

Proofs of Replicated Storage Without Timing Assumptions

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Abstract

In this paper we provide a formal treatment of *proof of replicated storage*, a novel cryptographic primitive recently proposed in the context of a novel cryptocurrency, namely Filecoin.

In a nutshell, proofs of replicated storage is a solution to the following problem: A user stores a file m on n different servers to ensure that the file will be available even if some of the servers fail. Using proof of retrievability, the user could check that every server is indeed storing the file. However, what if the servers collude and, in order to save on resources, decide to only store one copy of the file? A proof of replicated storage guarantees that, unless the server is indeed storing n copies of the file, the user will not accept the proof.

While some candidate proofs of replicated storage have already been proposed, their soundness relies on timing assumptions i.e., the user must reject the proof if the prover does not reply within a certain time-bound.

In this paper we provide the first construction of a proof of replication which does not rely on any timing assumptions.

1 Introduction

Consider a scenario where a user A wants to store a file m in the cloud or on some other decentralized network of servers. To make sure the file is accessible to both A and other users, A stores several replicas of m in different locations. However, A suspects that the servers she is using are adversarial and may collude, for instance to save on costs by using less space than they are supposed to. So she will be interested in checking that indeed unique space has been dedicated to each replica, and it is natural to require that this can be verified, even if all servers are controlled by an adversary. We will call this *proof of replication*.

A first issue to note is that the well-known notions of proof of retrievability or proof of space (which we discuss in more detail below) do not solve the problem if each replica is simply a copy of m . Such proofs allow a user to check that a given file is retrievable from a server, much more efficiently than by simply retrieving the file. However, even if A asks for a proof of retrievability of m from each of the servers and all these proofs are successful, this may simply be because the user is actually talking to the adversary who stores only a single copy of m .

Another idea that comes to mind is that A could let each replica be an encryption of m under some key K , but with fresh randomness for each replica. If the encryption is IND-CPA secure, the adversary cannot distinguish this from encryptions of random independent messages, and hence it seems he is forced to store all replicas in order for them to be retrievable later. While this intuition can in fact be proved, this would not be a satisfactory solution: recall that we want that anyone,

not just A , can retrieve the original file, so A would have to share K with other users. However, if any of these users collude with the adversary, the security breaks down. Besides, a solution that does not require A to store secret information for later is clearly more practical.

The idea of proof of replication was introduced in Filecoin [Lab17a, Lab17b], a decentralized storage network. They articulate a list of properties that they desired from such a notion. They define a *Sybil attack* which is exactly what we discussed above: if an honest client wishes to store the same file m in n different servers, an adversary can store these using sybil identities (all servers are controlled by one adversary) and successfully pass the storage audit, while essentially storing only one copy of the file.

A decentralized storage network defined in [Lab17a], as an abstraction is a network of independent storage providers that offer verifiable file storage and retrieval services. In the Filecoin protocol, miners earn protocol tokens by providing data storage services. In such a scenario, it is crucial that any security model allow the adversary to choose the file m . This is because an adversary could request for m to be stored, and then prove that m was stored to collect network rewards. One may consider such an attack in a case where the client is honest and is fooled into storing a particular file by the adversary, or one can consider a corrupt user working with a set of corrupted servers. The latter is what is referred to as the *generation attack* in the Filecoin paper, where the adversary can simply determine m such that it can be efficiently regenerated on demand. In this work we consider the case where the client is honest.

While the Filecoin paper does not give a formal treatment of proof of replication, they propose a construction for what they call a time-bounded proof of replication. In such a notion, the file to be stored is encoded so that the encoding process is slow: slow enough for a client to distinguish between honest proving time, and potentially adversarial proving time which includes the time to re-encode. Thus, the encoding process is, by design, distinguishably more expensive than honest proving time. This notion is realized by using a block-cipher and slowing it down by block chaining. A time-bounded proof of replication is a proof of storage of a replica that is encoded in this way. Even if the proof of storage scheme used offers public verifiability, this time-bounded proof of replication is publicly verifiable only if the encoding key (or the original file) is made publicly available by the client, or by computing the encodings within a scheme that proves computational integrity and privacy, for instance, a SNARK (Succinct Non-interactive ARGuments of Knowledge). As discussed earlier, a solution that does not need to store any secret information is desirable, and using generic SNARKs would be expensive.

In any case, the basic problem with all time-bounded schemes is the handling of recomputing attack: the encoding has to be made so slow that even a powerful server cannot encode faster than the time a proof takes. This is harder than it may seem at first: even if we know for sure how many operations are needed to encode for a given value of security parameter, the actual time it takes depends on the hardware held by the adversarial parties, and so is beyond the control of honest users. This makes a concrete choice of parameters very difficult: should we compensate for the adversary being more powerful than we expect and choose a very slow encoding thus making life harder also for honest clients when they encode? Or should we choose parameters more aggressively and run a bigger risk of being cheated? It would clearly be better if we did not have to make such a choice.

We ask if we can do better in all the above aspects: can we have a proof of replication scheme that provably resists sybil attacks, offers public verifiability and is not time-bounded?

Our Results. We give a formal treatment of proofs of replication, by giving a definition that captures the desired properties as well as a construction which we prove secure according to the definition. The construction works in the random oracle model and requires a primitive we call a *hard encoding* which can be instantiated from any one-way permutation. Each replica of the file m to be stored in our construction has size $O(|m| + \kappa)$, where $|m|$ is the length of m and κ is the security parameter. To verify replication, the user conducts a proof of retrievability with each server. Any such proof can be used, so we inherit whatever communication complexity that proof has.

We concentrate on the case where the client doing the encoding is honest, as this seems to be the most important case in practice, and is in line with the definitions of proof of retrievability and storage. But we also give at the end of the paper a solution that works for a corrupt client, at the expense of using more communication.

Very roughly speaking, the idea in our solution is that the adversary first receives each of the replicas to store, where each replica is a special encoding of m . He may now store a state for later use, which in the honest case would contain all replicas. What we show is that, no matter how the adversary computes the state, if it is significantly smaller than the combined size of all replicas, then some of the proofs of retrievability will fail, unless the adversary breaks a computational assumption. What exactly does this guarantee us? Since the proofs of retrievability are *extractable*, the above guarantees us that the replicas cannot be compressed i.e., the adversary must reserve enough space to store all replicas, and this space must contain some data which is equivalent (up to polynomial time computation) to the replicas of the file. But is this good enough? Why don't we ask that the adversary in fact stores a concatenation of all the replicas? We argue that this goal is in fact impossible to achieve: even in practice, different systems will locally store the same information in different formats (think of *little-* vs. *big-endian* representation), and therefore, from a security point of view, it does not matter whether the adversary stores exactly the same data that was received from the client, or an equivalent representation of it, as long as the original data can be efficiently recovered.

Significantly, compared with previous proposed solutions to the problem, our solution does not require the use of *time*: while the original, informal definition of proof of replication states that it should be hard for the server to recompute the encodings of the file in the time it takes to verify the proof, our definition is much stronger as it rules out that any polynomially bounded attacker who uses less storage than claimed can pass the verification. As discussed above, this makes implementing proof of replications much easier, since one does not need to worry about finding an appropriate value for the verifier timeout.

To avoid misunderstandings, we emphasize that even if our definitions and proof work with a single adversary that handles all replicas, the actual use case includes several servers that each store one replica (if they are honest). Since it is clearly impossible to check if a server stores a replica without talking to that server, the communication complexity of our protocol must be proportional to the number of servers¹.

¹Of course, if a single server would store all replicas, we can optimize the communication needed, this is also easy to see for our protocol, but this hardly seems like an interesting use case.

1.1 Related Work

Proofs of retrievability. A lot of user data today is outsourced for storage on the cloud both because of large volumes of data, and for reliability in case of failure of local storage. The problem with cloud storage is that of maintaining integrity of data and enforcing accountability of the storage provider. Proofs of retrievability, first formalized by Juels and Kaliski in [JK07] address this problem by allowing for audits. In a proof of retrievability, a client can store a file on the server, while storing (a short) verification string locally. In an audit protocol, the client acts as the verifier and the server proves that it possesses the client’s file. The property that the server “possesses” a file is formalized by the existence of an extractor that retrieves the client’s file from a server that makes a client accept in the audit protocol. Since their introduction, there have been several works [SW08, DVW09] constructing proof of retrievability schemes with a proof of security and efficient audit procedures. One property we prioritize in this work is *public verifiability* where any party can take the role of the verifier in the audit protocol, not just the client who originally stored the file. This means the client’s state storing any verification information for the file should not contain any secrets. The construction of [SW08] gives a proof of retrievability scheme secure in the random oracle model that allows public verifiability.

