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Witness-Indistinguishable Arguments with Σ -Protocols for Bundled Witness Spaces and its Application to Global Identities

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Abstract. We propose a generic construction of a Σ -protocol of commit-and-prove type, which is an AND-composition of Σ -protocols on statements that include a common commitment. Our protocol enables a prover to convince a verifier that the prover knows a bundle of witnesses that have a common component which we call a base witness point. When the component Σ -protocols are of witness-indistinguishable argument systems, our Σ -protocol is also a witness-indistinguishable argument system as a whole. As an application, we propose a decentralized multi-authority anonymous authentication scheme. We first give a syntax and security definitions of the scheme. Then we give a generic construction of the scheme. There a witness is a bundle of witnesses each of which consists of a common global identity string and a digital signature on it. We mention an instantiation in the setting of bilinear groups.

Keywords: interactive proof, sigma protocol, witness indistinguishability, decentralized, anonymity, collusion resistance

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1 Introduction

Global identities such as Passport Numbers (PNs), Social Security Numbers (SSNs) and e-mail addresses as global identifiers are currently common for identification. They are used not only for governmental identification but also for commercial services; that is, when we want to use a commercial service, we often ask the service administration authority for issuing an attribute certificate at the registration phase. In the phase, the authority confirms our identities by verifying the global identity string such as PN or SSN. Once the attribute certificate is issued, we become to be accepted at the authentication phase of the service. Hence the global identity strings work for us to be issued our attribute certificates. It is notable that recently multi-factor authentication schemes are utilized to prevent misauthentication. In the scheme a user of a service is granted access only after presenting several separate pieces of evidence. Actually the multi-factor authentication of using both a laptop PC, which is connected to Internet by a service provider, and a smartphone, which is activated by a cellular carrier, is getting usual. Thus, there is a compound model that involves independent administration authorities for us to be authenticated and receive benefit of a service.

Privacy protection is a function to be pursued in authentication. The growth of companies in the areas of the IT infrastructures made protecting privacy more critical for involved users because we use search engines, digital devices, social networking services and e-shopping services everyday. Considering this change of circumstance, the authentication framework using identity strings and passwords should be evolved into a framework where anonymity is guaranteed at the authentication phase. For example, when a smart household machine sends a report about the situation of a house via Internet as a query for useful suggestion (such as air conditioning or cooking recipes), the identity information is often unnecessary. A further example is connected vehicles which are connected to Internet and which use a combination of plural services like a local traffic information system and the passenger's web-scheduler, where the identity information should not be leaked even when the memberships should be made in the registration phase. In this example a user should be authenticated by the service providers simultaneously in the authentication phase, anonymously. This is an authentication framework in which plural attributes of a single user are authenticated. However, there is a threat on such anonymous authentication frameworks; *collusion attack*. For example, two malicious users with different identities bring together their private attribute certificates, and try to make a verifier accept anonymously by using the merged attribute certificates. Here the vary anonymity is a critical potential drawback from the view point of the collusion attack.

1.1 Our Contribution and Related Work

In this paper, we propose a new notion of a proof system; a witness-indistinguishable argument system (WIA) with Σ -protocols for a *bundled witness space*. It is known that WIA is a natural building block to achieve anonymity in cryptographic primitives ([Gol01]). However, there is no previous work for the multi-prover setting executed by a *hidden single prover* who is able to convince a verifier that she is certainly a single prover, though she is anonymous. By employing a commitment scheme as one of the building blocks we construct the kind of WIA as a kind of commit-and-prove scheme [CLOS02,EG14].

As an application, we give a generic construction of a decentralized multi-authority anonymous authentication scheme, which can be converted into a decentralized multi-authority attribute-based signature scheme (DMA-ABS) [OT13]. In an ABS scheme, a signer has certificates, which are also keys, on her attributes. The signer is able to sign a message which is associated with a signing policy expressed as a boolean formula on attributes if and only if her attributes satisfy the boolean formula. There are assignment patterns to satisfy the boolean formula, and the attribute privacy of an ABS scheme should assure that the signatures do not leak any information on the assignment pattern which she used. It should be noted that the attribute privacy implies the anonymity of the signer's identity. On the other hand, decentralized multi-authorities mean that there are independent key-issuing authorities each of which generates each private secret key for her. Our WIA can actually be converted into a DMA-ABS scheme if a prover chooses a monotone boolean formula instead of an all-AND formula, and if we apply the Fiat-Shamir transform [FS86] to the Σ -protocol of our authentication scheme.

The difference between the previous DMA-ABS schemes and our DMA-ABS scheme is that, in our DMA-ABS scheme, when a signer wants the authorities to issue private secret keys for her, the authorities *simply generate digital signatures on her single global identity string*. This feature is useful when her global identity string is easy to be validated.

1.2 Organization of the Paper

In Section 2, we prepare for needed notions and notations. In Section 3, we describe building blocks and give a generic construction of our witness-indistinguishable argument system with a Σ -protocol for the bundled witness space. In Section 4, we first define a syntax and security notions of our decentralized multi-authority anonymous authentication scheme. Then, we give a generic construction of the scheme. In Section 5, we conclude our work. In Appendix A, we briefly show an instantiation of the scheme in the setting of bilinear groups.

2 Preliminaries

The set of natural numbers is denoted by \mathbb{N} . We put $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$. The residue class ring of integers modulo a prime number p is denoted by \mathbb{Z}_p . The security parameter is denoted by λ . The bit length of a string a is denoted by $|a|$. The number of elements of a set S is denoted by $|S|$. A uniform random sampling of an element a from a set S is denoted as $a \leftarrow_R S$. The expression $a \stackrel{?}{=} b$ returns a boolean 1 (TRUE) when $a = b$, and otherwise 0 (FALSE). When an algorithm A with an input a returns z , we denote it as $z \leftarrow A(a)$, or, $A(a) \rightarrow z$. When a probabilistic algorithm A with an input a and a randomness r on a random tape returns z , we denote it as $z \leftarrow A(a; r)$. When an algorithm A with an input a and an algorithm B with an input b interact with each other and B returns z , we denote it as $z \leftarrow \langle A(a), B(b) \rangle$. The transcript of all the messages of the interaction is denoted by $\text{transc}(\langle A(a), B(b) \rangle)$. When an algorithm A accesses an oracle \mathbf{O} , we denote it by $A^{\mathbf{O}}$. When A accesses n oracles $\mathbf{O}_1, \dots, \mathbf{O}_n$ concurrently, i.e. in arbitrarily interleaved order of messages, we denote it by $A^{\mathbf{O}_i}_{i=1}^n$. “Probabilistic polynomial-time” is abbreviated as PPT. The probability of an event E is denoted by $\Pr[E]$. The conditional probability of an event E given events F_1, \dots, F_n in this order is denoted by $\Pr[E|F_1, \dots, F_n]$. The distribution of a random variable X is denoted by $\text{dist}(X)$. The distribution of a random variable X whose probability is given by a joint probability of random variables X, Y_1, \dots, Y_n is denoted by $\text{dist}(X|X, Y_1, \dots, Y_n)$. A probability P is said to be negligible in λ if for any given positive polynomial $\text{poly}(\lambda)$ $P < 1/\text{poly}(\lambda)$ for sufficiently large $\lambda \in \mathbb{N}$. Two probabilities P and Q are said to be computationally indistinguishable if $|P - Q|$ is negligible in λ , which is denoted as $P \approx_c Q$. St means the inner state of an algorithm.

2.1 Interactive Argument System, Σ -protocol and Witness-Indistinguishability

Suppose that there exists a predicate Φ that defines the membership of a binary relation R ; i.e., Φ maps $(x, w) \in (\{0, 1\}^*)^2$ to TRUE or FALSE. The relation R is defined as $R \stackrel{\text{def}}{=} \{(x, w) \in (\{0, 1\}^*)^2 \mid \Phi(x, w) = \text{TRUE}\}$. We consider relations parametrized by the security parameter λ . That is, R in our sense is actually a family $(R_\lambda)_{\lambda \in \mathbb{N}}$ of relations $R_\lambda \subseteq (\{0, 1\}^*)^2$. We say that R is polynomially bounded if there exist a constant c and a polynomial $\ell(\cdot)$ such that for any λ , $|x| \leq c \cdot \lambda$ and $|w| \leq \ell(|x|)$ for any $(x, w) \in R_\lambda$. We say that R is an NP relation if R is polynomially bounded and Φ is computable within polynomial-time in $|x|$ as an algorithm. For a pair $(x, w) \in R$ we call x a statement and w a witness of x . We call R the witness relation, and $\Phi(\cdot, \cdot)$ the predicate of the witness relation R . An NP language L for an NP relation R is defined as the set of all possible statements: $L \stackrel{\text{def}}{=} \{x \in \{0, 1\}^*; \exists w \in \{0, 1\}^*, (x, w) \in R\}$. We denote the set of witnesses of a statement x by $W(x)$: $W(x) \stackrel{\text{def}}{=} \{w \in \{0, 1\}^* \mid (x, w) \in R\}$. We call the union W of all the sets $W(x)$ for $x \in L$ the *witness space* of L : $W \stackrel{\text{def}}{=} \bigcup_{x \in L} W(x)$. We denote an interactive proof system on an NP relation R [Bab85, GMR85] by $\Pi = (\Pi.\text{Setup}, \mathbf{P}, \mathbf{V})$, where $\Pi.\text{Setup}$ is a set up algorithm for a set of public parameter pp , and \mathbf{P} and \mathbf{V} are a pair of interactive algorithms. \mathbf{P} , which is called a prover, is probabilistic and unbounded, and \mathbf{V} , which is called a verifier, is probabilistic polynomial-time (PPT). If \mathbf{P} is also limited to PPT, then Π is called an interactive *argument* system.

Σ -protocol [Cra96, Dam10] Let R be an NP relation. A Σ -protocol Σ on the relation R is a 3-move public-coin protocol of an interactive argument system $\Pi = (\Pi.\text{Setup}, \mathbf{P}, \mathbf{V})$ [Cra96, Dam10]. We introduce six PPT algorithms for a Σ -protocol: $\Sigma = (\Sigma_{\text{com}}, \Sigma_{\text{cha}}, \Sigma_{\text{res}}, \Sigma_{\text{vrf}}, \Sigma_{\text{ext}}, \Sigma_{\text{sim}})$. The first algorithm Σ_{com} is executed by \mathbf{P} . On input a pair of a statement and a witness $(x, w) \in R$, it generates a commitment message

COM and outputs its inner state St . It returns them as $\Sigma_{\text{com}}(x, w) \rightarrow (\text{COM}, St)$. The second algorithm Σ_{cha} is executed by V . On input the statement x , it reads out the size of the security parameter as 1^λ and chooses a challenge message $\text{CHA} \in_R \text{CHASP}(1^\lambda)$ from the challenge space $\text{CHASP}(1^\lambda) := \{0, 1\}^{\omega(\lambda)}$, where $\omega(\cdot)$ is a super-log function [BP02]. It returns the message as $\Sigma_{\text{cha}}(x) \rightarrow \text{CHA}$. The third algorithm Σ_{res} is executed by P . On input the state St and the challenge message CHA , it generates a response message RES . It returns the message as $\Sigma_{\text{res}}(St, \text{CHA}) \rightarrow \text{RES}$. The fourth algorithm Σ_{vrf} is executed by V . On input the statement x and the messages COM , CHA and RES , it computes a boolean decision d . It returns the decision as $\Sigma_{\text{vrf}}(x, \text{COM}, \text{CHA}, \text{RES}) \rightarrow d$. If $d = 1$, then we say that P is accepted by V on x . Otherwise, we say that P is rejected by V on x . The vector of all the messages $(\text{COM}, \text{CHA}, \text{RES})$ is called a transcript of the interaction on x .

These four algorithms $(\Sigma_{\text{com}}, \Sigma_{\text{cha}}, \Sigma_{\text{res}}, \Sigma_{\text{vrf}})$ must satisfy the following property.

Completeness For any $(x, w) \in R$, a prover $P(x, w)$ has a verifier $V(x)$ accept with probability 1: $\Pr[\Sigma_{\text{vrf}}(x, \text{COM}, \text{CHA}, \text{RES}) = 1 \mid \Sigma_{\text{com}}(x, w) \rightarrow (\text{COM}, St), \Sigma_{\text{cha}}(x) \rightarrow \text{CHA}, \Sigma_{\text{res}}(St, \text{CHA}) \rightarrow \text{RES}]$.

The fifth algorithm Σ_{ext} concerns with the following property.

Special Soundness There is a PPT algorithm Σ_{ext} called a *knowledge extractor*, which, on input a statement x and two accepting transcripts $(\text{COM}, \text{CHA}, \text{RES})$ and $(\text{COM}, \text{CHA}', \text{RES}')$, $\text{CHA} \neq \text{CHA}'$, computes a witness \hat{w} satisfying $(x, \hat{w}) \in R$ with an overwhelming probability in $|x|$:

$$\hat{w} \leftarrow \Sigma_{\text{ext}}(x, \text{COM}, \text{CHA}, \text{RES}, \text{CHA}', \text{RES}'). \quad (1)$$

Note here that commitment messages are common and challenge messages are different.

The sixth algorithm Σ_{sim} concerns with the following property.

Honest-Verifier Zero-Knowledge There is a PPT algorithm called a *simulator* Σ_{sim} , which, on input a statement x , computes an accepting transcript on x :

$$(\text{C}\tilde{\text{O}}\text{M}, \text{C}\tilde{\text{H}}\text{A}, \text{R}\tilde{\text{E}}\text{S}) \leftarrow \Sigma_{\text{sim}}(x), \quad (2)$$

where the distribution of the simulated transcripts $\text{dist}(\text{C}\tilde{\text{O}}\text{M}, \text{C}\tilde{\text{H}}\text{A}, \text{R}\tilde{\text{E}}\text{S})$ is identical to the distribution of the real accepting transcripts $\text{dist}(\text{COM}, \text{CHA}, \text{RES})$.

Note 1: Our Use Case In a Σ -protocol the challenge message CHA is a public coin. This property enables us in this paper to use the following variant of the simulator $\Sigma_{\text{sim}}(x)$: On input a simulated challenge message $\text{C}\tilde{\text{H}}\text{A}$ that is chosen uniformly at random, the variant generates a commitment $\text{C}\tilde{\text{O}}\text{M}$ and a response $\text{R}\tilde{\text{E}}\text{S}$:

$$\text{C}\tilde{\text{H}}\text{A} \in_R \text{CHASP}(1^\lambda), \quad (\text{C}\tilde{\text{O}}\text{M}, \text{R}\tilde{\text{E}}\text{S}) \leftarrow \Sigma_{\text{sim}}(x, \text{C}\tilde{\text{H}}\text{A}). \quad (3)$$

Witness-Indistinguishability [FS90, Gol01] Let R be an NP relation. Suppose that an interactive argument system $\Pi = (\Pi.\text{Setup}, P, V)$ with a Σ -protocol Σ on the relation R is given. In this paper we focus on the following property.

Perfect Witness Indistinguishability For any PPT algorithm V^* , any sequences of witnesses $\mathbf{w} = (w_x)_{x \in L}$ and $\mathbf{w}' = (w'_x)_{x \in L}$ s.t. $w_x, w'_x \in W(x)$, any string $x \in L$ and any string $z \in \{0, 1\}^*$, the two distributions $\text{dist}(x, z, \text{transc}(P(x, w_x), V^*(x, z)))$ and $\text{dist}(x, z, \text{transc}(P(x, w'_x), V^*(x, z)))$ are identical.

