# PGC: Decentralized Confidential Payment System with Auditability

Yu Chen \* Xuecheng Ma † Cong Tang ‡ Man Ho Au §

#### Abstract

Modern cryptocurrencies such as Bitcoin and Ethereum achieve decentralization by replacing a trusted center with a distributed and append-only ledger (known as blockchain). However, removing this trusted center comes at significant cost of privacy due to the public nature of blockchain. Many existing cryptocurrencies fail to provide transaction anonymity and confidentiality, meaning that addresses of sender, receiver and transfer amount are publicly accessible. As the privacy concerns grow, a number of academic work have sought to enhance privacy by leveraging cryptographic tools. Though strong privacy is appealing, it might be abused in some cases. Particularly, anonymity poses great challenges to auditability, which is a crucial property for the adoption of decentralized payment systems.

Aiming for a middle ground between privacy and auditability, we introduce the notion of auditable decentralized confidential payment (ADCP) system. In addition to offering transaction confidentiality, ADCP system supports two levels of auditability, namely regulation compliance and global supervision. We present a generic construction of ADCP system from integrated signature and encryption scheme and non-interactive zero-knowledge proof systems. We then instantiate our generic construction by carefully designing the underlying building blocks, yielding a standalone cryptocurrency called PGC. In PGC, the setup procedure is semi-transparent, and transaction cost is independent of system scale, which is roughly 1.4KB and takes under 28ms to generate and 9ms to verify.

At the core of PGC is an additively homomorphic public-key encryption scheme that we introduce, twisted ElGamal, which is not only as secure as standard exponential ElGamal, but also friendly to Sigma protocols and range proofs. This enables us to easily devise zero-knowledge proofs for basic correctness of transactions as well as various application-dependent policies in a modular fashion. Moreover, it is very efficient. Compared with the most efficient reported implementation of Paillier PKE, twisted ElGamal is an order of magnitude better in key and ciphertext size and decryption speed (for small message space), two orders of magnitude better in encryption speed. We believe twisted ElGamal is of independent interest on its own right. Along the way of designing and reasoning zero-knowledge proofs for PGC, we also obtain two interesting results. One is weak forking lemma which is a useful tool to prove computational knowledge soundness. The other is a method to prove no-knowledge of discrete logarithm, which is a complement of standard proof of discrete logarithm knowledge.

**Keywords:** cryptocurrencies, decentralized payment system, confidential transactions, auditable, twisted ElGamal, weak forking lemma, proof of no-knowledge

<sup>\*</sup>School of Cyber Science and Technology, Shandong University, Qingdao 266237, China. Email: <a href="mailto:yuchen.prc@gmail.com">yuchen.prc@gmail.com</a>
†State Key Laboratory of Information Security, Institute of Information Engineering, Chinese Academy of Sciences.
School of Cyber Security, University of Chinese Academy of Sciences. Email: <a href="maxwecheng@iie.ac.cn">maxwecheng@iie.ac.cn</a>

<sup>&</sup>lt;sup>‡</sup>Beijing Temi Co., Ltd, pgc.info. Email: congtang.cn@gmail.com

<sup>§</sup>Department of Computer Science, The University of Hong Kong. Email: allenau@cs.hku.hk

# Contents

1	Introduction	1
	1.1 Motivation	1
	1.3 Technical Overview	2
	1.4 Related Work	6
	1.5 Additions to Conference Version	8
2	Preliminaries	8
	2.1 Cryptographic Assumptions	
	2.2 Commitments	
	V VI	10
	2.4 Signature	
	2.6 Zero-Knowledge Protocols	
3	Definition of Auditable Decentralized Confidential Payment System 1	14
J	3.1 Data Structures	
	3.2 Entities	
	3.3 Auditable Decentralized Confidential Payment System	
	3.4 Correctness	
	3.5 Security Model	16
4	DCP with Auditability	18
	4.1 A Generic Construction of Auditable DCP from ISE and NIZK	
	4.2 Analysis of Generic Auditable DCP Construction	20
5	PGC: an Efficient Instantiation	<b>2</b> 4
	5.1 Instantiating ISE	
	5.2 Instantiating NIZK	27
6		33
	6.1 Transparent Setup	
	6.2 Necessity of Composing Sigma Protocol with Bulletproof	
	0.5 Ciphiertext Refreshing vs. Rey Switching	90
7	±	34
	7.1 Faster Decryption for Twisted ElGamal	
	7.2 More Efficient Assembly of NIZK	34 35
	7.3 Eliminate Explicit Signature	Эє
8		35
	•	35 26
		36 37
A		<b>42</b>
	A.1 Security Proof of Twisted ElGamal	42 10
	TILE THE DITE TO THE COORSE OF THE OPEN TO THE TWO SHOULD THE SHIP THE SHIP THE COORSE OF THE COORSE	14

# 1 Introduction

Unlike traditional centralized banking systems, decentralized payment systems such as Bitcoin [Nak08] and Ethereum [Woo14] realize peer-to-peer payment by maintaining an append-only public ledger known as blockchain, which is globally distributed and synchronized by consensus protocols. In blockchain-based decentralized payment systems (cryptocurrencies), validity of each transaction must be publicly verified by the validators<sup>1</sup> before being packed into a block. To facilitate public verification, the first generation of cryptocurrencies such as Bitcoin and Ethereum simply make all transaction details (sender, receiver and transaction amount) publicly accessible and thus are not privacy-preserving. According to [BBB+18], privacy of decentralized payment system consists of two axes: (1) anonymity, the identities of sender and receiver in a transaction tx is hidden from an external observer, and (2) confidentiality, the value in a transaction tx is hidden from an external observer (only known by the associated sender and receiver). While Bitcoin-like and Ethereum-like cryptocurrencies provide some weak anonymity through unlinkability of account addresses to real world identities, they lack confidentiality, which is of paramount importance.

#### 1.1 Motivation

Auditability is a crucial property in all financial systems. In transaction-based payment system, auditability roughly means that an auditor is able to check whether transactions are legal. Roughly, we consider two levels of auditability as follows:

- Regulation compliance: An external auditor can verify if a given set of transactions comply with regulation policies by inquiring involved participants. In this type of auditing, auditor usually does not process extra power over ordinary users, and thus auditing is interactive in nature (requiring auditee's consent). We refer to this kind of auditor as regulator.
- Global supervision: An internal auditor can inspect all the information in any transaction at his will. In this type of auditing, auditor typically owns some extra privilege over ordinary users (e.g. secret of system-wide parameters), and auditing is non-interactive (without auditee's consent). We refer to this kind of auditor as supervisor.

In centralized payment system where there exists a trusted center such as bank, Paypal or Alipay, auditability is a built-in property since the center knows details of all transactions by default. However, it is challenging to provide the same level of auditability in decentralized payment systems with strong privacy guarantee.

In our point of perspective, strong privacy is a double-edged sword. While confidentiality is arguably the primary concern of privacy for any payment system, anonymity might be abused or even prohibitive for applications that require auditability, because anonymity provides plausible deniability<sup>2</sup>, which allows participants to deny their involvements in given transactions. Particularly, anonymity denies the feasibility of enforcing regulation compliance, since a regulator is unable to determine who is involved in which transaction. Based on the above discussion, we conjecture that it may be impossible to construct a decentralized payment system maintaining the same level of auditability as centralized payment system while offering confidentiality and anonymity simultaneously without introducing some degree of centralization or trust assumption. We are thus motivated to find a sweet balance between privacy and auditability in decentralized payment system.

#### 1.2 Our Contributions

In this work, we stick to confidentiality, but trade anonymity for regulation compliance, and show further how to enable supervision. We summarize our contributions as follows.

Auditable decentralized confidential payment system. We introduce the notion of auditable decentralized confidential payment (DCP) system with a formal security model. For the sake of simplicity, we take an account-based approach and focus only on the transaction layer, and treat the

<sup>&</sup>lt;sup>1</sup>In cryptocurrencies jargon, validators are known as miners.

<sup>&</sup>lt;sup>2</sup>Plausible deniability [FMMO19]: no outsider observer can tell if a user meant to be involved in a transaction.

network/consensus-level protocols as black-box and ignore attacks against them. We first define security requirement for confidential payment system, i.e., authenticity, confidentiality and soundness. Authenticity stipulates that only the owner of an account can spend his coin. Confidentiality requires the account balance and transfer amount are hidden. Soundness captures that no adversary is able to generate an invalid transaction that passes verification. We then formalize security requirements for auditing. The weaker form of auditability — regulation compliance requires soundness and minimal information disclosure, namely, no malicious auditee participant can convince the regulator to accept false claims and the regulator learns nothing other than the auditing results. The stronger form of auditability — supervision requires consistency, namely for any transaction the view of supervisor is consistent with that of the intended receiver. Meanwhile, even the supervisor cannot potentially break the authenticity and soundness of confidential payment system (cause loss of property).

We then present a generic construction of ADCP from two building blocks, namely, integrated signature and encryption (ISE) schemes and non-interactive zero-knowledge (NIZK) proof systems. In our generic ADCP construction, ISE plays an important role. First, it guarantees that each account can safely use a single keypair for both encryption and signing. This feature greatly simplifies the overall design from both conceptual and practical aspects. Second, the encryption component of ISE ensures that the resulting ADCP system is *complete*, meaning that as soon as a transaction is recorded on the blockchain, the payment is settled and takes effect immediately – receiver's balance increases with the same amount that sender's balance decreases, and the receiver can spend his coin on his will. This is in contrast to many commitment-based cryptocurrencies that require *out-of-band* transfer, which are thus not complete. The effect of NIZK is three folds: (1) enables a sender to prove that transactions satisfies the default monetary invariants (validity policy) of a payment system; (2) allows an auditee to prove that any set of confidential transactions he participated comply with a wide range of application-dependent regulation policies; (3) guarantees that no user can evade the oversight of the supervisor. In summary, our generic ADCP construction is simple, complete, and supports flexible auditing.

PGC: a simple and efficient instantiation. While the generic ADCP construction is relatively simple and intuitive, an efficient realization is more technically involved. We instantiate the generic construction by designing a new ISE and carefully devising suitable NIZK proof system. We refer to the resulting decentralized payment system as PGC (stands for Pretty Good Confidentiality). Notably, in addition to the advantages inherited from the generic construction, PGC admits transparent setup, and its security is based solely on the widely-used discrete logarithm assumption. To demonstrate the efficiency and usability of PGC, we implement it as a standalone cryptocurrency, and also deploy it as smart contracts. We report the experimental results in Section 8.

#### 1.3 Technical Overview

We discuss our design choice in the generic construction of ADCP, followed by techniques and tools towards a secure and efficient instantiation, namely, PGC.

#### 1.3.1 Design Choice in Generic Construction of ADCP

Pseudonymity vs. Anonymity. Early blockchain-based cryptocurrencies offers pseudonymity, that is, addresses are assumed to be unlinkable to their real world identities. However, a variety of deanonymization attacks [RS13, BKP14] falsified this assumption. On the other hand, a number of cryptocurrencies such as Monero and Zcash sought to provide strong privacy guarantee, including both anonymity and confidentiality. In this work, we aim for finding a sweet balance between privacy and auditability. As indicated in [GGM16], identity is crucial to any regulatory system. Therefore, we choose to offer privacy in terms of confidentiality, and still stick to pseudonymous system. Interestingly, we view pseudonymity as a feature rather than a weakness, and assume that an auditor is able to link account addresses to real world identities. This opens the possibility to conduct auditing. We believe this is the most promising avenue for real deployment of DCP that requires auditability.

**PKE vs. Commitment.** A typical approach to achieve transaction confidentiality is to commit the balance and transaction amount using a global homomorphic commitment scheme (e.g., the Pedersen commitment [Ped91]), then derive a secret from blinding randomnesses to prove correctness of transaction and authorize transfer. The seminal DCP systems [Max, Poe] follow this approach.

Nevertheless, commitment-based approach suffers from several drawbacks. First, the resulting DCP systems are not *complete*. Due to lack of decryption capability, senders are required to honestly transmit the openings of outgoing commitments (includes randomness and amount) to receivers in an *out-of-band* manner. This issue makes the system much more complicated, as it must be assured that the out-of-band transfer is correct and secure. Second, users must be stateful since they have to keep track of the randomness and amount of each incoming commitment. Otherwise, failure to open a single incoming commitment will render an account totally unusable, due to either incapable of creating the NIZK proofs (lack of witness), or generating the signature (lack of signing key). This incurs extra security burden to the design of wallet (guarantee the openings must be kept in a safe and reliable way). Last but not the least, as stated by Bünz et al. [BBB+18], as Pedersen commitment is only computational binding based on discrete logarithm assumption, an adversary will be able to open a given commitment to an arbitrary value when quantum computers are available, which is a serious issue in any payment system.

Observe that homomorphic PKE can be viewed as a computationally hiding and perfectly binding commitment, in which the secret key serves as a natural trapdoor to recover message. With these factors in mind, our design equips each user with a PKE keypair rather than making all users share a global commitment.

Integrated Signature and Encryption vs. SIG+PKE. Intuitively, to secure a DCP system, we need a PKE scheme to provide confidentiality, and a signature scheme to provide authenticity. If we follow the principle of *key separation*, i.e., use different keypairs for encryption and signing operations respectively, the overall design would be complicated. In that case, each account will be associated with two keypairs, and consequently account address generation turns out to be very tricky. If we derive the address from one public key, which one should be chosen? If we derive the address from the two public keys, then additional mechanism is needed to link the two public keys together.

A better solution is to adopt *key reuse* strategy, i.e., use the same keypair for both encryption and signing. This will greatly simplify the design of overall system. However, reusing keypairs may create new security problems. As pointed out in [PSST11], the two operations may interact with one another badly, in such a way as to undermine the security of one or both of the primitives, e.g., the case of textbook RSA encryption and signature. In this work, we propose to use integrated signature and encryption (ISE) scheme with *joint security*, wherein a single keypair is used for both signature and encryption components in a secure manner (cf. Section 2.5 for definition), to replace the naive combination of signature and encryption. To the best of our knowledge, this is the first time that ISE is used in DCP system to ensure provable security. We remark that the existing proposal Zether [BAZB20] essentially adopts the key reuse strategy, employing a signature scheme and an encryption scheme with same keypair. Nevertheless, they do not explicitly identify the usage of key reuse strategy and formally address joint security.

#### 1.3.2 Overview of Our Generic Construction of ADCP

We present a generic construction of ADCP system from ISE and NIZK. We choose the account-based model for simplicity and usability. In our generic ADCP construction, user creates an account by generating a keypair of ISE, in which the public key is used as account address and secret key is used to control the account. The state of an account consists of a serial number (a counter that increments with every outgoing transaction) and an encrypted balance (encryption of plaintext balance under account public key). State changes are triggered by transactions from one account to another. A publicly accessible and append-only ledger called blockchain records monetary transactions in an immutable manner, and account states can always be deduced from history transactions on the blockchain.

For ease of exposition, we describe our ADCP construction in an incremental manner. We first outline the design of DCP that support regulation compliance, then sketch how to enable global supervision.

Basic DCP. Let  $\tilde{C}_s$  and  $\tilde{C}_r$  be the encrypted balances of two accounts controlled by Alice and Bob respectively. Suppose Alice wishes to transfer v coins to Bob. She constructs a confidential transaction via the following steps. First, she encrypts v under her public key  $pk_s$  and Bob's public key  $pk_r$  respectively to obtain  $C_s$  and  $C_r$ , and sets memo =  $(pk_s, pk_r, C_s, C_r)$ . Then, she produces a NIZK proof  $\pi_{\text{legal}}$  for the system-wide legality policy: (i)  $C_s$  and  $C_r$  are two encryptions of the same transaction amount under  $pk_s$  and  $pk_r$ ; (ii) the transaction amount lies in a right range; (iii) her remaining balance is not negative. Finally, she signs serial number sn together with memo and  $\pi_{\text{legal}}$  under her secret key, obtaining a signature  $\sigma$ . The entire transaction is of the form (sn, memo,  $\pi_{\text{legal}}$ ,  $\sigma$ ). In this way, validity

of a transaction is publicly verifiable by checking the signature and NIZK proof. If the transaction is valid, it will be recorded on the blockchain. Accordingly, Alice's balance (resp. Bob's balance) will be updated as  $\tilde{C}_s = \tilde{C}_s - C_s$  (resp.  $\tilde{C}_r = \tilde{C}_r + C_r$ ), and Alice's serial number increments. Such balance update operation implicitly requires that the underlying PKE scheme is additively homomorphic.

In summary, the signature component is used to provide authenticity (proving ownership of an account), the encryption component is used to hide the balance and transfer amount, while zero-knowledge proofs are used to prove the legality of transactions in a privacy-preserving manner.

Regulation compliance. A trivial solution to enforce auditability is to make the relevant participants reveal their secret keys. However, this approach will expose all the related transactions to the auditor, and thus does not satisfy minimal information disclosure. Note that the relationship between plain account states/transaction amounts and their encrypted form can be expressed as  $\mathcal{NP}$  relations. Regulation compliance can thus be easily achieved by leveraging NIZK. Moreover, the symmetric structure of memo allows either the sender or the receiver can prove that transactions comply with a variety of application-dependent policies. We provide examples of several regulation policies as below:

- Anti-money laundering: the sum of a collection of outgoing/incoming transactions from/to a particular account is limited.
- Tax payment: a user pays the tax according to a prior-fixed tax rate.
- Selectively disclosure: the transfer amount of some transaction is indeed some value.

Global supervision. A naive solution to enable supervision is to have the supervisor maintain a big database for all users. However, this naive solution comes with three deficiencies: (i) the complexity of key management grows linearly with the number of keys, and thus being inadequate for large-scale applications; (ii) collecting decryption keys require consent of all users, which could be difficult to conduct in practice; (iii) in the key reuse setting, the supervisor can easily compromise the authenticity with decryption keys. Our solution is to use PKE with global key escrow property [CTW21]. In more details, the supervisor plays the role of global key escrow center, and the sender encrypts transaction amount using global escrow PKE. The consistency and security of supervision follow from that of global escrow PKE and the fact that the global decryption key is independent of each individual user's key pair. Note that Chen et al. [CTW21] shows two generic approaches of building global escrow PKE. One is from PKE and NIZK, the other is from three-party non-interactive key exchange. We choose the first approach since it can compile any PKE in-use to a global escrow one, and therefore global supervision can be designed as an add-on mechanism.

#### 1.3.3 PGC: a secure and efficient instantiation

A secure and efficient instantiation of the above ADCP construction turns out to be more technical involved. Before proceeding, it is instructive to list the desirable features in mind:

- 1. Transparent setup do not require a trusted setup. This property is of uttermost importance in the setting of cryptocurrencies.  $^3$
- 2. Efficient only employ lightweight cryptographic schemes based on well-studied assumptions.
- 3. Modular build the whole scheme from reusable gadgets.

The above desirable features suggest us to devise efficient NIZK that admits transparent setup, rather than resorting to general-purpose zk-SNARKs, which are heavyweight or require trusted setup. Besides, the encryption component of ISE should be zero-knowledge proofs friendly.

We begin with the instantiation of ISE. A common choice is to select Schnorr signature [Sch91] as the signature component and exponential ElGamal PKE [CGS97] as the encryption component.<sup>4</sup> This choice brings us at least three benefits: (i) Schnorr signature and ElGamal PKE share the same discrete-logarithm (DL) keypair (i.e.,  $pk = g^{sk} \in \mathbb{G}, sk \in \mathbb{Z}_p$ ), thus we can simply use the common public

<sup>&</sup>lt;sup>3</sup>We argue that in some cases semi-transparent setup is also acceptable. Looking ahead, when adding global escrow property, the parameter of supervisor's part does not admit transparent setup, but even with the associated trapdoor, the supervisor is still unable to break authenticity and soundness.

<sup>&</sup>lt;sup>4</sup>In the remainder of this paper, we simply refer to the exponential ElGamal PKE as ElGamal PKE for ease of exposition.

key as account address. (ii) The signing operation of Schnorr's signature is largely independent to the decryption operation of ElGamal PKE, which indicates that the resulting ISE scheme could be jointly secure. (iii) ElGamal PKE is additively homomorphic. Efficient Sigma protocols can thus be employed to prove linear relations on algebraically-encoded values.

Obstacle of working with Bulletproof. To design efficient NIZK proofs for the basic correctness policies and various application-dependent policies, we also need range proofs to prove the encrypted values lie in the right interval. In more details, we need to prove that the value v encrypted in  $C_s$  and the value encrypted in  $\tilde{C}_s - C_s$  (the current balance subtracts v) lies in the right range. State-of-the-art range proof is Bulletproof [BBB+18], which enjoys efficient proof generation/verification, logarithmic proof size, and transparent setup. As per the desirable features of our instantiation, Bulletproof is an obvious choice. Recall that Bulletproof only accepts statements of the form of Pedersen commitment  $g^rh^v$ . To guarantee soundness, the DL relation between commitment key (g,h) must be unknown to the prover. Note that an ElGamal ciphertext C of v under pk is of the form  $(g^r, pk^rg^v)$ , in which the second part ciphertext can be viewed as a commitment of v under commitment key (pk,g). To prove v lies in the right range, it seems that we can simply run the Bulletproof on  $pk^rg^v$ . However, this usage is insecure since the prover owns an obvious trapdoor, say sk, of such commitment key.

