Tight reduction for generic construction of certificateless signature and tightly-secure scheme without pairing

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Abstract. Certificateless signature was proposed by Al-Riyami and Paterson to eliminate the certificate management in the public-key infrastructures and solve the key escrow problem in the identity-based signature. In 2007, Hu et al. proposed a generic construction of certificateless signature. They construct certificateless signature scheme from any standard identity-based signature and signature scheme. However, their security reduction is loose; the security of the constructed scheme depends on the number of users. In this paper, we show that their construction can achieve tight security if the underlying signature scheme is existentially unforgeable under adaptive chosen-message attacks in the multi-user setting with adaptive corruptions. Moreover, we instantiate a tightly-secure certificateless signature scheme without pairing, whose security is independent of the number of users. Best of our knowledge, this scheme is the first tightly-secure certificateless signature scheme.

Keywords: certificateless signatures, key escrow, tight security

1 Introduction

1.1 Background

Signature scheme, IBS, CLS. Digital signatures ensure the validity of a message based on a public key. However, verifiers cannot confirm the owner of the message from the public key alone because verifiers have no information about the issuer of the public key. To confirm the issuer of a public key, we must certify the relationship between the public key and the signer in an external way.

In public-key infrastructure (PKI) setting, a certificate authority issues the certificate that proves the connection between a public key and its owner. However, it is known that certificate management is laborious work.

In 1984, Shamir [Sha84] introduced the concept of identity-based signature (IBS) to eliminate the certificate management. In IBS setting, the user's identity, such as email address, is used as the public key. The corresponding secret key is generated by a trusted key generation center (KGC) and sent to its owner. Although IBS no longer requires certificates, it suffers from the key escrow problem; KGC knows all user's secret keys.

Certificateless signature (CLS) was proposed by Al-Riyami and Paterson [AP03] to solve both certificate management load in PKI and the key escrow problem in IBS. Unlike IBS, KGC only provides a partial private key, which is a part of the full secret key. The other part comes from the user's own choice and is kept secret. Therefore KGC does not have knowledge of the full signing key and key escrow is no longer a problem.

The first CLS scheme and the security definition of CLS were presented in [AP03]. In 2004, Yum and Lee proposed a generic construction of CLS from an IBS and a standard signature [YL04]. Later Hu et al. [HWZ+07] pointed out the security flaw on Yum and Lee construction and fixed it. Since the proposal of [AP03], a lot of CLS scheme using pairings have been proposed [HSM+05; ZWX+06; ZZ08; HHC13; Shi19]. However, these schemes are less efficient because the computational cost of a pairing operation is higher than that of an addition or a scalar multiplication. To improve performance, He et al. proposed the first pairing-free CLS scheme [HCZ12]. Later Tian and Huang revealed that their scheme is insecure [TH13]. In 2014, Gong and Li [GL14] proposed a new CLS scheme. Yeh et al. [YSC+17] proposed a new CLS scheme, but Jia et al. [JHL+18] pointed out the vulnerability of Yeh et al.'s scheme and developed an improved scheme. At present, the secure paring-free CLS schemes are [GL14] and [JHL+18].

Tight security. To prove the security of a cryptographic scheme, we generally construct a reduction algorithm, which turns an efficient attacker on the scheme into an algorithm solving some assumed-to-be-hard computational problem. If the reduction has about the same success probability as the attacker, we say that the reduction is tight and the scheme is tightly-security. If the cryptographic scheme is tightly-secure, we are easy to decide the parameter size because the security of the scheme is independent of other factors such as the number of users or that of hash function evaluations. Besides, we can use the smallest parameters that achieve the desired security level. As a result, data size (e.g. key length or signature length) and computation cost (e.g. signature generation and verification) are reduced. Therefore, tight reductions have been actively studied for many cryptographic primitives.

1.2 Motivation and contribution

Conventional provably secure CLS schemes come with a reduction which loses factors that depends on the number of users or the number of hash function evaluations. For example, the security reductions of [GL14] and [JHL+18] are very loose because they use rewinding technique. The security of CLS schemes from Hu et al. generic construction [HWZ+07] is dependent on the number of users even if underlying IBS and signature are tightly-secure.