2.2 Commit-and-Prove Scheme [CLOS02, EG14]

A commit-and-prove scheme CmtPrv consists of five PPT algorithms: $\text{CmtPrv} = (\text{CmtPrv}.\text{Setup}, \text{Cmt} = (\text{Cmt}.\text{Com}, \text{Cmt}.\text{Vrf}), \Pi = (P, V))$.

$\text{CmtPrv}.\text{Setup}(1^\lambda) \rightarrow pp$. On input the security parameter 1^λ , it generates a set of public parameter pp . It returns pp .

$\text{Cmt}.\text{Com}(m) \rightarrow (c, \kappa)$. On input a message m in the message space $\text{Msg}(1^\lambda)$, this PPT algorithm generates a commitment c . It also generates an opening key κ . It returns (c, κ) .

$\text{Cmt}.\text{Vrf}(c, m, \kappa) \rightarrow d$. On input a commitment c , a message m and an opening key κ , this deterministic polynomial-time algorithm generates a boolean decision d . It returns d .

The correctness should hold for the commitment part Cmt of the scheme: For any security parameter 1^λ , any set of public parameter pp and any message $m \in \text{Msg}(1^\lambda)$, $\Pr[d = 1 \mid (c, \kappa) \leftarrow \text{Cmt}.\text{Com}(m), d \leftarrow \text{Cmt}.\text{Vrf}(c, m, \kappa)] = 1$.

We denote by Φ a predicate that returns the boolean decision: $\Phi(c, (m, \kappa)) \stackrel{\text{def}}{=} (\text{Cmt.Vrf}(c, m, \kappa))$. In the scheme there is an interactive argument system $\Pi = (\text{P}, \text{V})$ for the following relation R :

$$R := \{(c, (m, \kappa)) \in \{0, 1\}^* \times (\{0, 1\}^*)^2 \mid \Phi(c, (m, \kappa)) = \text{TRUE}\}. \quad (4)$$

In this paper we focus on the following properties for the commitment part Cmt .

Perfectly Hiding For any security parameter 1^λ , any set of public parameter pp and any two messages $m, m' \in \text{Msg}(1^\lambda)$, the two distributions $\text{dist}(c \mid (c, \kappa) \leftarrow \text{Cmt.Com}(m))$ and $\text{dist}(c \mid (c, \kappa) \leftarrow \text{Cmt.Com}(m'))$ are identical.

Computationally Binding The attack of breaking binding property of Cmt by an algorithm \mathbf{A} is defined by the following experiment.

$$\text{Exp}_{\text{Cmt}, \mathbf{A}}^{\text{bind}}(1^\lambda) : \quad (5)$$

$$pp \leftarrow \text{Cmt.Priv.Setup}(1^\lambda), (c, m, \kappa, m', \kappa') \leftarrow \mathbf{A}(pp) \quad (6)$$

$$\text{If } \text{Cmt.Vrf}(c, m, \kappa) = \text{Cmt.Vrf}(c, m', \kappa') = 1 \wedge m \neq m', \text{ then Return WIN else Return LOSE} \quad (7)$$

The advantage of \mathbf{A} over Cmt is defined as $\text{Adv}_{\text{Cmt}, \mathbf{A}}^{\text{bind}}(\lambda) := \Pr[\text{Exp}_{\text{Cmt}, \mathbf{A}}^{\text{bind}}(1^\lambda) \text{ returns WIN}]$. The commitment scheme Cmt is said to be *computationally binding* if for any set of public parameter pp and any PPT algorithm \mathbf{A} , the advantage $\text{Adv}_{\text{Cmt}, \mathbf{A}}^{\text{bind}}(\lambda)$ is negligible in λ .

Note 2: Our Use Case The commitment generation algorithm Cmt.Com uses random tapes [Gol01]. In this paper we are in the case that a randomness $r \in \{0, 1\}^\lambda$ is used to generate a commitment c , and the opening key κ is the randomness: $\kappa := r$. That is, $\text{Cmt.Com}(m; r) \rightarrow (c, r)$.

2.3 Digital Signature Scheme [FS86]

A digital signature scheme Sig consists of four PPT algorithms: $\text{Sig} = (\text{Sig.Setup}, \text{Sig.KG}, \text{Sig.Sign}, \text{Sig.Vrf})$. $\text{Sig.Setup}(1^\lambda) \rightarrow pp$. On input the security parameter 1^λ , it generates a set of public parameter pp . It returns pp .

$\text{Sig.KG}(1^\lambda) \rightarrow (\text{PK}, \text{SK})$. On input the security parameter 1^λ , this PPT algorithm generates a signing key SK and the corresponding public key PK . It returns (PK, SK) .

$\text{Sig.Sign}(\text{PK}, \text{SK}, m) \rightarrow \sigma$. On input the public key PK , the secret key SK and a message m in the message space $\text{Msg}(1^\lambda)$, this PPT algorithm generates a signature σ . It returns σ .

$\text{Sig.Vrf}(\text{PK}, m, \sigma) \rightarrow d$. On input the public key PK , a message m and a signature σ , it returns a boolean d .

The correctness should hold for the scheme Sig : For any security parameter 1^λ and any message $m \in \text{Msg}(1^\lambda)$, $\Pr[d = 1 \mid pp \leftarrow \text{Sig.Setup}(1^\lambda), (\text{PK}, \text{SK}) \leftarrow \text{Sig.KG}(1^\lambda), \sigma \leftarrow \text{Sig.Sign}(\text{PK}, \text{SK}, m), d \leftarrow \text{Sig.Vrf}(\text{PK}, m, \sigma)] = 1$.

An adaptive chosen-message attack on the scheme Sig by a forger algorithm \mathbf{F} is defined by the following experiment.

$$\text{Exp}_{\text{Sig}, \mathbf{F}}^{\text{euf-cma}}(1^\lambda) : \quad (8)$$

$$pp \leftarrow \text{Sig.Setup}(1^\lambda), (\text{PK}, \text{SK}) \leftarrow \text{Sig.KG}(1^\lambda), (m^*, \sigma^*) \leftarrow \mathbf{F}^{\text{SignO}(\text{PK}, \text{SK}, \cdot)}(\text{PK}) \quad (9)$$

$$\text{If } m^* \notin \{m_j\}_{1 \leq j \leq q_s} \text{ and } \text{Sig.Vrf}(\text{PK}, m^*, \sigma^*) = 1, \text{ then Return WIN else Return LOSE} \quad (10)$$

In the experiment, \mathbf{F} issues a signing query to its signing oracle $\text{SignO}(\text{PK}, \text{SK}, \cdot)$ by sending a message m_j at most q_s times ($1 \leq j \leq q_s$). As a reply, \mathbf{F} receives a valid signature σ_j on m_j . After receiving replies, \mathbf{F} returns a message and a signature (m^*, σ^*) . A restriction is imposed on the algorithm \mathbf{F} : The set of queried messages $\{m_j\}_{1 \leq j \leq q_s}$ should not contain the message m^* . The advantage of \mathbf{F} over Sig is defined as $\text{Adv}_{\text{Sig}, \mathbf{F}}^{\text{euf-cma}}(\lambda) := \Pr[\text{Exp}_{\text{Sig}, \mathbf{F}}^{\text{euf-cma}}(1^\lambda) \text{ returns WIN}]$. The digital signature scheme Sig is said to be *existentially unforgeable against adaptive chosen-message attacks* if for any given PPT algorithm \mathbf{F} , the advantage $\text{Adv}_{\text{Sig}, \mathbf{F}}^{\text{euf-cma}}(\lambda)$ is negligible in λ .

3 Witness-Indistinguishable Argument with Σ -Protocol for Bundled Witness Space

In this section, we propose a generic construction of an interactive argument system that is a witness-indistinguishable argument system for a newly introduced *bundled witness space*. Our protocol of the interactive argument system is an AND-composition of Σ -protocols together with a commitment scheme, which is to prove the knowledge of witness pairs each of which consists of two components; one is a common component (such as a global identity string) and the other is an individual component (such as a digital signature issued by an individual authority on the global identity). We prove that our protocol is certainly a Σ -protocol. Finally, we prove that our interactive argument system with the protocol is perfectly witness-indistinguishable under the condition that the employed commitment scheme is perfectly hiding and the component Σ -protocols are perfectly witness-indistinguishable.

3.1 Building Blocks

Component Interactive Argument Systems with Σ -protocols For a polynomially bounded integer n , let A be the set of indices: $A := \{1, \dots, n\}$. We start with an efficiently computable predicate Φ^a for each $a \in A$, which determines an NP witness relation R^a :

$$R^a = \{(x^a, w^a) \in \{0, 1\}^* \times \{0, 1\}^* \mid \Phi^a(x^a, w^a) = \text{TRUE}\}, a \in A. \quad (11)$$

We suppose for each $a \in A$ that there is an interactive argument system $\Pi^a = (\Pi.\text{Setup}, \text{P}^a, \text{V}^a)$ which is executed in accordance with a Σ -protocol for the relation R^a :

$$\Sigma^a = (\Sigma_{\text{com}}^a, \Sigma_{\text{cha}}^a, \Sigma_{\text{res}}^a, \Sigma_{\text{vrf}}^a, \Sigma_{\text{ext}}^a, \Sigma_{\text{sim}}^a). \quad (12)$$

We suppose further that the witness space W^a decomposes into two components $W^a = W_0^a \times W_1^a$ for each $a \in A$. *In this paper, our interest is in the case that all the 0th components $W_0^a, a \in A$, are equal*, which we denote by W_0 . We call the common set W_0 the *base witness space* of the witness spaces W^a for $a \in A$, and an element $w_0 \in W_0$ a *base witness point*. Further, a witness $w^a \in W^a$ should consist of w_0 and w_1^a , where *the base witness point w_0 is common for $a \in A$* . That is, we will study the following type of witnesses w^a for $a \in A$;

$$\begin{aligned} W^a &= W_0 \times W_1^a, \\ \cup & \quad \cup \\ w^a &= (\exists w_0, \exists w_1^a). \end{aligned} \quad (13)$$

Commit-and-Prove Scheme with Σ -protocol To construct an interactive argument system for the relations $(R^a)^{a \in A}$ with the base witness space W_0 , we employ a commit-and-prove scheme with a Σ -protocol: $\text{CmtPrv} = (\text{CmtPrv}.\text{Setup}, \text{Cmt} = (\text{Cmt}.\text{Com}, \text{Cmt}.\text{Vrf}), \Pi_0 = (\text{P}_0, \text{V}_0))$, where the predicate Φ_0 and the relation R_0 is defined as follows, and Π_0 is executed in accordance with a Σ -protocol Σ_0 :

$$\Phi_0(c_0, (w_0, r_0)) \stackrel{\text{def}}{=} (\text{Cmt}.\text{Com}(w_0; r_0) =? (c_0, r_0)), \quad (14)$$

$$R_0 \stackrel{\text{def}}{=} \{(c_0, (w_0, r_0)) \in \{0, 1\}^* \times (\{0, 1\}^*)^2 \mid \Phi_0(c_0, (w_0, r_0)) = \text{TRUE}\}, \quad (15)$$

$$\Sigma_0 = (\Sigma_{0,\text{com}}, \Sigma_{0,\text{cha}}, \Sigma_{0,\text{res}}, \Sigma_{0,\text{vrf}}, \Sigma_{0,\text{ext}}, \Sigma_{0,\text{sim}}). \quad (16)$$

Note that a message m to be committed is a base witness point w_0 .

3.2 On the Construction of a Σ -protocol for Simultaneous Satisfiability

We introduce for each $a \in A$ the following composed relation determined by the two predicates Φ^a and Φ_0 . That is, the relation R_0^a is for *simultaneous satisfiability* of the two predicates Φ^a and Φ_0 on the base witness point w_0 :

$$R_0^a := \left\{ (x_0^a = (x^a, c_0), w_0^a = (w_0, w_1^a, r_0)) \mid \left\{ \begin{array}{l} \Phi^a(x^a, (w_0, w_1^a)) = \text{TRUE} \text{ and} \\ \Phi_0(c_0, (w_0, r_0)) = \text{TRUE} \end{array} \right. \right\}, a \in A. \quad (17)$$

We require here that the Σ -protocols Σ^a and Σ_0 can be merged into a single Σ -protocol Σ_0^a of an interactive argument system $\Pi_0^a = (\Pi.\text{Setup}, \text{CmtPrv}.\text{Setup}, \text{P}_0^a, \text{V}_0^a)$ for the above relation R_0^a :

$$\Sigma_0^a = (\Sigma_{0,\text{com}}^a, \Sigma_{0,\text{cha}}^a, \Sigma_{0,\text{res}}^a, \Sigma_{0,\text{vrf}}^a, \Sigma_{0,\text{ext}}^a, \Sigma_{0,\text{sim}}^a). \quad (18)$$