There are two approaches to circumvent this obstacle. The first approach is to commit v with randomness r under commitment key (g,h), then prove (v,r) in  $g^vh^r$  is consistent with that in the ciphertext  $(g^r,pk^rg^v)$ . Similar idea was used in Quisquis [FMMO19]. The drawback of this approach is that it brings extra overhead of proof size as well as proof generation/verification. The second approach is due to Bünz et al. [BAZB20] used in Zether. They extend Bulletproof to  $\Sigma$ -Bullets, which enables the interoperability between Sigma protocols and Bulletproof. Though  $\Sigma$ -Bullets is flexible, it requires custom design and analysis from scratch for each new Sigma protocols.

Twisted ElGamal - our customized solution. We are motivated to directly use Bulletproof in a black-box manner, without introducing Pedersen commitment as a bridge or dissecting Bulletproof. Our idea is to twist standard ElGamal, yielding the twisted ElGamal. We sketch twisted ElGamal below. The setup algorithm picks two random generators (g,h) of  $\mathbb G$  as global parameters, while the key generation algorithm is same as that of standard ElGamal. To encrypt a message  $m \in \mathbb Z_p$  under pk, the encryption algorithm picks  $r \stackrel{\mathbb R}{\leftarrow} \mathbb Z_p$ , then computes ciphertext as  $C = (X = pk^r, Y = g^rh^m)$ . The crucial difference to standard ElGamal is that the roles of key encapsulation and session key are switched and the message m is lifted on a new generator  $h.^5$  Clearly, twisted ElGamal retains additive homomorphism, and is as efficient and secure as the standard exponential ElGamal (as we will rigorously prove later). More importantly, it is zero-knowledge friendly. Note that the second part of twisted ElGamal ciphertext (even encrypted under different public keys) can be viewed as Pedersen commitment under the same commitment key (g,h), whose DL relation is unknown to all users. Such structure makes twisted ElGamal compatible with all zero-knowledge proofs whose statement is of the form Pedersen commitment. In particular, one can directly employ Bulletproof to generate range proofs for values encrypted by twisted ElGamal in a black-box manner, and these proofs can be easily aggregated. Next, we abstract two distinguished cases.

Prover knows the randomness. In our generic DCP construction, to create a transaction, sender encrypts transfer amount under his public key, then proves the encrypted amount lies in the right range. This case generalizes scenarios in which the prover is the producer of ciphertexts. Concretely, consider a twisted ElGamal ciphertext  $C = (X = pk^r, Y = g^rh^v)$ , to prove m lies in the right range, the prover first executes a Sigma protocol on C to prove the knowledge of (r, v), then invokes a Bulletproof on Y to prove V lies in the right range. The proof of knowledge property follows since the output of Sigma protocol's extractor equals that of Bulletproof's extractor with overwhelming probability based on the hardness of DLP related to (g, h). See Section 5.2.2 for the details.

Prover knows the secret key. In our generic DCP construction, sender also needs to prove the amount encrypted by  $\tilde{C}_s - C_s$  lies in the right range. Here,  $\tilde{C}_s$  is the encryption of his current balance, which is the accumulation of all previous incoming and outgoing transfers. Note that the randomness beneath  $\tilde{C}_s - C_s$  is generally unknown to the sender, even assuming homomorphism on randomness.<sup>6</sup> This case

<sup>&</sup>lt;sup>5</sup>As indicated in [CZ14], the essence of ElGamal is  $F_{sk}(g^r) = pk^r$  forms a publicly evaluable pseudorandom functions over  $\mathbb{G}$ . The key insight of switching is that  $F_{sk}$  is in fact a permutation.

<sup>&</sup>lt;sup>6</sup>Most encryption schemes are not randomness recovering.

generalizes scenarios where the prover is the recipient of ciphertexts. Due to lack of randomness as witness, the prover cannot directly invoke Bulletproof as above. We solve this problem by developing ciphertext refreshing approach. The prover first decrypts  $\tilde{C}_s - C_s$  to v using sk, then generates a new ciphertext  $C_s^*$  of v under fresh randomness  $r^*$ , and proves that  $\tilde{C}_s - C_s$  and  $C_s^*$  do encrypt the same message under his public key. As we will see later, this can be efficiently done by a Sigma protocol using sk as witness. Now, the prover is able to prove that v encrypted by  $C_s^*$  lies in the right range by composing a Sigma protocol and a Bulletproof, via the same way as the first case. See Section 5.2.3 for the details.

The above range proofs provide two specialized "proof gadgets" for proving encrypted values lie in the right range. We refer to them as Gadget-1 and Gadget-2 hereafter. As we will see, all the NIZK proofs used in this work can be built from these two gadgets and simple Sigma protocols. Such modular design helps to reduce the footprint of overall cryptographic code, and have the potential to admit parallel proof generation/verification. We highlight the two "gadgets" are interesting on their own right as privacy-preserving tools, which may find applications in other domains as well, e.g., secure machine learning [TMZC17].

Additional results. Along the way of designing the zero-knowledge proofs for PGC, we obtain two interesting results. The first one is weak forking lemma, which is a relaxed version of standard forking lemma. Weak forking lemma provides a useful tool of rigorously proving argument of knowledge based on average-case hard assumption embedded in public parameters, for example, Bulletproof and inner product argument. See Theorem 2.3 for details. The second one is a simple trick of proving no-knowledge of discrete logarithm, which is an interesting complement of the classical protocol of proving knowledge of discrete logarithm. See the discussion in Section 6.3 for details.

**Enforcing regulation policies.** In the above, we have discussed how to employ Sigma protocols and Bulletproof to enforce the basic system-wide legality policy for transactions. In fact, we are able to enforce a variety of regulation polices that can be expressed as linear constraints over transfer amounts.

- Limit policy: let  $\mathsf{ctx}_1, \dots, \mathsf{ctx}_n$  be a collection of confidential transaction sent from/to a particular account pk, and let  $C_i$  be the corresponding encryptions of transfer amounts  $v_i$  under pk. This policy stipulates that  $\sum_{i=1}^n v_i \leq a_{\max}$ , where  $a_{\max}$  is an upper bound depending on application. The prover could be either the sender or the receiver of  $\mathsf{ctx}_i$ , who are not required to keep track of history randomness. To enforce this policy, the prover utilizes additive homomorphism to compute to compute  $C = \sum_{i=1}^n C_i$ , then uses Gadget-2 to generate a proof for compliance of this policy. The auditor can enforce limit policy anti-laundering money.
- Rate policy: let  $\operatorname{ctx}_1$  be an incoming transaction sent to pk,  $\operatorname{ctx}_2$  be an transaction sent from pk, in which  $C_1$  and  $C_2$  are the corresponding encryptions of transfer amounts  $v_1$  and  $v_2$  under pk. This policy stipulates  $v_1/v_2 = \rho$ , where  $\rho$  is an application-dependent rate. Without much loss of generality, we assume  $\rho = \alpha/\beta$ , where  $\alpha$  and  $\beta$  are two integers. To demonstrate this policy, the prover utilizes additive homomorphism to compute  $C_1' = \beta \cdot C_1$  and  $C_2' = \alpha \cdot C_2$ , then prove that  $C_1$  and  $C_2$  encrypt the same value under pk. The auditor can enforce rate policy to ensure taxpayer paid tax according to the rules.
- Open policy: let ctx be a confidential transaction from  $pk_s$  to  $pk_r$ , and  $C_u$  be the encryption of transfer amount under  $pk_u$ , where the subscript u could be either s or r. This policy stipulates that the underlying transfer amount is indeed v, where v is an application-dependent value. Either the sender or receiver of ctx prove this via a classic Sigma protocol for discrete logarithm equality, using his secret key sk as witness. This policy can be used to enforce selective-disclosure.

#### 1.4 Related Work

The seminal cryptocurrencies such as Bitcoin and Ethereum do not provide sufficient level of privacy. In the past years privacy-enhancements have been developed along several lines of research. We provide a brief overview below.

The first direction aims to provide confidentiality. Maxwell [Max] initiates the study of *confidential transaction*. He proposes a DCP system by employing Pedersen commitment to hide transfer amount

and using range proofs to prove correctness of transaction. Mimblewimble/Grin [Poe, Gri] further improve Maxwell's construction by reducing the cost of signatures. The second direction aims to enhance anonymity. A large body of works enhance anonymity via utilizing mixing mechanisms. For instance, Coinjoin [Max13], CoinShuffle [RMK14], TumbleBit [HAB+17], Dash [Das], and Mixcoin [BNM+14] for Bitcoin and Möbius [MM] for Ethereum. The third direction aims to attain both confidentiality and anonymity. Monero [Noe15] achieves confidentiality via similar techniques used in Maxwell's DCP system, and provides anonymity by employing linkable ring signature and stealth address. Zcash [ZCa] achieves strong privacy by equipping two types of addresses and leveraging key-private PKE and zk-SNARK.

Despite great progress in privacy-enhancement, the aforementioned schemes are not without their limitations. In terms of reliability, most of them require out-of-band transfer and thus are not complete. In terms of efficiency, some of them suffer from slow transaction generation due to the use of heavy advanced cryptographic tools. In terms of security, some of them are not proven secure based on well-studied assumption, or rely on trusted setup.

Another related work is zkLedger [NVV18], which offers strong transaction privacy and privacy-preserving auditing. To attain anonymity, zkLedger uses a novel table-based ledger in combination with the OR proof techniques. As a consequence, the transaction size is linear in the total number of users in the system, and the efficiency of auditing is not only determined by the complexity of policy, but may also depends on the number of total transactions, which could be prohibitively expensive. This makes zkLedger only suitable for a small scale of participants (e.g. consortium of banks). Besides, zkLedger does not present formal threat model and security proof for the whole system. For instance, the claimed privacy seems questionable due to the use of correlated randomnesses. In comparison, our work focuses on confidentiality. We present a generic construction of DCP system with formal threat model and security proof as well as an efficient instantiation. As a bonus of dropping anonymity, our DCP system enjoys good scalability. The transaction size depends on the number of participants rather than the total number of users in the system, and the efficiency of auditing depends only on the complexity of policies.

Concurrent and independent work. Fauzi et al. [FMMO19] put forward a new design of cryptocurrency with strong privacy called Quisquis in the UTXO model. They employ updatable public keys to achieve anonymity and a slight variant of standard ElGamal encryption to achieve confidentiality. Bünz et al. [BAZB20] propose a confidential payment system called Zether, which is compatible with Ethereum-like smart contract platforms. They use standard ElGamal to hide the balance and transfer amount, and sketch how to acquire anonymity using one-out-of-many proof. (Recently, Diamond [Dia21] points out that one-out-of-many proof does not lead to anonymity and proposes many-out-of-many proof instead.) Both Quisquis and Zether design accompanying zero-knowledge proofs from Sigma protocols and Bulletproof, but take different approaches to tackle the incompatibility between ElGamal encryption and Bulletproof. Quisquis introduces ElGamal commitment to bridge ElGamal encryption, and uses Sigma protocol to prove consistency of bridging. Finally, Quisquis invokes Bulletproof on the second part of ElGamal commitment, which is exactly a Pedersen commitment. Zether develops a custom ZKP called  $\Sigma$ -Bullets, which is a dedicated integration of Bulletproof and Sigma protocol. Given an arithmetic circuit,  $\Sigma$ -Bullets ensures that a public linear combination of the circuit's wires is equal to some witness of a Sigma protocol. This enhancement in turn enables proofs on algebraically-encoded values such as ElGamal encryptions or Pedersen commitments in different groups or using different commitment keys.

We highlight the following crucial differences of our work to Quisquis and Zether: (i) We focus on confidentiality, and trade anonymity for auditability. (ii) We use jointly secure ISE, rather than ad-hoc combination of signature and encryption, to build DCP system in a provably secure way. (iii) As for instantiation, PGC employs our newly introduced twisted ElGamal rather than standard ElGamal to hide balance and transfer amount. The nice structure of twisted ElGamal enables the sender to generate range proof for encrypted transfer amount by directly invoking Bulletproof in a black-box manner, without any extra bridging overhead as Quisquis. The final proof for transaction correctness is obtained by assembling small "proof gadgets" together in a simple and modular fashion, which is flexible and reusable. This is opposed to Zether's approach, in which zero-knowledge proof is produced by a  $\Sigma$ -Bullets as a whole, while building case-tailored  $\Sigma$ -Bullets requires to dissect Bulletproof and skillfully design its interface to Sigma protocol.

#### 1.5 Additions to Conference Version

Since the initial publication of this work in [CMTA20], twisted ElGamal and related zero-knowledge proofs have been used in the construction of several DCP systems, such as in MiniLedger [CB21] and SPL ZK-Token [Sol]. Compared to the conference version [CMTA20], we add the following contributions:

**Enhanced security notion.** We enhance the confidentiality notion for basic DCP by allowing the adversary to make reveal query. By a careful analysis, we demonstrate our efficient instantiation PGC already satisfies standard authenticity.

**Support global supervision.** We refine the concept of auditing by considering two types of audit, namely regulation compliance and global supervision. Particularly, we show how to leverage global escrow PKE to support global supervision in a generic and flexible manner. This work wins the first prize in the 2020 Financial Cryptography contest in China [Fin].

**Optimizations.** We optimize the code implementation of PGC, bringing roughly  $2 \times$  speed up.

# 2 Preliminaries

Basic Notations. Throughout the paper, we denote the security parameter by  $\lambda \in \mathbb{N}$ . A function is negligible in  $\lambda$ , written  $\operatorname{negl}(\lambda)$ , if it vanishes faster than the inverse of any polynomial in  $\lambda$ . Let  $X = \{X_{\lambda}\}_{{\lambda} \in \mathbb{N}}$  and  $Y = \{Y_{\lambda}\}_{{\lambda} \in \mathbb{N}}$  be two distribution ensembles indexed by  $\lambda$ . We say that X and Y are statistically indistinguishable, written  $X \approx_s Y$ , if the statistical distance between  $X_{\lambda}$  and  $Y_{\lambda}$  is negligible in  $\lambda$ . We say that X and Y are computationally indistinguishable, written  $X \approx_c Y$ , if the advantage of any PPT algorithm in distinguishing  $X_{\lambda}$  and  $Y_{\lambda}$  is  $\operatorname{negl}(\lambda)$ . A probabilistic polynomial time (PPT) algorithm is a randomized algorithm that runs in time  $\operatorname{poly}(\lambda)$ . If A is a randomized algorithm, we write  $z \leftarrow A(x_1, \ldots, x_n; r)$  to denote that A outputs z on inputs  $(x_1, \ldots, x_n)$  and randomness r. For notational clarity we usually omit r and write  $z \leftarrow A(x_1, \ldots, x_n)$ . For a positive integer n, we use [n] to denote the set of numbers  $\{1, \ldots, n\}$ . For a set X, we use |X| to denote its size and use  $x \stackrel{\mathbb{R}}{\leftarrow} X$  to denote sampling x uniformly at random from X. We use  $U_X$  to denote the uniform distribution over X.

Below, we review the cryptographic assumptions and primitives that will be used in this work.

#### 2.1 Cryptographic Assumptions

Let GroupGen be a PPT algorithm that on input a security parameter  $\lambda$ , outputs description of a cyclic group  $\mathbb{G}$  of prime order  $p = \Theta(2^{\lambda})$ , and a random generator g for  $\mathbb{G}$ . In what follows, we describe the discrete-logarithm based assumptions related to  $(\mathbb{G}, g, p) \leftarrow \mathsf{GroupGen}(1^{\lambda})$ .

**Definition 2.1** (Discrete Logarithm Assumption). The discrete logarithm assumption holds if for any PPT adversary, we have:

$$\Pr[\mathcal{A}(g,h) = a \text{ s.t. } g^a = h] \le \mathsf{negl}(\lambda),$$

where the probability is over the randomness of  $\mathsf{GroupGen}(1^{\lambda})$ ,  $\mathcal{A}$ 's random tape, and the random choice of  $h \stackrel{\mathbb{R}}{\leftarrow} \mathbb{G}$ .

**Definition 2.2** (Decisional Diffie-Hellman Assumption). The DDH assumption holds if for any PPT adversary, we have:

$$\left|\Pr[\mathcal{A}(g, g^a, g^b, g^{ab}) = 1] - \Pr[\mathcal{A}(g, g^a, g^b, g^c) = 1]\right| \le \mathsf{negl}(\lambda),$$

where the probability is over the randomness of  $\mathsf{GroupGen}(1^{\lambda})$ ,  $\mathcal{A}$ 's random tape, and the random choices of  $a, b, c \overset{\mathtt{R}}{\leftarrow} \mathbb{Z}_p$ .

**Definition 2.3** (Divisible Decisional Diffie-Hellman Assumption). The divisible DDH assumption holds if for any PPT adversary, we have:

$$\left|\Pr[\mathcal{A}(g,g^a,g^b,g^{a/b})=1] - \Pr[\mathcal{A}(g,g^a,g^b,g^c)=1]\right| \leq \mathsf{negl}(\lambda),$$

where the probability is over the randomness of  $\mathsf{GroupGen}(1^{\lambda})$ ,  $\mathcal{A}$ 's random tape, and the random choices of  $a, b, c \overset{\mathtt{R}}{\leftarrow} \mathbb{Z}_p$ .

As proved in [BDZ03], the divisible DDH assumption is equivalent to the standard DDH assumption.

#### 2.2 Commitments

A non-interactive commitment scheme is a two-party protocol between a sender and a receiver with two stages. At the committing stage, the sender commits to some value m by sending a commitment to the receiver. At the opening stage, the sender can open the commitment by providing m and some auxiliary information, by which the receiver can verify that the value it received is indeed the value committed by the sender during the committing stage. Formally, a commitment scheme consists of three polynomial time algorithms as below:

- Setup( $1^{\lambda}$ ): on input a security parameter  $\lambda$ , output public parameters pp. We assume that pp includes the descriptions of message space M, randomness space R, and commitment space C. pp will be used as implicit input of the following two algorithms.
- $\mathsf{Com}(m;r)$ : the sender commits a message m by choosing uniform random coins r, and computing  $c \leftarrow \mathsf{Com}(m;r)$ , then sends c to receiver.
- Open(c, m, r): the sender can later decommit c by sending m, r to the receiver; the receiver outputs  $\mathsf{Com}(m; r) \stackrel{?}{=} c$ .

**Correctness.** For all  $pp \leftarrow \mathsf{Setup}(1^{\lambda})$ , all  $m \in M$  and all  $r \in R$ , we have  $\mathsf{Open}(\mathsf{Com}(m;r), m, r) = 1$ . For security, we require hiding and binding.

**Hiding.** A commitment Com(m; r) should not reveal anything about its committed value of m. Let  $\mathcal{A}$  be an adversary against hiding, we define its advantage via the following experiment:

$$\mathsf{Adv}_{\mathcal{A}}(\lambda) = \Pr \left[ \begin{matrix} pp \leftarrow \mathsf{Setup}(\lambda); \\ \beta' = \beta: & (m_0, m_1) \leftarrow \mathcal{A}_1(pp); \\ \beta \xleftarrow{\leftarrow} \{0, 1\}, r \xleftarrow{\leftarrow} R, c \leftarrow \mathsf{Com}(m_\beta; r); \\ \beta' \leftarrow \mathcal{A}_2(c); \end{matrix} \right] - \frac{1}{2}.$$

A commitment scheme is perfectly hiding if  $\mathsf{Adv}_{\mathcal{A}}(\lambda) = 0$  even for unbounded adversary, is statistical hiding (resp. computational hiding) if  $\mathsf{Adv}_{\mathcal{A}}(\lambda) = \mathsf{negl}(\lambda)$  w.r.t. unbounded adversary (resp. PPT adversary).

**Binding.** A commitment c cannot be opened to two different messages. Let  $\mathcal{A}$  be an adversary against binding, we define its advantage via the following experiment:

$$\mathsf{Adv}_{\mathcal{A}}(\lambda) = \Pr \left[ \begin{array}{c} m_0 \neq m_1 \wedge \\ c = \mathsf{Com}(m_0; r_0) = \mathsf{Com}(m_1; r_1) \end{array} \right. \colon \begin{array}{c} pp \leftarrow \mathsf{Setup}(\lambda); \\ (c, m_0, r_0, m_1, r_1) \leftarrow \mathcal{A}(pp); \end{array} \right].$$

A commitment scheme is perfectly binding if  $\mathsf{Adv}_{\mathcal{A}}(\lambda) = 0$  even for unbounded adversary (a.k.a.  $\forall m_0 \neq m_1$ , their commitment values are disjoint), is statistical binding (resp. computational binding) if  $\mathsf{Adv}_{\mathcal{A}}(\lambda) = \mathsf{negl}(\lambda)$  w.r.t. unbounded adversary (resp. PPT adversary).

Pedersen Commitment. Below we recall the Pedersen commitment [Ped91]:

- Setup(1 $^{\lambda}$ ): run ( $\mathbb{G}, p, g$ )  $\leftarrow$  GroupGen(1 $^{\lambda}$ ), pick  $h \stackrel{\mathbb{R}}{\leftarrow} \mathbb{G}^*$ , then output  $pp = (\mathbb{G}, p, g, h)$ . Here,  $M = R = \mathbb{Z}_p$ ,  $C = \mathbb{G}$ .
- $\mathsf{Com}(m;r)$ : on input message  $m \in \mathbb{Z}_p$  and randomness  $r \overset{\mathtt{R}}{\leftarrow} \mathbb{Z}_p$ , output  $c \leftarrow g^m h^r$ .
- $\mathsf{Open}(c,m,r)$ : output "1" if  $c=g^mh^r$  and "0" otherwise.

The Pedersen commitment is perfectly hiding and computational binding under the discrete logarithm assumption.

# 2.3 Public-Key Encryption

A PKE scheme consists of four polynomial time algorithms as follows.