Proofs of space. A proof of space is a protocol where a prover convinces a verifier that it has dedicated a significant amount of disk-space. Proofs of space were introduced in [DFKP15] as an alternative to proof of work (PoW), and further studied in [RD16, AAC⁺17]. There have been proposals based on proof of space like chia network [chi17] and Spacemint [PPK⁺15]. Very roughly, a proof of space gives the guarantee that it is more “expensive” for a malicious server that dedicates less space than an honest server to successfully pass an audit.

Data replication. Curtmola et al. [CKBA08] and Barsoum et al. [BH11] propose protocols that enable proofs of data replication in the private verifier model, where the client stores a secret key that is used for verification. The work of Hao and Yu [HY10] allows public verifiability but nevertheless requires the client to store a secret. Finally, the protocol of Armknecht et al. [ABBK16] is also in the private verification model, and in addition, uses RSA time-lock puzzles which results in a protocol with a time-bounded property that we elaborate on below.

Time-bounded Proofs of Replication. In a recent work by Pietrzak [Pie18], a construction for proof of replication based on proof of space is given. A proof of replication is not formally defined, and therefore it is not clear what is the replication property that the construction satisfies. In addition, since a proof of space is the starting point of the construction, it has the same “time-bounded” property as the Filecoin construction, since a malicious server can pass the audit by recomputing data. More recently, [FBBG18, BBBF18] construct proofs of replication based on slow encodings. They have the same time-bounded flavour of other recent works and is thus significantly different from ours.

1.2 Technical Overview

The existing time-bounded proofs use a public deterministic encoding function. The problem is that this always allow a malicious server to recompute encoded data and this may lead to a successful recomputation attack if the server has sufficient computational resources. Our observation

is that one can instead make the encoding be probabilistic. Now the adversary will only see the encoded data but not the randomness that the client used to encode. One may therefore hope that recomputing an encoding is not only slow, but completely unfeasible. On the other hand, decoding must still be easy for anyone.

To illustrate the idea of our solution, we start with a toy example: we assume that we are given oracle access to a random permutation T , and its inverse, acting on strings $\{0,1\}^n$. As is well known (and discussed in detail later) we can instantiate such an oracle in the standard random oracle model. In order to create replicas of a file, A will generate an instance of a one-way trapdoor permutation $f : \{0,1\}^n \mapsto \{0,1\}^n$, with trapdoor t_f . For simplicity, we assume that the file m to store is an $(n - \log n)$ -bit string. Then the i 'th replica is defined to be $(f, f^{-1}(T(m||i)))$, where $||$ denotes concatenation and f is a specification of the 1-way permutation. Clearly, anyone can easily compute m from a replica by computing f in the forward direction and calling T^{-1} . It turns out that this construction is secure if the adversary computes the state to store for later in a very restricted way, namely he forgets all information about at least one replica, say the i 'th one. Namely, the adversary forgets both the encoding $(f, f^{-1}(T(m||i)))$ and the intermediate value $T(m||i)$.

We can now argue that if the adversary is nevertheless able to produce the i 'th replica, he will have to invert the one-way permutation: from the output of the adversary $(f, f^{-1}(T(m||i)))$ we can (as the encoding can be decoded efficiently), extract $T(m||i)$. But, we assumed that the state did not contain any information about this value (except for a negligible amount following from the fact that it must be different from other outputs). Hence he must call the oracle to get $T(m||i)$. Therefore, in a security reduction, we can take a challenge value y and reprogram T such that $T(m||i) = y$. Now, the i 'th replica (that we assumed the adversary could produce) is exactly the preimage of y under f .

Of course, we cannot reasonably assume that the adversary behaves in this simple-minded way. As mentioned, we only want to assume that the state stored is smaller than the combined size of the replicas, say by a constant factor. To overcome this problem, we iterate the above construction several times, so that T is called several times while preparing a replica. Now there are many more outputs from T than the adversary can remember, and we show that by the setting the parameters right, at least one of these is almost uniform in the view of the adversary. Now we can place a challenge value for the one-way permutation in this position by an argument similar to the above.

To highlight exactly which properties of the one-way permutation we use, we introduce a notion we call a hard encoding. A hard encoding scheme `hardEnc` consists of algorithms $(\text{Gen}, \text{Enc}, \text{Dec})$, where $(\text{ek}, \text{dk}) \leftarrow \text{Gen}(1^\kappa)$ outputs an encoding and a decoding key; Enc takes a message $m \in \{0,1\}^*$ and the encoding key, and outputs an encoding, $c \leftarrow \text{Enc}(m, \text{ek})$; Dec takes an encoding, the decoding key and returns a message. Of course, encoding followed by decoding should return the correct message, but otherwise the main assumption is that computing the encoding of a given message should be hard unless you are given the encoding key. On the other hand, anyone can decode (in practice, the decoding key would be a part of the coded message).

This can trivially be instantiated from a trapdoor 1-way permutation by using the trapdoor as the encoding key. Note, however, that we actually do not need the full power of trapdoor 1-way permutations: in our application, we encode once and after this we only need decoding. So the encoding key can be deleted after use. In fact, we can go a step further: if we only want to encode a single message, we can collapse the Gen and Enc algorithms to a single one that takes the message m as input, and outputs a decoding key dk and an encoding c such that $\text{Dec}(c, \text{dk}) = m$. To

instantiate this we just need, very informally speaking, a kind of “punctured” function f that is one-way, but can be generated based on a given value m , such that one learns x with $f(x) = m$. This trivially generalizes to encoding of more than one message at once. We do not formalize this line of reasoning in this paper, nor do we know any instantiations other than 1-way permutation. We just mention this as a hint that our scheme might be instantiable from a weaker assumption and leave this as an interesting open problem.

2 Preliminaries

Notation. We denote the concatenation of two bit strings x and y by $x||y$. Throughout, we use κ to denote the security parameter. We denote a probabilistic polynomial time algorithm by PPT. A function is negligible if for all large enough values of the input, it is smaller than the inverse of any polynomial. We use negl to denote a negligible function. We use $[1, n]$ to represent the set of numbers $\{1, 2, \dots, n\}$. For a randomized algorithm Alg , we use $y \leftarrow \text{Alg}(x)$ to denote that y is the output of Alg on x . We write $y \xleftarrow{R} \mathcal{Y}$ to mean sampling a value y uniformly from the set \mathcal{Y} .

2.1 Trapdoor permutations

A collection of trapdoor permutations is a family $\mathcal{F} = \{f_i : \mathcal{D} \rightarrow \mathcal{R}\}$ such that:

- $\forall i, f_i$ is a permutation.
- There exists a PPT algorithm KeyGen such that $(i, t) \leftarrow \text{KeyGen}(1^\kappa)$
- There exists a PPT algorithm that given $i \in \mathcal{I}$ samples uniformly from \mathcal{D}_i .
- There exists a PPT algorithm that on input $i \in \mathcal{I}, x \in \mathcal{D}_i$, computes $f_i(x)$.
- For all PPT algorithms A , there exists a negligible function negl such that

$$\Pr[f_i(z) = y : i \leftarrow \mathcal{I}, x \leftarrow \mathcal{D}_i, y \leftarrow f_i(x), z \leftarrow A(i, y)] \leq \text{negl}(\kappa)$$

- There exists a PPT algorithm B such that $B(i, t, f_i(x)) = x$.

We call the above family a B -leakage trapdoor permutation if the following holds: For all PPT algorithms A , and all functions $L : \mathcal{D} \rightarrow \{0, 1\}^B$, there exists a negligible function negl such that

$$\Pr[f_i(z) = y : i \leftarrow \mathcal{I}, L \leftarrow A(i), x \leftarrow \mathcal{D}_i, y \leftarrow f_i(x), z \leftarrow A(y, L(x))] \leq \text{negl}(\kappa)$$

Note that every trapdoor function family is also a B -leakage trapdoor permutation family for $B = \log \kappa$.

