c_{ik})$ was received from P_i during Round 1 of $\mathsf{fair}_j, \mathsf{fair}_k$, cert_k (assuming $k = (i+1) \mod 3$) and fair respectively. Similar step is taken by P_k.
- If $\mathbf{cert}_i = (\gamma'_j, \gamma'_k)$, P_i sets $\mathcal{C}_i = P_l$ if $\gamma'_l \neq \{\mathcal{D}_i^j, \mathcal{D}_i^k, \mathcal{W}_k, \mathsf{pp}_i, c_{ij}, c_{ik}\}$ for $l \in \{j, k\}$.
- If $P_i \notin \mathcal{C}_j$, P_j participates in **Round 2** of fair_i. When $P_k \notin \mathcal{C}_j$, P_j additionally sends the ciphertexts $\mathsf{ct}_{j\alpha}^\beta$ for $\beta \in \{0,1\}$ and $\alpha \in [\ell]$ created as follows. Let $\{\mathbf{X}_{l(\ell+\alpha)}^0, \mathbf{X}_{l(\ell+\alpha)}^1\}$, denote the encoding information of co-garbler P_k's input wire α corresponding to $\mathbf{C}_l$ $(l \in \{j,k\})$. Then $\mathsf{ct}_{j\alpha}^\beta = \mathsf{Enc}_{\mathsf{sk}_\alpha^\beta}(o_{jk}, o_{kj})$ for $\mathsf{sk}_\alpha^0 = \mathbf{X}_{j(\ell+\alpha)}^0 \oplus \mathbf{X}_{k(\ell+\alpha)}^1$ and $\mathsf{sk}_\alpha^1 = \mathbf{X}_{j(\ell+\alpha)}^1 \oplus \mathbf{X}_{k(\ell+\alpha)}^0$. P_k takes similar steps.
- (Local Computation by P_i) Include P_l in $\mathcal{C}_i$ if $m_{ll} \neq x_{li}$ for $l \in \{j,k\}$. If $\mathcal{C}_i = \emptyset$, $\mathsf{flag}_j = 0, \mathsf{flag}_k = 0$, then use key $\mathbf{X}_{j(\ell+\alpha)}^{m_{jk}^\alpha} \oplus \mathbf{X}_{k(\ell+\alpha)}^{m_{kk}^\alpha}$ $(\alpha \in [\ell])$ to decrypt the ciphertexts $\mathsf{ct}_{j\alpha}^0$ or $\mathsf{ct}_{j\alpha}^1$ obtained from P_j. If the decryption succeeds, retrieve o_{kj}, o_{jk}. Execute $x_{kj} \leftarrow \mathsf{Open}(c_{kj}, o_{kj})$ and $x_{jk} \leftarrow \mathsf{Open}(c_{jk}, o_{jk})$. If the opening succeeds, then evaluate f on $(x_i, x_{ji} \oplus x_{jk}, x_{ki} \oplus x_{kj})$ to obtain y. Similarly, steps are taken with respect to P_j's input, using the key $\mathbf{X}_{j\alpha}^{m_{jj}^\alpha} \oplus \mathbf{X}_{k\alpha}^{m_{kj}^\alpha}$ to decrypt the ciphertexts $\mathsf{ct}_{k\alpha}^0$ or $\mathsf{ct}_{k\alpha}^1$ obtained from P_k.

Fig. 3: A Three-Round Fair 3PC protocol

Proof. An honest P_i is in $\{\mathtt{st}_1, \mathtt{st}_2\}$ only when $\mathcal{C}_i = \emptyset, \mathsf{flag}_j = 0, \mathsf{flag}_k = 0$ hold at the end of Round 2. Assume P_k is corrupt. P_k has not committed to a unique x_k implies either it has distributed different copies of commitments (c_{ki}, c_{kj}) to the honest parties

A party P_i is said to be in $\mathtt{st}_\alpha$ for $\alpha \in [4]$ if the following conditions are satisfied. Let $(\mathbf{Y}_i, C_j^{\mathsf{dec}}, C_k^{\mathsf{dec}})$, O_j^{dec} and O_k^{dec} denote the output of P_i in fair_i, fair_j and fair_k, respectively. Let $\mathbf{cert}_i$, $\mathbf{key}_j$, and $\mathbf{key}_k$ denotes the output of P_i in cert_i, cert_j and cert_k respectively.

(i) $\mathtt{st}_1$(output is already computed): If y and proofs (o_{jk}, o_{kj}) are computed in Round 2.
(ii) $\mathtt{st}_2$ (no corruption and no conflict detected): If $((\mathcal{C}_i = \emptyset) \wedge (\mathsf{flag}_j = 0) \wedge (\mathsf{flag}_k = 0))$ (which implies $\mathbf{Y}_i \neq \perp$ and $\mathbf{cert}_i \neq \perp$)
(iii) $\mathtt{st}_3$ (corruption detected): If $(\mathcal{C}_i \neq \emptyset)$
(iv) $\mathtt{st}_4$ (conflict detected, but no corruption detected): If $(\mathsf{flag}_j = 1) \vee (\mathsf{flag}_k = 1)$

Round 3: Each P_i for $i \in [3]$ does the following based one of the four states that it belongs to.

- If in $\mathtt{st}_1$, then send y to P_j, P_k. Send o_{jk} to P_j and o_{kj} to P_k as proofs.
- If in $\mathtt{st}_2$, then send $(\mathbf{Y}_i, \mathbf{cert}_i, O_l^{\mathsf{dec}})$ to P_l for $l \in \{j, k\}$.
- If in $\mathtt{st}_3$, then send O_l^{dec} to P_l for $l \in \{j, k\}$ only if $P_l \notin \mathcal{C}_i$.
- If in $\mathtt{st}_4$, then send $z_l = \mathsf{Enc}_{\mathbf{key}_l}(O_l^{\mathsf{dec}})$ to P_l only if $\mathsf{flag}_l = 1$. If $\mathsf{flag}_j = 1$ and $\mathbf{cert}_j$ received from P_j is same as $\mathbf{key}_j$, then set $\mathcal{C}_i = P_k$. Similar steps are taken to check and identify if P_j is corrupt. Update state from $\mathtt{st}_4$ to $\mathtt{st}_3$ if corrupt is identified.
- If in $\mathtt{st}_1$, then output y.
- If in $\{\mathtt{st}_2, \mathtt{st}_3, \mathtt{st}_4\}$ and if any other party is identified to be in $\mathtt{st}_1$, namely if y is received from P_j or P_k with o_{ki} or o_{ji} respectively such that $\mathsf{Open}(\mathsf{pp}_i, c_{li}, o_{li}) \neq \perp$ for $l \in \{j, k\}$, then output the received y.
- If in $\mathtt{st}_2$, then compute y as follows: Retrieve O_i^{dec} from either z_i (with $\mathbf{cert}_i$ as the key) received from P_j or from direct communication of P_j. If $d \leftarrow \mathsf{eOpen}(\mathsf{epp}_k, C_k^{\mathsf{dec}}, O_i^{\mathsf{dec}})$ is not $\perp$, then use d to compute $y \leftarrow \mathsf{De}(\mathbf{Y}_i^k, d)$. Similar steps are executed with respect to P_k's communication if y is not computed yet.
- If in $\mathtt{st}_3$, then output $y \leftarrow \mathsf{De}(\mathbf{Y}_l^i, d)$ where $\mathbf{Y}_l$ is received from (honest) $P_l \notin \mathcal{C}_i$ and decoding information d is known as garbler during fair_l. Otherwise output $y = \perp$.
- If in $\mathtt{st}_4$, output $y = \perp$.

Fig. 4: A Three-Round Fair 3PC protocol

or distributed incorrect opening information to some honest party. In the former case, flag_k will be set by P_i. In the latter case, at least one honest party will identify P_k to be corrupt by the end of Round 1. If it is P_i, then $\mathcal{C}_i \neq \emptyset$. Otherwise, P_j populates its corrupt set with P_k, leading to P_i setting $\mathsf{flag}_k = 1$ in Round 2. □

Lemma 8. *If an honest party is in* $\mathtt{st}_1$*, then its output* y *corresponds to the unique input committed by the corrupt party.*

Proof. An honest P_i is in $\mathtt{st}_1$ only when $\mathcal{C}_i = \emptyset, \mathsf{flag}_j = 0, \mathsf{flag}_k = 0$ hold at the end of Round 2 and it computes y via decryption of the ciphertexts ct sent by either P_j or P_k. Assume P_k is corrupt. By Lemma 7, P_k has committed to its input. The condition $\mathsf{flag}_j = 0$ implies that P_k exchanges the commitments on the shares of P_j's input, namely $\{c_{ji}, c_{jk}\}$, honestly. Now if P_i opens honest P_j's ciphertext, then it unlocks the opening information for the missing shares, namely (o_{kj}, o_{jk}) corresponding to common and agreed commitments (c_{kj}, c_{jk}). Using these it opens the missing shares $x_{kj} \leftarrow \mathsf{Open}(c_{kj}, o_{kj})$ and $x_{jk} \leftarrow \mathsf{Open}(c_{jk}, o_{jk})$ and finally computes output on $(x_i, x_{ji} \oplus x_{jk}, x_{ki} \oplus x_{kj})$. Next, we consider the case when P_i computes y by decrypting

a ct sent by corrupt P_k. In this case, no matter how the ciphertext is created, the binding property of NICOM implies that P_k will not be able to open c_{jk}, c_{kj} to anything other than x_{jk}, x_{kj} except with negligible probability. Thus, the output computed is still as above and the claim holds. □

Lemma 9. *If an honest party is in* $\mathtt{st}_2$*, then its encoded output* $\mathbf{Y}$ *corresponds to the unique input committed by the corrupt party.*

Proof. An honest P_i is in $\mathtt{st}_2$ only when $\mathcal{C}_i = \emptyset$, $\mathsf{flag}_j = 0, \mathsf{flag}_k = 0$ hold at the end of Round 2. The conditions also imply that P_i has computed $\mathbf{Y}_i$ successfully (due to Lemma 2) and P_k has committed to its input (due to Lemma 7). Now we show that $\mathbf{Y}_i$ correspond to the unique input committed by the corrupt P_k. We first note that P_k must have used the same input for both the circuits $\mathbf{C}_j$ and $\mathbf{C}_k$ in fair_i. Otherwise one of the ciphertexts prepared by honest P_j must have been opened and y would be computed, implying P_i belongs to $\mathtt{st}_1$ and not in $\mathtt{st}_2$ as assumed. We are now left to show that the input of P_k for its circuit $\mathbf{C}_k$ in fair_i is the same as the one committed.

In fair, honest P_j would use permutation string $p_{kk} = x_{kj}$ for permuting the commitments in $\mathcal{D}_k$ corresponding to x_k. Therefore, one can conclude that the commitments in $\mathcal{D}_k$ are constructed correctly and ordered as per x_{kj}. Now the only way P_k can decommit x'_k is by giving $m_{kk} = p_{kk} \oplus x'_k$. But in this case honest P_i would add P_k to $\mathcal{C}_i$ as the check $m_{kk} = x_{ki}$ would fail ($m_{kk} = p_{kk} \oplus x'_k \neq p_{kk} \oplus x_k$) and will be in $\mathtt{st}_3$ and not in $\mathtt{st}_2$ as assumed.

□

Lemma 10. *If an honest party is in* $\mathtt{st}_2$*, then its output* y *corresponds to the unique input committed by the corrupt party.*

Proof. Note that an honest party P_i in $\mathtt{st}_2$ either uses y of another party in $\mathtt{st}_1$ or computes output from its encoded output $\mathbf{Y}_i$. The proof for the former case goes as follows. By Lemma 6, a corrupt P_k can never be in $\mathtt{st}_1$. The correctness of y computed by an honest P_j follows directly from Lemma 8. For the latter case, Lemma 9 implies that $\mathbf{Y}_i$ corresponds to the unique input committed by the corrupt party. All that needs to be ensured is that P_i gets the correct decoding information. The condition $\mathsf{flag}_j = \mathsf{flag}_k = 0$ implies that the commitment to the decoding information is computed and distributed correctly for both $\mathbf{C}_j$ and $\mathbf{C}_k$. Now the binding property of eNICOM ensures that the decoding information received from either P_j (for $\mathbf{C}_k$) or P_k (for $\mathbf{C}_j$) must be correct implying correctness of y (by correctness of the garbling scheme). □

Lemma 11. *If an honest party is in* $\mathtt{st}_3$ *or* $\mathtt{st}_4$*, then its output* y *corresponds to the unique input committed by the corrupt party.*

Proof. An honest party P_i in $\mathtt{st}_3$ either uses y of another party in $\mathtt{st}_1$ or computes output from encoded output $\mathbf{Y}_j$ of P_j who it identifies as honest. For the latter case note that an honest P_j will never be identified as corrupt by P_i, due to Lemma 5. The claim now follows from Lemma 6, Lemma 8 and the fact that corrupt P_k cannot forge the 'proof' o_{ij} (binding of NICOM) for the former case and from Lemma 9 and the fact that it possesses correct decoding information as a garbler for $\mathbf{Y}_j$ for the latter case. An honest party P_i in $\mathtt{st}_4$ only uses y of another party in $\mathtt{st}_1$. The lemma follows in this case via the same argument as before. □

Theorem 1. *Protocol* fair *is correct.*

Proof. In order to prove the theorem, we show that if an honest party, say P_i outputs y that is not $\perp$, then it corresponds to x_1, x_2, x_3 where x_j is the input committed by P_j (Definition 1). We note that an honest P_i belong to one among $\{\mathtt{st}_1, \mathtt{st}_2, \mathtt{st}_3, \mathtt{st}_4\}$ at the time of output computation. The proof now follows from Lemmas 7,8,10,11. □

Fairness implies: (a) if a corrupt party gets the output then so does the honest parties; (b) if an honest party gets the output then so does the other parties. We give the intuition for both below starting with (a). The formal proof appears in Appendix D.2.

A corrupt P_k cannot be in $\mathtt{st}_1$ (due to Lemma 6). The only way it can retrieve the output is by having an honest party in $\mathtt{st}_1$ or $\mathtt{st}_2$. An honest party in $\mathtt{st}_3$ only releases the decoding information and it never release it to a corrupt party (Lemma 5 implies it identifies the honest party correctly). An honest party in $\mathtt{st}_4$ releases the encrypted decoding information z_k under key $\mathbf{key}_k$ to P_k conditionally when $\mathsf{flag}_k = 1$. The condition $\mathsf{flag}_k = 1$ implies that P_k must have distributed the common information incorrectly and so γ_i and γ_j are not same. This further implies $\mathbf{cert}_k$ is not same as $\mathbf{key}_k$ and so P_k does not have access to the key to open z_k and cannot recover the decoding information. So the corrupt P_k getting the output implies that at least one honest party is in $\{\mathtt{st}_1, \mathtt{st}_2\}$. Lemma 7 implies that in this case, P_k must have committed to a unique input. By Lemma 8 and Lemma 10, the y and encoded output $\mathbf{Y}$ computed by any honest party in $\mathtt{st}_1$ and in $\mathtt{st}_2$ respectively will correspond P_k's committed input. Further, if P_k computes encoded output $\mathbf{Y}_k$, it also correspond to P_k's committed input. So no matter how the corrupt party compute the output, it will be with respect to unique (x_1, x_2, x_3). We need to show that both honest parties receive the same output. This easily follows when at least one honest party is in $\mathtt{st}_1$. We now prove the lemma based on the following cases. (a) Both P_i, P_j are in $\mathtt{st}_2$: They receive the decoding information from each other on the clear and use their respective computed encoded output to compute the output y. (b) P_i is in $\mathtt{st}_2$ and P_j in $\mathtt{st}_3$: P_i uses the decoding information sent exclusively to him by P_j and decode the output as in the previous case. P_j uses the encoded output of P_i, $\mathbf{Y}_i$ and its decoding information (held as a garbler) to compute the output. (c) P_i is in $\mathtt{st}_2$ and P_j in $\mathtt{st}_4$: P_j must be in $\mathtt{st}_4$ because of $\mathsf{flag}_i = 1$. If $\mathsf{flag}_k = 1$, P_i will have the same status for this flag and would belong to $\mathtt{st}_4$. Now since $\mathsf{flag}_i = 1$, P_j sends encryption of the decoding information z_i to P_i who can use $\mathbf{cert}_i$ to decrypt z_i and compute the output as in the previous two cases. P_j, on noting that $\mathsf{flag}_i = 1$, yet P_i obtained $\mathbf{cert}_i = \mathbf{key}_i$, will identify P_k to be corrupt, upgrade to $\mathtt{st}_3$ and compute the output as in the previous case.

Next, we argue for part (b). For an honest party to compute the output y, at least one honest party must be in $\{\mathtt{st}_1, \mathtt{st}_2\}$. If both belong to $\{\mathtt{st}_3, \mathtt{st}_4\}$, then neither P_k has committed any input (due to Lemma 7) nor anyone gets the output. The latter follows by the argument below. An honest party in $\mathtt{st}_3$ only outputs based on the encoded output of the other honest party. But since the other honest party is in $\{\mathtt{st}_3, \mathtt{st}_4\}$, it will output $\perp$. An honest party in $\mathtt{st}_4$ outputs $\perp$, except for the case it finds one in $\mathtt{st}_1$ which is not true for both P_j and P_k (Lemma 6). The corrupt P_k does not get the output too following the fact that it cannot be in $\mathtt{st}_1$ (Lemma 6) and it does not receive decoding information from an honest party. An honest party P_i in $\mathtt{st}_3$ sends the decoding information only to the *identified* honest party. An honest party P_i in $\mathtt{st}_4$ may send the encrypted decoding

information z_k under key $\mathbf{key}_k$ to P_k when $\mathsf{flag}_k = 1$. But the condition $\mathsf{flag}_k = 1$ implies that P_k must have distributed the common information incorrectly and so γ_i and γ_j are not same. This further implies $\mathbf{cert}_k$ is not same as $\mathbf{key}_k$ and so P_k does not have access to the key to open z_k and cannot recover the opening information. Now we are left to show that when at least one honest party is in $\{\mathtt{st}_1, \mathtt{st}_2\}$, then everyone gets the output. This already follows from the argument given for the other direction.

4 2-round 3PC with Unanimous Abort

This section presents a tight upper bound for 3PC achieving unanimous abort in the setting with pair-wise private channels and a broadcast channel. The impossibility of one-round protocol in the same setting follows from "residual function" attack [HLP11]. Our result from Section 6.2 rules out the possibility of achieving unanimous abort in the absence of a broadcast channel in two rounds. This protocol can be used to yield a round-optimal fair protocol with broadcast (lower bound in Section 6.1) by application of the transformation of [IKP+16] that compiles a protocol with unanimous abort to a fair protocol via evaluating the circuits that compute shares (using error-correcting secret sharing) of the function output using the protocol with unanimous abort and then uses an additional round for reconstruction of the output.

In an attempt to build a protocol with unanimous abort, we note that any protocol with unanimous abort must be robust to any potential misbehaviour launched via the private communication in the second round. Simply because, there is no way to report the abort to the other honest party who may have seen honest behaviour from the corrupt party all along and has got the output, leading to selective abort. Our construction achieves unanimity by leveraging the availability of the broadcast channel to abort when a corrupt behaviour is identified either in the first round or in the broadcast communication in the second round, and behaving robustly otherwise. In summary, if the corrupt party does not strike in the first round and in the broadcast communication of the second round, then our construction achieves robustness.

Turning to the garbled circuit based constructions such as the two-round protocol of [IKKP15] achieving selective abort or the composition of three copies of the sub-protocol fair_i of fair, we note that the second round private communication that involves encoding information for inputs is crucial for computing the output and cannot transit via broadcast because of input privacy breach. A bit elaborately, the transfer of the encoding information for the inputs of the garblers can be completed in the first round itself and any inconsistency can be handled via unanimous abort in the second round. However, a similar treatment for the encoding information of the shares of the evaluator seems impossible as they are transferred to garblers only in the first round. We get past this seemingly impossible task via a clever 'two-part release mechanism' for the encoding information of the shares of the evaluator. Details follow.

Similar to protocol fair, we build our protocol ua upon three parallel executions of a sub-protocol ua_i ($i \in [3]$), each comprising of two rounds and with each party P_i enacting the role of the evaluator once. With fair_i as the starting point, each sub-protocol ua_i allows the parties to reach agreement on whether the run was successful and the evaluator got the output or not. A flag flag_i is used as an indicator. The protocol ua then

decides on unanimous abort if at least one of the flags from the three executions ua_i for $i \in [3]$ is set to true. Otherwise, the parties must have got the output. Input consistency checks ensure that the outputs are identical. Intra-execution input consistency is taken care by cheat-recovery mechanism (similar and simplified version of what protocol fair uses), while inter-execution input consistency is taken care by the same trick that we use in our fair protocol. Now looking inside ua_i, the challenge goes back to finding a mechanism for the honest evaluator to get the output when a corrupt party behaves honestly in the first round and in the broadcast communication of the second round. In other words, its private communication in the second round should not impact robustness. This is where the 'two-part release mechanism' for the encoding information of the shares of the evaluator kicks in. It is realized by tweaking the function to be evaluated as $f(x_j, x_k, (z_j \oplus r_j) \oplus (z_k \oplus r_k))$ in the instance ua_i where P_i enacts the role of the evaluator. Here r_j, r_k denote random pads chosen by the garblers P_j, P_k respectively in the first round. The encoding information for these are released to P_i *privately* in the first round itself. Any inconsistent behaviour in the first round is detected, the flag is set and the the protocol exits with $\perp$ unanimously. Next, z_j and z_k are the offsets of these random pads with the actual shares of P_i's input and are available only at the end of first round. The encoding information for these offsets and these offsets themselves are transferred via broadcast in the second round for public verification. As long as the pads are privately communicated, the offsets do not affect privacy of the shares of P_i's input. Lastly, note that the encoding information for a garbler's input for its own generated circuit can be transferred in the first round itself. This ensures that a corrupt garbler misbehaves either in the first round or in the broadcast communication in the second round or lets the evaluator get the output via its own GC. We describe execution ua_i, assuming input consistency. Entwining the three executions, tackling the input consistency and the final presentation of protocol ua are done next. Lastly, we present the security proof.

4.1 Protocol ua_i

With the goal to achieve agreement among the honest parties regarding whether the evaluator got the output or not, ua_i starts with fair_i and makes the following changes. First, the broadcast channel is used to reach agreement on the commitments to GCs and the encoding information. Second, a garbling scheme with soft decoding property is used to allow immediate output decoding. Third, a garbler opens its encoded input for its own GC in the first round itself. In addition, we implement the two-part release mechanism for P_i's shares where apart from the garblers, P_i too broadcasts the offsets in the second round. A flag flag_i is used to keep track if a complaint is raised for the first round communication by broadcast in the second round or the offsets broadcasted in parallel by both P_i and respective garblers do not match or the opening of the encoded input for the offsets fails. When flag_i remains to be false for the honest parties, an honest P_i must be able to evaluate and output from the GC prepared by the corrupt garbler. Because, the commitments to that GC and encoding information has been scrutinized by the honest co-garbler, the encoded input of the corrupt party has been verified by the evaluator, the release of the encoded inputs for the shares of the evaluator has been verified publicly and the offsets themselves matched. Lastly, since the flag when set to

be true by any honest party in the end of first round can be propagated to all in the second round and is only set based on the broadcasts in the second round, all honest parties exit ua_i with an agreement on flag_i. We now present our protocol in Figures 5-6 assuming input consistency and prove its properties needed later.

Protocol $\mathsf{ua}_i()$

Inputs: Party P_α has x_α for $\alpha \in [3]$.

Common Inputs: The circuit $C((x_j, r_j, z_j), (x_k, r_k, z_k), \perp)$ that computes $f(x_j, x_k, (z_j \oplus r_j) \oplus (z_k \oplus r_k))$ such that $z_j \oplus r_j = x_{ij}$, $z_k \oplus r_k = x_{ik}$ and $x_{ij} \oplus x_{ik} = x_i$ and where the inputs belong to $\{0,1\}^\ell$. For distinct $i, j, k \in [3]$, P_i acts as the evaluator and (P_j, P_k) as the garblers.

Output: All parties output boolean flag_i, initially set to 0. P_i outputs (y_j, y_k).