- Setup( $1^{\lambda}$ ): on input a security parameter  $\lambda$ , output public parameters pp. We assume pp also includes the descriptions of message space M, ciphertext space C, and randomness space R.
- KeyGen(pp): on input pp, output a public key pk and a secret key sk.
- $\mathsf{Enc}(pk, m)$ : on input a public key pk and a plaintext m, output a ciphertext c. When emphasizing the randomness r used for encryption, we denote this by  $c \leftarrow \mathsf{Enc}(pk, m; r)$ .
- $\mathsf{Dec}(sk,c)$ : on input a secret key sk and a ciphertext c, output a plaintext m or a distinguished symbol  $\bot$  indicating that c is invalid.

**Correctness.** For all  $pp \leftarrow \mathsf{Setup}(1^{\lambda})$ , all  $(pk, sk) \leftarrow \mathsf{KeyGen}(pp)$  and all  $m \in M$  (here M is the message space), we have  $\mathsf{Dec}(sk, \mathsf{Enc}(pk, m)) = m$ .

**Homomorphism.** For any public key pk, any  $(m_1, r_1), (m_2, r_2) \in M \times R$ , we have  $\mathsf{Enc}(pk, m_1; r_1) + \mathsf{Enc}(pk, m_2; r_2) = \mathsf{Enc}(pk, m_1 + m_2; r_1 + r_2)$ . Here, we slight abuse the symbol "+" to denote componentwise operation over space  $C \times C$ . Additive homomorphism on message already suffices for most applications:  $\mathsf{Enc}(pk, m_1) + \mathsf{Enc}(pk, m_2)$  is an encryption of  $(m_1 + m_2)$  of pk under some appropriate randomness. When both ciphertext space and message space are finite cyclic groups, we can naturally define scalar multiplication, which is a special case of additive homomorphism: for all  $k \in \mathbb{Z}_p$ ,  $k \cdot \mathsf{Enc}(pk, m)$  is an encryption of km under pk.

# 2.4 Signature

A signature scheme consists of four polynomial time algorithms as follows.

- Setup( $1^{\lambda}$ ): on input a security parameter  $\lambda$ , output public parameters pp. We assume pp also includes the descriptions of message space M, signature space  $\Sigma$ , and randomness space R.
- KeyGen(pp): on input pp, output a public key vk and a secret key sk.
- Sign(sk, m): on input sk and a message m, output a signature  $\sigma$ .
- Verify $(pk, m, \sigma)$ : on input pk, a message m, and a purported signature  $\sigma$ , output "1" indicating acceptance or "0" indicating rejection.

**Correctness.** For all  $pp \leftarrow \mathsf{Setup}(1^{\lambda})$ , all  $(vk, sk) \leftarrow \mathsf{KeyGen}(pp)$  and all  $m \in M$  (here M is the message space), it holds that  $\mathsf{Verify}(pk, m, \mathsf{Sign}(sk, m)) = 1$ .

# 2.5 Integrated Signature and Encryption Scheme

Haber and Pinkas [HP01] introduced the concept of combined public key (CPK) scheme, which is a combination of a signature scheme (Setup, KeyGen, Sign, Verify) and encryption scheme (Setup, KeyGen, Enc, Dec): the existing Sign, Verify, Enc, Dec algorithms are preserved, while the two sets of Setup, KeyGen algorithms are combined into a single one. Paterson et al. [PSST11] revisited this topic and gave a generic construction of combined public key scheme from identity-based encryption.

In this work, we use a special case of combined public key scheme which we refer to as integrated signature and encryption scheme (ISE). Compared to general CPK, ISE stipulates that the keypair is non-splittable, i.e., the keypair cannot be divided into two parts so that one is used for the encryption component and the other one is used for the signature component. This requirement excludes the naive Cartesian product construction of CPK. ISE uses the same keypair for both signing and encryption, thus the two operations may interfere with one another, in such a way to undermine the security of one or both of the components. For this reason, joint security must be considered when ISE is used, which captures potentially dangerous interactions. Please refer to [PSST11] for more detailed discussion on this point.

In the context of our DCP construction, the joint security of ISE stipulates that the PKE component is IND-CPA secure in the single-plaintext/two-recipient setting even in the presence of a signing oracle, while the signature component is sEUF-CMA secure even in the presence of two encryption oracles. Note that in the public key setting the adversary can always perfectly simulate encryption oracles itself, thus standard sEUF-CMA security implies that the signature component is secure in the joint sense and we only need to enhance the IND-CPA security for the PKE component. Let ISE = (Setup, KeyGen, Sign, Verify, Enc, Dec). We formally define the case-tailored joint security model used in this work.

**Definition 2.4** (Joint Security for ISE). We say an ISE is jointly secure (case-tailored for DCP setting) if its encryption and signature components satisfy the following security notions respectively.

IND-CPA security (1-plaintext/2-recipient) in the presence of a signing oracle. Let  $\mathcal{A}$  be an adversary against the PKE component and define its advantage in the following experiment:

Here,  $\mathcal{O}_{\text{sign}}$  provides unlimited access to signing oracle with respect to  $sk_1$  and  $sk_2$ . More precisely,  $\mathcal{O}_{\text{sign}}$  returns  $\mathsf{Sign}(sk_i, m)$  on input  $i \in \{1, 2\}$  and  $m \in M$ . The encryption component is IND-CPA secure in 1-plaintext/2-recipient setting if no PPT adversary  $\mathcal{A}$  has non-negligible advantage in the above security experiment.

**sEUF-CMA** security. The security requirement for the signature component is exactly the standard sEUF-CMA security. Let  $\mathcal{A}$  be an adversary against the signature component and define its advantage in the following experiment:

$$\mathsf{Adv}_{\mathcal{A}}(\lambda) = \Pr \left[ \begin{array}{cc} \mathsf{Verify}(pk, m^*, \sigma^*) = 1 & pp \leftarrow \mathsf{Setup}(\lambda); \\ \wedge (m^*, \sigma^*) \notin \mathcal{Q} & : & (pk, sk) \leftarrow \mathsf{KeyGen}(pp); \\ & (m^*, \sigma^*) \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{sign}}}(pp, pk); \end{array} \right].$$

The set  $\mathcal{Q}$  records the historical message/signature pairs obtained via  $\mathcal{O}_{\mathsf{sign}}$ , which returns  $\sigma \leftarrow \mathsf{Sign}(sk, m)$  on input m. The signature component is sEUF-CMA if no PPT adversary  $\mathcal{A}$  has non-negligible advantage in the above security experiment.

# 2.6 Zero-Knowledge Protocols

We first recall the definition of interactive proof systems.

**Definition 2.5** (Interactive Proof System). An interactive proof system is a two party protocol in which a prover can convince a verifier that some statement is true without revealing any knowledge about why it holds. Formally, it consists of three PPT algorithms (Setup, P, V) as below.

• The Setup algorithm on input  $1^{\lambda}$ , outputs public parameters pp. Let  $\mathsf{R}_{pp} \subseteq X \times W$  be an  $\mathcal{NP}$  relation indexed by pp. We say  $w \in W$  is a witness for a statement x iff  $(x, w) \in \mathsf{R}_{pp}$ .  $\mathsf{R}_{pp}$  naturally defines a family of public-parameters-dependent  $\mathcal{NP}$  languages:

$$L_{pp} = \{x \mid \exists w \in W \text{ s.t. } (x, w) \in \mathsf{R}_{pp}\}$$

From now on, we will drop the subscript pp occasionally when the context is clear.

• P and V are a pair of interactive algorithms, which both take pp as implicit input and the statement x as common input. We use the notation  $tr \leftarrow \langle P(x), V(y) \rangle$  to denote the transcript of an execution between P and V, where P has input x and V has input y. We write  $\langle P(x), V(y) \rangle = b$  depending on whether V accepts, b = 1, or rejects, b = 0. When the context is clear, we will also slightly abuse the notation of  $\langle P(x), V(y) \rangle$  to denote V's view (View $_V$ ) in the interaction, which consists of V's input tape, random tape and incoming messages sent by P.

An interactive proof system satisfies the following two properties:

**Completeness.** For any  $(x, w) \in \mathsf{R}_{pp}$  where  $pp \leftarrow \mathsf{Setup}(1^{\lambda})$ , we have:

$$\Pr[\langle P(x, w), V(x) \rangle = 1] = 1$$

**Statistical soundness.** For any  $x \notin L_{pp}$  where  $pp \leftarrow \mathsf{Setup}(1^{\lambda})$ , any cheating prover  $P^*$ , we have:

$$\Pr[\langle P^*(x), V(x) \rangle = 1] \le \mathsf{negl}(\lambda)$$

If we restrict  $P^*$  to be a PPT cheating prover in the above definition, we obtain an interactive argument system.

**Public coin.** An interactive proof/argument system is public-coin if all messages sent from V are chosen uniformly at random and independent of P's message.

We then recall several zero-knowledge extensions of interactive zero knowledge proof systems that will be used in this work.

**Definition 2.6** (Zero-Knowledge Argument of Knowledge). We say an interactive proof system is a zero-knowledge argument of knowledge if it satisfies standard completeness, argument of knowledge and zero knowledge.

Following [BCC<sup>+</sup>16], we use computational witness-extended emulation to define argument of knowledge. Intuitively, this definition says that whenever a malicious prover produces an accepting argument with some probability, there exists an emulator producing a similar argument with roughly the same probability together with a witness.

Computational witness-extended emulation. For all deterministic polynomial time  $P^*$  there exists an expected PPT emulator E such that for all PPT interactive adversaries  $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ 

$$\Pr\left[\begin{array}{l} pp \leftarrow \mathsf{Setup}(\lambda); \\ (x,w) \leftarrow \mathcal{A}_1(pp); \\ tr \leftarrow \langle P^*(x,w), V(x) \rangle; \\ tr \text{ is accepting}; \\ \mathcal{A}_2(tr) = 1 \end{array}\right] - \Pr\left[\begin{array}{l} pp \leftarrow \mathsf{Setup}(\lambda); \\ (x,w) \leftarrow \mathcal{A}_1(pp); \\ (tr,w') \leftarrow E^{\mathcal{O}}(x); \\ (x,w') \in \mathsf{R}_{pp}; \\ \mathcal{A}_2(tr) = 1 \end{array}\right] \leq \mathsf{negl}(\lambda)$$

where the oracle  $\mathcal{O} = \langle P^*(x, w), V(x) \rangle$  permits rewinding to a specific point and resuming with fresh randomness for the verifier from this point onwards. In the definition, s is interpreted as the state of  $P^*$ , including the randomness and auxiliary input. According to the definition, whenever  $P^*$  makes a convincing argument in state s, E can extract a witness. This is why it defines argument of knowledge.

Computational zero-knowledge. For any malicious PPT  $V^*$  there exists an expected PPT simulator S such that for any  $pp \leftarrow \mathsf{Setup}(1^{\lambda})$  and  $(x, w) \in \mathsf{R}_{pp}$ , we have:

$$\langle P(x,w), V^*(x) \rangle \approx_c \mathcal{S}(x)$$

Computational zero-knowledge can be strengthened to statistical (resp. perfect) zero-knowledge by requiring the views are statistically indistinguishable (resp. identical).

**Definition 2.7** (Sigma Protocol ( $\Sigma$ -protocol) [Dam]). An interactive proof system is called a Sigma protocol if it follows the following communication pattern (3-round public-coin):

- 1. (Commit) P sends a first message a to V;
- 2. (Challenge) V sends a random challenge e to P;
- 3. (Response) P replies with a second message z.

and satisfies standard completeness and the variants of soundness and zero-knowledge as below:

**Special soundness.** For any x and any pair of accepting transcripts (a, e, z), (a, e', z') with  $e \neq e'$ , there exists a PPT extractor outputs a witness w for x.

**Special honest-verifier zero-knowledge (SHVZK).** There exists a PPT simulator S such that for any  $(x, w) \in R_{pp}$  and randomness e, we have:

$$\langle P(x, w), V(x, e) \rangle \equiv \mathcal{S}(x, e)$$

**Definition 2.8** (Range Proof). For a commitment scheme over total ordering message space M and randomness space R, a range proof is a zero-knowledge argument of knowledge for the following language:

$$L = \{c \mid \exists m \in M, r \in R \text{ s.t. } c = \mathsf{Com}(m; r) \land x \in [a, b]\}$$

Interaction is typically expensive or impossible in some cases. Thus, removing interaction is of particular interest in practice. However, non-interactive zero-knowledge (NIZK) proofs for non-trivial languages are impossible in the plain model. We need to consider NIZK in the common reference string (CRS) model, wherein a string is generated after setup phase, and made available to everyone to prove/verify statement.

**Definition 2.9** (Non-Interactive Zero-Knowledge Proof [BFM88, FLS90]). A NIZK proof system in the CRS model consists of four PPT algorithms (Setup, CRSGen, P, V):

- Setup(1 $^{\lambda}$ ): same as that of ordinary zero-knowledge proof system, which on input a security parameter  $\lambda$ , output public parameters pp.
- CRSGen(pp): on input pp, output a common reference string crs.
- P(crs, x, w): on input crs and a statement-witness pair  $(x, w) \in \mathsf{R}_{pp}$ , output a proof  $\pi$ . We also denote this process by  $\pi \leftarrow \mathsf{Prove}(crs, x, w)$ .
- $V(crs, x, \pi)$ : on input crs, a statement x, and a proof  $\pi$ , outputs "0" if rejects and "1" if accepts. We also denote this process by  $b \leftarrow \mathsf{Verify}(crs, x, \pi)$ .

A NIZK proof system in the CRS model satisfies the following requirements:

Completeness. For any  $(x, w) \in \mathsf{R}_{pp}$  where  $pp \leftarrow \mathsf{Setup}(1^{\lambda})$ ,

$$\Pr\left[V(crs,x,\pi) = 1: \begin{array}{c} crs \leftarrow \mathsf{CRSGen}(pp); \\ \pi \leftarrow P(crs,x,w); \end{array}\right] = 1.$$

Statistical soundness. For any cheating prover  $P^*$ ,

$$\Pr = \left[ \begin{array}{cc} x \notin L \land & pp \leftarrow \mathsf{Setup}(1^\lambda); \\ V(crs, x, \pi) = 1 & : crs \leftarrow \mathsf{CRSGen}(pp); \\ (x, \pi) \leftarrow P^*(crs); \end{array} \right] \leq \mathsf{negl}(\lambda).$$

This definition is in the adaptive sense in that  $P^*$  may choose x after seeing crs. Statistical soundness can be relaxed to computational soundness by restricting  $P^*$  to be a PPT algorithm. In this case, a proof system degenerates to an argument system.

Computational zero-knowledge. For any  $pp \leftarrow \mathsf{Setup}(1^{\lambda})$ , any PPT adversary  $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ , there exists a PPT simulator  $\mathcal{S} = (\mathcal{S}_1, \mathcal{S}_2)$  such that:

$$\Pr\left[\begin{array}{c} crs \leftarrow \mathsf{CRSGen}(pp); \\ (x,w) \leftarrow \mathcal{A}_1(crs); \\ \pi \leftarrow P(crs,x,w); \\ \mathcal{A}_2(crs,x,\pi) = 1 \end{array}\right] - \Pr\left[\begin{array}{c} (crs,\tau) \leftarrow \mathcal{S}_1(pp); \\ (x,w) \leftarrow \mathcal{A}_1(crs); \\ \pi \leftarrow \mathcal{S}_2(crs,x,\tau); \\ \mathcal{A}_2(crs,x,\pi) = 1 \end{array}\right] \leq \mathsf{negl}(\lambda).$$

This definition is also in the adaptive sense in that  $A_1$  may adaptively choose x after seeing crs. Computational zero-knowledge can be strengthened to statistical zero-knowledge by requiring the above holds even for unbounded A.

Interestingly, in the random oracle model NIZK is possible without explicitly relying on common reference string. The celebrated Fiat-Shamir transform [FS86] shows how to compile a Sigma protocol into a NIZK by modeling cryptographic hash function as random oracle. It not only removes interaction, but also strengthens honest-verifier zero-knowledge to full zero-knowledge (against malicious verifiers). Fiat-Shamir transform actually applies to any public-coin SHVZK argument of knowledge. Formally, we have the following theorem.

**Theorem 2.1** (Fiat-Shamir Transform [BR93, FKMV12]). Let (Setup, P, V) be a (2k+1)-move public-coin SHVZK argument of knowledge, x be the statement,  $a_i$  be prover's ith round message and  $e_i$  be verifier's ith round challenge, H is a hash function whose range equal to verifier's challenge space. By setting  $e_i = H(a_1, \ldots, a_i)$  in (Setup, P, V), we obtain (Setup, P, V), which is a NIZK assuming H is a random oracle.

General Forking Lemma. We recall the general forking lemma of [BCC+16, BBB+18] that will be used in our proofs. Suppose that we have a (2k+1)-move public-coin argument with k challenges,  $e_1, \ldots, e_k$  in sequence. Let  $n_i \geq 1$  for  $i \in [k]$ . Consider  $\prod_{i=1}^k n_i$  accepting transcripts whose challenges satisfying the following tree format. The tree has depth k and  $\prod_{i=1}^k n_i$  leaves. The root of the tree is labeled with the statement. Each node of depth i < k has exactly  $n_i$  children, each labeled with a distinct value of the ith challenge  $e_i$ . This can be referred to as an  $(n_1, \ldots, n_k)$ -tree of accepting transcripts, which is a natural generalization of special soundness for Sigma protocols where k = 1 and n = 2. For the simplicity in the following lemma, we assume that the challenge space is  $\mathbb{Z}_p$  and  $p = \Theta(2^{\lambda})$ .

**Theorem 2.2** (Forking Lemma [BCC<sup>+</sup>16, BBB<sup>+</sup>18]). Let (Setup, P, V) be a (2k+1)-move, public-coin interactive protocol. Let E be a PPT witness extraction algorithm that succeeds with probability  $1 - \mu(\lambda)$  for some negligible function  $\mu(\lambda)$  in extracting a witness from an  $(n_1, \ldots, n_k)$ -tree of accepting transcripts. If  $\prod_{k=1}^k n_i$  is bounded by a polynomial in  $\lambda$ , then (Setup, P, V) has witness-extended emulation.

Forking lemma is a useful tool to prove witness-extended emulation, a.k.a. proof of knowledge. We note that the condition of standard forking lemma is required to hold regardlessly of the distributions of public parameters and instance. Such a strong form of condition is impossible to achieve in the computational setting when we need to argue that the output of extractor is indeed the witness based on average-case assumptions over public parameters. For example, the cases of inner product argument and Bulletproof [BBB+18]. We are thus motivated to introduce a weaker form of forking lemma by relaxing the condition, which still suffices to imply computational witness-extended emulation. In what follows, we formulate weak forking lemma by capturing the relaxed condition via an interactive game.

**Theorem 2.3** (Weak Forking Lemma). Let (Setup, P, V) be a (2k + 1)-move, public-coin interactive protocol. Let E be a PPT witness extraction algorithm that succeeds with overwhelming probability in extracting a witness from an  $(n_1, \ldots, n_k)$ -tree of accepting transcripts in the following game:

$$\Pr\left[\begin{array}{c} pp \leftarrow \mathsf{Setup}(1^{\lambda}); \\ x \leftarrow \mathcal{A}(crs); \\ w \leftarrow E(crs, x, tr); \\ tr \text{ is accepting } \Rightarrow (x, w) \in \mathsf{R}_{pp} \end{array}\right] \geq 1 - \mathsf{negl}(\lambda)$$

If such E exists for any PPT adversary  $\mathcal{A}$  and  $\Pi_{i=1}^k n_i$  is bounded by a polynomial in  $\lambda$ , then (Setup, P, V) has computational witness-extended emulation.

# 3 Definition of Auditable Decentralized Confidential Payment System

We formalize the notion of auditable decentralized confidential payment system in account-based model, adapting the notion of decentralized anonymous payment system [ZCa].

#### 3.1 Data Structures

We begin by describing the data structures used by an auditable DCP system.

**Blockchain.** An auditable DCP system operates on top of a blockchain B. The blockchain is publicly accessible, i.e., at any given time t, all users have access to  $B_t$ , the ledger at time t, which is a sequence of transactions. The blockchain is also append-only, i.e., t < t' implies that  $B_t$  is a prefix of  $B_{t'}$ .

 $<sup>^7\</sup>mathrm{To}$  get a unifying syntax of NIZK, one can also interpret the description of H as common reference string.

**Public parameters.** A trusted party generate public parameters pp at the setup time of system, which is used by system's algorithms. We assume that pp always include an integer  $v_{\text{max}}$ , which specifies the maximum possible number of coins that the system can handle. Any balance and transfer below must lie in the integer interval  $\mathcal{V} = [0, v_{\text{max}}]$ .

**Account.** Each account is associated with a keypair (pk, sk), an encoded balance  $\tilde{C}$  (which encodes plaintext balance  $\tilde{v}$ ), as well as an incremental serial number  $\mathsf{sn}$  (used to prevent replay attacks). Both  $\mathsf{sn}$ ,  $\tilde{C}$ , and pk are made public. The public key pk serves as account address, which is used to receive transactions from other accounts. The secret key sk is kept privately, which is used to direct transactions to other accounts and decodes encoded balance.

Confidential transaction. A confidential transaction ctx consists of three parts, i.e., sn, memo and aux. Here, sn is the current serial number of sender account  $pk_s$ , memo =  $(pk_s, pk_r, C)$  records basic information of a transaction from sender account  $pk_s$  to receiver account  $pk_r$ , where C is the encoding of transfer amount, and aux denotes the auxiliary information, which is application-dependent.