- $\Sigma_{0,\text{com}}^a(x_0^a, w_0^a) \rightarrow (\text{COM}^a, \text{COM}_0^a, \overline{St}_0^a)$. This PPT algorithm is executed by P_0^a . On input a statement $x_0^a = (x^a, c_0)$ and a witness $w_0^a = (w_0, w_1^a, r_0)$, it executes the algorithms $\Sigma_{\text{com}}^a(x^a, (w_0, w_1^a))$ and $\Sigma_{0,\text{com}}(c_0, (w_0, r_0))$. It obtains the commitment messages and the inner states, (COM^a, St^a) and (COM_0^a, St_0^a) , respectively. There is a constraint that the knowledge extractor $\Sigma_{0,\text{ext}}^a$ should return a witness which simultaneously satisfies the two predicates Φ^a and Φ_0 on the base witness point w_0 . It sets the state as $\overline{St}_0^a := (St^a, St_0^a)$. It returns $(\text{COM}^a, \text{COM}_0^a, \overline{St}_0^a)$. P_0^a sends $(\text{COM}^a, \text{COM}_0^a)$ to V_0^a as a commitment message, and keeps the state \overline{St}_0^a .
- $\Sigma_{0,\text{cha}}^a(x_0^a) \rightarrow \text{CHA}$. This PPT algorithm is executed by V_0^a . On input the statement x_0^a , it reads out the size of the security parameter as 1^λ and chooses a challenge message $\text{CHA} \in_R \text{CHASP}(1^\lambda)$. It returns CHA . V_0^a sends CHA to P_0^a as a challenge message.
- $\Sigma_{0,\text{res}}^a(\overline{St}_0^a, \text{CHA}) \rightarrow (\text{RES}^a, \text{RES}_0^a)$. This PPT algorithm is executed by P_0^a . On input the state \overline{St}_0^a and the challenge message CHA , it executes the algorithms $\Sigma_{\text{res}}^a(St^a, \text{CHA})$ and $\Sigma_{0,\text{res}}(St_0^a, \text{CHA})$. It obtains the response messages RES^a and RES_0^a , respectively. There is the constraint that the knowledge extractor $\Sigma_{0,\text{ext}}^a$ should return a witness which simultaneously satisfies Φ^a and Φ_0 on w_0 . It returns $(\text{RES}^a, \text{RES}_0^a)$. P_0^a sends $(\text{RES}^a, \text{RES}_0^a)$ to V_0^a as a response message.
- $\Sigma_{0,\text{vrf}}^a(x_0^a, (\text{COM}^a, \text{COM}_0^a), \text{CHA}, (\text{RES}^a, \text{RES}_0^a)) \rightarrow d$. This deterministic polynomial-time algorithm is executed by V_0^a . On input the statement $x_0^a = (x^a, c_0)$ and all the messages $(\text{COM}^a, \text{COM}_0^a)$, CHA and $(\text{RES}^a, \text{RES}_0^a)$, it executes the algorithms $\Sigma_{\text{vrf}}^a(x^a, \text{COM}^a, \text{CHA}, \text{RES}^a)$ and $\Sigma_{0,\text{vrf}}(c_0, \text{COM}_0^a, \text{CHA}, \text{RES}_0^a)$. It obtains two boolean decisions d^a and d_0^a . If the both d^a and d_0^a are 1, then it returns $d := 1$, and otherwise $d := 0$. V_0^a returns d as the decision of the interactive protocol on x_0^a .
- $\Sigma_{0,\text{ext}}^a(x_0^a, (\text{COM}^a, \text{COM}_0^a), \text{CHA}, (\text{RES}^a, \text{RES}_0^a), \text{CHA}', (\text{RES}^{a'}, \text{RES}_0^{a'})) \rightarrow (\hat{w}_0^a, \hat{w}_1^a, \hat{r}_0^a)$. This PPT algorithm is for knowledge extraction. On input the statement $x_0^a = (x^a, c_0)$ and two accepting transcripts $((\text{COM}^a, \text{COM}_0^a), \text{CHA}, (\text{RES}^a, \text{RES}_0^a))$ and $((\text{COM}^a, \text{COM}_0^a), \text{CHA}', (\text{RES}^{a'}, \text{RES}_0^{a'}))$, $\text{CHA} \neq \text{CHA}'$, it executes the algorithms $\Sigma_{\text{ext}}^a(x^a, \text{COM}^a, \text{CHA}, \text{RES}^a, \text{CHA}', \text{RES}^{a'})$ and $\Sigma_{0,\text{ext}}(c_0, \text{COM}_0^a, \text{CHA}, \text{RES}_0^a, \text{CHA}', \text{RES}_0^{a'})$. It obtains witnesses $(\hat{w}_0^a, \hat{w}_1^a)$ and $(\bar{w}_0^a, \hat{r}_0^a)$ which satisfy $(x^a, (\hat{w}_0^a, \hat{w}_1^a)) \in R^a$ and $(c_0, (\bar{w}_0^a, \hat{r}_0^a)) \in R_0$ with an overwhelming probability in $|x^a|$ and $|c_0|$, respectively. Note here that the commitment messages are common and the challenge messages are different. The simultaneous satisfiability on w_0 should assure the following equality:

$$\hat{w}_0^a = \bar{w}_0^a \text{ with probability one.} \quad (19)$$

It returns $(\hat{w}_0^a, \hat{w}_1^a, \hat{r}_0^a)$.

- $\Sigma_{0,\text{sim}}^a(x_0^a, \tilde{\text{CHA}}) \rightarrow ((\tilde{\text{COM}}^a, \tilde{\text{COM}}_0^a), (\tilde{\text{RES}}^a, \tilde{\text{RES}}_0^a))$. This PPT algorithm is for the simulation of an accepting transcript. On input a statement $x_0^a = (x^a, c_0)$ and a uniform random string $\tilde{\text{CHA}} \in_R \text{CHASP}(1^\lambda)$, it executes the algorithms $\Sigma_{\text{sim}}^a(x^a, \tilde{\text{CHA}})$ and $\Sigma_{0,\text{sim}}(c_0, \tilde{\text{CHA}})$. It obtains the remaining part of the transcripts $(\tilde{\text{COM}}^a, \tilde{\text{RES}}^a)$ and $(\tilde{\text{COM}}_0^a, \tilde{\text{RES}}_0^a)$, respectively. The simulated messages $((\tilde{\text{COM}}^a, \tilde{\text{COM}}_0^a), \tilde{\text{CHA}}, (\tilde{\text{RES}}^a, \tilde{\text{RES}}_0^a))$ should form a distribution $\text{dist}((\tilde{\text{COM}}^a, \tilde{\text{COM}}_0^a), \tilde{\text{CHA}}, (\tilde{\text{RES}}^a, \tilde{\text{RES}}_0^a) \mid \text{generated by } \text{CHASP}(1^\lambda) \text{ and } \Sigma_{0,\text{sim}}^a(x_0^a, \tilde{\text{CHA}}))$ which is identical to the distribution $\text{dist}((\text{COM}^a, \text{COM}_0^a), \text{CHA}, (\text{RES}^a, \text{RES}_0^a) \mid \text{real accepting transcript})$.

Remark To construct the algorithm $\Sigma_{0,\text{com}}^a$ of commitment message and the algorithm $\Sigma_{0,\text{res}}^a$ of response message is a non-trivial task. That is, we have to construct $\Sigma_{0,\text{com}}^a$ and $\Sigma_{0,\text{res}}^a$ so that the knowledge extractor $\Sigma_{0,\text{ext}}^a$ returns a witness which *simultaneously* satisfies Φ^a and Φ_0 on a base witness point w_0 . The idea of the construction is to use a common random tape to generate commitment messages COM^a and COM_0^a , but we do not describe the inner treatment of the random tapes in $\Sigma_{0,\text{com}}^a$ and $\Sigma_{0,\text{res}}^a$ for generality. Hence our approach is to show the construction when we instantiate the Σ -protocol Σ_0^a . In Section A we actually demonstrate the construction of Σ_0^a in an algebraic setting.

3.3 Bundled Witness Space

We now introduce an NP witness relation for our *bundled witness spaces*. We first fix the base witness point w_0 in the base witness space W_0 and consider a subset $R_{w_0}^a$ for each NP witness relation $R^a, a \in A$:

$$R_{w_0}^a := \{(x^a, w^a) \in R^a \mid w^a = (w_0, w_1^a) \text{ for some } w_1^a\} \subset R^a, a \in A. \quad (20)$$

Then we run the base witness point w_0 to claim the following property.

Claim 1 *For a polynomially bounded integer n , let A be the set of indices $\{1, \dots, n\}$. Then we have:*

$$\bigcup_{w_0 \in W_0} \left(\prod_{a \in A} R_{w_0}^a \right) \subset \prod_{a \in A} \left(\bigcup_{w_0 \in W_0} R_{w_0}^a \right) = \prod_{a \in A} R^a. \quad (21)$$

Proof. The equality of the right-hand side is because $\bigcup_{w_0 \in W_0} R_{w_0}^a = R^a$. An element of the left hand side is of the form $(x^1, (w_0, w_1^1)), \dots, (x^n, (w_0, w_1^n))$ where $w_0 \in W_0$ and $(x^a, (w_0, w_1^a)) \in R^a$ for each $a \in A$. This is an element of $\prod_{a \in A} R^a$, and hence the inclusion follows. \square

Deleting the redundancy, we obtain the following one-to-one correspondence as sets (\simeq):

$$R_{\text{bund}}^{a \in A} \stackrel{\text{def}}{=} \{((x^a)^{a \in A}, w_0, (w_1^a)^{a \in A}) \in \{0, 1\}^* \times (\{0, 1\}^*)^2 \mid (x^a, (w_0, w_1^a)) \in R^a, a \in A\} \quad (22)$$

$$\simeq \bigcup_{w_0 \in W_0} \left(\prod_{a \in A} R_{w_0}^a \right). \quad (23)$$

Claim 2 *For a polynomially bounded integer n , let A be the set of indices $\{1, \dots, n\}$. Then the relation $R_{\text{bund}}^{a \in A}$ is an NP relation.*

Proof. We first note that the number of indices $|A|$ is polynomially bounded. To bound the bit lengths of witnesses by a fixed polynomial, let $\text{poly}^a(\cdot)$ denote for each $a \in A$ the polynomial which bounds the bit lengths of witnesses: $|w^a| < \text{poly}^a(|x^a|)$ for $(x^a, w^a) \in R^a$. Let a polynomial $\text{poly}(\cdot)$ be the sum: $\text{poly}(\cdot) := \sum_{a \in A} \text{poly}^a(\cdot)$. Then $\text{poly}(\cdot)$ bounds the bit length of the witness as

$$|w_0, (w_1^a)^{a \in A}| \leq |(w_0, w_1^a)^{a \in A}| = |(w^a)^{a \in A}| \leq \sum_{a \in A} \text{poly}^a(|x^a|) \leq \sum_{a \in A} \text{poly}^a(|(x^a)^{a \in A}|) = \text{poly}(|(x^a)^{a \in A}|). \quad (24)$$

As for efficiency of deciding the membership of the relation $R_{\text{bund}}^{a \in A}$, we just remember that the number of indices $|A|$ is polynomially bounded. \square

Definition 1 (Relation for Bundled Witness Spaces) *For a polynomially bounded integer n , an NP witness relation for the bundled witness spaces is defined as $R_{\text{bund}}^{a \in A}$.*

Definition 2 (Bundled Witness Spaces) *For a polynomially bounded integer n , let A be the set of indices $\{1, \dots, n\}$. Let $R^a, a \in A$ be NP witness relations where each witness space decomposes $W^a = W_0 \times W_1^a, a \in A$. Then the bundled witness spaces is defined as follows.*

$$W_{\text{bund}}^{a \in A} \stackrel{\text{def}}{=} W_0 \times (W_1^a)^{a \in A}. \quad (25)$$

3.4 Generic Construction of a Σ -protocol for the Bundled Witness Space

By using the above Σ -protocols $(\Sigma_0^a)^{a \in A}$ and a commitment generation algorithm **Cmt.Com**, we construct an interactive argument system $\Pi_{\text{bund}}^{a \in A} = (\text{P}, \text{V})$ for the witness relation $R_{\text{bund}}^{a \in A}$ with a protocol $\Sigma_{\text{bund}}^{a \in A} \cdot \Sigma_{\text{bund}}^{a \in A}$ is actually a Σ -protocol, which consists of the six PPT algorithms described below (see also Fig.1):

$$\Sigma_{\text{bund}}^{a \in A} = (\Sigma_{\text{bund}, \text{com}}^{a \in A}, \Sigma_{\text{bund}, \text{cha}}^{a \in A}, \Sigma_{\text{bund}, \text{res}}^{a \in A}, \Sigma_{\text{bund}, \text{verf}}^{a \in A}, \Sigma_{\text{bund}, \text{ext}}^{a \in A}, \Sigma_{\text{bund}, \text{sim}}^{a \in A}). \quad (26)$$

- $\Sigma_{\text{bnd,com}}^{a \in A}((x^a)^{a \in A}, (w_0, (w_1^a)^{a \in A})) \rightarrow (c_0, (\text{COM}^a, \text{COM}_0^a)^{a \in A}, St)$. This PPT algorithm is executed by P. On input a statement that is a vector $(x^a)^{a \in A}$ and a witness that is a vector $(w_0, (w_1^a)^{a \in A})$, it computes a commitment c_0 to the base witness point w_0 with a randomness $r_0 \in_R \{0, 1\}^\lambda$ by running the commitment generation algorithm of Cmt: $(c_0, r_0) \leftarrow \text{Cmt.Com}(w_0; r_0)$. It sets the extended statement as $x_0^a := (x^a, c_0)$ and the extended witness as $w_0^a := (w_0, w_1^a, r_0)$ for each $a \in A$. It executes for each $a \in A$ the algorithm $\Sigma_{0,\text{com}}^a(x_0^a, w_0^a)$. It obtains $(\text{COM}^a, \text{COM}_0^a, \overline{St}_0^a)$. It sets the state as $St := (\overline{St}_0^a)^{a \in A}$. It returns $(c_0, (\text{COM}^a, \text{COM}_0^a)^{a \in A}, St)$. P sends $(c_0, (\text{COM}^a, \text{COM}_0^a)^{a \in A})$ to V as a commitment message, and keeps the state St .
- $\Sigma_{\text{bnd,cha}}^{a \in A}((x^a)^{a \in A}) \rightarrow \text{CHA}$. This PPT algorithm is executed by V. On input the statement $(x^a)^{a \in A}$, it reads out the size of the security parameter as 1^λ and chooses a challenge message $\text{CHA} \in_R \text{CHASP}(1^\lambda)$. It returns CHA. V_0^a sends CHA to P_0^a as a challenge message.
- $\Sigma_{\text{bnd,res}}^{a \in A}(St, \text{CHA}) \rightarrow (\text{RES}^a, \text{RES}_0^a)^{a \in A}$. This PPT algorithm is executed by P. On input the state St and the challenge message CHA, it executes for each $a \in A$ the algorithm $\Sigma_{0,\text{res}}^a(\overline{St}_0^a, \text{CHA})$. It obtains $(\text{RES}^a, \text{RES}_0^a)$. It returns $(\text{RES}^a, \text{RES}_0^a)$. P sends $(\text{RES}^a, \text{RES}_0^a)^{a \in A}$ to V as a response message.
- $\Sigma_{\text{bnd,vrf}}^{a \in A}((x^a)^{a \in A}) \rightarrow d$. This deterministic polynomial-time algorithm is executed by V. On input the statement $(x^a)^{a \in A}$ and all the messages $(c_0, (\text{COM}^a, \text{COM}_0^a)^{a \in A})$, CHA and $(\text{RES}^a, \text{RES}_0^a)^{a \in A}$, it first sets the extended statement as $x_0^a := (x^a, c_0)$ for each $a \in A$. Then it executes for each $a \in A$ the algorithm $\Sigma_{0,\text{vrf}}^a(x_0^a, \text{COM}^a, \text{COM}_0^a, \text{CHA}, \text{RES}^a, \text{RES}_0^a)$. It obtains boolean decisions. If all the decisions are 1, then V returns 1, and otherwise, 0.

These four algorithms $(\Sigma_{\text{bnd,com}}^{a \in A}, \Sigma_{\text{bnd,cha}}^{a \in A}, \Sigma_{\text{bnd,res}}^{a \in A}, \Sigma_{\text{bnd,vrf}}^{a \in A})$ must satisfy the following property.