Primitives: A correct, private and authentic garbling scheme $\mathcal{G} = (\mathsf{Gb}, \mathsf{En}, \mathsf{Ev}, \mathsf{De})$ with soft decoding, an eNICOM $(\mathsf{eGen}, \mathsf{eCom}, \mathsf{eOpen}, \mathsf{Equiv})$, a PRG G and a NICOM $(\mathsf{Com}, \mathsf{Open})$

Round 1:

- P_i randomly secret shares his input x_i as $x_i = x_{ij} \oplus x_{ik}$ and sends x_{ij} to P_j and x_{ik} to P_k.
- P_l for $l \in \{j, k\}$ samples $s_l \in_R \{0,1\}^\kappa$, epp_l and pp_l for G, eNICOM and NICOM resp. and:
  - compute garbled circuit $(\mathbf{C}_l, e_l, d_l) \leftarrow \mathsf{Gb}(1^\kappa, C)$ using randomness from $\mathsf{G}(s_l)$. Assume $\{e^0_{l\alpha}, e^1_{l\alpha}\}_{\alpha\in[\ell]}$, $\{e^0_{l(\ell+\alpha)}, e^1_{l(\ell+\alpha)}\}_{\alpha\in[\ell]}$, $\{e^0_{l(2\ell+\alpha)}, e^1_{l(2\ell+\alpha)}\}_{\alpha\in[2\ell]}$, $\{e^0_{l(4\ell+\alpha)}, e^1_{l(4\ell+\alpha)}\}_{\alpha\in[2\ell]}$ correspond to the encoding information for the input of P_j, P_k (i.e. x_j, x_k), the random inputs chosen by P_j, P_k (i.e. r_j, r_k) and the offsets between the random pads and the secret shares of P_i (i.e. z_j, z_k) respectively.
  - compute commitment for the GC as $(c_l, o_l) \leftarrow \mathsf{eCom}(\mathsf{epp}_l, \mathbf{C}_l)$
  - sample permutation strings $p_{lj}, p_{lk} \in_R \{0,1\}^\ell$ for the inputs of P_j and P_k and compute commitments of encoding information as: For $b \in \{0,1\}$, $(c^b_{l\alpha}, o^b_{l\alpha}) \leftarrow \mathsf{Com}(\mathsf{pp}_l, e^{p^\alpha_{lj}\oplus b}_{l\alpha})$, $(c^b_{l(\ell+\alpha)}, o^b_{l(\ell+\alpha)}) \leftarrow \mathsf{Com}(\mathsf{pp}_l, e^{p^\alpha_{lk}\oplus b}_{l(\ell+\alpha)})$ when $\alpha \in [\ell]$, $(c^b_{l(2\ell+\alpha)}, o^b_{l(2\ell+\alpha)}) \leftarrow \mathsf{Com}(\mathsf{pp}_l, e^b_{l(2\ell+\alpha)})$, $(c^b_{l(4\ell+\alpha)}, o^b_{l(4\ell+\alpha)}) \leftarrow \mathsf{Com}(\mathsf{pp}_l, e^b_{l(4\ell+\alpha)})$ when $\alpha \in [2\ell]$.
  - broadcast $\mathcal{D}_l = (\mathsf{epp}_l, \mathsf{pp}_l, c_l, \{c^b_{l\alpha}\}_{\alpha\in[6\ell], b\in\{0,1\}})$ and send $\{s_l, p_{lj}, p_{lk}, o_l, \{o^b_{l\alpha}\}_{\alpha\in[6\ell], b\in\{0,1\}}\}$ privately to the co-garbler $P_{\{j,k\}\setminus l}$.
- P_j computes indicator string $m_{jj} = p_{jj} \oplus x_j$, picks its share of pad $r_j \in_R \{0,1\}^\ell$ and sends $\left(\{o^{m^\alpha_{jj}}_{j\alpha}, o^{r^\alpha_j}_{j(2\ell+\alpha)}\}_{\alpha\in[\ell]}, m_{jj}, r_j\right)$ to P_i. Similarly, P_k computes m_{kk}, picks r_k and sends $\left(\{o^{m^\alpha_{kk}}_{k(\ell+\alpha)}, o^{r^\alpha_k}_{k(3\ell+\alpha)}\}_{\alpha\in[\ell]}, m_{kk}, r_k\right)$ to P_i.
- (Local Computation by garblers) P_j sets $\mathsf{flag}_i = 1$ if $\mathcal{D}_k$ and $\{s_k, p_{kj}, p_{kk}, o_k, \{o^b_{k\alpha}\}_{\alpha\in[6\ell], b\in\{0,1\}}\}$ received from P_k are not consistent. P_k performs similar steps with respect to the values received from P_j.
- (Local Computation by evaluator) P_i sets $\mathsf{flag}_i = 1$ if (a) the openings of the input labels sent by P_j fail to open some commitment in $\mathcal{D}_j$ i.e $\mathsf{Open}(\mathsf{pp}_j, c^{m^\alpha_{jj}}_{j\alpha}, o^{m^\alpha_{jj}}_{j\alpha}) = \perp$ or $\mathsf{Open}(\mathsf{pp}_j, c^{r^\alpha_j}_{j(2\ell+\alpha)}, o^{r^\alpha_j}_{j(2\ell+\alpha)}) = \perp$ for some $\alpha \in [\ell]$ OR (b) the openings for the input labels sent by P_k fail to open some commitment in $\mathcal{D}_k$.

Fig. 5: Protocol ua_i

Contd. Protocol $\mathsf{ua}_i()$

Round 2:

- P_j broadcasts `abort` if $\mathsf{flag}_i = 1$. Else, it computes its indicator string $m_{kj} = p_{kj} \oplus x_j$ for P_k's circuit and the offset $z_j = x_{ij} \oplus r_j$, sends $\left(\mathtt{OK}, o_k, \{o_{k\alpha}^{m_{kj}^\alpha}, o_{k(2\ell+\alpha)}^{r_j^\alpha}, o_{k(4\ell+\alpha)}^{z_j^\alpha}\}_{\alpha\in[\ell]}, m_{kj}\right)$ privately to P_i and broadcasts $\mathcal{W}_j = \left(z_j, \{o_{j(4\ell+\alpha)}^{z_j^\alpha}\}_{\alpha\in[\ell]}\right)$. P_k performs similar steps.
- P_i broadcasts `abort` if $\mathsf{flag}_i = 1$. Else, it broadcasts $z_j = x_{ij} \oplus r_j$ and $z_k = x_{ik} \oplus r_k$
- Every party sets $\mathsf{flag}_i = 1$ if **(a)** `abort` was received or sent via broadcast in Round 2 OR **(b)** either z_j broadcast by (P_j, P_i) or z_k broadcast by (P_k, P_i) do not match OR **(c)** $\mathcal{D}_j, \mathcal{W}_j$ is not consistent i.e $\mathsf{Open}(\mathsf{pp}_j, c_{j(4\ell+\alpha)}^{z_j^\alpha}, o_{j(4\ell+\alpha)}^{z_j^\alpha}) = \perp$ or similarly $\mathcal{D}_k, \mathcal{W}_k$ is not consistent.
- (Local Computation by P_i) Output $y_j = y_k = \perp$ if $\mathsf{flag}_i = 1$. Else, with respect to $\mathbf{C}_j$:
  - open $\mathbf{C}_j \leftarrow \mathsf{eOpen}(\mathsf{epp}_j, c_j, o_j)$ where the opening is received from P_k.
  - open $\mathbf{X}_j^\alpha = \mathsf{Open}(\mathsf{pp}_j, c_{j\alpha}^{m_{jj}^\alpha}, o_{j\alpha}^{m_{jj}^\alpha})$, $\mathbf{R}_j^\alpha = \mathsf{Open}(\mathsf{pp}_j, c_{j(2\ell+\alpha)}^{r_j^\alpha}, o_{j(2\ell+\alpha)}^{r_j^\alpha})$, and $\mathbf{Z}_j^\alpha = \mathsf{Open}(\mathsf{pp}_j, c_{j(4\ell+\alpha)}^{z_j^\alpha}, o_{j(4\ell+\alpha)}^{z_j^\alpha})$, for the openings received from P_j.
  - open $\mathbf{X}_k^\alpha = \mathsf{Open}(\mathsf{pp}_j, c_{j(\ell+\alpha)}^{m_{jk}^\alpha}, o_{j(\ell+\alpha)}^{m_{jk}^\alpha})$, $\mathbf{R}_k^\alpha = \mathsf{Open}(\mathsf{pp}_j, c_{j(3\ell+\alpha)}^{r_k^\alpha}, o_{j(3\ell+\alpha)}^{r_k^\alpha})$ and $\mathbf{Z}_k^\alpha = \mathsf{Open}(\mathsf{pp}_j, c_{j(5\ell+\alpha)}^{z_k^\alpha}, o_{j(5\ell+\alpha)}^{z_k^\alpha})$ for $\alpha \in [\ell]$, for openings are received from P_k.
  - If any of the above openings fail, set $y_j = \perp$. Else set $\mathbf{X} = \mathbf{X}_j|\mathbf{X}_k|\mathbf{R}_j|\mathbf{R}_k|\mathbf{Z}_j|\mathbf{Z}_k$, run $\mathbf{Y}_j \leftarrow \mathsf{Ev}(\mathbf{C}_j, \mathbf{X})$ and $y_j \leftarrow \mathsf{sDe}(\mathbf{Y}_j)$.

  Similar steps as above with respect to $\mathbf{C}_k$ is executed to compute $\mathbf{Y}_k$ and y_k.

Fig. 6: Protocol ua_i

Lemma 12. *At the end of protocol* ua_i*, all honest parties output the same* flag_i*.*

Proof. We have two cases based on whether atleast one honest party set $\mathsf{flag}_i = 1$ at the end of Round 1. If this is true, then the honest party would broadcast `abort` in Round 2 and all honest parties would output $\mathsf{flag}_i = 1$. Otherwise, an honest party sets flag_i based on the following conditions (a) `abort` was broadcast in Round 2 or (b) either z_j broadcast by (P_j, P_i) or z_k broadcast by (P_k, P_i) do not match or (c) $(\mathcal{D}_j, \mathcal{W}_j)$ or $(\mathcal{D}_k, \mathcal{W}_k)$ is inconsistent. All these checks are with respect to broadcast messages. Therefore, we can conclude that every honest party will output identical flag_i. □

Lemma 13. *Assuming input consistency, if* $\mathsf{flag}_i = 0$*, then* $y_k \neq \perp$ *where* P_k *is corrupt.*

Proof. First, Lemma 12 implies that both P_i, P_j output identical $\mathsf{flag}_i = 0$. Now $\mathsf{flag}_i = 0$ implies that: **(a)** $\mathbf{C}_k$ and the commitments to the encoding information are computed correctly; **(b)** the opening of encoding information $\mathbf{X}_k, \mathbf{R}_k$ for $\mathbf{C}_k$ is correct in Round 1 with high probability due to binding property of eNICOM and NICOM; **(c)** the opening of the remaining encoding information $\mathbf{Z}_k$ is correct with high probability due to binding property of NICOM. P_j being honest would open the encoding relevant to his input for $\mathbf{C}_k$, namely, $\mathbf{X}_j, \mathbf{R}_j, \mathbf{Z}_j$. So P_i has got complete encoded input $\mathbf{X}$ for $\mathbf{C}_k$ and will evaluate $\mathbf{C}_k$ to obtain y_k. Thus, if $\mathsf{flag}_i = 0$, then y_k will not be $\perp$. □

4.2 Protocol ua

Our two-round 3PC protocol ua achieving unanimous abort composes ua_i for $i \in [3]$ in parallel. Assuming input consistency, entwining the three executions requires tapping all the flags returned by the three executions and outputting the result computed as an evaluator when none of them are set to true and $\perp$, otherwise. This works since when a flag for an execution ua_i is false, then the evaluator P_i is guaranteed to get the output. The challenge that remains to handle is input consistency within and across executions which ensures the outputs computed are the same irrespective of the execution and GC. The inter-execution input consistency, i.e the consistency of the input committed by P_i in ua_i and the inputs given to the GCs constructed by P_i as garbler in the remaining two executions are enforced using the same trick that we use in fair via setting the permutation strings as the shares of the parties' input.

Dealing with the input consistency within an execution ua_i to make sure the garblers provide the same input for both the GCs without inflating the round complexity constitutes yet another challenge. Noting that this misbehaviour has no way to show up in the common flag as this is targeted via the private communication in the second round, the evaluator must find a way to robustly compute the output when conflicted outputs are computed from the two garbled circuits. This output must be based on the input of the corrupt garbler that it has committed as an evaluator and received output based on. We use the trick of "proof-of-cheating" mechanism [Lin13] to enable an (honest) evaluator with conflicting outputs to retrieve the inputs committed by both garblers in their respective instances. To be specific, the output keys corresponding to the mismatched output bit in the two garbled circuits, say $\mathbf{C}_1$ and $\mathbf{C}_3$ in ua_2, enables the evaluator P_2 to unlock the missing shares, namely, x_{31} and x_{13} of the two garblers from ua_3 and ua_1 respectively. To ensure that the recovered missing shares are as distributed in ua_1 and ua_3, the shares are committed via NICOM by the input owners and the openings are encrypted by the holders (as in fair). The binding of NICOM, prevents a corrupt P_1 to lie on (x_{13}, x_{31}). This allows the honest party to compute the same output that P_1 gets from ua_1. Lastly, the flag in execution ua_i also takes into account consistent dealing of the commitments by its evaluator P_i. Our protocol appears in Figure 7, the proof of correctness and the proof of security below. We use Definition 1 for input commitment.

Lemma 14. *If a corrupt party P_k has not committed its input or does not use the committed input in its GCs in $\{\mathsf{ua}_i, \mathsf{ua}_j\}$, then each honest party outputs $y = \perp$.*

Proof. P_k has not committed to a unique input implies it has not dealt correct opening to one or both the honest parties. In either case, `abort` is raised in the second round, leading to an output that is $\perp$. Now assume P_k uses input $x'_k \neq x_k$ during ua_i for its own GC. P_k should use x_{kj} as the permutation string p_{kk} in execution ua_i for permuting the commitments corresponding to x_k. If it does not, then honest P_j sets $\mathsf{flag}_i = 1$ in Round 1 and broadcasts `abort` in Round 2. Otherwise, the commitments are constructed correctly and ordered as per x_{kj}. Now the only way P_k can decommit x'_k is by giving $m_{kk} = p_{kk} \oplus x'_k$. But in this case honest P_i would set $\mathsf{flag}_i = 1$ in Round 1 and broadcast `abort` in Round 2 as the check $m_{kk} = x_{ki}$ would fail ($m_{kk} = p_{kk} \oplus x'_k \neq p_{kk} \oplus x_k$). Thus, every honest party outputs $y = \perp$. □

Protocol $\mathsf{ua}()$

Inputs: Party P_i has x_i for $i \in [3]$.
Output: $y = f(x_1, x_2, x_3)$ or $\perp$.
Sub-protocols: ua_i for $i \in [3]$ (Figure 5), a NICOM $(\mathsf{Com}, \mathsf{Open})$, CPA-secure SKE Enc.

Round 1: For $i \in [3]$ and for distinct indices $j, k \in [3] \setminus \{i\}$

- **Round 1** of ua_i are run parallel. In ua_i, P_j and P_k work with the permutation strings p_{jj} and p_{kk} respectively as x_{jk} and x_{kj}.
- P_i samples pp_i, generates $(c_{ij}, o_{ij}) \leftarrow \mathsf{Com}(\mathsf{pp}_i, x_{ij})$, $(c_{ik}, o_{ik}) \leftarrow \mathsf{Com}(\mathsf{pp}_i, x_{ik})$, broadcasts $\{\mathsf{pp}_i, c_{ij}, c_{ik}\}$ and sends o_{ij}, o_{ik} to P_j, P_k respectively.
- (Local Computation) P_i sets $\mathsf{flag}_i = 1$ if $\mathsf{Open}(c_{li}, o_{li}) \neq x_{li}$ or $m_{ll} \neq x_{li}$ for $l \in \{j, k\}$. P_j sets $\mathsf{flag}_i = 1$ if: **(a)** p_{kk} not taken as x_{kj} or **(b)** the check in ua_i fails. **(c)** $\mathsf{Open}(c_{ij}, o_{ij}) \neq x_{ij}$. P_k sets $\mathsf{flag}_i = 1$ if: **(a)** p_{jj} not taken as x_{jk} or **(b)** the check in ua_i fails. **(c)** $\mathsf{Open}(c_{ik}, o_{ik}) \neq x_{ik}$.

Round 2:

- **Round 2** of ua_i for $i \in [3]$ are run parallel. In ua_i, the garbler P_j (similar steps will be taken by P_k) does the following additionally if $\mathsf{flag}_i \neq 1$. Let $\{\mathbf{Y}_l^0, \mathbf{Y}_l^1\}$, denote the encoding information for output wire corresponding to $\mathbf{C}_l$ $(l \in \{j, k\})$. It sends two ciphertexts $(\mathsf{ct}_j^0, \mathsf{ct}_j^1)$ where $\mathsf{ct}_j^0 = \mathsf{Enc}_{\mathbf{Y}_j^0 \oplus \mathbf{Y}_k^1}(o_{jk}, o_{kj})$ and $\mathsf{ct}_j^1 = \mathsf{Enc}_{\mathbf{Y}_j^1 \oplus \mathbf{Y}_k^0}(o_{jk}, o_{kj})$.
- For $i \in [3]$, party P_i computes output as follows:
  - If $\mathsf{flag}_\alpha = 1$ for some $\alpha \in [3]$, then output $y = \perp$.
  - Otherwise, output y as y_j when $y_j = y_k$ or $y_k = \perp$, as y_k when $y_j = \perp$ where (y_j, y_k) are output from ua_i.
  - Otherwise, let the encoded outputs corresponding to $\mathbf{C}_j, \mathbf{C}_k$ in ua_i are $\mathbf{Y}_j, \mathbf{Y}_k$. It uses key $\mathbf{Y}_j^{y_j} \oplus \mathbf{Y}_k^{y_k}$ to decrypt the ciphertext $\mathsf{ct}_j^{y_j}$ obtained from P_j to retrieve (o_{jk}, o_{kj}). It executes $x_{kj} \leftarrow \mathsf{Open}(c_{kj}, o_{kj})$ and $x_{jk} \leftarrow \mathsf{Open}(c_{jk}, o_{jk})$. If x_{kj} or $x_{jk} = \perp$, then they are recomputed as above using $\mathsf{ct}_k^{y_j}$ obtained from P_k. Then P_i evaluates f on inputs $(x_i, x_{ji} \oplus x_{jk}, x_{ki} \oplus x_{kj})$ to obtain y.

Fig. 7: A Two-Round 3PC protocol achieving unanimous abort

Theorem 2. *Protocol* ua *is correct.*

Proof. In order to prove the theorem, we show that if an honest party, say P_i outputs y that is not $\perp$, then it corresponds to (x_1, x_2, x_3) where x_j is the input committed by P_j. Assume that P_k is corrupt. Recall that P_i outputs y_j and y_k in ua_i on evaluating the GCs of the garblers P_j and P_k respectively. We have the following cases.

- $y = y_k$. Follows from Lemma 13, 14.
- $y \neq y_k$. In this case, $y \neq y_j$ either as y is set to y_j when $y_j = y_k$ or $y_k = \perp$. Following Lemma 13, y_k cannot be $\perp$. So it must be that P_i retrieves the output via opening the ciphertexts. If the output is computed just from the ciphertext of honest P_j, then y is computed as $f(x_i, x_{ji} \oplus x_{jk}, x_{ki} \oplus x_{kj})$ using openings o_{kj}, o_{jk} given by P_j. Since an honest P_j correctly reveals the opening o_{kj} of the share of P_k's input given to P_j and o_{jk} corresponding to his input share, $f(x_i, x_{ji} \oplus x_{jk}, x_{ki} \oplus$

x_{kj}) corresponds to the correct value. If the output is computed from the ciphertext of corrupt P_k, then y computed must be still as above as a corrupt P_k cannot open the shares x_{jk}, x_{kj} in an incorrect way (following binding property of NICOM).

□

The intuition for achieving unanimous abort follows from the correctness and Lemma 12 that implies the honest parties will be on the same page for all flags. The formal proof appears in Appendix E.

5 3-round 3PC with Guaranteed Output Delivery

In this section, we present a three-round 3PC protocol, given access to pairwise-private channels and a broadcast channel. The protocol is round-optimal following 3-round lower bound for fair 3PC proven in Section 6.1. The necessity of the broadcast channel for achieving guaranteed output delivery with strict honest majority follows from [CHOR16].

Our tryst starts with the known generic transformations that are relevant such as the transformations from the unanimous abort to (identifiable) fair protocol [IKP+16] or identifiable fair to guaranteed output delivery [CL14]. However, these transformations being non-round-preserving do not turn out to be useful. Turning a 2-round protocol offering unanimous (or even selective) abort with identifiability (when the honest parties learn about the identity of the corrupt when deprived of the output) to a 3-round protocol with guaranteed output delivery in a black-box way show some promise. The third round can be leveraged by the honest parties to exchange their inputs and compute output on the clear. We face two obstacles with this approach. First, there is neither any known 2-round construction for selective / unanimous abort with identifiability nor do we see how to transform our unanimous abort protocol to one with identifiability in two rounds. Second, when none of the parties (including the corrupt) receive output from the selective / unanimous abort protocol and the honest parties compute it on the clear in the third round by exchanging their inputs and taking a default value for the input of the corrupt party, it is not clear how the corrupt party can obtain the same output (note that the ideal functionality demands delivering the output to the adversary).

We get around the above issues by taking a non-blackbox approach and tweaking ua_i and fair_i to get yet another sub-protocol god_i that achieves a form of local identifiability. Namely, the evaluator P_i in god_i either successfully computes the output or identifies the corrupt party. As usual, our final protocol god is built upon three parallel executions of god_i ($i \in [3]$), each comprising of two rounds and with each party P_i enacting the role of the evaluator once. Looking ahead, the local identifiability helps in achieving guaranteed output delivery as follows. In a case when both honest parties identify the corrupt party and the corrupt party received the output by the end of Round 2, the honest parties can exchange their inputs and reconstruct the corrupt party's input using the shares received during one of the executions of god_i and compute the function on clear inputs in the third round. Otherwise, the honest party who identifies the corrupt can simply accept the output computed and forwarded by the other honest party. The issue of the corrupt party getting the same output as that of the honest parties when it fails to obtain any in its instance of god_i is taken care as follows. First, the only reason

a corrupt party in our protocol does not receive its output in its instance of god_i is due to denial of committing its input. In this case it is detected early and the honest parties exchange inputs in the second round itself so that at least one honest party computes the output using a default input of the corrupt party by the end of Round 2 and hands it over to others in Round 3.

In the following, we describe one execution god_i. Entwining the three executions, tackling the input consistency and the final presentation of protocol god are done next. The security proof appears in Appendix F.

5.1 Protocol $\mathbf{god}_i$

Recall that the goal of god_i for $i \in [3]$ comprising of two rounds, is either successful computation of output or successful identification of the corrupt party by the evaluator P_i. Starting with the ideas of ua_i, we note that ua_i only ensures detection of the corrupt party by *s*ome honest party that is not necessarily the evaluator in case of a failed output computation. Specifically, a garbler would identify his co-garbler to be corrupt when the broadcast communication of co-garbler is not consistent with the privately shared randomness. In such a case, the evaluator neither gets the output nor has any clue on the identity of the corrupt, which is not in accordance with the goal of god_i. In the absence of broadcast, fair_i gives even weaker guarantee where the best any party gets to know is a conflict. The above is handled by having the garblers send their inputs on clear to the evaluator on finding inconsistent behaviour of the fellow garbler in the first round. If both the garblers are in conflict with each other, the evaluator gets their inputs and computes the function on clear. Otherwise, the evaluator can either evaluate at least one of the GCs or identify the corrupt. Lastly, as we do not require unanimity of any form at the end of two rounds, we simplify god_i by removing the two-part release mechanism and the flag altogether. Like ua_i, we do not take care of the possibility of a corrupt garbler handing out inconsistent input for the two GCs in god_i. This is taken care in the main protocol god via the input consistency. P_i outputs $(y = (y_j, y_k), \mathbf{Y}_i = (\mathbf{Y}_i^j, \mathbf{Y}_i^k), \mathcal{C}_i)$, the outputs computed from two GCs, the encoded outputs and its corrupt set, all initially set to $\perp$ and to be used in the main construction. If both (y_j, y_k) are $\perp$, then the corrupt set will be non-empty. The garblers output their corrupt set. We now prove a few lemmas. The protocol god_i appears in Figure 8.

Lemma 15. *$P_\beta \notin \mathcal{C}_\alpha$ holds for honest P_α, P_β.*

Proof. An honest P_α would include P_β in $\mathcal{C}_\alpha$ only if one of the following holds: (a) Both P_α, P_β are garblers and P_β broadcasts $\mathcal{D}_\beta$ inconsistent with values privately shared with P_α (b) P_α is an evaluator and P_β is a garbler and P_β's opening of a committed encoded input or garbled circuit approved by him fails. It is easy to verify that the cases will never occur for honest (P_α, P_β). □

Lemma 16. *Assuming input consistency, at the end of protocol god_i, an honest evaluator P_i either computes the output or identifies the corrupt party.*

Proof. Assume that P_k is the corrupt garbler. We have two cases.