Our main goal is to construct a tightly-secure CLS scheme. First, we show Hu et al. generic construction [HWZ+07] achieves tight security by changing the security of the underlying signature scheme. We prove if the underlying signature scheme is existentially unforgeable under adaptive chosen-message attacks in a

multi-user setting with adaptive corruptions (MU-EUF-CMA^{Corr}) [BHJ+15], the security of CLS schemes from Hu et al. generic construction is reduced to the security of the underlying signature scheme tightly. This generic construction makes the user easy to construct a new tightly-secure CLS scheme from another assumption without complicated proofs. Second, we instantiate the first tightly-secure CLS scheme without pairing from the tightly-secure IBS scheme [FH18] and the tightly-MU-EUF-CMA^{Corr}-secure signature scheme [GJ18]. Best of our knowledge, the instantiated scheme is the first tightly-secure CLS scheme.

1.3 Paper organization

The rest of the paper is organized as follows. Section 2 introduces notations and definitions of signature, IBS, and CLS. We review the existing generic construction of certificateless signature [HWZ+07] in Section 3. Section 4 presents a tight reduction for the generic construction. In Section 5, we instantiate a tightly-secure certificateless signature scheme without pairing and compare it with other pairing-free CLS schemes. The conclusion of this paper is given in Section 6.

2 Preliminaries

2.1 Notation

If x_1, \ldots, x_n are strings, we denote $x_1 \| \cdots \| x_n$ by the concatenation of x_1, \ldots, x_n . If x is a string, then |x| denotes its length. For a PPT algorithm \mathcal{A} ,

$$\mathcal{A}(x_1, x_2, \ldots; O_1, O_2, \ldots)$$

means A has inputs x_1, x_2, \ldots and access to oracles O_1, O_2, \ldots

A function f is said to be negligible on λ , if, for any polynomial ν , there exists a natural number λ_0 such that $f(\lambda) < 1/\nu(\lambda)$ for any $\lambda > \lambda_0$.

2.2 Digital signature

A signature scheme Sig consists of three algorithms:

- Sig.KGen(1 $^{\lambda}$): On input the security parameter 1 $^{\lambda}$, the key generation algorithm outputs a key pair (sk, pk).
- Sig.Sign(sk, M): On input a private key sk and a message M, the signing algorithm outputs a signature σ .
- Sig.Vrfy (pk, M, σ) : On input a public key pk, a message M, and a signature σ , the verification algorithm outputs 0 or 1.

For correctness, we require that for all $\lambda \in \mathbb{N}$ and $M \in \{0,1\}^*$, if $(sk,pk) \leftarrow \mathsf{Sig.KGen}(1^{\lambda})$ and $\sigma \leftarrow \mathsf{Sig.Sign}(sk,M)$, then $\Pr[\mathsf{Sig.Vrfy}(pk,M,\sigma)=1]=1$ holds.

For security, we define standard existential unforgeability under adaptive chosen-message attacks, called EUF-CMA security in [GMR88].

Fig. 1. Experiment used to define EUF-CMA security for signature scheme.

Definition 1. Let Sig is a signature scheme, \mathcal{A} an adversary, and $\lambda \in \mathbb{N}$ a security parameter. Define the experiment $\mathsf{Exp}^{euf\text{-}cma}_{\mathsf{Sig},\mathcal{A}}(\lambda)$ as shown in Fig. 1. The EUF-CMA advantage of \mathcal{A} in attacking Sig is

$$\mathsf{Adv}^{\mathit{euf-cma}}_{\mathit{Sig},\mathcal{A}}(\lambda) = \Pr \left[\mathsf{Exp}^{\mathit{euf-cma}}_{\mathit{Sig},\mathcal{A}}(\lambda) = 1 \right].$$

We say that Sig is an EUF-CMA-secure signature scheme if $Adv_{Sig,A}^{euf-cma}(\lambda)$ is negligible for any PPT adversary A.

Next we define multi-user existential unforgeability under adaptive chosen-message attacks with adaptive corruptions, called MU-EUF-CMA^{Corr} in [BHJ+15].

Definition 2. Let Sig is a signature scheme, \mathcal{A} an adversary, and $\lambda \in \mathbb{N}$ a security parameter. Define the experiment $\mathsf{Exp}^{mu-euf-cma-corr}_{\mathsf{Sig},\mathcal{A}}(\lambda)$ as shown in Fig. 2. The MU-EUF-CMA^{Corr} advantage of \mathcal{A} in attacking Sig is

$$\mathsf{Adv}^{mu\text{-}euf\text{-}cma\text{-}corr}_{\mathsf{Sig},\mathcal{A}}(\lambda) = \Pr\left[\mathsf{Exp}^{mu\text{-}euf\text{-}cma\text{-}corr}_{\mathsf{Sig},\mathcal{A}}(\lambda) = 1\right].$$

We say that Sig is a MU-EUF-CMA^{Corr}-secure signature scheme if $Adv^{mu-euf-cma-corr}_{Sig,\mathcal{A}}(\lambda)$ is negligible for any PPT adversary \mathcal{A} .