RSA trapdoor permutation. The RSA trapdoor permutation is given by:

- $\text{KeyGen}(1^\kappa)$: Choose κ -bit primes p, q , let $N = pq$. Choose e such that $\text{gcd}(e, (p-1)(q-1)) = 1$, let d be such that $ed = 1 \pmod{(p-1)(q-1)}$. Return $(pk = (e, N), sk = d)$
- For $x \in \mathbb{Z}_N^*$, given $pk = (e, N)$, compute $f_{pk}(x) = y = x^e \pmod{N}$.

- For $y \in \mathbb{Z}_N^*$, and $sk = d$, compute $f_{sk}^{-1}(y) = y^d \bmod N$

Define the advantage of \mathcal{A} as

$$\text{Adv}_{\mathcal{A}}^{rsa} = \Pr[f_{pk}(z) = y : (pk, sk) \leftarrow \text{KeyGen}(1^\kappa), x \xleftarrow{R} \mathbb{Z}_N^*, z \leftarrow \mathcal{A}(pk, f_{pk}(x))]$$

Definition 1. *The RSA inversion problem is said to be hard if for any \mathcal{A} running in time polynomial in κ , the $\text{Adv}_{\mathcal{A}}^{rsa}$ is negligible; there exists a negligible function negl such that*

$$\text{Adv}_{\mathcal{A}}^{rsa} \leq \text{negl}(\kappa)$$

Define the leakage-advantage of \mathcal{A} as

$$\text{Adv}_{\mathcal{A}}^{\text{L-rsa}} = \Pr[f_{pk}(z) = y : (pk, sk) \leftarrow \text{KeyGen}(1^\kappa), L \leftarrow \mathcal{A}(pk), x \xleftarrow{R} \mathbb{Z}_N^*, \\ z \leftarrow \mathcal{A}(pk, f_{pk}(x), L(x))]$$

Definition 2. *The RSA inversion is said to be B -leakage hard if for any \mathcal{A} running in time polynomial in κ , any function $L : \mathbb{Z}_N^* \rightarrow \{0, 1\}^B$, the $\text{Adv}_{\mathcal{A}}^{\text{L-rsa}}$ is negligible; there exists a negligible function negl such that*

$$\text{Adv}_{\mathcal{A}}^{\text{L-rsa}} \leq \text{negl}(\kappa)$$

2.2 Proof of retrievability

Proofs of retrievability, introduced by Juels and Kaliski [JK07] allow a client to store data on a server that is untrusted, and admit an *audit* protocol in which the server proves to the client that it is still storing all of the data. A scheme without random oracle was given in [DVW09], whereas [SW08] allows public verifiability. A proof of retrievability (PoR) scheme consists of three algorithms, $\text{Gen}, \mathcal{P}, \mathcal{V}$. We recall the definition from [SW08, DVW09] below.

- The generation algorithm takes as input a file $F \in \{0, 1\}^*$ and outputs a file to be stored on the server and a tag (verification information) for the client.

$$(F^*, \tau) \leftarrow \text{Gen}(F)$$

- The \mathcal{P}, \mathcal{V} algorithms define an audit protocol to prove retrievability of the file. The \mathcal{P} algorithm takes as input the processed file F^* and the \mathcal{V} algorithm takes the tag τ . At the end of the audit protocol, the verifier outputs a bit indicating whether the proof succeeds or not.

$$\{0, 1\} \leftarrow \langle \mathcal{P}(F^*), \mathcal{V}(\tau) \rangle$$

A PoR scheme needs to satisfy correctness and soundness. Correctness requires that for all file $F \in \{0, 1\}^*$, and for all (F^*, τ) output by $\text{Gen}(F)$, an honest prover will make the verifier accept in the audit protocol.

$$\langle \mathcal{P}(F^*), \mathcal{V}(\tau) \rangle = 1$$

Informally, a PoR scheme is sound if for any prover that convinces the verifier that it is storing the file, there exists an algorithm called the extractor that interacts with the prover and extracts the file. We give the formal definition below.

Experiment $\text{Expt}_{\mathcal{A}}^{\text{PoR-sound}}(\kappa)$

- The adversary \mathcal{A} picks a file $F \in \{0, 1\}^n$.
- The challenger creates $(F^*, \tau) \leftarrow \text{Gen}(F)$ and returns F^* to \mathcal{A} .
- \mathcal{A} can interact with $\mathcal{V}(\tau)$ by running many proofs and seeing whether \mathcal{V} outputs 0 or 1.
- \mathcal{A} outputs a prover algorithm (ITM) \mathcal{P}^* and returns this to the challenger.
- The challenger runs $b \leftarrow \langle \mathcal{P}^*, \mathcal{V}(\tau) \rangle$, and runs the extractor, $\tilde{F} = \text{ext}^{\mathcal{P}^*}(\tau, n, \kappa)$
- Output 1 if $b = 1 \wedge \tilde{F} \neq F$, or 0 otherwise.

Figure 1: Soundness for Proofs of Retrievability.

Definition 3 (Soundness for Proof of Retrievability). *A proof of retrievability (PoR) $\text{Gen}, \mathcal{P}, \mathcal{V}$ satisfies soundness if for any PPT adversary \mathcal{A} , there exists an extractor ext such that the advantage of \mathcal{A}*

$$\text{Adv}_{\mathcal{A}}^{\text{PoR-Sound}}(\kappa) = \Pr[\text{Expt}_{\mathcal{A}}^{\text{PoR-Sound}}(\kappa) = 1]$$

in the experiment described in Figure 1 is negligible in κ .

The definition in [DVW09] discusses the notion of knowledge soundness versus information soundness. If the definition holds for the class of efficient extractors, the scheme satisfies knowledge soundness. A somewhat weaker notion is that of information soundness where the running time of the extractor is not restricted.

2.3 Min entropy

Recall that the predictability of a random variable X is $\max_x \Pr[X = x]$ and its min-entropy $H_{\infty}(X)$ is $-\log(\max_x \Pr[X = x])$. The average case min-entropy is defined as follows. Let X and Y be random variables.

$$\tilde{H}_{\infty}(X|Y) = -\log\left(\mathbb{E}_{y \leftarrow Y}\left(2^{-H_{\infty}(X|Y=y)}\right)\right)$$

We make use of the following lemma which states that the average min-entropy of a variable (from the point of view of an adversary) does not go down by more than the number of bits (correlated with the variable) observed by the adversary. We recall the entropy weak chain rule for average case min entropy below in Lemma 1.

Lemma 1. ([DORS08]) *Let X and Y be random variables. If Y has at most 2^{λ} values, then*

$$\tilde{H}_{\infty}(X|Y) \geq H_{\infty}(X) - H_0(Y) = H_{\infty}(X) - \lambda$$

where $H_0(Y) = \log|\text{support}(Y)|$

3 Defining Proof of Replication

While several candidates of proof of replication have already been proposed, we are not aware of any formal definition of the security properties that such a proof should satisfy. It is indeed non-trivial to come up with the “right” definition, due to the fact that we ask the adversary to store many copies *of the same file*. Thus simply requiring the existence of an extractor algorithm (as in proof of knowledge or proof of storage) is not sufficient: it is not enough that the adversary knows the file, the adversary should know multiple replicas of the same file. But what does it mean for an extractor to extract replicas of the same file? Before providing our definition, we introduce some notions of encodings which will be used to build up our solution.

3.1 Hard Encoding

We introduce the notion of a hard encoding which is useful in the context of replicated storage. A hard encoding encodes a message such that the decoding procedure is easy to compute for everyone, but the encoding procedure is only easy if one has access to the encoding key. Hard encodings can be realized using trapdoor permutations, where the secret key of the permutation is erased after computing the encoding. A hard encoding scheme `hardEnc` consists of algorithms (Enc, Dec) , where $(\text{ek}, \text{dk}) \leftarrow \text{Gen}(1^\kappa)$ outputs an encoding and a decoding key; `Enc` takes a message $m \in \{0, 1\}^*$ and the encoding key, and outputs an encoding, $c \leftarrow \text{Enc}(m, \text{ek})$; `Dec` takes an encoding, the decoding key and returns a message. For the sake of concreteness, throughout the paper, we use the RSA trapdoor permutation to implement hard encodings.

