BlockMaze: An Efficient Privacy-Preserving Account-Model Blockchain Based on zk-SNARKs

Zhangshuang Guan, Zhiguo Wan, Yang Yang, Yan Zhou, and Butian Huang

***Abstract*—The disruptive blockchain technology is expected to have broad applications in many areas due to its advantages of transparency, fault tolerance, and decentralization, but the open nature of blockchain also introduces severe privacy issues. Since anyone can deduce private information about relevant accounts, different privacy-preserving techniques have been proposed for cryptocurrencies under the UTXO model, e.g., Zerocash and Monero. However, it is more challenging to protect privacy for account-model blockchains (e.g., Ethereum) since it is much easier to link accounts in the account-model blockchain. In this paper, we propose *BlockMaze*, an efficient privacy-preserving account-model blockchain based on zk-SNARKs. Along with dual-balance model, BlockMaze achieves strong privacy guarantees by hiding account balances, transaction amounts, and linkage between senders and recipients. Moreover, we provide formal security definitions and prove the security of *BlockMaze*. Finally, we implement a prototype of *BlockMaze* based on Libsnark and Go-Ethereum, and conduct extensive experiments to evaluate its performance. The experiment results demonstrate BlockMaze has high efficiency in computation and transaction throughput.**

***Index Terms*—Blockchain, Zero-Knowledge Proof, Account-Model, Privacy-Preserving, zk-SNARK**

I. Introduction

With the rise of cryptocurrencies, the blockchain has attracted tremendous interests worldwide, from IT industries, financial institutions, as well as academia. According to the statistics from BitInfoCharts [1], there are thousands of cryptocurrencies based on blockchain in the world. Among them, the market capitalization of Bitcoin [2] has exceeded $156 billion and Ethereum [3] has exceeded $28 billion. IT giants (e.g. Amazon, Alibaba, Google, and Facebook) and international financial institutions (e.g., JP Morgan and Goldman Sachs) also start to invest heavily on blockchains. In academia, researchers have started to investigate various issues of blockchain systems, e.g., consensus mechanisms, sharding mechanisms, and privacy protection.

From the perspective of balance representation, there are two popular models in blockchain networks: UTXO (Unspent Transaction Output) model and account model. The former is a directed graph of assets moving among users, in which the balance of a user is represented by all UTXOs related to that user in the blockchain system. The latter maintains a global state of all accounts and account balances that are updated whenever relevant transactions are executed. Bitcoin [2] is the first cryptocurrency based on the UTXO model, which is distributed through a proof of work "mining" process. Ethereum [3] adopts the account model with smart contract functionality to achieve Turing completeness. As shown in Table I, we briefly compare the advantages and disadvantages we have found with these two models. It shows that the primary advantages of the account model are superior programmability (i.e., smart contract) and fungibility of cryptocurrencies.

Both models achieve balance management for blockchains in different ways, and they both suffer from privacy leakage due to the openness of blockchains. Any adversary can obtain all transaction data, which contains too much privacy of the accounts. For example, the current transaction data of the Ethereum system is about 242 GB [1], which includes all transaction records from 30 July 2015 to 27 May 2019. By analyzing the transaction data in the blockchain [8], attackers can analyze the transaction relationship among different accounts. Since transactions are permanently recorded on the blockchain, which may cause an issue: once a historical transaction discloses the real identity of a user, the information of this user in all relevant transaction records will be revealed. Moreover, attackers can also use off-chain auxiliary information to infer the identity of accounts in the blockchain [9].

To address privacy issues in the blockchain, researchers have proposed several effective techniques in the literature, such as mixers, ring signatures, zero-knowledge proofs, and homomorphic encryption. For the UTXO model, there are existing projects such as Zerocoin [10], Zerocash [4], Monero [5], Dash [11], and CoinJoin [12] solving privacy issues. For the account model, however, there are very few proposals such as [13] providing limited privacy protection. Obviously, it is more challenging to protect the privacy of the blockchain in the account model than the UTXO model. The main reason is that the account model implicitly restricts that each user has only one account, which is required for smart contracts. Moreover, the account model updates the new balance of relevant accounts whenever a transaction is confirmed on the blockchain. On the contrary, the UTXO model does not have such a restriction, so it is easier to preserve privacy for a UTXO-model blockchain.

Inspired by the design of the UTXO-model blockchain Zerocash, we design BlockMaze, a privacy-preserving account-model blockchain based on zk-SNARKs (zero-knowledge Succinct Non-interactive ARguments of Knowledge) [14]–[17]. To the best of our knowledge, BlockMaze is the first privacy-preserving account-model blockchain that protects both trans-

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TABLE I: Comparison of balance models

Model	Typical projects	Smart contract	Advantages	Disadvantages
UTXO	Bitcoin [2] Zerocash [4] Monero [5]	No	Allow multi-threads for computations; support complete transparency of asset movements; process transactions easily in parallel.	Hard to work with smart contracts; increase the computational and storage burdens.
Account	Ethereum [3] Fabric [6] EOS [7]	Yes	Provide an intuitively clear approach of balances; give a simple implementation of smart contracts; achieve Turing-complete.	Need to store all accounts states; hard to track assets; analyze the states more easily.

action amounts and sender/recipient relationship. More specifically, we propose a *dual-balance* model for account-model blockchains, which is composed of a plaintext balance and a zero-knowledge balance for each account. Along with the zero-knowledge balance, we employ zk-SNARKs to construct privacy-preserving transactions to hide transaction amounts and account balances. To disconnect the linkage between senders and recipients, we design a two-step fund transfer procedure based on the privacy-preserving transactions.

Contributions. In summary, the main contributions of this paper are as follows:

- We present *BlockMaze*, to the best of our knowledge, the first privacy-preserving account-model blockchain, hiding both transaction amounts and the linkage between a transaction sender and its recipient. Using zk-SNARKs, we design a *dual-balance* model and a two-step fund transfer procedure for account-model blockchains.
- We provide a formal security model for *BlockMaze*, under which we prove its security. Moreover, we give a comprehensive discussion on practical issues regarding its compatibility and scalability.
- We implement a prototype of *BlockMaze* based on Libsnark [18] and Go-Ethereum [19], and conduct comprehensive experiments to evaluate its performances.

A. Related Work

Privacy-preserving technology. In the blockchain and cryptocurrency, techniques that have been considered for achieving privacy protection are as follows:

- Mixer: A specialized mix serves as the intermediary between multiple senders and recipients, hiding the relationship between senders and recipients (e.g., [11], [12]), which is similar to Chaum's mix in the setting of untraceable emails.
- Ring signature: The genuine sender of a transaction can be hidden among several decoys using ring signature (e.g., [5]). Although miners can verify the signature, they cannot identify the genuine sender.
- Zero-knowledge proof: A transaction sender can employ zero-knowledge proofs to prove that he/she is entitled to spend some cryptocurrencies without leaking identity information and transaction amounts (e.g., [4]).
- Homomorphic encryption: Homomorphic encryption is potent and can be used in blockchain to preserve transaction amounts (e.g., [13]).

We analyze the privacy-preserving cryptocurrencies mentioned above in Appendix A. Moreover, we refer the reader to [20] for some typical privacy-preserving mechanisms for blockchains.

Zero-knowledge proof systems for blockchains. The aim of zero-knowledge proof technology is for the prover to convince the verifier that he/she does know a secret without disclosing it. There are several major zero-knowledge proof systems for the blockchain as follows:

- Non-Interactive Zero-Knowledge (NIZK): The Zerocoin protocol [10] utilizes NIZK proof to preserve privacy for the UTXO-model blockchain. To implement privacy-preserving smart contracts, DAP [13] has been proposed as a tool to protect the inputs of complex smart contracts. Compared with other solutions, the main issues with NIZK include limited functionalities and high computation cost.
- Zero-Knowledge Succinct Non-interactive ARguments of Knowledge (zk-SNARK): Many zk-SNARK constructions [14]–[17] have been proposed in the literature. Due to its versatility, zk-SNARK has been successfully applied to preserve privacy for cryptocurrencies such as Zcash [4].
- Bulletproof [21]: Bulletproof is an efficient zero knowledge proof without trusted setup. Monero [5] has used it as the range proof to reduce transaction fee costs and transaction sizes. However, the prover's computation cost and verification cost are much more than zk-SNARK.
- Zero-Knowledge Scalable Transparent ARgument of Knowledge (zk-STARK) [22]: Compared with others, zk-STARK does not need a trusted setup, and it is much more efficient in computation than zk-SNARK. However, zk-STARK in the current state is not suitable for blockchains because of its large proof size.

Concerning privacy issues in smart contracts, there have been several proposals [23]–[25], which achieve privacy protection for smart contracts using zero-knowledge proofs.

B. Organization.

The remainder of the paper is structured as follows: we first provide preliminaries on our proposal, including cryptographic building blocks in Section II. Then, we give an intuition of the idea, data structures used in the system, and an overview of the system architecture in Section III. After that, we describe and construct our BlockMaze scheme with security proof in detail in Section IV. Furthermore, we give a comprehensive discussion and analysis of BlockMaze in Section V. In Section VI, we describe details on the implementation of the prototype of BlockMaze, evaluate its performance, and compare it with Zerocash. Finally, Section VII concludes this paper.

II. Preliminaries

In this section, we recall zk-SNARKs and public key encryption used throughout this paper.

A. zk-SNARK

A zk-SNARK scheme [16], [17] can be represented by a tuple of polynomial-time algorithms $\Pi_{\mathcal{Z}}$ = (Setup, KeyGen, GenProof, VerProof).

$\text{Setup}(1^\lambda) \rightarrow \mathsf{pp}_{\mathcal{Z}}$. Given a security parameter λ, this algorithm generates a list of public parameters $\mathsf{pp}_{\mathcal{Z}} = (p, e, \mathbb{G}_1, \mathcal{P}_1, \mathbb{G}_2, \mathcal{P}_2, \mathbb{G}_T, \mathbb{F}_p)$, where p is a prime; e is a bilinear map: $\mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$; $(\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T)$ are three cyclic groups of order p; $\mathcal{P}_1$ and $\mathcal{P}_2$ are generators of $\mathbb{G}_1$ and $\mathbb{G}_2$ respectively; $\mathbb{F}_p$ is a finite field. All algorithms utilize $\mathsf{pp}_{\mathcal{Z}}$ as default input public parameters.

$\text{KeyGen}(C) \rightarrow (\mathsf{pk}_{\mathcal{Z}}, \mathsf{vk}_{\mathcal{Z}})$. Given a circuit C, this algorithm utilizes the public parameters $\mathsf{pp}_{\mathcal{Z}}$ to generate a key pair $(\mathsf{pk}_{\mathcal{Z}}, \mathsf{vk}_{\mathcal{Z}})$. $\mathsf{pk}_{\mathcal{Z}}$ is a proving key for proof generation, while $\mathsf{vk}_{\mathcal{Z}}$ is a verification key for proof verification.

$\text{GenProof}(\mathsf{pk}_{\mathcal{Z}}, \vec{x}, \vec{a}) \rightarrow \pi$. The algorithm generates a zero-knowledge proof π (or $\perp$ if GenProof fails). $\mathsf{pk}_{\mathcal{Z}}$ is a proving key, $\vec{x}$ is a public statement which is an input of circuit C, $\vec{a}$ is a private witness which is an auxiliary input of circuit C, and π is a zero-knowledge proof proving the relation constructed by circuit C between $\vec{x}$ and $\vec{a}$. Note that $\vec{x}$ and π are published and made available to anyone.

$\text{VerProof}(\mathsf{vk}_{\mathcal{Z}}, \vec{x}, \pi) \rightarrow b$. According to this algorithm, anyone can check and verify a zero-knowledge proof. The algorithm outputs $b = 1$ if the check is successful; otherwise it outputs $b = 0$. $\mathsf{vk}_{\mathcal{Z}}$ is a verification key, π is a zero-knowledge proof generated in GenProof, and $\vec{x}$ is public data used to generate π in GenProof.

A zk-SNARK satisfies completeness, succinctness, proof of knowledge, and perfect zero-knowledge properties [15], [17]. Given a security parameter λ and any circuit C with a relation $\mathcal{R}_C$, the honest prover can utilize a proof π to convince the verifier for every pair $(\vec{x}, \vec{a}) \in \mathcal{R}_C$ where $\vec{x}$ is a statement and $\vec{a}$ is a witness.

Completeness. For each pair $(\vec{x}, \vec{a}) \in \mathcal{R}_C$, we have

$$\Pr\left[\text{VerProof}(\mathsf{vk}_{\mathcal{Z}}, \vec{x}, \pi) \rightarrow 1 \,\middle|\, \begin{matrix}\text{KeyGen}(C) \rightarrow (\mathsf{pk}_{\mathcal{Z}}, \mathsf{vk}_{\mathcal{Z}}) \\ \text{GenProof}(\mathsf{pk}_{\mathcal{Z}}, \vec{x}, \vec{a}) \rightarrow \pi\end{matrix}\right] = 1.$$

Succinctness. An honestly-generated proof π has $O_\lambda(1)$ bits and $\text{VerProof}(\mathsf{vk}_{\mathcal{Z}}, \vec{x}, \pi)$ runs in time $O_\lambda(|x|)$.

Proof of knowledge (and Soundness). For every probabilistic polynomial-time adversary $\mathcal{A}$, there is a probabilistic polynomial-time witness extractor $\mathcal{E}$, we have

$$\Pr\left[\begin{matrix}(\vec{x}, \vec{a}) \notin \mathcal{R}_C \\ \text{VerProof}(\mathsf{vk}_{\mathcal{Z}}, \vec{x}, \pi) \rightarrow 1\end{matrix} \,\middle|\, \begin{matrix}\text{KeyGen}(C) \rightarrow (\mathsf{pk}_{\mathcal{Z}}, \mathsf{vk}_{\mathcal{Z}}) \\ \mathcal{A}(\mathsf{pk}_{\mathcal{Z}}, \mathsf{vk}_{\mathcal{Z}}) \rightarrow (\vec{x}, \pi) \\ \mathcal{E}(\mathsf{pk}_{\mathcal{Z}}, \mathsf{vk}_{\mathcal{Z}}) \rightarrow \vec{a}\end{matrix}\right] \leq \mathsf{negl}(\lambda).$$

Perfect zero-knowledge. The proof is perfect zero-knowledge if there is a simulator $\mathcal{S}$ such that, for every pair $(\vec{x}, \vec{a}) \in \mathcal{R}_C$ and all non-uniform polynomial time adversaries $\mathcal{A}$, the following two probabilities are equal:

$$\Pr\left[\mathcal{A}(\mathsf{pk}_{\mathcal{Z}}, \mathsf{vk}_{\mathcal{Z}}, \pi) = 1 \,\middle|\, \begin{matrix}\text{KeyGen}(C) \rightarrow (\mathsf{pk}_{\mathcal{Z}}, \mathsf{vk}_{\mathcal{Z}}) \\ \text{GenProof}(\mathsf{pk}_{\mathcal{Z}}, \vec{x}, \vec{a}) \rightarrow \pi\end{matrix}\right],$$

$$\Pr\left[\mathcal{A}(\mathsf{pk}_{\mathcal{Z}}, \mathsf{vk}_{\mathcal{Z}}, \pi) = 1 \,\middle|\, \begin{matrix}\mathcal{S}(\mathcal{R}_C) \rightarrow (\mathsf{pk}_{\mathcal{Z}}, \mathsf{vk}_{\mathcal{Z}}, \mathsf{trap}) \\ \mathcal{S}(\mathsf{pk}_{\mathcal{Z}}, \vec{x}, \mathsf{trap}) \rightarrow \pi\end{matrix}\right].$$

B. Encryption

A public key encryption scheme can be represented by a tuple of polynomial-time algorithms $\Pi_{\mathcal{E}}$ = (Setup, KeyGen, Enc, Dec).

$\text{Setup}(1^\lambda) \rightarrow \mathsf{pp}_{\mathcal{E}}$. On input a security parameter λ, this algorithm generates a list of public parameters $\mathsf{pp}_{\mathcal{E}}$.

$\text{KeyGen}(\mathsf{pp}_{\mathcal{E}}) \rightarrow (sk_{\mathcal{E}}, pk_{\mathcal{E}})$. On input a list of public parameters $\mathsf{pp}_{\mathcal{E}}$, this algorithm generates a private/public key pair $(sk_{\mathcal{E}}, pk_{\mathcal{E}})$ for an account.

$\text{Enc}_{pk_{\mathcal{E}}}(m) \rightarrow c$. Given a public key $pk_{\mathcal{E}}$, this algorithm encrypts an input plaintext m to output a ciphertext c.

$\text{Dec}_{sk_{\mathcal{E}}}(c) \rightarrow m$. Given a private key $sk_{\mathcal{E}}$, this algorithm decrypts an input ciphertext c to get back the plaintext m.

For the sake of privacy, the pubic encryption scheme $\Pi_{\mathcal{E}}$ used in our scheme needs to satisfy the following two security properties: (i) *key indistinguishability under chosen-ciphertext attack* (IK-CCA security) [26], and (ii) *ciphertext indistinguishability under chosen-ciphertext attack* (IND-CCA security) [27].

III. System Architecture and Models

In this section, we firstly introduce an intuition of the idea, and then give data structures used in the system, and finally describe the system model to show how the proposed BlockMaze works.

A. Intuition of the idea

In the context of cryptocurrency, three types of privacy information need to be protected in the account-model blockchain: account balances, transaction amounts, and sender/recipient relationship. To preserve these types of privacy information, we present a *dual-balance* model that divides the balance of an account into two parts: a plaintext balance and a zero knowledge balance. Essentially, the zero-knowledge balance is associated with a commitment over the corresponding value, and it represents one part of the user's balance without disclosing the amount. The two types of balance can be converted to each other, and the zero-knowledge balance can be used to transfer fund to another account using zk-SNARKs.

However, the sender/recipient relationship cannot be hidden using the above approach. This problem does not exist in Zerocash [4] because Zerocash creates new accounts for each fund transfer transaction. Since the account model does not allow a user having multiple accounts, we design a two-step fund transfer procedure in BlockMaze. In the first step, the sender makes the fund transfer commitment with a `Send` transaction. To enforce the zero-knowledge balance update, we utilize its serial number to prevent double-spending issues based on the `Send` transaction. After the `Send` transaction is confirmed on the blockchain, the recipient collates its fund transfer commitment with other fund transfer commitments to form a Merkle tree. Then the recipient generates a zero-knowledge proof to receive the transferred fund without leaking from which transaction he/she receives the fund.

In summary, BlockMaze makes the following changes: (i) account balances and transaction amounts are hidden with a

secure commitment scheme; (ii) the linkage between transactions is obscured by a two-step procedure: first, a sender computes a commitment on the transferred amount and generates a zero-knowledge transaction to transfer funds; then, the recipient recovers the transfer commitment from sender and generates a zero-knowledge transaction to deposit funds from the sender; (iii) zk-SNARK is employed to guarantee that a zero-knowledge transaction is valid and account balance can be updated legally without creating new money.

B. Data structures

Ledger. Given any time $\mathtt{T}$, all users can access $\mathtt{Ledger}_{\mathtt{T}}$, which is a sequence of blocks including all transactions. For convenience, we assume that $\mathtt{Ledger}_{\mathtt{T}}$ stores $\mathtt{Ledger}_{\mathtt{T}'}$ for all $\mathtt{T}' \leq \mathtt{T}$. Each block in the ledger includes transactions and a new data set $\mathtt{TCMSet}$ described below. And the transactions in the ledger include both basic transactions as well as four zero-knowledge transactions. For convenience, we assume that $\mathtt{Ledger}_{\mathtt{T}}$ has stored $\mathtt{block}_{\mathtt{N}}$ before the given time $\mathtt{T}$.

Address key pair. There is an address key pair (sk, pk) instantiated for each account. sk is a private key for decrypting shared parameters and accessing private data, pk is a public key encrypting shared parameters, and $addr := \mathtt{CRH}(pk)$ ($\mathtt{CRH}$ is a collision-resistant hash function) is an account address sending and receiving payments.

Commitments. There are two types of commitments: balance commitments and fund transfer commitments.

A balance commitment is a commitment of the account balance as follows:

$$cmt_A := \mathtt{COMM}_{\mathtt{bc}}(addr_A, value_A, sn_A, r_A),$$

where $\mathtt{COMM}_{\mathtt{bc}}$ is a statistically-hiding non-interactive commitment scheme for account balance, cmt_A is the commitment of current account balance for user A, $addr_A$ is the account address of A, $value_A$ is the plaintext balance corresponding to cmt_A, sn_A is the serial number associated with cmt_A, and r_A is a random number masking sn_A. Moreover, $sn_A := \mathtt{PRF}(sk_A, r_A)$, where $\mathtt{PRF}$ is a pseudorandom function and sk_A is the private key of account A. Note that cmt_A is published and recorded on the blockchain as A's balance commitment.