Proposition 1 (Completeness) *If Cmt is correct, and if Σ_0^a is complete for each $a \in A$, then our $\Sigma_{\text{bnd}}^{a \in A}$ is complete.*

Proof. The completeness of our $\Pi_{\text{bnd}}^{a \in A}$ comes from the correctness of Cmt and the completeness of Π_0^a for each $a \in A$. \square

- $\Sigma_{\text{bnd,ext}}^{a \in A}((x^a)^{a \in A}, (c_0, (\text{COM}^a, \text{COM}_0^a)^{a \in A}), \text{CHA}, (\text{RES}^a, \text{RES}_0^a)^{a \in A}, \text{CHA}', ((\text{RES}^a)', (\text{RES}_0^a)')^{a \in A}) \rightarrow (\hat{w}_0, (\hat{w}_1^a)^{a \in A})$. This PPT algorithm is for knowledge extraction. On input the statement $(x^a)^{a \in A}$ and two accepting transcripts $((c_0, (\text{COM}^a, \text{COM}_0^a)^{a \in A}), \text{CHA}, (\text{RES}^a, \text{RES}_0^a)^{a \in A})$ and $((c_0, (\text{COM}^a, \text{COM}_0^a)^{a \in A}), \text{CHA}', (\text{RES}^{a'}, \text{RES}_0^{a'})^{a \in A})$, $\text{CHA} \neq \text{CHA}'$, it first sets the extended statement as $x_0^a := (x^a, c_0)$ for each $a \in A$. Note here that commitment messages are common and challenge messages are different. Then it executes for each $a \in A$ the algorithm $\Sigma_{0,\text{ext}}^a(x_0^a, (\text{COM}^a, \text{COM}_0^a), \text{CHA}, (\text{RES}^a, \text{RES}_0^a), \text{CHA}', (\text{RES}^{a'}, \text{RES}_0^{a'}))$. It obtains $(\hat{w}_0^a, \hat{w}_1^a, \hat{r}_0^a)$. If this event does not occur (i.e. at least at one a $\Sigma_{0,\text{ext}}^a$ fails to extract a witness), then it returns \perp . Otherwise, if $\hat{w}_0^a = \hat{w}_0^{a'}$ for any $a, a' \in A$, then it sets the common value $\hat{w}_0 := \hat{w}_0^a$ and returns $(\hat{w}_0, (\hat{w}_1^a)^{a \in A})$. Otherwise it returns \perp^* . The binding property of the commitment scheme Cmt assures that the former case holds with an overwhelming probability, as claimed in the following proposition.

Proposition 2 (Special Soundness) *If Cmt is correct and computationally binding, and if Σ_0^a has the special soundness for each $a \in A$, then our $\Sigma_{\text{bnd}}^{a \in A}$ has the special soundness.*

Proof. By employing $(\Sigma_{\text{bnd,com}}^{a \in A}, \Sigma_{\text{bnd,cha}}^{a \in A}, \Sigma_{\text{bnd,res}}^{a \in A}, \Sigma_{\text{bnd,vrf}}^{a \in A}, \Sigma_{\text{bnd,ext}}^{a \in A})$ as subroutines, we construct a PPT algorithm **A** that breaks the binding property of Cmt in accordance with the experiment $\text{Exp}_{\text{Cmt,A}}^{\text{bind}}(1^\lambda)$. **A** is given as input the set of public parameter pp_{CmtPrv} . **A** first reads out the security parameter 1^λ from pp_{CmtPrv} , and executes the setup algorithms $\Pi.\text{Setup}(1^\lambda)$. It obtains the set of public parameter pp_Π . **A** merges the sets of public parameter as $pp := (pp_\Pi, pp_{\text{CmtPrv}})$. Then **A** executes $\Pi_{\text{bnd}}^{a \in A} = (\text{P}, \text{V})$. If the decision d of V is 1, then **A** rewinds P back to the timing at which P had sent the challenge message CHA of the protocol $\Sigma_{\text{bnd}}^{a \in A}$. If the decision d of V is again 1, **A** executes the knowledge extractor $\Sigma_{\text{bnd,ext}}^{a \in A}$ on input $((x^a)^{a \in A}, (c_0, (\text{COM}^a, \text{COM}_0^a)^{a \in A}), \text{CHA}, (\text{RES}^a, \text{RES}_0^a)^{a \in A}, \text{CHA}', ((\text{RES}^a)', (\text{RES}_0^a)')^{a \in A})$. If $\Sigma_{\text{bnd,ext}}^{a \in A}$ outputs \perp^* , then there must be a pair $a, a' \in A^*, a \neq a'$ such that $(\hat{w}_0^a, \hat{w}_1^a, \hat{r}_0^a)$ and $(\hat{w}_0^{a'}, \hat{w}_1^{a'}, \hat{r}_0^{a'})$ pass the verification Cmt.Vrf and $\hat{w}_0^a \neq \hat{w}_0^{a'}$. The vector $(c_0, \hat{w}_0^a, \hat{r}_0^a, \hat{w}_0^{a'}, \hat{r}_0^{a'})$ breaks the binding property to yields

WIN in $\text{Exp}_{\text{Cmt}, \mathbf{A}}^{\text{bind}}(1^\lambda)$. This completes the description of \mathbf{A} , and the following equality holds.

$$\mathbf{Adv}_{\text{Cmt}, \mathbf{A}}^{\text{bind}}(\lambda) = \Pr[\Sigma_{\text{bind}, \text{ext}}^{a \in A} \text{ returns } \perp^*] \quad (27)$$

$$= 1 - (\Pr[\Sigma_{\text{bind}, \text{ext}}^{a \in A} \text{ returns } (\hat{w}_0, (\hat{w}_1^a)^{a \in A})] + \Pr[\Sigma_{\text{bind}, \text{ext}}^{a \in A} \text{ returns } \perp]). \quad (28)$$

Therefore,

$$\Pr[\Sigma_{\text{bind}, \text{ext}}^{a \in A} \text{ returns } (\hat{w}_0, (\hat{w}_1^a)^{a \in A})] = 1 - (\mathbf{Adv}_{\text{Cmt}, \mathbf{A}}^{\text{bind}}(\lambda) + \Pr[\Sigma_{\text{bind}, \text{ext}}^{a \in A} \text{ returns } \perp]) \quad (29)$$

$$= 1 - (\mathbf{Adv}_{\text{Cmt}, \mathbf{A}}^{\text{bind}}(\lambda) + (1 - \prod_{a \in A} \Pr[\Sigma_{0, \text{ext}}^a \text{ returns a witness}])). \quad (30)$$

The right-hand side is an overwhelming probability because $\Pr[\Sigma_{0, \text{ext}}^a \text{ returns a witness}]$ is an overwhelming probability for each $a \in A$ and $|A|$ is bounded by a polynomial in $|x|$. \square

Note 3: Our Use Case For simplicity of the later discussion, we hereafter assume that, for all $a \in A$, $\Pr[\Sigma_{0, \text{ext}}^a \text{ returns a witness}] = 1$. That is, we assume that $\Pr[\Sigma_{0, \text{ext}}^a \text{ returns } \perp] = 0$ for each $a \in A$.

- $\Sigma_{\text{bind}, \text{sim}}^{a \in A}((x^a)^{a \in A}, \text{c}\check{\text{H}}\text{A}) \rightarrow ((\tilde{c}_0, (\text{C}\check{\text{O}}\text{M}^a, \text{C}\check{\text{O}}\text{M}_0^a)^{a \in A}), (\text{R}\check{\text{E}}\text{S}^a, \text{R}\check{\text{E}}\text{S}_0^a)^{a \in A})$. This PPT algorithm is for the simulation of an accepting transcript. On input a statement $(x^a)^{a \in A}$ and a uniform random string $\text{c}\check{\text{H}}\text{A} \in_R \text{CHASP}(1^\lambda)$, it first chooses a base witness point $\tilde{w}_0 \in_R W_0$ uniformly at random, and executes the commitment generation algorithm with a randomness \tilde{r}_0 , $\text{Cmt.Com}(\tilde{w}_0; \tilde{r}_0) \rightarrow (\tilde{c}_0, \tilde{r}_0)$. It obtains a commitment \tilde{c}_0 . Then it sets the extended statement as $x_0^a := (x^a, \tilde{c}_0)$ for each $a \in A$. Then it executes for each $a \in A$ the algorithm $\Sigma_{0, \text{sim}}^a(x_0^a, \text{c}\check{\text{H}}\text{A})$. It obtains $((\text{C}\check{\text{O}}\text{M}^a, \text{C}\check{\text{O}}\text{M}_0^a), (\text{R}\check{\text{E}}\text{S}^a, \text{R}\check{\text{E}}\text{S}_0^a))$. It returns $((\tilde{c}_0, (\text{C}\check{\text{O}}\text{M}^a, \text{C}\check{\text{O}}\text{M}_0^a)^{a \in A}), (\text{R}\check{\text{E}}\text{S}^a, \text{R}\check{\text{E}}\text{S}_0^a)^{a \in A})$.

Proposition 3 (Honest-Verifier Zero-Knowledge) *If Cmt is perfectly hiding, and if Σ_0^a is honest-verifier zero-knowledge for each $a \in A$, then our $\Sigma_{\text{bind}}^{a \in A}$ is honest-verifier zero-knowledge.*

Proof. The perfectly hiding property assures that the distribution of simulated commitment \tilde{c}_0 is the same as the real. Then on input $(x_0^a, \text{c}\check{\text{H}}\text{A})$, the simulator $\Sigma_{0, \text{sim}}^a$ works to return the remaining part of the simulated transcript, $((\text{C}\check{\text{O}}\text{M}^a, \text{C}\check{\text{O}}\text{M}_0^a), (\text{R}\check{\text{E}}\text{S}^a, \text{R}\check{\text{E}}\text{S}_0^a))$ for each $a \in A$. Then, the merged transcripts $((\tilde{c}_0, (\text{C}\check{\text{O}}\text{M}^a, \text{C}\check{\text{O}}\text{M}_0^a)^{a \in A}), (\text{R}\check{\text{E}}\text{S}^a, \text{R}\check{\text{E}}\text{S}_0^a)^{a \in A})$ is identically distributed to the real. \square

Theorem 1 *If Cmt is correct, computationally binding and perfectly hiding, and if Σ_0^a is a Σ -protocol for each $a \in A$, then our protocol $\Sigma_{\text{bind}}^{a \in A}$ is a Σ -protocol.*

Proof. Propositions 1, 2 and 3 deduces that $\Sigma_{\text{bind}}^{a \in A}$ is a Σ -protocol. \square

Theorem 2 *If the component interactive proof system Π_0^a with Σ_0^a is perfectly witness-indistinguishable for each $a \in A$, and if Cmt is perfectly hiding, then our interactive argument system $\Pi_{\text{bind}}^{a \in A}$ with $\Sigma_{\text{bind}}^{a \in A}$ is perfectly witness-indistinguishable.*

Proof. The transcripts form a distribution $\text{dist}^{a \in A} := \text{dist}((c_0, (\text{COM}^a, \text{COM}_0^a)^{a \in A}), \text{CHA}, (\text{RES}^a, \text{RES}_0^a)^{a \in A})$, where the challenge message CHA is chosen by any given PPT verifier \mathbf{V}^* on input a set of statements $(x^a)^{a \in A}$, any given auxiliary input z and a commitment message $(c_0, (\text{COM}^a, \text{COM}_0^a)^{a \in A})$. If Cmt is perfectly hiding, then the distribution of the commitment c_0 is identical even if the committed element w_0 varies. For each $a \in A$, if Π_0^a is perfectly witness-indistinguishable, then the distribution of the commitment message and the response message $\text{dist}((\text{COM}^a, \text{COM}_0^a), (\text{RES}^a, \text{RES}_0^a))$ are identical even if the witness (w_0, w_1^a) varies and even if CHA chosen by $\mathbf{V}^*((x^a)^{a \in A}, z, (c_0, (\text{COM}^a, \text{COM}_0^a)^{a \in A}))$ deviates from the uniform random distribution. Therefore, for all $a \in A$, the distribution $\text{dist}^{a \in A}$ is identical even if the witness $(w_0, (w_1^a)_{a \in A})$ varies. \square

Note. OR-composition and Boolean Formulas The OR-proof, and more generally the proof for monotone formulas, are also possible for our Σ -protocol $\Sigma_{\text{bind}}^{a \in A}$ (see [CDS94, AAS14]).

4 Decentralized Multi-Authority Anonymous Authentication Scheme

In this section, we give a syntax and security definitions of an interactive anonymous authentication scheme in a decentralized multi-authority setting on key generation.

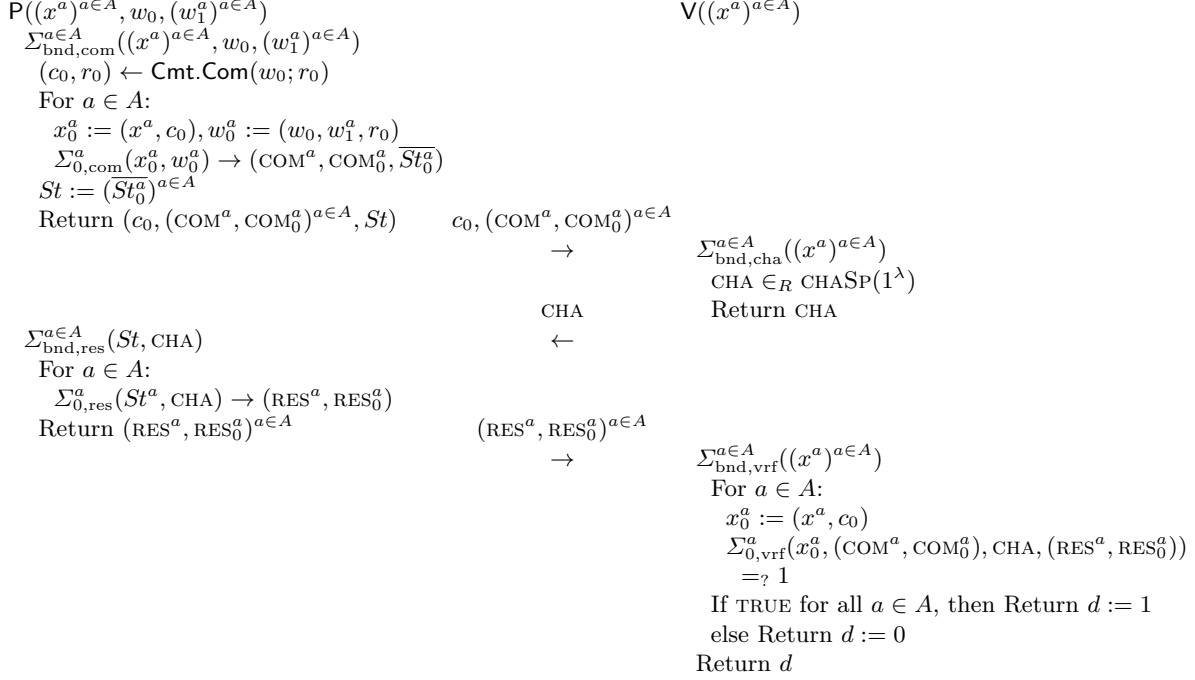


Fig. 1. The protocol $\Sigma_{\text{bnd}}^{a \in A}$ of our proof system $\Pi_{\text{bnd}}^{a \in A}$ for the NP witness relation $R_{\text{bnd}}^{a \in A}$.

4.1 Syntax and Security Definitions

Our scheme **a-auth** consists of five PPT algorithms, (**Setup**, **AuthKG**, **SKG**, **P**, **V**).