- `Gen`: $((e, N), d) \leftarrow \text{KeyGen}(1^\kappa)$ Set $\text{ek} = d, \text{dk} = (e, N)$
- `Enc`: For a message $m \in \mathbb{Z}_N^*$, given ek , compute $c = m^d \bmod N$
- `Dec`: Given $c \in \mathbb{Z}_N^*$, and dk , output $m = c^e \bmod N$

3.2 Replica Encodings

We now define `ReplicaEncoding` as a tuple of algorithms $(\text{rEnc}, \text{rDec})$ where `rEnc` takes a message $m \in \{0, 1\}^*$ and outputs a replica encoding of $m \in \{0, 1\}^*$,

$$y \leftarrow \text{rEnc}(\kappa, m)$$

The `rDec` algorithm takes a replica encoding and returns a message i.e., $m \leftarrow \text{rDec}(y)$.

Definition 4 (Replica encoding). *A pair $(\text{rEnc}, \text{rDec})$ is a secure replica encoding if the following holds:*

- *Completeness: The probability of incorrect decoding is negligible i.e.,*

$$\Pr[\text{rDec}(\text{rEnc}(\kappa, m)) \neq m] < \text{negl}(\kappa)$$

- *Soundness: Consider the game $\text{sound}_{\mathcal{A}_1, \mathcal{A}_2}$ between an adversary and a challenger defined in Figure 2. A replica encoding scheme is c -sound (for a constant $c, 0 < c < 1$) if for any $(\mathcal{A}_1, \mathcal{A}_2)$, there exists a negligible function negl such that the following holds.*

$$\Pr[|\text{state}| < cvk | v \leftarrow \text{sound}_{\mathcal{A}_1, \mathcal{A}_2}] \leq \text{negl}(\kappa)$$

Experiment $\text{sound}_{\mathcal{A}_1, \mathcal{A}_2}$

- The adversary \mathcal{A}_1 chooses a file $m \in \{0, 1\}^k$
- The challenger outputs n encodings of m

$$y^{(i)} \leftarrow \text{rEnc}(\kappa, m)$$

for $i \in [1, n]$ and returns $(y^{(1)}, \dots, y^{(n)})$ to \mathcal{A}_1 .

- \mathcal{A}_1 outputs a state $\text{state} \leftarrow \mathcal{A}_1(y^{(1)}, \dots, y^{(n)})$
- The challenger runs \mathcal{A}_2 on state.

$$(\tilde{y}^{(1)}, \dots, \tilde{y}^{(n)}) \leftarrow \mathcal{A}_2(\kappa, \text{state})$$

- Let $v_i = 1$ if $\tilde{y}^{(i)} = y^{(i)}$, and 0 otherwise. Output $v = \sum_{i=1}^n v_i$.

Figure 2: Soundness of a Replica Encoding scheme

3.3 Proof of Replication

We now use the notion of encodings to meaningfully capture the replication property. A proof of replication scheme consists of a tuple of algorithms `create`, `retrieve` and an audit protocol defined by two algorithms, \mathcal{P}, \mathcal{V} for the prover and verifier respectively. `create` is a randomized algorithm that takes as input a file $m \in \{0, 1\}^*$, that is to be replicated and stored, a replication factor n ; and produces n replicas $y^{(1)}, \dots, y^{(n)}$ together with verification information `ver`. Each replica $y^{(i)}$ is sent the server i to be stored, and `ver` with the client to be used for verification in the audit protocol. `retrieve` is a deterministic algorithm run by anyone that takes as input a replica $y^{(i)}$ and outputs a file m^* .

In the audit protocol, each server (prover) has a replica $y^{(i)}$, and the client (verifier) has `ver`. At the end of the audit, the verifier outputs a bit b indicating whether the audit was successful or not. We denote the protocol executing the prover and verifier algorithms by $\langle \mathcal{P}_i(\tilde{y}^{(i)}), \mathcal{V}(\text{ver}, i) \rangle$.

We require the scheme to satisfy completeness and soundness properties as defined below. Note that when considering the honest usage of the our protocol (e.g., completeness), each server is able to prove to the client that they are storing the file independently². On the other hand, when considering adversarial behaviour (e.g., soundness), we assume that all servers are under the control of a monolithic adversary.

All our algorithms below are parametrized by a security parameter, even when omitted as in the description below.

²For instance, an honest server does not need to communicate with the other servers, nor know that they exist.

Experiment $\text{sound}_{\mathcal{A}_1, \mathcal{A}_2}^{\mathcal{E}}$

- The adversary \mathcal{A}_1 chooses a file $m \in \{0, 1\}^k$
- The challenger runs $(y^{(1)}, \dots, y^{(n)}, \text{ver}) \leftarrow \text{create}(m, n)$ and returns $(y^{(1)}, \dots, y^{(n)})$ to \mathcal{A}_1 .
- \mathcal{A}_1 outputs a state $\text{state} \leftarrow \mathcal{A}_1(y^{(1)}, \dots, y^{(n)})$
- The challenger runs $\langle \mathcal{A}_2(\text{state}), \mathcal{V}(\text{ver}, i) \rangle$, let v_i be the output of \mathcal{V} for all $i \in [1, n]$ and $v = \sum_{i=1}^n v_i$.
- The challenger runs the extractor.

$$(\tilde{y}^{(1)}, \dots, \tilde{y}^{(n)}) = \mathcal{E}^{\mathcal{A}_2}(\kappa, \text{ver}, k)$$

- For all $i \in [1, n]$, define $u_i = 1$ if $\tilde{y}^{(i)} = y^{(i)}$, and $u = \sum_{i=1}^n u_i$
- The output of the game $\text{sound}_{\mathcal{A}_1, \mathcal{A}_2}^{\mathcal{E}}$ is (u, v) .

Figure 3: Soundness of a proof of replication scheme

Definition 5 (Proof of Replication). *A scheme $\text{PoRep} = (\text{create}, \text{retrieve}, \mathcal{P}, \mathcal{V})$ where,*

$$(y^{(1)}, \dots, y^{(n)}, \text{ver}) \leftarrow \text{create}(m, n), \text{ for } m \in \{0, 1\}^*, n \in \mathbb{Z}$$

$$m_i^* = \text{retrieve}(y^{(i)}), i \in [1, n]$$

$$\{0, 1\} \leftarrow \langle \mathcal{P}_i(\tilde{y}^{(i)}), \mathcal{V}(\text{ver}, i) \rangle$$

is a proof of replication scheme if the following properties are satisfied.

- *Completeness. For an honest client and honest server,*
 - *for $(y^{(1)}, \dots, y^{(n)}, \text{ver}) \leftarrow \text{create}(m, n), m_i^* = \text{retrieve}(y^{(i)}), m_i^* = m \forall i \in [1, n]$*
 - *The audit protocol interaction between honest client and honest server succeeds, that is, the client accepts and outputs $b = 1$.*

$$\langle \mathcal{P}_i(\tilde{y}^{(i)}), \mathcal{V}(\text{ver}, i) \rangle = 1$$

- *Soundness. We define the soundness game $\text{sound}_{\mathcal{A}_1, \mathcal{A}_2}^{\mathcal{E}}$ between an adversary and a challenger in Figure 3. The scheme PoRep is c -sound (for a constant $c, 0 < c < 1$) if for any $(\mathcal{A}_1, \mathcal{A}_2)$, there exists an extractor \mathcal{E} and a negligible function negl such that the following holds.*

$$\Pr [u < v \vee |\text{state}| < cvk \mid (u, v) \leftarrow \text{sound}_{\mathcal{A}_1, \mathcal{A}_2}^{\mathcal{E}}] \leq \text{negl}(\kappa)$$

The definition above guarantees that the malicious servers, even colluding, cannot make the verifier accept more proofs than the storage they have used.