Protocol $\mathsf{god}_i()$

Inputs: Party P_α has x_α for $\alpha \in [3]$.
Common Inputs: Same as fair_i (Figure 1).
Output: A garbler P_l for $l \in \{j,k\}$ outputs $\mathcal{C}_l$. P_i outputs $(y = (y_j, y_k), \mathbf{Y}_i = (\mathbf{Y}_i^j, \mathbf{Y}_i^k), \mathcal{C}_i)$ where y is the output (initially set to $\perp$) and $\mathcal{C}_i$ denotes the corrupt set maintained by P_i.
Primitives: A garbling scheme $\mathcal{G} = (\mathsf{Gb}, \mathsf{En}, \mathsf{Ev}, \mathsf{De})$ that is correct, private and authentic, an eNICOM $(\mathsf{eGen}, \mathsf{eCom}, \mathsf{eOpen}, \mathsf{Equiv})$ and a PRG G.

Round 1: Same as Round 1 of fair_i (Figure 1) except that the garblers do not commit to the decoding information and $\mathcal{D}_l$ computed by garbler P_l $(l \in \{j,k\})$ is communicated via broadcast.
Round 2:

– P_j computes indicator string $m_{jj} = p_{jj} \oplus x_j, m_{kj} = p_{kj} \oplus x_j$. If $P_k \notin \mathcal{C}_j$, then send $\left(\mathtt{OK}, (o_k, \{o_{k\alpha}^{m_{kj}^\alpha}, o_{k(2\ell+\alpha)}^{x_{ij}^\alpha}\}_{\alpha\in[\ell]}, m_{kj}), (\{o_{j\alpha}^{m_{jj}^\alpha}, o_{j(2\ell+\alpha)}^{x_{ij}^\alpha}\}_{\alpha\in[\ell]}, m_{jj})\right)$ to P_i. Else, it sends $\left(\mathtt{nOK}, x_j, (\{o_{j\alpha}^{m_{jj}^\alpha}, o_{j(2\ell+\alpha)}^{x_{ij}^\alpha}\}_{\alpha\in[\ell]}, m_{jj})\right)$. P_k performs similar steps.
– (Local Computation) If $\mathtt{nOK}$ is received from both P_j, P_k, then compute $y_j = y_k = f(x_1, x_2, x_3)$ using x_j, x_k. Otherwise, one of the parties has sent $\mathtt{OK}$. Assume, for simplicity that P_k has sent $\mathtt{OK}$. Then, compute $\mathbf{Y}_i^j$ as in fair_i. If $\mathbf{Y}_i^j \neq \perp$, then $y_j \leftarrow \mathsf{sDe}(\mathbf{Y}_i^j)$. If P_j sent $\mathtt{OK}$, then similar steps as above for $\mathbf{C}_k$ will be executed and y_k will be set.

Fig. 8: Protocol god_i

– P_k **sends** $\mathtt{nOK}$**:** If P_j sends $\mathtt{nOK}$ too, P_i receives x_l from P_l for $l \in \{j,k\}$ (else P_k is identified to be corrupt) and computes f on inputs x_i, x_j, x_k. If P_j sends $\mathtt{OK}$, then the garbled circuit $\mathbf{C}_k$ is correctly constructed and the corresponding encoding information is correctly committed. The only way a corrupt garbler P_k can stop P_i from evaluating $\mathbf{C}_k$ (and avoid being caught by P_i) is by sending encoded inputs corresponding to (x_k, x_{ik}) that are inconsistent with $\mathbf{C}_k$ via breaching the binding property of NICOM which happens only with negligible probability.
– P_k **sends** $\mathtt{OK}$**:** In this case, the binding property of eNICOM ensures that with high probability the correct $\mathbf{C}_j$ is opened (otherwise P_k is caught). The arguments now follow as the previous case where the probability that P_i does not get the output and does not detect P_k reduces to the probability of breaching binding of NICOM. □

5.2 Protocol god

Our three-round 3PC protocol achieving guaranteed output delivery composes god_i for $i \in [3]$, with each party acting as the evaluator in parallel. At a high level, the protocol assures that every party either outputs y that is not $\perp$ or identifies the corrupt by end of second round. In the third round, a party simply sends his output if it is non-$\perp$, else it sends its input and share of the corrupt party's input to the honest party alone. A party outputs its own output computed in second round if it is not $\perp$. Otherwise, it outputs the non-$\perp$ output received from the non-faulty party or computes the output using the input and share sent by the non-faulty party. The input consistency is handled exactly as in ua. Additionally every party maintains a corrupt set and populates it when it identifies the

corrupt. The overall composition maintains guaranteed output delivery as below based on when a corrupt party chooses to expose itself.

The cases when a corrupt P_i is detected by the end of first round itself, the honest party who makes the identification, halts the execution where it plays the evaluator with the corrupt set as the output and also halts god_i to stop letting P_i get output in god_i. Since the detection may be owing to non-commitment of any input by P_i in god_i, the unique input of P_i has to be set to the one that it commits in the running execution or as a default value when either there is no running execution or P_i does not commit to anything in the running execution. Specifically, if both the honest parties identify P_i to be corrupt by the end of first round, both would have exchanged their input as per the code of god protocols and a default common value is taken as the input of P_i to compute the function output by the end of second round itself and the output is handed over to P_i in third round. Handing the output to corrupt P_i is necessary to technically realise the functionality correctly where the corrupt party also gets the output. If just one of the parties detects the corrupt party P_i, say P_j, it stops its execution as the evaluator in god_j and as garbler in god_i to prevent P_i getting any output in god_i. Now P_i has two options: either it passes on its input on clear to P_k or it lets P_k to evaluate the garbled circuit of P_j by giving its encoded input. In either case, this input of P_i is taken as his committed input and the output computed by P_k is the one to be outputted by all. (Note that P_i's own GC will not be approved by its co-garbler who has identified it as corrupt by the end of first round.) P_k can simply pass on the output to P_i and P_j in the third round and P_j simply takes the output of P_k who it knows to be honest. Our protocol appears in Figure 9. The proof of correctness appear below and the full proof in Appendix F.

Lemma 17. *$P_\beta \notin \mathcal{C}_\alpha$ holds for honest P_α, P_β in protocol god_i, where $i \in [3]$.*

Proof. This lemma follows from Lemma 15 and the fact that the following will not be true for honest (P_α, P_β): (a) P_β sends $o_{\beta\alpha}, x_{\beta\alpha}$ to P_α such that $\mathsf{Open}(c_{\beta\alpha}, o_{\beta\alpha}) \neq x_{\beta\alpha}$ (b) Both P_α, P_β are garblers and $p_{\beta\beta} \neq x_{\beta\alpha}$. (c) P_β is the garbler, P_α is an evaluator and $m_{\beta\beta} \neq x_{\beta\alpha}$ □

Lemma 18. *Every party P_i uses its 'committed' input x_i (Definition 1) in its GCs in $\{\mathsf{god}_j, \mathsf{god}_k\}$. Otherwise, it is identified by at least one of the honest parties.*

Proof. P_i has not committed to its input implies it has not dealt correct opening to one or both the honest parties. In either case, at least one of the honest parties identify him. Now assume P_i has committed to input x_i but uses input $x_i' \neq x_i$ during god_j for the garbled circuit constructed by P_i. P_i should use x_{ik} as the permutation string p_{ii} in execution god_j for permuting the commitments corresponding to x_i. If P_i does otherwise, then it is identified by honest P_k. Otherwise, the commitments are constructed correctly and ordered as per x_{ik}. Now the only way P_i can decommit x_i' is by giving $m_{ii} = p_{ii} \oplus x_i'$. But P_j identifies P_i as corrupt as $m_{ii} = p_{ii} \oplus x_i' \neq p_{ii} \oplus x_i$. □

We now prove correctness of the protocol accounting exhaustively all the scenarios: the corrupt party

- belongs to the corrupt set of both the honest parties,
- belongs to the corrupt set of exactly one of the honest parties and

Protocol $\mathsf{god}()$

Inputs: Party P_i has x_i for $i \in [3]$.
Output: $y = f(x_1, x_2, x_3)$.
Sub-protocols Used: god_i, $i \in [3]$ (Figure 8), a NICOM $(\mathsf{Com}, \mathsf{Open})$, CPA-secure SKE Enc.

Round 1: For $i \in [3]$ and for distinct indices $j, k \in [3] \setminus \{i\}$

- **Round 1** of god_i are run parallel. In god_i, P_j and P_k work with the permutation strings p_{jj} and p_{kk} respectively as x_{jk} and x_{kj}.
- P_i samples pp_i, generates $(c_{ij}, o_{ij}) \leftarrow \mathsf{Com}(\mathsf{pp}_i, x_{ij})$, $(c_{ik}, o_{ik}) \leftarrow \mathsf{Com}(\mathsf{pp}_i, x_{ik})$, broadcasts $\{\mathsf{pp}_i, c_{ij}, c_{ik}\}$ and sends o_{ij}, o_{ik} to P_j, P_k respectively.
- (Local Computation) P_i adds P_ℓ in $\mathcal{C}_i$ if $\mathsf{Open}(c_{li}, o_{li}) \neq x_{li}$. P_j adds P_k in $\mathcal{C}_j$ if: **(a)** p_{kk} not taken as x_{kj} or **(b)** the check in god_i fails. P_k adds P_j in $\mathcal{C}_k$ if: **(a)** p_{jj} not taken as x_{jk} or **(b)** the check in god_i fails.

Round 2: For $i \in [3]$ and for distinct indices $j, k \in [3] \setminus \{i\}$

- If $P_i \notin \mathcal{C}_j$, then P_j participates in god_i. If $P_k \notin \mathcal{C}_j$, it additionally sends the following ciphertexts $\{\mathsf{ct}_j^0, \mathsf{ct}_j^1\}$ created as below. Let $\{\mathbf{Y}_l^0, \mathbf{Y}_l^1\}$, denote the encoding information for output wire corresponding to $\mathbf{C}_l$ $(l \in \{j, k\})$. Then $\mathsf{ct}_j^0 = \mathsf{Enc}_{\mathbf{Y}_j^0 \oplus \mathbf{Y}_k^1}(o_{jk}, o_{kj})$ and $\mathsf{ct}_j^1 = \mathsf{Enc}_{\mathbf{Y}_j^1 \oplus \mathbf{Y}_k^0}(o_{jk}, o_{kj})$.
- P_i includes P_l in $\mathcal{C}_i$ if $m_{ll} \neq x_{li}$ for $l \in \{j, k\}$.
- (Local Computation by P_i) If $\mathcal{C}_i = \emptyset$, then compute $y = (y_j, y_k)$ as in god_i. If $y_j = y_k$ $(\neq \perp)$ or one of them is non-$\perp$, set y to one of them in the former and to the not-$\perp$ in the latter. If $y_j \neq y_k$, use key $\mathbf{Y}_j^{y_j} \oplus \mathbf{Y}_k^{y_k}$ to decrypt the ciphertexts $\mathsf{ct}_j^{y_j}$ obtained from P_j to retrieve (o_{jk}, o_{kj}). Execute $x_{kj} \leftarrow \mathsf{Open}(c_{kj}, o_{kj})$ and $x_{jk} \leftarrow \mathsf{Open}(c_{jk}, o_{jk})$. If x_{kj} or $x_{jk} = \perp$, then they are recomputed as above using $\mathsf{ct}_k^{y_j}$ obtained from P_k. Then evaluate f on inputs $(x_i, x_{ji} \oplus x_{jk}, x_{ki} \oplus x_{kj})$ to obtain y. If $\mathcal{C}_i \neq \emptyset$, $y = \perp$ and P_i receives x_l from $P_l \notin \mathcal{C}_i$, compute y as the value of f on x_i, x_l and a default value for the remaining party's input.

Round 3: Each P_i for $i \in [3]$ either has $y \neq \perp$ or $\mathcal{C}_i \neq \emptyset$. It does the following

- If $y \neq \perp$, send y to P_j, P_k. Send (x_i, x_{ji}) to P_k when $P_j \in \mathcal{C}_i$ or (x_i, x_{ki}) to P_j when $P_k \in \mathcal{C}_i$.
- If $y \neq \perp$, output y. Else if $P_l \notin \mathcal{C}_i$ sends y, output y. Else if $P_j \notin \mathcal{C}_i$ sends (x_j, x_{kj}), then compute y as the output of f on $(x_i, x_j, x_{kj} \oplus x_{ki})$. Similar steps are executed when $P_k \notin \mathcal{C}_i$ and it sends (x_k, x_{jk}) i.e y is derived from $(x_i, x_k, x_{jk} \oplus x_{ji})$.

Fig. 9: A Three-Round 3PC protocol achieving guaranteed output delivery

– does not belong to the corrupt set of the honest parties

by the end of the first round. For simplicity, we assume that P_k is the corrupt party and P_i, P_j are the honest parties.

Lemma 19. *Assuming that the corrupt party belongs to the corrupt set of both the honest parties by the end of the first round, protocol* god *is correct.*

Proof. In this case, P_i and P_j does not communicate at all in the second round of god_k preventing P_k to compute an output. In god_i and god_j, P_j and P_i, respectively send

their inputs on clear to each other along with nOK signal. Both compute y on the inputs x_i, x_j that are exchanged and a default common value for x_k by the end of round 2. In the third round, P_k receives y from the honest parties and the honest parties output y. In this case the unique input of the corrupt party taken for computation is the default commonly-agreed value. □

Lemma 20. *Assuming that the corrupt party belongs to the corrupt set of exactly one of the honest parties by the end of the first round, protocol* god *is correct.*

Proof. For simplicity $P_k \in \mathcal{C}_i$ at the end of first round. (The proof follows in a similar way when $P_k \in \mathcal{C}_j$.) This implies P_i, as an evaluator, ignores communication from both the garblers in its execution god_i and will conclude the second round with $y = \perp$ and $\mathcal{C}_i = P_k$. P_i does not participate in god_k as a garbler making sure P_k cannot compute an output by the end of second round. In god_j, P_i sends x_i on clear to P_j with nOK signal which implies evaluation of the GC created by P_k is ruled out. Now based on whether P_k commits to any input or not, P_j computes the output in the following way. If nOK signal is sent along with its input x_k, then P_j computes $y = y_i = y_k$ using its own input x_j and the inputs sent by P_i and P_k. If P_k sends OK with its encoded input which verifies correctly with respect to the committed encoded information, P_j obtains $y = y_i$ upon GC ($\mathbf{C}_i$) evaluation. In the case when P_k does not commit to any input either on clear or in encoded form (namely, the encoded input does not verify against the committed encoded input), P_j must have identified P_k to be corrupt and computes y using its own input x_j, the input sent by P_i and using a default value for x_k. The third round is finally used by P_i and P_k to obtain the output of P_j and correctness follows. The unique input of P_k is taken as the one that it sends either on clear or in encoded form to P_j in the former case and a default value in the latter. □

Lemma 21. *Assuming that the corrupt party does not belong to the corrupt set of both the honest parties by the end of the first round, protocol* god *is correct.*

Proof. In this case, P_k must have 'committed' (Definition 1) to his input (else would be identified by atleast one of the honest parties at end of Round 1) and obtained output y based on its committed input during god_k. Further, P_k is not detected yet by the end of first round, implies that it has played the role of the garblers in god_i and god_j honestly in the first round. In this case, we prove that no matter how P_k behaves in the second round, the honest parties will obtain y based on their inputs and P_k's committed input. We present the argument for honest P_i. Similar argument holds for P_j. Based on the observation that P_i must have attempted to evaluate $\mathbf{C}_k$ since P_j must have sent OK signal in god_i, we consider the following cases:

- P_i is unsuccessful in evaluating the circuit $\mathbf{C}_k$ of garbler P_k in god_i. This implies P_k has given inconsistent encoded input for its circuit to P_i. So P_i concludes the second round with $y = \perp$ and $\mathcal{C}_i = P_k$.
- P_i is successful in evaluating the circuit $\mathbf{C}_k$ of garbler P_k in god_i. By Lemma 18, P_k must have given encoded input corresponding to its committed input x_k for $\mathbf{C}_k$. This implies the output obtained via $\mathbf{C}_k$ (i.e y_k) is the desired y in this case. Now we have two cases based on whether P_k approves the garbled circuit constructed by

P_j or not. In each case we show that, P_i outputs the desired y by the end of second round itself. If P_k disapproves, then $y_j = \perp$ and P_i outputs the value $y = y_k$ obtained via the GC $\mathbf{C}_k$ as per the specification of god_i. Otherwise, P_i evaluates both circuits, namely $\mathbf{C}_j$ and $\mathbf{C}_k$. If the outputs are the same, then the guarantee provided by Lemma 18 implies P_i outputs the desired y. Else if P_i has got conflicting outputs ($y_j \neq y_k$), then it gets access to the key $\mathbf{Y}_j^{y_j} \oplus \mathbf{Y}_k^{y_k}$ and uses it to decrypt at least one of the ciphertexts $\{\mathsf{ct}_j^{y_j}, \mathsf{ct}_k^{y_j}\}$ generated by P_j and P_k. If the decryption of only the honest party P_j's ciphertext succeeds, then P_i obtains (o_{jk}, o_{kj}), retrieves his missing shares x_{jk}, x_{kj} and computes y using x_i, $x_j = x_{ji} \oplus x_{jk}$ and $x_k = x_{ki} \oplus x_{kj}$ where P_i and P_j receives x_{ki} and x_{kj} respectively from P_k in god_k. Even if corrupt P_k's ciphertext is decrypted successfully, the y computed is still as above due the fact that P_k cannot open a different value for x_{jk}, x_{kj} due to the binding property of NICOM. P_i retains this output in the third round.

In the former case, if both P_i and P_j outputs $\perp$ in the end of second round, then the third round is used by P_i and P_j to exchange their inputs and the shares of x_k that they possess. By the end of third round P_i (and P_j as well) outputs the desired y. If P_j was successful in computing y in god_j, then P_j sends the output directly in third round which P_i takes as the output. In the latter case, P_i retains his output in the third round. □

Theorem 3. *Protocol* god *is correct.*

Proof. The proof follows from Lemma 19,20,21 as we have considered all the cases exhaustively based on whether the corrupt party P_k is identified by none, exactly one or both the honest parties by the end of first round. □

6 Lower Bounds

In this paper, we present two lower bounds– **(a)** three rounds are necessary for achieving fairness in the presence of pair-wise private channels and a broadcast channel; **(b)** three rounds are necessary for achieving unanimous abort in the presence of just pair-wise private channels (and no broadcast). The second result holds even if broadcast was allowed in the first round. Our results extend for any n and t with $3t \geq n > 2t$ via standard player-partitioning technique [Lyn96]. Our results imply the following. First, selective abort is the best amongst the four notions (considered in this work) that we can achieve in two rounds without broadcast (from **(b)**). Second, unanimous abort as well as fairness require 3 rounds in the absence of broadcast (from **(b)**). Third, broadcast does not help to improve the round complexity of fairness (from **(a)**). Lastly, guaranteed output delivery requires 3 rounds with broadcast (from **(a)**).

6.1 The Impossibility of 2-round Fair 3PC

In this section, we show that it is impossible to construct a fair 2-round 3PC for general functions. [GLS15] presents a lower bound of three rounds assuming *non-private*

point-to-point channels and a broadcast channel (their proof crucially relies on the assumption of non-private channels). [GIKR02] presents a three-round lower bound for fair MPC with $t \geq 2$ (arbitrary number of parties) in the same network setting as ours. Similar to the lower bounds of [GLS15] and [GIKR02] (for the function of conjunction of two input bits), our lower bound result does not exploit the rushing nature of the adversary and hence holds for non-rushing adversary as well. Finally, we observe that the impossibility of 2-round 3PC for the information-theoretic setting follows from the impossibility of 2-round 3-party statistical VSS of [PCRR09] (since VSS is a special case of MPC). We now prove the impossibility formally.

Theorem 4. *There exist functions f such that no two-round fair 3PC protocol can compute f, even in the honest majority setting and assuming access to pairwise-private and broadcast channel.*

Proof. Let $\mathcal{P} = \{P_1, P_2, P_3\}$ denote the set of 3 parties and the adversary $\mathcal{A}$ may corrupt any one of them. We prove the theorem by contradiction. We assume that there exists a two-round fair 3PC protocol π that can compute $f(x_1, x_2, x_3)$ defined below for P_i's input x_i:

$$f(x_1, x_2, x_3) = \begin{cases} 1 & \text{if } x_2 = x_3 = 1 \\ 0 & \text{otherwise} \end{cases}$$

At a high level, we discuss two adversarial strategies $\mathcal{A}_1$ and $\mathcal{A}_2$ of $\mathcal{A}$. We consider party P_i launching $\mathcal{A}_i$ in execution Σ_i ($i \in [2]$) of π. Both the executions are assumed to be run for the same input tuple (x_1, x_2, x_3) and the same random inputs (r_1, r_2, r_3) of the three parties. (Same random inputs are considered for simplicity and without loss of generality. The same arguments hold for distribution ensembles as well.) When strategy $\mathcal{A}_1$ is launched in execution Σ_1, we would claim that by correctness of π, $\mathcal{A}$ corrupting P_1 should learn the output $y = f(x_1, x_2, x_3)$. Here, we note that the value of $f(x_1, x_2, x_3)$ depends only on the inputs of honest P_2, P_3 (i.e input values x_2, x_3) and is thus well-defined. We refer to $f(x_1, x_2, x_3)$ as the value determined by this particular combination of inputs (x_2, x_3) henceforth. Now, since $\mathcal{A}$ corrupting P_1 learnt the output, due to fairness, P_2 should learn the output too in Σ_1. Next strategy $\mathcal{A}_2$ is designed so that P_2 in Σ_2 can obtain the same view as in Σ_1 and therefore it gets the output too. Due to fairness, we can claim that P_3 receives the output in Σ_2. A careful observation then lets us claim that P_3 can, in fact, learn the output at the end of Round 1 itself in π. Lastly, using the above observation, we show a strategy for P_3 that explicitly allows P_3 to breach privacy.

We use the following notation: Let $\mathsf{p}^r_{i \to j}$ denote the pairwise communication from P_i to P_j in round r and b^r_i denote the broadcast by P_i in round r, where $r \in [2], \{i, j\} \in [3]$. V_i denotes the view of party P_i at the end of execution of π. Below we describe the strategies $\mathcal{A}_1$ and $\mathcal{A}_2$.

$\mathcal{A}_1$: P_1 behaves honestly during Round 1 of the protocol. In Round 2, P_1 waits to receive the messages from other parties, but does not communicate at all.

$\mathcal{A}_2$: P_2 behaves honestly towards P_3 in Round 1, i.e sends the messages $\mathsf{p}^1_{2 \to 3}, \mathsf{b}^1_2$ according to the protocol specification. However P_2 does not communicate to P_1 in Round 1. In Round 2, P_2 waits to receive messages from P_3, but does not communicate to the other parties.

Next we present the views of the parties in the two executions Σ_1 and Σ_2 in Table 1. The communications that could potentially be different from the communications in an honest execution (where all parties behave honestly) with the considered inputs and random inputs of the parties are appended with $\star$ (e.g. $\mathsf{p}^2_{1\to3}(\star)$). We now prove a sequence of lemmas to complete our proof.