A fund transfer commitment is a commitment of an amount being transferred between two parties as follows:

$$cmt_v := \mathtt{COMM}_{\mathtt{tc}}(addr_A, v, pk_B, sn_v, r_v, sn_A),$$

where $\mathtt{COMM}_{\mathtt{tc}}$ is a statistically-hiding non-interactive commitment scheme for fund transfer, cmt_v is a fund transfer commitment from sender A to recipient B, $addr_A$ is the account address of sender A, v is the plaintext amount to be transferred, pk_B is the public key of recipient B, $sn_v := \mathtt{PRF}(sk_A, r_v)$ is the serial number associated with cmt_v, r_v is a random number masking cmt_v, and sn_A is the serial number associated with the balance commitment cmt_A of sender A.

Balance. Account-model blockchains adopt Merkle Patricia Tree (MPT) [3] to record the new account balance (e.g., Ethereum). Using MPT, we design a *dual-balance* model for account balances. There are two forms of account balance: one is a plaintext balance, denoted as $\mathtt{pt\_balance}$, which is public and made accessible to anyone; the other one is a zero-knowledge balance, denoted as $\mathtt{zk\_balance}$, which hides the corresponding plaintext value using balance commitments. For example, the balance of account A is as follows:

$$\mathtt{pt\_balance}_A := \text{the amount of plaintext balance},$$
$$\mathtt{zk\_balance}_A := (cmt_A, addr_A, value_A, sn_A, r_A).$$

The asset of account A is the sum of both $\mathtt{pt\_balance}_A$ and $\mathtt{zk\_balance}_A.value_A$. Note that $\mathtt{zk\_balance}_A$ is only stored secretly by the owner of account A. $value_A$, sn_A and r_A are private data of account A, while $\mathtt{pt\_balance}_A$ and cmt_A are publicly recorded on MPT and made accessible to anyone. As shown in Fig. 1, the MPT realizes both fast search and authentication of an account balance. As each path from the root to a leaf node represents the account address, MPT records account balances at the corresponding leaf nodes for all existing accounts. One can not only quickly find the balance of a given account by searching MPT, but also authenticate the account balance by verifying whether the corresponding leaf node is on the MPT.

Our scheme realizes two-way balance transformation between plaintext balance and zero-knowledge balance. Although the transformed amount is public in the initial transformation transaction, the zero-knowledge balance is no longer visible after a fund transfer transaction.

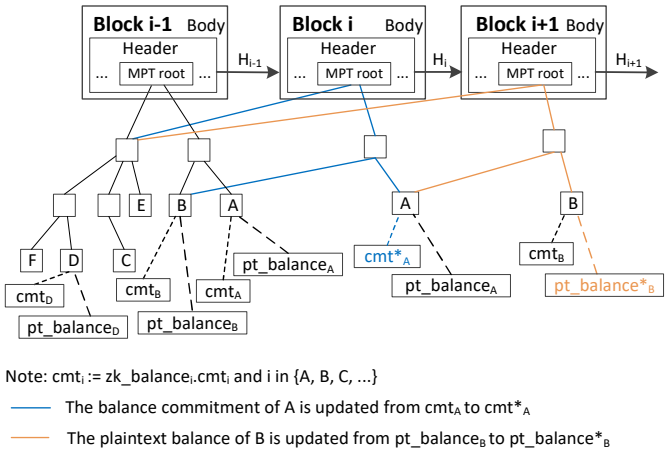


Fig. 1: Merkle Patricia Tree (MPT)

Data Set. There are two new types of data set to store the published one-time variables. Since that $\mathtt{Ledger}_{\mathtt{T}}$ has stored $\mathtt{block}_{\mathtt{N}}$ before the given time $\mathtt{T}$, it is convenient to deduce these sets from $\mathtt{Ledger}_{\mathtt{T}}$.

- $\mathtt{TCMSet}$. For any given block number $\mathtt{N}$, $\mathtt{TCMSet}_{\mathtt{N}}$ denotes the set of all transfer commitments cmt_v appearing in transactions in $\mathtt{block}_{\mathtt{N}}$.
- $\mathtt{SNSet}$. For any given time $\mathtt{T}$, $\mathtt{SNSet}_{\mathtt{T}}$ denotes the set of all serial numbers sn_A and sn_v appearing in transactions in $\mathtt{Ledger}_{\mathtt{T}}$, and these serial numbers are allowed to be used only once.

Merkle tree. Given any time $\mathtt{T}$, a user selects a set of block sequence numbers $\mathtt{SEQ}$ corresponding to randomly selected blocks. Each block contains one or more fund transfer commitments cmt_v. Based on the block whose number belongs to $\mathtt{SEQ}$, all $\mathtt{TCMSet}$ of these blocks are set as leaf nodes t form a Merkle tree. We utilize $\mathtt{MT}_{\mathtt{T}}$ to denote a Merkle tree

over $\bigcup_{n\in\mathtt{SEQ}}\mathtt{TCMSet_n}$ and $rt_{\mathtt{T}}$ to denote its root. Furthermore, $\mathtt{Path_T}(cmt_v)$ is an authentication path from the specified fund transfer commitment cmt_v appearing in $\mathtt{TCMSet_N}$ to $rt_{\mathtt{T}}$ at any given time $\mathtt{T}$.

On-Chain transactions. Apart from basic transactions, BlockMaze defines four new types of zero-knowledge transactions using zk-SNARK as below. More details about the format of these transactions are described in Section IV-C.

- `Mint`. This transaction $\mathtt{tx_{Mint}}$ converts a plaintext amount into a zero-knowledge amount and merges the zero-knowledge amount into the current zero-knowledge balance of an account.
- `Redeem`. This transaction $\mathtt{tx_{Redeem}}$ converts a zero-knowledge amount back into a plaintext amount and merges the plaintext amount into the current plaintext balance of an account.
- `Send`. This transaction $\mathtt{tx_{Send}}$ is built to send a zero-knowledge amount from a sender to a recipient. And the transaction $\mathtt{tx_{Send}}$ hides the recipient's address and the transaction amount.
- `Deposit`. This transaction $\mathtt{tx_{Deposit}}$ allows a recipient to deposit a received payment into his/her account.

C. System Model

Definition 1 (Account-Model Blockchain)**.** An account-model blockchain stores and maintains an append-only ledger `LEDGER`, an account list `ACCOUNT` and a stateful tree `BALANCE` as below:

$$\mathtt{LEDGER} \stackrel{\text{def}}{=} List[\mathtt{transaction}]$$

$$\mathtt{ACCOUNT} \stackrel{\text{def}}{=} List[\mathtt{user} \Rightarrow \mathtt{address}]$$

$$\mathtt{BALANCE} \stackrel{\text{def}}{=} List[\mathtt{address} \Rightarrow \mathtt{balance}]$$

Given an account-based transaction tx, we can parse it as $tx \stackrel{\text{def}}{=} \{sender, recipient, value, *\}$ where $sender$ and $recipient$ are the account addresses, $value$ is the amount to be transferred and $*$ denotes other useful fields. Note that each user has only one account address. If tx is a valid transaction, $tx.value$ is deducted from the balance of $tx.sender$ while $tx.value$ is added to the balance of $tx.recipient$.

Given any time $\mathtt{T}$ and a transaction list `TxList` containing all transactions which are confirmed during $(\mathtt{T}-1, \mathtt{T}]$, for each $tx \in \mathtt{TxList}$ and $(u, addr)$ in tx, the above data structures change as follows:

$$\mathtt{LEDGER_T} = \mathtt{LEDGER_{T-1}} \cup \mathtt{TxList}$$

$$\mathtt{ACCOUNT_T} = \mathtt{ACCOUNT_{T-1}}.updateAddress(u \Rightarrow addr)$$

$$\mathtt{BALANCE_T} = \mathtt{BALANCE_{T-1}}.updateBalance(addr \Rightarrow (bal \pm tx.value))$$

Definition 2 (Privacy-preserving Account-Model Blockchain)**.** A privacy-preserving account-model blockchain stores and maintains an append-only ledger `LEDGER`, an account list `ACCOUNT` and a modified tree combining `PT_BALANCE` and `ZK_BALANCE` defined as below:

$$\mathtt{LEDGER} \stackrel{\text{def}}{=} List[\mathtt{zk\_transaction}]$$

$$\mathtt{ACCOUNT} \stackrel{\text{def}}{=} List[\mathtt{user} \Rightarrow \mathtt{address}]$$

$$\mathtt{PT\_BALANCE} \stackrel{\text{def}}{=} List[\mathtt{address} \Rightarrow \mathtt{pt\_balance}]$$

$$\mathtt{ZK\_BALANCE} \stackrel{\text{def}}{=} List[\mathtt{address} \Rightarrow \mathtt{zk\_balance}]$$

Given a zero-knowledge account-based transaction tx, we can parse it as $tx \stackrel{\text{def}}{=} \{sender, cmt, proof, *\}$ where $sender$ is an account address, cmt is a commitment of value to be transferred, $proof$ is a zero-knowledge proof and $*$ denotes other useful fields. Note that each user has only one account address. `LEDGER`, `ACCOUNT`, `PT_BALANCE` are identical to the one defined in Definition 1.

Given any time $\mathtt{T}$ and a transaction list `TxList` containing all zero-knowledge transactions which are confirmed during $(\mathtt{T}-1, \mathtt{T}]$, for each $tx \in \mathtt{TxList}$ and $(u, addr)$ in tx, the `ZK_BALANCE` changes as follows:

$$\mathtt{ZK\_BALANCE_T} = \mathtt{ZK\_BALANCE_{T-1}}.updateZKBalance(addr \Rightarrow cmt).$$

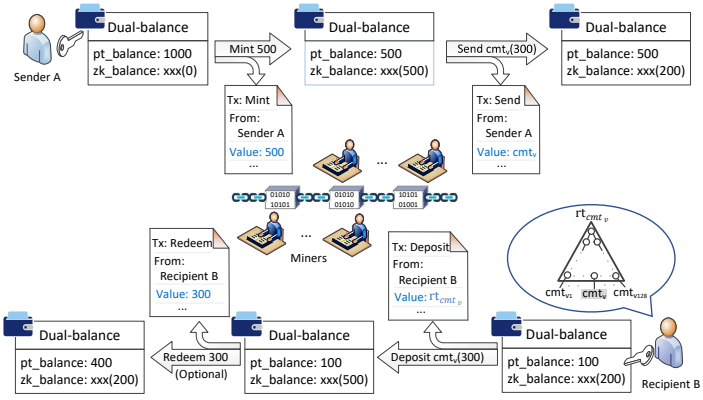


Fig. 2: System architecture and workflow of BlockMaze

As shown in Fig. 2, the system is composed of a ledger and nodes including users (i.e., senders and recipients) and miners. Users can convert plaintext amounts into zero-knowledge balances by generating `Mint` transactions, and vice versa (resp., `Redeem` transactions). Senders can transfer money to others using `Send` transactions, while recipients can deposit received payment from others to their accounts by building `Deposit` transactions with a Merkle tree. Miners are responsible for processing and confirming these zero-knowledge transactions, utilize the consensus algorithm to pack the transactions into a block, and maintain the ledger. After confirming the zero-knowledge transactions, miners update the balance of accounts as shown in Fig. 1.

IV. BlockMaze Scheme

In this section, we firstly describe a general BlockMaze scheme, and then give its security definition and security proof, and finally give a specific construction of BlockMaze.

For the sake of convenience, we suppose that there are two accounts: sender A (Alice) and recipient B (Bob), their key pairs are (sk_A, pk_A) and (sk_B, pk_B) respectively, their account addresses are $addr_A$ and $addr_B$, their plaintext balances are denoted as $\mathtt{pt\_balance}_A$ and $\mathtt{pt\_balance}_B$, and their current zero-knowledge balances can be represented as follows:

$$\mathtt{zk\_balance}_A := (cmt_A, addr_A, value_A, sn_A, r_A),$$
$$\mathtt{zk\_balance}_B := (cmt_B, addr_B, value_B, sn_B, r_B).$$

A. Description of BlockMaze

A BlockMaze scheme Π is composed of polynomial-time algorithms Π = (**Setup**, **CreateAccount**, **Mint**, **Redeem**, **Send**, **Deposit**, **VerTx**). It has three properties: ledger indistinguishability, transaction unlinkability, and balance.

1) $\mathbf{Setup}(1^\lambda) \rightarrow \mathsf{pp}$. Given a security parameter λ, this algorithm generates a list of public parameters pp, which are published and made available to anyone. Note that **Setup** is just executed only once by a trusted party.

2) $\mathbf{CreateAccount}(\mathsf{pp}) \rightarrow \{addr, (sk, pk)\}$. Given public parameters pp, this algorithm creates an account address $addr$ and key pair (sk, pk) for a user, where sk is a private key for accessing private data (also decrypting transaction data), pk is a public key for encrypting transaction data, and $addr$ is an account address for sending and receiving payments.

3) $\mathbf{Mint}(\mathsf{pp}, \mathtt{zk\_balance}_A, \mathtt{pt\_balance}_A, sk_A, v) \rightarrow \{\mathtt{zk\_balance}^*_A, \mathtt{tx}_{\mathtt{Mint}}\}$. This algorithm enables an account (say A) to convert a plaintext amount v into a zero-knowledge amount and merge it with the current zero-knowledge balance. Given public parameters pp, the current zero-knowledge balance $\mathtt{zk\_balance}_A$, the current plaintext balance $\mathtt{pt\_balance}_A$, the account private key sk_A, and a plaintext amount v to be converted into a zero-knowledge amount, account A utilizes this algorithm to mint her new zero-knowledge balance $\mathtt{zk\_balance}^*_A$ and generate a transaction $\mathtt{tx}_{\mathtt{Mint}}$.

Once the $\mathtt{tx}_{\mathtt{Mint}}$ is recorded on blockchain successfully, the state change of A is as follows:

A's state recorded on MPT before **Mint** : $\{\mathtt{pt\_balance}_A, cmt_A\}$,
A's state recorded on MPT after **Mint** : $\{\mathtt{pt\_balance}_A - v, cmt^*_A\}$.

4) $\mathbf{Redeem}(\mathsf{pp}, \mathtt{zk\_balance}_A, sk_A, v) \rightarrow \{\mathtt{zk\_balance}^*_A, \mathtt{tx}_{\mathtt{Redeem}}\}$. This algorithm enables an account (say A) to convert a zero-knowledge amount back into a plaintext balance. Given public parameters pp, the current zero-knowledge balance $\mathtt{zk\_balance}_A$, the account private key sk_A, and a plaintext amount v to be converted back from the zero-knowledge balance, account A utilizes this algorithm to redeem a plaintext amount v from her new zero-knowledge balance $\mathtt{zk\_balance}^*_A$ and generate a transaction $\mathtt{tx}_{\mathtt{Redeem}}$.

After that the $\mathtt{tx}_{\mathtt{Redeem}}$ is recorded on blockchain successfully, the state change of A is as follows:

A's state recorded on MPT before **Redeem** : $\{\mathtt{pt\_balance}_A, cmt_A\}$,
A's state recorded on MPT after **Redeem** : $\{\mathtt{pt\_balance}_A + v, cmt^*_A\}$.

5) $\mathbf{Send}(\mathsf{pp}, \mathtt{zk\_balance}_A, sk_A, pk_B, v) \rightarrow \{\mathtt{zk\_balance}^*_A, \mathtt{tx}_{\mathtt{Send}}\}$. This algorithm enables sender A to send a zero-knowledge amount to recipient B. Given public parameters pp, the current zero-knowledge balance $\mathtt{zk\_balance}_A$, the account private key sk_A, recipient's public key pk_B, and a plaintext amount v to be transferred, account A calls this algorithm to obtain her new zero-knowledge balance $\mathtt{zk\_balance}^*_A$ and generate a transaction $\mathtt{tx}_{\mathtt{Send}}$.

After building transaction $\mathtt{tx}_{\mathtt{Send}}$, A should inform B with a hash $h_{\mathtt{tx}_{\mathtt{Send}}} := \mathtt{CRH}(\mathtt{tx}_{\mathtt{Send}})$ such that B can retrieve and parse $\mathtt{tx}_{\mathtt{Send}}$ to construct $\mathtt{tx}_{\mathtt{Deposit}}$. When the $\mathtt{tx}_{\mathtt{Send}}$ is recorded on blockchain successfully, the state change of A is as follows:

A's state recorded on MPT before **Send** : $\{\mathtt{pt\_balance}_A, cmt_A\}$,
A's state recorded on MPT after **Send** : $\{\mathtt{pt\_balance}_A, cmt^*_A\}$.

6) $\mathbf{Deposit}(\mathtt{Ledger}_\mathtt{T}, \mathsf{pp}, (sk_B, pk_B), h_{\mathtt{tx}_{\mathtt{Send}}}, \mathtt{zk\_balance}_B) \rightarrow \{\mathtt{zk\_balance}^*_B, \mathtt{tx}_{\mathtt{Deposit}}\}$. This algorithm enables recipient B to check and deposit a received payment into his account. Given the current ledger $\mathtt{Ledger}_\mathtt{T}$, public parameters pp, account key pair (sk_B, pk_B), the hash of a Send transaction $h_{\mathtt{tx}_{\mathtt{Send}}}$ and the current zero-knowledge balance $\mathtt{zk\_balance}_B$, recipient B calls **Deposit** to obtain his new zero-knowledge balance $\mathtt{zk\_balance}^*_B$ and build a transaction $\mathtt{tx}_{\mathtt{Deposit}}$.

Based on the hash of $\mathtt{tx}_{\mathtt{Send}}$ whose sender is A, recipient B can retrieve and parse $\mathtt{tx}_{\mathtt{Send}}$ to construct $\mathtt{tx}_{\mathtt{Deposit}}$. Once the $\mathtt{tx}_{\mathtt{Deposit}}$ is recorded on blockchain successfully, the state change of B is as follows:

B's state recorded on MPT before **Deposit** : $\{\mathtt{pt\_balance}_B, cmt_B\}$,
B's state recorded on MPT after **Deposit** : $\{\mathtt{pt\_balance}_B, cmt^*_B\}$.

7) $\mathbf{VerTx}(\mathtt{Ledger}_\mathtt{T}, \mathsf{pp}, \mathtt{tx}) \rightarrow b$. Given the current ledger $\mathtt{Ledger}_\mathtt{T}$, public parameters pp and a zero-knowledge transaction tx, miners call this algorithm to check the validity of all zero-knowledge transactions. The algorithm outputs $b = 1$ if tx is valid, otherwise it outputs $b = 0$. Miners (or nodes maintaining the blockchain) are responsible for verifying all transactions and then update the state of related accounts.

B. Security of BlockMaze

Definition 3 (Security)**.** A BlockMaze scheme is *secure* if it satisfies ledger indistinguishability, transaction unlinkability, and balance as defined below.

Ledger Indistinguishability [4]. A ledger is indistinguishable if it does not reveal new information to the adversary besides the publicly-revealed information. We say a BlockMaze scheme Π is ledger indistinguishable if, for every probabilistic polynomial-time (PPT) adversary $\mathcal{A}$ and sufficiently large λ, we have $\Pr\left[\mathsf{BlockMaze}^{\mathtt{L\text{-}IND}}_{\Pi,\mathcal{A}}(\lambda) = 1\right] \leq \frac{1}{2} + \mathsf{negl}(\lambda)$, where $\Pr\left[\mathsf{BlockMaze}^{\mathtt{L\text{-}IND}}_{\Pi,\mathcal{A}}(\lambda) = 1\right]$ is $\mathcal{A}$'s winning probability in the L-IND experiment.

Transaction Unlinkability. A transaction is unlinkable if it does not leak the linkage between its sender and recipient during fund transfers. We say a BlockMaze scheme Π is transaction unlinkable if, for every PPT adversary $\mathcal{A}$ and sufficiently large λ, we have $\Pr\left[\mathsf{BlockMaze}^{\mathtt{TR\text{-}UL}}_{\Pi,\mathcal{A}}(\lambda) = 1\right] \leq \mathsf{negl}(\lambda)$, where $\Pr\left[\mathsf{BlockMaze}^{\mathtt{TR\text{-}UL}}_{\Pi,\mathcal{A}}(\lambda) = 1\right]$ is $\mathcal{A}$'s winning probability in the TR-UL experiment.

Balance [4]. A ledger is balanced if the adversary cannot spend the money more than that in his account. We say a BlockMaze scheme Π is balanced if, for every probabilistic polynomial-time (PPT) adversary $\mathcal{A}$ and sufficiently large λ, we have $\Pr\left[\mathsf{BlockMaze}^{\mathtt{BAL}}_{\Pi,\mathcal{A}}(\lambda) = 1\right] \leq \mathsf{negl}(\lambda)$, where $\Pr\left[\mathsf{BlockMaze}^{\mathtt{BAL}}_{\Pi,\mathcal{A}}(\lambda) = 1\right]$ is $\mathcal{A}$'s winning probability in the BAL experiment.