4 Constructing Proof of Replication

We begin by giving a high-level overview of our construction. Following the idea behind our definition, we create many independent encodings, and use a proof of retrievability on the encodings. Even though each encoding can independently be decoded to the same file without any secret information, the proof of retrievability on the encodings enforces that the server stores each encoding and therefore dedicates space for *each* replica. Recall that in Section 1.2 we have already described a simple solution which works in a restricted model in which the adversary is restricted to either store or delete entire replicas. Of course this is not a realistic threat model and a malicious server could choose to forget arbitrary parts of each encoding (say, a constant fraction). Now, to pass the audit, the server would have to compute a preimage of the underlying trapdoor permutation, but *given* a constant fraction of bits of the preimage. Unfortunately, the definition of security for trapdoor permutation does not allow us to say that this is not possible; in other words, we cannot construct a reduction for this kind of adversaries.

To address this problem, we use the following approach: we start by applying a random permutation on the message concatenated with a short seed (which is different for each replica), and then we use the hard encoding on the result. We then iterate the permutation plus encoding as a round function sufficiently many times. Intuitively the trapdoor permutation of the round function ensures that the adversary has to do something “hard” in every round, the random permutation of the round function is used to make sure that the “hard tasks” are all independent.

Again, in any given round, we cannot rule out that the adversary might have stored some (small) information that allows to easily invert the trapdoor permutation. However, since we repeat this for many rounds and the adversary must store some preimage information at every round to potentially break the trapdoor permutation, eventually the total information that the adversary would have to store will exceed the bound necessary for replicated storage.

Note that our combination of the RSA trapdoor permutation with a random oracle is reminiscent of full domain hash-signatures and, to a greater extent, CCA secure encryption via RSA and OAEP. Note however that, for our proof to go through, we need T to be indistinguishable from a random oracle, thus the two Feistel rounds of OAEP are not enough. Moreover, in our construction, we apply the oracle and the trapdoor permutation for multiple rounds, and the domain of the random oracle is a vector of blocks for the RSA permutation. The idea of iterating a combination of RSA with a random oracle was used before in [VDJO⁺12], however (apart from their work having a less in-depth treatment) there are two major differences, namely that they did not consider replication as an application, and that they use a strictly weaker notion of security, namely “near-incompressibility”.

4.1 Replica Encodings

We now proceed to describe our construction in detail, and first construct a replica encoding scheme $\text{ReplicaEncoding} = (\text{rEnc}, \text{rDec})$ in Figures 4 and 5.

Soundness of the scheme. Before formally proving the soundness of the scheme, we give an overview of the proof idea. If the state `state` that is passed from \mathcal{A}_1 to \mathcal{A}_2 is small, then the adversary \mathcal{A}_2 cannot “remember” all the answers to the queries that \mathcal{A}_1 made in the first part of the game since the outputs of a random oracle are incompressible. However, we can extract the outputs of the random oracle used during the encoding from the adversary since the replica encodings can be efficiently decoded. This implies that the adversary must make some queries to

Let $m \in \{0, 1\}^{k'}$ be a message to be encoded.

- Choose a string r^i uniformly at random from $\{0, 1\}^\kappa$, and let $y_0^{(i)} = m || r^i$. Let $k = k' + \kappa$.
- Let $\text{hardEnc} = (\text{Gen}, \text{Enc}, \text{Dec})$ be a hard encoding scheme. $(\text{ek}^{(i)}, \text{dk}^{(i)}) = \text{hardEnc.Gen}(\kappa)$. Divide $y_0^{(i)}$ into s blocks such that each block is in \mathbb{Z}_N , where $\text{dk}^{(i)} = (e, N)$. Let $T_N : \mathcal{D} \rightarrow \mathcal{D}$ be viewed as an invertible RO, where the domain and range \mathcal{D} is $(\mathbb{Z}_N)^s$. Iterate the following round function: For each round j from 1 to r , define
 - Apply the RO T_N ,

$$z_j = T_N(y_{j-1}^{(i)})$$
 - Parse z_j as $Z_{1j} || \dots || Z_{sj}$ where each $Z_{tj} \in \mathbb{Z}_N$. Apply the hard encoding block-wise. For each $t \in [1, s]$

$$Y_{tj}^{(i)} = \text{hardEnc.Enc}(Z_{tj}, \text{ek}^{(i)})$$
 - Let $y_j^{(i)} = Y_{1j}^{(i)} || \dots || Y_{sj}^{(i)}$
- Let $R^{(i)} = (y^{(i)}, \text{dk}^{(i)})$, where $y^{(i)} = y_r^{(i)}$
- Return $R^{(i)}$

Figure 4: The Replica Encoding Algorithm $\text{rEnc}(\kappa, m)$

For a replica $R^{(i)} = (y^{(i)}, \text{dk}^{(i)})$, let $y_r = y^{(i)}$, $(e, N) = \text{dk} = \text{dk}^{(i)}$. For each round j from r down to 1, compute

- Round j :
 - Parse $y_j^{(i)}$ as $Y_{1j}^{(i)} || \dots || Y_{sj}^{(i)}$. Decode the hard encoding block-wise, for each $t \in [1, s]$

$$Z_{tj} = \text{hardEnc.Dec}(Y_{tj}^{(i)}, \text{dk})$$
 - Let $z_j = Z_{1j} || \dots || Z_{sj}$

$$y_{j-1}^{(i)} = T_N^{-1}(z_j)$$
- Parse $y_0 = m || r$ where m is the first k' bits of y_0 . Return m .

Figure 5: The replica decoding algorithm $\text{rDec}(R^{(i)})$

the random oracle in phase 2 of the game. Now, for each of the queries the adversary makes, there are two options: either the response to the query has full entropy in the view of the adversary, or it doesn't. If it has full entropy, i.e., if the state contained no (or very little) information about what the oracle would answer, then we are done, as we will elaborate next. But first let us consider if not; that is, the response to the query made did not have full entropy in \mathcal{A}_2 's view. This means that the state must have contained some information about the answer to the query. Now, since the encoding uses the random oracle in each round, and since the state that the adversary is allowed to remember is small, by carefully accounting for the entropy budget for each query made, we argue that after a certain round, the entropy in the state is exhausted and therefore there is at least one query that the adversary had to make to the oracle whose response has full entropy in order to win the game. Finally, once we have found such a query for which we know that the output of the oracle has full entropy that \mathcal{A}_2 had to make to win the game, we can reprogram the random oracle with a challenge for the trapdoor permutation. Thus since \mathcal{A}_2 is nevertheless able to produce the replicas, we use it to break the assumption and reach a contradiction.

Theorem 1. *Assuming T is an invertible random oracle, the construction `ReplicaEncoding` = `(rEnc, rDec)` is a secure replica encoding scheme for message length k' , replication parameter n as per Definition 4. For number of rounds $r > \frac{(cn+1)k}{B}$, it is complete and c -sound with soundness error $\epsilon \leq (\epsilon' + 2^{-nk(1-c)})nr$ where $k = k' + \kappa$, the advantage of any adversary in B -leakage inversion of the RSA permutation as per Definition 2 is at most ϵ' .*

Proof.

Completeness. For n encodings that are created honestly, $R^{(i)} \leftarrow \text{rEnc}(m, \kappa)$, $m^* = \text{rDec}(R^{(i)})$. Since T is a permutation and the hard encoding is the RSA permutation, $m^* = m, \forall i$.

Soundness. Assume there exists an adversary $(\mathcal{A}_1, \mathcal{A}_2)$ such that

$$\Pr[|\text{state}| < cvk | v \leftarrow \text{sound}_{\mathcal{A}_1, \mathcal{A}_2}] > \epsilon .$$

Therefore, the adversary \mathcal{A}_2^T outputs $R^{(i_1)}, \dots, R^{(i_v)}$, where each $i_j \in [1, n]$. Let $I \subset [1, n], |I| = v$ be the set of indices indicating the replicas that \mathcal{A}_2 outputs correctly. We argue that if the state is too small, it does not have enough entropy to store information about $R^{(i)}, \forall i \in I$ and therefore one of the $R^{(i)}$ must have been recomputed, (by making relevant queries to T), which we use to invert the B -leakage trapdoor permutation of the hard encoding. The proof idea is that since the state is small, in round r , \mathcal{A}_2 must have learned some of the z values of round r from responses of T . If the response of T to these queries do not have full entropy in \mathcal{A}_2 's view given the state, then this deficit must be accounted for in the size of the state. We continue this argument for every round going backwards from the last round, by reasoning about the set of relevant queries made in each round, and accounting for every query made that did not have entropy in \mathcal{A}_2 's view with B bits in the state. We then hit a round where the response of T for one of \mathcal{A}_2 's queries must have high enough entropy from \mathcal{A}_2 's point of view. Such a query is guaranteed to exist since the state size is used up after enough queries of the former kind. We use the response to this query to embed a challenge and invert the trapdoor permutation. We now proceed to give the reduction.