**Policies.** Except system-wide legality policy that all transaction must satisfy, transactions under auditing should also been proved to comply with application-dependent regulation policies. Observe that the value-encoding tuples naturally constitute polynomially decidable relations, policies over transfer amounts and balances can thus be expressed by  $\mathcal{NP}$  relations. Looking ahead, this allow participants to prove compliance with specified policies by leveraging NIZK.

We formally capture polices as predicates over public key and related transactions. Let  $\{\mathsf{ctx}_i\}_{i=0}^n$  be a set of confidential transactions related to pk, i.e., for each  $\mathsf{ctx}_i, pk$  is either the sender or the receiver, and  $v_i$  be the transfer amount of  $\mathsf{ctx}_i$ . A policy over  $\{v_i\}_{i=1}^n$  is satisfied if and only if  $f(pk, \{\mathsf{ctx}_i\}_{i=0}^n) = 1$ . The basic correctness policy for a single transaction in any payment system requires the transfer amount lies in the correct range and the sender account is solvent, we denote the associated predicate as  $f_{\text{legal}}(pk, \mathsf{ctx})$ . We list more useful policies as below: (i) limit policy  $-\sum_i^n v_i \leq a_{\text{max}}$ , the associated predicate is  $f_{\text{limit}}(pk, \{\mathsf{ctx}_i\}_{i=1}^n)$ ; (i) rate policy  $-v_1/v_2 = \rho$ , the associated predicate is  $f_{\text{rate}}(pk, (\mathsf{ctx}_1, \mathsf{ctx}_2))$ ; (i) open policy  $-v = v^*$ , the associated predicate is  $f_{\text{open}}(pk, \mathsf{ctx})$ .

#### 3.2 Entities

In an ADCP system, there are the following types of entities:

Users: each user may control several accounts.

Validator: checking the validity of proposed transactions.

**Regulator:** checking if a given set of transactions satisfies regulation policies by interacting with involved users. Typically, regulators are authorities in the real world, and do not hold any secret,

**Supervisor:** inspecting any confidential transaction without interaction with involved users. Typically, supervisor owns a global trapdoor.

## 3.3 Auditable Decentralized Confidential Payment System

An auditable DCP system is a tuple of polynomial-time algorithms defined as below:

- Setup( $\lambda$ ): on input a security parameter  $\lambda$ , output public parameter pp and possibly an associated secret parameter sp. A trusted party executes this algorithm once-for-all to setup the whole system. pp will be used as an implicit input in the rest algorithms.
- CreateAccount( $\tilde{v}$ , sn): on input an initial balance  $\tilde{v}$  and a serial number sn, output a keypair (pk, sk) and an encoded balance  $\tilde{C}$ . A user runs this algorithm to create an account.
- RevealBalance $(sk, \hat{C})$ : on input a secret key sk and an encoded balance  $\hat{C}$ , output the balance  $\tilde{v}$  in plaintext. A user runs this algorithm to reveal the balance.
- CreateCTx $(sk_s, pk_s, pk_r, v)$ : on input a keypair  $(sk_s, pk_s)$  of sender account, a receiver account address  $pk_r$ , and a transfer amount v, output a confidential transaction ctx. A user runs this algorithm to transfer v coins from account  $pk_s$  to account  $pk_r$ .

- VerifyCTx(ctx): on input a confidential transaction ctx, output "0" denotes valid and "1" denotes invalid. Validators run this algorithm to check the validity of proposed confidential transaction ctx. If ctx is valid, it will be recorded on the blockchain B. Otherwise, it is discarded.
- UpdateCTx(ctx): for each fresh ctx appearing on the blockchain B, the corresponding sender and receiver update their encoded balances to reflect the change, i.e., the sender account decreases with v coins while the receiver account increases with v coins.
- JustifyCTx(pk, sk, {ctx}, f): on input a user's keypair (pk, sk), a set of confidential transactions he participated and a policy f, output a proof  $\pi$  for  $f(pk, \{\text{ctx}\}) = 1$ . A user runs this algorithm to generate a proof for auditing.
- AuditCTx(pk, {ctx}, f,  $\pi$ ): on input a user's public key, a set of confidential transactions he participated, a policy f and a proof, output "0" denotes accept and "1" denotes reject. A regulator runs this algorithm to check if  $f(pk, \{\text{ctx}\}) = 1$ .
- OpenCTx(sp, ctx): on input secret parameter sp, output the transaction amount of ctx. A supervisor runs this algorithm to inspect confidential transactions.

#### 3.4 Correctness

Correctness of basic DCP functionality requires that a valid ctx will always be accepted and recorded on the blockchain, and the states of associated accounts will be updated properly, i.e., the balance of sender account decreases the same amount as the balance of receiver account increases. Correctness of regulation compliance requires honestly generated proofs for transactions complying with policies will always be accepted. Correctness of supervision requires that for any honestly generated transactions the views of supervisor and intended receiver are identical.

## 3.5 Security Model

Following the treatment of Quisquis [FMMO19], we focus solely on the *transaction layer* of a cryptocurrency, and assume network-level or consensus-level attacks are out of scope.

Intuitively, an auditable DCP system should provide authenticity, confidentiality and soundness as standard DCP system, and also support secure auditing. Authenticity requires that the sender can only be the owner of an account, nobody else (who does not know the secret key) is able to make a transfer from this account. Confidentiality requires that other than the sender and receiver (who does not know the secret keys of sender and receiver), no one can learn the value hidden in a confidential transaction. While the former two notions address security against outsider adversary, soundness addresses security against insider adversary (e.g. the sender himself). It requires that no PPT adversary is able to generate a  $\mathsf{ctx} = (\mathsf{sn}, \mathsf{memo}, \mathsf{aux})$  such that  $\mathsf{VerifyCTx}(\mathsf{ctx}) = 1$  but  $\mathsf{memo}$  does not satisfy  $f_{\mathsf{legal}}$ , even it knows the secret key of sender. Secure auditing requires the audit is sound and zero-knowledge.

We formalize the notions of authenticity, confidentiality and soundness via security experiments. Let  $\mathcal{A}$  be an adversary attacking a DCP system. We assume that  $\mathcal{A}$  can not only induce honest parties to perform DCP operations, but can also corrupt some honest parties. Formally, we capture attack behaviors as adversarial queries to oracles implemented by a challenger  $\mathcal{CH}$ . We list the oracles available to the adversary as below.

- $\mathcal{O}_{\mathsf{regH}}$ :  $\mathcal{A}$  queries this oracle for registering an honest account.  $\mathcal{CH}$  keeps track of this type of queries by maintaining a list  $T_{\mathsf{honest}}$ , which is initially empty. Upon receiving a fresh query,  $\mathcal{CH}$  responds as below: picks a random balance  $\tilde{v}$  and a serial number  $\mathsf{sn}$ , runs  $\mathsf{CreateAccount}(\tilde{v},\mathsf{sn})$  to obtain (pk,sk) and  $\tilde{C}$ .  $\mathcal{CH}$  records  $(pk,sk,\tilde{C},\tilde{v},\mathsf{sn})$  in  $T_{\mathsf{honest}}$ , then returns  $(pk,\tilde{C},\mathsf{sn})$  to  $\mathcal{A}$ . This oracle captures that  $\mathcal{A}$  can observe the public information of honest accounts in the system. Hereafter, let  $Q_{\mathsf{honest}}$  be the maximum number of honest account registration queries that  $\mathcal{A}$  makes.
- $\mathcal{O}_{\text{regC}}$ :  $\mathcal{A}$  queries this oracle with a public key pk, an initial encoded balance  $\tilde{C}$  as well as an initial serial number  $\mathsf{sn}$ .  $\mathcal{CH}$  keeps track of this type of queries by maintaining a list  $T_{\text{corrupt}}$ , which is initially empty. Upon receiving a fresh query,  $\mathcal{CH}$  records  $(pk, \perp, \tilde{C}, \perp, \mathsf{sn})$  in  $T_{\text{corrupt}}$ . We stress that pk and  $\tilde{C}$  submitted by  $\mathcal{A}$  may not be honestly generated, and pk is not allowed to be the one in  $T_{\text{honest}}$ . This oracle captures that  $\mathcal{A}$  can fully control some accounts in the system.

- $\mathcal{O}_{\text{extH}}$ :  $\mathcal{A}$  queries this oracle with a public key pk that was registered as an honest account, a.k.a. in  $T_{\text{honest}}$ .  $\mathcal{CH}$  returns the associated sk to  $\mathcal{A}$  and moves this entry to  $T_{\text{corrupt}}$ . This oracle captures that  $\mathcal{A}$  can corrupt some honest accounts.
- $\mathcal{O}_{\text{trans}}$ :  $\mathcal{A}$  queries this oracle with  $(pk_s, pk_r, v)$  to conduct a confidential transaction, subject to the restriction that  $pk_s \in T_{\text{honest}}$ . This restriction is natural because for  $pk_s \in T_{\text{corrupt}}$ ,  $\mathcal{A}$  can generate the confidential transaction itself.  $\mathcal{CH}$  keeps track of this type of queries by maintaining a list  $T_{\text{ctx}}$ , which is initially empty. Upon receiving a fresh query,  $\mathcal{CH}$  responds as below: If v does not constitute a valid transaction originated from  $pk_s$  with balance  $\tilde{v}_s$ , i.e.,  $v \notin \mathcal{V}$  or  $(\tilde{v}_s v) \notin \mathcal{V}$ ,  $\mathcal{CH}$  returns  $\bot$ . Else,  $\mathcal{CH}$  runs  $\text{ctx} \leftarrow \text{CreateCTx}(sk_s, pk_s, pk_r, v)$ , updates the state of associated accounts, records (ctx, v) to  $T_{\text{ctx}}$ , then sends ctx to  $\mathcal{A}$ . This oracle captures that  $\mathcal{A}$  can instruct any honest account to generate specific transactions as its will.
- $\mathcal{O}_{\text{reveal}}$ :  $\mathcal{A}$  queries this oracle with ctx to reveal a confidential transaction, subject to the restriction that  $pk_s, pk_r \in T_{\text{honest}}$ . This restriction is natural because if either  $pk_s \in T_{\text{corrupt}}$  or  $pk_r \in T_{\text{corrupt}}$ ,  $\mathcal{A}$  can open the confidential transaction itself. Upon receiving a fresh query,  $\mathcal{CH}$  first checks if ctx is valid. If not,  $\mathcal{CH}$  returns  $\bot$ . Else,  $\mathcal{CH}$  decrypts the memo part using either  $sk_s$  or  $sk_r$ , then sends the resulting value to  $\mathcal{A}$ . This oracle captures a rather strong "CCA-style" attack, a.k.a.  $\mathcal{A}$  can instruct any honest account to reveal confidential transactions and observe the results.
- $\mathcal{O}_{\text{inject}}$ :  $\mathcal{A}$  submits a confidential transaction ctx, subject to the restriction that ctx has not been recorded in  $T_{\text{ctx}}$ . If VerifyCTx(ctx) = 1,  $\mathcal{CH}$  updates the states associated account state and records  $(\text{ctx}, \bot)$  to  $T_{\text{ctx}}$ . Otherwise,  $\mathcal{CH}$  ignores. This oracle captures that  $\mathcal{A}$  can dishonestly generate any (possible malformed) transactions itself.

**Authenticity.** We define authenticity via the following security experiment between  $\mathcal{A}$  and  $\mathcal{CH}$ .

$$\mathsf{Adv}_{\mathcal{A}}(\lambda) = \Pr \left[ \begin{array}{c} \mathsf{VerifyCTx}(\mathsf{ctx}^*) = 1 \; \wedge \\ pk_s^* \in T_{\mathsf{honest}} \; \wedge \; \mathsf{ctx}^* \notin T_{\mathsf{ctx}}(pk_s^*) \end{array} \right. : \begin{array}{c} pp \leftarrow \mathsf{Setup}(\lambda); \\ \mathsf{ctx}^* \leftarrow \mathcal{A}^{\mathcal{O}}(pp); \end{array} \right].$$

Here  $\mathsf{ctx}^* = (\mathsf{sn}^*, \mathsf{memo}^* = (pk_s^*, pk_r^*, C^*), \mathsf{aux}^*)$  is a confidential transaction from  $pk_s^*$ ,  $T_{\mathsf{ctx}}(pk_s^*)$  denotes the set of all the confidential transactions originated from  $pk_s^*$  in  $T_{\mathsf{ctx}}$ . A DCP system satisfies authenticity if no PPT adversary has non-negligible advantage in the above experiment.

Confidentiality. We define confidentiality via the following security experiment between A and CH.

$$\mathsf{Adv}_{\mathcal{A}}(\lambda) = \Pr \left[ \begin{array}{c} pp \leftarrow \mathsf{Setup}(\lambda); \\ (state, pk_s^*, pk_r^*, v_0, v_1) \leftarrow \mathcal{A}_1^{\mathcal{O}}(pp); \\ \beta = \beta': \; \beta \xleftarrow{\mathbb{R}} \{0, 1\}; \\ \mathsf{ctx}^* \leftarrow \mathsf{CreateCTx}(sk_s^*, pk_s^*, pk_r^*, v_\beta); \\ \beta' \leftarrow \mathcal{A}_2^{\mathcal{O}}(state, \mathsf{ctx}^*); \end{array} \right] - \frac{1}{2}.$$

To prevent trivial attacks,  $\mathcal{A}$  is subject to the following restrictions: (i)  $pk_s^*, pk_r^*$  chosen by  $\mathcal{A}$  are required to be honest accounts, and  $\mathcal{A}$  is not allowed to make corrupt queries to either  $pk_s^*$  or  $pk_r^*$ ; (ii)  $\mathcal{A}_2$  is not allowed to query  $\mathcal{O}_{\text{reveal}}$  with ctx\*; (iii) let  $\tilde{v}_s$  be the balance of  $pk_s^*$ ,  $v_{\text{sum}}$  be the accumulation of the transfer amounts in  $\mathcal{O}_{\text{trans}}$  queries related to  $pk_s^*$  after ctx\* ( $v_{\text{sum}}$  is a variable with initial value 0), both  $\tilde{v}_s - v_0 - v_{\text{sum}}$  and  $\tilde{v}_s - v_1 - v_{\text{sum}}$  are required to lie in  $\mathcal{V}$ .

Restrictions (i), (ii) prevents trivial attack by decryption, while restriction (iii) prevents inferring  $\beta$  by testing whether overdraft happens. A DCP system satisfies confidentiality if no PPT adversary has non-negligible advantage in the above experiment.

**Soundness.** We define soundness via the following security experiment between  $\mathcal{A}$  and  $\mathcal{CH}$ .

$$\mathsf{Adv}_{\mathcal{A}}(\lambda) = \Pr \left[ \begin{array}{cc} \mathsf{VerifyCTx}(\mathsf{ctx}^*) = 1 \\ \wedge f_{\mathrm{legal}}(\mathsf{memo}^*) = 0 \end{array} \right. \colon \begin{array}{c} pp \leftarrow \mathsf{Setup}(\lambda); \\ \mathsf{ctx}^* \leftarrow \mathcal{A}^{\mathcal{O}}(pp); \end{array} \right].$$

Here,  $\mathsf{ctx}^* = (\mathsf{sn}^*, \mathsf{memo}^*, \mathsf{aux}^*)$ . A DCP system satisfies soundness if no PPT adversary has non-negligible advantage in the above experiment.

**Secure auditing.** We then sketch the security requirements of auditing for ADCP system. For regulation compliance, we require:

- Soundness: no PPT adversary can fool the regulator to accept a false auditing result.
- Minimal information disclosure: the regulator learns nothing other than the auditing result.

For global supervision, we require:

- Consistency: no PPT adversary can generate a transaction such that supervisor's view is different from the real receiver's view.
- Robustness: even the supervisor with sp cannot break the authenticity and soundness.

# 4 DCP with Auditability

# 4.1 A Generic Construction of Auditable DCP from ISE and NIZK

We present a generic construction of auditable DCP from ISE and NIZK in an incremental manner. In a nutshell, we use homomorphic PKE to encode the balance and transfer amount, use NIZK to enforce senders to build confidential transactions honestly and make validity publicly verifiable, and use digital signature to authenticate transactions. Let ISE = (Setup, KeyGen, Sign, Verify, Enc, Dec) be an ISE scheme whose PKE component is additively homomorphic on message space  $\mathbb{Z}_p$ . Let NIZK = (Setup, CRSGen, Prove, Verify)<sup>8</sup> be a NIZK proof system for  $L_{\text{legal}}$  (which will be specified later). The construction is as below.

- Setup(1 $^{\lambda}$ ): on input a security parameter  $\lambda$ , runs  $pp_{\text{ise}} \leftarrow \text{ISE.Setup}(1^{\lambda})$ ,  $pp_{\text{nizk}} \leftarrow \text{NIZK.Setup}(1^{\lambda})$ ,  $crs \leftarrow \text{NIZK.CRSGen}(pp_{\text{nizk}})$ , outputs  $pp = (pp_{\text{ise}}, pp_{\text{nizk}}, crs)$ .
- CreateAccount( $\tilde{v}$ , sn): on input an initial balance  $\tilde{v} \in \mathbb{Z}_p$  and a serial number sn  $\in \{0,1\}^n$  (e.g., n=256), runs ISE.KeyGen( $pp_{ise}$ ) to generate a keypair (pk,sk), computes  $\tilde{C} \leftarrow \text{ISE.Enc}(pk,\tilde{v};r)$  as the initial encrypted balance, sets sn as the initial serial number<sup>9</sup>, outputs public key pk and secret key sk. Fix the public parameters, the KeyGen algorithm naturally induces an  $\mathcal{NP}$  relation  $\mathsf{R}_{key} = \{(pk,sk) : \exists r \text{ s.t. } (pk,sk) = \mathsf{KeyGen}(r)\}.$
- RevealBalance $(sk, \tilde{C})$ : on input secret key sk and encrypted balance  $\tilde{C}$ , outputs  $\tilde{v} \leftarrow \text{ISE.Dec}(sk, \tilde{C})$ .
- CreateCTx $(sk_s, pk_s, v, pk_r)$ : on input sender's keypair  $(pk_s, sk_s)$ , the transfer amount v, and receiver's public key  $pk_r$ , the algorithm first checks if  $(\tilde{v}_s v) \in \mathcal{V}$  and  $v \in \mathcal{V}$  (here  $\tilde{v}_s$  is the current balance of sender account  $pk_s$ ). If not, returns  $\bot$ . Otherwise, it creates a confidential transaction ctx via the following steps:
  - 1. compute  $C_s \leftarrow \text{ISE.Enc}(pk_s, v; r_1)$ ,  $C_r \leftarrow \text{ISE.Enc}(pk_r, v; r_2)$ , set  $\mathsf{memo} = (pk_s, pk_r, C_s, C_r)$ , here  $(C_s, C_r)$  serve as the encoding of transfer amount;
  - 2. run NIZK.Prove with witness  $(sk_s, r_1, r_2, v)$  to generate a zero-knowledge proof  $\pi_{\text{legal}}$  for  $\mathsf{memo} = (pk_s, pk_r, C_s, C_r) \in L_{\text{legal}}$ , where  $L_{\text{legal}}$  is defined as below:

$$\begin{split} L_{\text{legal}} &= \{ (pk_s, pk_r, C_s, C_r) \mid \exists sk_s, r_1, r_2, v \text{ s.t. } C_s = \mathsf{Enc}(pk_s, v; r_1) \land C_r = \mathsf{Enc}(pk_r, v; r_2) \\ &\land v \in \mathcal{V} \ \land (pk_s, sk_s) \in \mathsf{R}_{\text{key}} \land \mathsf{Dec}(sk_s, \tilde{C}_s - C_s) \in \mathcal{V} \} \end{split}$$

 $L_{\text{legal}}$  can be decomposed as  $L_{\text{equal}} \wedge L_{\text{right}} \wedge L_{\text{solvent}}$ :

$$\begin{split} L_{\text{equal}} &= \{ (pk_s, C_s, pk_r, C_r) \mid \exists r_1, r_2, v \text{ s.t. } C_s = \mathsf{Enc}(pk_s, v; r_1) \land C_r = \mathsf{Enc}(pk_r, v; r_2) \} \\ &\qquad \qquad L_{\text{right}} = \{ (pk_s, C_s) \mid \exists r_1, v \text{ s.t. } C_s = \mathsf{Enc}(pk_s, v; r_1) \land v \in \mathcal{V} \} \\ &\qquad \qquad L_{\text{solvent}} = \{ (pk_s, \tilde{C}_s, C_s) \mid \exists sk_s \text{ s.t. } (pk_s, sk_s) \in \mathsf{R}_{\text{key}} \land \mathsf{Dec}(sk_s, \tilde{C}_s - C_s) \in \mathcal{V} \} \end{split}$$

<sup>&</sup>lt;sup>8</sup>We describe our generic DCP construction using NIZK in the CRS model. The construction and security proof carries out naturally if using NIZK in the random oracle model instead.

<sup>&</sup>lt;sup>9</sup>By default,  $\tilde{v}$  and sn should be zero, r should be a fixed and publicly known randomness, say the zero string  $0^{\lambda}$ . This settlement guarantees that the initial account state is publicly auditable. Here, we do not make it as an enforcement for flexibility.