We provide the formal definitions in Appendix B. Ledger indistinguishability, which is defined by the L-IND experiment, means that no PPT adversary $\mathcal{A}$ can distinguish between two ledgers L_0 and L_1, which are constructed by $\mathcal{A}$ requesting queries to two separated BlockMaze oracles. Transaction unlinkability, which is formalized by the $\mathtt{TR-UL}$ experiment, means that given a valid fund transfer transaction, the adversary cannot tell: (i) who is the recipient of the transaction

and (ii) who is the sender of the payment in the transaction. Balance, which is defined by the BAL experiment, means that the adversary cannot obtain more money than what he minted or received via payments from others.

Theorem 1. The tuple Π = (**Setup**, **CreateAccount**, **Mint**, **Redeem**, **Send**, **Deposit**, **VerTx**), as defined in Section IV-C, is a *secure* BlockMaze scheme.

Proof. Refer to Appendix C. □

C. Construction of BlockMaze

In the following description, we utilize gray background variables to denote private data (or hidden variables), which is combined into a witness. Each witness $\vec{a}$ is taken as an auxiliary input in $\Pi_{\mathcal{Z}}.\mathrm{GenProof}$ to generate proofs. Note that hidden variables are accessible only to related account owners. For the sake of convenience, we summarize the construction of BlockMaze in Appendix D.

Setup. This algorithm generates a list of public parameters. To satisfy the requirements of zero-knowledge transactions, we build specific circuits C_i to prove the validity of these transactions (e.g., `Mint`, `Redeem`, `Send`, `Deposit`). These circuits are taken to generate a key pair $(\mathsf{pk}_{\mathcal{Z}_i}, \mathsf{vk}_{\mathcal{Z}_i})$ for proof generation and verification. Note that this algorithm is executed only once to output a list of public parameters. The detailed process proceeds as follows:

Setup
- inputs: a security parameter λ
- outputs: public parameters pp

1) Compute $\mathsf{pp}_{\mathcal{E}} := \Pi_{\mathcal{E}}.\mathrm{Setup}(1^{\lambda})$.
2) Compute $\mathsf{pp}_{\mathcal{Z}} := \Pi_{\mathcal{Z}}.\mathrm{Setup}(1^{\lambda})$.
3) For each $i \in \{\texttt{Mint}, \texttt{Redeem}, \texttt{Send}, \texttt{Deposit}\}$
   a) Construct a circuit C_i.
   b) Compute $(\mathsf{pk}_{\mathcal{Z}_i}, \mathsf{vk}_{\mathcal{Z}_i}) := \Pi_{\mathcal{Z}}.\mathrm{KeyGen}(C_i)$.
4) Set $\mathsf{PK}_{\mathcal{Z}} := \bigcup \mathsf{pk}_{\mathcal{Z}_i}$ and $\mathsf{VK}_{\mathcal{Z}} := \bigcup \mathsf{vk}_{\mathcal{Z}_i}$.
5) Output $\mathsf{pp} := (\mathsf{pp}_{\mathcal{E}}, \mathsf{pp}_{\mathcal{Z}}, \mathsf{PK}_{\mathcal{Z}}, \mathsf{VK}_{\mathcal{Z}})$.

CreateAccount. Based on $\Pi_{\mathcal{E}}.\mathrm{KeyGen}$ and a collision-resistant hash function `CRH`, this algorithm creates an account address and key pair for each user as follows:

CreateAccount
- inputs: public parameters pp
- outputs: $(addr, (sk, pk))$

1) Compute $(sk, pk) := \Pi_{\mathcal{E}}.\mathrm{KeyGen}(\mathsf{pp}_{\mathcal{E}})$.
2) Compute an account address $addr := \texttt{CRH}(pk)$.
3) Output $addr$ and an account key pair (sk, pk).

Mint. This algorithm builds a `Mint` transaction to convert a plaintext amount into the current zero-knowledge balance of an account (say A). The transaction $\texttt{tx}_{\texttt{Mint}}$ of account A is composed of these variables:

- An account address denoted as $addr_A$: the sender of $\texttt{tx}_{\texttt{Mint}}$.
- A plaintext value denoted as v: a plaintext amount to be converted into a zero-knowledge amount.
- A serial number of the balance commitment denoted as sn_A: a unique string associated with the current balance commitment cmt_A.
- The new balance commitment denoted as cmt_A^*: the new balance commitment to be updated.
- A zero-knowledge proof denoted as prf_m: a proof generated in $\Pi_{\mathcal{Z}}.\mathrm{GenProof}$ proving that the following equations hold for the circuit of $\texttt{tx}_{\texttt{Mint}}$, as shown in Fig. 3(a):
  - $cmt_A = \texttt{COMM}_{\texttt{bc}}(addr_A, value_A, sn_A, r_A)$;
  - $sn_A = \texttt{PRF}(sk_A, r_A)$;
  - $cmt_A^* = \texttt{COMM}_{\texttt{bc}}(addr_A, value_A + v, sn_A^*, r_A^*)$;
  - $sn_A^* = \texttt{PRF}(sk_A, r_A^*)$.

The detailed process proceeds as follows:

Mint
This algorithm merges a plaintext amount with the current zero-knowledge balance of an account (say A).
- inputs:
  - public parameters pp
  - the current zero-knowledge balance $\texttt{zk\_balance}_A$
  - the current plaintext balance $\texttt{pt\_balance}_A$
  - account private key sk_A
  - a plaintext amount v to be converted into a zero-knowledge amount
- outputs:
  - the new zero-knowledge balance $\texttt{zk\_balance}_A^*$
  - a `Mint` transaction $\texttt{tx}_{\texttt{Mint}}$

1) Return fail if $\texttt{pt\_balance}_A < v$.
2) Parse $\texttt{zk\_balance}_A$ as $(cmt_A, addr_A, value_A, sn_A, r_A)$.
3) Generate a new random number r_A^*.
4) Sample a new serial number $sn_A^* := \texttt{PRF}(sk_A, r_A^*)$.
5) Compute $cmt_A^* := \texttt{COMM}_{\texttt{bc}}(addr_A, value_A + v, sn_A^*, r_A^*)$.
6) Set $\vec{x}_1 := (cmt_A, addr_A, sn_A, cmt_A^*, v)$.
7) Set $\vec{a}_1 := (value_A, r_A, sk_A, sn_A^*, r_A^*)$.
8) Compute $prf_m := \Pi_{\mathcal{Z}}.\mathrm{GenProof}(\mathsf{PK}_{\mathcal{Z}}, \vec{x}_1, \vec{a}_1)$.
9) Set $\texttt{tx}_{\texttt{Mint}} := (addr_A, v, sn_A, cmt_A^*, prf_m)$, $\texttt{zk\_balance}_A^* := (cmt_A^*, addr_A, value_A + v, sn_A^*, r_A^*)$.
10) Output $\texttt{zk\_balance}_A^*$ and $\texttt{tx}_{\texttt{Mint}}$.

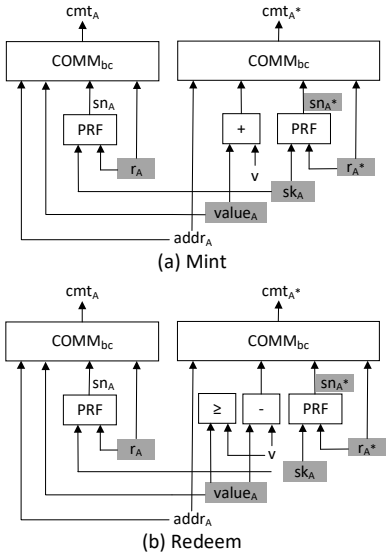


Fig. 3: Mint and Redeem circuits

Redeem. This algorithm generates a `Redeem` transaction to convert a zero-knowledge amount back into the plaintext balance of an account (say A). Some variables are the same as the transaction $\texttt{tx}_{\texttt{Mint}}$ except the proof prf_r, which proves

that the following equations hold for the circuit of $\mathtt{tx}_{\mathtt{Redeem}}$, as shown in Fig. 3(b):

- $cmt_A = \mathtt{COMM}_{\mathtt{bc}}(addr_A, value_A, sn_A, r_A)$;
- $sn_A = \mathtt{PRF}(sk_A, r_A)$;
- $value_A \geq v$;
- $cmt^*_A = \mathtt{COMM}_{\mathtt{bc}}(addr_A, value_A - v, sn^*_A, r^*_A)$;
- $sn^*_A = \mathtt{PRF}(sk_A, r^*_A)$.

The detailed process proceeds as follows:

Redeem
This algorithm converts a zero-knowledge amount back into the plaintext balance of an account (say A).

- inputs:
  - public parameters $\mathtt{pp}$
  - the current zero-knowledge balance $\mathtt{zk\_balance}_A$
  - account private key sk_A
  - a plaintext amount v to be converted back from the zero-knowledge balance
- outputs:
  - the new zero-knowledge balance $\mathtt{zk\_balance}^*_A$
  - a $\mathtt{Redeem}$ transaction $\mathtt{tx}_{\mathtt{Redeem}}$

1) Parse $\mathtt{zk\_balance}_A$ as $(cmt_A, addr_A, value_A, sn_A, r_A)$.
2) Return fail if $value_A < v$.
3) Generate a new random number r^*_A.
4) Sample a new serial number $sn^*_A := \mathtt{PRF}(sk_A, r^*_A)$.
5) Compute $cmt^*_A := \mathtt{COMM}_{\mathtt{bc}}(addr_A, value_A - v, sn^*_A, r^*_A)$.
6) Set $\vec{x}_2 := (cmt_A, addr_A, sn_A, cmt^*_A, v)$.
7) Set $\vec{a}_2 := (value_A, r_A, sk_A, sn^*_A, r^*_A)$.
8) Compute $prf_r := \Pi_{\mathcal{Z}}.\mathrm{GenProof}(\mathsf{PK}_{\mathcal{Z}}, \vec{x}_2, \vec{a}_2)$.
9) Set $\mathtt{tx}_{\mathtt{Redeem}} := (addr_A, v, sn_A, cmt^*_A, prf_r)$, $\mathtt{zk\_balance}^*_A := (cmt^*_A, addr_A, value_A - v, sn^*_A, r^*_A)$.
10) Output $\mathtt{zk\_balance}^*_A$ and $\mathtt{tx}_{\mathtt{Redeem}}$.

Send. This algorithm sends a zero-knowledge amount from a sender (say A) to a recipient (say B). The transaction $\mathtt{tx}_{\mathtt{Send}}$ of account A consists of these variables as follows:

- An account address denoted as $addr_A$: the owner of $\mathtt{tx}_{\mathtt{Send}}$.
- A serial number of the balance commitment denoted as sn_A: a unique string associated with the current balance commitment cmt_A.
- The new balance commitment denoted as cmt^*_A: the new balance commitment to be updated.
- A fund transfer commitment denoted as cmt_v: a commitment of the zero-knowledge amount corresponding to v.
- A ciphertext denoted as aux_A: a ciphertext of sharing parameters using the public key of the recipient.
- An authorization for a ciphertext denoted as $auth_{enc}$: an authorization guarantees the integrity of the ciphertext.
- A zero-knowledge proof denoted as prf_s: a proof generated in $\Pi_{\mathcal{Z}}.\mathrm{GenProof}$ proving that the following equations hold for the circuit of $\mathtt{tx}_{\mathtt{Send}}$, as shown in Fig. 4(a):
  - $cmt_A = \mathtt{COMM}_{\mathtt{bc}}(addr_A, value_A, sn_A, r_A)$;
  - $sn_A = \mathtt{PRF}(sk_A, r_A)$;
  - $value_A \geq v$;
  - $cmt_v = \mathtt{COMM}_{\mathtt{tc}}(addr_A, v, pk_B, sn_v, r_v, sn_A)$;
  - $sn_v = \mathtt{PRF}\ sk_A, r_v)$;
  - $cmt^*_A = \mathtt{COMM}_{\mathtt{bc}}(addr_A, value_A - v, sn^*_A, r^*_A)$;
  - $sn^*_A = \mathtt{PRF}(sk_A, r^*_A)$;
  - $auth_{enc} = \mathtt{PRF}(sk_A, h_{enc})$.

The detailed process proceeds as follows:

Send
This algorithm sends a zero-knowledge amount from sender A to recipient B.

- inputs:
  - public parameters $\mathtt{pp}$
  - the current zero-knowledge balance $\mathtt{zk\_balance}_A$
  - account private key sk_A
  - recipient's public key pk_B
  - a plaintext amount v to be transferred
- outputs:
  - the new zero-knowledge balance $\mathtt{zk\_balance}^*_A$
  - a $\mathtt{Send}$ transaction $\mathtt{tx}_{\mathtt{Send}}$

1) Parse $\mathtt{zk\_balance}_A$ as $(cmt_A, addr_A, value_A, sn_A, r_A)$.
2) Generate a new random number r_v.
3) Sample a new serial number $sn_v := \mathtt{PRF}(sk_A, r_v)$.
4) Compute $cmt_v := \mathtt{COMM}_{\mathtt{tc}}(addr_A, v, pk_B, sn_v, r_v, sn_A)$.
5) Set $aux_A := \Pi_{\mathcal{E}}.\mathrm{Enc}_{pk_B}(\{v, sn_v, r_v, sn_A\})$.
6) Generate a new random number r^*_A.
7) Sample a new serial number $sn^*_A := \mathtt{PRF}(sk_A, r^*_A)$.
8) Compute $cmt^*_A := \mathtt{COMM}_{\mathtt{bc}}(addr_A, value_A - v, sn^*_A, r^*_A)$.
9) Compute $h_{enc} := \mathtt{CRH}(aux_A)$.
10) Compute $auth_{enc} := \mathtt{PRF}(sk_A, h_{enc})$.
11) Set $\vec{x}_3 := (cmt_A, addr_A, sn_A, cmt_v, cmt^*_A, h_{enc}, auth_{enc})$.
12) Set $\vec{a}_3 := (value_A, r_A, sk_A, v, pk_B, sn_v, r_v, sn^*_A, r^*_A)$.
13) Compute $prf_s := \Pi_{\mathcal{Z}}.\mathrm{GenProof}(\mathsf{PK}_{\mathcal{Z}}, \vec{x}_3, \vec{a}_3)$.
14) Set $\mathtt{zk\_balance}^*_A := (cmt^*_A, addr_A, value_A - v, sn^*_A, r^*_A)$ $\mathtt{tx}_{\mathtt{Send}} := (addr_A, sn_A, cmt^*_A, cmt_v, aux_A, auth_{enc}, prf_s)$.
15) Output $\mathtt{zk\_balance}^*_A$ and $\mathtt{tx}_{\mathtt{Send}}$.

The new plaintext value in cmt^*_A is determined by the plaintext value in both cmt_A and cmt_v. The cmt_v in the above transaction is associated with cmt_A via sn_A, i.e., the same variable sn_A is used in both cmt_v and cmt_A. Note that pk_B is visible only between two negotiating parties during **Send**.

Deposit. This algorithm enables a recipient (say B) to check and deposit a received payment into his account. The transaction $\mathtt{tx}_{\mathtt{Deposit}}$ of B is composed of these variables:

- A sequence set denoted as seq: a set of block numbers to get transfer commitments and construct a Merkle tree $\mathtt{MT}$.
- A Merkle root denoted as rt_{cmt}: the root of $\mathtt{MT}$.
- A serial number of the balance commitment denoted as sn_B: a unique string associated with the current balance commitment cmt_B.
- The new balance commitment denoted as cmt^*_B: the new balance commitment to be updated.
- A serial number of the fund transfer commitment denoted as sn_v: a unique string associated with the transfer commitment cmt_v.
- A public key denoted as pk_B: the public key of B.
- A zero-knowledge proof denoted as prf_d: a proof generated in $\Pi_{\mathcal{Z}}.\mathrm{GenProof}$ proving that the following conditions hold for the circuit of $\mathtt{tx}_{\mathtt{Deposit}}$, as shown in Fig. 4(b):
  - $cmt_v = \mathtt{COMM}_{\mathtt{tc}}(addr_A, v, pk_B, sn_v, r_v, sn_A)$;
  - a $path$ from cmt_v to rt_{cmt} recorded on a Merkle tree;
  - $cmt_B = \mathtt{COMM}_{\mathtt{bc}}(addr_B, value_B, sn_B, r_B)$;
  - $sn_B = \mathtt{PRF}(sk_B, r_B)$;
  - $cmt^*_B = \mathtt{COMM}_{\mathtt{bc}}(addr_B, value_B + v, sn^*_B, r^*_B)$;
  - $sn^*_B = \mathtt{PRF}(sk_B, r^*_B)$.

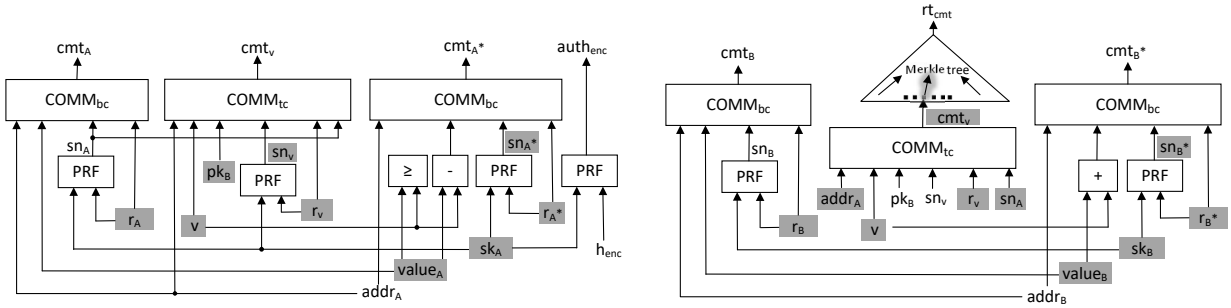


Fig. 4: Send and Deposit circuits

The detailed process proceeds as follows:

Deposit
This algorithm enables a recipient (say B) to check and deposit a received payment into his account.

- inputs:
  - the current ledger $\texttt{Ledger}_\texttt{T}$
  - public parameters pp
  - account key pair (sk_B, pk_B)
  - the hash of a Send transaction $h_{\texttt{tx}_\texttt{Send}}$
  - the current zero-knowledge balance $\texttt{zk\_balance}_B$
- outputs:
  - the new zero-knowledge balance $\texttt{zk\_balance}_B^*$
  - a Deposit transaction $\texttt{tx}_\texttt{Deposit}$

1) Parse $\texttt{zk\_balance}_B$ as $(cmt_B, addr_B, value_B, sn_B, r_B)$.
2) Obtain transaction information of $h_{\texttt{tx}_\texttt{Send}}$ from $\texttt{Ledger}_\texttt{T}$
   - the Send transaction is $\texttt{tx}_\texttt{Send}$,
   - the block number of $\texttt{tx}_\texttt{Send}$ is N.
3) Parse a Send transaction $\texttt{tx}_\texttt{Send}$ as $(addr_A, sn_A, cmt_A^*, cmt_v, aux_A, auth_{enc}, prf_s)$.
4) Compute $(v, sn_v, r_v, sn_A) := \Pi_\mathcal{E}.\text{Dec}_{sk_B}(aux_A)$.
5) Return fail if sn_v appears in $\texttt{SNSet}_\texttt{T}$.
6) Return fail if $cmt_v \neq \texttt{COMM}_\texttt{tc}(addr_A, v, pk_B, sn_v, r_v, sn_A)$
7) Randomly select a set $seq := \{n_1, n_2, ..., N, ..., n_9\}$ from existed block numbers.
8) Construct a Merkle tree MT over $\bigcup_{n\in seq}\texttt{TCMSet}_\texttt{n}$.
9) Compute $path := \texttt{Path}(cmt_v)$ and rt_{cmt} over MT.
10) Generate a new random number r_B^*.
11) Sample a new serial number $sn_B^* := \texttt{PRF}(sk_B, r_B^*)$.
12) Compute $cmt_B^* := \texttt{COMM}_\texttt{bc}(addr_B, value_B + v, sn_B^*, r_B^*)$.
13) Set $\vec{x}_4 := (pk_B, sn_v, rt_{cmt}, cmt_B, addr_B, sn_B, cmt_B^*)$,
14) Set $\vec{a}_4 := (\#, value_B, r_B, sk_B, sn_B^*, r_B^*, path)$ and # as $(cmt_v, addr_A, v, r_v, sn_A)$.
15) Compute $prf_d := \Pi_\mathcal{Z}.\text{GenProof}(\mathsf{PK}_\mathcal{Z}, \vec{x}_4, \vec{a}_4)$.
16) Set $\texttt{tx}_\texttt{Deposit} := (seq, rt_{cmt}, sn_B, cmt_B^*, sn_v, pk_B, prf_d)$, $\texttt{zk\_balance}_B^* := (cmt_B^*, addr_B, value_B + v, sn_B^*, r_B^*)$.
17) Output $\texttt{zk\_balance}_B^*$ and $\texttt{tx}_\texttt{Deposit}$.