Table 1: Views of P_1, P_2, P_3 in Σ_1 and Σ_2

	Σ_1			Σ_2		
	V_1	V_2	V_3	V_1	V_2	V_3
Initial Input	(x_1, r_1)	(x_2, r_2)	(x_3, r_3)	(x_1, r_1)	(x_2, r_2)	(x_3, r_3)
Round 1	$\mathsf{p}^1_{2\to1}$, $\mathsf{p}^1_{3\to1}$ $\mathsf{b}^1_2, \mathsf{b}^1_3$	$\mathsf{p}^1_{1\to2}$, $\mathsf{p}^1_{3\to2}$, $\mathsf{b}^1_1, \mathsf{b}^1_3$	$\mathsf{p}^1_{1\to3}$, $\mathsf{p}^1_{2\to3}$, $\mathsf{b}^1_1, \mathsf{b}^1_2$	$-$, $\mathsf{p}^1_{3\to1}$, $\mathsf{b}^1_2, \mathsf{b}^1_3$	$\mathsf{p}^1_{1\to2}$, $\mathsf{p}^1_{3\to2}$, $\mathsf{b}^1_1, \mathsf{b}^1_3$	$\mathsf{p}^1_{1\to3}$, $\mathsf{p}^1_{2\to3}$, $\mathsf{b}^1_1, \mathsf{b}^1_2$
Round 2	$\mathsf{p}^2_{2\to1}$, $\mathsf{p}^2_{3\to1}$, $\mathsf{b}^2_2, \mathsf{b}^2_3$	$-$, $\mathsf{p}^2_{3\to2}$, b^2_3	$-$, $\mathsf{p}^2_{2\to3}$, b^2_2	$-$, $\mathsf{p}^2_{3\to1}$, b^2_3	$\mathsf{p}^2_{1\to2}(\star)$, $\mathsf{p}^2_{3\to2}$, $\mathsf{b}^2_1(\star), \mathsf{b}^2_3$	$-$, $\mathsf{p}^2_{1\to3}(\star)$, $\mathsf{b}^2_1(\star)$

Lemma 22. *A corrupt P_1 launching $\mathcal{A}_1$ in Σ_1 should learn the output $y = f(x_1, x_2, x_3)$.*

Proof. The proof follows easily. Since P_1 behaved honestly during Round 1, it received all the desired communication from honest P_2 and P_3 in Round 2 (refer to Table 1 for the view of P_1 in Σ_1 in the end of Round 2). So it follows from the correctness property that his view at the end of the protocol i.e V_1 should enable P_1 to learn the correct function output $f(x_1, x_2, x_3)$. □

Lemma 23. *A corrupt P_2 launching $\mathcal{A}_2$ in Σ_2 should learn the output y.*

Proof. We prove the lemma with the following two claims. First, the view of P_2 in Σ_2 subsumes the view of honest P_2 in Σ_1. Second, P_2 learns the output in Σ_1 due to the fact that the corrupt P_1 learns it and π is fair. We now prove our first claim. In Σ_1, we observe that P_2 has received communication from both P_1 and P_3 in the first round, and only from P_3 in the second round. So $\mathsf{V}_2 = \{x_2, r_2, \mathsf{p}^1_{1\to2}, \mathsf{b}^1_1, \mathsf{p}^1_{3\to2}, \mathsf{b}^1_3, \mathsf{p}^2_{3\to2}, \mathsf{b}^2_3\}$ (refer to Table 1). We now analyze P_2's view in Σ_2. Both P_1 and P_3 are honest and must have sent $\{\mathsf{p}^1_{1\to2}, \mathsf{b}^1_1, \mathsf{p}^1_{3\to2}, \mathsf{b}^1_3\}$ according to the protocol specifications in Round 1. Since P_3 received the expected messages from P_2 in Round 1, P_3 must have sent $\{\mathsf{p}^2_{3\to2}, \mathsf{b}^2_3\}$ in Round 2. Note that we can rule out the possibility of P_3's messages in this round having been influenced by P_1 possibly reporting P_2's misbehavior towards P_1. This holds since P_3 would send the messages in the beginning of Round 2. We do not make any assumption regarding P_1's communication to P_2 in Round 2 since P_1 has not received the expected message from P_2 in Round 1. Thus, overall, P_2's view V_2 comprises of $\{x_2, r_2, \mathsf{p}^1_{1\to2}, \mathsf{b}^1_1, \mathsf{p}^1_{3\to2}, \mathsf{b}^1_3, \mathsf{p}^2_{3\to2}, \mathsf{b}^2_3\}$ (refer to Table 1). Note that there may also be some additional messages from P_1 to P_2 in Round 2 which can be ignored by P_2. These are marked with '$(\star)'$ in Table 1. A careful look shows that the view of P_2 in Σ_2 subsumes the view of honest P_2 in Σ_1. This concludes our proof. □

Lemma 24. *P_3 in Σ_2 should learn the output y by the end of Round 1.*

Proof. According to the previous lemma, P_2 should learn the function output in Σ_2. Due to fairness property, it must hold that an honest P_3 learns the output as well (same as obtained by P_2 i.e y with respect to x_2). First, we note that as per strategy $\mathcal{A}_2$, P_2 only communicates to P_3 in Round 1. Second, we argue that the second round communication from P_1 does not impact P_3's output computation as follows.

We observe that the function output depends only on (x_2, x_3). Clearly, Round 1 messages $\{\mathsf{p}^1_{1\to 3}, \mathsf{b}^1_1\}$ of P_1 does not depend on x_2. Next, since there is no private communication to P_1 from P_2 as per strategy $\mathcal{A}_2$, the only information that can possibly hold information on x_2 and can impact the round 2 messages of P_1 is b^1_2. However, since this is a broadcast message, P_3 holds this by the end of Round 1 itself. □

Lemma 25. *A corrupt P_3 violates the privacy property of π.*

Proof. The adversary corrupting P_3 participates in the protocol honestly by fixing input $x_3 = 0$. Since P_3 can get the output from P_2's and P_1's round 1 communication (Lemma 24), it must be true that P_3 can evaluate the function f locally by plugging in any value of x_3. (Note that P_2 and P_1's communication in round 1 are independent of the communication of P_3 in the same round.) Now a corrupt P_3 can plug in $x_3 = 1$ locally and learn x_2 (via the output $x_2 \wedge x_3$). In the ideal world, corrupt P_3 must learn nothing beyond the output 0 as it has participated in the protocol with input 0. But in the execution of π (in which P_3 participated honestly with input $x_3 = 0$), P_3 has learnt x_2. This is a clear breach of privacy as P_3 learns x_2 regardless of his input. □
Hence, we have arrived at a contradiction, completing the proof of Theorem 4. □

6.2 The Impossibility of 2-round 3PC with Unanimous Abort

Theorem 5. *There exist functions f such that no two-round 3PC protocol achieving security with unanimous abort can compute f assuming access to pairwise-private channels, even in the honest majority setting.*

Proof. We prove the theorem by contradiction. We assume that there exists a two-round 3PC protocol π achieving security with unanimous abort that can compute the same function $f(x_1, x_2, x_3)$ considered in the proof of Theorem 4.

At a high level, we discuss three adversarial strategies $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3$ of $\mathcal{A}$. We consider party P_1 launches $\mathcal{A}_1$ in execution Σ_1, and P_2 launches $\mathcal{A}_2, \mathcal{A}_3$ in executions Σ_2, Σ_3 of π respectively. For the sake of simplicity, the executions are assumed to be run for the same input tuple (x_1, x_2, x_3) and the same random inputs (r_1, r_2, r_3) (without loss of generality) of the three parties. We use the notation V^j_i to denote the view of party P_i at the end of execution Σ_j of π. The skeleton of the proof goes as follows: We first claim that strategy $\mathcal{A}_1$ leads to honest P_2 computing the output $y = f(x_1, x_2, x_3)$. Here, we note that the value of $f(x_1, x_2, x_3)$ depends only on the inputs of honest P_2, P_3 (i.e input values x_2, x_3) and is thus well-defined. We refer to $f(x_1, x_2, x_3)$ as the value determined by this particular combination of inputs (x_2, x_3) henceforth. Since the protocol achieves unanimous abort, honest P_3's view V^1_3 at the end of Σ_1 must lead to output computation of y by P_3. Next, strategy $\mathcal{A}_2$ executed by P_2 during Σ_2 results in P_3 having the same view as in Σ_1 i.e $\mathsf{V}^1_3 = \mathsf{V}^2_3$. Thus, honest P_3 computes the output

and to preserve the property of unanimous abort, honest P_1 with view V_1^2 must also compute the output. Finally, we present a strategy $\mathcal{A}_3$ by P_2 during Σ_3 that results in P_1 having the same view as in Σ_2 i.e $\mathsf{V}_1^2 = \mathsf{V}_1^3$. It follows that honest P_1 computes the output and therefore honest P_3 with view V_3^3 must be able to compute the output too. This results in a contradiction as we conclude that if P_3's view V_3^3 enables output computation, P_3 must be able to compute the output at the end of Round 1 itself which violates privacy as proved in Lemma 25.

Let $\mathsf{p}_{i\to j}^r$ denote the pairwise communication from P_i to P_j in round r, where $r \in [2], \{i,j\} \in [3]$. Below we describe the strategies $\mathcal{A}_1, \mathcal{A}_2$ and $\mathcal{A}_3$.

$\mathcal{A}_1$**:** P_1 behaves honestly during Round 1 of the protocol. In Round 2, P_1 behaves honestly towards P_2. P_1's communication to P_3 in Round 2 is according to the protocol specification for the scenario when P_1 didn't receive the expected message (or nothing) from P_2 in Round 1. In more detail, suppose $\overline{\mathsf{p}_{1\to 3}^2}$ is the message that should be sent by P_1 to P_3 according to the protocol incase P_1 didn't receive anything from P_2 in Round 1. Then as per $\mathcal{A}_1$, corrupt P_1 sends $\overline{\mathsf{p}_{1\to 3}^2}$ to P_3 in Round 2.

$\mathcal{A}_2$**:** P_2 does not communicate at all to P_1 but behaves honestly to P_3 throughout π.

$\mathcal{A}_3$**:** In Round 1, P_2 does not communicate to P_1 but behaves honestly to P_3. In Round 2, P_2 does not communicate at all.

Next we present the views of the parties in Σ_1, Σ_2 and Σ_3 in Table 2. Here, $\overline{\mathsf{p}_{1\to 3}^2}$ is the message that should be sent by P_1 to P_3 according to the protocol incase P_1 didn't receive anything from P_2 in Round 1. Besides this, the communications that could potentially be different from the communications in an honest execution with the considered inputs and random inputs of the parties are appended with $\star$ (e.g. $\mathsf{p}_{1\to 2}^2(\star)$). We now prove a sequence of lemmas to complete our proof.

Table 2: Views of P_1, P_2, P_3 in Σ_1, Σ_2, Σ_3

	Σ_1			Σ_2			Σ_3		
	V_1	V_2	V_3	V_1	V_2	V_3	V_1	V_2	V_3
Initial Input	(x_1, r_1)	(x_2, r_2)	(x_3, r_3)	(x_1, r_1)	(x_2, r_2)	(x_3, r_3)	(x_1, r_1)	(x_2, r_2)	(x_3, r_3)
Round 1	$\mathsf{p}_{2\to 1}^1, \mathsf{p}_{3\to 1}^1$	$\mathsf{p}_{1\to 2}^1, \mathsf{p}_{3\to 2}^1$,	$\mathsf{p}_{1\to 3}^1, \mathsf{p}_{2\to 3}^1$,	$-, \mathsf{p}_{3\to 1}^1$,	$\mathsf{p}_{1\to 2}^1, \mathsf{p}_{3\to 2}^1$,	$\mathsf{p}_{1\to 3}^1, \mathsf{p}_{2\to 3}^1$,	$-, \mathsf{p}_{3\to 1}^1$,	$\mathsf{p}_{1\to 2}^1, \mathsf{p}_{3\to 2}^1$,	$\mathsf{p}_{1\to 3}^1, \mathsf{p}_{2\to 3}^1$,
Round 2	$\mathsf{p}_{2\to 1}^2, \mathsf{p}_{3\to 1}^2$,	$\mathsf{p}_{1\to 2}^2, \mathsf{p}_{3\to 2}^2$,	$\overline{\mathsf{p}_{1\to 3}^2}, \mathsf{p}_{2\to 3}^2$,	$-, \mathsf{p}_{3\to 1}^2$,	$\mathsf{p}_{1\to 2}^2(\star), \mathsf{p}_{3\to 2}^2$,	$\overline{\mathsf{p}_{1\to 3}^2}, \mathsf{p}_{2\to 3}^2$,	$-, \mathsf{p}_{3\to 1}^2$,	$\mathsf{p}_{1\to 2}^2(\star), \mathsf{p}_{3\to 2}^2$,	$\overline{\mathsf{p}_{1\to 3}^2}, -$

Lemma 26. *P_3 computes the output $y = f(x_1, x_2, x_3)$ at the end of Σ_1.*

Proof. The proof follows easily. During Σ_1, as per strategy $\mathcal{A}_1$, corrupt P_1 behaved honestly to P_2 throughout π. Therefore P_2 would compute the output $y = f(x_1, x_2, x_3)$. Due to property of unanimous abort, honest P_3 must learn the output as well. □

Lemma 27. *P_3 computes the output $y = f(x_1, x_2, x_3)$ at the end of Σ_2.*

Proof. We observe that the view of P_3 during Σ_1, Σ_2 is same. As per both strategies $\mathcal{A}_1$, and $\mathcal{A}_2$, P_3 receives communication from P_1, P_2 as per honest execution in Round

1. In Round 2, according to $\mathcal{A}_1$, corrupt P_1 sends $\overline{\mathsf{p}^2_{1\to 3}}$ as per protocol specification for case when P_1 receives nothing from P_2 in Round 1. A similar message would be sent by honest P_1 to P_3 who did not receive anything from P_2 in Round 1 (as per $\mathcal{A}_2$) during Σ_2. It is now easy to check (refer Table 2) that $\mathsf{V}^1_3 = \mathsf{V}^2_3$. Finally, since V^1_3 leads to output computation of y as per Lemma 26, P_3's view at the end of Σ_2 i.e V^2_3 must result in P_3 computing the output y. □

Lemma 28. *P_3 learns the output at the end of Σ_3.*

Proof. Firstly, it follows from lemma 27 and property of unanimous abort that honest P_1 must compute the output at the end of Σ_2. Next, it is easy to check that $\mathsf{V}^2_1 = \mathsf{V}^3_1$ (refer Table 2). We can thus conclude that honest P_1 computes the output at the end of Σ_3. Therefore, honest P_3 must also be able to compute the output at the end of Σ_3 (by assumption that π achieves unanimous abort). □

Finally, we now prove that P_3 learns the output at the end of Round 1 (similar to Lemma 24).

Lemma 29. *P_3 in Σ_3 should learn the output y by the end of Round 1.*

Proof. According to lemma 28, P_3 should learn the function output in Σ_3. First, we note that as per strategy $\mathcal{A}_3$, corrupt P_2 only communicates to P_3 in Round 1. Second, we argue that the second round communication from P_1 does not impact P_3's output computation as follows.

We observe that the function output depends only on (x_2, x_3). Clearly, the first round messages $\{\mathsf{p}^1_{1\to 3}\}$ of P_1 does not depend on x_2. Next, since there is no communication to P_1 from P_2 as per strategy $\mathcal{A}_3$, round 2 messages of P_1 hold no information about x_2. □

If P_3 is able to compute output at the end of Round 1, we know that protocol π violates privacy (proved in Lemma 25). We have thus arrived at a contradiction, concluding the proof of Theorem 5. □

We observe that even if broadcast was allowed in the first round, all the above arguments would still hold. We state this as a corollary below.

Corollary 1. *There exist functions f such that no two-round 3PC protocol achieving security with unanimous abort can compute f assuming access to pairwise-private and broadcast channels in Round 1 and only pairwise-private channels in Round 2; even in the honest majority setting.*

Proof. We observe that the following minor tweaks to the proof of Theorem 5 imply Corollary 1: We redefine $\overline{\mathsf{p}^2_{1\to 3}}$ to be the message that should be sent by P_1 to P_3 in Round 2 according to the protocol incase P_1 didn't receive anything *privately* (over pairwise-private channel) from P_2 in Round 1 (if Round 1 includes broadcast communication from P_2, then we assume P_1 has received P_2's broadcast communication). $\mathcal{A}_1$ remains the same with $\overline{\mathsf{p}^2_{1\to 3}}$ defined as above. We emphasize that there is no broadcast channel available in Round 2 and $\overline{\mathsf{p}^2_{1\to 3}}$ is communicated via pairwise-private channel between P_1 and P_3. Strategies $\mathcal{A}_2$ and $\mathcal{A}_3$ are tweaked to include honest behavior of P_2 in broadcast communication of Round 1. It is now easy to check that the arguments

of Lemma 26 - 28 hold. We can now conclude that P_3 learns the output at the end of Σ_3 where the only communication from P_2 throughout the protocol includes broadcast communication in Round 1 and private communication to P_3 in Round 1. Finally, similar to Lemma 29 we can argue that P_3 learns the output at the end of Round 1 itself which violates privacy. This completes the proof. □

Alternative functions. While it suffices to show impossibility with respect to a particular function to rule out the possibility of having generic protocols, we cite yet another function that can lead to the same conclusion. Consider a function f' that outputs the message m which is the decryption of ciphertext c (P_2's input) where the decryption key k constitutes P_3's input. All our arguments still hold except Lemma 25: Instead of the argument of how privacy could be breached by corrupt P_3 who gets access to output at the end of Round 1, in the context of this function f', a corrupt P_3 (who gets access to the output at the end of Round 1 itself) would be able to get decryptions of the ciphertext c corresponding to multiple keys k of his choice which violates correctness.

7 Conclusion

In this paper, we settle exact round complexity of 3PC with selective abort, unanimous abort, fairness and guaranteed output delivery in a setting with private pairwise channels and with or without broadcast channel. Our lower bounds extend for any n and t with $3t > n > 2t$. Our protocols rely on injective OWF.

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Supplementary Material

A Primitives

A.1 Properties of Garbling Scheme

Definition 2. *(Correctness) A garbling scheme $\mathcal{G}$ is* correct *if for all input lengths $n \leq \mathsf{poly}(\kappa)$, circuits $C : \{0,1\}^n \to \{0,1\}^m$ and inputs $x \in \{0,1\}^n$, the following probability is negligible in κ:*

$$\Pr\left[\mathsf{De}(\mathsf{Ev}(\mathbf{C}, \mathsf{En}(e,x)), d) \neq C(x) : (\mathbf{C}, e, d) \leftarrow \mathsf{Gb}(1^\kappa, C)\right].$$

Definition 3. *(Privacy) A garbling scheme $\mathcal{G}$ is* private *if for all input lengths $n \leq \mathsf{poly}(\kappa)$, circuits $C : \{0,1\}^n \to \{0,1\}^m$, there exists a PPT simulator $\mathcal{S}_{\mathbf{priv}}$ such that for all inputs $x \in \{0,1\}^n$, for all probabilistic polynomial-time adversaries $\mathcal{A}$, the following two distributions are computationally indistinguishable:*

- $\textsc{Real}(C, x)$ *: run* $(\mathbf{C}, e, d) \leftarrow \mathsf{Gb}(1^\kappa, C)$*, and output* $(\mathbf{C}, \mathsf{En}(x, e), d)$.
- $\textsc{Ideal}_{\mathcal{S}_{\mathbf{priv}}}(C, C(x))$*: output* $(\mathbf{C}', \mathbf{X}, d') \leftarrow \mathcal{S}_{\mathbf{priv}}(1^\kappa, C, C(x))$

Definition 4. *(Authenticity) A garbling scheme $\mathcal{G}$ is* authentic *if for all input lengths $n \leq \mathsf{poly}(\kappa)$, circuits $C : \{0,1\}^n \to \{0,1\}^m$, inputs $x \in \{0,1\}^n$, and all PPT adversaries $\mathcal{A}$, the following probability is negligible in κ:*

$$\Pr\left[\begin{array}{c} \widehat{\mathbf{Y}} \neq \mathsf{Ev}(\mathbf{C}, \mathbf{X}) \\ \wedge \mathsf{De}(\widehat{\mathbf{Y}}, d) \neq \bot \end{array} : \begin{array}{c} \mathbf{X} = \mathsf{En}(x, e),\ (\mathbf{C}, e, d) \leftarrow \mathsf{Gb}(1^\kappa, C) \\ \widehat{\mathbf{Y}} \leftarrow \mathcal{A}(\mathbf{C}, \mathbf{X}) \end{array}\right].$$

Definition 5. *(Obliviousness) A garbling scheme $\mathcal{G}$ achieves* obliviousness *if for all input lengths $n \leq \mathsf{poly}(\kappa)$, circuits $C : \{0,1\}^n \to \{0,1\}^m$, there exists a PPT simulator $\mathcal{S}_{\mathbf{obv}}$ such that for all inputs $x \in \{0,1\}^n$, for all probabilistic polynomial-time adversaries $\mathcal{A}$, the following two distributions are computationally indistinguishable:*

- $\textsc{Real}(C, x)$ *: run* $(\mathbf{C}, e, d) \leftarrow \mathsf{Gb}(1^\kappa, C)$*, and output* $(\mathbf{C}, \mathsf{En}(x, e))$.
- $\textsc{Ideal}_{\mathcal{S}_{\mathbf{obv}}}(C)$*: output* $(\mathbf{C}', \mathbf{X}) \leftarrow \mathcal{S}_{\mathbf{obv}}(1^\kappa, C)$

A.2 Non-Interactive Commitment Schemes (NICOM)

Properties.

- *Correctness:* For all pp, $x \in \mathcal{M}$ and $r \in \mathcal{R}$, if $(c, o) \leftarrow \mathsf{Com}(x; r)$ then $\mathsf{Open}(c, o) = x$.
- *Binding:* For all PPT adversaries $\mathcal{A}$ and all pp, it is with negligible probability that $\mathcal{A}(\mathsf{pp})$ outputs (c, o, o') such that $\mathsf{Open}(c, o) \neq \mathsf{Open}(c, o')$ and $\bot \notin \{\mathsf{Open}(c, o), \mathsf{Open}(c, o')\}$
- *Hiding:* For all PPT adversaries $\mathcal{A}$, the following difference is negligible (over uniform choice of pp and the random coins of $\mathcal{A}$) for all $x, x' \in \mathcal{M}$:

$$\left|\mathsf{Pr}_{(c,o)\leftarrow\mathsf{Com}(x)}[\mathcal{A}(c) = 1] - \mathsf{Pr}_{(c,o)\leftarrow\mathsf{Com}(x')}[\mathcal{A}(c) = 1]\right|$$

Instantiations. Here we present two instantiations of NICOM. In the random oracle model, commitment is $(c, o) = (\mathsf{H}(x||r), x||r) = \mathsf{Com}(x; r)$. The pp can in fact be empty. In the standard model, we can use the following bit-commitment scheme from any injective one-way function. Let $f : \{0,1\}^n \to \{0,1\}^n$ be a one-way permutation and $h : \{0,1\}^n \to \{0,1\}$ a hard core predicate for $f(\cdot)$. Then the commitment scheme for a single bit x is:

- $\mathsf{Com}(x; r)$: set $c = (f(r), x \oplus h(r))$; where $r \in_R \{0,1\}^n$; set $o = (r, x)$.
- $\mathsf{Open}(c, o = (r, x))$: return x if $c = (f(r), x \oplus h(r))$; otherwise return $\perp$.

For commitment of multi-bit string, the Goldreich-Goldwasser-Micali [GGM86] construction from a one-way permutation f can be used. Recall the GGM construction: given one-way permutation $f : \{0,1\}^k \to \{0,1\}^k$ with hard-core predicate $h : \{0,1\}^k \to \{0,1\}$, first construct a length-doubling pseudorandom generator $\mathsf{G} : \{0,1\}^k \to \{0,1\}^k$ via: $\mathsf{G}(s) = f^k(s)\ h(f^{k-1}(s)) \dots h(s)$. Let $\mathsf{G}_0(s)$ denote the first k bits of $\mathsf{G}(s)$, and let $\mathsf{G}_1(s)$ denote the last k bits of $\mathsf{G}(s)$. For a binary string s, the commitment c can be defined as $c = \mathsf{F}(s, 0^\ell) = \mathsf{G}_0(\dots(\mathsf{G}_0(\mathsf{G}_0(s)))\dots)$ with $o = (s)$. It is shown in [GGM86] that the function family $\mathcal{F} = \{F^\kappa\}$ with $F^\kappa = \{\mathsf{F}(s)\}_{s \in \{0,1\}^\kappa}$ is pseudorandom. Now, note that $\mathsf{F}(s, 0^\ell) = f^{\ell \cdot \kappa}(s)$. Since f is a permutation, this means that the function $g(x) = \mathsf{F}(x, 0^\ell)$ is a permutation, and hence the commitment scheme has the binding property. Hiding follows from the property of PRF F [KS05].

A.3 Equivocal Non-interactive Commitment Schemes (eNICOM)

An eNICOM comprises of the following algorithms, apart from the ones needed in NICOM:

- $\mathsf{eGen}(1^\kappa)$ returns a public parameter and a corresponding trapdoor (epp, t), where epp is used by both eCom and eOpen. The trapdoor t is used for equivocation.
- $\mathsf{Equiv}(c, o', x, t)$ is invoked on a certain commitment c and its corresponding opening o', given message x and the trapdoor t and returns o such that $x \leftarrow \mathsf{eOpen}(\mathsf{epp}, c, o)$.

An eNICOM satisfies correctness, hiding and binding properties much like the NICOM does. The hiding property of eNICOM is slightly changed compared to that of NICOM taking the equivocation property into account. This new definition implies the usual hiding definition.

Properties.

- *Correctness:* For all $(\mathsf{epp}, t) \leftarrow \mathsf{eGen}(1^\kappa)$, $x \in \mathcal{M}$ and $r \in \mathcal{R}$, if $(c, o) \leftarrow \mathsf{eCom}(x; r)$ then $\mathsf{eOpen}(c, o) = x$.
- *Binding:* For all $(\mathsf{epp}, t) \leftarrow \mathsf{eGen}(1^\kappa)$ and for all PPT adversaries $\mathcal{A}$, it is with negligible probability that $\mathcal{A}(\mathsf{epp})$ outputs (c, o, o') such that $\mathsf{eOpen}(c, o) \neq \mathsf{eOpen}(c, o')$ and $\perp \notin \{\mathsf{eOpen}(c, o), \mathsf{eOpen}(c, o')\}$
- *Hiding:* For all $(\mathsf{epp}, t) \leftarrow \mathsf{eGen}(1^\kappa)$ and for all PPT adversaries $\mathcal{A}$, and all $x, x' \in \mathcal{M}$, the following difference is negligible:

$$\left|\mathsf{Pr}_{(c,o)\leftarrow\mathsf{eCom}(x)}[\mathcal{A}(c, o) = 1] - \mathsf{Pr}_{(c,o)\leftarrow\mathsf{eCom}(x'),o\leftarrow\mathsf{Equiv}(c,x,t)}[\mathcal{A}(c, o) = 1]\right|$$

Instantiations. We first present the equivocal bit commitment scheme of [CIO98], which is based on Naor's commitment scheme [Nao91] for single bit message. This scheme avoids the use of public-key primitives. Let $\mathsf{G} : \{0,1\}^n \to \{0,1\}^{4n}$ be a pseudorandom generator.