- **Setup**(1^λ) $\rightarrow pp$. This PPT algorithm is needed to generate a set of public parameter pp . On input the security parameter 1^λ , it generates the set pp . It returns pp .
- **AuthKG**($1^\lambda, a$) $\rightarrow (\text{PK}^a, \text{MSK}^a)$. This PPT algorithm is executed by a key-issuing authority indexed by a positive integer a . On input the security parameter 1^λ and the authority index a , it generates the a -th public key PK^a of the authority and the corresponding a -th master secret key MSK^a . It returns $(\text{PK}^a, \text{MSK}^a)$.
- **SKG**($\text{PK}^a, \text{MSK}^a, \text{gid}$) $\rightarrow \text{sk}_{\text{gid}}^a$. This PPT algorithm is executed by the a -th key-issuing authority. On input the a -th public and master secret keys $(\text{PK}^a, \text{MSK}^a)$ and a string gid of a prover (a global identity string), it generates a private secret key sk_{gid}^a of a prover. It returns sk_{gid}^a .
- $\langle \mathbf{P}((\text{PK}^a, \text{sk}_{\text{gid}}^a)^{a \in A'}), \mathbf{V}((\text{PK}^a)^{a \in A'}) \rangle \rightarrow d$. These two interactive PPT algorithms are a prover who is to be authenticated, and a verifier who confirms that the prover certainly knows the secret keys for indices $a \in A'$, respectively, where A' denotes a subset of all indices at which the prover is issued her private secret keys by authorities. On input the public keys $(\text{PK}^a)^{a \in A}$ to **P** and **V** and the corresponding private secret keys $(\text{sk}_{\text{gid}}^a)^{a \in A}$ to **P**, **P** and **V** interact with each other. After at most polynomially many (in λ) moves of messages between **P** and **V**, **V** returns $d := 1$ (“accept”) or $d := 0$ (“reject”).

We discuss two security notions for our authentication scheme **a-auth**; security against concurrent and collusion attacks that yield misauthentication, and anonymity for privacy of provers’ global identities.

Security against Concurrent and Collusion Attack of Misauthentication One of the possible attacks to cause misauthentication is the concurrent and collusion attack on our **a-auth**. For a formal treatment we define the

following experiment on **a-auth** and an adversary algorithm **A**.

$$\text{Exp}_{\mathbf{a-auth}, \mathbf{A}}^{\text{conc-coll}}(1^\lambda) : \quad (31)$$

$$(q_A, St) \leftarrow \mathbf{A}(1^\lambda), A := \{1, \dots, q_A\}, pp \leftarrow \text{Setup}(1^\lambda), \text{For } a \in A : (\text{PK}^a, \text{MSK}^a) \leftarrow \text{AuthKG}(1^\lambda, a) \quad (32)$$

$$(q_I, St) \leftarrow \mathbf{A}(St, pp, (\text{PK}^a)^{a \in A}), I := \{1, \dots, q_I\}, ((\text{gid}_i)_{i \in I}, St) \leftarrow \mathbf{A}(St) \quad (33)$$

$$\text{For } a \in A : \text{For } i \in I : \text{sk}_{\text{gid}_i}^a \leftarrow \text{SKG}(\text{PK}^a, \text{MSK}^a, \text{gid}_i) \quad (34)$$

$$(A^*, St) \leftarrow \mathbf{A}^{\text{P}((\text{PK}^a, \text{sk}_{\text{gid}_i}^a)^{a \in A})_{i \in I}, \text{PrivKO}(\text{PK}^{\cdot}, \text{MSK}^{\cdot}, \cdot)}(St) \quad (35)$$

$$\langle \mathbf{A}(St), \text{V}((\text{PK}^a)^{a \in A^*}) \rangle \rightarrow d, \text{If } d = 1 \text{ then Return WIN else Return LOSE} \quad (36)$$

Intuitively, the above experiment describes the attack as follows. The adversary algorithm **A**, on input the security parameter 1^λ , first outputs the number q_A of key-issuing authorities. Then, on input the set of public parameter pp and the issued public keys $(\text{PK}^a)^{a \in A}$, **A** outputs the number q_I of provers with which **A** interacts concurrently (i.e. in arbitrarily interleaved order of messages). In addition, **A** collects at most q_{sk} private secret keys by issuing queries to the private secret key oracle **PrivKO**($\text{PK}^{\cdot}, \text{MSK}^{\cdot}, \cdot$) with an authority index $a \in A$ and a global identity string $\text{gid}_j \in \{0, 1\}^\lambda$ for $j = q_I + 1, \dots, q_I + q_{\text{sk}}$. We denote by A_j the set of authority indices for which the queries with the global identity string gid_j were issued. That is,

$$A_j := \{a \in A \mid \mathbf{A} \text{ receives } \text{sk}_{\text{gid}_j}^a\}, j = q_I + 1, \dots, q_I + q_{\text{sk}}. \quad (37)$$

We here require that the numbers q_A , q_I and q_{sk} are bounded by a polynomial in λ . At the last of this “learning phase”, **A** outputs a target set of authority indices A^* and its inner state St . Next, in the “attacking phase”, on input the inner state St , the adversary **A** interacts with the verifier $\text{V}((\text{PK}^a)^{a \in A^*})$. If the decision d of **V** is 1, then the experiment returns WIN and otherwise, returns LOSE.

A restriction is imposed on the adversary **A**: The target set of authority indices A^* should not be a subset of any single set A_j :

$$A^* \not\subseteq A_j, j = q_I + 1, \dots, q_I + q_{\text{sk}}. \quad (38)$$

This restriction is because, otherwise, **A** is given private secret keys for A^* on a single gid_{i^*} for some i^* , $q_I < i^* \leq q_I + q_{\text{sk}}$, and then **A** can trivially be accepted in the attacking phase.

The advantage of an adversary **A** over our authentication scheme **a-auth** in the experiment is defined as: $\text{Adv}_{\mathbf{a-auth}, \mathbf{A}}^{\text{conc-coll}}(\lambda) \stackrel{\text{def}}{=} \Pr[\text{Exp}_{\mathbf{a-auth}, \mathbf{A}}^{\text{conc-coll}}(1^\lambda) = \text{WIN}]$. An authentication scheme **a-auth** is called secure against concurrent and collusion attacks of misauthentication if, for any given PPT algorithm **A**, the advantage $\text{Adv}_{\mathbf{a-auth}, \mathbf{A}}^{\text{conc-coll}}(\lambda)$ is negligible in λ .

Anonymity As is explained in Section 1, a critical feature to be attained is provers’ anonymity on global identities when the provers are authenticated. For a formal treatment we define the following experiment on **a-auth** and an adversary algorithm **A**.

$$\text{Exp}_{\mathbf{a-auth}, \mathbf{A}}^{\text{ano}}(1^\lambda) : \quad (39)$$

$$(q_A, St) \leftarrow \mathbf{A}(1^\lambda), A := \{1, \dots, q_A\}, pp \leftarrow \text{Setup}(1^\lambda), \text{For } a \in A : (\text{PK}^a, \text{MSK}^a) \leftarrow \text{AuthKG}(1^\lambda, a) \quad (40)$$

$$\text{gid}_0, \text{gid}_1 \leftarrow \mathbf{A}(St, pp, (\text{PK}^a)^{a \in A}), \text{For } a \in A : \text{For } i \in \{0, 1\} : \text{sk}_{\text{gid}_i}^a \leftarrow \text{SKG}(\text{PK}^a, \text{MSK}^a, \text{gid}_i) \quad (41)$$

$$b \in_R \{0, 1\}, b^* \leftarrow \mathbf{A}^{\text{P}((\text{PK}^a, \text{sk}_{\text{gid}_b}^a)^{a \in A})}(St, (\text{sk}_{\text{gid}_0}^a, \text{sk}_{\text{gid}_1}^a)^{a \in A}) \quad (42)$$

$$\text{If } b = b^*, \text{ then Return WIN, else Return LOSE} \quad (43)$$

Intuitively, the above experiment describes the attack as follows. The adversary algorithm **A**, on input the security parameter 1^λ , first outputs the number q_A of key-issuing authorities. Then, on input the issued public keys $(\text{PK}^a)^{a \in A}$, **A** designates two identity strings gid_0 and gid_1 (as is usual in the indistinguishability games). Next, **A** interacts with a prover **P** on input the private secret keys $(\text{sk}_{\text{gid}_b}^a)^{a \in A}$, where the index b is chosen uniformly at random. If the decision b^* of **A** is equal to b , then the experiment returns WIN and otherwise, returns LOSE.

The advantage of an adversary **A** over our authentication scheme **a-auth** in the experiment is defined as: $\text{Adv}_{\mathbf{a-auth}, \mathbf{A}}^{\text{ano}}(\lambda) \stackrel{\text{def}}{=} |\Pr[\text{Exp}_{\mathbf{a-auth}, \mathbf{A}}^{\text{ano}}(1^\lambda) = \text{WIN}] - (1/2)|$. An authentication scheme **a-auth** is called to have anonymity if, for any PPT algorithm **A**, the advantage $\text{Adv}_{\mathbf{a-auth}, \mathbf{A}}^{\text{ano}}(\lambda)$ is negligible in λ .

Setup(1^λ)	AuthKG($1^\lambda, a$)	SKG(PK^a, MSK^a, gid)
$pp_{\text{Sig}} \leftarrow \text{Sig.Setup}(1^\lambda)$	$(SK, PK) \leftarrow \text{Sig.KG}(1^\lambda)$	$\sigma_{\text{gid}}^a \leftarrow \text{Sig.Sign}(PK^a, MSK^a, \text{gid})$
$pp_{\Pi} \leftarrow \Pi.\text{Setup}(1^\lambda)$	$PK^a := PK, MSK^a := SK$	$sk_{\text{gid}}^a := \sigma_{\text{gid}}^a$
$pp_{\text{CmtPrv}} \leftarrow \text{CmtPrv.Setup}(1^\lambda)$	Return (PK^a, MSK^a)	Return sk_{gid}^a
$pp := (pp_{\Pi}, pp_{\text{CmtPrv}}, pp_{\text{Sig}})$		
Return pp		
(Execute $\Sigma_{\text{bnd}}^{a \in A}$)		
$P((PK^a)^{a \in A}, (sk_{\text{gid}}^a)^{a \in A})$		$V((PK^a)^{a \in A})$
For $a \in A$: $x^a := PK^a, w_1^a := sk_{\text{gid}}^a$		For $a \in A$: $x^a := PK^a$
$w_0 := \text{gid}$		Return $(d \leftarrow \Sigma_{\text{bnd}, \text{vrf}}^{a \in A})$

Fig. 2. Generic construction of our decentralized multi-authority anonymous authentication scheme **a-auth**.

4.2 Generic Construction

We give a generic construction of our authentication scheme **a-auth**. The building blocks are the interactive proof system $\Pi_{\text{bnd}}^{a \in A}$ with our Σ -protocol $\Sigma_{\text{bnd}}^{a \in A}$ and a digital signature scheme **Sig**. We note that a commit-and-prove scheme **CmtPrv** is employed in $\Sigma_{\text{bnd}}^{a \in A}$.

- **Setup**(1^λ) $\rightarrow pp$. On input the security parameter 1^λ , this PPT algorithm generates a set of public parameter by running the setup algorithms **Sig.Setup**(1^λ), $\Pi.\text{Setup}(1^\lambda)$ and **CmtPrv.Setup**(1^λ). These algorithms are for the digital signature scheme **Sig**, the interactive argument systems $(\Pi_0^a)^{a \in A}$, and the commitment generation algorithm **Cmt.Com**. They generate pp_{Sig} , pp_{Π} and pp_{Cmt} , respectively. It merges them as $pp := (pp_{\text{Sig}}, pp_{\Pi}, pp_{\text{Cmt}})$. It returns pp .
- **AuthKG**($1^\lambda, a$) $\rightarrow (PK^a, MSK^a)$. On input the security parameter 1^λ and an authority index a , this PPT algorithm executes the key generation algorithm **Sig.KG**(1^λ). It obtains a signing key **SK** and the corresponding public key **PK**. It sets the master secret key as $MSK^a := SK$ and the corresponding public key as $PK^a := PK$. It returns (PK^a, MSK^a) .
- **SKG**(PK^a, MSK^a, gid) $\rightarrow sk_{\text{gid}}^a$. On input a public key PK^a , the corresponding master secret key MSK^a and a string **gid**, this PPT algorithm executes the signing algorithm **Sig.Sign**(PK^a, MSK^a, gid). It obtains a digital signature σ_{gid}^a on the message **gid**. It puts a private secret key sk_{gid}^a as $sk_{\text{gid}}^a := \sigma_{\text{gid}}^a$. It returns sk_{gid}^a .
- $P((PK^a)^{a \in A}, (sk_{\text{gid}}^a)^{a \in A})$ and $V((PK^a)^{a \in A})$. On input the public keys $(PK^a)^{a \in A}$ to the prover **P** and the verifier **V**, and the corresponding private secret keys $(sk_{\text{gid}}^a)^{a \in A}$ to **P**, PPT algorithms **P** and **V** first set the statements as $x^a := PK^a$ for each $a \in A$ and **P** sets the witness as $w_0 := \text{gid}$ and $w_1^a := sk_{\text{gid}}^a$ for each $a \in A$. The witness spaces $W^a, a \in A$ are described as follows.

$$W^a = W_0 \times W_1^a, \quad (44)$$

$$W_0 = \{\text{gid} \mid \text{string of length } \lambda\} = \{0, 1\}^\lambda, \quad (45)$$

$$W_1^a = \{\sigma_{\text{gid}}^a \mid \sigma_{\text{gid}}^a \leftarrow \text{Sig.Sign}(PK^a, MSK^a, \text{gid}) \text{ for some } \text{gid} \in W_0\}. \quad (46)$$

P and **V** execute the Σ protocol $\Sigma_{\text{bnd}}^{a \in A}$. **V** returns the returned boolean d of the verifier algorithm $\Sigma_{\text{bnd}, \text{vrf}}^{a \in A}$.

4.3 Properties

Theorem 3 *If the component proof system Π_0^a is perfectly witness-indistinguishable for each $a \in A$, if the commitment scheme **Cmt** is perfectly hiding and computationally binding, and if the digital signature scheme **Sig** is existentially unforgeable against adaptive chosen-message attacks, then our **a-auth** is secure against concurrent and collusion attacks. More precisely, let q_A denote the maximum number of authorities, q_I denote the maximum number of provers and q_{sk} denote the maximum number of private secret key queries. For any given PPT algorithm **A** that executes a concurrent and collusion attack on our **a-auth** in accordance with the experiment $\text{Exp}_{\text{a-auth}, \mathbf{A}}^{\text{conc-coll}}(1^\lambda)$, there exist a PPT algorithm **F** that generates an existential forgery on **Sig** in accordance with the experiment $\text{Exp}_{\text{Sig}, \mathbf{F}}^{\text{uf-cma}}(1^\lambda)$ and a PPT algorithm **B** that breaks the binding property of*

Cmt in accordance with the experiment $\text{Exp}_{\text{Cmt},\mathbf{B}}^{\text{bind}}(1^\lambda)$ which satisfy the following inequality.

$$\mathbf{Adv}_{a\text{-auth},\mathbf{A}}^{\text{conc-coll}}(\lambda) \leq \frac{1}{|\text{CHASP}(1^\lambda)|} + \sqrt{\frac{2^\lambda}{2^\lambda - q_I - q_{\text{sk}}} \cdot q_A \cdot \mathbf{Adv}_{\text{Sig},\mathbf{F}}^{\text{euf-cma}}(\lambda) + \mathbf{Adv}_{\text{Cmt},\mathbf{B}}^{\text{bind}}(\lambda)}. \quad (47)$$

Proof. Given any PPT algorithm \mathbf{A} on $\text{Exp}_{a\text{-auth},\mathbf{A}}^{\text{conc-coll}}(1^\lambda)$, we construct a PPT algorithm \mathbf{F} that generates an existential forgery on Sig in accordance with the experiment $\text{Exp}_{\text{Sig},\mathbf{F}}^{\text{euf-cma}}(1^\lambda)$. \mathbf{F} is given as input the set of public parameter pp_{Sig} and a public key PK_{Sig} . \mathbf{F} first reads out the security parameter 1^λ from pp_{Sig} , and executes the setup algorithms $\Pi.\text{Setup}(1^\lambda)$ and $\text{CmtPrv}.\text{Setup}(1^\lambda)$. It obtains the sets of public parameter pp_Π and pp_{CmtPrv} , respectively. \mathbf{F} merges the sets of public parameter as $pp := (pp_{\text{Sig}}, pp_\Pi, pp_{\text{CmtPrv}})$. Then \mathbf{F} invokes the algorithm \mathbf{A} with 1^λ . It obtains the number q_A of key-issuing authorities. \mathbf{F} chooses a *target index* a^* from the set $A := \{1, \dots, q_A\}$ uniformly at random. For $a \in A$ *except* the target index a^* , \mathbf{F} executes the authority key generation algorithm honestly. As for a^* , \mathbf{F} uses the input public key:

$$\text{For } a \in A \text{ s.t. } a \neq a^* : (\text{PK}^a, \text{MSK}^a) \leftarrow \text{AuthKG}(1^\lambda, a), \quad (48)$$

$$\text{For } a = a^* : \text{PK}^{a^*} := \text{PK}_{\text{Sig}}. \quad (49)$$

\mathbf{F} inputs the set of public parameter pp and the public keys $(\text{PK}^a)^{a \in A}$ into \mathbf{A} . It obtains the number q_I of concurrent provers. \mathbf{F} sets I as $I := \{1, \dots, q_I\}$.