Only after generating a valid proof prf_d with pk_B can B deposit the received payment into his account address $addr_B := \texttt{CRH}(pk_B)$. Moreover, cmt_v associated with pk_B is organized and hidden as a Merkle tree for proof generation, thus outsiders do not know which cmt_v is taken to generate prf_d in a Deposit transaction.

VerTx. This algorithm checks all zero-knowledge transactions. Once these transactions are packed into a candidate block, each transaction must be rechecked to confirm that its related account information (e.g., serial number and new transfer commitment) has not become published, and its Merkle root is valid. If all the checks above are satisfied, then miners will: (i) replace the balance commitment of an account with the new balance commitment (i.e., changing from cmt_A to cmt_A^*); (ii) append a published serial number (e.g., sn_A, sn_v and sn_B) into $\texttt{SNSet}_\texttt{T}$; and (iii) add a transfer commitment (e.g., cmt_v) into $\texttt{TCMSet}_\texttt{N}$ stored in the new block $\texttt{block}_\texttt{N}$. The detailed process proceeds as follows:

VerTx
The algorithm checks all zero-knowledge transactions.

- inputs:
  - the current ledger $\texttt{Ledger}_\texttt{T}$
  - public parameters pp
  - a zero-knowledge transaction tx
- outputs: bit b

1) If given a transaction tx is $\texttt{tx}_\texttt{Mint}$
   a) Parse $\texttt{tx}_\texttt{Mint}$ as $(addr_A, v, sn_A, cmt_A^*, prf_m)$.
   b) Obtain related information of $addr_A$ from $\texttt{Ledger}_\texttt{T}$
      - the current plaintext balance is $\texttt{pt\_balance}_A$,
      - the current balance commitment is cmt_A.
   c) Return 0 if $\texttt{pt\_balance}_A < v$.
   d) Return 0 if sn_A appears in $\texttt{SNSet}_\texttt{T}$.
   e) Set $\vec{x}_1 := (cmt_A, addr_A, sn_A, cmt_A^*, v)$.
   f) Output $b := \Pi_\mathcal{Z}.\text{VerProof}(\mathsf{VK}_\mathcal{Z}, \vec{x}_1, prf_m)$.
2) If given a transaction tx is $\texttt{tx}_\texttt{Redeem}$
   a) Parse $\texttt{tx}_\texttt{Redeem}$ as $(addr_A, v, sn_A, cmt_A^*, prf_r)$.
   b) Obtain related information of $addr_A$ from $\texttt{Ledger}_\texttt{T}$
      - the current balance commitment is cmt_A.
   c) Return 0 if sn_A appears in $\texttt{SNSet}_\texttt{T}$.
   d) Set $\vec{x}_2 := (cmt_A, addr_A, sn_A, cmt_A^*, v)$.
   e) Output $b := \Pi_\mathcal{Z}.\text{VerProof}(\mathsf{VK}_\mathcal{Z}, \vec{x}_2, prf_r)$.
3) If given a transaction tx is $\texttt{tx}_\texttt{Send}$
   a) Parse $\texttt{tx}_\texttt{Send}$ as $(addr_A, sn_A, cmt_A^*, cmt_v, aux_A, auth_{enc}, prf_s)$.
   b) Obtain related information of $addr_A$ from $\texttt{Ledger}_\texttt{T}$
      - the current balance commitment is cmt_A.
   c) Return 0 if sn_A appears in $\texttt{SNSet}_\texttt{T}$.
   d) Compute $h_{enc} := \texttt{CRH}(aux_A)$.
   e) Set $\vec{x}_3 := (cmt_A, addr_A, sn_A, cmt_v, cmt_A^*, h_{enc}, auth_{enc})$.
   f) Output $b := \Pi_\mathcal{Z}.\text{VerProof}(\mathsf{VK}_\mathcal{Z}, \vec{x}_3, prf_s)$.
4) If given a transaction tx is $\texttt{tx}_\texttt{Deposit}$
   a) Parse $\texttt{tx}_\texttt{Deposit}$ as $(seq, rt_{cmt}, sn_B, cmt_B^*, sn_v, pk_B, prf_d)$.
   b) Compute $addr_B := \texttt{CRH}(pk_B)$.
   c) Obtain related information of $addr_B$ from $\texttt{Ledger}_\texttt{T}$
      - the current balance commitment is cmt_B.
   d) Return 0 if sn_B or sn_v appears in $\texttt{SNSet}_\texttt{T}$.
   e) Return 0 if rt_{cmt} is not the root over $\bigcup_{n\in seq}\texttt{TCMSet}_\texttt{n}$.

f) Set $\vec{x}_4 := (pk_B, sn_v, rt_{cmt}, cmt_B, addr_B, sn_B, cmt_B^*)$.
g) Output $b := \Pi_{\mathcal{Z}}.\text{VerProof}(\mathsf{VK}_{\mathcal{Z}}, \vec{x}_4, prf_d)$.

Note that confirming a `Deposit` transaction, miners first compute $addr_B := \mathtt{CRH}(pk_B)$ and then update the new balance commitment cmt_B^* for $addr_B$, in other words, only the owner of pk_B can deposit the received payment.

V. Discussion and Analysis

In this section, we first discuss details of BlockMaze, then analyze its privacy protection, and finally consider attacks on BlockMaze.

A. Discussion

Serial number. In BlockMaze, we utilize a unique serial number to prevent the same balance from being spent more than once, which is to be discussed in the next subsection. A unique serial number sn_A associated with the balance commitment cmt_A is generated by hashing related data such as the account private key, balance, current time, random number, etc. Once cmt_A is spent and changed, sn_A is published and deemed as used. It should be replaced by a new serial number sn_A^* to unlock the account. Each account stores its sn_A^* privately until it is used. Then miners append sn_A into `SNSet` after confirming corresponding transactions. In the two-step fund transfer procedure, sn_v associated with the fund transfer commitment cmt_v is processed in the same manner.

Off-Chain communication. Like most blockchain systems, the public key of recipient pk_B and the hash of `Send` transaction $h_{\mathtt{tx}_{\mathtt{Send}}}$ are transmitted off-chain between both parties. Additionally, the network communication used to broadcast zero-knowledge transactions should be anonymous, to avoid leaking identity information. For instance, these issues can be addressed with I2P and Tor [28].

Sharing parameters. In BlockMaze, both parties share parameters to construct a same fund transfer commitment. When a sender A wants to share parameters with a recipient B, A can utilize B's public key pk_B to encrypt parameters (e.g., v, sn_v, r_v, and sn_A mentioned in **Send**), and put the ciphertext into $\mathtt{tx}_{\mathtt{Send}}$. Then B can retrieve and parse $\mathtt{tx}_{\mathtt{Send}}$ with its hash $h_{\mathtt{tx}_{\mathtt{Send}}}$ obtained from A, and decrypt to get back the relevant parameters with his private key sk_B. Of course, B can also utilize sk_B to scan the ledger decrypting the ciphertext in $\mathtt{tx}_{\mathtt{Send}}$ paid to him. Finally, both parties use the same parameters to compute the fund transfer commitment.

Merkle tree. Since a Merkle proof in zk-SNARK needs a Merkle tree with a fixed depth, it is particularly important to set an appropriate depth of Merkle tree. There are two ways to maintain a Merkle tree. One is organized by all cmt_v which are published in $\mathtt{tx}_{\mathtt{Send}}$ recorded on $\mathtt{LEDGER}_{\mathtt{T}}$. The other method is based on a random block number sequence set including $\mathtt{block}_{\mathtt{N}}$, which includes a transfer commitment cmt_v to be proved. The former must set a large depth (e.g., 32) in advance, which may bring about a long time because of proof generation. Considering efficiency and scalability, the latter applies a flexible method to construct a Merkle tree with a smaller depth (e.g., 8) using cmt_v contained in randomly selected block mentioned in Section III.

Transaction fee. Since we design a dual-balance model for BlockMaze, each account has a plaintext balance and a zero-knowledge balance. When a user generates a zero-knowledge transaction, he can utilize an amount from the plaintext balance to pay for the zero-knowledge transaction fee, which is as a reward for the miners checking the validity of the zero-knowledge transaction during the "mining" process.

B. Privacy protection

In BlockMaze, the privacy of transaction amount and zero-knowledge balance amount is preserved. Meanwhile, the attacker cannot get the linkage of transactions.

Transaction amount. There are two types of transaction amounts: one is a fund transfer commitment in `Send` (and `Deposit`) transactions, and the other is a plaintext amount in `Mint` (and `Redeem`) transactions. Obviously, the transaction amount in `Send` (and `Deposit`) is an unknown value to outsiders due to that the transfer commitment satisfies the hiding property (see Lemma 3). Although an adversary can monitor an amount in `Mint` (and `Redeem`), he cannot utilize it to infer the specific amount associated with the zero-knowledge balance of an account since he does not know other secret data. Moreover, since the users pay with a zero-knowledge commitment in `Send` (and `Deposit`) transactions, an adversary cannot deduce the transaction amount from the transfer commitment.

Zero-knowledge balance amount. Based on a dual-balance model, each account has a plaintext balance and a zero-knowledge balance. The zero-knowledge balance is only stored secretly by its owner, while its balance commitment is publicly recorded on MPT and made accessible to anyone. After publishing its serial number, the zero-knowledge balance is spent and replaced by a new one; then its balance commitment is also replaced by the new one. Since the balance commitment satisfies the hiding property (see Lemma 3, an adversary cannot infer the specific amount from both the balance commitment and published serial numbers.

The linkage of transactions. In contrast to traditional transactions in account-model blockchains, each zero-knowledge transaction (e.g., `Mint`, `Redeem`, `Send` and `Deposit`) has a sender and no recipients. Moreover, we hide the linkage of a transaction sender and recipient with the two-step fund transfer procedure: a sender executes the **Send** algorithm to publish a fund transfer commitment cmt_v, and then a recipient runs **Deposit** algorithm to spend the corresponding cmt_v in a Merkle tree and deposit it into his account. Therefore, an adversary cannot obtain the linkage between a transaction sender and its recipient (see Appendix C.2.

C. Attacks

Here, we neglect the common attacks such as selfish mining attack, Sybil attack, etc. That is because these attacks are for consensus algorithms, which are not suitable for BlockMaze. In section IV-B, we define properties of BlockMaze, including *ledger indistinguishability*, *transaction unlinkability*,

and *balance*, where *transaction unlinkability* resists linkability attack, and *balance* resists both double-spending attack and over-spending attack. These attacks are discussed as follows.

Double-spending attack. Double-spending attack means that the same balance is spent more than once. In traditional account-model based blockchains, a nonce (i.e., a transaction counter in each account) is set in each transaction to resist this attack. In BlockMaze, we utilize a unique serial number (instead of a nonce) to address this attack. More specifically, we associate each zero-knowledge balance with a unique serial number. When each zero-knowledge balance is spent, its serial number must be published in a corresponding transaction and replaced by a new one. Then the miners confirm this transaction and insert the old serial number into the used serial number set (i.e., `SNSet`) to resist double-spending attack.

Over-spending attack. Over-spending attack means that an adversary spends more money than what he owns. In BlockMaze, each zero-knowledge balance must be spent with a valid zero-knowledge proof. If the adversary attempts to spend a specific amount of money that does not belong to him, he has to forge a zero-knowledge proof from the public data. The security of zk-SNARK guarantees that the success probability of such attack is negligible.

Linkability attack. Linkability attack means that an adversary identifies the linkage between a `Send` transaction and its corresponding `Deposit` transaction. To resist this attack, we design a two-step fund transfer procedure, which we have discussed in Section V-B.

VI. Implementation and Performance Evaluation

In this section, we first implement a prototype of BlockMaze, and then introduce how to conduct comprehensive experiments evaluating its performance.

A. Implementation

We implement our BlockMaze scheme based on Libsnark [18] and Ethereum [3], where Libsnark is a library that implements zk-SNARK schemes in C++. The source code of BlockMaze is available at our GitHub repository[1], which is composed of the following two components.

zk-SNARKs for BlockMaze Transactions. For zero-knowledge transactions in BlockMaze (i.e., `Mint`, `Redeem`, `Send` and `Deposit`), we construct four zk-SNARKs based on their respective circuits, as shown in Fig. 3 and Fig. 4. These circuits are composed by combining addition, subtraction, less-than, hash, merkle_tree components according to their specifications. The key pair $(\mathsf{PK}_{\mathcal{Z}}, \mathsf{VK}_{\mathcal{Z}})$ for zk-SNARK proof generation/verification is generated and pre-installed at each node. For our zk-SNARKs, we take ALT_BN128 as the default elliptic curve and instantiate the `CRH` function with SHA-256 hash function, same as the one used in the Merkle tree. Regarding the Merkle tree used in `Deposit`, it is constructed by taking 128 cmt_v's from the set of blocks (e.g., $block_n$ and $n \in seq$) as tree leaves.

The Underlying Account-Model Blockchain. Our implementation is based on Go-Ethereum [19] v1.8.12 and we have made the following revisions to it: (i) add four new types of transactions (i.e., `Mint`, `Redeem`, `Send` and `Deposit`) besides the original Ethereum transactions; (ii) add functions to generate and verify different zero-knowledge transactions using the Libsnark module; (iii) implement two global tables (e.g., `TCMSet` and `SNSet`) for double spending prevention. All zero-knowledge transactions are verified with the **VerTx** algorithm when they are received to full nodes, while Ethereum's original transactions are processed as usual. Moreover, we adopt ECIES, a hybrid encryption scheme providing semantic security against chosen plaintext attack (CPA) and chosen ciphertext attack (CCA), to instantiate public key encryption $\Pi_{\mathcal{E}}$, and instantiate `PRF`, $\mathtt{COMM}_{\mathtt{bc}}$, $\mathtt{COMM}_{\mathtt{tc}}$ and `CRH` with SHA-256 hash function.

[1] https://github.com/Agzs/BlockMaze

TABLE II: Performance of our zk-SNARKs

Operations		**Mint**	**Redeem**	**Send**	**Deposit**
Setup	time+	17.251s	17.180s	29.131s	46.727s
	$\mathsf{pp}_{\mathcal{Z}}$	802B			
	$\mathsf{pk}_{\mathcal{Z}_i}$	42.6MB	42.6MB	76.3MB	164.1MB
	$\mathsf{vk}_{\mathcal{Z}_i}$	596B	596B	633B	633B
	pp	326MB			
$\Pi_{\mathcal{Z}}$.GenProof	time	4.173s	4.152s	7.803s	12.152s
	prf#	288B			
$\Pi_{\mathcal{Z}}$.VerProof	time	21.6ms			

Remark The results are the average values for 15 runs.
+ It denotes the time executing $\Pi_{\mathcal{Z}}$.Setup and $\Pi_{\mathcal{Z}}$.KeyGen for each of zk-SNARKs.
# prf denotes a zk-SNARK proof in a transaction.

B. Experiment Setup and Results

We conduct comprehensive experiments to evaluate the performance of the proposed scheme. First of all, we evaluate the performance of our four zk-SNARKs for zero knowledge transactions on a desktop. Then we compare BlockMaze with Zerocash in terms of computation and storage costs. At last, we establish a 270-node test network for BlockMaze, and evaluate the performance in a practical scenario.

Performance of zk-SNARKs. We compile and execute our zk-SNARKs on a desktop equipped with Intel Core i5-8500 CPU @3.00GHz and 16-GB RAM running 64bit Ubuntu 16.04 LTS. Then we evaluate the performance of the 4 zk-SNARKs in Libsnark module in terms of computation and storage costs. As shown in Table II, the running time of $\Pi_{\mathcal{Z}}$.Setup is from approximately 17 to 47 seconds, which is mainly caused by generating the key pair $(\mathsf{pk}_{\mathcal{Z}_i}, \mathsf{vk}_{\mathcal{Z}_i})$. The setup is executed only once, so it will not impact BlockMaze's performance afterwards. The time cost of $\Pi_{\mathcal{Z}}$.GenProof is between about 4 and 13 seconds while $\Pi_{\mathcal{Z}}$.VerProof takes about 22 milliseconds. The efficient proof verification enables miners to process transactions faster, which is of paramount importance for transaction throughput.

Moreover, it can be seen that the time required for executing $\Pi_{\mathcal{Z}}$.Setup is approximately linear to the size of the proving key $\mathsf{pk}_{\mathcal{Z}_i}$, which accounts for the vast majority size of public parameters pp. All these algorithms generate a zk-SNARK proof whose size is 288 bytes. It shows that $\Pi_{\mathcal{Z}}$.Setup and

TABLE III: Comparison with Zerocash

This work (Account-model)			Zerocash# (UTXO-model)		
Setup	time	110.289s	270.897s	time	**Setup**
	pp	326MB	1.87GB	pp	
Create Account	time	1ms	731ms	time	**Create Address**
	pk	64B	343B	$\mathtt{addr}_{\mathtt{pk}}$	
	sk	32B	319B	$\mathtt{addr}_{\mathtt{sk}}$	
Mint	time	4.195s	1μs	time	**Mint**
	$\mathtt{tx}_{\mathtt{Mint}}$	485B	72B	$\mathtt{tx}_{\mathtt{Mint}}$	
Redeem	time	4.207s	100.692s	time	**Pour**
	$\mathtt{tx}_{\mathtt{Redeem}}$	485B	1004B	$\mathtt{tx}_{\mathtt{Pour}}$	
Send	time	7.893s	100.692s	time	**Pour**
	$\mathtt{tx}_{\mathtt{Send}}$	637B	1004B	$\mathtt{tx}_{\mathtt{Pour}}$	
Deposit	time	12.431s	1.72ms	time	**Receive**
	$\mathtt{tx}_{\mathtt{Deposit}}$	561B			
VerTx	time+	23.5ms	8.6μs	mint	**Verify Transaction**
			24.779ms	pour	

Remark A common transaction except payload in Ethereum is about 198B in size. We exclude the storage cost of seq in `Deposit` transactions (as this cost depends on the number of cmt_v). Each zero-knowledge transaction is extended with extra fields (e.g., $code$, $price$, $gasLimit$ and $hash$).
+ It denotes average time cost for verifying a zero-knowledge transaction.

$\Pi_{\mathcal{Z}}.\mathrm{GenProof}$ of **Deposit** algorithm take more time than the other three algorithms, that is because the generation of Merkle proof takes the vast majority of the time.

Comparison with Zerocash. To compare with Zerocash, we deploy a test network consisting of 5 desktops, and each desktop is equipped with Intel Core i5-8500 CPU @3.00GHz and 8-GB RAM running 64bit Ubuntu 16.04 LTS. In this test blockchain network, each desktop runs two types of nodes: one is the miner for mining blocks, and the other is the public node only sends transactions. Note that each node connects with other nodes to form the test network, and the public node generates all types of transactions. In a duration of 1 hour, we run this test blockchain as follows: each public node generates zero-knowledge transactions and basic Ethereum transactions to transfer funds every 10 seconds, while each miner checks these transactions and mines blocks.

Then, we run and evaluate Zerocash v0.11.2.z0[2], which invokes a libzerocash[3] library, in the same manner. Table III compares the experiment results of two schemes from different functionality aspects. From the functional perspective, our **Redeem** and **Send** algorithms are equivalent to **Pour** algorithm of Zerocash. For the size of public parameters pp, we neglect basic parameters of $\mathsf{pp}_{\mathcal{E}}$ since the size of proving key dominates the size of pp in both schemes. Obviously, we can see that BlockMaze has clearly advantages in both time costs and storage sizes of **Setup** and **CreateAccount** algorithms comparing with Zerocash. Since each zero-knowledge transaction contains extra information to prevent double spending and be proved by zero-knowledge proof, its size is larger than a basic transaction in Ethereum. Combining with Table II, we can obtain that the time of zero-knowledge transactions generation is dominated by the running time of $\Pi_{\mathcal{Z}}.\mathrm{GenProof}$.

Processing Performance of BlockMaze. To evaluate the performance of BlockMaze, we deploy a test network consisting of 90 desktops equipped with Intel Core i7-6700 CPU @3.40GHz and 8-GB RAM running 64bit Ubuntu 17.10. In this test network, we utilize Docker to instantiate three types of nodes on each desktop: one is the miner for mining blocks, and the other two are public nodes for sending transactions. Note that each node connects with other 25 randomly selected nodes to form the test network, and each public node sends all types of transactions. In a 1-hour experiment, we run this 270-node test network as follows: (i) all public nodes generate basic Ethereum transactions and zero-knowledge transactions periodically (every 10 seconds) with variable percentage $\alpha \in \{0\%, 25\%, 50\%, 100\%\}$, which is the proportion of zero-knowledge transactions in all transactions; (ii) each miner checks these transactions and mines blocks. Although the test network is relatively small in size, it can still provide meaningful insights about the performance of BlockMaze.