- $\mathsf{eGen}(1^\kappa)$: set $(\mathsf{epp}, t) = (\sigma, (r_0, r_1))$, where $\sigma = \mathsf{G}(r_0) \oplus \mathsf{G}(r_1)$
- $\mathsf{eCom}(x; r)$: set $c = \mathsf{G}(r)$ if $x = 0$, else $c = \mathsf{G}(r) \oplus \sigma$; set $o = (r, x)$
- $\mathsf{eOpen}(c, o = (r, x))$: return x if $c = \mathsf{G}(r) \oplus x \cdot \sigma$ (where $(\cdot)$ denotes multiplication by constant); otherwise return $\perp$.
- $\mathsf{Equiv}(c = \mathsf{G}(r_0), \perp, x, t)$: return $o = (r, x)$ where $r = r_0$ if $x = 0$, else $r = r_1$.

Next, we present the instantiation based on Pedersen commitment scheme [Ped91]. Let p, q denote large primes such that q divides $(p-1)$, G_q is the unique subgroup of $\mathbb{Z}_p^*$ of order q and g is a generator of G_q.

- $\mathsf{eGen}(1^\kappa)$: set $(\mathsf{epp}, t) = ((g, h), \alpha)$ where $\alpha \in \mathbb{Z}_q$; $h = g^\alpha$
- $\mathsf{eCom}(x; r)$: set $c = g^x h^r$; set $o = (r, x)$.
- $\mathsf{eOpen}(c, o = (r, x))$: return x if $c = g^x h^r$; otherwise return $\perp$.
- $\mathsf{Equiv}((c = \mathsf{eCom}(x'; r'))\,, (x', r'), x, t)$: return $o = (r, x)$ where $r = r' + \frac{x'-x}{t}$

While in Naor-based instantiation, a specific commitment $c = \mathsf{G}(r_0)$ can be decommitted to either 0 or 1, the Pedersen commitment scheme allows equivocation of *any* commitment. For the purpose of our protocols, even the former weaker property suffices and hence our protocols can be based on symmetric key operations alone.

A.4 Symmetric-Key Encryption with Special Correctness

Definition 6. *A CPA-secure symmetric-key encryption scheme $\pi = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ satisfies special correctness if there is some negligible function ϵ such that for any message m we have:* $\Pr[\mathsf{Dec}_{k_2}(\mathsf{Enc}_{k_1}(m)) \neq \perp : k_1, k_2 \leftarrow \mathsf{Gen}(1^\kappa)] \leq \epsilon(\kappa)$

Instantiation. Here we present an instantiation borrowed from [JW16, LP09]. Let $\mathcal{F} = \{f_k\}$ be a family of pseudorandom functions where $f_k = \{0,1\}^\kappa \to \{0,1\}^{\kappa+s}$, for $k \in \{0,1\}^\kappa$ and s is a parameter denoting message length.

- $\mathsf{Enc}_k(m) = (r, f_k(r) \oplus m0^\kappa)$ where $m \in \{0,1\}^s, r \leftarrow \{0,1\}^\kappa$ and $m0^\kappa$ denotes the concatenation of m with a string of 0s of length κ.
- $\mathsf{Dec}_k(c)$ which parses $c = (r, z)$, computes $w = z \oplus f_k(r)$ and if the last κ bits of w are 0's, it outputs the first s bits of w, else it outputs $\perp$

B The Security Model

We prove the security of our protocols based on the standard real/ideal world paradigm. Essentially, the security of a protocol is analyzed by comparing what an adversary can do in the real execution of the protocol to what it can do in an ideal execution, that is considered secure by definition (in the presence of an incorruptible trusted party). In an

ideal execution, each party sends its input to the trusted party over a perfectly secure channel, the trusted party computes the function based on these inputs and sends to each party its respective output. Informally, a protocol is secure if whatever an adversary can do in the real protocol (where no trusted party exists) can be done in the above described ideal computation. We refer to [Can00, Gol01, Lin17, CL14] for further details regarding the security model.

The "ideal" world execution involves parties P_1, P_2, P_3, an ideal adversary $\mathcal{S}$ who may corrupt one of the parties, and a functionality $\mathcal{F}$. The "real" world execution involves the PPT parties P_1, P_2, P_3, and a real world adversary $\mathcal{A}$ who may corrupt one of the parties. We let $\text{IDEAL}_{\mathcal{F},\mathcal{S}}(1^\kappa, z)$ denote the output pair of the honest parties and the ideal-world adversary $\mathcal{S}$ from the ideal execution with respect to the security parameter 1^κ and auxiliary input z. Similarly, let $\text{REAL}_{\Pi,\mathcal{A}}(1^\kappa, z)$ denote the output pair of the honest parties and the adversary $\mathcal{A}$ from the real execution with respect to the security parameter 1^κ and auxiliary input z.

Definition 7. *For $n \in \mathbb{N}$, let $\mathcal{F}$ be a functionality and let Π be a 3-party protocol. We say that Π securely realizes $\mathcal{F}$ if for every PPT real world adversary $\mathcal{A}$, there exists a PPT ideal world adversary $\mathcal{S}$, corrupting the same parties, such that the following two distributions are computationally indistinguishable:*

$$\text{IDEAL}_{\mathcal{F},\mathcal{S}} \overset{c}{\approx} \text{REAL}_{\Pi,\mathcal{A}}.$$

Target Functionalities. Taking motivation from [CL14, GLS15], we define ideal functionalities $\mathcal{F}_{\mathsf{sa}}$, $\mathcal{F}_{\mathsf{ua}}$, $\mathcal{F}_{\mathsf{fair}}$, $\mathcal{F}_{\mathsf{god}}$ in Figures 10, 11, 12, 13 for secure 3PC of a function f with selective abort, unanimous abort, fairness and guaranteed output delivery respectively.

$\mathcal{F}_{\mathsf{sa}}$

Input: On message $(\mathsf{sid}, \mathsf{Input}, x_i)$ from a party P_i $(i \in [3])$, do the following: if $(\mathsf{sid}, \mathsf{Input}, *)$ message was received from P_i, then ignore. Otherwise record $x_i' = x_i$ internally. If x_i' is outside of the domain for P_i $(i \in [3])$, consider $x_i' = \texttt{abort}$.

Output to adversary: If there exists $i \in [3]$ such that $x_i' = \texttt{abort}$, send $(\mathsf{sid}, \texttt{Output}, \perp)$ to all the parties. Else, send $(\mathsf{sid}, \texttt{Output}, y)$ to the adversary, where $y = f(x_1', x_2', x_3')$.

Output to selected honest parties: Receive $(\texttt{select}, \{I\})$ from adversary, where $\{I\}$ denotes a subset of the honest parties. If an honest party belongs to I, send $(\mathsf{sid}, \texttt{Output}, y)$, else send $(\mathsf{sid}, \texttt{Output}, \perp)$.

Fig. 10: Ideal Functionality for selective abort

$\mathcal{F}_{\mathsf{ua}}$

Input: On message $(\mathsf{sid}, \mathsf{Input}, x_i)$ from a party P_i $(i \in [3])$, do the following: if $(\mathsf{sid}, \mathsf{Input}, *)$ message was received from P_i, then ignore. Otherwise record $x_i' = x_i$ internally. If x_i' is outside of the domain for P_i $(i \in [3])$, consider $x_i' = \texttt{abort}$.

Output to adversary: If there exists $i \in [3]$ such that $x_i' = \texttt{abort}$, send $(\mathsf{sid}, \texttt{Output}, \perp)$ to all the parties. Else, send $(\mathsf{sid}, \texttt{Output}, y)$ to the adversary, where $y = f(x_1', x_2', x_3')$.

Output to honest parties: Receive either `continue` or `abort` from adversary. In case of `continue`, send y to honest parties, whereas in case of `abort` send $\perp$ to all honest parties.

Fig. 11: Ideal Functionality for unanimous abort

$\mathcal{F}_{\mathsf{fair}}$

Input: On message $(\mathsf{sid}, \mathsf{Input}, x_i)$ from a party P_i $(i \in [3])$, do the following: if $(\mathsf{sid}, \mathsf{Input}, *)$ message was received from P_i, then ignore. Otherwise record $x_i' = x_i$ internally. If x_i' is outside of the domain for P_i $(i \in [3])$, consider $x_i' = \texttt{abort}$.

Output: If there exists $i \in [3]$ such that $x_i' = \texttt{abort}$, send $(\mathsf{sid}, \texttt{Output}, \perp)$ to all the parties. Else, send $(\mathsf{sid}, \texttt{Output}, y)$ to party P_i for every $i \in [3]$, where $y = f(x_1', x_2', x_3')$.

Fig. 12: Ideal Functionality for fairness

$\mathcal{F}_{\mathsf{god}}$

Input: On message $(\mathsf{sid}, \mathsf{Input}, x_i)$ from a party P_i $(i \in [3])$, do the following: if $(\mathsf{sid}, \mathsf{Input}, *)$ message was received from P_i, then ignore. Otherwise record $x_i' = x_i$ internally. If x_i' is outside of the domain for P_i, set x_i' to be some predetermined default value.

Output: Compute $y = f(x_1', x_2', x_3')$ and send $(\mathsf{sid}, \texttt{Output}, y)$ to party P_i for every $i \in [3]$.

Fig. 13: Ideal Functionality for guaranteed output delivery

C Optimizations

In this section, we propose some optimizations to our protocols fair, ua and god that will reduce their communication. To reduce total communication, the transmission of garbled circuits should be kept minimal since they constitute the dominant part of communication. We note that the protocols already ensure that each distinct GC is communicated only once to the evaluator, namely when a garbler sends the opening of the co-garbler's circuit. Next, a proposed optimization to reduce communication is that H of the GC could be committed rather than the GC itself, where H denotes a collision-resistant hash function. Infact since broadcast communication is considered more expensive than private communication, corresponding to broadcast of a message, say m, let $\mathsf{H}(m)$ be the message broadcast by the sender while m is sent privately over pairwise channels. The same trick can be applied on the redundant common messages sent over pairwise channels as well i.e if both P_1, P_2 are supposed to send m to P_3, then have P_1 send m and P_2 send $\mathsf{H}(m)$. P_3 can locally compute the hash of the message which would suffice to verify if P_1 and P_2 agree on a common m. The above techniques reduce total communication and makes the broadcast communication complexity of the

protocol independent of the circuit size. Lastly, an optimization with respect to protocol fair is that the inputs to the subprotocol cert_i can be modified to hash of the relevant inputs instead, reducing considerably the size of the equality-checking circuit in cert_i.

D Round Optimal 3PC with fairness

D.1 Schematic Diagram

We present the schematic diagram of the 3-round fair protocol in Figures 14 - 15.

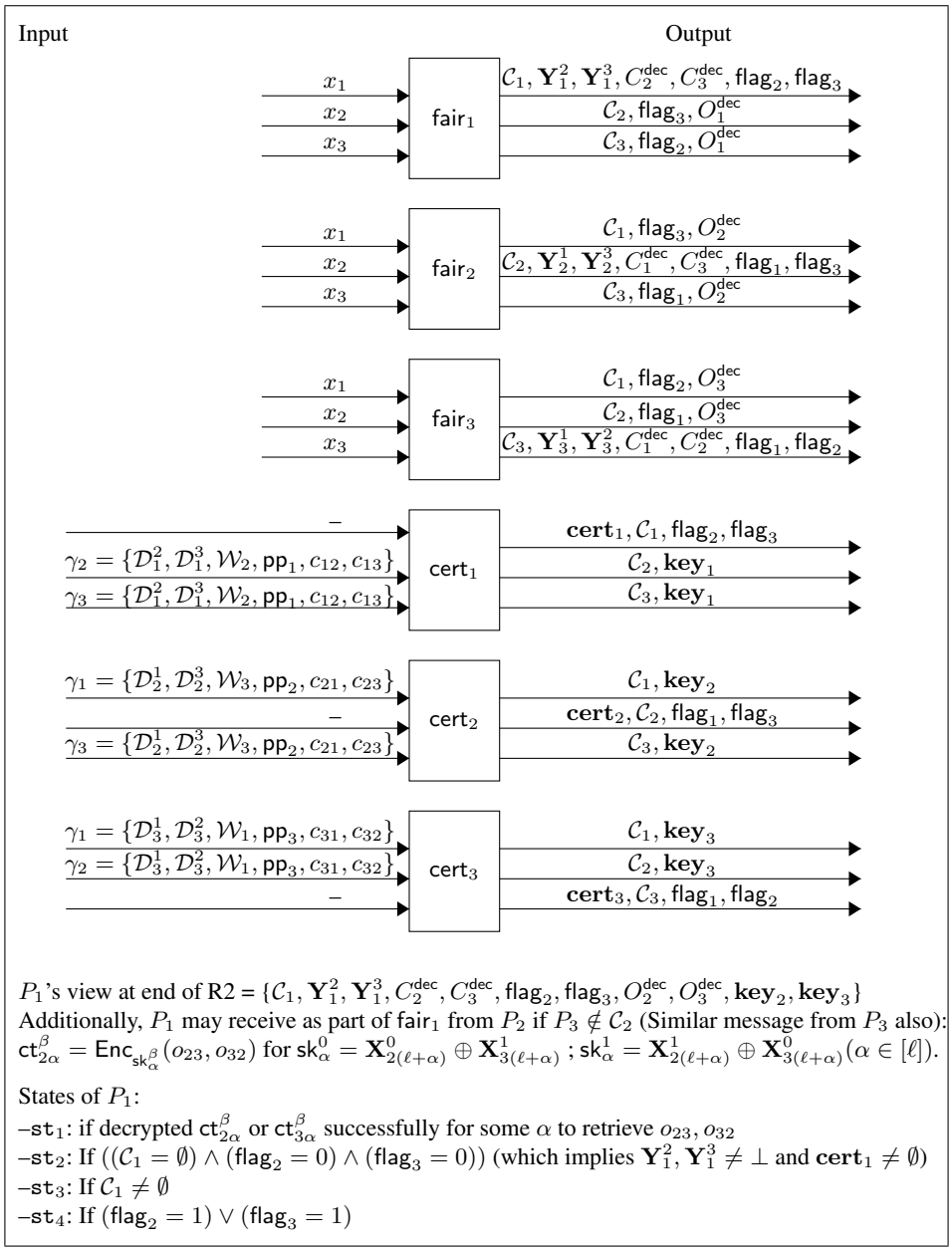
D.2 Formal Proof of Security for Protocol fair

In this section, we present the proof of security of fair relative to the ideal functionality for fairness shown in Appendix B. For better clarity, we assume without loss of generality that P_1 is corrupt (denoted as P_1^*) and describe the simulator $\mathcal{S}_{\mathsf{fair}}$. Since the roles of the parties are symmetric in fair, similar proof would hold in case of corrupt P_2, P_3 as well. The simulator plays the role of the honest parties P_2, P_3 and simulates each step of the protocol fair. Recall that during the first two rounds of fair, the two round protocols fair_i $(i \in [3])$ and cert_i $(i \in [3])$ run in parallel. We divide the description of $\mathcal{S}_{\mathsf{fair}}$ as follows: We describe $\mathcal{S}_{\mathsf{fair}}$ during $\mathsf{fair}_1, \mathsf{cert}_1$ where corrupt P_1^* is the evaluator and during $\mathsf{fair}_2, \mathsf{cert}_2$ when corrupt P_1^* acts as a garbler. The steps corresponding to fair_3, and cert_3 would follow symmetrically from that described corresponding to $\mathsf{fair}_2, \mathsf{cert}_2$. Finally, we describe the steps corresponding to the third round. The simulator $\mathcal{S}_{\mathsf{fair}}$ appears in Figure 16-17 with **R1/R2/R3** indicating simulation for round 1, 2 and 3 respectively and **f/c/F** denoting the steps corresponding to subprotocol $\mathsf{fair}_i, \mathsf{cert}_i, \mathsf{fair}$ respectively.

When simulating fair_1, the simulator does not have access to the inputs of the honest parties. Further, it does not know if and what P_1 commits as its input in Round 1, when simulating and sending the commitments for GC and encoding information in parallel in Round 1. Nor does it know if all the parties will get the output (relative to corrupt P_1's committed input from Round 1) or not, when it opens the encoded input and GC in Round 2. The decision comes from P_1's behaviour in Round 2. A privacy simulator $\mathcal{S}_{\mathsf{prv}}$ cannot be invoked for emulating Round 2 message, as $\mathcal{F}_{\mathsf{fair}}$ cannot be invoked yet and so y is not available. Instead oblivious simulator $\mathcal{S}_{\mathbf{obv}}$ is invoked that works without y. Later if and when $\mathcal{F}_{\mathsf{fair}}$ is invoked and y is known, $\mathcal{S}_{\mathsf{prv}}$ is invoked which simply returns the decoding information that makes the fake GC returned by $\mathcal{S}_{\mathbf{obv}}$ output y.

We now argue that $\text{IDEAL}_{\mathcal{F}_{\mathsf{fair}}, \mathcal{S}_{\mathsf{fair}}} \overset{c}{\approx} \text{REAL}_{\mathsf{fair}, \mathcal{A}}$, when $\mathcal{A}$ corrupts P_1. The views are shown to be indistinguishable via a series of intermediate hybrids.

- HYB_0: Same as $\text{REAL}_{\mathsf{fair}, \mathcal{A}}$.
- HYB_1: Same as HYB_0, except that P_2, P_3 in fair_1 use uniform randomness rather than pseudo-randomness for the garbled circuit construction.
- HYB_2: Same as HYB_1, except that some of the commitments of encoded inputs which will not be sent to P_1 during fair_1 are replaced with commitments on dummy values. Specifically, these are corresponding to indices not equal to $m_{22}, m_{23}, x_{12}, x_{13}$ for $\mathbf{C}_2$ and not equal to $m_{32}, m_{33}, x_{12}, x_{13}$ for $\mathbf{C}_3$.



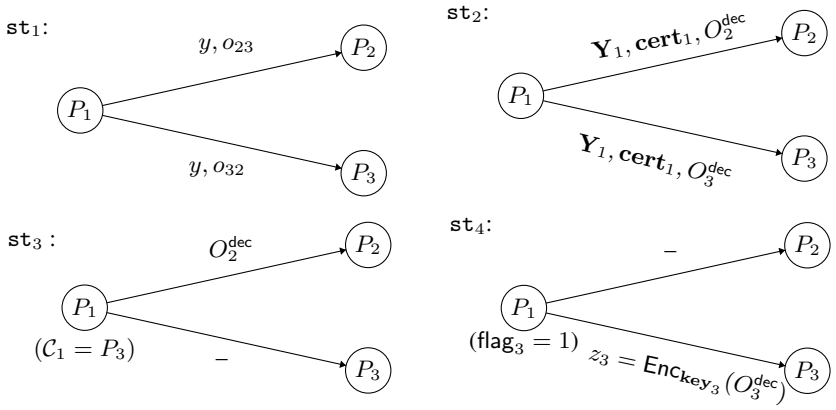
P_1's view at end of R2 = $\{\mathcal{C}_1, \mathbf{Y}_1^2, \mathbf{Y}_1^3, C_2^{\mathsf{dec}}, C_3^{\mathsf{dec}}, \mathsf{flag}_2, \mathsf{flag}_3, O_2^{\mathsf{dec}}, O_3^{\mathsf{dec}}, \mathbf{key}_2, \mathbf{key}_3\}$
Additionally, P_1 may receive as part of fair_1 from P_2 if $P_3 \notin \mathcal{C}_2$ (Similar message from P_3 also): $\mathsf{ct}_{2\alpha}^{\beta} = \mathsf{Enc}_{\mathsf{sk}_\alpha^\beta}(o_{23}, o_{32})$ for $\mathsf{sk}_\alpha^0 = \mathbf{X}_{2(\ell+\alpha)}^0 \oplus \mathbf{X}_{3(\ell+\alpha)}^1$; $\mathsf{sk}_\alpha^1 = \mathbf{X}_{2(\ell+\alpha)}^1 \oplus \mathbf{X}_{3(\ell+\alpha)}^0 (\alpha \in [\ell])$.

States of P_1:
–$\mathtt{st}_1$: if decrypted $\mathsf{ct}_{2\alpha}^{\beta}$ or $\mathsf{ct}_{3\alpha}^{\beta}$ successfully for some α to retrieve o_{23}, o_{32}
–$\mathtt{st}_2$: If $((\mathcal{C}_1 = \emptyset) \wedge (\mathsf{flag}_2 = 0) \wedge (\mathsf{flag}_3 = 0))$ (which implies $\mathbf{Y}_1^2, \mathbf{Y}_1^3 \neq \perp$ and $\mathbf{cert}_1 \neq \emptyset$)
–$\mathtt{st}_3$: If $\mathcal{C}_1 \neq \emptyset$
–$\mathtt{st}_4$: If $(\mathsf{flag}_2 = 1) \vee (\mathsf{flag}_3 = 1)$

Fig. 14: Schematic Diagram of fair protocol (Round 1 and 2)

– HYB_3 : Same as HYB_2, except the following:
  - $\text{HYB}_{3.1}$: When the execution results in P_1 evaluating GCs during fair_1 but results in $\mathtt{abort}$, $\mathbf{C}_2$ is created as $\mathbf{C}_2' \leftarrow \mathcal{S}_{\mathsf{obv}}(1^\kappa, C, \mathbf{X}_2 = \{e_{2\alpha}^{m_{22}^\alpha}, e_{2(\ell+\alpha)}^{m_{23}^\alpha}, e_{2(2\ell+\alpha)}^{x_{12}^\alpha}, e_{2(3\ell+\alpha)}^{x_{13}^\alpha}\}_{\alpha\in[\ell]})$. The commitment c_2 is later equiv-

Fig. 15: Schematic Diagram of Protocol fair (Round 3 wrt P_1)



Output Computation:
– $\mathtt{st}_1$: Output y
– $\{\mathtt{st}_2, \mathtt{st}_3, \mathtt{st}_4\}$: Output y if y received from P_2 or P_3 with valid o_{13} or o_{12} respectively
– $\mathtt{st}_2$: Retrieve O_1^{dec} either directly or using z_1 (with $\mathbf{cert}_1$ as the key) from P_2.
If $d \leftarrow \mathsf{eOpen}(\mathsf{epp}_3, C_3^{\mathsf{dec}}, O_1^{\mathsf{dec}})$ is not $\perp$, compute $y \leftarrow \mathsf{De}(\mathbf{Y}_1^3, d)$. (Similar steps wrt P_3)
– $\mathtt{st}_3$: (Let $\mathcal{C}_1 = P_3$) If received $\mathbf{Y}_2$ from P_2, output $y \leftarrow \mathsf{De}(\mathbf{Y}_2^1, d)$. Else output $\perp$
– $\mathtt{st}_4$: (Let $\mathsf{flag}_3 = 1$) If $\mathbf{cert}_3 = \mathbf{key}_3$ received from P_3, update $\mathcal{C}_1 = P_2$ and go to $\mathtt{st}_3$. Else output $\perp$.

ocated to $\mathbf{C}'_2$ using o_2 computed via $o_2 \leftarrow \mathsf{Equiv}(c_2, \mathbf{C}'_2, t_2)$. The commitment to the decoding information is created for a dummy value. Since the encoding information are committed in round 1 using committing commitments that cannot be equivocated, we invoke $\mathcal{S}_{\mathbf{obv}}$ using an $\mathbf{X}$ that corresponds to the correct shares of P_1 and it returns a fake GC (consistent with the labels in $\mathbf{X}$) such that indistinguishability holds. We note that most of the known garbling schemes based on Yao and optimizations [Yao82, ZRE15, KS08] have simulators that comply with the above.

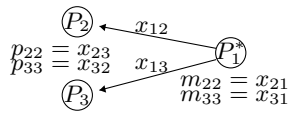
- $\text{HYB}_{3.2}$: When the execution results in P_1 evaluating GCs during fair_1 and output y, the GC is created as $(\mathbf{C}'_2, d_2) \leftarrow \mathcal{S}_{\mathsf{prv}}(1^\kappa, C, y, \mathbf{X}_2 = \{e_{2\alpha}^{m_{22}^\alpha}, e_{2(\ell+\alpha)}^{m_{23}^\alpha}, e_{2(2\ell+\alpha)}^{x_{12}^\alpha}, e_{2(3\ell+\alpha)}^{x_{13}^\alpha}\}_{\alpha\in[\ell]})$. The commitment c_2 is later equivocated to $\mathbf{C}'_2$ using o_2 computed via $o_2 \leftarrow \mathsf{Equiv}(c_2, \mathbf{C}'_2, t_2)$. The commitment c_2^{dec} to the decoding information is created for a dummy value and later equivocated to d_2 using o_{d_2} computed via $o_{d_2} \leftarrow \mathsf{Equiv}(c_2^{\mathsf{dec}}, d_2, t_2)$. The set of ciphertexts ct and z_1 (if) generated use d_2 .