Experiment $Exp^{\mathrm{mu-euf-cma-corr}}_{Sig,\mathcal{A}}(\lambda)$	$\underline{\text{Oracle }\mathbf{CORR}(i)}$
for 1 to μ do	$CU \leftarrow CU \cup \{i\}$
$(pk_i, sk_i) \leftarrow Sig.KGen(1^\lambda)$	$\mathbf{return} \ sk_i$
$CU \leftarrow \emptyset; MSG[1] \leftarrow \emptyset,, MSG[\mu] \leftarrow \emptyset$ $(i^*, M^*, \sigma^*) \leftarrow \mathcal{A}(pk_1,, pk_\mu; \mathbf{CORR}, \mathbf{SIGN})$	Oracle $\mathbf{SIGN}(M, i)$
$\mathbf{if}\ i^* \notin CU \wedge M^* \notin MSG[i^*] \wedge Sig.Vrfy(pk_{i^*}, M^*, \sigma^*) = 1$	$\sigma \leftarrow Sig.Sign(sk_i, M)$
return 1	$MSG[i] \leftarrow MSG[i] \cup \{M\}$
else return 0	$\textbf{return} \sigma$

Fig. 2. Experiment used to define MU-EUF-CMA^{Corr} security for signature scheme.

2.3 Identity-based signature

An identity-based signature scheme IBS consists of four algorithms:

- IBS.Setup(1^{λ}): On input 1^{λ} , the setup algorithm outputs a key pair (msk, mpk).
- IBS.Extract(mpk, msk, ID): On input a master public key mpk, a master secret key msk, and an identity ID, the key extraction algorithm outputs a key usk.
- IBS.Sign(mpk, ID, usk, M): On input mpk, ID, usk, and a message M, the signing algorithm outputs a signature σ .
- IBS.Vrfy (mpk, ID, M, σ) : On input mpk, ID, a message M, and a signature σ , the verification algorithm outputs 0 or 1.

For correctness, we require that for all $\lambda \in \mathbb{N}$, $ID \in \{0,1\}^*$, and $M \in \{0,1\}^*$, if $(msk, mpk) \leftarrow \mathsf{IBS.Setup}(1^{\lambda})$, $usk \leftarrow \mathsf{IBS.Extract}(mpk, msk, ID)$, and $\sigma \leftarrow \mathsf{IBS.Sign}(mpk, ID, usk, M)$, then $\Pr[\mathsf{IBS.Vrfy}(mpk, ID, M, \sigma) = 1] = 1$ holds.

The existential unforgeability under adaptive chosen message attacks (ID-EUF-CMA) of IBS is defined as follows [BNN09]:

Definition 3. Let IBS is an identity-based signature scheme, \mathcal{A} an adversary, and $\lambda \in \mathbb{N}$ a security parameter. Define the experiment $\mathsf{Exp}^{id\text{-}euf\text{-}cma}_{\mathsf{IBS},\mathcal{A}}(\lambda)$ as shown in Fig. 3. The ID-EUF-CMA advantage of \mathcal{A} in attacking IBS is

$$\mathsf{Adv}^{id\text{-}euf\text{-}cma}_{\mathit{IBS},\mathcal{A}}(\lambda) = \Pr\left[\mathsf{Exp}^{id\text{-}euf\text{-}cma}_{\mathit{IBS},\mathcal{A}}(\lambda) = 1\right].$$

We say that IBS is an ID-EUF-CMA-secure IBS scheme if $Adv_{IBS,A}^{id-euf-cma}(\lambda)$ is negligible for any PPT adversary A.