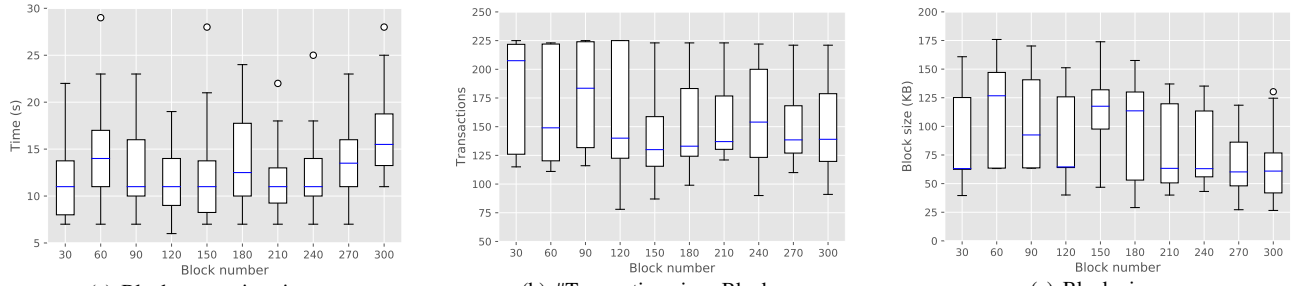
We provide results in Fig. 5, where each box plot represents the data distribution per 30 blocks. Comparing with the data collected by Ethstats[4], we can see: (i) *block generation time* is between 8 to 19 seconds, and its mean value is about 13 seconds (resp., 15.35 seconds in the official Ethereum), which is caused by the proof-of-work mining process; (ii) *#transactions* in a block is between 120 to 225, and its average is about 160 (resp., 225 in the official Ethereum), which is determined by *#transactions* the miner has received before mining a new block; and (iii) *block size* is between 40 to 150 KB, and its mean is about 90KB (resp., 25KB in the official Ethereum), which is determined by *#transactions* in a block (mainly caused by the number of zero-knowledge transactions). Using TPS := $\frac{\textit{\#transactions in a block}}{\textit{block generation time}}$, we can estimate that TPS (Transactions Per Second) of BlockMaze is about $160/13 \approx 13$ in our test network, which is close to $225/15.35 \approx 15$ of the official Ethereum.

Meanwhile, we measure average *transaction latency* and *block verification time* of BlockMaze by setting different α. *Transaction latency* denotes the time of a transaction from its creation to being recorded on the blockchain, and *block verification time* denotes the average time required for each node validating a block. As can be seen from Fig. 6, *transaction latency* and *block verification time* are approximately linear to α. The maximal *transaction latency* is about 46 seconds, which means users must wait for approximately 46 seconds before spending received payments. The maximal *block verification time* is roughly 1350 ms, which is larger than the one in Ethereum. That is because it takes a long time for miners to validate zero-knowledge transactions and update the state of the dual-balance model.

VII. Conclusion

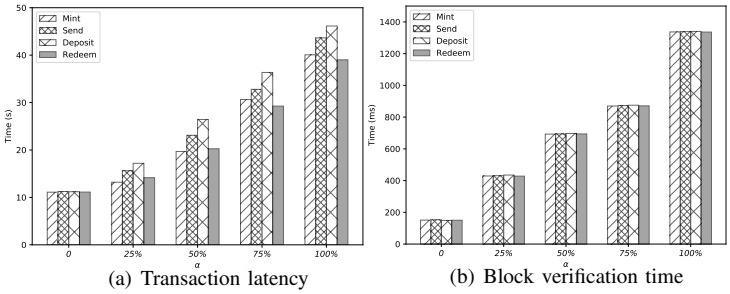
In this paper, we have proposed BlockMaze, a privacy-preserving account-model blockchain that allows users to pay each other with strong privacy guarantees. It shows the practicability to resolve two privacy issues: transaction amounts and the linkage between a transaction sender and its recipient. To solve the above issues, we present a *dual-balance* model

[2]https://github.com/Agzs/zcash/tree/v0.11.2.z0
[3]https://github.com/Agzs/libzerocash
[4]https://ethstats.net/



(a) Block generation time (b) #Transactions in a Block (c) Block size

Fig. 5: The performance of BlockMaze. Each box plot represents the data distribution per 30 blocks, the inner (blue) line shows the median, the dots represent outliers, the ends of the box denote the first and third quartiles, and horizontal lines above and below the box represent the maximum and minimum.



(a) Transaction latency (b) Block verification time

Fig. 6: The average time of different operations

and design a two-step fund transfer procedure combining zk-SNARK. More importantly, we provide a concrete construction of BlockMaze to show the compatibility to account-model blockchains, and give a formal security proof. Finally, we implement BlockMaze based on Go-Ethereum and Libsnark and design comprehensive experiments to evaluate its performance. A future direction is to utilize a new non-interactive zero-knowledge scheme to achieve higher efficiency over account-model blockchains without the trusted setup.

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APPENDIX A
SUMMARY OF PRIVACY-PRESERVING CRYPTOCURRENCIES

Quite a few privacy-preserving cryptocurrencies have been proposed in the literature, and they allow a user to hide transaction amounts and/or obscure the linkage between a transaction and its sender and recipient. We now survey typical privacy-preserving cryptocurrencies and compare them in Table IV.

As shown in Table IV, we can see that only Verge [29] supports hiding IP address using Tor and I2P [28], and others obscure the linkage between a transaction and the public wallet addresses of the involved parties. Zcash [4], Komodo [30] and ZenCash [31] offer privacy guarantees to hide the sender address, recipient address, and transaction amounts using zk-SNARK. Monero [5] achieves the same goal using CryptoNote, which is a protocol based on ring signatures. Grin [32] also solves it using MimbleWimble, which is based on elliptic curve cryptography and derived from confidential transactions, while NavCoin [33] achieves it using ZeroCT based on Zerocoin protocol and confidential transactions.

Moreover, Zcoin [10] utilizes Zerocoin, a protocol based on zero- knowledge proof, to achieve unlinkability and untraceability. As a Dash [11] fork, PIVX [34] disconnects senders and recipients using the zPIV protocol, which is based on Zerocoin protocol with custom Proof of Stake (PoS). Dash [11] and CoinShuffle [35] employ CoinJoin, a method based on a mixer idea, to obscure the linkage between a transaction and its sender and recipient.

APPENDIX B
SECURITY OF BLOCKMAZE SCHEMES

A BlockMaze scheme Π = ($\mathbf{Setup}$, $\mathbf{CreateAccount}$, $\mathbf{Mint}$, $\mathbf{Redeem}$, $\mathbf{Send}$, $\mathbf{Deposit}$, $\mathbf{VerTx}$) is *secure* if it satisfies ledger indistinguishability, transaction unlinkability, and balance. We now formally define them here.

Following a similar model defined in Zerocash [4], we design an experiment as a game between an adversary $\mathcal{A}$ and a challenger $\mathcal{C}$. Each experiment is employed depending on a stateful BlockMaze oracle $\mathcal{O}^{\text{BM}}$, which provides an interface for executing the algorithms defined in a BlockMaze scheme Π. The oracle $\mathcal{O}^{\text{BM}}$ stores and maintains a ledger L, a set of accounts ACCOUNT, a set of plaintext balance PT_BALANCE, and a set of zero-knowledge balance ZK_BALANCE, where the sets are initialized to be empty at the beginning. The oracle $\mathcal{O}^{\text{BM}}$ responses to queries for different algorithms.

- $Query$(***CreateAccount***). Receiving ***CreateAccount*** query, $\mathcal{C}$ computes an account address $addr$ and a key pair (sk, pk) by calling $\mathbf{CreateAccount}$ algorithm, which are added to ACCOUNT. Then, $\mathcal{C}$ outputs $(addr, pk)$.
- $Query$(***Mint***, $addr_A, v$). Receiving ***Mint*** query, $\mathcal{C}$ computes $(\texttt{zk\_balance}^*_A, \texttt{tx}_{\texttt{Mint}})$ for user A by calling $\mathbf{Mint}$ algorithm. Add $\texttt{zk\_balance}^*_A$ to ZK_BALANCE, $\texttt{pt\_balance}^*_A$ to PT_BALANCE, and $\texttt{tx}_{\texttt{Mint}}$ to L, where $\texttt{zk\_balance}^*_A.value = \texttt{zk\_balance}_A.value + v$ and $\texttt{pt\_balance}^*_A = \texttt{pt\_balance}_A - v$.
- $Query$(***Redeem***, $addr_A, v$). Receiving ***Redeem*** query, $\mathcal{C}$ computes $(\texttt{zk\_balance}^*_A, \texttt{tx}_{\texttt{Redeem}})$ for user A by calling $\mathbf{Redeem}$ algorithm. Add $\texttt{zk\_balance}^*_A$ to ZK_BALANCE, $\texttt{pt\_balance}^*_A$ to PT_BALANCE, and $\texttt{tx}_{\texttt{Redeem}}$ to L, where $\texttt{zk\_balance}^*_A.value = \texttt{zk\_balance}_A.value - v$ and $\texttt{pt\_balance}^*_A = \texttt{pt\_balance}_A + v$.
- $Query$(***Send***, $addr_A, addr_B, v$). Receiving ***Send*** query, $\mathcal{C}$ computes $(\texttt{zk\_balance}^*_A, \texttt{tx}_{\texttt{Send}})$ by calling $\mathbf{Send}$ algorithm. Then, add $\texttt{zk\_balance}^*_A$ to ZK_BALANCE, and $\texttt{tx}_{\texttt{Send}}$ to L, where $\texttt{zk\_balance}^*_A.value$ is equal to $(\texttt{zk\_balance}_A.value - v)$.
- $Query$(***Deposit***, $addr_B, h_{\texttt{tx}_{\texttt{Send}}}$). Receiving ***Deposit*** query, $\mathcal{C}$ computes $\texttt{zk\_balance}^*_B$ and generates $\texttt{tx}_{\texttt{Deposit}}$ for user B by executing $\mathbf{Deposit}$ algorithm. Then, add $\texttt{tx}_{\texttt{Deposit}}$ to L, and $\texttt{zk\_balance}^*_B$ to ZK_BALANCE, where $\texttt{zk\_balance}^*_B.value$ is equal to $(\texttt{zk\_balance}_B.value+v)$.
- $Query$(***Insert***, tx). Receiving ***Insert*** query, $\mathcal{C}$ verifies the output of $\mathbf{VerTx}$ algorithm: if the output is 1, add the Mint/Redeem/Send/Deposit transaction tx to L; otherwise, it aborts.

B.1 Ledger Indistinguishability

We utilize the method employed in the extended version of [4] to describe ledger indistinguishability. It can be represented by an experiment L-IND, which involves a probabilistic polynomial-time adversary $\mathcal{A}$ trying to attack a BlockMaze scheme. Before giving a formal experiment, we first define public consistency for a pair of queries.

Definition 4 (Public consistency)**.** A pair of queries (Q, Q') is *publicly consistent* if both queries are of the *same* type and consistent in $\mathcal{A}$'s view. The public information contained in (Q, Q') must be equal including: (i) the value to be transformed; (ii) the account address; (iii) the balance commitment; (iv) the transfer commitment; and (v) published serial number. Moreover, both queries must satisfy the following restrictions for different query types:

For ***CreateAccount*** type, (Q, Q') are always publicly consistent since that the ledgers remain unchanged during the queries. Moreover, we require that both oracles generate the same account to reply to both queries.

For ***Mint*** and ***Redeem*** type, (Q, Q') must be mutually independent and meet the following restrictions:

TABLE IV: Comparison of privacy-preserving blockchain systems based on UTXO-model

Cryptocurrencies	Consensus	Privacy guarantees IP/Sender/Recipient/Amount				Techniques	Pros/Cons
Zcash [4]	PoW (Equihash)	×	√	√	√	zk-SNARK	Provide privacy protection of transaction amount and sender/recipient, but require a trust setup.
Monero [5]	PoW (CryptoNight)	×	√	√	√	Ring Signature	Achieve unlinkability of transactions but with limited privacy protection and large transaction size.
Zcoin [10]	PoW	×	√	√	×	Zero Knowledge Proof	Provide privacy protection of sender/recipient but with large proof size and high verification latency.
Dash [11]	PoW (X11)	×	√	√	×	Mix	Achieve untraceability of transactions, but mixing process is slow.
CoinShuffle [35]	PoW	×	√	√	×	Mix	Allow users to utilize Bitcoin in a truly anonymous manner without any trusted third party.
PIVX [34]	PoS	×	√	√	×	Zero knowledge proof	Be forked from Dash, and provide private instant verified transactions.
Komodo [30]	dPoW	×	√	√	√	zk-SNARK	Be forked from Zcash, and focus on security, scalability, interoperability, and adaptability.
NavCoin [33]	PoW/PoS (X13)	×	√	√	√	ZeroCT [36]	Be forked from Bitcoin, and provide affordable and fast digital payments focused on privacy and simplicity.
ZenCash [31]	PoW	×	√	√	√	zk-SNARK	Be forked from Zcash, and focus on usability of the end-to-end encrypted system.
Verge [29]	PoW	√	×	×	×	Tor and I2P	Provide end-user identity obfuscation, but repeatedly suffer from 51% attacks.
Grin [32]	PoW (Cuckoo Cycle)	×	√	√	√	MimbleWimble	Focus on privacy, fungibility, and scalability, but require a secure communication channel.

- the balance commitment cmt_A in Q must correspond to $\texttt{zk\_balance}_A$ that appears in ZK_BALANCE;
- the zero-knowledge balance $\texttt{zk\_balance}_A$ must be valid, i.e., its serial number must never be published before;
- the account address $addr_A$ contained in Q must match with that in $\texttt{zk\_balance}_A$ and $\texttt{zk\_balance}^*_A$;
- $(\texttt{zk\_balance}_A.value + \texttt{pt\_balance}_A)$ is equal to $(\texttt{zk\_balance}^*_A.value + \texttt{pt\_balance}^*_A)$.

For ***Send*** type, (Q, Q') must be mutually independent and meet the following restrictions:

- the balance commitment cmt_A in Q must correspond to $\texttt{zk\_balance}_A$ that appears in ZK_BALANCE;
- the zero-knowledge balance $\texttt{zk\_balance}_A$ must be valid, i.e., its serial number must never be published before;
- the account address $addr_A$ specified in Q must match with that in $\texttt{zk\_balance}_A$, $\texttt{zk\_balance}^*_A$ and cmt_v;
- $\texttt{zk\_balance}_A.value - v = \texttt{zk\_balance}^*_A.value$.

For ***Deposit*** type, (Q, Q') must be mutually independent and meet the following restrictions:

- the balance commitment cmt_B in Q must correspond to $\texttt{zk\_balance}_B$ that appears in ZK_BALANCE;
- the zero-knowledge balance $\texttt{zk\_balance}_B$ must be valid, i.e., its serial number must never be published before;
- the transfer commitment cmt_v must be valid, i.e., it must appear in a valid transaction $\texttt{tx}_{\texttt{Send}}$ on the corresponding oracle's ledger, and its serial number must never be published before;
- the account address $addr_B$ in Q must match with that in $\texttt{zk\_balance}_B$ and $\texttt{zk\_balance}^*_B$;
- $\texttt{zk\_balance}_B.value + v = \texttt{zk\_balance}^*_B.value$.

Formally, let Π = (**Setup**, **CreateAccount**, **Mint**, **Redeem**, **Send**, **Deposit**, **VerTx**) be a BlockMaze scheme. Let $\mathcal{A}$ be an adversary, which is formally just a (stateful) algorithm. Let λ be a security parameter. We define the ledger indistinguishability experiment $\mathsf{BlockMaze}^{\texttt{L-IND}}_{\Pi,\mathcal{A}}(\lambda)$ as follows:

1) The public parameters $\mathsf{pp} := \mathbf{Setup}(1^\lambda)$ is computed and given to $\mathcal{A}$. Two *independent* BlockMaze oracles $\mathcal{O}^{\text{BM}}_0$ and $\mathcal{O}^{\text{BM}}_1$ are initialized. A uniform bit $b \in \{0, 1\}$ is chosen.
2) Whenever $\mathcal{A}$ sends a pair of *publicly consistent* queries (Q, Q'), answer the queries in the following way:
   a) Provide two separate ledgers (L_b, L_{1-b}) to $\mathcal{A}$ in each step. L_b is the current ledger in $\mathcal{O}^{\text{BM}}_b$, and L_{1-b} is the one in $\mathcal{O}^{\text{BM}}_{1-b}$.
   b) Send Q to $\mathcal{O}^{\text{BM}}_b$ and Q' to $\mathcal{O}^{\text{BM}}_{1-b}$ to obtain two oracle answers (a_b, a_{1-b}).
   c) Return (a_b, a_{1-b}) to $\mathcal{A}$.
3) Continue answering *publicly consistent* queries of $\mathcal{A}$ until $\mathcal{A}$ outputs a bit b'.
4) The game outputs 1 if $b' = b$, and 0 otherwise. If $\mathsf{BlockMaze}^{\texttt{L-IND}}_{\Pi,\mathcal{A}}(\lambda) = 1$, we say that $\mathcal{A}$ succeeds.

Definition 5 (L-IND Security)**.** A BlockMaze scheme Π = (**Setup**, **CreateAccount**, **Mint**, **Redeem**, **Send**, **Deposit**, **VerTx**) is L-IND secure, if for all probabilistic polynomial-time adversaries $\mathcal{A}$ there is a negligible function negl such that, for security parameter λ,

$$\Pr\left[\mathsf{BlockMaze}^{\texttt{L-IND}}_{\Pi,\mathcal{A}}(\lambda) = 1\right] \leq \frac{1}{2} + \mathsf{negl}(\lambda).$$

B.2 Transaction Unlinkability

Formally, let Π = (**Setup**, **CreateAccount**, **Mint**, **Redeem**, **Send**, **Deposit**, **VerTx**) be a BlockMaze scheme, $\mathcal{A}$ an adversary, and λ the security parameter. Let $\mathcal{T}$

be the table of zero-knowledge transactions (i.e., $\texttt{tx}_{\texttt{Send}}$ and $\texttt{tx}_{\texttt{Depoist}}$) generated by $\mathcal{O}^{\text{BM}}$ in response to ***Send*** and ***Deposit*** queries. We define the transaction unlinkability experiment $\mathsf{BlockMaze}^{\texttt{TR-UL}}_{\Pi,\mathcal{A}}(\lambda)$ as follows:

1) The public parameters $\mathsf{pp} := \mathbf{Setup}(1^\lambda)$ is computed and given to $\mathcal{A}$. A BlockMaze oracle $\mathcal{O}^{\text{BM}}$ is initialized.
2) Whenever $\mathcal{A}$ queries $\mathcal{O}^{\text{BM}}$, answer this query along with the ledger L at each step.
3) Continue answering queries of $\mathcal{A}$ until $\mathcal{A}$ sends a pair of zero-knowledge transactions $(\texttt{tx}, \texttt{tx}')$ with the requirements: (i) $(\texttt{tx}, \texttt{tx}') \in \mathcal{T}$ are of the *same* type; (ii) $\texttt{tx} \neq \texttt{tx}'$; (iii) the senders of $(\texttt{tx}, \texttt{tx}')$ are not $\mathcal{A}$ if $\texttt{tx} = \texttt{tx}_{\texttt{Send}}$; (iv) the recipients of $(\texttt{tx}, \texttt{tx}')$ are not $\mathcal{A}$ if $\texttt{tx} = \texttt{tx}_{\texttt{Deposit}}$.
4) The experiment outputs 1 (indicating $\mathcal{A}$ wins the game) if one of the following conditions holds: (i) the recipients of payments contained in $(\texttt{tx}, \texttt{tx}')$ are the same if $\texttt{tx} = \texttt{tx}_{\texttt{Send}}$; and (ii) the senders of payments contained in $(\texttt{tx}, \texttt{tx}')$ are the same if $\texttt{tx} = \texttt{tx}_{\texttt{Deposit}}$. Otherwise, it outputs 0.

Definition 6 (TR-UL Security)**.** A BlockMaze scheme Π = (**Setup**, **CreateAccount**, **Mint**, **Redeem**, **Send**, **Deposit**, **VerTx**) is TR-UL secure, if for all probabilistic polynomial-time adversaries $\mathcal{A}$ there is a negligible function negl such that, for security parameter λ,

$$\Pr\left[\mathsf{BlockMaze}^{\texttt{TR-UL}}_{\Pi,\mathcal{A}}(\lambda) = 1\right] \leq \mathsf{negl}(\lambda).$$

B.3 Balance

We employ an experiment BAL, which involves a probabilistic polynomial-time adversary $\mathcal{A}$ trying to attack a given BlockMaze scheme, where a similar definition is given in [4]. Firstly, we define eight variables for the security model of balance.