– HYB_4 : Same as HYB_2, except the following:

- $\text{HYB}_{4.1}$: When the execution results in P_1 evaluating GCs during fair_1 but results in $\mathtt{abort}$, $\mathbf{C}_3$ is created as $\mathbf{C}'_3 \leftarrow \mathcal{S}_{\mathsf{obv}}(1^\kappa, C, \mathbf{X}_3 =$

Fig. 16: Description of $\mathcal{S}_{\mathsf{fair}}$

(a) $\mathcal{S}_{\mathsf{fair}}$ during $\mathsf{fair}_1, \mathsf{cert}_1, \mathsf{fair}$



R1 **f:** Sample $\mathsf{epp}_2, \mathsf{epp}_3$ for eCom, having trapdoor t_2, t_3. Choose $m_{22} = x_{21}$ (sent during fair_2), $m_{33} = x_{31}$ (sent during fair_3), m_{23}, m_{32} at random. On behalf of P_i ($i \in \{2,3\}$) compute $(\mathbf{C}_i, e_i, d_i) \leftarrow \mathsf{Gb}(1^\kappa, C)$ using uniform randomness. Send $\mathcal{D}_i = (\mathsf{epp}_i, \mathsf{pp}_i, c_i, \{c_{i\alpha}^b\}_{\alpha\in[4\ell],b\in\{0,1\}}, c_i^{\mathsf{dec}})$ to P_1^* where c_i, $\{c_{i\alpha}^{m_{i2}^\alpha}, c_{i(\ell+\alpha)}^{m_{i3}^\alpha}, c_{i(2\ell+\alpha)}^0, c_{i(2\ell+\alpha)}^1, c_{i(3\ell+\alpha)}^0, c_{i(3\ell+\alpha)}^1\}_{\alpha\in[\ell]}$ be computed as per the protocol. Let c_i^{dec} and remaining $\{c_{i\alpha}^b\}$ commit to dummy values. (For Naor-based eNICOM, set c_i, c_i^{dec} to the specific commitment supporting equivocation)

R1 **c:** As per the protocol, compute and send $\mathcal{W}_1$ to P_1^* on behalf of P_3.

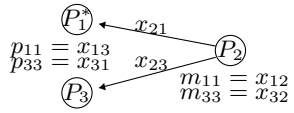
R1 **f:** Receive x_{12}, x_{13} from P_1^* on behalf of P_2, P_3 respectively.

R1 **F:** Receive $(\mathsf{pp}_1, c_{12}, c_{13}, o_{12})$ on behalf of P_2 and $(\mathsf{pp}_1, c_{12}, c_{13}, o_{13})$ on behalf of P_3 from P_1^*. Set $\mathcal{C}_i = \{P_1\}$, $i \in \{2,3\}$ if $\mathsf{Open}(\mathsf{pp}_i, c_{1i}, o_{1i}) \neq x_{1i}$.

R2 **f:** If $P_1 \notin \mathcal{C}_2, \mathcal{C}_3$, run $\mathbf{C}_i' \leftarrow \mathcal{S}_{\mathsf{obv}}(1^\kappa, C, \mathbf{X}_i = \{e_{i\alpha}^{m_{i2}^\alpha}, e_{i(\ell+\alpha)}^{m_{i3}^\alpha}, e_{i(2\ell+\alpha)}^{x_{12}^\alpha}, e_{i(3\ell+\alpha)}^{x_{13}^\alpha}\}_{\alpha\in[\ell]})$. Using trapdoor t_i, compute $o_i = \mathsf{Equiv}(c_i, \mathbf{C}_i', t_i)$. Send $\mathtt{OK}$ message on behalf of P_2, P_3 as per protocol using computed o_2, o_3.

R2 **f:** Else if $P_1 \notin \mathcal{C}_i$, then act on behalf of P_i as per the protocol (For Naor-based eNICOM equivocate c_i to $\mathbf{C}_i$ using t_i.)

(b) $\mathcal{S}_{\mathsf{fair}}$ during $\mathsf{fair}_2, \mathsf{cert}_2, \mathsf{fair}$



R1 **f:** Choose x_{21} at random and send to P_1^* on behalf of P_2.

R1 **F:** Let $p_{33} = x_{31}$ (sent during fair_3).

R1 **F:** Sample pp_2 to compute $(c_{21}, o_{21}) \leftarrow \mathsf{Com}(\mathsf{pp}_2, x_{21})$. Send $\{\mathsf{pp}_2, c_{21}, c_{23}, o_{21}\}$ to P_1^* where c_{23} is dummy commitment.

R1 **f:** Compute and send $\mathcal{D}_3$ and the information associated with $\mathcal{D}_3$ to P_1^* on behalf of P_3 according to the protocol.

R1 **f:** Receive $\mathcal{D}_1$ and associated information privately from P_1^* on behalf of P_3. Do all the verifications as an honest P_3 would perform for P_1 and update $\mathcal{C}_3$.

R1 **F:** Add P_1 to $\mathcal{C}_3$ if $p_{11} \neq x_{13}$ (received in fair_1).

R1 **c:** Receive $\mathcal{W}_2$ from P_1^* on behalf of P_2.

R1 **c:** Receive $(s_2, \mathcal{W}_2)$ from P_1^* on behalf of P_3. Do all the verifications and update $\mathcal{C}_3$ as per the protocol.

R2 **f:** Send $\mathcal{D}_3$ (as computed on behalf of P_3) to P_1^* on behalf of P_2.

R2 **f:** Set $\mathsf{flag}_1 = 1$ and $\mathbf{Y}_2^1 = \perp$ on behalf of P_2 if $P_1 \in \mathcal{C}_3$ (equivalent to receiving $\mathtt{nOK}$ from P_3) or $P_1 \notin \mathcal{C}_3$ but P_1 sends something other than $\mathcal{D}_1$ (known to P_2 as simulator runs on behalf of P_3)

R2 **f:** Set $\mathsf{flag}_3 = 1$ and $\mathbf{Y}_2^3 = \perp$ on behalf of P_2 if P_1 sends $\mathtt{nOK}$ or sends $\mathtt{OK}$ with something other than $\mathcal{D}_3$ (known to P_2 as simulator runs on behalf of P_3).

R2 **f:** If $\mathsf{flag}_1 = \mathsf{flag}_3 = 0$ wrt P_2, set $\mathcal{C}_2 = P_1$ if any of the decommitments (corresponding to $\mathbf{C}_3$ or encoded inputs corresponding to $\mathbf{C}_1, \mathbf{C}_3$) sent by P_1^* opens to something other than what was originally committed (known on behalf of P_3)

Fig. 17: Description of $\mathcal{S}_{\mathsf{fair}}$ (contd.)

R2 **F:** Set $\mathsf{flag}_1 = 1$ on behalf of both P_2, P_3 if either $P_1 \in \mathcal{C}_2$ or $P_1 \in \mathcal{C}_3$ or $\{\mathsf{pp}_1, c_{12}, c_{13}\}$ received on behalf of P_2, P_3 are not identical.

R2 **F:** Send ciphertext $\mathtt{ct}$ on dummy message on behalf of P_i if $P_1 \notin \mathcal{C}_i$ ($i \in \{2,3\}$).

R2 **F:** If $P_1 \notin \mathcal{C}_i, \gamma_i = \{\mathcal{D}_1^1, \mathcal{D}_1^3, \mathcal{W}_2, \mathsf{pp}_1, c_{12}, c_{13}\}$ received from P_1 on behalf of P_i ($i \in \{2,3\}$).

R2 **c:** If $P_1 \notin \mathcal{C}_2$, send $\mathtt{o}_1, \mathcal{W}_1$ (same as computed on behalf of P_3 in Round 1) and (opening of) encoding of γ_2 to P_1 on behalf of P_2 as per the protocol.

R2 **c:** If $P_1 \notin \mathcal{C}_3$, send (opening of) encoding of γ_3 to P_1 on behalf of P_3.

R2 **F:** Set $\mathcal{C}_2 = P_1$ if $m_{11} \neq x_{12}$

R2 **F:** Send $\{\mathsf{pp}_2, c_{21}, c_{23}\}$ (as sent on behalf of P_2) to P_1^* on behalf of P_3. Set $\mathsf{flag}_2 = 1$ wrt P_3 if nothing / other than $\{\mathsf{pp}_2, c_{21}, c_{23}\}$ received from P_1^*.

R2 **c:** Set $\mathbf{cert}_2 = \perp$ and $\mathsf{flag}_1 = 1$ on behalf of P_2 if either $P_1 \in \mathcal{C}_3$ (equivalent to receiving $\mathtt{nOK}$ from P_3) or $\mathcal{C}_3 = \emptyset$ but P_1^* sends $\mathcal{W}_2$ different from one received on behalf of P_3 in Round 1.

R2 **F:** Else, set $\mathcal{C}_2 = P_1$ if P_1 sends opening of encoded input (known on behalf of P_3) that opens to anything other than the encoding of value $\gamma_1 = \{\mathcal{D}_2^1, \mathcal{D}_2^3, \mathcal{W}_3, (\mathsf{pp}_2, c_{21}, c_{23})\}$ sent on behalf of P_2 during $\mathsf{fair}_1, \mathsf{fair}_3, \mathsf{cert}_3$ and fair respectively.

$\mathcal{S}_{\mathsf{fair}}$ during Round 3:

R3 Suppose $\mathcal{C}_2 = \emptyset, \mathsf{flag}_1 = \mathsf{flag}_3 = 0$ wrt P_2: If P_1 sends encoded inputs corresponding to mismatched input bit across $\mathbf{C}_1, \mathbf{C}_3$ during fair_2 (known on behalf of P_3), mark P_2 as being in $\mathtt{st}_1$. Invoke $\mathcal{F}_{\mathsf{fair}}$ with $(\mathsf{sid}, \mathsf{Input}, x_1)$ to obtain y where $x_1 = x_{12} \oplus x_{13}$. Send (y, o_{13}) to P_1 on behalf of P_2. Similar steps are executed on behalf of P_3 if $\mathcal{C}_3 = \emptyset, \mathsf{flag}_1 = \mathsf{flag}_2 = 0$.

R3 For every input bit of P_3, choose s bits uniformly at random, say $b_1, \ldots b_s$. Using key based on P_3's consistent input b_α ($\alpha \in [s]$) used in $\mathbf{C}_1, \mathbf{C}_3$ during fair_2, try to decrypt ciphertext $\mathsf{ct}_{1\alpha}^{\beta}$ for ($\beta \in \{0,1\}$) received from P_1 in Round 2. If the decryption is successful and the openings retrieved are same as (o_{13}, o_{31}), mark P_2 as being in $\mathtt{st}_1$ and do the following: invoke $\mathcal{F}_{\mathsf{fair}}$ with $(\mathsf{sid}, \mathsf{Input}, x_1)$ to obtain y where $x_1 = x_{12} \oplus x_{13}$. Send (y, o_{13}) to P_1 on behalf of P_2. Similar steps are executed by the simulator on behalf of P_3

R3 If P_2 or P_3 is in $\mathtt{st}_2$, let $x_1 = x_{12} \oplus x_{13}$. Invoke $\mathcal{F}_{\mathsf{fair}}$ with $(\mathsf{sid}, \mathsf{Input}, x_1)$ to obtain y.

R3 If P_2 in $\mathtt{st}_2$, to retrieve decoding information o_3^{dec}: Run $(\mathbf{C}_3, d_3^1) \leftarrow \mathcal{S}_{\mathbf{priv}}(1^\kappa, C, y, \mathbf{X} = \{e_{3\alpha}^{m_{i2}^\alpha}, e_{3(\ell+\alpha)}^{m_{i3}^\alpha}, e_{3(2\ell+\alpha)}^{x_{12}^\alpha}, e_{3(3\ell+\alpha)}^{x_{13}^\alpha}\}_{\alpha\in[\ell]})$. Equivocate the commitment on decoding information in fair_1 (c_3^{dec}) to get $o_3^{\mathsf{dec}} = \mathsf{Equiv}(c_3^{\mathsf{dec}}, d_3^1, t_3)$. Send $(\mathbf{Y}_2, \mathbf{cert}_2, o_3^{\mathsf{dec}})$ to P_1^* on behalf of P_2. Here $\mathbf{cert}_2$ is set as encoding of 1 on output wire of $\mathbf{C}_2$ during cert_2 and $\mathbf{Y}_2$ is the encoding corresponding to output y of $\mathbf{C}_1, \mathbf{C}_3$ during fair_2; both of which are known as simulator acts on behalf of P_3.

R3 Similar steps as above if P_3 is in $\mathtt{st}_2$.

R3 Invoke $\mathcal{F}_{\mathsf{fair}}$ with $(\mathsf{sid}, \mathsf{Input}, \mathtt{abort})$ if neither P_2 nor P_3 belong to $\{\mathtt{st}_1, \mathtt{st}_2\}$.

R3 Send dummy ciphertext z_1 to P_1^* on behalf of $P_i, i \in \{2,3\}$ if P_i in $\mathtt{st}_4$ with $\mathsf{flag}_1 = 1$.

$\{e_{3\alpha}^{m_{32}^\alpha}, e_{3(\ell+\alpha)}^{m_{33}^\alpha}, e_{3(2\ell+\alpha)}^{x_{12}^\alpha}, e_{3(3\ell+\alpha)}^{x_{13}^\alpha}\}_{\alpha\in[\ell]})$. The commitment c_3 is later equiv-

ocated to $\mathbf{C}'_3$ using o_3 computed via $o_3 \leftarrow \mathsf{Equiv}(c_3, \mathbf{C}'_3, t_3)$. The commitment to the decoding information is created for a dummy value.

- $\text{HYB}_{4.2}$: When the execution results in P_1 evaluating GCs during fair_1 and output y, the GC is created as $(\mathbf{C}'_3, d_3) \leftarrow \mathcal{S}_{\mathsf{prv}}(1^\kappa, C, y, \mathbf{X}_3 = \{e_{3\alpha}^{m_{32}^\alpha}, e_{3(\ell+\alpha)}^{m_{33}^\alpha}, e_{3(2\ell+\alpha)}^{x_{12}^\alpha}, e_{3(3\ell+\alpha)}^{x_{13}^\alpha}\}_{\alpha\in[\ell]})$. The commitment c_3 is later equivocated to $\mathbf{C}'_3$ using o_3 computed via $o_3 \leftarrow \mathsf{Equiv}(c_3, \mathbf{C}'_3, t_3)$. The commitment c_3^{dec} to the decoding information is created for a dummy value and later equivocated to d_3 using o_{d_3} computed via $o_{d_3} \leftarrow \mathsf{Equiv}(c_3^{\mathsf{dec}}, d_3, t_3)$. The set of ciphertexts ct and z_1 (if) generated uses d_3.

– HYB_5: Same as HYB_4, except that during fair_2, $\mathcal{C}_2$ is set to P_1 if P_2 receives o_3 that opens to a value other than the originally committed $\mathbf{C}_3$.
– HYB_6: Same as HYB_5, except that during fair_3, $\mathcal{C}_3$ is set to P_1 if P_3 receives o_2 that opens to a value other than the originally committed $\mathbf{C}_2$.
– HYB_7: Same as HYB_6, except that during fair_2, $\mathcal{C}_2$ is set to P_1 if P_2 accepts any encoded input not consistent with $\mathbf{C}_1, \mathbf{C}_3$
– HYB_8: Same as HYB_7, except that during fair_3, $\mathcal{C}_3$ is set to P_1 if P_3 accepts any encoded input not consistent with $\mathbf{C}_1, \mathbf{C}_2$
– HYB_9: Same as HYB_8, except that when the execution does not result in P_1 getting access to the opening of commitment c_{23} (corresponding to x_{23}) sent by P_2 during fair_2, the commitment is replaced with commitment of dummy value.
– HYB_{10}: Same as HYB_9, except that when the execution does not result in P_1 getting access to the opening of commitment c_{32} (corresponding to x_{32}) sent by P_3 during fair_3, the commitment is replaced with commitment of dummy value.
– HYB_{11}: Same as HYB_{10}, except that when the execution fair_1 does not result in P_1 getting encoded inputs corresponding to mismatched input bit across the two garbled circuits corresponding to any garbler, the set of ct is replaced by encryption of a dummy message.
– HYB_{12}: Same as HYB_{11}, except that during cert_2, P_2 (with $\mathsf{flag}_1 = 0$) adds P_1 to $\mathcal{C}_2$ if (opening of) encoded input sent by P_1 corresponding to $\mathbf{C}_2$ is anything other than the opening of the originally committed encoded information corresponding to value $\gamma = \{\mathcal{D}_2^1, \mathcal{D}_2^3, \mathcal{W}_3, (\mathsf{pp}_2, c_{21}, c_{23})\}$ sent by P_2 in Round 1.
– HYB_{13}: Same as HYB_{12}, except that during cert_3, P_3 (with $\mathsf{flag}_2 = 0$) adds P_1 to $\mathcal{C}_3$ if (opening of) encoded input sent by P_1 corresponding to $\mathbf{C}_3$ is anything other than the opening of the originally committed encoded information corresponding to value $\gamma = \{\mathcal{D}_3^1, \mathcal{D}_3^2, \mathcal{W}_1, (\mathsf{pp}_3, c_{31}, c_{32})\}$ sent by P_3 in Round 1.
– HYB_{14}: Same as HYB_{13}, except that during cert_1, when P_1's evaluation of $\mathbf{C}_1$ does not result in output 1, z_1 (if) sent to P_1 is replaced with encryption of dummy message.
– HYB_{15}: Same as HYB_{14}, except that $\mathbf{Y}_2^1, \mathbf{Y}_2^3$ is computed via $\mathsf{De}(\mathbf{Y}_2^1, d_1) = y$, $\mathsf{De}(\mathbf{Y}_2^3, d_3) = y$, (where d_1, d_3 correspond to decoding information of $\mathbf{C}_1, \mathbf{C}_3$ during fair_2) rather than $\mathbf{Y}_2^1 = \mathsf{Ev}(\mathbf{C}_1, \mathbf{X})$, $\mathbf{Y}_2^3 = \mathsf{Ev}(\mathbf{C}_3, \mathbf{X})$.
– HYB_{16}: Same as HYB_{15}, except that $\mathbf{Y}_3^1, \mathbf{Y}_3^2$ is computed via $\mathsf{De}(\mathbf{Y}_3^1, d_1) = y$, $\mathsf{De}(\mathbf{Y}_3^2, d_2) = y$ (where d_1, d_2 correspond to decoding information of $\mathbf{C}_1, \mathbf{C}_2$ during fair_3) rather than $\mathbf{Y}_3^1 = \mathsf{Ev}(\mathbf{C}_1, \mathbf{X})$, $\mathbf{Y}_3^2 = \mathsf{Ev}(\mathbf{C}_2, \mathbf{X})$.
– HYB_{17}: Same as HYB_{16}, except that during cert_2, if P_2 gets access to $\mathbf{Y}_2 \leftarrow (\mathbf{C}_2, \mathbf{X})$ such that $\mathsf{sDe}(\mathbf{Y}_2) = 1$, $\mathbf{cert}_2 = \mathbf{Y}_2$ is computed via $\mathsf{De}(\mathbf{Y}_2, d_2) = 1$

(where d_2 corresponds to decoding information of $\mathbf{C}_2$ during cert_2) rather than $\mathbf{Y}_2 = \mathsf{Ev}(\mathbf{C}_2, \mathbf{X})$

- HYB_{18}: Same as HYB_{17}, except that during cert_3, if P_3 gets access to $\mathbf{Y}_3 \leftarrow (\mathbf{C}_3, \mathbf{X})$ such that $\mathsf{sDe}(\mathbf{Y}_3) = 1$, $\mathbf{cert}_3 = \mathbf{Y}_3$ is computed via $\mathsf{De}(\mathbf{Y}_3, d_3) = 1$ (where d_3 corresponds to decoding information of $\mathbf{C}_3$ during cert_3) rather than $\mathbf{Y}_3 = \mathsf{Ev}(\mathbf{C}_3, \mathbf{X})$
- HYB_{19}: Same as HYB_{18}, except that P_2 sends (y, o_{13}) to P_1 if decryption of ct sent by P_1 during fair_2 is successful (and includes openings of x_{13}, x_{31} corresponding to original commitments) using P_3's encoding corresponding to random input.
- HYB_{20}: Same as HYB_{19}, except that P_3 sends (y, o_{12}) to P_1 if decryption of ct sent by P_1 during fair_3 is successful (and includes openings of x_{12}, x_{21} corresponding to original commitments) using P_2's encoding corresponding to random input.

Since $\text{HYB}_{20} := \text{IDEAL}_{\mathcal{F}_{\mathsf{fair}}, \mathcal{S}_{\mathsf{fair}}}$, we show that every two consecutive hybrids are computationally indistinguishable which concludes the proof.

$\text{HYB}_0 \overset{c}{\approx} \text{HYB}_1$: The difference between the hybrids is that P_2, P_3 in fair_1 use uniform randomness in HYB_1 rather than pseudorandomness as in HYB_0. The indistinguishability follows via reduction to the security of the PRG G.

$\text{HYB}_1 \overset{c}{\approx} \text{HYB}_2$: The difference between the hybrids is some of the commitments of encoded inputs which will not be sent to P_1 during fair_1 are replaced with commitments on dummy values. The indistinguishability between the hybrids follows from the hiding property of NICOM.

$\text{HYB}_2 \overset{c}{\approx} \text{HYB}_{3.1}$: The difference between the hybrids is in the way $(\mathbf{C}_2, \mathbf{X})$ is generated when the execution results in $\mathtt{abort}$. In HYB_2, $(\mathbf{C}_2, e, d) \leftarrow \mathsf{Gb}(1^\kappa, C)$ is run, which gives $(\mathbf{C}_2, \mathsf{En}(x, e))$. In $\text{HYB}_{3.1}$, it is generated as $\mathbf{C}_2' \leftarrow \mathcal{S}_{\mathsf{obv}}(1^\kappa, C, \mathbf{X}_2 = \{e_{2\alpha}^{m_{22}^\alpha}, e_{2(\ell+\alpha)}^{m_{23}^\alpha}, e_{2(2\ell+\alpha)}^{x_{12}^\alpha}, e_{2(3\ell+\alpha)}^{x_{13}^\alpha}\}_{\alpha\in[\ell]})$. The commitment to the garbled circuit is later equivocated to $\mathbf{C}_2'$ using o_2 computed via $o_2 \leftarrow \mathsf{Equiv}(c_2, \mathbf{C}_2', t_2)$. Additionally, the commitment to the decoding information is created for a dummy value in $\text{HYB}_{3.1}$. The indistinguishability follows via reduction to the obliviousness of the garbling scheme and the usual hiding property of commitment schemes which is implied by the hiding property of eCom.

$\text{HYB}_2 \overset{c}{\approx} \text{HYB}_{3.2}$: The difference between the hybrids is in the way $(\mathbf{C}_2, \mathbf{X}, d)$ is generated. In HYB_2, $(\mathbf{C}_2, e, d) \leftarrow \mathsf{Gb}(1^\kappa, C)$ is run, which gives $(\mathbf{C}_2, \mathsf{En}(x, e), d)$. In $\text{HYB}_{3.2}$, it is generated as $(\mathbf{C}_2', d_2^1) \leftarrow \mathcal{S}_{\mathsf{prv}}(1^\kappa, C, y, \mathbf{X}_2 = \{e_{2\alpha}^{m_{22}^\alpha}, e_{2(\ell+\alpha)}^{m_{23}^\alpha}, e_{2(2\ell+\alpha)}^{x_{12}^\alpha}, e_{2(3\ell+\alpha)}^{x_{13}^\alpha}\}_{\alpha\in[\ell]})$. The commitment to the garbled circuit is later equivocated to $\mathbf{C}_2'$ using o_2 computed via $o_2 \leftarrow \mathsf{Equiv}(c_2, \mathbf{C}_2', t_2)$. Additionally, the commitment to the decoding information is created for a dummy value and later equivocated to d_2^1 using o_2^{dec} computed via $o_2^{\mathsf{dec}} \leftarrow \mathsf{Equiv}(c_2^{\mathsf{dec}}, d_2^1, t_2)$. The indistinguishability follows via reduction to the privacy of the garbling scheme and the hiding property of eCom.

$\text{HYB}_3 \overset{c}{\approx} \text{HYB}_4$: Similar argument as above with respect to $\mathbf{C}_3$.

$\text{HYB}_4 \overset{c}{\approx} \text{HYB}_5$: The difference between the hybrids is that in HYB_4, P_2 sets $\mathcal{C}_2 = P_1$ if the o_3 sent by P_1 in fair_2 output $\perp$ while in HYB_5, P_2 sets $\mathcal{C}_2 = P_1$ if o_3 sent by P_1 in fair_2 opens to any value other than $\mathbf{C}_3$. Since the commitment scheme eCom

is binding, in HYB_4, P_1 could have decommitted successfully to a different garbled circuit than what was originally committed, only with negligible probability. Therefore, the hybrids are indistinguishable.