Oracle INIT(ID)	$\frac{\text{Oracle } \mathbf{EXT}(ID)}{}$	Oracle $\mathbf{SIGN}(ID, M)$
if $\mathit{ID} \in \mathit{CU} \cup \mathit{HU}$ then return \bot	$HU \leftarrow HU \setminus \{\mathit{ID}\}$	if $ID \notin HU$ then return \perp
$usk[\mathit{ID}] \leftarrow IBS.Extract(\mathit{mpk}, \mathit{msk}, \mathit{ID})$	$CU \leftarrow CU \cup \{\mathit{ID}\}$	$\sigma \leftarrow IBS.Sign(\mathit{mpk},\mathit{ID},\mathit{usk}[\mathit{ID}],M)$
$\mathit{MSG[ID]} \leftarrow \emptyset; HU \leftarrow HU \cup \{\mathit{ID}\}$	$\mathbf{return}\ usk[ID]$	$\mathit{MSG[ID]} \leftarrow \mathit{MSG[ID]} \cup \{M\}$
return 1		$\textbf{return} \sigma$

```
Experiment \operatorname{Exp}^{\operatorname{id-euf-cma}}_{\operatorname{IBS},\mathcal{A}}(\lambda)
(msk, mpk) \leftarrow \operatorname{IBS.Setup}(1^{\lambda})
HU \leftarrow \emptyset; CU \leftarrow \emptyset
(ID^*, M^*, \sigma^*) \leftarrow \mathcal{A}(mpk; \mathbf{INIT}, \mathbf{EXT}, \mathbf{SIGN})
if ID^* \in HU \wedge M^* \notin MSG[ID^*] \wedge \operatorname{IBS.Vrfy}(mpk, ID^*, M^*, \sigma^*) = 1
return 1
else return 0
```

Fig. 3. Experiment used to define ID-EUF-CMA security for identity-based signature scheme.

2.4 Certificateless signature

A certificateless signature scheme CLS consists of five algorithms:

- CLS.Setup(1^{λ}): On input 1^{λ} , the setup algorithm outputs a key pair (msk, mpk).
- CLS.PPKExtract(mpk, msk, ID): On input a master public key mpk, a master secret key msk, and an identity ID, the partial private key extraction algorithm outputs a partial-private-key psk.
- CLS.UserKeyGen(mpk, ID): On input mpk and ID, the user key generation algorithm outputs a key pair (sk, pk).
- CLS.Sign(mpk, ID, psk, sk, pk, M): On input mpk, ID, psk, sk, pk, and a message M, the signing algorithm outputs a signature σ .
- CLS.Vrfy (mpk, ID, pk, M, σ) : On input mpk, ID, pk, M, and σ , the verification algorithm outputs 0 or 1.

For correctness, we require that for all $\lambda \in \mathbb{N}$, $ID \in \{0,1\}^*$, $M \in \{0,1\}^*$, if $(msk, mpk) \leftarrow \mathsf{CLS}.\mathsf{Setup}(1^\lambda)$, $psk \leftarrow \mathsf{CLS}.\mathsf{PPKExtract}(mpk, msk, ID)$, $(sk, pk) \leftarrow \mathsf{CLS}.\mathsf{UserKeyGen}(mpk, ID)$, and $\sigma \leftarrow \mathsf{CLS}.\mathsf{Sign}(mpk, ID, usk, sk, pk, M)$, then $\mathsf{Pr}\left[\mathsf{CLS}.\mathsf{Vrfy}(mpk, ID, pk, M, \sigma) = 1\right] = 1$ holds.

In CLS setting, there exist two types of adversaries, \mathcal{A}_1 and \mathcal{A}_2 . Adversary \mathcal{A}_1 represents malicious users. \mathcal{A}_1 can compromise the target user's secret key or replace the public key but cannot obtain the master secret key nor the partial private keys. \mathcal{A}_2 represents a malicious KGC. \mathcal{A}_2 knows the master secret key and partial private keys but is not able to obtain the user secret key nor replace the public keys.

As in [HWZ+07], we define existential unforgeability under adaptive chosen-message attacks for both adversaries.

Definition 4. Let CLS is a certificateless signature scheme, \mathcal{A}_1 an adversary, and $\lambda \in \mathbb{N}$ a security parameter. Define the experiment $\mathsf{Exp}^{cl-euf-cma-1}_{\mathsf{CLS},\mathcal{A}_1}(\lambda)$ as shown in Fig. 4. The CL-EUF-CMA-1 advantage of \mathcal{A}_1 in attacking CLS is

$$\mathsf{Adv}^{\mathit{cl-euf-cma-1}}_{\mathit{CLS},\mathcal{A}_1}(\lambda) = \Pr \left[\mathsf{Exp}^{\mathit{cl-euf-cma-1}}_{\mathit{CLS},\mathcal{A}_1}(\lambda) = 1 \right].$$

We say that CLS is a CL-EUF-CMA-1-secure CLS scheme if $Adv_{CLS,A_1}^{cl-euf-cma-1}(\lambda)$ is negligible for any PPT adversary A_1 .