- 3. run  $\sigma \leftarrow \text{ISE.Sign}(sk_s, (\mathsf{sn}, \mathsf{memo}), \pi_{\text{legal}})$ , here  $\mathsf{sn}$  is the current serial number of  $pk_s$ ;
- 4. output the entire confidential transaction  $\mathsf{ctx} = (\mathsf{sn}, \mathsf{memo}, \mathsf{aux}), \text{ where } \mathsf{aux} = (\pi_{\mathsf{legal}}, \sigma).$
- VerifyCTx(ctx): on input ctx = (sn, memo, aux), first parses memo =  $(pk_s, pk_r, C_s, C_r)$ , aux =  $(\pi_{\text{legal}}, \sigma)$ , then checks its validity via the following steps:
  - 1. check if sn is a fresh serial number of  $pk_s$  (this can be done by inspecting the blockchain);
  - 2. check if ISE.Verify $(pk_s, (\mathsf{sn}, \mathsf{memo}, \pi_{\mathrm{legal}}), \sigma) = 1;$
  - 3. check if NIZK.Verify(crs, memo,  $\pi_{legal}$ ) = 1.

If all the above tests pass, outputs "1", validators confirm that ctx is valid and record it on the blockchain via consensus protocol, sender updates his balance as  $\tilde{C}_s = \tilde{C}_s - C_s$  and increments the serial number, and receiver updates his balance as  $\tilde{C}_r = \tilde{C}_r + C_r$ . Else, outputs "0" and validators discard ctx.

On the basis that all confidential transactions on the blockchain satisfying the system-wide legality policy, we further describe how to enforce a variety of regulation policies.

• JustifyCTx( $pk_s, sk_s, \{\text{ctx}_i\}_{i=1}^n, f_{\text{limit}}\}$ : on input  $pk_s, sk_s, \{\text{ctx}_i\}_{i=1}^n$  and  $f_{\text{limit}}$ , first parses  $\text{ctx}_i = (\text{sn}_i, \text{memo}_i = (pk_s, pk_{r_i}, C_{s,i}, C_{r_i}), \text{aux}_i)$ , then runs NIZK.Prove with witness  $sk_s$  to generate a proof  $\pi_{\text{limit}}$  for the statement  $(pk_s, \{C_{s,i}\}_{1 \leq i \leq n}, a_{\text{max}}) \in L_{\text{limit}}$ , where  $L_{\text{limit}}$  is defined as:

$$\{(pk, \{C_i\}_{1 \leq i \leq n}, a_{\max}) \mid \exists sk \text{ s.t. } (pk, sk) \in \mathsf{R}_{\text{key}} \land v_i = \text{ISE.Dec}(sk, C_i) \land \sum_{i=1}^n v_i \leq a_{\max}\}$$

A user runs this algorithm to prove compliance with limit policy, i.e., the sum of  $v_i$  sent from account  $pk_s$  is less than  $a_{\text{max}}$ . The same algorithm can be used to proving the sum of  $v_i$  sent to the same account is less than  $a_{\text{max}}$ .

- AuditCTx $(pk_s, \{\mathsf{ctx}_i\}_{i=1}^n, \pi_{\mathrm{limit}}, f_{\mathrm{limit}})$ : on input  $pk_s, \{\mathsf{ctx}_i\}_{i=1}^n, \pi_{\mathrm{limit}}$  and  $f_{\mathrm{limit}}$ , first parses  $\mathsf{ctx}_i = (\mathsf{sn}_i, \mathsf{memo}_i = (pk_s, pk_{r_i}, C_{s,i}, C_{r_i}), \mathsf{aux}_i)$ , outputs NIZK. Verify $(crs, (pk_s, \{C_{s,i}\}_{1 \leq i \leq n}, a_{\max}), \pi_{\mathrm{limit}})$ . The auditor runs this algorithm to check compliance with limit policy.
- JustifyCTx $(pk_u, sk_u, \{\text{ctx}\}_{i=1}^2, f_{\text{rate}})$ : on input  $pk_u, sk_u, \{\text{ctx}_i\}_{i=1}^2$  and  $f_{\text{rate}}$ , the algorithm parses  $\text{ctx}_1 = (\text{sn}_1, \text{memo}_1 = (pk_1, pk_u, C_1, C_{u,1}), \text{aux}_1)$  and  $\text{ctx}_2 = (\text{sn}_2, \text{memo}_2 = (pk_u, pk_2, C_{u,2}, C_2), \text{aux}_2)$ , then runs NIZK.Prove with witness  $sk_u$  to generate a zero-knowledge proof  $\pi_{\text{rate}}$  for the statement  $(pk_u, C_{u,1}, C_{u,2}, \rho) \in L_{\text{rate}}$ , where  $L_{\text{rate}}$  is defined as:

$$\{(pk, C_1, C_2, \rho) \mid \exists sk \text{ s.t. } (pk, sk) \in \mathsf{R}_{kev} \land v_i = \mathsf{ISE.Dec}(sk, C_i) \land v_1/v_2 = \rho\}$$

A user runs this algorithm to demonstrate compliance with tax rule, i.e., proving  $v_1/v_2 = \rho$ .

- AuditCTx $(pk_u, \{\mathsf{ctx}_i\}_{i=1}^2, \pi_{\mathsf{rate}}, f_{\mathsf{rate}})$ : on input  $pk_u, \{\mathsf{ctx}_i\}_{i=1}^2, \pi_{\mathsf{rate}}$  and  $f_{\mathsf{rate}}$ , first parses  $\mathsf{ctx}_1 = (\mathsf{sn}_1, \mathsf{memo}_1 = (pk_1, pk_u, C_1, C_{u,1}), \mathsf{aux}_1)$ ,  $\mathsf{ctx}_2 = (\mathsf{sn}_2, \mathsf{memo}_2 = (pk_u, pk_2, C_{u,2}, C_2), \mathsf{aux}_2)$ , outputs NIZK.Verify $(crs, (pk_u, C_{u,1}, C_{u,2}, \rho), \pi_{\mathsf{rate}})$ . An auditor runs this algorithm to check compliance with rate policy.
- JustifyCTx( $sk_u$ , ctx,  $f_{open}$ ): on input  $pk_u$ ,  $sk_u$ , ctx and  $f_{open}$ , parses ctx = (sn,  $pk_s$ ,  $pk_r$ ,  $C_s$ ,  $C_r$ , aux), then runs NIZK.Prove with witness  $sk_u$  (where the subscript u could be either s or r) to generate a proof  $\pi_{open}$  for the statement ( $pk_u$ ,  $C_u$ , v)  $\in L_{open}$ , where  $L_{open}$  is defined as:

$$\{(pk, C, v^*) \mid \exists sk \text{ s.t. } (pk, sk) \in \mathsf{R}_{kev} \land v^* = \mathsf{ISE}.\mathsf{Dec}(sk, C)\}$$

A user runs this algorithm to demonstrate compliance with open policy.

• AuditCTx( $pk_u$ , ctx,  $\pi_{\text{open}}$ ,  $f_{\text{open}}$ ): on input  $pk_u$ , ctx,  $\pi_{\text{open}}$  and  $f_{\text{open}}$ , the algorithm first parses ctx =  $(\text{sn}, pk_s, pk_r, C_s, C_r, \text{aux})$ , then outputs NIZK. Verify(crs,  $(pk_u, C_u, v^*), \pi_{\text{open}}$ ), where the subscript u could be either s or r. An auditor runs this algorithm to check compliance with open policy.

Remark 4.1. We note that sender's current balance can be inferred from the blockchain. Therefore, it is not necessary to include sender's current balance in the transaction. To speed up the process, validators can maintain a local database of validation state (including the set of mutable accounts), in the same way as most implementation of existing account-based cryptocurrencies (such as Nxt, Ethereum, Bitshares, NEM, Tezos).

We sketch show how to support the optional global supervision property. Follow the approach outlined in the introduction part, in the setup stage the supervisor generates an ISE keypair  $(pk_a, sk_a)$ , then includes  $pk_a$  as part of public parameter and sets  $sk_a$  as secret parameter. When sender creates a confidential transaction, he will encrypt the transfer value under  $(pk_s, pk_r)$  and  $pk_a$ , and generate the accompanying zero-knowledge proof for plaintext equality. The supervisor can inspect any confidential transaction by decrypting  $C_a$  using  $sk_a$ .

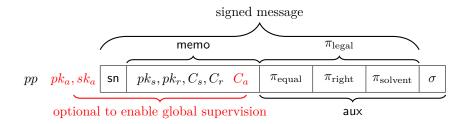


Figure 1: Data structure of confidential transaction in ADCP.

## 4.2 Analysis of Generic Auditable DCP Construction

Correctness of our generic DCP construction follows readily from the correctness of ISE and completeness of NIZK. For the security of our DCP construction, we have the following theorem.

**Theorem 4.1.** Assuming the security of ISE and NIZK, the above ADCP construction is secure.

*Proof.* We prove this theorem via the following three lemmas.

**Lemma 4.2.** Assuming the security of ISE's signature component and the adaptive zero-knowledge property of NIZK, our ADCP construction satisfies authenticity.

*Proof.* We proceed via a sequence of games. Let  $S_i$  be the probability that  $\mathcal{A}$  wins in Game i.

**Game 0.** The real experiment for authenticity.  $\mathcal{CH}$  interacts with  $\mathcal{A}$  as below.

- 1. Setup:  $\mathcal{CH}$  runs  $pp_{\text{ise}} \leftarrow \text{ISE.Setup}(1^{\lambda}), pp_{\text{nizk}} \leftarrow \text{NIZK.Setup}(1^{\lambda}), \text{ and } crs \leftarrow \text{NIZK.CRSGen}(pp_{\text{nizk}}),$  then sends  $pp = (pp_{\text{ise}}, pp_{\text{nizk}}, crs)$  to  $\mathcal{A}$ .
- 2. Queries: Throughout the experiment,  $\mathcal{A}$  can adaptively query  $\mathcal{O}_{regH}$ ,  $\mathcal{O}_{regC}$ ,  $\mathcal{O}_{extH}$ ,  $\mathcal{O}_{trans}$  and  $\mathcal{O}_{inject}$ , and  $\mathcal{CH}$  answers these queries as described above.
- 3. Forge:  $\mathcal{A}$  outputs  $\mathsf{ctx}^* = (\mathsf{sn}^*, \mathsf{memo}^* = (pk_s^*, pk_r^*, C_s^*, C_r^*), \pi_{\mathrm{legal}}^*, \sigma^*)$ , and wins if  $pk_s^* \in T_{\mathrm{honest}} \land \mathsf{VerifyCTx}(\mathsf{ctx}^*) = 1 \land (\mathsf{sn}^*, \mathsf{memo}^*) \notin T_{\mathsf{ctx}}(pk_s^*)$ .

According to the definition, we have:

$$Adv_{\mathcal{A}}(\lambda) = Pr[S_0]$$

**Game 1.** Game 1 is same as Game 0 except  $\mathcal{CH}$  makes a random guess for the index of target  $pk_s^*$  at the beginning, i.e., randomly picks an index  $j \in [Q_{\text{honest}}]$ . If  $\mathcal{A}$  makes an extraction query of  $pk_j$  in the learning stage, or  $\mathcal{A}$  picks  $pk_s^* \neq pk_j$  in the challenge stage,  $\mathcal{CH}$  aborts.

Let W be the event that  $\mathcal{CH}$  does not abort. It is easy to see that  $\Pr[W] = 1/Q_{\text{honest}}$ . Conditioned on  $\mathcal{CH}$  does not abort,  $\mathcal{A}$ 's view in Game 0 is identical to that in Game 1. Thereby, we have:

$$\Pr[S_1] = \Pr[S_0] \cdot \Pr[W]$$

Game 2. Same as Game 1 except that  $\mathcal{CH}$  runs  $(crs, \tau) \leftarrow \mathcal{S}_1(pp_{\text{nizk}})$  in the Setup stage, then generates the zero-knowledge proof by running  $\mathcal{S}_2$  in the simulation mode. We make this modification to avoid using secret key (part of the witness) to generate the zero-knowledge proofs. More precisely, in the Setup stage  $\mathcal{CH}$  runs  $(crs, \tau) \leftarrow \mathcal{S}_1(pp_{\text{nizk}})$ . When handling transaction queries,  $\mathcal{CH}$  runs  $\mathcal{S}_2(crs, \tau, \mathsf{memo})$  to generate  $\pi_{\text{legal}}$  for  $\mathsf{memo} \in L_{\text{legal}}$ . By a direct reduction to the adaptive zero-knowledge property of the underlying NIZK, we have:

$$|\Pr[S_2] - \Pr[S_1]| \le \mathsf{negl}(\lambda)$$

We now argue that no PPT adversary has non-negligible advantage in Game 2.

Claim 4.3. Assuming the sEUF-CMA security of ISE's signature component,  $\Pr[S_2] \leq \operatorname{negl}(\lambda)$  for all PPT adversary A.

*Proof.* Suppose there exists a PPT  $\mathcal{A}$  has non-negligible advantage in Game 2, we can build an adversary  $\mathcal{B}$  breaks the sEUF-CMA security of ISE with the same advantage. Given the challenge  $(pp_{ise}, pk^*)$ ,  $\mathcal{B}$  simulates Game 2 as follows:

- 1. Setup:  $\mathcal{B}$  runs  $pp_{\text{nizk}} \leftarrow \text{NIZK.Setup}(1^{\lambda}), (crs, \tau) \leftarrow \mathcal{S}_1(pp_{\text{nizk}}), \text{ then sends } pp = (pp_{\text{ise}}, pp_{\text{nizk}}, crs)$  to  $\mathcal{A}$ .  $\mathcal{B}$  also randomly picks an index  $j \in [Q_{\text{honest}}]$ .
- 2. Queries: Throughout the experiment,  $\mathcal{A}$  can query the following types of oracles adaptively.  $\mathcal{B}$  answers these queries by maintaining two lists  $T_{\text{honest}}$  and  $T_{\text{corrupt}}$ , which both are initially empty.
  - $\mathcal{O}_{\mathsf{regH}}$ : On the *i*-th query,  $\mathcal{B}$  picks a random balance  $\tilde{v}$  and a serial number  $\mathsf{sn}$ , then proceeds as below:
    - If  $i \neq j$ ,  $\mathcal{B}$  runs CreateAccount( $\tilde{v}$ , sn) to obtain (pk, sk) and the initial encrypted balance  $\tilde{C} \leftarrow \text{ISE.Enc}(pk, \tilde{v})$ , then records  $(pk, sk, \tilde{C}, \tilde{v}, \text{sn})$  in  $T_{\text{honest}}$ .
    - If i = j,  $\mathcal{B}$  sets  $pk = pk^*$ ,  $\mathcal{B}$  computes  $\tilde{C} \leftarrow \text{ISE.Enc}(pk, \tilde{v})$ , then records  $(pk, \perp, \tilde{C}, \tilde{v}, \mathsf{sn})$  in  $T_{\text{honest}}$ .

Finally,  $\mathcal{B}$  returns  $(pk, \tilde{C}, \mathsf{sn})$  to  $\mathcal{A}$ .

- $\mathcal{O}_{\mathsf{regC}}$ :  $\mathcal{A}$  makes this query with a public key pk, a serial number  $\mathsf{sn}$  and an initial encrypted balance  $\tilde{C}$ .  $\mathcal{B}$  records  $(pk, \perp, \tilde{C}, \perp, \mathsf{sn})$  in  $T_{\mathsf{corrupt}}$ .
- $\mathcal{O}_{\text{extH}}$ :  $\mathcal{A}$  makes a query with pk in  $T_{\text{honest}}$ . If  $pk = pk_j$ ,  $\mathcal{B}$  aborts. Else,  $\mathcal{B}$  returns sk to  $\mathcal{A}$ , then moves the corresponding entry to  $T_{\text{corrupt}}$ .
- $\mathcal{O}_{\text{trans}}$ :  $\mathcal{A}$  makes a transaction query  $(pk_s, pk_r, v)$  subject to the restriction that  $pk_s \in T_{\text{honest}}$ . Let  $\mathsf{sn}$  be the serial number,  $\tilde{C}_s$  be the encrypted balance of  $pk_s$ .  $\mathcal{B}$  proceeds as below:
  - (a) compute  $C_s \leftarrow \text{ISE.Enc}(pk_s, v), C_r \leftarrow \text{ISE.Enc}(pk_r, v);$
  - (b) set memo =  $(pk_r, pk_s, C_r, C_s)$ , compute  $\pi_{\text{legal}} \leftarrow \mathcal{S}_2(crs, \tau, \text{memo})$ ;
  - (c) if  $pk_s \neq pk_j$ , generate a signature  $\sigma$  for (sn, memo,  $\pi_{legal}$ ) with  $sk_s$ ; else, generate such signature  $\sigma$  by querying the signing oracle.

 $\mathcal{B}$  updates the states of associated accounts, inserts ctx to  $T_{\text{ctx}}$ , then returns ctx to  $\mathcal{A}$ .

- $\mathcal{O}_{\text{reveal}}$ :  $\mathcal{A}$  makes a reveal query ctx subject to the restriction that  $pk_s, pk_r \in T_{\text{honest}}$ .  $\mathcal{B}$  proceeds as below:
  - (a) check if VerifyCTx(ctx) = 1, if not, return  $\bot$ ;
  - (b) parse  $\mathsf{memo} = (pk_s, pk_r, C_s, C_r)$ , suppose  $pk_{\kappa} \neq pk^*$  (here the subscript  $\kappa$  could be r or s), then use  $sk_{\kappa}$  to decrypt  $C_{\kappa}$ , and send the decryption result to  $\mathcal{A}$ .
- $\mathcal{O}_{\text{inject}}$ :  $\mathcal{A}$  submits a confidential transaction ctx. If VerifyCTx(ctx) = 1,  $\mathcal{B}$  inserts (ctx,  $\perp$ ) to  $T_{\text{ctx}}$  and updates the states of associated accounts. Otherwise,  $\mathcal{B}$  ignores.
- 3. Forge: Finally,  $\mathcal{A}$  outputs  $\mathsf{ctx}^* = (\mathsf{sn}^*, \mathsf{memo}^*, \mathsf{aux}^*)$ , where  $\mathsf{memo}^* = (pk_s^*, pk_r^*, C_s^*, C_r^*)$  and  $\mathsf{aux}^* = (\pi_{\mathsf{legal}}^*, \sigma^*)$ .

If  $pk_s^* \neq pk_j$ ,  $\mathcal{B}$  aborts. Otherwise,  $\mathcal{B}$  forwards  $(\mathsf{sn}^*, \mathsf{memo}^*, \sigma^*)$  to its own challenger. It is easy to see that  $\mathcal{B}$ 's simulation for Game 2 is perfect. Thus,  $\mathcal{B}$  wins with the same advantage as  $\mathcal{A}$  wins. This proves Claim 4.3.

This proves Lemma 4.2.

**Lemma 4.4.** Assuming the security of ISE's signature component, the security of ISE's PKE component, and the zero-knowledge property of NIZK, our ADCP construction satisfies confidentiality.

*Proof.* We proceed via a sequence of games. Let  $S_i$  be the probability that  $\mathcal{A}$  wins in Game i.

**Game 0.** The real experiment for confidentiality.  $\mathcal{CH}$  interacts with  $\mathcal{A}$  as below.

- 1. Setup:  $\mathcal{CH}$  runs  $pp_{\text{ise}} \leftarrow \text{ISE.Setup}(1^{\lambda}), pp_{\text{nizk}} \leftarrow \text{NIZK.Setup}(1^{\lambda}), \text{ and } crs \leftarrow \text{NIZK.CRSGen}(pp_{\text{nizk}}),$  then sends  $pp = (pp_{\text{ise}}, pp_{\text{nizk}}, crs, pk_a)$  to  $\mathcal{A}$ .
- 2. Pre-challenge queries: Throughout the experiment,  $\mathcal{A}$  can adaptively query  $\mathcal{O}_{regH}$ ,  $\mathcal{O}_{regC}$ ,  $\mathcal{O}_{extH}$ ,  $\mathcal{O}_{trans}$ ,  $\mathcal{O}_{reveal}$  and  $\mathcal{O}_{inject}$ ,  $\mathcal{CH}$  answers these queries as prescribed.
- 3. Challenge:  $\mathcal{A}$  picks sender  $pk_s^*$ , receiver  $pk_r^*$  and two transfer values  $v_0, v_1$  as the challenge, subject to the restriction that  $pk_s^*, pk_r^* \in T_{\text{honest}}$  and both  $v_0$  and  $v_1$  constitute valid transactions from  $pk_s^*$ .  $\mathcal{CH}$  picks a random bit  $\beta$ , computes  $\mathsf{ctx}^* \leftarrow \mathsf{CreateCTx}(sk_s^*, pk_r^*, pk_r^*, v_\beta)$ , inserts  $(\mathsf{ctx}^*, \bot)$  to  $T_{\mathsf{ctx}}$ , updates the states of accounts  $pk_s^*$  and  $pk_r^*$ , then sends  $\mathsf{ctx}^*$  to  $\mathcal{A}$  as the challenge.
- 4. Post-challenge queries: After receiving the challenge,  $\mathcal{A}$  can continue query  $\mathcal{O}_{\text{regH}}$ ,  $\mathcal{O}_{\text{regC}}$ ,  $\mathcal{O}_{\text{extH}}$ ,  $\mathcal{O}_{\text{trans}}$  and  $\mathcal{O}_{\text{inject}}$ ,  $\mathcal{CH}$  responds the same way as in pre-challenge stage, but with the following exceptions: (i) reject  $\mathcal{A}$ 's  $\mathcal{O}_{\text{extH}}$  query with  $pk_s^*$  or  $pk_r^*$ ; (ii) reject  $\mathcal{A}$ 's  $\mathcal{O}_{\text{trans}}$  query with  $pk_s^*$  if it will make either  $(\tilde{v}_s^* v_0 v_{\text{sum}})$  or  $(\tilde{v}_s^* v_1 v_{\text{sum}})$  fall outside of  $\mathcal{V}$ . Here,  $v_{\text{sum}}$  is the accumulation of transfer amounts related to  $pk_s^*$  in post-challenge  $\mathcal{O}_{\text{trans}}$  queries.
- 5. Guess:  $\mathcal{A}$  outputs a guess  $\beta'$  for  $\beta$  and wins if  $\beta' = \beta$ .