Simulation of Concurrent Provers When \mathbf{A} invokes a prover with $\text{gid}_i \in \{0, 1\}^\lambda$ ($1 \leq i \leq q_I$), for $a \in A$ where $a \neq a^*$, \mathbf{F} executes the generation algorithm of private secret key with gid_i honestly, and for $a = a^*$, \mathbf{F} issues a signing query with gid_i :

$$\text{For } a \in A \text{ s.t. } a \neq a^* : \text{sk}_{\text{gid}_i}^a \leftarrow \text{SKG}(\text{PK}^a, \text{MSK}^a, \text{gid}_i),$$

$$\text{For } a = a^* : \text{sk}_{\text{gid}_i}^{a^*} \leftarrow \text{SignO}(\text{PK}, \text{SK}, \text{gid}_i).$$

In the simulation of concurrent provers $\mathcal{P}((\text{PK}^a, \text{sk}_{\text{gid}_i}^a)^{a \in A})_{|i \in I}$, \mathbf{F} uses the private secret keys $(\text{sk}_{\text{gid}_i}^a)^{a \in A}$, $i \in I$. Note that this is a perfect simulation.

Simulation of Private Secret Key Oracle When \mathbf{A} issues a private secret key query with $A_j \subset A$ and $\text{gid}_j \in \{0, 1\}^\lambda$ ($q_I + 1 \leq j \leq q_I + q_{\text{sk}}$), for $a \in A$ such that $a \neq a^*$ \mathbf{F} executes the generation algorithm of private secret key with gid_j honestly, and for $a = a^*$ \mathbf{F} issues a signing query with gid_j :

$$\text{For } a \in A \text{ s.t. } a \neq a^* : \text{sk}_{\text{gid}_j}^a \leftarrow \text{SKG}(\text{PK}^a, \text{MSK}^a, \text{gid}_j),$$

$$\text{For } a = a^* : \text{sk}_{\text{gid}_j}^{a^*} \leftarrow \text{SignO}(\text{PK}, \text{SK}, \text{gid}_j).$$

\mathbf{F} replies to \mathbf{A} with the secret key $\text{sk}_{\text{gid}_j}^a$. This is a perfect simulation.

At the end of the “learning phase” \mathbf{A} outputs a target set of authority indices A^* and its inner state St . *Generating Existential Forgery* Next, in the “attacking phase”, on input the inner state St , the adversary \mathbf{A} interacts with the verifier. That is, \mathbf{F} executes a verifier \mathbf{V} with an input $((\text{PK}^a)^{a \in A^*})$. If the decision d of \mathbf{V} is 1, then \mathbf{F} *rewinds* (Bellare-Palacio [BP02]) \mathbf{A} back to the timing at which \mathbf{A} had sent the challenge message of the Σ -protocol $\Sigma_{\text{bnd}}^{a \in A}$. If the decision d of \mathbf{V} is again 1, \mathbf{F} executes the knowledge extractor $\Sigma_{\text{bnd}, \text{ext}}^{a \in A}$ on input $((x^a)^{a \in A}, (c_0, (\text{COM}^a, \text{COM}_0^a)^{a \in A}))$, CHA , $(\text{RES}^a, \text{RES}_0^a)^{a \in A}$, CHA' , $((\text{RES}^a)', (\text{RES}_0^a)')^{a \in A}$. If $\Sigma_{\text{bnd}, \text{ext}}^{a \in A}$ outputs a witness $\hat{w} := (\hat{w}_0, (\hat{w}_1^a)^{a \in A})$, then \mathbf{F} sets a message gid^* as $\text{gid}^* := \hat{w}_0$. The restriction (38) of the experiment assures that there exists an authority index \hat{a} such that $\hat{a} \in A^*$ and $\hat{a} \notin A_j$, $q_I \leq \forall j \leq q_I + q_{\text{sk}}$. \mathbf{F} chooses such an \hat{a} at random and sets a signature σ^* as $\sigma^* := \hat{w}_1^{\hat{a}}$. \mathbf{F} returns a forgery pair of a message and a signature (gid^*, σ^*) . This completes the description of \mathbf{F} .

Probability Evaluation The probability that the returned value (gid^*, σ^*) is actually an existential forgery

is evaluated as follows. We name the events in the above as:

$$\text{ACC} : \mathbf{V} \text{ accepts } \mathbf{A}, \quad (50)$$

$$\text{RST} : \mathbf{V} \text{ accepts } \mathbf{A} \text{ both before and after the rewinding with different } \text{CHA}, \quad (51)$$

$$\text{TGTIDX} : \hat{a} = a^*, \quad (52)$$

$$\text{EXT} : \Sigma_{\text{bind,ext}}^{a \in A} \text{ returns a witness } \hat{w} := (\hat{w}_0, (\hat{w}_1^a)^{a \in A^*}), \quad (53)$$

$$\text{NEWID} : \text{gid}^* \notin \{\text{gid}_i\}_{i=1}^{q_I + q_{sk}}, \quad (54)$$

$$\text{FORGE} : (\text{gid}^*, \sigma^*) \text{ is an existential forgery on } \text{Sig}. \quad (55)$$

We have the following inequality by Reset Lemma [BP02].

$$\Pr[\text{ACC}] \leq \frac{1}{|\text{CHASP}(1^\lambda)|} + \sqrt{\Pr[\text{RST}]}. \quad (56)$$

Besides, the above discussion as well as the definitions deduce the following equalities.

$$\text{Adv}_{\mathbf{a}\text{-auth}, \mathbf{A}}^{\text{conc-coll}}(\lambda) = \Pr[\text{ACC}], \quad (57)$$

$$\Pr[\text{TGTIDX}, \text{RST}, \text{EXT}, \text{NEWID}] = \Pr[\text{FORGE}], \quad (58)$$

$$\Pr[\text{FORGE}] = \text{Adv}_{\text{Sig}, \mathbf{F}}^{\text{euf-cma}}(\lambda). \quad (59)$$

The left-hand side of the equality (58) is expanded as follows.

$$\Pr[\text{TGTIDX}, \text{RST}, \text{EXT}, \text{NEWID}] = \Pr[\text{TGTIDX}] \cdot \Pr[\text{RST}, \text{EXT}, \text{NEWID}] \quad (60)$$

$$= \Pr[\text{TGTIDX}] \cdot \Pr[\text{RST}, \text{EXT}] \cdot \Pr[\text{NEWID} \mid \text{RST}, \text{EXT}]. \quad (61)$$

Lemma 1

$$\Pr[\text{TGTIDX}] = 1/q_A. \quad (62)$$

Proof. The restriction (38) of the experiment assures that there exists an authority index \hat{a} such that $\hat{a} \in A^*$ and $\hat{a} \notin A_j$, $q_I \leq \forall j \leq q_I + q_{sk}$. Besides, \hat{a} coincides with a^* with probability $1/q_A$. \square

Lemma 2

$$\Pr[\text{NEWID} \mid \text{RST}, \text{EXT}] \geq \frac{2^\lambda - q_I - q_{sk}}{2^\lambda}. \quad (63)$$

Proof. gid^* is not in $\{\text{gid}_i\}_{i=1}^{q_I + q_{sk}}$ with probability at least $\frac{2^\lambda - q_I - q_{sk}}{2^\lambda}$. \square

Lemma 3 *For any given PPT algorithm \mathbf{A} that executes a concurrent and collusion attack on our \mathbf{a} -auth in accordance with the experiment $\text{Exp}_{\mathbf{a}\text{-auth}, \mathbf{A}}^{\text{conc-coll}}(1^\lambda)$, there exists a PPT algorithm \mathbf{B} that breaks the binding property of Cmt in accordance with the experiment $\text{Exp}_{\text{Cmt}, \mathbf{B}}^{\text{bind}}(1^\lambda)$ satisfying the following equality.*

$$\Pr[\text{RST}, \overline{\text{EXT}}] = \text{Adv}_{\text{Cmt}, \mathbf{B}}^{\text{bind}}(\lambda). \quad (64)$$

Proof. Given any PPT algorithm \mathbf{A} on $\text{Exp}_{\mathbf{a}\text{-auth}, \mathbf{A}}^{\text{conc-coll}}(1^\lambda)$, we construct a PPT algorithm \mathbf{B} that breaks the binding property of Cmt in accordance with the experiment $\text{Exp}_{\text{Cmt}, \mathbf{B}}^{\text{bind}}(1^\lambda)$. \mathbf{B} is given as input the set of public parameter pp_{CmtPrv} . \mathbf{B} first reads out the security parameter 1^λ from pp_{CmtPrv} , and executes the setup algorithms $\text{II.Setup}(1^\lambda)$ and $\text{Sig.Setup}(1^\lambda)$. It obtains the sets of public parameter pp_Π and pp_{Sig} , respectively. \mathbf{B} merges the sets of public parameter as $pp := (pp_{\text{Sig}}, pp_\Pi, pp_{\text{CmtPrv}})$. Then \mathbf{B} invokes the algorithm \mathbf{A} with 1^λ . It obtains the number q_A of key-issuing authorities. The simulation of concurrent provers and the simulation of the private secret key oracle are done in the same way. (Note that \mathbf{B} does not need to choose a^* .) In the “attacking phase”, \mathbf{B} executes a verifier \mathbf{V} with an input $((\text{PK}^a)^{a \in A^*})$. If the decision d of \mathbf{V} is 1, then \mathbf{B} rewinds \mathbf{A} back to the timing at which \mathbf{A} had sent the challenge message

of the Σ -protocol $\Sigma_{\text{bind}}^{a \in A}$. If the decision d of \mathbf{V} is again 1, \mathbf{B} executes the knowledge extractor $\Sigma_{\text{bind,ext}}^{a \in A}$ on input $((x^a)^{a \in A}, (c_0, (\text{COM}^a, \text{COM}_0^a)^{a \in A}), \text{CHA}, (\text{RES}^a, \text{RES}_0^a)^{a \in A}, \text{CHA}', ((\text{RES}^a)', (\text{RES}_0^a)')^{a \in A})$. If $\Sigma_{\text{bind,ext}}^{a \in A}$ outputs \perp^* , then there must be a pair $a, a' \in A^*, a \neq a'$ such that $(\hat{w}_0^a, \hat{w}_1^a, \hat{r}_0^a)$ and $(\hat{w}_0^{a'}, \hat{w}_1^{a'}, \hat{r}_{a',0})$ pass the verification Cmt.Vrf and $\hat{w}_0^a \neq \hat{w}_0^{a'}$. The vector $(c_0, \hat{w}_0^a, \hat{r}_0^a, \hat{w}_0^{a'}, \hat{r}_{a',0})$ breaks the binding property to yields WIN in $\text{Exp}_{\text{Cmt}, \mathbf{B}}^{\text{bind}}(1^\lambda)$. This completes the description of \mathbf{B} , and \mathbf{B} satisfies (64). \square

Note that we have the equality:

$$\Pr[\text{RST}] = \Pr[\text{RST}, \text{EXT}] + \Pr[\text{RST}, \overline{\text{EXT}}]. \quad (65)$$

Combining (58), (61), (62), (63), (64) and (65), we have:

$$\Pr[\text{RST}] = \frac{2^\lambda}{2^\lambda - q_I - q_{\text{sk}}} \cdot q_A \cdot \Pr[\text{FORGE}] + \text{Adv}_{\text{Cmt}, \mathbf{B}}^{\text{bind}}(\lambda). \quad (66)$$

Combining (56), (57), (66) and (59), we have:

$$\text{Adv}_{\text{a-auth}, \mathbf{A}}^{\text{conc-coll}}(\lambda) \leq \frac{1}{|\text{CHASP}(1^\lambda)|} + \sqrt{\frac{2^\lambda}{2^\lambda - q_I - q_{\text{sk}}} \cdot q_A \cdot \text{Adv}_{\text{Sig}, \mathbf{F}}^{\text{euf-cma}}(\lambda) + \text{Adv}_{\text{Cmt}, \mathbf{B}}^{\text{bind}}(\lambda)}. \quad (67)$$

\square

Theorem 4 *If the component proof system Π_0^a is perfectly witness-indistinguishable for each $a \in A$, and if the commitment scheme Cmt is perfectly hiding, then our a-auth has anonymity. More precisely, for any given algorithm \mathbf{A} that is not necessarily bounded and that executes the anonymity game on our a-auth in accordance with the experiment $\text{Exp}_{\text{a-auth}, \mathbf{A}}^{\text{ano}}(1^\lambda)$, the following equality holds.*

$$\text{Adv}_{\text{a-auth}, \mathbf{A}}^{\text{ano}}(\lambda) = 0. \quad (68)$$

Proof. The perfect witness-indistinguishability of Π_0^a for each $a \in A$ and the perfectly hiding property of the commitment scheme Cmt assure that our proof system $\Pi_{\text{bind}}^{a \in A}$ is perfectly witness-indistinguishable by Theorem 2. Then the two distribution $\text{dist}^{a \in A} := \text{dist}((c_0, (\text{COM}^a, \text{COM}_0^a)^{a \in A}), \text{CHA}, (\text{RES}^a, \text{RES}_0^a)^{a \in A})$ is identical even if the auxiliary input z is private secret keys $(\text{sk}_{\text{gid}_0}^a, \text{sk}_{\text{gid}_1}^a)^{a \in A}$. Therefore, the advantage $\text{Adv}_{\text{a-auth}, \mathbf{A}}^{\text{ano}}(\lambda)$ is zero. \square

Note. Relation to Attribute-Based Identifications and Signatures Using a monotone formula instead of the AND-composition, a decentralized multi-authority attribute-based authentication scheme [AAHI13] is obtained over a small universe A . Moreover, the Fiat-Shamir transform [FS86] gives a decentralized multi-authority attribute-based signature scheme [OT13]. Note here that the security against the collusion attacks means the security against the passive attacks, and therefore the unforgeability is derived (see [AABN02]).