Definition 5. Let CLS is a certificateless signature scheme, A_2 an adversary, and $\lambda \in \mathbb{N}$ a security parameter. Define the experiment $\mathsf{Exp}^{cl-euf\text{-}cma-2}_{\mathsf{CLS},A_2}(\lambda)$ as shown in Fig. 4. The CL-EUF-CMA-2 advantage of A_2 in attacking CLS is

$$\mathsf{Adv}^{\mathit{cl-euf-cma-2}}_{\mathit{CLS},\mathcal{A}_2}(\lambda) = \Pr\left[\mathsf{Exp}^{\mathit{cl-euf-cma-2}}_{\mathit{CLS},\mathcal{A}_2}(\lambda) = 1\right].$$

We say that CLS is a CL-EUF-CMA-2-secure CLS scheme if $Adv_{CLS,A_2}^{cl-euf-cma-2}(\lambda)$ is negligible for any PPT adversary A_2 .

```
Oracle CU(ID)
                                                                         Oracle PPK(ID)
if ID \in CU \cup HU then return \bot
                                                                         if ID \notin HU then return \bot
usk[ID] \leftarrow \mathsf{CLS.PPKExtract}(mpk, msk, ID)
                                                                         HU \leftarrow HU \setminus \{ID\}
(sk[ID], pk[ID]) \leftarrow \mathsf{CLS.UserKeyGen}(mpk, ID)
                                                                         CU \leftarrow CU \cup \{ID\}
MSG[ID] \leftarrow \emptyset; HU \leftarrow HU \cup \{ID\}
                                                                         return usk[ID]
return pk[ID]
Oracle SK1(ID)
                                                 Oracle SK2(ID)
if \mathit{ID} \notin \mathit{HU} then return \bot
                                                if ID \notin HU then return \bot
                                                 HU \leftarrow HU \setminus \{ID\}
return sk[ID]
                                                CU \leftarrow CU \cup \{ID\}
                                                 return sk[ID]
Oracle PKR(ID, pk')
                                                          Oracle SIGN(ID, M)
if ID \notin HU \cup CU then return \bot
                                                         if ID \notin HU \vee sk[ID] = \bot then return \bot
pk[ID] \leftarrow pk', sk[ID] \leftarrow \bot
                                                          \sigma \leftarrow \mathsf{CLS.Sign}(mpk, ID, usk[ID], sk[ID], pk[ID], M)
return 1
                                                          MSG[ID] \leftarrow MSG[ID] \cup \{M\}
                                                          return \sigma
           Experiment \mathsf{Exp}^{\mathrm{cl-euf-cma-1}}_{\mathsf{CLS},\mathcal{A}_1}(\lambda)
           (msk, mpk) \leftarrow \mathsf{CLS}.\mathsf{Setup}(1^{\lambda})
           HU \leftarrow \emptyset; CU \leftarrow \emptyset
           (ID^*, M^*, \sigma^*) \leftarrow \mathcal{A}_1(mpk; \mathbf{CU}, \mathbf{PPK}, \mathbf{SK1}, \mathbf{PKR}, \mathbf{SIGN})
           if ID^* \in HU \land M^* \notin MSG[ID^*] \land \mathsf{CLS.Vrfy}(mpk, ID^*, pk[ID^*], M^*, \sigma^*) = 1
              return 1
           else return 0
Experiment \mathsf{Exp}^{\mathrm{cl-euf-cma-2}}_{\mathsf{CLS},\mathcal{A}_2}(\lambda)
(msk, mpk) \leftarrow \mathsf{CLS}.\mathsf{Setup}(1^{\lambda})
HU \leftarrow \emptyset; CU \leftarrow \emptyset
(ID^*, M^*, \sigma^*) \leftarrow \mathcal{A}_2(msk, mpk; \mathbf{CU}, \mathbf{PKR}, \mathbf{SK2}, \mathbf{SIGN})
if ID^* \in HU \land sk[ID^*] \neq \bot \land M^* \notin MSG[ID^*] \land \mathsf{CLS.Vrfy}(mpk, ID^*, pk[ID^*], M^*, \sigma^*) = 1
   return 1
else return 0
```

Fig. 4. Experiment used to define CL-EUF-CMA security of the scheme.