According to the definition, we have:

$$\mathsf{Adv}_{\mathcal{A}}(\lambda) = |\Pr[S_0] - 1/2|$$

**Game 1.** Same as Game 0 except  $\mathcal{CH}$  makes a random guess for index of target accounts  $(pk_s^*, pk_r^*)$  at the beginning, i.e., randomly picks two distinct indices  $j, k \in [Q_{\text{honest}}]$ . If  $\mathcal{A}$  makes an extraction query to  $pk_j$  or  $pk_k$  in pre-challenge queries, or  $\mathcal{A}$  picks  $(pk_s^*, pk_r^*) \neq (pk_j, pk_k)$  in the challenge stage,  $\mathcal{CH}$  aborts.

Let W be the event that  $\mathcal{CH}$  does not abort. It is easy to see that  $\Pr[W] = 1/Q_{\text{honest}}(Q_{\text{honest}} - 1)$ . Conditioned on  $\mathcal{CH}$  does not abort,  $\mathcal{A}$ 's view in Game 0 is identical to that in Game 1. Therefore, we have:

$$\Pr[S_1] = \Pr[S_0] \cdot \Pr[W]$$

Game 2. Same as Game 1 except that  $\mathcal{CH}$  runs  $(crs, \tau) \leftarrow \mathcal{S}_1(pp_{\text{nizk}})$  in the Setup stage, then generates all zero-knowledge proofs by running  $\mathcal{S}_2$  in the simulation mode. More precisely, when handling transaction queries and challenge queries,  $\mathcal{CH}$  runs  $\mathcal{S}_2(crs, \tau, \mathsf{memo})$  to generate  $\pi_{\text{legal}}$  for  $\mathsf{memo} \in L_{\text{legal}}$ . By a direct reduction to the adaptive zero-knowledge property of the underlying NIZK, we have:

$$|\Pr[S_2] - \Pr[S_1]| \le \mathsf{negl}(\lambda)$$

Game 3. Let E be the event that  $\mathcal{A}$  makes a valid reveal query ctx with  $pk_j$  and  $pk_k$  as participants while ctx is not recorded in  $T_{\text{ctx}}$  or ctx is recorded in  $T_{\text{ctx}}$  but the corresponding value is  $\bot$ . In other words, E denotes the event that  $\mathcal{A}$  generates a fresh valid confidential transaction between  $pk_j$  and  $pk_k$  itself. Game 3 is same as Game 2 except that  $\mathcal{CH}$  aborts if E happens. By the difference lemma, we have:

$$|\Pr[S_3] - \Pr[S_2]| \le \Pr[E]$$

By a direct reduction to the sEUF-CMA security of the ISE's signature component (two user setting), we conclude that  $\Pr[E] \leq \mathsf{negl}(\lambda)$ .

We now argue that no PPT adversary has non-negligible advantage in Game 3.

Claim 4.5. Assuming the IND-CPA security (1-plaintext, 2-recipient) of the ISE's encryption component,  $|\Pr[S_3] - \Pr[W]/2| \leq \operatorname{negl}(\lambda)$  for all PPT adversary A.

*Proof.* Suppose there exists a PPT adversary  $\mathcal{A}$  has non-negligible advantage in Game 2, we can build an adversary  $\mathcal{B}$  breaks the IND-CPA security (1-plaintext, 2-recipient) of ISE's encryption component with the same advantage. Given the challenge  $(pp_{ise}, pk_a, pk_b)$ ,  $\mathcal{B}$  simulates Game 3 as follows:

- 1. Setup:  $\mathcal{B}$  runs  $pp_{\text{nizk}} \leftarrow \text{NIZK.Setup}(1^{\lambda}), \mathcal{S}_1(pp_{\text{nizk}}) \rightarrow (crs, \tau), \text{ sends } pp = (pp_{\text{ise}}, pp_{\text{nizk}}, crs) \text{ to } \mathcal{A}.$   $\overline{\mathcal{B}}$  randomly picks two indices  $j, k \in [Q_{\text{honest}}].$
- 2. Pre-challenge queries: Throughout the experiment,  $\mathcal{A}$  can query  $\mathcal{O}_{\text{regH}}$ ,  $\mathcal{O}_{\text{regC}}$ ,  $\mathcal{O}_{\text{extH}}$ ,  $\mathcal{O}_{\text{trans}}$  and  $\overline{\mathcal{O}}_{\text{inject}}$  adaptively.  $\mathcal{B}$  answers these queries by maintaining two lists  $T_{\text{honest}}$  and  $T_{\text{corrupt}}$ , which both are initially empty.
  - $\mathcal{O}_{\mathsf{regH}}$ : On the *i*-th query,  $\mathcal{B}$  randomly picks an initial balance  $\tilde{v}$  and serial number  $\mathsf{sn}$  and proceeds as below:
    - If  $i \neq j$  and k,  $\mathcal{B}$  runs CreateAccount $(\tilde{v}, \mathsf{sn})$  to obtain (pk, sk) and encrypted balance  $\tilde{C} \leftarrow \mathsf{ISE}.\mathsf{Enc}(pk, \tilde{v})$ , then records  $(pk, sk, \tilde{C}, \tilde{v}, \mathsf{sn})$  in  $T_{\mathsf{honest}}$ .
    - If i = j or k,  $\mathcal{B}$  sets  $pk = pk_a$  or  $pk_b$ ,  $\mathcal{B}$  computes  $\tilde{C} \leftarrow \text{ISE.Enc}(pk, \tilde{v})$ , then records  $(pk, \perp, \tilde{C}, \tilde{v}, \mathsf{sn})$  in  $T_{\text{honest}}$ .

Finally,  $\mathcal{B}$  returns  $(pk, \tilde{C}, \mathsf{sn})$  to  $\mathcal{A}$ .

- $\mathcal{O}_{\mathsf{regC}}$ :  $\mathcal{A}$  makes this query with a public key pk, a serial number  $\mathsf{sn}$  and an initial encrypted balance  $\tilde{C}$ .  $\mathcal{B}$  records  $(pk, \perp, \tilde{C}, \perp, sn)$  in  $T_{\mathsf{corrupt}}$ .
- $\mathcal{O}_{\text{extH}}$ :  $\mathcal{A}$  makes this query with pk in  $T_{\text{honest}}$ . If  $pk = pk_j$  or  $pk_k$ ,  $\mathcal{B}$  aborts. Else,  $\mathcal{B}$  returns the associated sk to  $\mathcal{A}$ , then moves the corresponding entry to  $T_{\text{corrupt}}$ .
- $\mathcal{O}_{\mathsf{trans}}$ :  $\mathcal{A}$  makes a transaction query  $(pk_s, pk_r, v)$  subject to the restriction that  $pk_s \in T_{\mathsf{honest}}$ . Let  $\mathsf{sn}$  and  $\tilde{C}_s$  be the serial number and encrypted balance of  $pk_s$ ,  $\mathcal{B}$  proceeds as below:
  - (a) compute  $C_s \leftarrow \text{ISE.Enc}(pk_s, v), C_r \leftarrow \text{ISE.Enc}(pk_r, v);$
  - (b) set memo =  $(pk_r, pk_s, C_r, C_s)$ , compute  $\pi_{legal} \leftarrow \mathcal{S}_2(crs, \tau, memo)$ ;
  - (c) if  $pk_s \neq \{pk_j, pk_k\}$ , generate a signature  $\sigma$  for (sn, memo,  $\pi_{legal}$ ) with  $sk_s$ ; else, generate such signature  $\sigma$  by querying the signing oracle of the underlying ISE.

 $\mathcal{B}$  updates the associated account state, then returns ctx to  $\mathcal{A}$ .

- $\mathcal{O}_{\text{reveal}}$ :  $\mathcal{A}$  makes a reveal query ctx = (sn, memo =  $(pk_s, pk_r, C_s, C_r)$ , aux) subject to the restriction that  $pk_s, pk_r \in T_{\text{honest}}$ . If event E happens,  $\mathcal{B}$  directly aborts. Otherwise,  $\mathcal{B}$  proceeds as below:
  - (a) if the participants are  $pk_j$  and  $pk_k$  and  $\mathsf{ctx} \in T_{\mathsf{ctx}}$ ,  $\mathcal{B}$  returns the corresponding v;
  - (b) else let  $pk_{\kappa} \neq pk_{j}$ ,  $pk_{k}$  (where the subscript  $\kappa$  could be either s or r),  $\mathcal{B}$  decrypts  $C_{\kappa}$  with  $sk_{\kappa}$  and sends the result to  $\mathcal{A}$ .
- $\mathcal{O}_{\text{inject}}$ :  $\mathcal{A}$  submits a confidential transaction ctx. If VerifyCTx(ctx) = 1,  $\mathcal{B}$  inserts (ctx,  $\perp$ ) to  $T_{\text{ctx}}$  and updates the states of associated accounts. Otherwise,  $\mathcal{B}$  ignores.
- 3. Challenge:  $\mathcal{A}$  picks sender  $pk_s^*$ , receiver  $pk_r^*$  and two transfer values  $v_0, v_1$  as the challenge, subject to the restriction that  $pk_s^*, pk_r^* \in T_{\text{honest}}$  and both  $v_0$  and  $v_1$  constitute a valid transaction from  $pk_s^*$ . If  $(pk_s^*, pk_r^*) \neq (pk_j, pk_k)$ ,  $\mathcal{B}$  aborts. Else,  $\mathcal{B}$  submits  $(v_0, v_1)$  to its own challenger and receives back  $C^* = (C_s^*, C_r^*)$ , which is an encryption of  $v_\beta$  under  $(pk_s^*, pk_r^*)$ . Let  $\mathsf{sn}^*$  and  $\tilde{C}_s^*$  be the serial number and encrypted balance of  $pk_s^*$ ,  $\mathcal{B}$  then prepares  $\mathsf{ctx}^*$  as follows:
  - (a) set memo\* =  $(pk_s^*, pk_r^*, C_s^*, C_r^*)$ .
  - (b) generate  $\pi_{\text{legal}}^* \leftarrow \mathcal{S}_2(crs, \tau, \mathsf{memo}^*)$ .
  - (c) query the signing oracle of ISE to obtain a signature  $\sigma^*$  of  $(sn^*, memo^*)$  under  $sk_s^*$ .

 $\mathcal{B}$  inserts (ctx\*,  $\perp$ ) to  $T_{\text{ctx}}$ , updates the states of accounts  $pk_s^*$  and  $pk_r^*$ , then sends ctx\* to  $\mathcal{A}$  as the challenge.

- 4. Post-challenge queries: After receiving the challenge,  $\mathcal{A}$  can continue query to  $\mathcal{O}_{\mathsf{regH}}$ ,  $\mathcal{O}_{\mathsf{regC}}$ ,  $\mathcal{O}_{\mathsf{extH}}$ ,  $\mathcal{O}_{\mathsf{trans}}$ ,  $\mathcal{O}_{\mathsf{reveal}}$  and  $\mathcal{O}_{\mathsf{inject}}$ ,  $\mathcal{B}$  responds the same way as in pre-challenge stage, but with the following exceptions: (i) reject  $\mathcal{A}$ 's  $\mathcal{O}_{\mathsf{extH}}$  query with  $pk_s^*$  or  $pk_r^*$ ; (ii) reject  $\mathcal{A}$ 's  $\mathcal{O}_{\mathsf{trans}}$  query with  $pk_s^*$  if it will make either  $(\tilde{v}_s^* v_0 v_{\mathsf{sum}})$  or  $(\tilde{v}_s^* v_1 v_{\mathsf{sum}})$  fall outside of  $\mathcal{V}$ . Here,  $v_{\mathsf{sum}}$  is the accumulation of transfer amounts related to  $pk_s^*$  in post-challenge  $\mathcal{O}_{\mathsf{trans}}$  queries; (iii) reject  $\mathcal{A}$ 's  $\mathcal{O}_{\mathsf{reveal}}$  query with  $\mathsf{ctx}^*$ .
- 5. Guess: Finally,  $\mathcal{A}$  outputs its guess  $\beta'$  for  $\beta$ .

If  $\mathcal{B}$  aborts during the simulation for  $\mathcal{A}$ 's challenger in Game 3, it will proceed to interact with its own challenger and outputs a random guess for  $\beta$ . Otherwise, it will forward  $\mathcal{A}$ 's output to its own challenger. It is easy to see that  $\mathcal{B}$ 's simulation for Game 2 is perfect. Thus,  $\mathcal{B}$ 's advantage against the ISE's encryption component is  $|(1 - \Pr[W])/2 + \Pr[S_3] - 1/2| = |\Pr[S_3] - \Pr[W]/2|$ , which is negligible in  $\lambda$  assuming the IND-CPA security of ISE. This proves Claim 4.5.

We further have  $|\Pr[S_1] - \Pr[W]/2| \le \mathsf{negl}(\lambda)$ . Note that  $\Pr[W]$  is noticeable in  $\lambda$ . Putting all the above together, we have  $|\Pr[S_0] - 1/2| = |\Pr[S_1] - \Pr[W]/2| / \Pr[W] = \mathsf{negl}(\lambda)$ . This proves Lemma 4.4.

**Lemma 4.6.** Assuming the soundness of NIZK, our DCP construction satisfies soundness.

*Proof.* Note that  $\mathsf{VerifyCTx}(\mathsf{ctx}^*) = 1 \land f_{\mathrm{legal}}(\mathsf{memo}^*) = 0$  implies that  $\mathsf{NIZK}.\mathsf{Verify}(\mathit{crs},\mathsf{memo}^*,\pi^*_{\mathrm{legal}}) = 1 \land \mathsf{memo}^* \notin L_{\mathrm{legal}}$ . The reduction is thus straightforward. We omit the details here. □

Putting Lemma 4.2, 4.4 and 4.6 together, we proves the theorem.

The secure auditing property follows directly from the soundness and zero-knowledge properties of the used NIZK, and the fact that supervisor's keypair is independent of users. We omit the details for its triviality.

# 5 PGC: an Efficient Instantiation

We now present an efficient realization of our generic DCP construction. We first instantiate ISE from our newly introduced twisted ElGamal PKE and Schnorr signature, then devise NIZK proofs from Sigma protocols and Bulletproof.

# 5.1 Instantiating ISE

We first describe twisted ElGamal encryption and recall Schnorr signature, which share the same setup and key generation algorithms. We then formally prove their integration constitutes ISE that satisfies joint security.

**Twisted ElGamal.** We propose twisted ElGamal encryption as the PKE component. Formally, twisted ElGamal consists of four algorithms as below:

- Setup(1 $^{\lambda}$ ): run ( $\mathbb{G}, g, p$ )  $\leftarrow$  GroupGen(1 $^{\lambda}$ ), pick  $h \stackrel{\mathbb{R}}{\leftarrow} \mathbb{G}^*$ , set  $pp = (\mathbb{G}, g, h, p)$  as global public parameters. The randomness and message spaces are  $\mathbb{Z}_p$ .
- KeyGen(pp): on input pp, choose  $sk \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$ , set  $pk = g^{sk}$ .
- $\operatorname{Enc}(pk, m; r)$ : compute  $X = pk^r$ ,  $Y = q^rh^m$ , output C = (X, Y).
- Dec(sk, C): parse C = (X, Y), compute  $h^m = Y/X^{sk^{-1}}$ , recover m from  $h^m$ .

Remark 5.1. As with the standard exponential ElGamal, decryption can only be efficiently done when the message is small. However, it suffices to instantiate our generic DCP framework with small message space (say, 32-bits). In this setting, our implementation shows that decryption can be very efficient. Indeed, it is actually 30 times faster than the well-known homomorphic encryption scheme Paillier at the same security level. See Section 7.1 for details.

Correctness and additive homomorphism are obvious. The standard IND-CPA security can be proved in the standard model based on the divisible DDH assumption. We provide the proof in Appendix A for the sake of completeness.

In this work, we require twisted ElGamal is also secure in the single plaintext, two recipient setting. A natural solution is simply concatenating independently encryptions for two recipients, a.k.a.,  $\operatorname{Enc}(pk_1,m;r_1)||\operatorname{Enc}(pk_2,m;r_2)|$ . Security of this solution is implied by the results independently proved by Bellare et al. [BBM00] and Baudron et al. [BPS00]. Kurosawa [Kur02] showed that for standard El-Gamal PKE, randomness can be reused in the single-plaintext, multi-recipient setting. Zether [BAZB20] utilizes Kurosawa's result to make their zero-knowledge component more efficient. Thanks to the random self-reducibility of the divisible DDH problem, our twisted ElGamal PKE is also secure in the single-plaintext, multi-recipient setting even after implementing the randomness reusing trick. This not only shortens the overall transaction size, but also improves the efficiency of the associated zero-knowledge proofs. The security proof is summarized in the following theorem.

**Theorem 5.1.** Twisted ElGamal is IND-CPA secure (1-plaintext/2-recipient) based on the divisible DDH assumption.

*Proof.* We proceed via two games. Let  $S_i$  be the probability that  $\mathcal{A}$  wins in Game i.

**Game 0.** The real IND-CPA security experiment in the 1-plaintext/2-recipient setting. Challenger  $\mathcal{CH}$  interacts with  $\mathcal{A}$  as below:

- 1. Setup:  $\mathcal{CH}$  runs  $pp \leftarrow \mathsf{Setup}(1^{\lambda})$ , runs  $\mathsf{KeyGen}(pp)$  twice independently to obtain  $(pk_1 = g^{sk_1}, sk_1)$  and  $(pk_2 = g^{sk_2}, sk_2)$ , then sends  $(pp, pk_1, pk_2)$  to  $\mathcal{A}$ .
- 2. Challenge:  $\mathcal{A}$  submits  $m_0, m_1$  to  $\mathcal{CH}$  as the target messages.  $\mathcal{CH}$  picks a random bit  $\beta$  and fresh randomness r, computes  $X_1 = pk_1^r$ ,  $X_2 = pk_2^r$ ,  $Y = g^r h^{m_\beta}$ , sends  $C = (X_1, X_2, Y)$  to  $\mathcal{A}$ .
- 3. Guess:  $\mathcal{A}$  outputs its guess  $\beta'$  for  $\beta$  and wins if  $\beta' = \beta$ .

According to the definition of Game 0, we have:

$$\mathsf{Adv}_{\mathcal{A}}(\lambda) = \Pr[S_0] - 1/2.$$

Game 1. Same as Game 0 except  $\mathcal{CH}$  generates the challenge ciphertext in a different way:

2. Challenge:  $\mathcal{A}$  submits  $m_0, m_1$  to  $\mathcal{CH}$  as the target messages.  $\mathcal{CH}$  picks a random bit  $\beta$  and two independent randomness r and s, computes  $X_1 = pk_1^r$ ,  $X_2 = pk_2^r$ ,  $Y = g^sh^{m_{\beta}}$ , sends  $C = (X_1, X_2, Y)$  to  $\mathcal{A}$ .

In Game 1, the distribution of C is independent of  $\beta$ , thus the following holds even for unbounded A:

$$\Pr[S_1] = 1/2$$

It remains to prove  $\Pr[S_1]$  and  $\Pr[S_0]$  are negligibly close. We prove this by showing if not so, we can build an adversary  $\mathcal{B}$  breaks the divisible DDH assumption with the same advantage. Let the public parameter be  $(\mathbb{G}, g, p)$ , given the divisible DDH challenge instance  $(g, g^a, g^b, g^c)$ ,  $\mathcal{B}$  is asked to decide if it is a divisible DDH tuple or a random tuple. To do so,  $\mathcal{B}$  interacts with  $\mathcal{A}$  by simulating  $\mathcal{A}$ 's challenger in the following IND-CPA experiment.

- 1. <u>Setup</u>:  $\mathcal{B}$  picks  $t \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$ , picks  $h \stackrel{\mathbb{R}}{\leftarrow} \mathbb{G}^*$ , sends  $pp = (\mathbb{G}, g, h, p)$  and  $pk_1 = g^b$  and  $pk_2 = g^{bt}$  to  $\mathcal{A}$ . Here, b and bt serve as  $sk_1$  and  $sk_2$ , which are unknown to  $\mathcal{B}$ .
- 2. Challenge:  $\mathcal{A}$  submits  $m_0, m_1$  to  $\mathcal{B}$ .  $\mathcal{B}$  picks a random bit  $\beta$  and sets  $X_1 = g^a, X_2 = g^{at}, Y = g^c h^{m_{\beta}}$ , sends  $C = (X_1, X_2, Y)$  to  $\mathcal{A}$ .
- 3. Guess:  $\mathcal{A}$  outputs a guess  $\beta'$  for  $\beta$ .  $\mathcal{B}$  outputs "1" if  $\beta' = \beta$  and "0" otherwise.

If  $(g, g^a, g^b, g^c)$  is a divisible DDH tuple,  $\mathcal{B}$  simulates Game 0 perfectly (with randomness c = a/b). Else,  $\mathcal{B}$  simulates Game 1 perfectly (with two independent randomness a/b and c). Thereby, we have  $\mathsf{Adv}_{\mathcal{B}} = |\Pr[S_0] - \Pr[S_1]|$ , which is negligible in  $\lambda$  by the divisible DDH assumption.