- $v_{\texttt{zk\_unspent}}$, the spendable amount in $\texttt{zk\_balance}^*$, i.e., $\texttt{zk\_balance}^*.value$. The challenger $\mathcal{C}$ can check that $\texttt{zk\_balance}^*$ is valid by accessing to $\mathcal{A}$'s balance commitment recorded on MPT on L.
- $v_{\texttt{pt\_unspent}}$, the spendable amount of plaintext balance $\texttt{pt\_balance}^*$. The challenger $\mathcal{C}$ can check that $\texttt{pt\_balance}^*$ is valid by accessing to $\mathcal{A}$'s account plaintext balance recorded on MPT on L.
- $v_{\texttt{Mint}}$, the total value of all plaintext amount minted by $\mathcal{A}$. To compute $v_{\texttt{Mint}}$, the challenger $\mathcal{C}$ looks up all Mint transactions placed on L via ***Mint*** queries and sums up the values that were transformed to $\mathcal{A}$.
- $v_{\texttt{Redeem}}$, the total value of all zero-knowledge amount redeemed by $\mathcal{A}$. To compute $v_{\texttt{Redeem}}$, the challenger $\mathcal{C}$ looks up all Redeem transactions placed on L via ***Redeem*** queries and sums up the values that were transformed to $\mathcal{A}$.
- $v_{\texttt{zk:ACCOUNT}\to\mathcal{A}}$, the total value of payments received by $\mathcal{A}$ from account addresses in ACCOUNT. To compute $v_{\texttt{zk:ACCOUNT}\to\mathcal{A}}$, the challenger $\mathcal{C}$ looks up all Deposit transactions placed on L via ***Deposit*** queries and sums up the values in S_{cmt_v} whose recipient is $\mathcal{A}$.
- $v_{\texttt{pt:ACCOUNT}\to\mathcal{A}}$, the total value of payments received by $\mathcal{A}$ from account addresses in ACCOUNT. To compute $v_{\texttt{pt:ACCOUNT}\to\mathcal{A}}$, the challenger $\mathcal{C}$ looks up all plaintext transactions placed on L and sums up the values that were transferred to $\mathcal{A}$.
- $v_{\texttt{zk:}\mathcal{A}\to\texttt{ACCOUNT}}$, the total value of payments sent by $\mathcal{A}$ to account addresses in ACCOUNT. To compute $v_{\texttt{zk:}\mathcal{A}\to\texttt{ACCOUNT}}$, the challenger $\mathcal{C}$ looks up all Send transactions placed on L via ***Send*** queries and sums up the values in S_{cmt_v} whose sender is $\mathcal{A}$.
- $v_{\texttt{pt:}\mathcal{A}\to\texttt{ACCOUNT}}$, the total value of payments sent by $\mathcal{A}$ to account addresses in ACCOUNT. To compute $v_{\texttt{pt:}\mathcal{A}\to\texttt{ACCOUNT}}$, the challenger $\mathcal{C}$ looks up all plaintext transactions placed on L and sums up the values whose sender is $\mathcal{A}$.

For an honest account u, the following equations hold:

$$v_{\texttt{zk\_unspent}} + v_{\texttt{Redeem}} + v_{\texttt{zk:}u\to\texttt{ACCOUNT}} = v_{\texttt{Mint}} + v_{\texttt{zk:ACCOUNT}\to u},$$
$$v_{\texttt{pt\_unspent}} + v_{\texttt{Mint}} + v_{\texttt{pt:}u\to\texttt{ACCOUNT}} = v_{\texttt{Redeem}} + v_{\texttt{pt:ACCOUNT}\to u}.$$

Add these two equations together, and we can obtain that

$$v_{\texttt{zk\_unspent}} + v_{\texttt{pt\_unspent}} + v_{\texttt{zk:}u\to\texttt{ACCOUNT}} + v_{\texttt{pt:}u\to\texttt{ACCOUNT}} = v_{\texttt{zk:ACCOUNT}\to u} + v_{\texttt{pt:ACCOUNT}\to u}.$$

Formally, let Π = (**Setup**, **CreateAccount**, **Mint**, **Redeem**, **Send**, **Deposit**, **VerTx**) be a BlockMaze scheme, $\mathcal{A}$ an adversary, and λ the security parameter. We define the balance experiment $\mathsf{BlockMaze}^{\texttt{BAL}}_{\Pi,\mathcal{A}}(\lambda)$ as follows:

1) The public parameters $\mathsf{pp} := \mathbf{Setup}(1^\lambda)$ is computed and given to $\mathcal{A}$. A BlockMaze oracle $\mathcal{O}^{\text{BM}}$ is initialized.
2) Whenever $\mathcal{A}$ queries $\mathcal{O}^{\text{BM}}$, answer this query along with the ledger L in each step.
3) Continue answering queries of $\mathcal{A}$ until $\mathcal{A}$ sends a table of transfer commitments S_{cmt_v}, the new account balance $\texttt{zk\_balance}^*$ and $\texttt{pt\_balance}^*$.
4) Compute the eight variables mentioned above.
5) The experiment outputs 1 if $(v_{\texttt{zk\_unspent}} + v_{\texttt{pt\_unspent}} + v_{\texttt{zk:}\mathcal{A}\to\texttt{ACCOUNT}} + v_{\texttt{pt:}\mathcal{A}\to\texttt{ACCOUNT}})$ is greater than $(v_{\texttt{zk:ACCOUNT}\to\mathcal{A}} + v_{\texttt{pt:ACCOUNT}\to\mathcal{A}})$. Otherwise, it outputs 0.

Definition 7 (BAL Security)**.** A BlockMaze scheme Π = (**Setup**, **CreateAccount**, **Mint**, **Redeem**, **Send**, **Deposit**, **VerTx**) is BAL secure, if for all probabilistic polynomial-time adversaries $\mathcal{A}$ there is a negligible function negl such that, for security parameter λ,

$$\Pr\left[\mathsf{BlockMaze}^{\texttt{BAL}}_{\Pi,\mathcal{A}}(\lambda) = 1\right] \leq \mathsf{negl}(\lambda).$$

APPENDIX C
PROOF OF SECURITY

A BlockMaze scheme Π = (**Setup**, **CreateAccount**, **Mint**, **Redeem**, **Send**, **Deposit**, **VerTx**) is *secure* if it satisfies ledger indistinguishability, transaction unlinkability, and balance.

C.1 Proof of Ledger Indistinguishability

We now give a formal proof to prove Theorem 1, which is proved using game-based frameworks. The notations used in this proof are listed below. The adversary $\mathcal{A}$ interacts with a challenger $\mathcal{C}$ as in the L-IND experiment. After receiving a

pair of *publicly consistent* queries (Q, Q') from $\mathcal{A}$, $\mathcal{C}$ answers (Q, Q') as in the simulation $\ni_{\mathsf{sim}}$. Thus, $\mathcal{A}$'s advantage in $\ni_{\mathsf{sim}}$ (represented by $\mathsf{Adv}^{\ni_{\mathsf{sim}}}$) is 0. We now prove that $\mathsf{Adv}^{\texttt{L-IND}}_{\Pi,\mathcal{A}}(\lambda)$ (i.e., $\mathcal{A}$'s advantage in the L-IND experiment) is at most negligibly different than $\mathsf{Adv}^{\ni_{\mathsf{sim}}}$.

TABLE V: Notations

$\ni_{\mathsf{real}}$	The original L-IND experiment
$\ni_i$	A hybrid game with a modification of the $\ni_{\mathsf{real}}$
q_{CA}	The total number of ***CreateAccount*** queries issued by $\mathcal{A}$
q_{M}	The total number of ***Mint*** queries issued by $\mathcal{A}$
q_{R}	The total number of ***Redeem*** queries issued by $\mathcal{A}$
q_{S}	The total number of ***Send*** queries issued by $\mathcal{A}$
q_{D}	The total number of ***Deposit*** queries issued by $\mathcal{A}$
$\mathsf{Adv}^{\ni}$[5]	$\mathcal{A}$'s advantage in game $\ni$
$\mathsf{Adv}^{\Pi\mathcal{E}}$	$\mathcal{A}$'s advantage in $\Pi_{\mathcal{E}}$'s IND-CCA and IK-CCA experiments
$\mathsf{Adv}^{\texttt{PRF}}$	$\mathcal{A}$'s advantage in distinguishing PRF from a random one
$\mathsf{Adv}^{\texttt{COMM}}$	$\mathcal{A}$'s advantage against the hiding property of COMM

Firstly, we describe a simulation $\ni_{\mathsf{sim}}$ in which the adversary $\mathcal{A}$ interacts with a challenger $\mathcal{C}$ as in the queries defined in Appendix B, with the following modification: for each $i \in \{\texttt{Mint}, \texttt{Redeem}, \texttt{Send}, \texttt{Deposit}\}$, the zk-SNARK keys are generated as $(\mathsf{pk}_{\mathcal{Z}_i}, \mathsf{vk}_{\mathcal{Z}_i}, \mathtt{trap}_i) := \mathcal{S}(C_i)$, to obtain the zero-knowledge trapdoor $\mathtt{trap}_i$. After checking each query from $\mathcal{A}$, the challenger $\mathcal{C}$ answers the queries as below.

- To answer ***CreateAccount*** queries, $\mathcal{C}$ does the same as in $Query(\textbf{\textit{CreateAccount}})$ with the following differences: $\mathcal{C}$ computes $(sk, pk) := \Pi_{\mathcal{E}}.\mathrm{KeyGen}(\mathsf{pp})$; then, $\mathcal{C}$ utilizes a random string to replace pk, and computes an account address $addr := \mathtt{CRH}(pk)$; finally, $\mathcal{C}$ stores (sk, pk) in a table and returns $(addr, pk)$ to $\mathcal{A}$.
- To answer ***Mint*** queries, $\mathcal{C}$ makes the following modifications in $Query(\textbf{\textit{Mint}}, addr_A, v)$: $\mathcal{C}$ samples a uniformly random sn_A; if $addr_A$ is an account address created by a previous query to ***CreateAccount***, then $\mathcal{C}$ samples a balance commitment cmt^*_A on a random input, otherwise, $\mathcal{C}$ computes cmt^*_A as in the **Mint** algorithm; moreover, $\mathcal{C}$ computes and obtains all remaining values as in the **Mint** algorithm; finally, $\mathcal{C}$ constructs a statement $\vec{x}_1$ and computes the Mint proof $prf_m := \mathcal{S}(\mathsf{pk}_{\mathcal{Z}_1}, \vec{x_1}, \mathtt{trap}_{\mathtt{Mint}})$.
- To answer ***Redeem*** queries, $\mathcal{C}$ makes the modifications in $Query(\textbf{\textit{Redeem}}, addr_A, v)$ as answering ***Mint*** queries, except for the following modification: $\mathcal{C}$ computes the Redeem proof $prf_r := \mathcal{S}(\mathsf{pk}_{\mathcal{Z}_2}, \vec{x_2}, \mathtt{trap}_{\mathtt{Redeem}})$, where $\vec{x}_2$ is a statement.
- To answer ***Send*** queries, $\mathcal{C}$ makes the following modifications in $Query(\textbf{\textit{Send}}, addr_A, addr_B, v)$: $\mathcal{C}$ first samples a uniformly random sn_A; if $addr_B$ is an account address created by a previous query to ***CreateAccount***, then $\mathcal{C}$ does as follows: (i) sample a transfer commitment cmt_v and a balance commitment cmt^*_A on random inputs, (ii) compute $(sk'_B, pk'_B) := \Pi_{\mathcal{E}}.\mathrm{KeyGen}(\mathsf{pp}_{\mathcal{E}})$; (iii) encrypt $aux_A := \Pi_{\mathcal{E}}.\mathrm{Enc}_{pk'_B}(r)$, where r is a random string of appropriate length in the plaintext space of the encryption scheme; otherwise, $\mathcal{C}$ computes (cmt_v, cmt^*_A, aux_A) as in the **Send** algorithm; moreover, $\mathcal{C}$ computes and obtains all remaining values as in the **Send** algorithm; finally, $\mathcal{C}$ constructs a statement $\vec{x}_3$ and computes the Send proof $prf_s := \mathcal{S}(\mathsf{pk}_{\mathcal{Z}_3}, \vec{x_3}, \mathtt{trap}_{\mathtt{Send}})$.
- To answer ***Deposit*** queries, $\mathcal{C}$ makes the following modifications in $Query(\textbf{\textit{Deposit}}, addr_B, h_{\mathtt{tx}_{\mathtt{Send}}})$: $\mathcal{C}$ first samples a uniformly random sn_B; if $addr_B$ is an account address created by a previous query to ***CreateAccount***, then $\mathcal{C}$ samples a balance commitment cmt^*_B on a random input, otherwise, $\mathcal{C}$ computes cmt^*_B as in the **Deposit** algorithm; moreover, $\mathcal{C}$ computes and obtains all remaining values as in the **Deposit** algorithm; finally, $\mathcal{C}$ computes the Deposit proof $prf_d := \mathcal{S}(\mathsf{pk}_{\mathcal{Z}_4}, \vec{x_4}, \mathtt{trap}_{\mathtt{Deposit}})$, where $\vec{x}_4$ is a statement.
- To answer ***Insert*** queries, $\mathcal{C}$ does the same as in $Query(\textbf{\textit{Insert}}, \mathsf{tx})$.

Note that the answer to $\mathcal{A}$ is computed independently of the bit b for each of the above cases. Thus, when $\mathcal{A}$ outputs a guess b', it must be the case that $\Pr[b = b'] = 1/2$, i.e., $\mathcal{A}$'s advantage in $\ni_{\mathsf{sim}}$ is 0.

Game $\ni_1$. This is the same as $\ni_{\mathsf{real}}$, except for one modification: $\mathcal{C}$ simulates each zk-SNARK proof. For each $i \in \{\texttt{Mint}, \texttt{Redeem}, \texttt{Send}, \texttt{Deposit}\}$, the zk-SNARK keys are generated as $(\mathsf{pk}_{\mathcal{Z}_i}, \mathsf{vk}_{\mathcal{Z}_i}, \mathtt{trap}_i) := \mathcal{S}(C_i)$ instead of $\Pi_{\mathcal{Z}}.\mathrm{KeyGen}$, to obtain the zero-knowledge trapdoor $\mathtt{trap}_i$. Then $\mathcal{C}$ computes $prf_i := \mathcal{S}(\mathsf{pk}_{\mathcal{Z}_i}, \vec{x_i}, \mathtt{trap}_i)$, without using any witnesses $\vec{a_i}$, instead of using $\Pi_{\mathcal{Z}}.\mathrm{GenProof}$ in the **Mint**, **Redeem**, **Send**, **Deposit** algorithms. Since the zk-SNARK is perfect zero-knowledge, the distribution of the simulated prf_i is identical to that of the proofs computed in $\ni_{\mathsf{real}}$. Moreover, we also modify $\ni_{\mathsf{real}}$ such that: each time $\mathcal{A}$ issues a ***CreateAccount*** query, the value pk associated with the returned $addr$ is substituted with a random string of the same length. Since the $(sk, pk) := \Pi_{\mathcal{E}}.\mathrm{KeyGen}(\mathsf{pp})$, the distribution of the simulated (sk, pk) is identical to that of the key pairs computed in $\ni_{\mathsf{real}}$. Hence $\mathsf{Adv}^{\ni_1} = 0$.

Game $\ni_2$. $\ni_2$ is the same as $\ni_1$ with one modification: $\mathcal{C}$ utilizes a random string of the suitable length to replace the ciphertext in a Send transaction. If $\mathcal{A}$ sends a ***Send*** query where the address $addr_B$ is an account address created by a previous query to ***CreateAccount***, then $\mathcal{C}$ invokes $\Pi_{\mathcal{E}}.\mathrm{KeyGen}(\mathsf{pp}_{\mathcal{E}})$ to compute (sk'_B, pk'_B) and obtains $aux_A := \Pi_{\mathcal{E}}.\mathrm{Enc}_{pk'_B}(r)$ for a random string r of suitable length; otherwise, $\mathcal{C}$ computes aux_A as in the **Send** algorithm. By Lemma 1 (see below), $|\mathsf{Adv}^{\ni_2} - \mathsf{Adv}^{\ni_1}| \leq 2 \cdot q_{\mathsf{S}} \cdot \mathsf{Adv}^{\Pi\mathcal{E}}$.

Game $\ni_3$. $\ni_3$ is the same as $\ni_2$ with one modification: $\mathcal{C}$ utilizes random strings of the suitable length to replace all serial numbers generated by PRF. As the subsequent results of the ***Mint***, ***Redeem***, ***Send***, ***Deposit*** queries, these serial numbers (e.g., sn_A and sn_v) are respectively placed in $\mathtt{tx}_{\mathtt{Mint}}$, $\mathtt{tx}_{\mathtt{Redeem}}$, $\mathtt{tx}_{\mathtt{Send}}$, $\mathtt{tx}_{\mathtt{Deposit}}$. By Lemma 2 (see below), $|\mathsf{Adv}^{\ni_3} - \mathsf{Adv}^{\ni_2}| \leq (q_{\mathsf{M}} + q_{\mathsf{R}} + q_{\mathsf{S}} + q_{\mathsf{D}}) \cdot \mathsf{Adv}^{\texttt{PRF}}$.

Game $\ni_{\mathsf{sim}}$. $\ni_{\mathsf{sim}}$ defined above is the same as $\ni_3$ with one modification: $\mathcal{C}$ replaces all balance commitments generated by COMM with commitments to random inputs. As the subsequent results of the ***Mint***, ***Redeem***, ***Send***, ***Deposit*** queries, these balance commitments (e.g., cmt_A, cmt^*_A and cmt_v) are respectively placed in $\mathtt{tx}_{\mathtt{Mint}}$, $\mathtt{tx}_{\mathtt{Redeem}}$, $\mathtt{tx}_{\mathtt{Send}}$, $\mathtt{tx}_{\mathtt{Deposit}}$. By Lemma 3 (see below), $|\mathsf{Adv}^{\ni_{\mathsf{sim}}} - \mathsf{Adv}^{\ni_3}| \leq (q_{\mathsf{M}} + q_{\mathsf{R}} + 2 \cdot q_{\mathsf{S}} + q_{\mathsf{D}}) \cdot \mathsf{Adv}^{\texttt{COMM}}$.

[5] We abuse $\mathsf{Adv}^{\ni}$ to denote the absolute value of the difference between (i) the L-IND advantage of $\mathcal{A}$ in $\ni$ and (ii) the L-IND advantage of $\mathcal{A}$ in $\ni_{\mathsf{real}}$.

As mentioned above, we can obtain $\mathcal{A}$'s advantage in the L-IND experiment (i.e., $\eth_{\mathsf{real}}$) by summing over $\mathcal{A}$'s advantages in all games as follows:

$$\mathsf{Adv}^{\text{L-IND}}_{\Pi,\mathcal{A}}(\lambda) \leq 2{\cdot}q_{\mathsf{S}}{\cdot}\mathsf{Adv}^{\Pi\mathcal{E}} + (q_{\mathsf{M}}{+}q_{\mathsf{R}}{+}2{\cdot}q_{\mathsf{S}}{+}q_{\mathsf{D}}){\cdot}(\mathsf{Adv}^{\mathtt{PRF}}{+}\mathsf{Adv}^{\mathtt{COMM}}).$$

Since $\mathsf{Adv}^{\text{L-IND}}_{\Pi,\mathcal{A}}(\lambda) := 2 \cdot \Pr\left[\mathsf{BlockMaze}^{\text{L-IND}}_{\Pi,\mathcal{A}}(\lambda) = 1\right] - 1$ and $\mathcal{A}$'s advantage in the L-IND experiment is negligible in λ, we can conclude that the proof of ledger indistinguishability.

Lemma 1. Let $\mathsf{Adv}^{\Pi\mathcal{E}}$ be $\mathcal{A}$'s advantage in $\Pi\mathcal{E}$'s IND-CCA and IK-CCA experiments. If $\mathcal{A}$ issues q_{S} ***Send*** queries, then $|\mathsf{Adv}^{\eth_2} - \mathsf{Adv}^{\eth_1}| \leq 2 \cdot q_{\mathsf{S}} \cdot \mathsf{Adv}^{\Pi\mathcal{E}}$.

Proof. We utilize a hybrid $\eth_{\mathsf{H}}$ as an intermediation between $\eth_1$ and $\eth_2$ to prove that $\mathsf{Adv}^{\eth_2}$ is at most negligibly different than $\mathsf{Adv}^{\eth_1}$.