$\frac{CLS.Setup(1^\lambda)}{}$	$\overline{CLS.PPKExtract(\mathit{msk},\mathit{ID})}$		(ID) CLS.UserKeyGen (mpk, ID)	
$(msk, mpk) \leftarrow IBS.Setup(1^{\lambda})$ return (msk, mpk)	$\begin{aligned} psk_{ID} \leftarrow IBS.Extract(r \\ \mathbf{return} \ \ psk_{ID} \end{aligned}$	nsk, ID)	$(sk_{ID}, pk_{ID}) \leftarrow Sig.KGen(1^{\lambda})$ $\mathbf{return}\ (sk_{ID}, pk_{ID})$	
$\overline{CLS.Sign(mpk,ID,psk_{ID},sk_{ID},}$	$pk_{ID}, M)$	CLS.Vrf	$fy(\mathit{mpk},\mathit{ID},\mathit{pk}_\mathit{ID},M,\sigma)$	
$\sigma_1 \leftarrow Sig.Sign(sk_{ID}, M \ mpk \ I)$	$D\ pk_{ID})$	Parse ($(\sigma_1,\sigma_2) \leftarrow \sigma$	
$\sigma_2 \leftarrow IBS.Sign(\mathit{mpk},\mathit{ID},\mathit{psk}_\mathit{ID},$	$M\ mpk\ ID\ pk_{ID}\ \sigma_1)$	if IBS.\	$Vrfy(mpk,ID,M\ mpk\ ID\ pk_{ID},\sigma_1)=0$	
$\textbf{return} (\sigma_1,\sigma_2)$		retu	rn 0	
		if Sig.√	$ extstyle extstyle V extstyle r extstyle fyllow for the following points for the first fill extstyle p k_{ID}, M \ mpk\ ID \ pk_{ID}\ \sigma_1, \sigma_2) = 0$	
		retu	rn 0	
		else re	turn 1	

Fig. 5. Generic construction of CLS.

3 Generic construction of certificateless signature

In this section, we review Hu et al. construction in [HWZ+07]. Let IBS = (IBS.Setup, IBS.Extract, IBS.Sign, IBS.Vrfy) be an ID-EUF-CMA-secure identity-based scheme and Sig = (Sig.KGen, Sig.Sign, Sig.Vrfy) be an EUF-CMA-secure signature scheme. CLS scheme from Hu et al. construction CLS = (CLS.Setup, CLS.PPKExtract, CLS.UserKeyGen, CLS.Sign, CLS.Vrfy) is described in Fig. 5.

The following propositions hold for the construction.

Proposition 1 ([HWZ+07, Theorem 1]). For any Type-I adversary A_1 that breaks the CL-EUF-CMA-1 security of CLS, there exists an algorithm \mathcal{B}_1 that breaks the ID-EUF-CMA security of IBS, where

$$\mathsf{Adv}^{\mathit{cl-euf-cma-1}}_{\mathit{CLS},\mathcal{A}_1}(\lambda) = \mathsf{Adv}^{\mathit{id-euf-cma}}_{\mathit{IBS},\mathcal{B}_1}(\lambda).$$

Proposition 2 ([HWZ+07, Theorem 2]). Let Q_{cu} be the number of queries for CU oracle, i.e. the number of users. For any Type-II adversary A_2 that breaks the CL-EUF-CMA-2 security of CLS, there exists an algorithm \mathcal{B}_2 that breaks the EUF-CMA security of Sig, where

$$\mathsf{Adv}^{\mathit{cl-euf-cma-2}}_{\mathit{CLS},\mathcal{A}_2}(\lambda) \leq Q_{\mathit{cu}} \mathsf{Adv}^{\mathit{euf-cma}}_{\mathit{Sig},\mathcal{B}_2}(\lambda).$$