Putting all the above together, Theorem 5.1 follows. It is easy to see that this theorem extends to 1-message/n-recipient setting as well.

Schnorr Signature. We choose Schnorr signature [Sch91] as the signature component. The choice is out of the following reasons. First, the setup and key generation algorithms of Schnorr signature are almost identical to those of twisted ElGamal. Second, the signing procedure of Schnorr signature is irrelevant to the decryption procedure of twisted ElGamal. This suggests that we are able to safely implement key reuse strategy to build ISE, as we will rigorously prove later.

Last but not the least, Schnorr signature is efficient and can be easily adapted to multi-signature scheme, which is particularly useful in cryptocurrencies setting [BN06], e.g. shrinking the size of the ledger [BDN18].

For the sake of completeness, we recall the Schnorr signature and sketch its security proof as below.

- Setup(1 $^{\lambda}$ ): run ( $\mathbb{G}, g, p$ )  $\leftarrow$  GroupGen(1 $^{\lambda}$ ), pick a cryptographic hash function  $H: M \times \mathbb{G} \to \mathbb{Z}_p$ , where M denotes the message space. Finally, output  $pp = (\mathbb{G}, g, p, H)$  as global public parameters.
- KeyGen(pp): on input pp, choose  $sk \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$ , set  $pk = g^{sk}$ .
- Sign(sk, m): pick  $r \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$ , set  $A = g^r$ , compute  $e = \mathsf{H}(m, A)$ ,  $z = r + sk \cdot e \mod p$ , output  $\sigma = (A, z)$ .
- Verify $(pk, \sigma, m)$ : parse  $\sigma = (A, z)$ , compute  $e = \mathsf{H}(m, A)$ , output "1" if  $g^z = A \cdot pk^e$  and "0" otherwise.

**Theorem 5.2** ([PS00]). Assume H is a random oracle, Schnorr signature is sEUF-CMA secure based on the discrete logarithm assumption.

We sketch the security proof due to Pointcheval and Stern [PS00] as follows. On a high level, the reduction  $\mathcal{R}$  (from DLP to the sEUF-CMA security of Schnorr signature) works as follows. Given the DLP challenge instance  $(g, g^a)$ ,  $\mathcal{R}$  embeds  $g^a$  into the public key (i.e., sets  $pk = g^a$ ), then simulates the signing oracle by programming the random oracle H without knowledge of a and uses the oracle-replay attack to obtain two different forgeries that share the signing randomness  $(A = g^r)$  from the forger. This enables  $\mathcal{R}$  to solve for a.

ISE from Schnorr signature and twisted ElGamal encryption. By merging the Setup and KeyGen algorithms of twisted ElGamal encryption and Schnorr signature, we obtain the ISE scheme, whose joint security is captured by the following theorem.

**Theorem 5.3.** The obtained ISE scheme is jointly secure if the twisted ElGamal is IND-CPA secure (1-plaintext/2-recipient) and the Schnorr signature is sEUF-CMA secure.

*Proof.* As we analyzed before, in our generic DCP construction we only require the PKE component of ISE satisfying passive security, thus the security of the signature component is implied by its standalone sEUF-CMA security. In what follows, we prove the security for the PKE component, namely IND-CPA security (in the 1-plaintext/2-recipient setting) in the presence of a signing oracle for Schnorr signature. We proceed via a sequence of games.

**Game 0.** The real security experiment for ISE's PKE component (cf. Definition 2.4). Challenger  $\mathcal{CH}$  interacts with  $\mathcal{A}$  as below:

- 1. Setup:  $\mathcal{CH}$  runs  $pp \leftarrow \mathsf{Setup}(1^{\lambda})$ , runs  $\mathsf{KeyGen}(pp)$  twice independently to obtain  $(pk_1 = g^{sk_1}, sk_1)$  and  $(pk_2 = g^{sk_2}, sk_2)$ , then sends  $(pp, pk_1, pk_2)$  to  $\mathcal{A}$ .
- 2. Queries: Throughout the experiment,  $\mathcal{A}$  can make hash queries and signing queries.  $\mathcal{CH}$  emulates random oracle by maintaining a list  $T_{\text{hash}}$  using standard lazy method.  $T_{\text{hash}}$  is initially empty, which stores triples  $(\cdot, \cdot, \cdot) \in M \times \mathbb{G} \times \mathbb{Z}_p$ , an entry (m, A, e) indicates that  $e := \mathsf{H}(m, A)$ . Concretely,  $\mathcal{CH}$  responds  $\mathcal{A}$ 's queries as below:
  - Hash queries: On query (m, A), if there is an entry (m, A, e) in  $T_{\text{hash}}$ ,  $\mathcal{CH}$  returns e. Else,  $\mathcal{CH}$  picks  $e \overset{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$  and inserts (m, A, e) in  $T_{\text{hash}}$ , then returns e.
  - Signing queries for  $sk_1$  or  $sk_2$ :  $\mathcal{CH}$  responds with the corresponding secret key. On query (b,m) where  $b \in \{0,1\}$  indicating which secret key is used,  $\mathcal{CH}$  picks a fresh randomness  $r \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$ , sets  $A = g^r$ , obtains  $e = \mathsf{H}(m,A)$  by accessing the hash oracle, then computes  $z = r + sk_b \cdot e \mod p$ , returns  $\sigma = (A, z)$ .

- 3. Challenge:  $\mathcal{A}$  submits  $m_0, m_1$  to  $\mathcal{CH}$  as the target messages.  $\mathcal{CH}$  picks a random bit  $\beta$  and fresh randomness r, computes  $X_1 = pk_1^r$ ,  $X_2 = pk_2^r$ ,  $Y = g^r h^{m_\beta}$ , sends  $C = (X_1, X_2, Y)$  to  $\mathcal{A}$ .
- 4. Guess:  $\mathcal{A}$  outputs its guess  $\beta'$  for  $\beta$  and wins if  $\beta' = \beta$ .

According to the definition of Game 0, we have:

$$\mathsf{Adv}_{\mathcal{A}}(\lambda) = \Pr[S_0] - 1/2.$$

**Game 1.** The same as Game 0 except that  $\mathcal{CH}$  simulates signing oracle by programming random oracle H, rather than using the real secret keys. To do so,  $\mathcal{CH}$  emulates random oracle by maintaining a list  $T_{\text{hash}}^*$ , which is initially empty.  $T_{\text{hash}}^*$  stores triples  $(\cdot, \cdot, \cdot, \cdot, \cdot) \in M \times \mathbb{G} \times \mathbb{Z}_p \times \{0, 1\}$ , an entry (m, A, e, \*, \*) indicates that  $e := \mathsf{H}(m, A)$ , where \* is the wild-card.

- 2. Queries:  $\mathcal{CH}$  handles hash queries and signing queries as below.
  - Hash queries: On query (m, A), if there is an entry (m, A, e, \*, \*) in  $T_{\text{hash}}^*$ ,  $\mathcal{CH}$  returns e. Else,  $\mathcal{CH}$  picks  $e \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$ , inserts  $(m, A, e, \bot, \bot)$  in  $T_{\text{hash}}^*$ , then returns e.
  - Signing queries for  $sk_1$  or  $sk_2$ : On query (b, m), if there is an entry (m, A, e, z, b) (with the same value of b and m) in the  $T^*_{\text{hash}}$  list,  $\mathcal{CH}$  simply returns  $\sigma = (A, z)$ . Otherwise,  $\mathcal{CH}$  picks  $z, e \overset{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$ , sets  $A = g^z/pk_b^e$ , if there is no entry indexed by (m, A) in  $T^*_{\text{hash}}$ , then inserts (m, A, e, z, b) in  $T^*_{\text{hash}}$  and returns  $\sigma = (A, z)$ . Else,  $\mathcal{CH}$  aborts to avoid possible inconsistence in programming.

Denote the event that  $\mathcal{CH}$  aborts in Game 1 by E. Conditioned on E does not occur,  $\mathcal{A}$ 's view in Game 0 and Game 1 are identical. This follows from the fact that  $\mathcal{CH}$  perfectly mimic the hash oracle and signing oracle. Let  $Q_{\text{hash}}$  and  $Q_{\text{sign}}$  be the maximum number of hash queries and signing queries that  $\mathcal{A}$  makes during security experiment. By the union bound, we conclude that  $\Pr[E] \leq (Q_{\text{hash}}Q_{\text{sign}})/p$ , which is negligible in  $\lambda$ . In summary, we have:

$$|\Pr[S_1] - \Pr[S_0]| \le \Pr[E] \le \mathsf{negl}(\lambda)$$

It remains to bound  $Pr[S_1]$ . We have the following claim.

Claim 5.4. Assume the IND-CPA security (1-plaintext/2-recipient) of twisted ElGamal PKE,  $Pr[S_1]$  is negligible in  $\lambda$  for any PPT adversary A.

*Proof.* We prove this claim by showing that if there exists a PPT adversary  $\mathcal{A}$  has non-negligible advantage in Game 1, we can build a PPT adversary  $\mathcal{B}$  that breaks the IND-CPA security (single-message, two-recipient) of twisted ElGamal PKE with the same advantage. Note that according to the definition of Game 1,  $\mathcal{CH}$  can simulate the signing oracles without using the secret keys. Thereby, given  $(pp, pk_1, pk_2)$ ,  $\mathcal{B}$  can perfectly simulate Game 1 by forwarding the encryption challenge to its own challenger. This proves the claim.

Putting all the above together, Theorem 5.3 immediately follows.

#### 5.2 Instantiating NIZK

Now, we design efficient NIZK proof systems for basic validity policy  $(L_{\text{legal}})$  and more extended policies  $(L_{\text{limit}}, L_{\text{rate}}, L_{\text{open}})$ .

As stated in Section 4.1,  $L_{\text{legal}}$  can be decomposed as  $L_{\text{equal}} \wedge L_{\text{right}} \wedge L_{\text{solvent}}$ . Let  $\Pi_{\text{equal}}$ ,  $\Pi_{\text{right}}$ ,  $\Pi_{\text{solvent}}$  be NIZK for  $L_{\text{equal}}$ ,  $L_{\text{right}}$ ,  $L_{\text{solvent}}$  respectively, and let  $\pi_{\text{legal}} := \Pi_{\text{equal}} \circ \Pi_{\text{right}} \circ \Pi_{\text{solvent}}$ , where  $\circ$  denotes sequential composition. By the property of NIZK for conjunctive statements [Gol06],  $\pi_{\text{legal}}$  is a NIZK proof system for  $L_{\text{legal}}$ . Now, the task breaks down to design  $\Pi_{\text{equal}}$ ,  $\Pi_{\text{right}}$ ,  $\Pi_{\text{solvent}}$ . We describe them one by one as below.

<sup>&</sup>lt;sup>10</sup>In the non-interactive setting, there is no distinction between sequential and parallel composition.

## 5.2.1 NIZK for $L_{\text{equal}}$

According to our DCP construction and definition of twisted ElGamal, L<sub>equal</sub> can be written as:

$$\{(pk_1, X_1, Y_1, pk_2, X_2, Y_2) \mid \exists r_1, r_2, v \text{ s.t. } X_i = pk_i^{r_i} \land Y_i = g^{r_i}h^v \text{ for } i = 1, 2\}.$$

As analyzed before, for twisted ElGamal randomness can be safely reused in the 1-plaintext/2recipient setting.  $L_{\text{equal}}$  can thus be simplified to:

$$\{(pk_1, pk_2, X_1, X_2, Y) \mid \exists r, v \text{ s.t. } Y = g^r h^v \land X_i = pk_i^r \text{ for } i = 1, 2\}.$$

Sigma protocol for  $L_{\text{equal}}$ . To obtain a NIZK for  $L_{\text{equal}}$ , we first design a Sigma protocol  $\Sigma_{\text{equal}}$  $(\mathsf{Setup}, P, V)$  for  $L_{\mathrm{equal}}$ . The  $\mathsf{Setup}$  algorithm of  $\Sigma_{\mathrm{equal}}$  is same as that of the twisted ElGamal. On statement  $(pk_1, pk_2, X_1, X_2, Y)$ , P and V interact as below:

- 1. P picks  $a, b \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$ , sends  $A_1 = pk_1^a$ ,  $A_2 = pk_2^a$ ,  $B = g^a h^b$  to V.
- 2. V picks  $e \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$  and sends it to P as the challenge.
- 3. P computes  $z_1 = a + er$ ,  $z_2 = b + ev$  using witness w = (r, v), then sends  $(z_1, z_2)$  to V. V accepts iff the following three equations hold simultaneously:

$$pk_1^{z_1} = A_1 X_1^e (1)$$

$$pk_2^{z_1} = A_2 X_2^e (2)$$

$$pk_2^{z_1} = A_2 X_2^e$$

$$g^{z_1} h^{z_2} = BY^e$$
(2)

$$(pk_1, pk_2, X_1, X_2, Y)$$

$$P$$

$$w = (r, v)$$

$$a, b \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$$

$$A_1 \leftarrow pk_1^a, A_2 \leftarrow pk_2^a$$

$$B \leftarrow g^a h^b$$

$$z = a + er$$

$$t = b + ev$$

$$A_1, A_2, B \rightarrow e$$

$$e \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$$

$$check if$$

$$pk_1^z = A_1 X_1^e$$

$$pk_2^z = A_2 X_2^e$$

$$g^z h^t = BY^e$$

Figure 2:  $\Sigma_{\rm equal}$  for  $L_{\rm equal}$ : proof of knowledge of two twisted ElGamal ciphertexts encrypting the same value under different public keys

**Lemma 5.5.**  $\Sigma_{\text{equal}}$  is a public-coin SHVZK proof of knowledge for  $L_{\text{equal}}$ .

*Proof.* We prove that all three properties required for Sigma protocol are met.

Perfect completeness is obvious from simple calculation.

To show special soundness, fix the initial message  $(A_1, A_2, B)$ , suppose there are two accepting transcripts (e, z, t) and (e', z', t') with  $e \neq e'$ , the witness can be extracted as below. From either (1) or (2), we have z = a + er and z' = a + e'r, which implies r = (z - z')/(e - e'). Further from (3), we have t = b + ev and t' = b + e'v, which imply v = (t - t')/(e - e').

To show special HVZK, for a fixed challenge e, the simulator  $\mathcal{S}$  works as below: picks  $z, t \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$ , computes  $A_1 = pk_1^z/X_1^e$ ,  $A_2 = pk_2^z/X_2^e$ ,  $B = g^zh^t/Y^e$ . Clearly,  $(A_1, A_2, B, e, z, t)$  is an accepting transcript, and it is distributed as in the real protocol.

This proves Lemma 5.5.

Applying Fiat-Shamir transform to  $\Sigma_{\rm equal}$ , we obtain  $\Pi_{\rm equal}$ , which is actually a NIZKPoK for  $L_{\rm equal}$ . To support global supervision, the send has to encrypt the transfer value using 1-message/3-recipient twisted ElGamal and then prove the correctness of encryption. For completeness, we include the accompanying NIZK in Appendix A.2.

#### 5.2.2 NIZK for $L_{right}$

According to our DCP construction and the definition of twisted ElGamal,  $L_{\text{right}}$  can be written as:

$$\{(pk, X, Y) \mid \exists r, v \text{ s.t. } X = pk^r \land Y = g^r h^v \land v \in \mathcal{V}\}.$$

For ease of analysis, we additionally define  $L_{\rm enc}$  and  $L_{\rm range}$  as below:

$$L_{\text{enc}} = \{ (pk, X, Y) \mid \exists r, v \text{ s.t. } X = pk^r \land Y = g^r h^v \}$$
  
$$L_{\text{range}} = \{ Y \mid \exists r, v \text{ s.t. } Y = g^r h^v \land v \in \mathcal{V} \}$$

It is straightforward to verify that  $L_{\text{right}} \subset L_{\text{enc}} \wedge L_{\text{range}}$ . Observe that each instance  $(pk, X, Y) \in L_{\text{right}}$  has a unique witness, while the last component Y can be viewed as a Pedersen commitment of value v under commitment key (g, h), whose discrete logarithm  $\log_g h$  is unknown to any users. To prove  $(pk, X, Y) \in L_{\text{right}}$ , we first prove  $(pk, X, Y) \in L_{\text{enc}}$  with witness (r, v) via a Sigma protocol  $\Sigma_{\text{enc}} = (\text{Setup}, P_1, V_1)$ , then prove  $Y \in L_{\text{range}}$  with witness (r, v) via a Bulletproof  $\Lambda_{\text{bullet}} = (\text{Setup}, P_2, V_2)$ .

Sigma protocol for  $L_{enc}$ . We begin with a Sigma protocol  $\Sigma_{enc} = (\mathsf{Setup}, P, V)$  for  $L_{enc}$ . The Setup algorithm is same as that of twisted ElGamal. On statement x = (pk, X, Y), P and V interact as below:

- 1. P picks  $a, b \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$ , sends  $A = pk^a$  and  $B = g^a h^b$  to V.
- 2. V picks  $e \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$  and sends it to P as the challenge.
- 3. P computes  $z_1 = a + er$ ,  $z_2 = b + ev$  using witness w = (r, v), then sends  $(z_1, z_2)$  to V. V accepts iff the following two equations hold simultaneously:

$$pk^{z_1} = AX^e (4)$$

$$g^{z_1}h^{z_2} = BY^e (5)$$

**Lemma 5.6.**  $\Sigma_{\rm enc}$  is a public-coin SHVZK proof of knowledge for  $L_{\rm enc}$ .

*Proof.* We prove that all three properties required for Sigma protocol are met.

Perfect completeness is obvious from simple calculation.

To show special soundness, fix the initial message (A, B), suppose there are two accepting transcripts  $(e, z = (z_1, z_2))$  and  $(e', z' = (z'_1, z'_2))$  with  $e \neq e'$ , we can extract the witness as below. From (4), we have  $z_1 = a + er$  and  $z'_1 = a + e'r$ , which implies  $r = (z_1 - z'_1)/(e - e')$ . Further from (5), we have  $z_2 = b + ev$  and  $z'_2 = b + e'v$ , which imply  $v = (z_2 - z'_2)/(e - e')$ .

To show special HVZK, for a fixed challenge e, the simulator S works as below: picks  $z_1, z_2 \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$ , computes  $A = pk^{z_1}/X^e$ ,  $B = g^{z_1}h^{z_2}/Y^e$ . Clearly,  $(A, B, e, z_1, z_2)$  is an accepting transcript, and it is distributed as in the real protocol.

This proves Lemma 5.6.

Bulletproofs for  $L_{\text{range}}$ . We employ the logarithmic size Bulletproof  $\Lambda_{\text{bullet}} = (\mathsf{Setup}, P, V)$  to prove  $L_{\text{range}}$ . To avoid repetition, we refer to [BBB<sup>+</sup>18, Section 4.2] for the details of the interaction between P and V.

**Lemma 5.7.** [BBB<sup>+</sup> 18, Theorem 3] Assuming the hardness of discrete logarithm problem,  $\Lambda_{\text{bullet}}$  is a public-coin SHVZK argument of knowledge for  $L_{\text{range}}$ .

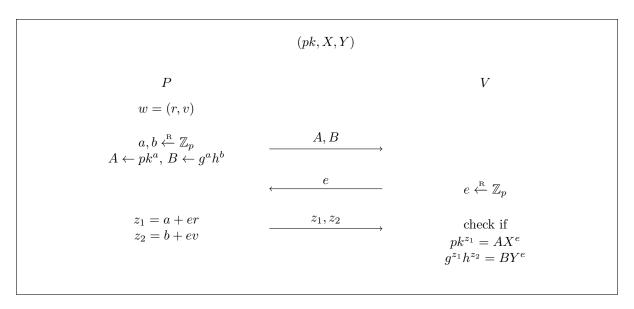


Figure 3:  $\Sigma_{\rm enc}$  for  $L_{\rm enc}$ : proof of knowledge of twisted ElGamal ciphertext

Sequential composition. Let  $\Gamma_{\text{right}} = \Sigma_{\text{enc}} \circ \Lambda_{\text{bullet}}$  be the sequential composition of  $\Sigma_{\text{enc}}$  and  $\Lambda_{\text{bullet}}$ . The Setup algorithm of  $\Gamma_{\text{right}}$  is a merge of that of  $\Sigma_{\text{enc}}$  and  $\Lambda_{\text{bullet}}$ . For range  $\mathcal{V} = [0, 2^{\ell} - 1]$ , it first generates a group  $\mathbb{G}$  of prime order p together with two random generators g and h, then picks independent generators g,  $\mathbf{h} \in \mathbb{G}^{\ell}$ . Let  $P_1 = \Sigma_{\text{enc}}.P$ ,  $V_1 = \Sigma_{\text{enc}}.V$ ,  $P_2 = \Lambda_{\text{bullet}}.P$ ,  $V_2 = \Lambda_{\text{bullet}}.V$ . We have  $\Gamma_{\text{right}}.P = (P_1, P_2)$ ,  $\Gamma_{\text{right}}.V = (V_1, V_2)$ .

**Lemma 5.8.** Assuming the discrete logarithm assumption,  $\Gamma_{right} = (Setup, P, V)$  is a public-coin SHVZK argument of knowledge for  $L_{right}$ .

*Proof.* The completeness and zero-knowledge properties of  $\Gamma_{\text{right}}$  follow from that of  $\Sigma_{\text{enc}}$  and  $\Lambda_{\text{bullet}}$ .