5 Conclusion

We proposed a generic construction of a Σ -protocol of commit-and-prove type, which is an AND-composition of Σ -protocols on the statements that include a common commitment. When the component Σ -protocols are of witness-indistinguishable argument systems, our Σ -protocol is also a witness-indistinguishable argument system as a whole. As an application, we gave a generic construction of a decentralized multi-authority anonymous authentication scheme. There a witness is a bundle of witnesses each of which decomposes into a fixed global identity string and a digital signature on it. We show an instantiation of the scheme in the setting of bilinear groups.

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Appendices

A Instantiation

We discuss an instantiation of our generic authentication scheme **a-auth** that was given in Section 4.

Basically, we can employ any three building blocks that satisfy requirements stated in Section 4. Below we briefly mention an instantiation in the setting of bilinear groups. We put the symbol pp at the subscript of the algorithms to indicate that the set of public parameters pp is the basis of transition functions of the underlying Turing machines that correspond to the algorithms.

The three building blocks are the pairing version of the Camenisch-Lysyanskaya digital signature scheme Sig^{CL} (See Appendix C) [Oka06,SNF11,TF12], the pairing version of the Camenisch-Lysyanskaya perfectly witness-indistinguishable argument of knowledge system Π^{CL} (See Appendix D) [Oka06,SNF11,TF12], and the Pedersen-Okamoto commit-and-prove scheme $\text{CmtPrv}^{\text{PO}}$ (See Appendix E) which is a combination of the perfectly hiding commitment scheme of Pedersen [Ped91] and the perfectly witness-indistinguishable argument of knowledge system by Okamoto [Oka92]. The five algorithms of our **a-auth** are instantiated as follows.

- $\text{Setup}(1^\lambda) \rightarrow pp$. On input the security parameter 1^λ , this PPT algorithm executes the setup algorithm $\text{Sig}^{\text{CL}}.\text{Setup}$. That is, it executes the group generation algorithm \mathcal{BG} to generate bilinear groups of a prime order p of length $|p| = \lambda$: $\mathcal{BG}(1^\lambda) \rightarrow \Lambda := (p, e, \mathbb{G}, \tilde{\mathbb{G}}, \mathbb{G}_T, G, \tilde{G})$. Here $e : \mathbb{G} \times \tilde{\mathbb{G}} \rightarrow \mathbb{G}_T$ is a bilinear map and $G \in_R \mathbb{G}, \tilde{G} \in_R \tilde{\mathbb{G}}$ with $e(G, \tilde{G}) \neq 1_{\mathbb{G}_T}$ are the generators, respectively. Then it chooses a set of base elements for Sig^{CL} and $\text{CmtPrv}^{\text{PO}}$ as $G_0, G_1, G_2, H \in_R \mathbb{G}, \tilde{G}_0 \in_R \tilde{\mathbb{G}}$. It returns the set of public parameter $pp := (\Lambda, G_0, G_1, G_2, H, \tilde{G}_0)$. Note that, in the case of Sig^{CL} , the setup algorithm $\text{Sig}^{\text{CL}}.\text{Setup}$ is also the setup algorithm $\Pi^{\text{CL}}.\text{Setup}$. As for $\text{CmtPrv}^{\text{PO}}.\text{Setup}$, we use the group \mathbb{G} for $\text{CmtPrv}^{\text{PO}}$.
- $\text{AuthKG}_{pp}(1^\lambda, a) \rightarrow (\text{PK}^a, \text{MSK}^a)$. On input pp and an authority index a , this PPT algorithm chooses exponents $\alpha_a \in_R \mathbb{Z}_p$ and computes $\tilde{G}_{a,1} := \tilde{G}_0^{\alpha_a}$. It sets $\text{PK}^a := \tilde{G}_{a,1}, \text{MSK}^a := \alpha_a$. It returns $(\text{PK}^a, \text{MSK}^a)$.
- $\text{SKG}_{pp}(\text{PK}^a, \text{MSK}^a, \text{gid}) \rightarrow \text{sk}_{\text{gid}}^a$. On input $\text{PK}^a, \text{MSK}^a$ and a string $\text{gid} \in \mathbb{Z}_p$, this PPT algorithm generates a CL signature on gid . That is, it chooses an exponent $\gamma_a, \delta_a \in_R \mathbb{Z}_p$ and computes $V_a := (G_0 G_1^{\text{gid}} G_2^{\gamma_a})^{1/(\delta_a + \alpha_a)}$. (We omit the index gid for simplicity.) It sets $\text{sk}_{\text{gid}}^a := (V_a, \gamma_a, \delta_a)$. The right-hand side is a CL signature on gid . It returns sk_{gid}^a .
- $\langle \text{P}_{pp}((\text{PK}^a)^{a \in A}, (\text{sk}_{\text{gid}}^a)^{a \in A}), \text{V}_{pp}((\text{PK}^a)^{a \in A}) \rangle \rightarrow 1/0$. P_{pp} , the prover, and V_{pp} , the verifier, take a common input $(\text{PK}^a)^{a \in A}$. P_{pp} also takes as input her set of private keys $(\text{sk}_{\text{gid}}^a)^{a \in A}$ that are signatures on gid . These PPT interactive algorithms execute the following protocol of an argument system. Note here that the statement is $x := (x^a)^{a \in A} := (\text{PK}^a)^{a \in A}$ and the witness is $w := (w_0, (w_1^a)^{a \in A}) := (\text{gid}, (\text{sk}_{\text{gid}}^a)^{a \in A})$. Hence, the first type of the predicate and relation is the following ones.

$$\Phi_{pp}^a(x^a, w^a) \stackrel{\text{def}}{=} (e(G_0 G_1^{\text{gid}} G_2^{\gamma_a}, \tilde{G}_0) =? e(V_a, \tilde{G}_0^{\delta_a} \tilde{G}_{a,1})), \quad (69)$$

$$R^a \stackrel{\text{def}}{=} \{(x^a, w^a) \in \tilde{\mathbb{G}} \times (\mathbb{Z}_p \times \mathbb{G} \times \mathbb{Z}_p^2) \mid \Phi_{pp}^a(x^a, w^a) = \text{TRUE}\}. \quad (70)$$

- $\Sigma_{\text{PO,com}}^{\text{CL},a \in A}, \Sigma_{\text{PO},0,\text{com}}^{\text{CL},a}$. To start the interactive argument, P_{pp} first chooses a randomness $u \in_R \mathbb{Z}_p$ and compute a Pedersen commitment to gid as $C_0 := G^{\text{gid}} H^u$. P_{pp} puts a statement and the witness as $x_0 := C_0, (w_0, r_0) := (\text{gid}, u)$. Hence, the second type of the predicate and relation is the following ones.

$$\Phi_{0,pp}(x_0, (w_0, r_0)) \stackrel{\text{def}}{=} (C_0 =? G^{\text{gid}} H^u), \quad (71)$$

$$R_0 \stackrel{\text{def}}{=} \{(x_0, (w_0, r_0)) \in \mathbb{G} \times \mathbb{Z}_p^2 \mid \Phi_{0,pp}(x_0, (w_0, r_0)) = \text{TRUE}\}. \quad (72)$$

$\text{P}_{pp}(x, w)$ and $\text{V}_{pp}(x)$ execute a three-move public coin protocol (like the Schnorr [Sch89]) as follows.

P_{pp} for each $a \in A$, chooses $v_a \in_R \mathbb{Z}_p$ and re-randomize the secret element V_a as $R_a := V_a G_2^{v_a}$, and puts $z_a := \gamma_a + v_a \delta_a$. Then P_{pp} chooses $r_{a,\text{gid}}, r_{a,z}, r_{a,v}, r_{a,\delta} \in_R \mathbb{Z}_p$ and computes $T_a := e(G_1, \tilde{G}_0)^{r_{a,\text{gid}}}$. $e(G_2, \tilde{G}_0)^{r_{a,z}} e(G_2, \tilde{G}_1)^{r_{a,v}} e(R_a, \tilde{G}_0)^{-r_{a,\delta}}$. Besides, P_{pp} chooses an exponent $r_{a,u} \in_R \mathbb{Z}_p$ and computes $A_a := G^{r_{a,\text{gid}}} H^{r_{a,u}}$. P_{pp} sends the commitment message $(C_0, (R_a, T_a, A_a)^{a \in A})$ to V_{pp} .

- $\Sigma_{\text{PO,cha}}^{\text{CL},a \in A}, \Sigma_{\text{PO},0,\text{cha}}^{\text{CL},a}$. V_{pp} computes a challenge message by choosing an exponent $c \in_R \mathbb{Z}_p$. V_{pp} sends c to P_{pp} .
- $\Sigma_{\text{PO,res}}^{\text{CL},a \in A}, \Sigma_{\text{PO},0,\text{res}}^{\text{CL},a}$. P_{pp} computes the response message as $s_{a,\text{gid}} := r_{a,\text{gid}} + c \text{gid}$, $s_{a,z} := r_{a,z} + c z_a$, $s_{a,v} := r_{a,v} + c v_a$, $s_{a,\delta} := r_{a,\delta} + c \delta_a$, $s_{a,u} := r_{a,u} + c u$. P_{pp} sends $(s_{a,\text{gid}}, s_{a,z}, s_{a,v}, s_{a,\delta}, s_{a,u})^{a \in A}$ to V_{pp} .
- $\Sigma_{\text{PO,vrf}}^{\text{CL},a \in A}, \Sigma_{\text{PO},0,\text{vrf}}^{\text{CL},a}$. V_{pp} checks whether the following equalities hold. If those hold, then return 1. Otherwise, 0.

For $a \in A$:

$$e(G_1, \tilde{G}_0)^{s_{a,\text{gid}}} e(G_2, \tilde{G}_0)^{s_{a,z}} e(G_2, \tilde{G}_{a,1})^{s_{a,v}} e(R_a, \tilde{G}_0)^{-s_{a,\delta}} =? T_a (e(R_a, \tilde{G}_{a,1}) / e(G_0, \tilde{G}_0))^c, \text{ and} \quad (74)$$

$$G^{s_{a,\text{gid}}} H^{s_{a,u}} =? A_a C_0^c. \quad (75)$$

We omit the description of the knowledge extractor $\Sigma_{\text{PO},0,\text{ext}}^{\text{CL},a}$ and the simulator $\Sigma_{\text{PO},0,\text{sim}}^{\text{CL},a}$. As has been explained at (18), we have to show that the $\Sigma_{\text{PO},0}^{\text{CL},a}$ is actually a Σ -protocol and $\Pi_{\text{PO},0}^{\text{CL},a}$ is perfectly witness indistinguishable.

We obtain the following propositions and theorems. (The details and proofs are omitted.)

Proposition 4 $\Sigma_{\text{PO},0}^{\text{CL},a}$ is a Σ -protocol.

Proposition 5 $\Pi_{\text{PO},0}^{\text{CL},a}$ is perfectly witness indistinguishable.

Theorem 5 If $\text{Cmt}_{pp}^{\text{Ped}}$ is perfectly hiding and computationally binding, and if Sig^{CL} is existentially unforgeable against adaptive chosen-message attacks, then our a -auth is secure against concurrent and collusion attacks.

Theorem 6 Our a -auth has anonymity.

B Algebraic Settings and Number-Theoretic Assumptions

Let (p, \mathbb{G}) denote a cyclic group of prime order p , where $|p| = \lambda$. Let G denote a generator chosen uniformly at random, $G \in_R \mathbb{G} \setminus \{1_{\mathbb{G}}\}$. Let \mathcal{G} denote a PPT algorithm which, on input 1^λ , returns the set of parameters $\Lambda := (p, \mathbb{G}, G)$. That is, $\Lambda := (p, \mathbb{G}, G) \leftarrow \mathcal{G}(1^\lambda)$.

Let $(p, e, \mathbb{G}, \tilde{\mathbb{G}}, \mathbb{G}_T)$ denote bilinear groups of prime order p and of Type 3 [GPS08, BB08], where $|p| = \lambda$. Here we require that the bilinear map $e : \mathbb{G} \times \tilde{\mathbb{G}} \rightarrow \mathbb{G}_T$ is efficiently computable (i.e., polynomial-time in λ). Let G and \tilde{G} denote generators chosen uniformly at random, $G \in_R \mathbb{G} \setminus \{1_{\mathbb{G}}\}, \tilde{G} \in_R \tilde{\mathbb{G}} \setminus \{1_{\tilde{\mathbb{G}}}\}$ with $e(G, \tilde{G}) \neq 1_{\mathbb{G}_T}$. Let \mathcal{BG} denote a PPT algorithm which, on input 1^λ , returns the set of parameters $\Lambda := (p, e, \mathbb{G}, \tilde{\mathbb{G}}, \mathbb{G}_T, G, \tilde{G})$. That is, $\Lambda := (p, e, \mathbb{G}, \tilde{\mathbb{G}}, \mathbb{G}_T, G, \tilde{G}) \leftarrow \mathcal{BG}(1^\lambda)$. Bilinear groups are widely recognized in the form of the pairing on elliptic curves [GPS08].

B.1 Discrete Logarithm Assumption (DL) [EG85]

The DL assumption is stated as follows. For any PPT algorithm \mathbf{S} , the advantage of \mathbf{S} over \mathcal{G} defined by the following equality is negligible in λ :

$$\text{Adv}_{\mathcal{G},\mathbf{S}}^{\text{dl}}(\lambda) := \Pr[\gamma = \gamma^* \mid \Lambda \leftarrow \mathcal{G}(1^\lambda), \gamma \in_R \mathbb{Z}_p, \gamma^* \leftarrow \mathbf{S}(\Lambda, G, G^\gamma)]. \quad (76)$$

The probability is taken over the random tape of \mathcal{G} , the uniform random sampling of γ , and the random tape of \mathbf{S} .

B.2 Strong Diffie-Hellman Assumption (SDH) [BB04]

The SDH assumption is stated as follows. Let q be a natural number that is a function of λ bounded by a polynomial in λ . For any PPT algorithm \mathbf{S} and for any q , the advantage of \mathbf{S} over \mathcal{BG} defined by the following equality is negligible in λ :

$$\text{Adv}_{\mathcal{BG},\mathbf{S}}^{\text{sdh}}(\lambda) := \Pr[V^{\gamma+e} = G \mid \Lambda \leftarrow \mathcal{BG}(1^\lambda), \gamma \in_R \mathbb{Z}_p, (V, e) \leftarrow \mathbf{S}(\Lambda, (\tilde{G}^\gamma, \tilde{G}^{\gamma^2}, \dots, \tilde{G}^{\gamma^q}))]. \quad (77)$$

The probability is taken over the random tape of \mathcal{G} , the uniform random sampling of γ , and the random tape of \mathbf{S} .

C Camenisch-Lysyanskaya Signatures, Pairing Version [Oka06,SNF11,TF12]

The pairing version of the Camenisch-Lysyanskaya signature scheme Sig^{CL} , which was originally in the RSA setting, was proposed by Okamoto [Oka06]. We summarize the digital signature scheme here in the form which is found in Sudarsono-Nakanishi-Funabiki [SNF11] and Teranishi and Furukawa [TF12]. Sig^{CL} consists of four PPT algorithms, $\text{Sig}^{\text{CL}} := (\text{Sig}^{\text{CL}}.\text{Setup}, \text{Sig}^{\text{CL}}.\text{KG}_{pp}, \text{Sig}^{\text{CL}}.\text{Sign}_{pp}, \text{Sig}^{\text{CL}}.\text{Vrf}_{pp})$.