As shown in Proposition 2, the reduction to EUF-CMA-secure signature scheme is not tight. Thus we cannot obtain tightly-secure schemes from the above reduction. In the next section, we show the new security reduction to construct tightly-secure CLS schemes.

```
Oracle CU(ID)
                                                     Oracle SK2(ID)
if ID \in HU \cup CU then return \bot
                                                     if ID \notin HU then return \bot
                                                     CU \leftarrow CU \cup \{ID\}; HU \leftarrow HU \setminus \{ID\}
pk[ID] \leftarrow pk_{ctr}; ctr[ID] \leftarrow ctr
psk[ID] \leftarrow \mathsf{IBS}.\mathsf{Extract}(msk, ID)
                                                     sk[ID] \leftarrow \mathbf{CORR}(ctr[ID])
MSG[ID] \leftarrow \emptyset
                                                     return sk[ID]
HU \leftarrow HU \cup \{ID\}; ctr \leftarrow ctr + 1
return pk[ID]
Oracle PKR(ID, pk')
                                                     Oracle SIGN(ID, M)
if ID \notin HU \cup CU then return \bot
                                                     if ID \notin HU \vee sk[ID] = \bot then return \bot
pk[ID] \leftarrow pk', sk[ID] \leftarrow \bot
                                                     \sigma_1 \leftarrow \mathbf{SIGN}(M \| mpk \| ID \| pk_{ID}, ctr[ID])
return 1
                                                     \sigma_2 \leftarrow \mathsf{IBS.Sign}(ID, psk[ID], M || mpk || ID || pk[ID] || \sigma_1)
                                                     MSG[ID] \leftarrow MSG[ID] \cup \{M\}
                                                     return (\sigma_1, \sigma_2)
```

Fig. 6. Oracle simulation performed by \mathcal{B}_2 .

4 Tight reduction for the generic construction

We show that Hu et al. construction is tightly secure if the underlying signature scheme is MU-EUF-CMA^{Corr}. Note that the security against Type-I adversary is the same as Proposition 1.

Theorem 1. For any Type-II adversary A_2 that breaks the CL-EUF-CMA-2 security of CLS, there exists an algorithm B_2 that breaks the MU-EUF-CMA^{Corr} security of Sig, where

$$\mathsf{Adv}^{\mathit{cl-euf-cma-2}}_{\mathit{CLS},\mathcal{A}_2}(\lambda) = \mathsf{Adv}^{\mathit{mu-euf-cma-corr}}_{\mathit{Sig},\mathcal{B}_2}(\lambda).$$

Proof. Let A_2 be a PPT adversary against CLS. We construct a PPT adversary B_2 which breaks the MU-EUF-CMA^{Corr} security of Sig by running A_2 .

 \mathcal{B}_2 takes input the security parameter 1^{λ} and Q_{CU} public keys $pk_1, \ldots, pk_{Q_{\text{CU}}}$ of Sig, where Q_{CU} is the number of CU queries. It has access to the corruption oracle CORR and signing oracle SIGN. \mathcal{B}_2 generates $(msk, mpk) \leftarrow \text{CLS.Setup}(1^{\lambda})$ and sets $HU \leftarrow \emptyset, CU \leftarrow \emptyset, ctr \leftarrow 1$. It runs \mathcal{A}_2 as subroutine and answers their oracle queries as shown in Fig. 6.

 \mathcal{A}_2 outputs ID^* , M^* , $\sigma^* = (\sigma_1^*, \sigma_2^*)$. Let ctr^* be the counter and pk_{ID^*} be the public key corresponding to ID^* . If \mathcal{A}_2 wins the game, $pk_{ID^*} = pk_{ctr^*}$ holds because $\mathbf{PKR}(ID^*, \cdot)$ has never been queried. Moreover, $(M^* \| mpk \| ID^* \| pk_{ID^*}, \sigma_1^*)$ is a valid signature with respect to the signature scheme Sig. In addition, $\mathbf{SK2}(ID^*)$ and $\mathbf{SIGN}(ID^*, M^*)$ has never been queried from \mathcal{A}_2 , i.e. $ID^* \notin CU$ and $M^* \notin MSG[ID^*]$. In other words, \mathcal{B}_2 has never queried $\mathbf{EXT}(ctr^*)$ and $\mathbf{SIGN}(M^* \| mpk \| ID^* \| pk_{ID^*}, ctr^*)$. Therefore $(ctr^*, M^* \| mpk \| ID^* \| pk_{ID^*}, \sigma_1^*)$ is a valid forgery for Sig.

If A_2 is successful, B_2 is also successful. Thus we get

$$\mathsf{Adv}^{\mathrm{cl-euf\text{-}cma\text{-}2}}_{\mathsf{CLS},\mathcal{A}_2}(\lambda) = \mathsf{Adv}^{\mathrm{mu\text{-}euf\text{-}cma\text{-}corr}}_{\mathsf{Sig},\mathcal{B}_2}(\lambda).$$

Theorem 1 indicates that the generic construction in Fig. 5 achieves tight security if the underlying signature scheme is tightly MU-EUF-CMA^{Corr}-security. Thus we are ready to construct a tightly-secure CLS scheme.