Let \mathcal{B} be an adversary whose task is to invert the trapdoor permutation of the underlying `hardEnc` scheme. \mathcal{B} receives (\hat{e}, \hat{N}) , a challenge \hat{x} , and wins if it outputs \hat{y} such that $\hat{y}^{\hat{e}} = \hat{x} \bmod \hat{N}$.

\mathcal{B} interacts with $(\mathcal{A}_1, \mathcal{A}_2)$ in the soundness game. \mathcal{B} receives a file $m \in \{0, 1\}^{k'}$ from \mathcal{A}_1 . \mathcal{B} creates encoded replicas honestly, except for the following. It chooses a random $i^* \in [1, n]$, and creates replica i^* in the following way. It chooses a random value y and computes the trapdoor permutation in the forward direction with the challenge key (\hat{e}, \hat{N}) for each block in y to obtain z . It then programs the oracle T such that $T(m) = z$. This is repeated for each round $j \in [1, r]$. That is, \mathcal{B} defines the i^* th replica in rEnc as follows.

- For $i = i^*$, let $y_0^{(i)}$ be a uniformly random string of length $k = k' + \kappa$. Let r^i be a uniformly random string of length κ . Parse $y_0^{(i)}$ as $Y_{10}^{(i)} || \dots || Y_{s0}^{(i)}$, $Y_{t0}^{(i)} \in Z_{\hat{N}}$. For each $t \in [1, s]$

$$Z_{t0} = (Y_{t0}^{(i)})^{\hat{e}} \bmod \hat{N}$$

- Let $z_0 = Z_{10} || \dots || Z_{s0}$
- Define

$$T_{\hat{N}}(m || r^{(i)}) = z_0$$

In each round j from 1 to r , define

- Choose random $y_j^{(i)} \in \{0, 1\}^k$. Parse $y_j^{(i)}$ as $Y_{1j}^{(i)} || \dots || Y_{sj}^{(i)}$. For each $t \in [1, s]$

$$Z_{tj} = (Y_{tj}^{(i)})^{\hat{e}} \bmod \hat{N}$$

- Let $z_j = Z_{1j} || \dots || Z_{sj}$
- Define

$$y_{j-1}^{(i)} = T_{\hat{N}}^{-1}(z_j)$$

- Let $y^{(i^*)} = y_r^{(i)}$ and $\text{dk}^{(i^*)} = (\hat{e}, \hat{N})$
- Return $R^{(i^*)} = (y^{(i^*)}, \text{dk}^{(i^*)})$

\mathcal{B} responds to any other oracle queries of \mathcal{A}_1 honestly, and finally gives $(R^{(1)}, \dots, R^{(n)})$ to \mathcal{A}_1 , where $R^{(i)}$ for $i \neq i^*$ is created honestly. \mathcal{A}_1 outputs a state state . Now, \mathcal{B} interacts with \mathcal{A}_2 . It runs \mathcal{A}_2 on state , and receives and responds to \mathcal{A}_2 's oracle queries in the following way. \mathcal{B} randomly chooses $j^* \in [1, r]$. If \mathcal{A}_2 queries T on $y_{j^*-1}^{(i^*)}$, \mathcal{B} chooses a random block $t^* \in [1, s]$, sets the response to embed its challenge \hat{x} in the following way. Set $Z_{t^*j^*} = \hat{x}$, and choose Z_{tj^*} for all $t \neq t^*$ uniformly, and for $z_j^* = Z_{1j^*} || \dots || Z_{sj^*}$,

$$T_{\hat{N}}(y_{j^*-1}^{(i^*)}) = z_j^*$$

The rest of the queries are answered honestly. If \mathcal{A}_2 makes a query $y_{j^*}^{(i^*)} = Y_{1j^*}^{(i^*)} || \dots || Y_{sj^*}^{(i^*)}$ with $Y_{tj^*}^{(i^*)} = \hat{y}$ such that, $\hat{y}^{\hat{e}} = \hat{x} \bmod \hat{N}$, \mathcal{B} outputs \hat{y} . If there is no such query, \mathcal{B} outputs \perp . We now argue that \mathcal{B} wins with probability at least $\frac{\epsilon}{cn^2} - 2^{-vk(1-c)}$.

We have $|\text{state}| < cvk, c < 1$. Consider the min-entropy of the random variable state , which is at most the bit length, $H_\infty(\text{state}) \leq cvk$. \mathcal{A}_2 on state returns,

$$\{R^{(i)}\}_{i \in I} \leftarrow \mathcal{A}_2(\text{state})$$

for $R^{(i)} = (y_r^{(i)}, \text{dk}^i)$. Let $\text{dk}_i = (e_i, N_i)$. Each $y_r^{(i)}$ is parsed as $Y_{1r} || \dots || Y_{sr}$, for $Y_{tr} \in Z_{N_i}$ and can be decoded block-wise to obtain $z_r^{(i)} = Z_{1r} || \dots || Z_{sr}$ such that, $Z_{tr} = \text{hardEnc.Dec}(Y_{tr}, \text{dk}^i)$ for each $t \in [1, s]$.

Let $\mathcal{Y}_r = y_r^{(i_1)} || \dots || y_r^{(i_v)}$ and $\mathcal{Z}_r = z_r^{(i_1)} || \dots || z_r^{(i_v)}$, $i_j \in I$. Each $z_r^{(i)}$ is the output of RO, and is therefore unpredictable. We have,

$$H_\infty(z_r^{(i)} | z_r^{(1)}, \dots, z_r^{(i-1)}, z_r^{(i+1)}, \dots, z_r^{(n)}) = k$$

and therefore, $H_\infty(\mathcal{Z}_r) = vk$. Since \mathcal{Z}_r can be extracted from $\mathcal{A}_2^T(\text{state})$, either the state contains information about each $z_r^{(i)}$ in $z_r^{(1)} || \dots || z_r^{(v)}$, or \mathcal{A}_2 must make relevant RO queries, that is, query the RO on the inputs corresponding to $z_r^{(i)}$. By the conditional rule for average case min-entropy (Lemma 1),

$$\tilde{H}_\infty(\mathcal{Z}_r | \text{state}) \geq H_\infty(\mathcal{Z}_r) - H_0(\text{state})$$

$$\tilde{H}_\infty(\mathcal{Z}_r | \text{state}) \geq H_\infty(\mathcal{Z}_r) - cvk = vk - cvk$$

\mathcal{Z}_r is extracted by making no RO queries only with probability $< 2^{-vk(1-c)}$. Therefore, there is at least one RO query. Let \mathcal{Q}_r be the indices in I that indicates the queries which are y -values of round r . That is, $\forall u \in \mathcal{Q}_r$, \mathcal{A}_2 queried T on $y_{r-1}^{(u)}$, and $T(y_{r-1}^{(u)}) = z_r^{(u)}$. Let $q_r = |\mathcal{Q}_r|$ denote the number of ‘‘relevant’’ r -round queries. For each query, either the state contains information about the response or not; we consider the two cases where the state stores $< B$ bits of information or $\geq B$ of information of a query response. If the state contains $< B$ bits of information about responses in round r , then \mathcal{B} wins if the challenge is embedded in that response and we are done. Otherwise, the state contains information about each $z_r^{(u)}$ which means the state stores at least B bits of information for each query.

$$H_\infty(\text{state}) \geq q_r B$$

Now, let us consider the set of queries made with indices in \mathcal{Q}_r . For each $y_{r-1}^{(u)}$, $u \in \mathcal{Q}_r$, we can extract $z_{r-1}^{(u)}$ by computing the decoding block-wise. That is, $z_{r-1}^{(u)} = Z_{1(r-1)} || \dots || Z_{s(r-1)}$ such that, $Z_{t(r-1)} = \text{hardEnc.Dec}(Y_{t(r-1)}, \text{dk}^u)$ for each $t \in [1, s]$, where $y_{r-1}^{(u)} = Y_{1(r-1)} || \dots || Y_{s(r-1)}$, $Y_{tr} \in Z_{N_u}$. These q_r elements are outputs of RO, and therefore have full entropy. Let $\mathcal{Z}_{r-1} = z_{r-1}^{(u_1)} || \dots || z_{r-1}^{(u_{q_r})}$ where each $u_i \in \mathcal{Q}_r$. We have $H_\infty(\mathcal{Z}_{r-1}) = q_r k$. If \mathcal{Z}_{r-1} can be extracted from $\mathcal{A}_2^T(\text{state})$, either the state contains information about $z_{r-1}^{(i)}$, $\forall i \in \mathcal{Q}_r$, or \mathcal{A}_2 must make more RO queries. If there are no more queries, then $H_\infty(\text{state}) \geq vk$. Therefore, there must be more queries on inputs corresponding to the indices in \mathcal{Q}_r .