$\text{HYB}_5 \overset{c}{\approx} \text{HYB}_6$: Similar argument as above with respect to P_3 in fair_3.

$\text{HYB}_6 \overset{c}{\approx} \text{HYB}_7$: The difference between the hybrids is that in HYB_6, P_2 sets $\mathcal{C}_2 = P_1$ if the encoded inputs sent by P_1 in fair_2 is inconsistent with $\mathcal{D}_1, \mathcal{D}_3$, while in HYB_7 $\mathcal{C}_2$ is set to P_1 if P_2 accepts any encoded input not consistent with $\mathbf{C}_1, \mathbf{C}_3$. It follows from the biding property of NICOM that in HYB_6, P_1 could have sent an encoded input not consistent with $\mathbf{C}_1, \mathbf{C}_3$ but consistent with $\mathcal{D}_1$, $\mathcal{D}_3$, only with negligible probability. Therefore, the hybrids are indistinguishable.

$\text{HYB}_7 \overset{c}{\approx} \text{HYB}_8$: Similar argument as above with respect to P_3 in fair_3.

$\text{HYB}_8 \overset{c}{\approx} \text{HYB}_9$: The difference between the hybrids is that when the execution does not result in P_1 getting access to the opening of commitment c_{23} (corresponding to x_{23}) sent by P_2, c_{23} corresponds to the actual input share x_{23} in HYB_8 while it corresponds to dummy value in HYB_9. The indistinguishability follows from the hiding property of NICOM.

$\text{HYB}_9 \overset{c}{\approx} \text{HYB}_{10}$: Similar argument as above with respect to commitment c_{32} sent by P_3.

$\text{HYB}_{10} \overset{c}{\approx} \text{HYB}_{11}$: The difference between the hybrids is that when the execution fair_1 does not result in P_1 getting encoded inputs corresponding to mismatched input bits of any garbler on two garbled circuits, in HYB_{10}, the set of ct is the encryption of a opening of input shares while in HYB_{11}, it is replaced with encryption of dummy message. Assuming the encryption key is unknown to P_1 (holds except with negligible probability due to privacy of garbling scheme), indistinguishability follows from the security of the encryption scheme with special correctness.

$\text{HYB}_{11} \overset{c}{\approx} \text{HYB}_{12}$: The difference between the hybrids is that while in HYB_{11}, during cert_2, P_2 adds P_1 to $\mathcal{C}_2$ if opening of encoded input sent by P_1 results in $\perp$ or $\mathbf{C}_2$ evaluates to 0 revealing P_1's input being not equal to $\gamma = \{\mathcal{D}_2^1, \mathcal{D}_2^3, \mathcal{W}_3, \mathsf{pp}_2, c_{21}, c_{23}\}$; while in HYB_{12} P_1 is added to $\mathcal{C}_2$ if he sends anything other than opening of the originally committed encoded information of $\mathbf{C}_2$ corresponding to value $\gamma = \{\mathcal{D}_2^1, \mathcal{D}_2^3, \mathcal{W}_3, \mathsf{pp}_2, c_{21}, c_{23}\}$. The indistinguishability follows from the binding of NICOM and the correctness of the privacy-free garbling scheme (used during cert_2).

$\text{HYB}_{12} \overset{c}{\approx} \text{HYB}_{13}$: Similar argument as above with respect to P_3 during cert_3.

$\text{HYB}_{13} \overset{c}{\approx} \text{HYB}_{14}$: The difference between the hybrids is that in HYB_{12}, z_1 is set as encryption of the decoding information of fair_1 while in HYB_{13}, z_1 is replaced with encryption of a dummy message when P_1's evaluation of $\mathbf{C}_1$ during cert_1 does not lead to output 1. Assuming the encryption key is unknown to P_1 (holds except with negligible probability due to authenticity of privacy-free garbling scheme used in cert_1), indistinguishability follows from the security of the encryption scheme.

$\text{HYB}_{14} \overset{c}{\approx} \text{HYB}_{15}$: The difference between the hybrids is that in HYB_{14}, P_2 computes $\mathbf{Y}_2 = (\mathbf{Y}_2^1, \mathbf{Y}_2^3)$ via $\mathsf{Ev}(\mathbf{C}_1, \mathbf{X})$, $\mathbf{Y}_2^3 = \mathsf{Ev}(\mathbf{C}_3, \mathbf{X})$, while in HYB_{15}, $\mathbf{Y}_2^1, \mathbf{Y}_2^3$ is computed such that $\mathsf{De}(\mathbf{Y}_2^1, d_1) = y$, $\mathsf{De}(\mathbf{Y}_2^3, d_3) = y$ (where d_1, d_3 is the decoding information corresponding to $\mathbf{C}_1, \mathbf{C}_3$ during fair_2). Due to the correctness of the garbling

scheme, the equivalence of $\mathbf{Y}_2^1, \mathbf{Y}_2^3$ computed via $\mathsf{Ev}(\mathbf{C}_1, \mathbf{X})$, $\mathsf{Ev}(\mathbf{C}_3, \mathbf{X})$ or such that $\mathsf{De}(\mathbf{Y}_2^1, d_1) = y$, $\mathsf{De}(\mathbf{Y}_2^3, d_3) = y$ holds.

$\text{HYB}_{15} \overset{c}{\approx} \text{HYB}_{16}$: Similar argument as above with respect to $\mathbf{Y}_3$ computed by P_3 during fair_3.

$\text{HYB}_{16} \overset{c}{\approx} \text{HYB}_{17}$: The difference between the hybrids is that in HYB_{16}, if P_2 obtains $\mathbf{Y}_2 \leftarrow \mathsf{Ev}(\mathbf{C}_2, \mathbf{X})$ such that $\mathsf{sDe}(\mathbf{Y}) = 1$, then P_2 sets $\mathbf{cert}_2 = \mathbf{Y}_2$ while in HYB_{15}, in this case $\mathbf{cert}_2$ is set to $\mathbf{Y}_2$ computed such that $\mathsf{De}(\mathbf{Y}_2, d_2) = 1$ (where d_2 is the decoding information corresponding to $\mathbf{C}_2$ during cert_2). Due to the correctness of the privacy-free garbling scheme, the equivalence of $\mathbf{Y}_2$ computed via $\mathsf{Ev}(\mathbf{C}_2, \mathbf{X})$ or such that $\mathsf{De}(\mathbf{Y}_2, d_2) = y$ holds.

$\text{HYB}_{17} \overset{c}{\approx} \text{HYB}_{18}$: Similar argument as above with respect to $\mathbf{cert}_3$ computed by P_3 during cert_3.

$\text{HYB}_{18} \overset{c}{\approx} \text{HYB}_{19}$: The difference between the hybrids is that in HYB_{18}, P_2 sends (y, o_{13}) to P_1 if decryption of ct sent by P_1 during fair_2 is successful using keys based on P_3's encoding of actual input, whereas in HYB_{19}, P_2 sends (y, o_{13}) to P_1 if decryption of ct sent by P_1 during fair_2 is successful using keys based on P_3's encoding of random input. The indistinguishability between the hybrids follows from the following claim: Consider single bit input for simplicity. For any two different inputs x and x' of P_3, the difference between the probability that P_2 sends (y, o_{13}) to P_1 when P_3's input is x and when P_3's input is x' is at most 2^{-s+1}. The argument can be divided into three cases (similar to [LP07]). **(1)** Suppose for some $\alpha \in [s]$, P_1 replaces both ciphertexts $\mathsf{ct}_{1\alpha}^0, \mathsf{ct}_{1\alpha}^1$: one based on consistent input 0 of P_3 and other based on consistent input 1 of P_3 (say, $\mathsf{sk}_\alpha^0 = \mathbf{X}_{1(s+\alpha)}^0 \oplus \mathbf{X}_{3(s+\alpha)}^0$ and $\mathsf{sk}_\alpha^1 = \mathbf{X}_{1(s+\alpha)}^1 \oplus \mathbf{X}_{3(s+\alpha)}^1$). In this case, P_2 would be able to decrypt the ciphertext successfully regardless of P_3's input with probability 1 and would send (y, o_{13}) to P_2. **(2)** Suppose P_1 replaces exactly one of the two ciphertexts with consistent input corresponding to $1 \leq j < s$. Since the values assigned (in encoding) by P_3 to any proper subset of the s bits are independent of P_3's actual input, P_2 would be able to decrypt the ciphertext successfully with probability $1 - 2^{-j}$ regardless of the actual value of its original input. **(3)** Suppose P_1 replaces one ciphertext based on consistent input for each of the $\alpha \in [s]$ (say all based on consistent value '1'). Then if x had encoding with any one such value ('1' in the example), the ciphertext would be decrypted successfully with probability 1, whereas decryption would be successful with probability $1 - 2^{-s+1}$ if x' had the other value (in the example, P_2 will be unable to decrypt if $x' = 0$ and the encoding of $x' = 0$ was chosen as $x'_\alpha = 0$ for all $\alpha \in [s]$ (where $x' = \oplus_{\alpha=1}^s x_\alpha$) which occurs with probability 2^{-s+1}).

$\text{HYB}_{19} \overset{c}{\approx} \text{HYB}_{20}$: Similar argument as above with respect to ct received by P_3 during fair_3.

E Proof of Security for Protocol ua

In this section, we present the proof of security of ua relative to the ideal functionality for unanimous abort (Figure 11) shown in Appendix B. For clarity, we assume without loss of generality that P_1 is corrupt (denoted as P_1^*) and describe the simulator $\mathcal{S}_{\mathsf{ua}}$. Since the roles of the parties are symmetric in ua, similar proof would hold in case of

corrupt P_2, P_3 as well. The simulator plays the role of the honest parties P_2, P_3 and simulates each step of the protocol ua.

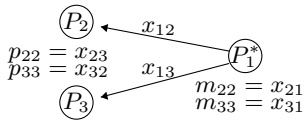
We divide the description of $\mathcal{S}_{\mathsf{ua}}$ as follows: We describe $\mathcal{S}_{\mathsf{ua}}$ during ua_1 where corrupt P_1^* is the evaluator and during ua_2 when corrupt P_1^* acts as a garbler. The steps corresponding to ua_3, would follow symmetrically from that described corresponding to ua_2. The simulator $\mathcal{S}_{\mathsf{ua}}$ appears in Figure 18-19 with **R1/R2** indicating simulation for round 1 and 2 respectively and **a/A** denoting the steps corresponding to subprotocol $\mathsf{ua}_i, \mathsf{ua}$ respectively. When simulating ua_1, the commitments for GC and encoding information need to be simulated and sent in Round 1 itself, while the privacy simulator $\mathcal{S}_{\mathsf{prv}}$ can only be invoked on noting the adversary's behaviour in Round 1 that decides what input it commits and whether it obtains output or $\perp$. Using equivocality of the commitment of GC, we can equivocate the GC as returned by the simulator. But since commitments on the encoding information are committing and the simulator didn't have access to $\mathbf{X}$ during simulation of Round 1, the encoded input $\mathbf{X}$ returned by $\mathcal{S}_{\mathsf{prv}}$ cannot be explained. So we use a slightly modified version of $\mathcal{S}_{\mathsf{prv}}$ which takes an encoded input (correspond to what will be opened to corrupt P_1) as parameter and returns just the fake GC compatible with it. Yao's privacy simulator can be made to work as above for any encoded input and the indistinguishability will hold with respect to the fake GC and given encoded input.

We now argue that $\text{IDEAL}_{\mathcal{F}_{\mathsf{ua}},\mathcal{S}_{\mathsf{ua}}} \overset{c}{\approx} \text{REAL}_{\mathsf{ua},\mathcal{A}}$, when $\mathcal{A}$ corrupts P_1. The views are shown to be indistinguishable via a series of intermediate hybrids.

- HYB_0: Same as $\text{REAL}_{\mathsf{ua},\mathcal{A}}$.
- HYB_1: Same as HYB_0, except that P_2, P_3 in ua_1 use uniform randomness rather than pseudo-randomness for the garbled circuit construction.
- HYB_2: Same as HYB_1, except that some of the commitments of encoded inputs which will not be sent to P_1 during ua_1 are replaced with commitment on dummy values. Specifically, these are corresponding to indices not equal to $m_{22}, m_{23}, r_2, r_3, z_2, z_3$ for $\mathbf{C}_2$ and not equal to $m_{32}, m_{33}, r_2, r_3, z_2, z_3$ for $\mathbf{C}_3$.
- HYB_3 : Same as HYB_2, except that when the execution results in P_1 evaluating GCs during ua_1, the GC $\mathbf{C}_2$ is created as $(\mathbf{C}_2', d_2) \leftarrow \mathcal{S}_{\mathsf{prv}}(1^\kappa, C, y, \mathbf{X}_2 = \{e_{2\alpha}^{m_{22}^\alpha}, e_{2(\ell+\alpha)}^{m_{23}^\alpha}, e_{2(2\ell+\alpha)}^{r_2^\alpha}, e_{2(3\ell+\alpha)}^{r_3^\alpha}, e_{2(4\ell+\alpha)}^{z_2^\alpha}, e_{2(5\ell+\alpha)}^{z_3^\alpha}\}_{\alpha\in[\ell]})$. The commitment c_2 is later equivocated to $\mathbf{C}_2'$ using o_2 computed via $o_2 \leftarrow \mathsf{Equiv}(c_2, \mathbf{C}_2', t_2)$. The set of ciphertexts ct generated uses d_2 in their keys.
- HYB_4 : Same as HYB_3, except that when the execution results in P_1 evaluating GCs during ua_1, the GC $\mathbf{C}_3$ is created as $(\mathbf{C}_3', d_3) \leftarrow \mathcal{S}_{\mathsf{prv}}(1^\kappa, C, y, \mathbf{X}_3 = \{e_{3\alpha}^{m_{32}^\alpha}, e_{3(\ell+\alpha)}^{m_{33}^\alpha}, e_{3(2\ell+\alpha)}^{r_2^\alpha}, e_{3(3\ell+\alpha)}^{r_3^\alpha}, e_{3(4\ell+\alpha)}^{z_2^\alpha}, e_{3(5\ell+\alpha)}^{z_3^\alpha}\}_{\alpha\in[\ell]})$. The commitment c_3 is later equivocated to $\mathbf{C}_3'$ using o_3 computed via $o_3 \leftarrow \mathsf{Equiv}(c_3, \mathbf{C}_3', t_3)$. The set of ciphertexts ct generated uses d_3 in their keys.
- HYB_5: Same as HYB_4, except that during ua_2, flag_2 is set to 1 if $\mathcal{W}_1$ broadcast by P_1 has anything other than (opening of) encoded input corresponding to z_1 in $\mathbf{C}_1$.
- HYB_6: Same as HYB_5, except that during ua_3, flag_3 is set to 1 if $\mathcal{W}_1$ broadcast by P_1 has anything other than (opening of) encoded input corresponding to z_1 in $\mathbf{C}_1$.
- HYB_7: Same as HYB_6, except that when the execution does not result in P_1 getting access to the opening of commitment c_{23} (corresponding to x_{23}) broadcast by P_2 during ua_2, the commitment is replaced with commitment of dummy value.

Fig. 18: Description of $\mathcal{S}_{\mathsf{ua}}$

(a) $\mathcal{S}_{\mathsf{ua}}$ during $\mathsf{ua}_1, \mathsf{ua}$



R1 **a:** Receive (x_{12}, x_{13}) privately from P_1^* on the behalf of P_2, P_3.

R1 **A:** Receive $(\mathsf{pp}_1, c_{12}, c_{13})$ via broadcast and o_{12}, o_{13} privately from P_1^* on behalf of P_2, P_3. Set $\mathsf{flag}_1 = 1$ on behalf of P_i if $\mathsf{Open}(c_{1i}, o_{1i}) \neq x_{1i}$ for $i \in \{2, 3\}$.

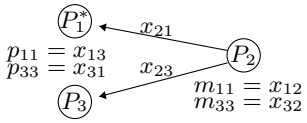
R1 **a:** Sample $\mathsf{epp}_2, \mathsf{epp}_3$ for eCom, having trapdoor t_2, t_3. Choose $m_{22} = x_{21}$ (sent during ua_2), $m_{33} = x_{31}$ (sent during ua_3), m_{23}, m_{32}, r_2, r_3 at random. On behalf of P_i $(i \in \{2, 3\})$ compute $(\mathbf{C}_i, e_i, d_i) \leftarrow \mathsf{Gb}(1^\kappa, C)$ using uniform randomness. Broadcast $\mathcal{D}_i = (\mathsf{pp}_i, \mathsf{epp}_i, c_i, \{c_{i\alpha}^b\}_{\alpha\in[6\ell],b\in\{0,1\}})$ where $c_i, \{c_{i\alpha}^{m_{i2}^\alpha}, c_{i(\ell+\alpha)}^{m_{i3}^\alpha}, c_{i(2\ell+\alpha)}^{r_2^\alpha}, c_{i(3\ell+\alpha)}^{r_3^\alpha}, c_{i(4\ell+\alpha)}^0, c_{i(4\ell+\alpha)}^1, c_{i(5\ell+\alpha)}^0, c_{i(5\ell+\alpha)}^1\}_{\alpha\in[\ell]}$ be computed as per the protocol. (If Naor-based eNICOM is used, c_i should be set to the specific commitment that suppports equivocation as per epp_i.) Let the remaining $\{c_{i\alpha}^b\}$ commit to dummy values.

R1 **a:** Send $\left(\{o_{2\alpha}^{m_{22}^\alpha}, o_{2(2\ell+\alpha)}^{r_2^\alpha}\}_{\alpha\in[\ell]}, m_{22}, r_2\right)$ to P_1^* on behalf of P_2. Send $\left(\{o_{3(\ell+\alpha)}^{m_{33}^\alpha}, o_{3(3\ell+\alpha)}^{r_3^\alpha}\}_{\alpha\in[\ell]}, m_{33}, r_3\right)$ privately to P_1^* on behalf of P_3.

R2 **a:** If $\mathsf{flag}_1 = 1$ for either P_2 or P_3, invoke $\mathcal{F}_{\mathsf{ua}}$ with $(\mathsf{sid}, \mathsf{Input}, \mathtt{abort})$ on behalf of P_1^*.

R2 **a:** Broadcast $\mathtt{abort}$ on behalf of P_i $(i \in \{2, 3\})$ if $\mathsf{flag}_1 = 1$ on behalf of P_i.

(b) $\mathcal{S}_{\mathsf{ua}}$ during $\mathsf{ua}_2, \mathsf{ua}$



R1 **A:**Set $p_{33} = x_{31}$ (sent during ua_3)

R1 **a:** Compute and broadcast $\mathcal{D}_3$ (using p_{33}) on behalf of P_3 according to the protocol. Send $\{s_3, p_{31}, p_{33}, o_3, \{o_{3\alpha}^b\}_{\alpha\in[6\ell],b\in\{0,1\}}\}$ to P_1^*.

R1 **a:** Choose x_{21} at random and send to P_1^* on behalf of P_2.

R1 **A:** Sample pp_2 to compute $(c_{21}, o_{21}) \leftarrow \mathsf{Com}(\mathsf{pp}_2, x_{21})$. Broadcast $\{\mathsf{pp}_2, c_{21}, c_{23}\}$ where c_{23} is commitment of dummy value and send o_{21} to P_1^* on behalf of P_2.

R1 **a:** Receive $\{s_1, p_{11}, p_{13}, o_1, \{o_{1\alpha}^b\}_{\alpha\in[6\ell],b\in\{0,1\}}\}$ from P_1^* on behalf of P_3. Do all the verifications as an honest P_3 would perform for P_1 and update flag_2 with respect to (wrt) P_3.

R1 **A:** Set $\mathsf{flag}_2 = 1$ on behalf of P_3 if $p_{11} \neq x_{13}$ (x_{13} received in ua_1)

R1 **a:** Set $\mathsf{flag}_2 = 1$ on behalf of P_2 if P_1^* sends encoded inputs inconsistent with $\mathcal{D}_1$

R1 **A:** Set $\mathsf{flag}_2 = 1$ on behalf of P_2 if $\mathsf{Open}(c_{12}, o_{12}) \neq x_{12}$ or $m_{11} \neq x_{12}$ (x_{12} received during ua_1).

– HYB_8: Same as HYB_7, except that when the execution does not result in P_1 getting access to the opening of commitment c_{32} (corresponding to x_{32}) broadcast by P_3 during ua_3, the commitment is replaced with commitment of dummy value.

– HYB_9: Same as HYB_8, except that when the execution ua_1 does not result in P_1 getting conflicting output on two garbled circuits, the set of ct is replaced by encryption of a dummy message.

Fig. 19: Description of $\mathcal{S}_{\mathsf{ua}}$

(a) $\mathcal{S}_{\mathsf{ua}}$ during $\mathsf{ua}_1, \mathsf{ua}$ (Contd.)

R2 **a:** If $\mathsf{flag}_1 = 0$ wrt P_i for exactly one $i \in \{2,3\}$, then act on behalf of P_i as per the protocol opening the garbled circuit (equivocate c_i to $\mathbf{C}_i$ in case of Naor-based eNICOM) and encoded input as per m_{23} or m_{32} accordingly chosen as above. Broadcast $\mathcal{W}_i$ using $z_i = r_i \oplus x_{1i}$ as per the protocol.

R2 **a:** If $\mathsf{flag}_1 = 0$ for both P_2 and P_3, invoke $\mathcal{F}_{\mathsf{ua}}$ with $(\mathsf{sid}, \mathsf{Input}, x_1)$ on behalf of P_1^* to obtain output y, where $x_1 = x_{12} \oplus x_{13}$. Let $z_2 = r_2 \oplus x_{12}$ and $z_3 = r_3 \oplus x_{13}$. For $(i \in \{2,3\})$, run $(\mathbf{C}'_i, \mathbf{X}_i, d_i) \leftarrow \mathcal{S}_{\mathsf{prv}}(1^\kappa, C, y, \{e_{i\alpha}^{m_{i2}^\alpha}, e_{i(\ell+\alpha)}^{m_{i3}^\alpha}, e_{i(2\ell+\alpha)}^{r_2^\alpha}, e_{i(3\ell+\alpha)}^{r_3^\alpha} e_{i(4\ell+\alpha)}^{z_2^\alpha}, e_{i(5\ell+\alpha)}^{z_3^\alpha}\}_{\alpha\in[\ell]})$. Using trapdoor t_i, compute $o_i = \mathsf{Equiv}(c_i, \mathbf{C}'_i, t_i)$. Send $\mathtt{OK}$ message privately to P_1^* on behalf of P_2, P_3 as per the protocol using computed o_2, o_3. Broadcast $\mathcal{W}_2, \mathcal{W}_3$ on behalf of P_2, P_3 as per protocol.

R2 **A:** If $\mathsf{flag}_1 \neq 1$ wrt P_i, send set of ct on behalf of P_i $(i \in \{2,3\})$ using a dummy message.

R2 **a:** Set $\mathsf{flag}_1 = 1$ on behalf of both P_2, P_3 if either (a) `abort` was sent or received via broadcast in Round 2 (b) P_1 broadcasts $z_2 \neq x_{12} \oplus r_2$ or $z_3 \neq x_{13} \oplus r_3$

(b) $\mathcal{S}_{\mathsf{ua}}$ during $\mathsf{ua}_2, \mathsf{ua}$ (Contd.)

R2 **a:** On behalf of P_3: If $\mathsf{flag}_2 = 0$ wrt P_3, choose random z_3 and broadcast $\mathcal{W}_3$ as per the protocol. Else broadcast `abort`.

R2 **a:** On behalf of P_2: If $\mathsf{flag}_2 = 0$ wrt P_2, broadcast z_1, z_3 where z_1 is computed as per the protocol as $z_1 = x_{21} \oplus r_1$, where x_{21} sent to P_1^* in Round 1 and r_1 received from P_1^*. z_3 is either same as chosen above (if $\mathsf{flag}_2 = 0$ wrt P_3) or random (if $\mathsf{flag}_2 = 1$ wrt P_3). Else broadcast `abort`.

R2 **a:** Set $\mathsf{flag}_2 = 0$ on behalf of both P_2, P_3 if (a) `abort` was sent or received via broadcast in Round 2 (b) P_1^* broadcasts anything other than $(z_1, o_{1(4\ell+\alpha)}^{z_1^\alpha})$ $(o_{1(4\ell+\alpha)}^{z_1^\alpha}$ known on behalf of $P_3)$ where $z_1 = x_{21} \oplus r_1$ $(r_1, x_{21}$ known to $P_2)$

$\mathcal{S}_{\mathsf{ua}}$ after $\mathsf{ua}_1, \mathsf{ua}_2, \mathsf{ua}_3$: If $\mathsf{flag}_i = 1$ (on behalf of both P_2, P_3) for any $i \in [3]$, invoke $\mathcal{F}_{\mathsf{ua}}$ with `abort` on behalf of P_1^*. Else invoke $\mathcal{F}_{\mathsf{ua}}$ with `continue` on behalf of P_1^*.