More precisely, $\eth_{\mathsf{H}}$ is the same as $\eth_1$ with one modification: $\mathcal{C}$ utilizes a new public key generated by the $\Pi_{\mathcal{E}}.\mathrm{KeyGen}(\mathsf{pp}_{\mathcal{E}})$, instead of the public key created by a previous query to ***CreateAccount***, to encrypt the same plaintext. After q_{CA} ***CreateAccount*** queries, $\mathcal{A}$ queries the IK-CCA challenger to obtain $pk_{\mathcal{E}} := pk_{\mathcal{E},0}$ where $pk_{\mathcal{E},0}$ is the public key in $(pk_{\mathcal{E},0}, pk_{\mathcal{E},1})$ provided by the IK-CCA challenger. At each ***Send*** query, the IK-CCA challenger encrypts the corresponding plaintext pt as $\mathsf{ct}^* := \Pi_{\mathcal{E}}.\mathrm{Enc}_{pk_{\mathcal{E},\bar{b}}}(\mathsf{pt})$, where $\bar{b}$ is the bit chosen by the IK-CCA challenger, in response to $\mathcal{A}$. Then $\mathcal{C}$ sets $\mathsf{ct} := \mathsf{ct}^*$ and adds $\mathtt{tx}_{\mathtt{Send}}$ (whose aux_A is set by ct) to the ledger L. Finally, $\mathcal{A}$ outputs a guess bit b', which is regarded as the guess in the IK-CCA experiment. Thus, if $\bar{b} = 0$ then $\mathcal{A}$'s view represents $\eth_1$, while $\mathcal{A}$'s view represents $\eth_{\mathsf{H}}$ if $\bar{b} = 1$. Note that $\mathcal{A}$ issues q_{S} ***Send*** queries, then he obtains the q_{S} ciphertexts at most. If the maximum adversarial advantage against the IK-CCA experiment is $\mathsf{Adv}^{\Pi\mathcal{E}}$, then we can conclude that $|\mathsf{Adv}^{\eth_{\mathsf{H}}} - \mathsf{Adv}^{\eth_1}| \leq q_{\mathsf{S}} \cdot \mathsf{Adv}^{\Pi\mathcal{E}}$.

In a similar vein, $\eth_2$ is the same as $\eth_{\mathsf{H}}$ with one modification: $\mathcal{C}$ utilizes a random string of the appropriate length in the plaintext space to replace the plaintext computed in the ***Send*** query. For simplicity, we omit the formal description for IND-CCA experiment, which has a similar pattern above. If the maximum adversarial advantage against the IND-CCA experiment is $\mathsf{Adv}^{\Pi\mathcal{E}}$, then we can conclude that $|\mathsf{Adv}^{\eth_2} - \mathsf{Adv}^{\eth_{\mathsf{H}}}| \leq q_{\mathsf{S}} \cdot \mathsf{Adv}^{\Pi\mathcal{E}}$. Thus, we can sum the above $\mathcal{A}$'s advantages to obtain $|\mathsf{Adv}^{\eth_2} - \mathsf{Adv}^{\eth_1}| \leq 2 \cdot q_{\mathsf{S}} \cdot \mathsf{Adv}^{\Pi\mathcal{E}}$. □

Lemma 2. Let $\mathsf{Adv}^{\mathtt{PRF}}$ be $\mathcal{A}$'s advantage in distinguishing PRF from a random function. If $\mathcal{A}$ issues q_{M} ***Mint*** queries, q_{R} ***Redeem*** queries, q_{S} ***Send*** queries and q_{D} ***Deposit*** queries, then $|\mathsf{Adv}^{\eth_3} - \mathsf{Adv}^{\eth_2}| \leq (q_{\mathsf{M}} + q_{\mathsf{R}} + 2 \cdot q_{\mathsf{S}} + q_{\mathsf{D}}) \cdot \mathsf{Adv}^{\mathtt{PRF}}$.

Proof. We utilize a hybrid $\eth_{\mathsf{H}}$ as an intermediation between $\eth_2$ and $\eth_3$ to prove that $\mathsf{Adv}^{\eth_3}$ is at most negligibly different than $\mathsf{Adv}^{\eth_2}$.

More precisely, $\eth_{\mathsf{H}}$ is the same as $\eth_2$ with one modification: $\mathcal{C}$ utilizes a random string of the appropriate length to replace the public key pk associated with the returned $addr$ for $\mathcal{A}$'s first ***CreateAccount*** query; then, on each subsequent ***Mint***, ***Redeem***, ***Send***, ***Deposit*** queries, $\mathcal{C}$ replaces sn_A respectively with a random string of appropriate length and simulates the respective zk-SNARK proof (e.g., $prf_m, prf_r, prf_s, prf_d$) with the help of a trapdoor by the simulator $\mathcal{S}$ for $\mathtt{tx}_{\mathtt{Mint}}$, $\mathtt{tx}_{\mathtt{Redeem}}$, $\mathtt{tx}_{\mathtt{Send}}$, $\mathtt{tx}_{\mathtt{Deposit}}$.

Let sk be the random, secret key created by the first ***CreateAccount*** query and employed in PRF to compute $sn^*_A := \mathtt{PRF}(sk_A, r^*_A)$ and $sn_v := \mathtt{PRF}(sk_A, r_v)$ in **Mint**, **Redeem**, **Send**, **Deposit** algorithm. Note that PRF takes different random number r to publish old serial number sn for different transactions (generated by the same account). Moreover, let $\mathcal{O}$ be an oracle that implements either PRF or a random function. Then, we utilize $\mathcal{O}$ to generate all random strings (i.e., sn) for the two cases of $\mathcal{O}$ in response to a distinguisher (as an experiment). If $\mathcal{O}$ implements a random function, then it represents $\eth_{\mathsf{H}}$, while the experiment represents $\eth_2$ if $\mathcal{O}$ implements PRF. Thus, $\mathcal{A}$'s advantage in distinguishing PRF from a random one is at most $\mathsf{Adv}^{\mathtt{PRF}}$.

In a similar vein, we extend the above pattern to all $q_{\mathsf{M}} + q_{\mathsf{R}} + 2 \cdot q_{\mathsf{S}} + q_{\mathsf{D}}$ oracle-generated serial numbers (corresponding to what happens in $\eth_3$). We can obtain that $\mathcal{A}$'s advantage in distinguishing PRF from a random one is at most $(q_{\mathsf{M}} + q_{\mathsf{R}} + 2 \cdot q_{\mathsf{S}} + q_{\mathsf{D}}) \cdot \mathsf{Adv}^{\mathtt{PRF}}$. Finally, we deduce that $|\mathsf{Adv}^{\eth_3} - \mathsf{Adv}^{\eth_2}| \leq (q_{\mathsf{M}} + q_{\mathsf{R}} + 2 \cdot q_{\mathsf{S}} + q_{\mathsf{D}}) \cdot \mathsf{Adv}^{\mathtt{PRF}}$. □

Lemma 3. Let $\mathsf{Adv}^{\mathtt{COMM}}$ be $\mathcal{A}$'s advantage against the hiding property of COMM. If $\mathcal{A}$ issues q_{M} ***Mint*** queries, q_{R} ***Redeem*** queries, q_{S} ***Send*** queries and q_{D} ***Deposit*** queries, then $|\mathsf{Adv}^{\eth_{\mathsf{sim}}} - \mathsf{Adv}^{\eth_3}| \leq (q_{\mathsf{M}} + q_{\mathsf{R}} + 2 \cdot q_{\mathsf{S}} + q_{\mathsf{D}}) \cdot \mathsf{Adv}^{\mathtt{COMM}}$.

Proof. This proof can be proved with the similar pattern used in Lemma 2 above. On each ***Mint***, ***Redeem***, ***Send***, ***Deposit*** query, $\mathcal{C}$ replaces the balance commitment $cmt^* := \mathtt{COMM}_{\mathtt{bc}}(addr, value, sn^*_A, r^*)$ and the transfer commitment $cmt_v := \mathtt{COMM}_{\mathtt{tc}}(addr, v, pk, sn_v, r_v, sn_A)$ with random strings of the appropriate length. Thus $\mathcal{A}$'s advantage in distinguishing this modified experiment from $\eth_3$ is at most $\mathsf{Adv}^{\mathtt{COMM}}$. If we extend it to all q_{M} ***Mint*** queries, all q_{R} ***Redeem*** queries, all q_{S} ***Send*** queries and all q_{D} ***Deposit*** queries, and utilize random strings of the suitable length to replace $q_{\mathsf{M}} + q_{\mathsf{R}} + 2 \cdot q_{\mathsf{S}} + q_{\mathsf{D}}$ commitments (i.e., cmt_A and cmt_v), then we obtain $\eth_{\mathsf{sim}}$, and conclude that $|\mathsf{Adv}^{\eth_{\mathsf{sim}}} - \mathsf{Adv}^{\eth_3}| \leq (q_{\mathsf{M}} + q_{\mathsf{R}} + 2 \cdot q_{\mathsf{S}} + q_{\mathsf{D}}) \cdot \mathsf{Adv}^{\mathtt{COMM}}$. □

C.2 Proof of Transaction Unlinkability

Letting $\mathcal{T}$ be the table of zero-knowledge transactions (i.e., $\mathtt{tx}_{\mathtt{Send}}$ and $\mathtt{tx}_{\mathtt{Depoist}}$) generated by $\mathcal{O}^{\mathrm{BM}}$ in response to ***Send*** and ***Deposit*** queries. $\mathcal{A}$ wins the TR-UL experiment whenever it outputs a pair of zero-knowledge transactions $(\mathtt{tx}, \mathtt{tx}')$ if one of the following conditions holds: (i) the recipients of payments contained in $(\mathtt{tx}, \mathtt{tx}')$ are the same if $\mathtt{tx} = \mathtt{tx}_{\mathtt{Send}}$; and (ii) the senders of payments contained in $(\mathtt{tx}, \mathtt{tx}')$ are the same if $\mathtt{tx} = \mathtt{tx}_{\mathtt{Deposit}}$.

Suppose $\mathcal{A}$ outputs a pair of Send transactions $(\mathtt{tx}_{\mathtt{Send}}, \mathtt{tx}'_{\mathtt{Send}})$, where $\mathtt{tx}_{\mathtt{Send}}$ satisfies the following equations:

$$\mathtt{tx}_{\mathtt{Send}} := (addr_A, sn_A, cmt^*_A, cmt_v, aux_A, auth_{enc}, prf_s)$$
$$cmt_v = \mathtt{COMM}_{\mathtt{tc}}(addr_A, v, pk_B, sn_v, r_v, sn_A)$$
$$aux_A := \Pi_{\mathcal{E}}.\mathrm{Enc}_{pk_B}(\{v, sn_v, r_v, sn_A\}),$$

and $\mathtt{tx}'_{\mathtt{Send}}$ satisfies the following equations:

$$\mathtt{tx}'_{\mathtt{Send}} := (addr'_A, sn'_A, cmt^{*}_A{}', cmt'_v, aux'_A, auth'_{enc}, prf'_s)$$
$$cmt'_v = \mathtt{COMM}_{\mathtt{tc}}(addr'_A, v', pk'_B, sn'_v, r'_v, sn'_A)$$
$$aux'_A := \Pi_{\mathcal{E}}.\mathrm{Enc}_{pk'_B}(\{v', sn'_v, r'_v, sn'_A\}).$$

$\mathcal{A}$ wins the TR-UL experiment if the recipients of payments contained in $(\mathtt{tx}_{\mathtt{Send}}, \mathtt{tx}'_{\mathtt{Send}})$ are the same, i.e., $pk_B = pk'_B$. There are three ways for $\mathcal{A}$ to distinguish whether $pk_B \stackrel{?}{=} pk'_B$: (i) distinguish the public keys from the ciphertexts; (ii) distinguish the public keys from the transfer commitments; (iii) distinguish the public keys from the zero-knowledge proofs.

For condition (i), $\mathcal{A}$ must distinguish (pk_B, pk'_B) based on the different ciphertexts (aux_A, aux'_A), which indicates that $\mathcal{A}$ should win the IK-CCA experiment in Lemma 1. For condition (ii), $\mathcal{A}$ must distinguish (pk_B, pk'_B) from the different commitments (cmt_v, cmt'_v) without knowing other secret values (i.e., hidden variables), which indicates that $\mathcal{A}$ should break the *hiding* property of COMM in Lemma 3. For condition (iii), $\mathcal{A}$ must distinguish (pk_B, pk'_B) from different zero-knowledge proof (prf_s, prf'_s), which indicates that $\mathcal{A}$ should break the *proof of knowledge* property of the zk-SNARK. However, since the security of Lemma 1, Lemma 3 and zk-SNARK, $\mathcal{A}$ cannot distinguish the two recipients (i.e., public keys) from (aux_A, aux'_A) and (cmt_v, cmt'_v).

Suppose $\mathcal{A}$ outputs a pair of Deposit transactions $(\mathtt{tx}_{\mathtt{Deposit}}, \mathtt{tx}'_{\mathtt{Deposit}})$, where $\mathtt{tx}_{\mathtt{Deposit}}$ satisfies the following equations:

$$\mathtt{tx}_{\mathtt{Deposit}} := (seq, rt_{cmt}, sn_B, cmt^{*}_B, sn_v, pk_B, prf_d)$$
$$cmt_v = \mathtt{COMM}_{\mathtt{tc}}(addr_A, v, pk_B, sn_v, r_v, sn_A),$$

and $\mathtt{tx}'_{\mathtt{Deposit}}$ satisfies the following equations:

$$\mathtt{tx}'_{\mathtt{Deposit}} := (seq', rt'_{cmt}, sn'_B, cmt^{*}_B{}', sn'_v, pk'_B, prf'_d)$$
$$cmt'_v = \mathtt{COMM}_{\mathtt{tc}}(addr'_A, v', pk'_B, sn'_v, r'_v, sn'_A)$$

$\mathcal{A}$ wins the TR-UL experiment if the senders of payments contained in $(\mathtt{tx}_{\mathtt{Deposit}}, \mathtt{tx}'_{\mathtt{Deposit}})$ are the same, i.e., $addr_A = addr'_A$. There are two ways for $\mathcal{A}$ to distinguish whether $addr_A \stackrel{?}{=} addr'_A$: (i) distinguish the address of senders from the zero-knowledge proof; (ii) obtain the transfer commitments used in Deposit transactions from $\mathcal{A}$'s view, and distinguish the address of senders combining with previous Send transactions.

For condition (i), $\mathcal{A}$ must distinguish $(addr_A, addr'_A)$ from different zero-knowledge proof (prf_d, prf'_d), which indicates that $\mathcal{A}$ should break the *proof of knowledge* property of the zk-SNARK. For condition (ii), if $\mathcal{A}$ can analyze (cmt_v, cmt'_v) without knowing other secret values (i.e., hidden variables), then, he can distinguish $(addr_A, addr'_A)$ by seeking senders of the previous transactions $(\mathtt{tx}_{\mathtt{Send}}, \mathtt{tx}'_{\mathtt{Send}})$ containing (cmt_v, cmt'_v) respectively. Note that (cmt_v, cmt'_v) are not visible for anyone during the generation of $(\mathtt{tx}_{\mathtt{Deposit}}, \mathtt{tx}'_{\mathtt{Deposit}})$. There are two ways for $\mathcal{A}$ to obtain different transfer commitments: (a) the Merkle roots; (b) the zero-knowledge proofs. For condition (a), $\mathcal{A}$ must analyze (cmt_v, cmt'_v) from different Merkle roots (rt_{cmt}, rt'_{cmt}), which indicates that $\mathcal{A}$ should break the collision resistance property of CRH. For condition (b), $\mathcal{A}$ must analyze (cmt_v, cmt'_v) from different zero-knowledge proofs (prf_d, prf'_d), which indicates that $\mathcal{A}$ should break the *proof of knowledge* property of the zk-SNARK. However, since the security of zk-SNARK and CRH, $\mathcal{A}$ cannot neither distinguish the two senders from (prf_d, prf'_d) nor analyze (cmt_v, cmt'_v) from (rt_{cmt}, rt'_{cmt}) and (prf_d, prf'_d).

To conclude, due to the security of zk-SNARK, CRH, COMM and encryption schemes, the adversary $\mathcal{A}$ cannot distinguish the unlinkability from $(\mathtt{tx}_{\mathtt{Send}}, \mathtt{tx}'_{\mathtt{Send}})$ nor $(\mathtt{tx}_{\mathtt{Deposit}}, \mathtt{tx}'_{\mathtt{Deposit}})$.

C.3 Proof of Balance

The BAL experiment is changed without affecting $\mathcal{A}$'s view as follows: for each Deposit transaction $\mathtt{tx}_{\mathtt{Deposit}}$ on the ledger L, $\mathcal{C}$ computes the zk-SNARK proof $prf_d := \Pi_{\mathcal{Z}}.\mathrm{GenProof}(\mathsf{PK}_{\mathcal{Z}}, \vec{x}_4, \vec{a}_4)$ where $\vec{x}_4$ is a statement corresponding to $\mathtt{tx}_{\mathtt{Deposit}}$ and $\vec{a}_4$ is a witness for the zk-SNARK instance $\vec{x}_4$, then $\mathcal{C}$ organizes all $(tx, \vec{a})$ as an augmented ledger $(L, \boldsymbol{A})$, which is a list of matched pairs $(\mathtt{tx}_{\mathtt{Deposit}}, \vec{a_4}_j)$ where $\mathtt{tx}_{\mathtt{Deposit}}$ is the j-th Deposit transaction in L and $\vec{a_4}_j \in \boldsymbol{A}$ is the witness of $\mathtt{tx}_{\mathtt{Deposit}}$ for $\vec{x_4}_j$.

Definition 8 (Balanced ledger)**.** An augmented ledger $(L, \boldsymbol{A})$ is balanced if the following conditions hold:

- *Condition I.* Each $(\mathtt{tx}_{\mathtt{Deposit}}, \vec{a_4}) \in (L, \boldsymbol{A})$ publishes openings (e.g., sn_B) of a unique balance commitment cmt_B, which is the output of a previous zero-knowledge transaction before $\mathtt{tx}_{\mathtt{Deposit}}$ on L.
- *Condition II.* Each $(\mathtt{tx}_{\mathtt{Deposit}}, \vec{a_4}) \in (L, \boldsymbol{A})$ publishes openings (e.g., sn_v and pk_B) of a unique transfer commitment cmt_v, which is the output of a previous Send transaction before $\mathtt{tx}_{\mathtt{Deposit}}$ on L.
- *Condition III.* For all $(\mathtt{tx}_{\mathtt{Deposit}}, \vec{a_4}) \in (L, \boldsymbol{A})$ publish distinct and unique openings (e.g., sn_v, sn_B and pk_B) of the commitment (i.e., cmt_v and cmt_B).
- *Condition IV.* Each cmt_B, cmt_v, cmt^{*}_B to values $value_B$, v, $value^{*}_B$ (respectively) contained in $(\mathtt{tx}_{\mathtt{Deposit}}, \vec{a_4}) \in (L, \boldsymbol{A})$ satisfies the condition that $value_B + v = value^{*}_B$.
- *Condition V.* The values (i.e., $addr_A$, v, pk_B, sn_v, r_v, sn_A) used to compute cmt_v are respectively equal to the values for cmt'_v, if $cmt_v = cmt'_v$ where cmt_v is employed in $(\mathtt{tx}_{\mathtt{Deposit}}, \vec{a_4}) \in (L, \boldsymbol{A})$, and cmt'_v is the output of a previous Send transaction before $\mathtt{tx}_{\mathtt{Deposit}}$.
- *Condition VI.* If $(\mathtt{tx}_{\mathtt{Deposit}}, \vec{a_4}) \in (L, \boldsymbol{A})$ was inserted by $\mathcal{A}$, and cmt_v used in $\mathtt{tx}_{\mathtt{Deposit}}$ is the output of a previous Send transaction $\mathtt{tx}'$, then $addr \notin \mathtt{ACCOUNT}$ where $addr := \mathtt{CRH}(pk_B)$ is the recipient's account address of $\mathtt{tx}'$.

One can prove that $(L, \boldsymbol{A})$ is balanced if the following equation holds: $v_{\mathtt{zk\_unspent}} + v_{\mathtt{pt\_unspent}} + v_{\mathtt{zk}:\mathcal{A}\to\mathtt{ACCOUNT}} + v_{\mathtt{pt}:\mathcal{A}\to\mathtt{ACCOUNT}} = v_{\mathtt{zk}:\mathtt{ACCOUNT}\to\mathcal{A}} + v_{\mathtt{pt}:\mathtt{ACCOUNT}\to\mathcal{A}}$.

For each of the above conditions, we utilize a contraction to prove that each case is in a negligible probability. Note that we suppose $\Pr\left[\mathcal{A}(\overline{Con\text{-}i}) = 1\right]$ to denote a non-negligible probability that $\mathcal{A}$ wins but violates *Condition i*.

Violating *Condition I*. Each $(\mathtt{tx}_{\mathtt{Deposit}}, \vec{a_4}) \in (L, \boldsymbol{A})$, where $\mathtt{tx}_{\mathtt{Deposit}}$ was not inserted by $\mathcal{A}$, must satisfy *Condition I* in $\mathcal{O}^{\mathrm{BM}}$; thus, $\Pr\left[\mathcal{A}(\overline{\textit{Con-I}}) = 1\right]$ is a probability that $\mathcal{A}$ inserts $\mathtt{tx}_{\mathtt{Deposit}}$ to construct a pair $(\mathtt{tx}_{\mathtt{Deposit}}, \vec{a_4}) \in (L, \boldsymbol{A})$ where cmt_B used in $\mathtt{tx}_{\mathtt{Deposit}}$ is not the output of any previous zero-knowledge transaction before $\mathtt{tx}_{\mathtt{Deposit}}$. However, each $\mathtt{tx}_{\mathtt{Deposit}}$ utilizes the witness $\vec{a_4}$, which must contain a balance commitment cmt_B for the serial number sn_B, to compute the proof proving the validity of $\mathtt{tx}_{\mathtt{Deposit}}$. Obviously, cmt_B is the output of an earlier zero-knowledge transaction and recorded on MPT in L. Therefore, if cmt_B to sn_B does not previously appear in L, then it means that this violation contradicts the binding property of COMM in Lemma 3.