Let q_{r-1} be the number of relevant round $(r-1)$ queries. Define a set of query indices \mathcal{Q}_{r-1} , from which we can extract $\mathcal{Z}_{r-2} = z_{r-2}^{(u_1)} || \dots || z_{r-2}^{(u_{q_{r-1}})}$, for $u_i \in \mathcal{Q}_{r-1}$. Again, for each query, either the state dedicates $< B$ of information, in which case \mathcal{B} wins if the challenge is embedded in that response and we are done. Otherwise, the state stores at least B bits of information about each $y_{r-1}^{(u)}$, and therefore we have,

$$H_\infty(\text{state}) \geq q_r B + q_{r-1} B$$

Making a similar argument as before, there must be more RO queries corresponding to the indices in set \mathcal{Q}_{r-1} . Thus, we have, after r rounds, $\mathcal{Z}_r, \dots, \mathcal{Z}_1$ are extracted from $\mathcal{A}_2(\text{state})$, and we have

$$H_\infty(\text{state}) \geq \sum_{i=1}^r q_i B$$

Let $\mathcal{Z}^j = \mathcal{Z}_j || \mathcal{Z}_{j+1} || \cdots || \mathcal{Z}_{r-1}$. Let \mathcal{A}_2 make RO queries in each round j for the replicas given by the indices in \mathcal{Q}_j such that the reduction could not successfully embed a challenge in any of the responses. Then after r rounds, for $\mathcal{Z}^1 = \mathcal{Z}_1 || \mathcal{Z}_2 || \cdots || \mathcal{Z}_{r-1}$,

$$\tilde{H}_\infty(\mathcal{Z}^1 | \text{state}) \geq \sum_{i=1}^r q_i B - cvk$$

Setting,

$$\sum_{i=1}^r q_i B - cvk = k$$

Since there is at least one query in each round, we get $\tilde{H}_\infty(\mathcal{Z}^1 | \text{state}) \geq k$, when $r > k(cv+1)/B$.

Therefore, at some round $\ell \leq r$, the entropy of the response is full when making an RO query at round ℓ . That is, $\exists \ell \in [1, r], w \in [1, n]$ such that,

$$\tilde{H}_\infty(z_\ell^{(w)} | \text{state}) = k$$

Thus $z_\ell^{(w)} = T(y_{\ell-1}^{(w)})$ with $Z_{t^* j^* - 1} = \hat{x}$ is the programmed RO response, is the probability that $i^* = w, j^* = \ell$ which is $1/nr$. Thus the probability that \mathcal{B} wins is at least $\frac{\epsilon}{nr} - 2^{-vk(1-c)}$. \square

The hard encoding may be realized using any trapdoor permutation. Since the proof does not rely on the trapdoor permutation of the replica encoding scheme being RSA, we get the following corollary stating the security of replica encoding scheme based on trapdoor permutation f .

Corollary 1. *Assuming T is an invertible random oracle, the construction $\text{ReplicaEncoding} = (\text{rEnc}, \text{rDec})$ when the underlying hardEnc uses trapdoor permutation f is a secure replica encoding scheme for message length k' , replication parameter n as per Definition 4. For number of rounds $r > \frac{(cn+1)k}{B}$, it is complete and c -sound with soundness error $\epsilon \leq (\epsilon' + 2^{-nk(1-c)})nr$ where $k = k' + \kappa$, the advantage of any adversary in B -leakage inversion of the permutation f is at most ϵ' .*

As any trapdoor function is also trivially B -leakage secure for $B = \log(k)$, we obtain the following corollaries.

Corollary 2. *Assuming T is an invertible random oracle, the construction $\text{ReplicaEncoding} = (\text{rEnc}, \text{rDec})$ is a secure replica encoding scheme for message length k' , replication parameter n as per Definition 4. For number of rounds $r > \frac{(cn+1)k}{\log k}$, it is complete and c -sound with soundness error $\epsilon \leq (\epsilon' + 2^{-nk(1-c)})nr$ where $k = k' + \kappa$, the advantage of any adversary in the inversion of the RSA permutation as per Definition 1 is at most ϵ' .*

Corollary 3. *Assuming T is an invertible random oracle, the construction $\text{ReplicaEncoding} = (\text{rEnc}, \text{rDec})$ when the underlying hardEnc uses trapdoor permutation f is a secure replica encoding scheme for message length k' , replication parameter n as per Definition 4. For number of rounds $r > \frac{(cn+1)k}{\log k}$, it is complete and c -sound with soundness error $\epsilon \leq (\epsilon' + 2^{-nk(1-c)})nr$ where $k = k' + \kappa$, the advantage of any adversary in inverting f is at most ϵ' .*

On instantiating the oracle T . As stated, each encoding key (e, N) defines a different random oracle T . We require that T be indifferentiable from a random permutation. The indifferentiability framework, first proposed by Maurer et al. [MRH04], informally says that given ideal primitives G and H , a construction C^G is indifferentiable from H , if there exists a simulator S with oracle access to H such that (C^G, G) is indistinguishable from (H, S^H) . Coron et al. [CHK⁺16] showed that a 14-round Feistel network where the round functions are independent random oracles is indifferentiable from a random permutation. A series of subsequent works [DKT16, DS16] show that 8 rounds is sufficient.

Note that the input and output of our random oracles T are vectors of elements in \mathbb{Z}_N , where N is part of the public decoding key. We note that we can instantiate the Feistel construction in this domain as well by replacing XOR with multiplication modulo N i.e., given a random oracle $H : (\mathbb{Z}_N)^{\ell/2} \rightarrow (\mathbb{Z}_N)^{\ell/2}$ we can define $\mathcal{F} : (\mathbb{Z}_N)^\ell \rightarrow (\mathbb{Z}_N)^\ell$ as follows:

$$\mathcal{F}^H(\vec{x}) = \vec{s} \parallel \vec{t}, \text{ where } \vec{s} = \vec{x} \circ H(\vec{r}) \bmod N, \vec{t} = \vec{r} \circ H(\vec{s}) \bmod N$$

where \circ is the Hadamard product. Note that \mathcal{F} is invertible except with negligible probability i.e., if \mathcal{F} is not invertible then a non-trivial factor of N is found.

Efficiency of decoding. We note that, when instantiating the construction with the RSA trapdoor permutation, it is possible to use a small exponent (i.e., $e = 3$). Now decoding would be much faster than encoding, which is a desirable property in applications where a single user uploads a file which is then retrieved by a large number of users.

4.2 From Replica Encodings to Proofs of Replication

We now construct a proof of replication scheme `create, retrieve`, \mathcal{P}, \mathcal{V} . The idea is very simple: to construct a proof of replication we use the replica encoding scheme from the previous section to create replicas, and then apply a proof of retrievability on the encoded replicas. The proof of security is also simple, as an adversary that breaks soundness for the proof of replication can be used to break the soundness property of the proof of retrievability scheme or the soundness of the replica encoding scheme.

The `create` procedure is formally described in Figure 6. The prover, and verifier algorithms \mathcal{P}, \mathcal{V} are the same as the prover and verifier in the proof of retrievability. Finally, the `retrieve` algorithm simply runs the replica decoding algorithm `rDec` if the proof of retrievability accepts.