5 Instantiation

5.1 Tightly-secure certificateless signature scheme without pairing

We can instantiate real tightly-secure CLS schemes using the generic construction. We choose the IBS scheme of Fukumitsu and Hasegawa [FH18] as the underlying IBS scheme, which is the most efficient and tightly-secure scheme in the DDH assumption. For the underlying MU-EUF-CMA^{Corr}-secure signature scheme, we choose the efficient scheme of Gjøsteen and Jager [GJ18] whose security is tightly reduced to the DDH assumption. Therefore, the instantiated CLS scheme also provides tight security in the DDH assumptions. As both [FH18] and [GJ18] are pairing-free, the constructed CLS scheme is also pairing-free.

5.2 Comparison

We compare the tightly-secure instantiation (Tight) with the two conventional pairing-free CLS scheme [GL14; JHL+18] and a non-tight instantiation (Non-Tight) from EUF-CMA-secure signature. To instantiate a CLS scheme from EUF-CMA-secure signature, we choose [GJK+07] as the underlying EUF-CMA-secure signature scheme, which is an efficient pairing-free and tightly-secure signature schemes in the DDH assumption.

We denote by (\mathbb{G}, q) a group \mathbb{G} of a prime order q. Table 1 shows the estimation of the bit length of the group element. We choose parameters that provide 128 bits security. Table 2 shows the number of elements in the secret key, public key, and signature and the actual signature size. Because both security reduction

Table 1. Evaluation of security level for each scheme and the bit length of group elements. ϵ denotes a success probability of an adversary against each scheme and we set the parameters so that $\epsilon = 2^{-256}$ for all schemes. ϵ' denotes a success probability of an algorithm that solves the underlying problem. The column "Tightness" shows the gap between ϵ and ϵ' . We assume the number of users is $\mu = 2^{50}$ and that of hash function evaluations is $h = 2^{80}$.

Scheme	Assumption	0		$ \mathbb{Z}_q $ and $ \mathbb{G} $ [bits]
Gong and Li [GL14]		$\epsilon \le h\sqrt[4]{h^6\epsilon'}$		
Jia et al. [JHL+18]	DL	$\epsilon \le \mu \sqrt[4]{h^6 \epsilon'}$		1554
Non-Tight (IBS: $[FH18] + Sig: [GJK+07]$)	DDH	$\epsilon \le \mu \epsilon'$	2^{-306}	306
Tight (IBS: $[FH18] + Sig:[GJ18]$)	DDH	$\epsilon \le 4\epsilon'$	2^{-258}	258

Table 2. Comparison on the number of group elements and the actual signature size.

Scheme	psk + sk	pk	$ \sigma $
Gong and Li [GL14]	$ \mathbb{G} + 2\mathbb{Z}_q $	G	$2 \mathbb{G} + \mathbb{Z}_q $
Jia et al. [JHL+18]	$2\left \mathbb{Z}_{q}\right $	$2 \mathbb{G} $	$ \mathbb{G} + \mathbb{Z}_q $
Non-Tight (IBS: $[FH18] + Sig: [GJK+07]$)	$2 \mathbb{G} +2 \mathbb{Z}$	$_{q} $ $3 \mathbb{G} $	$2\left \mathbb{G}\right +4\left \mathbb{Z}_{q}\right $
Tight (IBS: $[FH18] + Sig:[GJ18]$)	$2\left \mathbb{G}\right +2\left \mathbb{Z}_{q}\right $	+12 G	$4\left \mathbb{G}\right + 6\left \mathbb{Z}_q\right + \lambda$
Scheme	psk + sk	[bits] $ pk $	[bits] $ \sigma $ [bits]
Scheme Gong and Li [GL14]	$\frac{ psk + sk }{4752}$		$\frac{\text{[bits] } \sigma \text{ [bits]}}{584 4752}$
		1	
Gong and Li [GL14]	4752 3108	1 3	584 4752

in [GL14] and [JHL+18] is very loose, we need larger group order q. As a result, the actual signature size is very large in spite of the small number of group elements in signature. Both Non-Tight and Tight instantiation has a small order. In general, the smaller the group order, the better the computation efficiency. Comparing Tight instantiation with Non-Tight instantiation, Tight instantiation is better in key size and Non-Tight instantiation is better in signature size.

6 Conclusion

In this paper, we have improved the reduction cost for generic construction of certificateless signature proposed by Hu et al. [HWZ+07]. Using the construction, we have instantiated the first tightly-secure certificateless signature scheme without pairing.

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