Since $\text{HYB}_9 := \text{IDEAL}_{\mathcal{F}_{\mathsf{ua}}, \mathcal{S}_{\mathsf{ua}}}$, we show that every two consecutive hybrids are computationally indistinguishable which concludes the proof.

$\text{HYB}_0 \overset{c}{\approx} \text{HYB}_1$: The difference between the hybrids is that P_2, P_3 in ua_1 use uniform randomness in HYB_1 rather than pseudorandomness as in HYB_0. The indistinguishability follows via reduction to the security of the PRG G.

$\text{HYB}_1 \overset{c}{\approx} \text{HYB}_2$: The difference between the hybrids is some of the commitments of encoded inputs which will not be sent to P_1 during ua_1 are replaced with commitment on dummy messages. The indistinguishability follows from the hiding property of NICOM.

$\text{HYB}_2 \overset{c}{\approx} \text{HYB}_3$: The difference between the hybrids is in the way $(\mathbf{C}_2, \mathbf{X}, d_2)$ is generated. In HYB_2, $(\mathbf{C}_2, e_2, d_2) \leftarrow \mathsf{Gb}(1^\kappa, C)$ is run, which gives

$(\mathbf{C}_2, \mathsf{En}(x, e), d_2)$. In HYB_3, it is generated as $(\mathbf{C}_2', d_2) \leftarrow \mathcal{S}_{\mathsf{prv}}(1^\kappa, C, y, \mathbf{X}_2 = \{e_{2\alpha}^{m_{22}^\alpha}, e_{2(\ell+\alpha)}^{m_{23}^\alpha}, e_{2(2\ell+\alpha)}^{r_2^\alpha}, e_{2(3\ell+\alpha)}^{r_3^\alpha}, e_{2(4\ell+\alpha)}^{z_2^\alpha}, e_{2(5\ell+\alpha)}^{z_3^\alpha}\}_{\alpha\in[\ell]})$. The commitment to the garbled circuit is later equivocated to $\mathbf{C}_2'$ using o_2 computed via $o_2 \leftarrow \mathsf{Equiv}(c_2, \mathbf{C}_2', t_2)$. The indistinguishability follows via reduction to the privacy of the garbling scheme and the hiding property of eCom.

$\text{HYB}_3 \overset{c}{\approx} \text{HYB}_4$: Similar argument as above with respect to $\mathbf{C}_3$.

$\text{HYB}_4 \overset{c}{\approx} \text{HYB}_5$: The difference between the hybrids is that in HYB_4, flag_2 is set to 1 if $\mathcal{W}_1$ broadcast by P_1 during ua_2 has (opening of) encoded input that is inconsistent with commitment corresponding to z_1 in $\mathcal{D}_1$, while in HYB_5, flag_2 is set to 1 if $\mathcal{W}_1$ broadcast by P_1 has (opening of) encoded input anything other than encoding of z_1 corresponding to $\mathbf{C}_1$. It follows from the binding property of NICOM that P_1 could have sent an encoded input not consistent with $\mathbf{C}_1$ but consistent with $\mathcal{D}_1$, only with negligible probability. Therefore, the hybrids are indistinguishable.

$\text{HYB}_5 \overset{c}{\approx} \text{HYB}_6$: Similar argument as above with respect to $\mathcal{W}_1$ broadcast by P_1 during ua_3.

$\text{HYB}_6 \overset{c}{\approx} \text{HYB}_7$: The difference between the hybrids is that when the execution does not result in P_1 getting access to the opening of commitment c_{23} (corresponding to x_{23}) broadcast by P_2 during ua_2, c_{23} corresponds to the actual input share x_{23} in HYB_8 while it corresponds to dummy value in HYB_9. The indistinguishability follows from the hiding property of NICOM Com.

$\text{HYB}_7 \overset{c}{\approx} \text{HYB}_8$: Similar argument as above with respect to commitment c_{32} broadcast by P_3 during ua_3.

$\text{HYB}_8 \overset{c}{\approx} \text{HYB}_9$: The difference between the hybrids is that when the execution ua_1 does not result in P_1 getting conflicting output on two garbled circuits, in HYB_8, the set of ct is the encryption of opening of shares of input while in HYB_9, it is replaced with encryption of dummy message. Assuming the encryption key is unknown to P_1 (holds except with negligible probability due to authenticity), indistinguishability follows from the CPA security of the encryption scheme.

F Proof of Security for Protocol god

In this section, we present the proof of security of god relative to the ideal functionality for guaranteed output delivery shown in Appendix B. For better clarity, we assume without loss of generality that P_1 is corrupt (denoted as P_1^*) and describe the simulator $\mathcal{S}_{\mathsf{god}}$. Since the roles of the parties are symmetric in god, similar proof would hold in case of corrupt P_2, P_3 as well. The simulator plays the role of the honest parties P_2, P_3 and simulates each step of the protocol god.

Similar to $\mathcal{S}_{\mathsf{ua}}$, we divide the description of $\mathcal{S}_{\mathsf{god}}$ as follows: We describe $\mathcal{S}_{\mathsf{god}}$ during god_1 where corrupt P_1^* is the evaluator and during god_2 when corrupt P_1^* acts as a garbler. The steps corresponding to god_3, would follow symmetrically from that described corresponding to god_2. We then describe the steps of the simulator $\mathcal{S}_{\mathsf{god}}$ corresponding to the third round. In the protocol god, the behavior of corrupt P_1 in Round 1, 2 determines his committed input. Hence, the privacy simulator can only be invoked earliest after the simulation of the first round. Similar to $\mathcal{S}_{\mathsf{ua}}$, since the commitments on

encoding information is sent in the first round itself, we use a modified version of the privacy simulator of the garbling scheme which additionally takes an encoded input as parameter (see Section E). The simulator $\mathcal{S}_{\mathsf{god}}$ appears in Figure 20-21 with **R1/R2/R3** indicating simulation for round 1, 2 and 3 and and **g/G** denoting the steps corresponding to subprotocol god_i, god respectively.

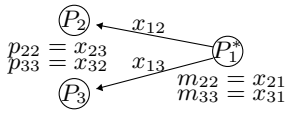
We now argue that $\text{IDEAL}_{\mathcal{F}_{\mathsf{god}},\mathcal{S}_{\mathsf{god}}} \overset{c}{\approx} \text{REAL}_{\mathsf{god},\mathcal{A}}$, when $\mathcal{A}$ corrupts P_1. The views are shown to be indistinguishable via a series of intermediate hybrids.

- HYB_0: Same as $\text{REAL}_{\mathsf{god},\mathcal{A}}$.
- HYB_1: Same as HYB_0, except that P_2, P_3 in god_1 use uniform randomness rather than pseudo-randomness for the garbled circuit construction.
- HYB_2: Same as HYB_1, except that some of the commitments that will not be opened by P_1 during god_1 are replaced with commitment on dummy values. Specifically, these are corresponding to indices not equal to $m_{22}, m_{23}, x_{12}, x_{13}$ for $\mathbf{C}_2$ and not equal to $m_{32}, m_{33}, x_{12}, x_{13}$ for $\mathbf{C}_3$.
- HYB_3 : Same as HYB_2, except that when the execution results in P_1 evaluating GCs during god_1, the GC $\mathbf{C}_2$ is created as $(\mathbf{C}_2', d_2) \leftarrow \mathcal{S}_{\mathsf{prv}}(1^\kappa, C, y, \mathbf{X}_2 = \{e_{2\alpha}^{m_{22}^\alpha}, e_{2(\ell+\alpha)}^{m_{23}^\alpha}, e_{2(2\ell+\alpha)}^{x_{12}^\alpha}, e_{2(3\ell+\alpha)}^{x_{13}^\alpha}\}_{\alpha\in[\ell]})$. The commitment c_2 is later equivocated to $\mathbf{C}_2'$ using o_2 computed via $o_2 \leftarrow \mathsf{Equiv}(c_2, \mathbf{C}_2', t_2)$. The set of ciphertexts ct generated uses d_2 in their keys.
- HYB_4 : Same as HYB_3, except that when the execution results in P_1 evaluating GCs during god_1, the GC $\mathbf{C}_3$ is created as $(\mathbf{C}_3', d_3) \leftarrow \mathcal{S}_{\mathsf{prv}}(1^\kappa, C, y, \mathbf{X}_3 = \{e_{3\alpha}^{m_{32}^\alpha}, e_{3(\ell+\alpha)}^{m_{33}^\alpha}, e_{3(2\ell+\alpha)}^{x_{12}^\alpha}, e_{3(3\ell+\alpha)}^{x_{13}^\alpha}\}_{\alpha\in[\ell]})$. The commitment c_3 is later equivocated to $\mathbf{C}_3'$ using o_3 computed via $o_3 \leftarrow \mathsf{Equiv}(c_3, \mathbf{C}_3', t_3)$. The set of ciphertexts ct generated uses d_3 in their keys.
- HYB_5: Same as HYB_4, except that during god_2, $\mathcal{C}_2$ is set to P_1 if P_2 receives o_3 that opens to a value other than the originally committed $\mathbf{C}_3$.
- HYB_6: Same as HYB_5, except that during god_3, $\mathcal{C}_3$ is set to P_1 if P_3 receives o_2 that opens to a value other than the originally committed $\mathbf{C}_2$.
- HYB_7: Same as HYB_6, except that during god_2, $\mathcal{C}_2$ is set to P_1 if P_2 accepts any encoded input not consistent with $\mathbf{C}_1, \mathbf{C}_3$
- HYB_8: Same as HYB_7, except that during god_3, $\mathcal{C}_3$ is set to P_1 if P_3 accepts any encoded input not consistent with $\mathbf{C}_1, \mathbf{C}_2$
- HYB_9: Same as HYB_8, except that when the execution does not result in P_1 getting access to the opening of commitment c_{23} (corresponding to x_{23}) broadcast by P_2 during god_2, the commitment is replaced with commitment of dummy value.
- HYB_{10}: Same as HYB_9, except that when the execution does not result in P_1 getting access to the opening of commitment c_{32} (corresponding to x_{32}) broadcast by P_3 during god_3, the commitment is replaced with commitment of dummy value.
- HYB_{11}: Same as HYB_{10}, except that when the execution god_1 does not result in P_1 getting conflicting output on two garbled circuits, the set of ct is replaced by encryption of a dummy message.

Since $\text{HYB}_{11} := \text{IDEAL}_{\mathcal{F}_{\mathsf{god}},\mathcal{S}_{\mathsf{god}}}$, we show that every two consecutive hybrids are computationally indistinguishable which concludes the proof.

Fig. 20: Description of $\mathcal{S}_{\mathsf{god}}$

(a) $\mathcal{S}_{\mathsf{god}}$ during $\mathsf{god}_1, \mathsf{god}$



R1 g: Receive x_{12}, x_{13} from P_1^* on behalf of P_2, P_3.

R1 G: Receive $(\mathsf{pp}_1, c_{12}, c_{13})$ via broadcast and (o_{12}, o_{13}) privately from P_1^* on behalf of P_2, P_3. Set $\mathcal{C}_i = \{P_1\}$ if $\mathsf{Open}(c_{1i}, o_{1i}) \neq x_{1i}$ for $i \in \{2, 3\}$

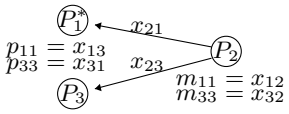
R1 g: Sample $\mathsf{epp}_2, \mathsf{epp}_3$ for eCom, having trapdoor t_2, t_3. Choose $m_{22} = x_{21}$ (sent during god_2), $m_{33} = x_{31}$ (sent during god_3), m_{23}, m_{32} at random. On behalf of P_i $(i \in \{2, 3\})$ compute $(\mathbf{C}_i, e_i, d_i) \leftarrow \mathsf{Gb}(1^\kappa, C)$ using uniform randomness. Broadcast $\mathcal{D}_i = (\mathsf{epp}_i, \mathsf{pp}_i, c_i, \{c^b_{i\alpha}\}_{\alpha\in[4\ell],b\in\{0,1\}})$ where c_i, $\{c^{m^\alpha_{i2}}_{i\alpha}, c^{m^\alpha_{i3}}_{i(\ell+\alpha)}, c^0_{i(2\ell+\alpha)}, c^1_{i(2\ell+\alpha)}, c^0_{i(3\ell+\alpha)}, c^1_{i(3\ell+\alpha)}\}_{\alpha\in[\ell]}$ be computed as as per the protocol. Let the remaining $\{c^b_{i\alpha}\}$ commit to dummy values. (For Naor-based eNICOM, c_i set to the specific commitment that suppports equivocation)

R2 g: If $P_1 \notin \mathcal{C}_2, \mathcal{C}_3$, invoke $\mathcal{F}_{\mathsf{god}}$ with $(\mathsf{sid}, \mathsf{Input}, x_1)$ on behalf of P_1^* to obtain output y, where $x_1 = x_{12} \oplus x_{13}$. For $(i \in \{2, 3\})$, run $(\mathbf{C}'_i, d_i) \leftarrow \mathcal{S}_{\mathsf{prv}}(1^\kappa, C, y, \mathbf{X}_i = \{e^{m^\alpha_{i2}}_{i\alpha}, e^{m^\alpha_{i3}}_{i(\ell+\alpha)}, e^{x^\alpha_{12}}_{i(2\ell+\alpha)}, e^{x^\alpha_{13}}_{i(3\ell+\alpha)}\}_{\alpha\in[\ell]})$. Using trapdoor t_i, compute $o_i = \mathsf{Equiv}(c_i, \mathbf{C}'_i, t_i)$. Send $\mathtt{OK}$ msg on behalf of P_2, P_3 as per the protocol using computed o_2, o_3.

R2 g: Else if $P_1 \notin \mathcal{C}_i$ for $i \in \{2, 3\}$, act on behalf of P_i as per the protocol opening the garbled circuit (equivocate c_i to $\mathbf{C}_i$ in case of Naor-based eNICOM) and encoded input as per m_{23} and m_{32}

R2 g: If $P_1 \notin \mathcal{C}_i$ $(i \in \{2, 3\})$, send set of ct on behalf of P_i using a dummy message.

(b) $\mathcal{S}_{\mathsf{god}}$ during $\mathsf{god}_2, \mathsf{god}$



R1 G: Set $p_{33} = x_{31}$ (sent during god_3) on behalf of P_3.

R1 g: Compute and broadcast $\mathcal{D}_3$ (using p_{33}) on behalf of P_3 and send private information to P_1^* as per protocol

R1 G: Compute $(c_{21}, o_{21}) \leftarrow \mathsf{Com}(\mathsf{pp}_2, x_{21})$ with randomly chosen x_{21}. Broadcast $\{\mathsf{pp}_2, c_{21}, c_{23}\}$ where c_{23} is commitment of dummy value

R1 g: Send $\{x_{21}, o_{21}\}$ to P_1^* on behalf of P_2.

R1 g: Do all the verifications wrt $\mathcal{D}_1$ as an honest P_3 would perform for P_1 and update $\mathcal{C}_3$.

R1 G: Add P_1 to $\mathcal{C}_2$ if $m_{11} \neq x_{12}$.

R2 g: Add P_1 to $\mathcal{C}_2$ if any of the openings sent by P_1 (for $\mathbf{C}_3$ or encoded inputs) is anything other than originally committed (known on behalf of P_3).

R2 g: If $P_1 \notin \mathcal{C}_2$ and $P_1 \in \mathcal{C}_3$: Extract P_1's input x_1 if committed: **(a)** either on clear with $\mathtt{nOK}$ **(b)** or in encoded form as $x_1 = m_{31} \oplus p_{31}$. Invoke $\mathcal{F}_{\mathsf{god}}$ with $(\mathsf{sid}, \mathsf{Input}, x_1)$ on behalf of P_1^* to obtain output y. Else, (P_1's input not committed) invoke $\mathcal{F}_{\mathsf{god}}$ with $(\mathsf{sid}, \mathsf{Input}, x_1)$ on behalf of P_1^* to obtain output y for default x_1.

$\text{HYB}_0 \overset{c}{\approx} \text{HYB}_1$: The difference between the hybrids is that P_2, P_3 in god_1 use uniform randomness in HYB_1 rather than pseudorandomness as in HYB_0. The indistinguishability follows via reduction to the security of the PRG G.

Fig. 21: Description of $\mathcal{S}_{\mathsf{god}}$ (contd.)

$\mathcal{S}_{\mathsf{god}}$ during **R3**: If $P_1 \in \mathcal{C}_2, \mathcal{C}_3$ at the end of round 1, invoke $\mathcal{F}_{\mathsf{god}}$ with $(\mathsf{sid}, \mathsf{Input}, x_1)$ on behalf of P_1^* to obtain y for a default x_1. Send y to P_1^* on behalf of both P_2 and P_3 if $P_1 \in \mathcal{C}_2, \mathcal{C}_3$ in the end of round one. Send y to P_1^* on behalf of only P_2 (P_3) if $P_1 \in \mathcal{C}_3$ ($\mathcal{C}_2$) in the end of round one.

$\text{HYB}_1 \overset{c}{\approx} \text{HYB}_2$: The difference between the hybrids is some of the commitments that will not be opened by P_1 during god_1 are replaced with commitments on dummy values. The indistinguishability follows from the hiding property of the commitment scheme.

$\text{HYB}_2 \overset{c}{\approx} \text{HYB}_3$: The difference between the hybrids is in the way $(\mathbf{C}_2, \mathbf{X}, d_2)$ is generated. In HYB_2, $(\mathbf{C}_2, e_2, d_2) \leftarrow \mathsf{Gb}(1^\kappa, C)$ is run, which gives $(\mathbf{C}_2, \mathsf{En}(x, e), d_2)$. In HYB_3, it is generated as $(\mathbf{C}_2', d_2) \leftarrow \mathcal{S}_{\mathsf{prv}}(1^\kappa, C, y, \mathbf{X}_2 = \{e_{2\alpha}^{m_{22}^\alpha}, e_{2(\ell+\alpha)}^{m_{23}^\alpha}, e_{2(2\ell+\alpha)}^{x_{12}^\alpha}, e_{2(3\ell+\alpha)}^{x_{13}^\alpha}\}_{\alpha\in[\ell]})$. The commitment to the garbled circuit is later equivocated to $\mathbf{C}_2'$ using o_2 computed via $o_2 \leftarrow \mathsf{Equiv}(c_2, \mathbf{C}_2', t_2)$. The indistinguishability follows via reduction to the privacy of the garbling scheme and the hiding property of eCom.

$\text{HYB}_3 \overset{c}{\approx} \text{HYB}_4$: Similar argument as above with respect to $\mathbf{C}_3$.

$\text{HYB}_4 \overset{c}{\approx} \text{HYB}_5$: The difference between the hybrids is that in HYB_4, P_2 sets $\mathcal{C}_2 = P_1$ if the o_3 sent by P_1 in god_2 output $\perp$ while in HYB_5, P_2 sets $\mathcal{C}_2 = P_1$ if o_3 sent by P_1 in god_2 opens to any value other than $\mathbf{C}_3$. Since the commitment scheme eCom is binding and epp was chosen uniformly at random by P_3, in HYB_4, P_1 could have decommitted successfully to a different garbled circuit than what was originally committed, only with negligible probability. Therefore, the hybrids are indistinguishable.

$\text{HYB}_5 \overset{c}{\approx} \text{HYB}_6$: Similar argument as above with respect to P_3 in god_3.

$\text{HYB}_6 \overset{c}{\approx} \text{HYB}_7$: The difference between the hybrids is that in HYB_6, P_2 sets $\mathcal{C}_2 = P_1$ if opening of commitment on the encoded inputs sent by P_1 in god_2 results in $\perp$ while in HYB_7, $\mathcal{C}_2$ is set to P_1 if P_2 accepts the opening of any commitment to a value other than what was originally committed. The indistinguishability between the hybrids follows from the binding property of NICOM.

$\text{HYB}_7 \overset{c}{\approx} \text{HYB}_8$: Similar argument as above with respect to P_3 in god_3.

$\text{HYB}_8 \overset{c}{\approx} \text{HYB}_9$: The difference between the hybrids is that when the execution does not result in P_1 getting access to the opening of commitment c_{23} (corresponding to x_{23}) broadcast by P_2 during god_2, c_{23} corresponds to the actual input share x_{23} in HYB_8 while it corresponds to dummy value in HYB_9. The indistinguishability follows from the hiding property of Com.

$\text{HYB}_9 \overset{c}{\approx} \text{HYB}_{10}$: Similar argument as above with respect to commitment c_{32} broadcast by P_3 during god_3.

$\text{HYB}_{10} \overset{c}{\approx} \text{HYB}_{11}$: The difference between the hybrids is that when the execution god_1 does not result in P_1 getting conflicting output on two garbled circuits, in HYB_{10}, the set

of ct is the encryption of an input and a share of input while in HYB_{11}, it is replaced with encryption of dummy message. Assuming the encryption key is unknown to P_1 (holds except with negligible probability due to authenticity), indistinguishability follows from the CPA security of the encryption scheme.

G Authenticated Conditional Disclosure of Secret

The subprotocol cert_i (Figure 2) used in our protocol fair is reminiscent of the notion of 'conditional disclosure of secrets (CDS)' which was first introduced in [GIKM00]. Informally, the problem of conditional disclosure of secrets involves two parties Alice and Bob, who hold inputs x and y respectively and wish to release a common secret s to Carol (who knows both x and y) if only if the input (x, y) satisfies some predefined predicate f. The model allows Alice and Bob to have access to shared random string (hidden from Carol) and the only communication allowed is a single unidirectional message sent from each player (Alice and Bob) to Carol. Traditionally, CDS involves two properties, namely *correctness* (if $f(x, y)$ is true, then Carol is always able to reconstruct s from her input and the messages she receives) and *privacy* (if $f(x, y)$ is false, Carol obtains no information about the secret s). Formally,

Definition 8 (Conditional Disclosure of Secret). *[AARV17] Let $f : \mathcal{X} \times \mathcal{Y} \to \{0, 1\}$ be a predicate. Let $F_1 : \mathcal{X} \times \mathcal{S} \times \mathcal{R} \to \mathcal{T}_1$ and $F_2 : \mathcal{Y} \times \mathcal{S} \times \mathcal{R} \to \mathcal{T}_2$ be deterministic encoding algorithms, where $\mathcal{S}$ is the secret domain. Then, the pair (F_1, F_2) is a CDS scheme for f if the function $F(x, y, s, r) = (F_1(x, s, r), F_2(y, s, r))$ that corresponds to the joint computation of F_1 and F_2 on a common s and r, satisfies the following:*

- *δ-correctness: There exists a deterministic algorithm Dec, called a decoder, such that for every 1-input (x, y) of f and any secret $s \in \mathcal{S}$, the following holds: $Pr_{r \leftarrow \mathcal{R}}[\mathsf{Dec}(x, y, F(x, y, s, r)) \neq s] \leq \delta$*
- *ϵ-privacy: There exists a simulator $\mathcal{S}$ such that for every 0-input (x, y) of f and any secret $s \in \mathcal{S}$, it holds that $|Pr[D(\mathcal{S}(x, y) = 1)] - Pr[D(F(x, y, s, r)) = 1]| \leq \epsilon$ for every distinguisher D. ($\mathcal{S}, D$ assumed to be poly-time or computationally unbounded depending on computational / information-theoretic setting).*

Interestingly, we find that the functionality realized by subprotocol cert_i subsumes the above properties under computational variant adapted to tolerate active corruption of single party and gives some stronger guarantees. We thus formally define a variant of CDS known as 'Authenticated Conditional Disclosure of Secret' below and show realization of the same by cert_i.

Definition 9 (Authenticated Conditional Disclosure of Secret). *Let A, B denote two parties holding inputs $x \in \mathcal{X}$ and $y \in \mathcal{Y}$ respectively and having access to common secret $s \in \mathcal{S}$ and C denote an external party. We assume a PPT adversary $\mathcal{A}$ who can actively corrupt at most 1 party among A, B and C. An authenticated CDS protocol is secure against $\mathcal{A}$ if the following properties hold:*

- *δ-correctness holds for honest A, B, and C where $\delta = \mathtt{negl}(\kappa)$.*
- *ϵ-privacy holds against $\mathcal{A}$ corrupt C, where $\epsilon = \mathtt{negl}(\kappa)$.*

– *Authenticity: For* 1*-input* (x, y) *of* f *and any secret* s*,* Dec *may result in* $\perp$ *when either* A *or* B *is corrupt, in which case* C *either identifies a corrupt party or a pair of parties in conflict that includes the corrupt party.*

Our cert_i gives an authenticated CDS as follows. The garblers P_j, P_k take the role of A and B and the evaluator takes the role of C. The common randomness r is the seed for the PRG used for generating the entire randomness for GC generation etc. The secret s is the key corresponding to 1 in the circuit. The predicate is the circuit that we garble in cert_i. While for the purpose of our 3-round fair protocol, the predicate is equality checking, in theory, we can garble any predicate. F_1 and F_2 are the codes of P_j and P_k respectively. Dec is the code that P_i executes. The correctness and privacy follow from the correctness and authenticity of the garbling scheme. The authenticity follows from the fact that P_i either receives the correct secret or detects a conflict or corrupt.