Theorem 2. *PoRep = (create, retrieve, \mathcal{P}, \mathcal{V}) is a proof of replication scheme for message length k' and replication parameter n secure as per Definition 5. It is complete and c -sound with soundness error $\gamma \leq \delta + \epsilon$ where the underlying PoR scheme has soundness error δ , and the replica encoding scheme has soundness error ϵ for message length k' .*

Proof. We first argue completeness: Given $R^{(i)}$ and `dk`, for encodings that are created honestly, an honest server can recover $m^* = \text{retrieve}(R^{(i)})$. By completeness of the replica encoding scheme `rEnc`, we have `rDec`($R^{(i)}$) = `rDec`($y^{(i)}, \text{dk}^{(i)}$) = m , $\forall i$.

We now argue the soundness of the construction. Let $(\mathcal{A}_1, \mathcal{A}_2)$ be an adversary, that wins the soundness game $\text{sound}_{\mathcal{A}_1, \mathcal{A}_2}^{\mathcal{E}}$ with advantage γ . Let $(u, v) \leftarrow \text{sound}_{\mathcal{A}_1, \mathcal{A}_2}^{\mathcal{E}}$. We consider the two cases:

Case 1. $u < v$. Let `ext` be the extractor of the PoR scheme, and let the file output by `ext` be $\{\tilde{R}^{(i)}\}_{i=1}^n$. By assumption that $u < v$ there must be an index $i \in [1, n]$ such that the

Let $\text{PoR} = (\text{Gen}, \mathcal{P}, \mathcal{V})$ be a proof of retrievability scheme. Given a file $m \in \{0, 1\}^{k'}$, and a replication factor n :

- For each $i \in [1, n]$

$$R^{(i)} \leftarrow \text{rEnc}(m, \kappa)$$

- $(\{\tilde{R}^{(i)}\}_i, \tau) = \text{PoR.Gen}(\{R^{(i)}\}_i)$
- Set $\text{ver} = \tau$
- $\tilde{R}^{(i)}$ is sent to the server i for storage and ver is returned to the client.

Figure 6: $\text{create}(m, n)$: Create replicated storage

adversary \mathcal{A}_2 succeeds in the audit protocol (i.e., $v_i = 1$), but $\tilde{R}^{(i)} \neq R^{(i)}$ (i.e., $u_i = 0$). By the soundness of the proof of retrievability scheme PoR , this happens only with probability δ .

Case 2. $|\text{state}| \leq cvk$. In this case the adversary \mathcal{A}_2^T succeeds in v audit protocols, and since $u \geq v$, the extractor \mathcal{E} outputs $\tilde{R}^{(i)} = R^{(i)}$ for $i \in I \subset [1, n]$, $|I| = v$. Let $(\mathcal{B}_1, \mathcal{B}_2)$ be an adversary whose task is to break the soundness of the replica encoding scheme rEnc . \mathcal{B}_1 interacts with $(\mathcal{A}_1, \mathcal{A}_2)$ in the soundness game. \mathcal{B}_1 receives a file $m \in \{0, 1\}^k$ from \mathcal{A}_1 , and outputs m to its challenger. \mathcal{B}_1 receives n replica encodings $(R^{(1)}, \dots, R^{(n)})$ from the challenger \mathcal{B}_1 runs the PoR on the replicas. $(\{\tilde{R}^{(i)}\}_i, \text{ver}) \leftarrow \text{PoR.Gen}\{R^{(i)}\}_i$ and returns $\{\tilde{R}^{(i)}\}_i$ to \mathcal{A}_1 . \mathcal{B}_1 outputs as state whatever \mathcal{A}_1 outputs with $|\text{state}| \leq cvk$. For every successful audit proof given by \mathcal{A}_2 , \mathcal{B}_2 runs the extractor $\mathcal{E}(\text{ver}, n, \kappa)$ of the scheme. Thus \mathcal{B}_2 outputs $\tilde{R}^{(i)} = R^{(i)}$ for each $i \in I$ with probability at least γ .

□

5 Dealing with Malicious Clients

We discuss here some limitations and possible extensions of our approach.

Our definition and construction so far has concentrated on the case where the client is honest. This is not a problem for our base use-case where a user wants to make sure they will be able to retrieve their files in the future, but it is a problem in the Filecoin use case where servers are rewarded for the files they store. In this case, we need to prevent against the so called *generation attack* and it is therefore important to have some security guarantees when the client is corrupt and might work with a set of corrupt servers to convince honest users that they store many replicas whereas in fact the replicas are generated “on-the-fly” for each proof.

Our solution from the previous section does not work in this case, as a corrupt user could share the RSA secret key with the servers and now they can indeed encode a replica on the fly. If the client who owns the file is corrupt and is the only user involved in the encoding process, then the

adversary knows everything about the encoding process, and a different solution is needed. One way to go is to involve several users in the encoding process and work under the assumption that at least one of them is honest.

In a Filecoin-like scenario one could implement such a solution by rewarding users for helping others to encode. We now describe two different approaches to such multi-user encoding.

Parallel Encoding. Given one-way trapdoor functions f_1, \dots, f_n that act on k -bit strings, we define a new function F on kn bit strings by $F(x_1, \dots, x_n) = (f_1(x_1), \dots, f_n(x_n))$, where each x_i is a k -bit string. It is clear that F is a one-way trapdoor function even with respect to an adversary who knows all but one of the trapdoors for f_1, \dots, f_n . Namely, if the j 'th trapdoor is unknown, we can take a challenge y_j , choose $y_i = f_i(x_i)$ for $i \neq j$ and random x_i and give (y_1, \dots, y_n) to the adversary. If he computes a preimage, then the j 'th component is the answer to challenge y_j .

Note that our main result gives a construction that is secure based on any one-way trapdoor function and so it also works for F .

Now, in the practical use-case, we will assume that n user are involved, such that user i has f_i as part of his public key and knows the corresponding trapdoor. A public-key infrastructure is one of several ways to realize this. Then the n users can collaborate to encode: whenever we need to compute F , we can assume the input to the current round is known (initially it will be the file to encode), so each user i can apply the permutation oracle and compute and broadcast f_i^{-1} on his part of the result.

It follows immediately from the above that if at least one user is honest, then this construction results in a secure replica encoding. Note also that the contributions of each user can be verified by computing his function in the forward direction. Moreover, the overhead in encoding size and the cost per bit of encoding and decoding is the same as for the single honest user case. On the other hand, we need a number of rounds for the encoding protocol that equals the number of rounds in the encoding process.

Sequential Encoding. An obvious alternative is to do the encoding sequentially. Namely, the first user does an encoding of the input file using his (set of) trapdoor functions and broadcasts the result. The second user encodes the output of the first, etc. In the end, we have an encoding that is essentially done just like our original construction, only with more rounds. Note that one can decode the output of each user and check the result is correct.

The intuition behind the security of this approach is the following: under the assumption that at least one user is honest, we have the same security as in the original construction assuming that at least one of the users is honest. This is simply because the adversary does not know the trapdoors for the honest member, and his encoding process involves the same number of random oracle responses that we considered in the original proof. This approach increases the size of the encodings but not significantly (the complexity would grow from $O(|m| + \kappa)$ to $O(|m| + n \cdot \kappa)$ with n users). The cost of encoding and decoding in this solution is a factor n larger than for the single honest user case. On the other hand, the number of rounds to encode is n , which may be better than parallel encoding, depending on the concrete scenario.

6 Conclusions and Future Work

We gave two possible solutions to multi-user encoding above. However, there is also a solution of a different nature that comes to mind: namely we can share a trapdoor (say, an RSA key) between a set of users and have them collaborate to compute the encoding using that trapdoor function securely. This has the advantage that an encoding looks just like what an honest client would produce, we are not forced to have larger block size when many users are involved. Also, decoding is as fast as in the honest client case, and one can set up the protocol such that just one participating client needs to be honest in order for the secret key to not leak. On the other hand, encoding requires more work. For the encoding protocol, one can take advantage of the huge body of literature on distributed RSA key generation and distributed signing. Finding the optimal solution for this approach is left for future work.

We also leave as an open question the problem of finding a secure replica encoding where the number of rounds in the encoding process does not depend on the number of replicas.

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