Violating *Condition II*. Each $(\mathtt{tx}_{\mathtt{Deposit}}, \vec{a_4}) \in (L, \boldsymbol{A})$, where $\mathtt{tx}_{\mathtt{Deposit}}$ was not inserted by $\mathcal{A}$, must satisfy *Condition II* in $\mathcal{O}^{\mathrm{BM}}$; thus, $\Pr\left[\mathcal{A}(\overline{\textit{Con-II}}) = 1\right]$ is a probability that $\mathcal{A}$ inserts $\mathtt{tx}_{\mathtt{Deposit}}$ to construct a pair $(\mathtt{tx}_{\mathtt{Deposit}}, \vec{a_4}) \in (L, \boldsymbol{A})$ where cmt_v used in $\mathtt{tx}_{\mathtt{Deposit}}$ is not the output of any previous Send transaction before $\mathtt{tx}_{\mathtt{Deposit}}$. However, each $\mathtt{tx}_{\mathtt{Deposit}}$ utilizes the witness $\vec{a_4}$, which must contain a authentication path $path$ for a Merkle tree, to compute the proof proving the validity of $\mathtt{tx}_{\mathtt{Deposit}}$. Obviously, the Merkle tree, whose root rt_{cmt} is published, is constructed using the transfer commitment cmt_v of any previous Send transactions on L. More precisely, if cmt_v does not previously appear in L, then it means that $path$ is invalid but with a valid rt_{cmt}. Therefore, this violation contradicts the collision resistance of CRH.

Violating *Condition III*. Each $(\mathtt{tx}_{\mathtt{Deposit}}, \vec{a_4}) \in (L, \boldsymbol{A})$ publishes a unique and distinct sn_v, (sn_B, pk_B) for cmt_v, cmt_B (respectively). Obviously, $\Pr\left[\mathcal{A}(\overline{\textit{Con-III}}) = 1\right]$ is a probability that $\mathcal{A}$ wins in the following two situations: (i) spending the same balance commitment cmt_B in two zero-knowledge transactions; or (ii) receiving the same transfer commitment cmt_v. In (i), it means that two Deposit transactions $\mathtt{tx}_{\mathtt{Deposit}}, \mathtt{tx}'$ recorded on L spend the same balance commitment cmt_B, but publish two different serial numbers sn_B and sn'_B, furthermore, their corresponding witnesses $\vec{a_4}, \vec{a_4}'$ must contain different openings of cmt_B since both transactions are valid Deposit transactions on L. Therefore, this violation contradicts the binding property of COMM. In a similar vein, for (ii), it means that two Deposit transactions $\mathtt{tx}_{\mathtt{Deposit}}, \mathtt{tx}'$ receive the same transfer commitment cmt_v but publish two different serial numbers sn_v and sn'_v. Therefore, this violation also contradicts the binding property of COMM in Lemma 3.

Violating *Condition IV*. Each $(\mathtt{tx}_{\mathtt{Deposit}}, \vec{a_4}) \in (L, \boldsymbol{A})$ contains a proof ensuring that cmt_B, cmt_v, cmt^*_B to values $value_B$, v, $value^*_B$ (respectively) satisfy the equation $value_B + v = value^*_B$. Obviously, $\Pr\left[\mathcal{A}(\overline{\textit{Con-IV}}) = 1\right]$ is a probability that the equation $value_B + v \neq value^*_B$ holds. Therefore, this violation contradicts the *proof of knowledge* property of the zk-SNARK.

Violating *Condition V*. Each $(\mathtt{tx}_{\mathtt{Deposit}}, \vec{a_4}) \in (L, \boldsymbol{A})$ contains values (i.e., $addr_A$, v, pk_B, sn_v, r_v, sn_A) of cmt_v, and cmt_v is a also transfer commitment to values (i.e., $addr_A$, v', pk_B, sn_v, r_v, sn_A) in a previous Send transaction. Obviously, $\Pr\left[\mathcal{A}(\overline{\textit{Con-V}}) = 1\right]$ is a probability that the equation $v \neq v'$ holds. Therefore, this violation also contradicts the binding property of COMM in Lemma 3.

Violating *Condition VI*. Each $(\mathtt{tx}_{\mathtt{Deposit}}, \vec{a_4}) \in (L, \boldsymbol{A})$ publishes the recipient's account address $addr := \mathtt{CRH}(pk_B)$ of a transfer commitment cmt_v. Obviously, $\Pr\left[\mathcal{A}(\overline{\textit{Con-VI}}) = 1\right]$ is a probability that an inserted Deposit transaction $\mathtt{tx}_{\mathtt{Deposit}}$ publishes $addr$ of cmt_v which is the output of a previous Send transaction $\mathtt{tx}'$ whose recipient's account address $addr$ lies in ACCOUNT; moreover, the witness associated to $\mathtt{tx}'$ contains pk such that $addr = \mathtt{CRH}(pk)$. Therefore, this violation contradicts the collision resistance of CRH.

Finally, we utilize the similar structure of the argument to prove balance for the other three transactions (i.e., Mint, Redeem and Send) generated by $\mathcal{O}^{\mathrm{BM}}$ in response to ***Mint***, ***Redeem*** and ***Send*** queries, and obtain that $\Pr\left[\mathsf{BlockMaze}^{\mathtt{BAL}}_{\Pi,\mathcal{A}}(\lambda) = 1\right]$ is negligible in λ.

APPENDIX D
ALGORITHMS OF BLOCKMAZE SCHEMES

For convenience, we summarize algorithms employed in a BlockMaze scheme Π = (**Setup**, **CreateAccount**, **Mint**, **Redeem**, **Send**, **Deposit**, **VerTx**) in Fig. 7.

Setup
The algorithm generates a list of public parameters.
- inputs: a security parameter λ
- outputs: public parameters pp

1) Compute $\mathsf{pp}_{\mathcal{E}} := \Pi_{\mathcal{E}}.\mathrm{Setup}(1^{\lambda})$.
2) Compute $\mathsf{pp}_{\mathcal{Z}} := \Pi_{\mathcal{Z}}.\mathrm{Setup}(1^{\lambda})$.
3) For each $i \in \{\texttt{Mint}, \texttt{Redeem}, \texttt{Send}, \texttt{Deposit}\}$
   a) Construct a circuit C_i.
   b) Compute $(\mathsf{pk}_{\mathcal{Z}_i}, \mathsf{vk}_{\mathcal{Z}_i}) := \Pi_{\mathcal{Z}}.\mathrm{KeyGen}(C_i)$.
4) Set $\mathsf{PK}_{\mathcal{Z}} := \bigcup \mathsf{pk}_{\mathcal{Z}_i}$ and $\mathsf{VK}_{\mathcal{Z}} := \bigcup \mathsf{vk}_{\mathcal{Z}_i}$.
5) Output $\mathsf{pp} := (\mathsf{pp}_{\mathcal{E}}, \mathsf{pp}_{\mathcal{Z}}, \mathsf{PK}_{\mathcal{Z}}, \mathsf{VK}_{\mathcal{Z}})$.

CreateAccount
The algorithm creates an account address and key pair for a user.
- inputs: public parameters pp
- outputs: $(addr, (sk, pk))$

1) Compute an account key pair $(sk, pk) := \Pi_{\mathcal{E}}.\mathrm{KeyGen}(\mathsf{pp}_{\mathcal{E}})$.
2) Compute an account address $addr := \texttt{CRH}(pk)$.
3) Output the $(addr, (sk, pk))$.

Mint
This algorithm merges a plaintext amount with the current zero-knowledge balance of an account (say A).
- inputs:
  - public parameters pp
  - the current zero-knowledge balance $\texttt{zk\_balance}_A$
  - the current plaintext balance $\texttt{pt\_balance}_A$
  - account private key sk_A
  - a plaintext amount v to be converted into a zero-knowledge amount
- outputs:
  - the new zero-knowledge balance $\texttt{zk\_balance}^*_A$
  - a `Mint` transaction $\texttt{tx}_{\texttt{Mint}}$

1) Return fail if $\texttt{pt\_balance}_A < v$.
2) Parse $\texttt{zk\_balance}_A$ as $(cmt_A, addr_A, value_A, sn_A, r_A)$.
3) Generate a new random number r^*_A.
4) Sample a new serial number $sn^*_A := \texttt{PRF}(sk_A, r^*_A)$.
5) Compute $cmt^*_A := \texttt{COMM}_{\texttt{bc}}(addr_A, value_A + v, sn^*_A, r^*_A)$.
6) Set $\vec{x}_1 := (cmt_A, addr_A, sn_A, cmt^*_A, v)$.
7) Set $\vec{a}_1 := (value_A, r_A, sk_A, sn^*_A, r^*_A)$.
8) Compute $prf_m := \Pi_{\mathcal{Z}}.\mathrm{GenProof}(\mathsf{PK}_{\mathcal{Z}}, \vec{x}_1, \vec{a}_1)$.
9) Set $\texttt{tx}_{\texttt{Mint}} := (addr_A, v, sn_A, cmt^*_A, prf_m)$,
   $\texttt{zk\_balance}^*_A := (cmt^*_A, addr_A, value_A + v, sn^*_A, r^*_A)$.
10) Output $\texttt{zk\_balance}^*_A$ and $\texttt{tx}_{\texttt{Mint}}$.

Redeem
This algorithm converts a zero-knowledge amount back into the plaintext balance of an account (say A).
- inputs:
  - public parameters pp
  - the current zero-knowledge balance $\texttt{zk\_balance}_A$
  - account private key sk_A
  - a plaintext amount v to be converted back from zero-knowledge balance
- outputs:
  - the new zero-knowledge balance $\texttt{zk\_balance}^*_A$
  - a `Redeem` transaction $\texttt{tx}_{\texttt{Redeem}}$

1) Parse $\texttt{zk\_balance}_A$ as $(cmt_A, addr_A, value_A, sn_A, r_A)$.
2) Return fail if $value_A < v$.
3) Generate a new random number r^*_A.
4) Sample a new serial number $sn^*_A := \texttt{PRF}(sk_A, r^*_A)$.
5) Compute $cmt^*_A := \texttt{COMM}_{\texttt{bc}}(addr_A, value_A - v, sn^*_A, r^*_A)$.
6) Set $\vec{x}_2 := (cmt_A, addr_A, sn_A, cmt^*_A, v)$.
7) Set $\vec{a}_2 := (value_A, r_A, sk_A, sn^*_A, r^*_A)$.
8) Compute $prf_r := \Pi_{\mathcal{Z}}.\mathrm{GenProof}(\mathsf{PK}_{\mathcal{Z}}, \vec{x}_2, \vec{a}_2)$.
9) Set $\texttt{tx}_{\texttt{Redeem}} := (addr_A, v, sn_A, cmt^*_A, prf_r)$,
   $\texttt{zk\_balance}^*_A := (cmt^*_A, addr_A, value_A - v, sn^*_A, r^*_A)$.
10) Output $\texttt{zk\_balance}^*_A$ and $\texttt{tx}_{\texttt{Redeem}}$.

Send
This algorithm sends a zero-knowledge amount from sender A to recipient B.
- inputs:
  - public parameters pp
  - the current zero-knowledge balance $\texttt{zk\_balance}_A$
  - account private key sk_A
  - recipient's public key pk_B
  - a plaintext amount v to be transferred
- outputs:
  - the new zero-knowledge balance $\texttt{zk\_balance}^*_A$
  - a `Send` transaction $\texttt{tx}_{\texttt{Send}}$

1) Parse $\texttt{zk\_balance}_A$ as $(cmt_A, addr_A, value_A, sn_A, r_A)$.
2) Generate a new random number r_v.
3) Sample a new serial number $sn_v := \texttt{PRF}(sk_A, r_v)$.
4) Compute $cmt_v := \texttt{COMM}_{\texttt{tc}}(addr_A, v, pk_B, sn_v, r_v, sn_A)$.
5) Set $aux_A := \Pi_{\mathcal{E}}.\mathrm{Enc}_{pk_B}(\{v, sn_v, r_v, sn_A\})$.
6) Generate a new random number r^*_A.
7) Sample a new serial number $sn^*_A := \texttt{PRF}(sk_A, r^*_A)$.
8) Compute $cmt^*_A := \texttt{COMM}_{\texttt{bc}}(addr_A, value_A - v, sn^*_A, r^*_A)$.
9) Compute $h_{enc} := \texttt{CRH}(aux_A)$.
10) Compute $auth_{enc} := \texttt{PRF}(sk_A, h_{enc})$.
11) Set $\vec{x}_3 := (cmt_A, addr_A, sn_A, cmt_v, cmt^*_A, h_{enc}, auth_{enc})$.
12) Set $\vec{a}_3 := (value_A, r_A, sk_A, v, pk_B, sn_v, r_v, sn^*_A, r^*_A)$.
13) Compute $prf_s := \Pi_{\mathcal{Z}}.\mathrm{GenProof}(\mathsf{PK}_{\mathcal{Z}}, \vec{x}_3, \vec{a}_3)$.
14) Set $\texttt{tx}_{\texttt{Send}} := (addr_A, sn_A, cmt^*_A, cmt_v, aux_A, auth_{enc}, prf_s)$,
    $\texttt{zk\_balance}^*_A := (cmt^*_A, addr_A, value_A - v, sn^*_A, r^*_A)$.
15) Output $\texttt{zk\_balance}^*_A$ and $\texttt{tx}_{\texttt{Send}}$.

Deposit
This algorithm makes a recipient (say B) check and deposit a received payment into his account.
- inputs:
  - the current ledger $\texttt{Ledger}_{\texttt{T}}$
  - public parameters pp
  - account key pair (sk_B, pk_B)
  - the hash of a send transaction $h_{\texttt{tx}_{\texttt{Send}}}$
  - the current zero-knowledge balance $\texttt{zk\_balance}_B$
- outputs:
  - the new zero-knowledge balance $\texttt{zk\_balance}^*_B$
  - a `Deposit` transaction $\texttt{tx}_{\texttt{Deposit}}$

1) Parse $\texttt{zk\_balance}_B$ as $(cmt_B, addr_B, value_B, sn_B, r_B)$.
2) Obtain transaction information of $h_{\texttt{tx}_{\texttt{Send}}}$ from $\texttt{Ledger}_{\texttt{T}}$
   - the send transaction is $\texttt{tx}_{\texttt{Send}}$,
   - the block number of $\texttt{tx}_{\texttt{Send}}$ is N.
3) Parse $\texttt{tx}_{\texttt{Send}}$ as $(addr_A, sn_A, cmt^*_A, cmt_v, aux_A, auth_{enc}, prf_s)$.
4) Decrypt shared parameters $(v, sn_v, r_v, sn_A) := \Pi_{\mathcal{E}}.\mathrm{Dec}_{sk_B}(aux_A)$.
5) Return fail if sn_v appears in $\texttt{SNSet}_{\texttt{T}}$.
6) Return fail if $cmt_v \neq \texttt{COMM}_{\texttt{tc}}(addr_A, v, pk_B, sn_v, r_v, sn_A)$.
7) Randomly select a set $seq := \{n_1, n_2, ..., N, ..., n_9\}$ from existed block numbers.
8) Construct a Merkle tree $\texttt{MT}$ over $\bigcup_{n \in seq} \texttt{TCMSet}_{\texttt{n}}$.
9) Compute $path := \texttt{Path}(cmt_v)$ and rt_{cmt} over $\texttt{MT}$.
10) Generate a new random number r^*_B.
11) Sample a new serial number $sn^*_B := \texttt{PRF}(sk_B, r^*_B)$.
12) Compute $cmt^*_B := \texttt{COMM}_{\texttt{bc}}(addr_B, value_B + v, sn^*_B, r^*_B)$.
13) Set $\vec{x}_4 := (pk_B, sn_v, rt_{cmt}, cmt_B, addr_B, sn_B, cmt^*_B)$,
    $\vec{a}_4 := (cmt_v, addr_A, v, r_v, sn_A, value_B, r_B, sk_B, sn^*_B, r^*_B, path)$.
14) Compute $prf_d := \Pi_{\mathcal{Z}}.\mathrm{GenProof}(\mathsf{PK}_{\mathcal{Z}}, \vec{x}_4, \vec{a}_4)$.
15) Set $\texttt{tx}_{\texttt{Deposit}} := (seq, rt_{cmt}, sn_B, cmt^*_B, sn_v, pk_B, prf_d)$,
    $\texttt{zk\_balance}^*_B := (cmt^*_B, addr_B, value_B + v, sn^*_B, r^*_B)$.
16) Output $\texttt{zk\_balance}^*_B$ and $\texttt{tx}_{\texttt{Deposit}}$.

VerTx
This algorithm checks the validity of all zero-knowledge transactions.
- inputs:
  - the current ledger $\texttt{Ledger}_{\texttt{T}}$
  - public parameters pp
  - a zero-knowledge transaction $\texttt{tx}$
- outputs: bit b

1) If given a transaction $\texttt{tx}$ is $\texttt{tx}_{\texttt{Mint}}$
   a) Parse $\texttt{tx}_{\texttt{Mint}}$ as $(addr_A, v, sn_A, cmt^*_A, prf_m)$.
   b) Obtain related information of $addr_A$ from $\texttt{Ledger}_{\texttt{T}}$
      - the current plaintext balance is $\texttt{pt\_balance}_A$,
      - the current balance commitment is cmt_A.
   c) Return 0 if $\texttt{pt\_balance}_A < v$.
   d) Return 0 if sn_A appears in $\texttt{SNSet}_{\texttt{T}}$.
   e) Set $\vec{x}_1 := (cmt_A, addr_A, sn_A, cmt^*_A, v)$.
   f) Output $b := \Pi_{\mathcal{Z}}.\mathrm{VerProof}(\mathsf{VK}_{\mathcal{Z}}, \vec{x}_1, prf_m)$.
2) If given a transaction $\texttt{tx}$ is $\texttt{tx}_{\texttt{Redeem}}$
   a) Parse $\texttt{tx}_{\texttt{Redeem}}$ as $(addr_A, v, sn_A, cmt^*_A, prf_r)$.
   b) Obtain related information of $addr_A$ from $\texttt{Ledger}_{\texttt{T}}$
      - the current balance commitment is cmt_A.
   c) Return 0 if sn_A appears in $\texttt{SNSet}_{\texttt{T}}$.
   d) Set $\vec{x}_2 := (cmt_A, addr_A, sn_A, cmt^*_A, v)$.
   e) Output $b := \Pi_{\mathcal{Z}}.\mathrm{VerProof}(\mathsf{VK}_{\mathcal{Z}}, \vec{x}_2, prf_r)$.
3) If given a transaction $\texttt{tx}$ is $\texttt{tx}_{\texttt{Send}}$
   a) Parse $\texttt{tx}_{\texttt{Send}}$ as $(addr_A, sn_A, cmt^*_A, cmt_v, aux_A, auth_{enc}, prf_s)$.
   b) Obtain related information of $addr_A$ from $\texttt{Ledger}_{\texttt{T}}$
      - the current balance commitment is cmt_A.
   c) Return 0 if sn_A appears in $\texttt{SNSet}_{\texttt{T}}$.
   d) Compute $h_{enc} := \texttt{CRH}(aux_A)$.
   e) Set $\vec{x}_3 := (cmt_A, addr_A, sn_A, cmt_v, cmt^*_A, h_{enc}, auth_{enc})$.
   f) Output $b := \Pi_{\mathcal{Z}}.\mathrm{VerProof}(\mathsf{VK}_{\mathcal{Z}}, \vec{x}_3, prf_s)$.
4) If given a transaction $\texttt{tx}$ is $\texttt{tx}_{\texttt{Deposit}}$
   a) Parse $\texttt{tx}_{\texttt{Deposit}}$ as $(seq, rt_{cmt}, sn_B, cmt^*_B, sn_v, pk_B, prf_d)$.
   b) Compute $addr_B := \texttt{CRH}(pk_B)$.
   c) Obtain related information of $addr_B$ from $\texttt{Ledger}_{\texttt{T}}$
      - the current balance commitment is cmt_B.
   d) Return 0 if sn_B or sn_v appears in $\texttt{SNSet}_{\texttt{T}}$.
   e) Return 0 if rt_{cmt} is not the Merkle root over $\bigcup_{n \in seq} \texttt{TCMSet}_{\texttt{n}}$.
   f) Set $\vec{x}_4 := (pk_B, sn_v, rt_{cmt}, cmt_B, addr_B, sn_B, cmt^*_B)$.
   g) Output $b := \Pi_{\mathcal{Z}}.\mathrm{VerProof}(\mathsf{VK}_{\mathcal{Z}}, \vec{x}_4, prf_d)$.

Fig. 7: BlockMaze main algorithms