- $\text{Sig}^{\text{CL}}.\text{Setup}(1^\lambda) \rightarrow pp$. On input the security parameter 1^λ , this PPT algorithm generates a set of public parameter. That is, it executes a group generation algorithm \mathcal{BG} to generate bilinear groups of a prime order p of length $|p| = \lambda$: $\Lambda := (p, e, \mathbb{G}, \tilde{\mathbb{G}}, \mathbb{G}_T, G, \tilde{G}) \leftarrow \mathcal{BG}(1^\lambda)$. Besides, it chooses a set of base elements of $G_0, G_1, G_2 \in_R \mathbb{G}, \tilde{G}_0 \in_R \tilde{\mathbb{G}}$. It returns $pp := (\lambda, G_0, G_1, G_2, \tilde{G}_0)$.
- $\text{Sig}^{\text{CL}}.\text{KG}_{pp}(1^\lambda) \rightarrow (\text{PK}, \text{SK})$. On input 1^λ this PPT algorithm chooses an exponent $\alpha \in_R \mathbb{Z}_p$ and computes $\tilde{G}_1 := \tilde{G}_0^\alpha$. It sets a public key and the corresponding secret key as $\text{PK} := \tilde{G}_1, \text{SK} := \alpha$, respectively. It returns (PK, SK) .
- $\text{Sig}^{\text{CL}}.\text{Sign}_{pp}(\text{PK}, \text{SK}, m) \rightarrow \sigma$. On input PK, SK and a message $m \in \mathbb{Z}_p$, this PPT algorithm chooses two randomnesses $\gamma, \delta \in \mathbb{Z}_p$. It computes $V := (G_0 G_1^m G_2^\gamma)^{1/(\delta+\alpha)}$. It sets a signature $\sigma := (V, \gamma, \delta)$. It returns σ .
- $\text{Sig}^{\text{CL}}.\text{Vrf}_{pp}(\text{PK}, m, \sigma) \rightarrow 1/0$. On input PK, m and σ , this deterministic polynomial time algorithm returns a boolean decision 1 if the following holds. Otherwise, 0: $e(G_0 G_1^m G_2^\gamma) \stackrel{?}{=} e(V, \tilde{G}_0^\delta \tilde{G}_1)$.

The pairing version of the Camenisch-Lysyanskaya signature scheme Sig^{CL} is known to be existentially unforgeable against adaptive chosen-message attacks under the Strong Diffie-Hellman assumption on \mathcal{BG} (see Appendix B.2) [Oka06,SNF11,TF12].

D Camenisch-Lysyanskaya WIAoK, Pairing Version [Oka06,SNF11,TF12]

The pairing version of the Camenisch-Lysyanskaya argument of knowledge system Π^{CL} , which was originally in the RSA setting, was first proposed by Okamoto [Oka06]. We summarize the argument system here in the form found in Sudarsono-Nakanishi-Funabiki [SNF11] and Teranishi and Furukawa [TF12]. $\Pi^{\text{CL}} = (\Pi^{\text{CL}}.\text{Setup}, \text{P}_{pp}, \text{V}_{pp})$ is executed in accordance with a Σ -protocol $\Sigma^{\text{CL}} = (\Sigma_{\text{com}}^{\text{CL}}, \Sigma_{\text{cha}}^{\text{CL}}, \Sigma_{\text{res}}^{\text{CL}}, \Sigma_{\text{vrf}}^{\text{CL}}, \Sigma_{\text{ext}}^{\text{CL}}, \Sigma_{\text{sim}}^{\text{CL}})$.

The setup algorithm $\Pi^{\text{CL}}.\text{Setup}$ is the same as $\text{Sig}^{\text{CL}}.\text{Setup}(1^\lambda)$. The set of public parameter pp is common.

For $\alpha \in_R \mathbb{Z}_p$, the statement is $x := \tilde{G}_1 := \tilde{G}_0^\alpha$. For a given string $\text{gid} \in \mathbb{Z}_p$, choose two randomnesses $\gamma, \delta \in \mathbb{Z}_p$ and compute $V := (G_0 G_1^{\text{gid}} G_2^\gamma)^{1/(\delta+\alpha)}$. The witness of the statement x is $w := (\text{gid}, V, \gamma, \delta)$. Note that $\sigma := (V, \gamma, \delta)$ is a Camenisch-Lysyanskaya signature on the message gid . The following equality holds.

$$e(G_0 G_1^{\text{gid}} G_2^\gamma, \tilde{G}_0) = e(V, \tilde{G}_0^\delta \tilde{G}_1). \quad (78)$$

It is notable that the statement x does not include any information on the witness w , and the number of elements in $W(x)$ is p^3 because there are three independent variable in w ; that is, $(\text{gid}, \gamma, \delta)$. In other words, this number is the number of the solutions of the *equation* (78) determined by pp and x .

The protocol between P_{pp} and V_{pp} is a Σ -protocol. It goes as follows.

- $\Sigma_{\text{com}}^{\text{CL}}(x, w) \rightarrow (\text{COM}, St)$. This PPT algorithm is executed by P_{pp} . On input a statement x and a witness w , it chooses $v \in_R \mathbb{Z}_p$ and re-randomize the secret element V as $R := VG_2^v$. It puts $z := \gamma + v\delta$. It chooses $r_{\text{gid}}, r_z, r_v, r_\delta \in_R \mathbb{Z}_p$ and computes $T := e(G_1, \tilde{G}_0)^{r_{\text{gid}}} e(G_2, \tilde{G}_0)^{r_z} e(G_2, \tilde{G}_1)^{r_v} e(R, \tilde{G}_0)^{-r_\delta}$. It puts the commitment message as $\text{COM} := (R, T)$. It returns COM and its inner state St . P_{pp} sends COM to V_{pp} . Note that the following equality holds after the re-randomization.

$$e(G_1, \tilde{G}_0)^{\text{gid}} e(G_2, \tilde{G}_0)^z e(G_2, \tilde{G}_1)^v e(R, \tilde{G}_0)^{-\delta} = e(R, \tilde{G}_1) / e(G_0, \tilde{G}_0). \quad (79)$$

- $\Sigma_{\text{cha}}^{\text{CL}}(x) \rightarrow \text{CHA}$. This PPT algorithm is executed by V_{pp} . On input the statement x , it reads out the size of the security parameter as 1^λ and chooses a challenge message $c \in_R \text{CHASP}(1^\lambda)$. It puts the challenge message as $\text{CHA} := c$. It returns CHA . V_{pp} sends CHA to P_{pp} .
- $\Sigma_{\text{res}}^{\text{CL}}(St, \text{CHA}) \rightarrow \text{RES}$. This PPT algorithm is executed by P_{pp} . On input the state St^a and the challenge message CHA , it computes $s_{\text{gid}} := r_{\text{gid}} + c\text{gid}, s_z := r_z + cz, s_v := r_v + cv, s_\delta := r_\delta + c\delta$. It sets the response message as $\text{RES} := (s_{\text{gid}}, s_z, s_v, s_\delta)$. It returns RES . P_{pp} sends RES to V_{pp} .
- $\Sigma_{\text{vrf}}^{\text{CL}}(x, \text{COM}, \text{CHA}, \text{RES}) \rightarrow d$. This deterministic polynomial-time algorithm is executed by V_{pp} . On input the statement x and all the messages $(\text{COM}, \text{CHA}, \text{RES})$, it checks whether the following equality holds. If it holds, then return 1 (“accept”), and otherwise, 0 (“reject”).

$$e(G_1, \tilde{G}_0)^{s_{\text{gid}}} e(G_2, \tilde{G}_0)^{s_z} e(G_2, \tilde{G}_1)^{s_v} e(R, \tilde{G}_0)^{-s_\delta} \stackrel{?}{=} T(e(R, \tilde{G}_1) / e(G_0, \tilde{G}_0))^c. \quad (80)$$

For the remaining two, $\Sigma_{\text{ext}}^{\text{CL}}$ and $\Sigma_{\text{sim}}^{\text{CL}}$, see [SNF11,TF12]. The protocol Σ^{CL} is known to be a Σ -protocol.

Π^{CL} is *perfectly witness-indistinguishable* [FS90]. This is because the distribution of transcripts is independent of the witness $w \in W(x)$ even if the distribution of CHA deviates from the uniform random distribution.

E Pedersen-Okamoto Commitment-and-Prove Scheme [Ped91,Oka92]

The Pedersen commitment scheme [Ped91] $\text{Cmt}_{pp}^{\text{Ped}}$ is a commitment scheme in the discrete logarithm setting. $\text{Cmt}_{pp}^{\text{Ped}}$ consists of three PPT algorithms, $\text{Cmt}_{pp}^{\text{Ped}} = (\text{Cmt}_{pp}^{\text{Ped}}.\text{Setup}, \text{Cmt}_{pp}^{\text{Ped}}.\text{Com}_{pp}, \text{Cmt}_{pp}^{\text{Ped}}.\text{Vrf}_{pp})$.

- $\text{Cmt}_{pp}^{\text{Ped}}.\text{Setup}(1^\lambda) \rightarrow pp$. On input the security parameter 1^λ , this PPT algorithm generates a set of public parameter. That is, it executes a group generation algorithm \mathcal{G} to generate a cyclic group of a prime order p of length $|p| = \lambda: A := (p, \mathbb{G}, G) \leftarrow \mathcal{G}(1^\lambda)$. In addition, it chooses $\rho \in_R \mathbb{Z}_p$ and computes $H := G^\rho$. It returns $pp := (p, \mathbb{G}, G, H)$.
- $\text{Cmt}_{pp}^{\text{Ped}}.\text{Com}_{pp}(m) \rightarrow (C, \kappa)$. On input a message $m \in \mathbb{Z}_p$, this PPT algorithm generates a commitment $c \in \mathbb{G}$ and an opening key $\kappa \in \mathbb{Z}_p$. That is, it chooses $u \in_R \mathbb{Z}_p$ and computes the commitment $C = G^m H^u$ to m , and it sets κ as $\kappa := u$. It returns (C, κ) .
- $\text{Cmt}_{pp}^{\text{Ped}}.\text{Vrf}_{pp}(C, m, \kappa) \rightarrow d$. On input C, m and κ , this deterministic polynomial-time algorithm generates a boolean decision d . That is, it checks whether $C = G^m H^\kappa$ holds or not. If it holds, then it returns $d := 1$, and otherwise, $d := 0$.

$\text{Cmt}_{pp}^{\text{Ped}}$ is *perfectly hiding*. the distribution of the commitment C is independent of the committed message m . $\text{Cmt}_{pp}^{\text{Ped}}$ is *computationally binding* under the discrete logarithm assumption on \mathcal{G} (see Appendix B.1). If a commitment C is opened in two different ways $(m, \kappa) \neq (m', \kappa')$ with non-negligible probability in λ , then a PPT algorithm \mathbf{S} is constructed and it solves instances of the discrete logarithm problem, $H = G^\rho$. with a non-negligible probability in λ .

The Okamoto interactive argument system $\Pi_{pp}^{\text{Oka}} = (\Pi_{pp}^{\text{Oka}}.\text{Setup}, P_{pp}, V_{pp})$ [Oka92] is executed in accordance with a Σ -protocol $\Sigma^{\text{Oka}} = (\Sigma_{\text{com}}^{\text{Oka}}, \Sigma_{\text{cha}}^{\text{Oka}}, \Sigma_{\text{res}}^{\text{Oka}}, \Sigma_{\text{vrf}}^{\text{Oka}}, \Sigma_{\text{ext}}^{\text{Oka}}, \Sigma_{\text{sim}}^{\text{Oka}})$.

The setup algorithm $\Pi_{pp}^{\text{Oka}}.\text{Setup}$ is the same as $\text{Cmt}_{pp}^{\text{Ped}}.\text{Setup}(1^\lambda)$. The set of public parameter pp is common.

For $t, u \in_R \mathbb{Z}_p$, the statement is $x := X := G^t H^u$. The witness of x is $w = (t, u)$. It is notable that the number of elements in $W(x)$ is p because there are one independent variable in w ; that is, one of t and u . In other words, this number is the number of the solutions of the *equation* $X = G^t H^u$ determined by pp and x .

The protocol between P_{pp} and V_{pp} is a Σ -protocol. It goes as follows.

- $\Sigma_{\text{com}}^{\text{Oka}}(x, w) \rightarrow (\text{COM}, St)$. This PPT algorithm is executed by P_{pp} . On input a statement x and a witness w , it chooses $r_t, r_u \in_R \mathbb{Z}_p$ and computes $A := G^{r_t} H^{r_u}$. It puts the commitment message as $\text{COM} := A$. It returns COM and its inner state St . P_{pp} sends COM to V_{pp} .
- $\Sigma_{\text{cha}}^{\text{Oka}}(x) \rightarrow \text{CHA}$. This PPT algorithm is executed by V_{pp} . On input the statement x , it reads out the size of the security parameter as 1^λ and chooses a challenge message $c \in_R \text{CHASP}(1^\lambda)$. It puts the challenge message as $\text{CHA} := c$. It returns CHA . V_{pp} sends CHA to P_{pp} .
- $\Sigma_{\text{res}}^{\text{Oka}}(St, \text{CHA}) \rightarrow \text{RES}$. This PPT algorithm is executed by P_{pp} . On input the state St^a and the challenge message CHA , it computes $s_t := r_t + ct, s_u := r_u + cu$. It sets the response message as $\text{RES} := (s_t, s_u)$. P_{pp} sends RES to V_{pp} .
- $\Sigma_{\text{vrf}}^{\text{Oka}}(x, \text{COM}, \text{CHA}, \text{RES}) \rightarrow d$. This deterministic polynomial-time algorithm is executed by V_{pp} . On input the statement x and all the messages $(\text{COM}, \text{CHA}, \text{RES})$, it checks whether the following equality holds: $G^{s_t} H^{s_u} \stackrel{?}{=} AX^c$.

For the remaining two, $\Sigma_{\text{ext}}^{\text{Oka}}$ and $\Sigma_{\text{sim}}^{\text{Oka}}$, see [Oka92]. The protocol Σ^{Oka} is known to be a Σ -protocol.

Π_{pp}^{Oka} is *perfectly witness-indistinguishable* [FS90]. This is because the distribution of transcripts is independent of the witness $w \in W(x)$ even if the distribution of CHA deviates from the uniform random distribution.

Combining the Pedersen commitment scheme $\text{Cmt}_{pp}^{\text{Ped}}$ and the Okamoto interactive argument system Π_{pp}^{Oka} with the Σ -protocol Σ^{Oka} , we obtain the Pedersen-Okamoto commit-and-prove scheme $\text{CmtPrv}^{\text{PO}} = (\text{CmtPrv}^{\text{PO}}.\text{Setup}, \text{Cmt}_{pp}^{\text{Ped}} = (\text{Cmt}_{pp}^{\text{Ped}}.\text{Com}_{pp}, \text{Cmt}_{pp}^{\text{Ped}}.\text{Vrf}_{pp}), \Pi_{pp}^{\text{Oka}} = (P_{pp}, V_{pp}))$.