In order to prove argument of knowledge, it suffices to construct a PPT witness extraction algorithm E. Let  $E_1$  and  $E_2$  be the witness extraction algorithm for  $\Sigma_{\rm enc}$  and  $\Lambda_{\rm bullet}$  respectively. Note that  $\Gamma_{\rm right}$  is a sequential composition of  $\Sigma_{\rm enc}$  and  $\Lambda_{\rm bullet}$ , thus an  $(n_0, n_1, \ldots, n_k)$ -tree of accepting transcripts for  $\Gamma_{\rm right}$  is a composition of an  $n_0$ -subtree of accepting transcripts for  $\Sigma_{\rm enc}$  (where  $n_0=2$ ) and an  $(n_1, \ldots, n_k)$ -subtree of accepting transcripts for  $\Lambda_{\rm bullet}$  (see [BBB+18] for the concrete values of  $n_1, \ldots, n_k$ ).

E first runs  $E_1$  on  $n_0$ -subtree of accepting transcripts to extract w=(r,v), then runs  $E_2$  on  $(n_1,\ldots,n_k)$ -subtree of accepting transcripts to extract w'=(r',v'). Finally, E outputs w if w=w' and aborts otherwise. Note that  $\prod_{i=0}^k n_i$  is still bounded by a polynomial of  $\lambda$ .

According to Lemma 5.6,  $E_1$  outputs a witness of statement  $(pk, X, Y) \in L_{\text{enc}}$  with probability 1. According to Lemma 5.7,  $E_2$  outputs a witness of statement  $Y \in L_{\text{range}}$  with overwhelming probability assuming the hardness of DLP. We argue that  $E_2$ 's output w' = (r', v') equals  $E_1$ 's output w = (r, v) with overwhelming probability, since otherwise the relation  $g^v h^r = Y = g^{v'} h^{r'}$  immediately yields a solution  $\log_g h$  to the DLP instance (g, h). We thus conclude that E outputs a witness for  $x = (pk, X, Y) \in L_{\text{right}}$  with overwhelming probability. Thus,  $\Gamma_{\text{right}}$  satisfies computational witness-extended emulation.

Putting all the above together, Lemma 5.8 immediately follows.

Applying Fiat-Shamir transform to  $\Gamma_{\text{right}}$ , we obtain  $\Pi_{\text{right}}$ , which is a NIZKAoK for  $L_{\text{right}}$ .

#### 5.2.3 NIZK for $L_{\text{solvent}}$

According to our generic DCP construction,  $L_{\text{solvent}}$  is defined as:

$$\{(pk, \tilde{C}, C) \mid \exists sk \text{ s.t. } (pk, sk) \in \mathsf{R}_{\text{key}} \land \mathsf{ISE.Dec}(sk, \tilde{C} - C) \in \mathcal{V}\}.$$

In the above,  $\tilde{C}=(\tilde{X}=pk^{\tilde{r}},\tilde{Y}=g^{\tilde{r}}h^{\tilde{v}})$  is the encryption of current balance  $\tilde{v}$  of account pk under randomness  $\tilde{r}$ ,  $C=(X=pk^r,Y=g^rh^v)$  encrypts the transfer amount v under randomness r. Let

$$C' = (X' = pk^{r'}, Y' = g^{r'}h^{m'}) = \tilde{C} - C$$
,  $L_{\text{solvent}}$  can be rewritten as:

$$\{(pk, C') \mid \exists r', m' \text{ s.t. } C' = \text{ISE.Enc}(pk, m'; r') \land m' \in \mathcal{V}\}.$$

By the additive homomorphism of twisted ElGamal, we have  $r' = \tilde{r} - r$ ,  $m' = \tilde{v} - v$ . It seems that we can prove  $L_{\text{solvent}}$  using the same protocol for  $L_{\text{right}}$ . However, while the sender (playing the role of prover) learns  $\tilde{v}$  (by decrypting  $\tilde{C}$  with sk), v and r, it generally does not know the randomness  $\tilde{r}$ . This is because  $\tilde{C}$  is the sum of all the incoming and outgoing transactions of pk, whereas the randomness behind incoming transactions is unknown. The consequence is that r' (the first part of witness) is unknown, which renders us unable to directly invoke the Bulletproof on instance Y'.

Our trick is encrypting the value  $m' = (\tilde{v} - v)$  under a fresh randomness  $r^*$  to obtain a new ciphertext  $C^* = (X^*, Y^*)$ , where  $X^* = pk^{r^*}$ ,  $Y^* = g^{r^*}h^{m'}$ .  $C^*$  could be viewed as a refreshment of C'. In a nutshell, we express  $L_{\text{solvent}}$  as  $L_{\text{equal}} \wedge L_{\text{right}}$ , a.k.a. we have:

$$(pk, C') \in L_{\text{solvent}} \iff (pk, C', C^*) \in L_{\text{equal}} \land (pk, C^*) \in L_{\text{right}}$$

To prove that C' and  $C^*$  encrypt the same value under public key pk, we cannot simply use a Sigma protocol like Protocol 2, in which the prover uses the message and randomness as witness, as the randomness r' behind C' is typically unknown. Luckily, we are able to prove this more efficiently by using the secret key as witness. Generally,  $L_{\text{equal}}$  can be written as:

$$\{(pk, C_1, C_2) \mid \exists sk \text{ s.t. } (pk, sk) \in \mathsf{R}_{kev} \land \mathsf{ISE.Dec}(sk, C_1) = \mathsf{ISE.Dec}(sk, C_2)\}$$

When instantiated with twisted ElGamal,  $C_1 = (X_1 = pk^{r_1}, Y_1 = g^{r_1}h^m)$  and  $C_2 = (X_2 = pk^{r_2}, Y_2 = g^{r_2}h^m)$  are ciphertexts under the same public key pk, then proving membership of  $L_{\rm equal}$  is equivalent to proving  $\log_{Y_1/Y_2} X_1/X_2$  equals  $\log_g pk$ . This can be efficiently done by utilizing the Sigma protocol  $\Sigma_{\rm ddh} = ({\sf Setup}, P, V)$  for discrete logarithm equality due to Chaum and Pedersen [CP92]. For the sake of completeness, we describe it as below.

The Setup algorithm generates a cyclic group  $\mathbb{G}$  of prime order p. Define the language  $L_{\rm ddh}$  as below:

$$L_{\text{ddh}} = \{(g_1, h_1, g_2, h_2) \mid \exists w \in \mathbb{Z}_p \text{ s.t. } \log_{g_1} h_1 = w = \log_{g_2} h_2\}$$

It is easy to see that  $\log_{g_1} h_1 = \log_{g_2} h_2$  iff  $(g_1, h_1, g_2, h_2)$  constitutes a DDH tuple. On statement  $x = (g_1, h_1, g_2, h_2)$ , P interacts with V as below:

- 1. P picks  $a \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$ , sends  $A_1 = g_1^a$ ,  $A_2 = g_2^a$  to V;
- 2. V picks  $e \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$  and sends it to P as the challenge;
- 3. P computes z = a + we with witness w and sends it to V. V accepts iff:

$$g_1^z = A_1 h_1^e \wedge g_2^z = A_2 h_2^e$$

Lemma 5.9 ([CP92]).  $\Sigma_{ddh}$  is a public-coin SHVZK proof of knowledge for  $L_{ddh}$ .

Applying Fiat-Shamir transform to  $\Sigma_{\rm ddh}$ , we obtain a NIZKPoK  $\Pi_{\rm ddh}$  for  $L_{\rm ddh}$ . We then prove  $(pk, C^* = (X^*, Y^*)) \in L_{\rm right}$  using the NIZKPoK  $\Pi_{\rm right}$  as we described before. Let  $\Pi_{\rm solvent} = \Pi_{\rm ddh} \circ \Pi_{\rm right}$ , we conclude that  $\Pi_{\rm solvent}$  is a NIZKPoK for  $L_{\rm solvent}$  by the properties of AND-proofs.

Putting all the sub-protocols described above, we obtain  $\pi_{\text{legal}} = \Pi_{\text{equal}} \circ \Pi_{\text{right}} \circ \Pi_{\text{solvent}}$ .

**Theorem 5.10.**  $\Pi_{\text{valid}}$  is a NIZKAoK for  $L_{\text{valid}} = L_{\text{equal}} \wedge L_{\text{right}} \wedge L_{\text{solvent}}$ .

*Proof.* The proof of this theorem follows from the properties of AND-proofs.

Two useful proof gadgets. The above construction contributes two useful "proof gadgets" for generating range proofs for values in encrypted form.  $\Pi_{\text{right}}$  constitutes Gadget-1, which admits a prover to generate range proofs with knowledge of message and randomness.  $\Pi_{\text{right}}$  constitutes Gadget-2, which admits a prover to generate range proofs with knowledge of secret key.

$$P(w)$$

$$a \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_{p}$$

$$A_{1} \leftarrow g_{1}^{a}, A_{2} \leftarrow g_{2}^{a}$$

$$\leftarrow e$$

$$check if$$

$$g_{1}^{z} = A_{1}h_{1}^{e}$$

$$g_{2}^{z} = A_{2}h_{2}^{e}$$

$$\downarrow V$$

$$e \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_{p}$$

$$check if$$

$$g_{1}^{z} = A_{1}h_{1}^{e}$$

$$g_{2}^{z} = A_{2}h_{2}^{e}$$

Figure 4:  $\Sigma_{\rm ddh}$  for  $L_{\rm ddh}$ : proof of knowledge of discrete logarithm equality/DDH tuple

#### 5.2.4 NIZK for $L_{\text{limit}}$

According to our DCP construction and the definition of twisted ElGamal,  $L_{\text{limit}}$  can be written as:

$$\{(pk, \{C_i\}_{1 \leq i \leq n}, a_{\max}) \mid \exists sk \text{ s.t. } (pk, sk) \in \mathsf{R}_{key} \land v_i = \mathsf{ISE.Dec}(sk, C_i) \land \sum_{i=1}^n v_i \leq a_{\max}\}$$

Let  $C_i = (X_i = pk^{r_i}, Y_i = g^{r_i}h^{v_i})$ . By additive homomorphism of twisted ElGamal, the prover first computes  $C = \sum_{i=1}^n C_i = (X = pk^r, Y = g^rh^v)$ , where  $r = \sum_{i=1}^n r_i$ ,  $v = \sum_{i=1}^n v_i$ . As aforementioned, in PGC users are not required to maintain history state, which means that users may forget the related random coins when cooperating with audit. Nevertheless, this is not a problem. It is equivalent to prove  $(pk, C) \in L_{\text{solvent}}$ , which can be done by using Gadget-2.

# 5.2.5 NIZK for $L_{\text{rate}}$

According to our DCP construction and the definition of twisted ElGamal,  $L_{\text{rate}}$  can be written as:

$$\{(pk, C_1, C_2, \rho) \mid \exists sk \text{ s.t. } (pk, sk) \in \mathsf{R}_{key} \land v_i = \mathsf{ISE.Dec}(sk, C_i) \land v_1/v_2 = \rho\}$$

Without much loss of generality, we assume  $\rho = \alpha/\beta$ , where  $\alpha$  and  $\beta$  are two positive integers that are much smaller than p. Let  $C_1 = (pk^{r_1}, g^{r_1}h^{v_1})$ ,  $C_2 = (pk^{r_2}, g^{r_2}h^{v_2})$ . By additive homomorphism of twisted ElGamal, we compute  $C_1' = \beta \cdot C_1 = (X_1' = pk^{\beta r_1}, Y_1' = g^{\beta r_1}h^{\beta v_1})$ ,  $C_2' = \alpha \cdot C_2 = (X_2' = pk^{\alpha r_2}, Y_2' = g^{\alpha r_2}h^{\alpha v_2})$ . Note that  $v_1/v_2 = \rho = \alpha/\beta$  if and only if  $h^{\beta v_1} = h^{\alpha v_2}$ ,  $h^{\gamma v_1} = h^{\gamma v_2}$ . In the is equivalent to  $(Y_1'/Y_2', X_1'/X_2', g, pk) \in L_{ddh}$ , which in turn can be efficiently proved via  $\Pi_{ddh}$  for discrete logarithm equality using sk as witness, as already described in Protocol 4.

#### 5.2.6 NIZK for $L_{\text{open}}$

According to our DCP construction and the definition of twisted ElGamal,  $L_{\rm open}$  can be written as:

$$\{(pk, C = (X, Y), v) \mid \exists sk \text{ s.t. } X = (Y/h^v)^{sk} \land pk = g^{sk}\}$$

The above language is equivalent to  $(Y/h^v, X, g, pk) \in L_{ddh}$ , which in turn can be efficiently proved via  $\Pi_{ddh}$  for discrete logarithm equality, as already described in Protocol 4.

<sup>&</sup>lt;sup>11</sup>Since both  $v_1, v_2, \alpha, \beta$  are much smaller than p, no overflow will happen.

# 6 Further Discussions

## 6.1 Transparent Setup

Benefit from our dedicate design, the global parameters of PGC are exactly the public parameters of Bulletproofs, a.k.a. simply random generators  $(g, h, \mathbf{g}, \mathbf{h})$  without special structure. We can thus use hash function to dismiss trusted setup. We refer to [BBB+18] for more details on this point.

# 6.2 Necessity of Composing Sigma Protocol with Bulletproof

We prove the membership of  $L_{\text{right}}$  by sequentially composing a Sigma protocol for  $L_{\text{enc}}$  and a Bulletproof for  $L_{\text{range}}$ . One may wonder if we can only use the Bulletproof for  $L_{\text{range}}$  to fulfill this task. Actually, that will be problematic. Consider a malicious prover  $P^*$  generates ciphertext dishonestly as below: picks random r, r' from  $\mathbb{Z}_q$  and  $v' \in \mathcal{V}$ , sets  $X = pk^r$ ,  $Y = g^{r'}h^{v'}$ .  $P^*$  can thus use r' and v' as witness to generate a valid Bulletproof for  $Y \in L_{\text{range}}$ . However, the real message v encrypted by (X, Y) falls outside the range  $\mathcal{V}$  with overwhelming probability, and thus  $(pk, X, Y) \notin L_{\text{right}}$ . This compromises proof of knowledge. It is also instructive to examine where the security proof will break. Let (pk, X, Y) be the statement and (r, v) be the corresponding randomness-message pair. To prove witness-extended emulation via weak forking lemma, we hope to argue that the output of Bulletproof's extractor (r', v') equals (r, v) overwhelmingly based on the collision resistance of hash function  $H(x, y) = g^x h^y$ , which in turn is implied the hardness of DLP related to (g, h). However, the condition of weak forking lemma is required to hold for any PPT adversary  $\mathcal{A}$ , and thus there is no way to embed the collision into the input instance.

# 6.3 Ciphertext Refreshing vs. Key Switching

In order to safely use Bulletproof to prove the membership of  $L_{\text{solvent}}$ , we refresh  $C' = (pk^{r'}, g^{r'}h^{m'})$  to  $C^* = (pk^{r^*}, g^{r^*}h^{m'})$ , then directly invoking the Bulletproof on the second component of  $C^*$  with witness  $(r^*, m')$ . To ensure the soundness of the overall protocol, we also need another protocol to prove that C' and  $C^*$  encrypt the same message.

One may attempt to use "key switching" trick to avoid the overhead of refreshing. Note that  $pk = g^{sk}$ , thus  $g^{r'}h^{m'}$  can be rewritten as  $(pk^{r'})^{sk^{-1}}h^{m'}$ . It seems that now we are able to invoke Bulletproof on  $(pk^{r'})^{sk^{-1}}h^{m'}$  by interpreting  $(pk^{r'},h)$  as Pedersen commitment key and using  $(sk^{-1},m')$  as the witness. The soundness of the Bulletproof's requires that the prover does not know the DL relation between  $(pk^{r'},h)$ . Is it possible for the prover to rigorously prove this intuition? It is well-known that one can generate a proof of knowledge of discrete logarithm using the Schnorr's protocol. But, how can the prover give a proof of no-knowledge? Interestingly, the prover can prove that he indeed does not know  $\log_h pk^{r'}$  by proving his knowledge of  $\log_g pk^{r'}$ . This makes sense because otherwise he can solve the DL relation between (g,h), which contradicts to the assumption. Inevitably, an honest prover falls into a deadlocked setting — he needs to know r' to create a proof of knowledge of  $\log_g pk^{r'}$ , which is exactly the obstacle he want to overcome at the beginning. Even the soundness of the Bulletproof's part holds, additional mechanism is needed to prove that the extractor's output is consistent to the message fixed by the ciphertext, which seems difficult.

We distill the above reasoning to a trick of proving no-knowledge of discrete logarithm, which might be of independent interest. Let y, g and h be group elements, where the DL relation between (g,h) is unknown. A prover can prove that he does not know the knowledge of  $\alpha = \log_h y$  by proving knowledge of  $\beta = \log_g y$ . The soundness follows because if the prover cheated (he knows  $\alpha = \log_h y$ ), then he must be able to solve the discrete logarithm between g and h by computing  $\beta\alpha^{-1}$ .

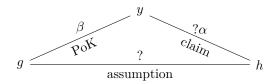


Figure 5: How to prove no-knowledge of  $x = \log_h y$  by assuming the DL relation between (g, h) is intractable

# 7 Optimizations

# 7.1 Faster Decryption for Twisted ElGamal

As in standard exponential ElGamal, to ensure additive homomorphism on message space  $\mathbb{Z}_p$ , message in twisted ElGamal is also encoded to the exponent. This makes decryption inefficient even with knowledge of secret key. This seems problematic because the exact balance is required to be known when generating the range proof for solvency. Similar problem also occurs in other confidential transaction systems [FMMO19, BAZB20].

Fortunately, this is not an issue. First, each user can learn its current balance by keeping track of the incoming and outgoing transfers. While the outgoing transfer amounts are always known to the senders themselves by default, the incoming transfer amounts are also known to the recipients in most times. This makes sense because senders typically communicate the transfer amount to their recipients off-chain in real-world applications. It remains to consider the worst case, i.e., the receivers do not know the exact value of incoming transfer amounts. Suppose the transfer amount is restricted to the range  $[0, 2^{\ell} - 1]$  by protocol specification. Related works [FMMO19, BAZB20] suggested that the receiver can figure out m from  $g^m$  by brute-force enumeration with time complexity  $O(2^{\ell})$ . Here, we recommend employing dedicated algorithms to compute a discrete logarithm in an interval more efficiently. For instance, Pollard's kangaroo method [Pol78] and Galbraith-Ruprai algorithm [GR10] run in time  $O(2^{\ell/2})$  with constant memory cost, Shanks's algorithm [Sha71] runs in time  $O(2^{\ell/2})$  using a table of size  $O(2^{\ell/2})$ , and Bernstein-Lange algorithm [BL12] runs in time  $O(2^{\ell/3})$  using a table of size  $O(2^{\ell/3})$ .

In this work, we choose to implement the Shanks's algorithm, since it admits flexible time/space tradeoff and is amenable to parallelization. In more details, by building a key-value map of size  $O(2^s)$  (with EC
point as key and its discrete logarithm as value), the running time complexity drops to  $O(2^{\ell-s})$ . Assume
each EC point on the underlying curve can be represented by m bits, then a direct implementation
requires at least  $O(2^s m)$  bits to store the key-value map. Very recently, Chatzigiannis et al. [CCN21]
observes that when the set of keys is much smaller than the set of all EC points, the size of key-value map
can be greatly reduced. In this work, we use non-cryptographic hash (such as Murmurhash) to encoding
the keys in a more compact form. On elliptic curve P-256 (also known as secp256r1 and prime256v1),
when  $\ell = 32$ , s = 23, the average decryption time is approximately 0.6ms by using a key-value map of
size 64MB. This demonstrates that Shanks's algorithm is more preferable in practice for small range. If
larger range are needed, say  $[0, 2^{64})$ , the sender can represent a 64-bit value v by a concatenation of two
32-bit values  $v_1||v_0$ , then encrypts  $v_1$  and  $v_0$  independently under receiver's public key, i.e., replacing  $C_r = \text{Enc}(pk_r, v)$  with  $C_{r,1} = \text{Enc}(pk_r, v_1)$  and  $C_{r,0} = \text{Enc}(pk_r, v_0)$ . In this way, time complexity of
decryption only grows linearly in the size of bit-length rather than exponentially.

## 7.2 More Efficient Assembly of NIZK

Towards a modular design of NIZK for  $L_{\rm legal}$ , we break this task to designing NIZK proofs for  $L_{\rm equal}$ ,  $L_{\rm right}$  and  $L_{\rm solvent}$  separately. Particularly, to build NIZK for  $L_{\rm right}$  (asserting a twisted ElGamal ciphertext encrypts a message in expected range), we compose a Sigma protocol  $\Sigma_{\rm enc}$  for  $L_{\rm enc}$  and a Bulletproof  $\Lambda_{\rm bullet}$  for  $L_{\rm range}$  sequentially, then apply the Fiat-Shamir transform. The resulting  $\Pi_{\rm right}$  for  $L_{\rm right}$  can be used as a sub-protocol when proving membership in  $L_{\rm solvent}$ .

 $<sup>^{12}</sup>$ We remark that extra zero-knowledge proofs are needed to ensure that the values are encrypted in a correct form. Similar idea was mentioned in Zether [BAZB20] without details.

When comes to instantiation, we can reassemble the sub-protocols in a more efficient manner. Recall that  $\Pi_{\text{equal}}$  is a NIZKoK for  $L_{\text{equal}}$ ,  $\Pi_{\text{enc}} \circ \Lambda_{\text{bullet}}$  is a NIZKPoK for  $L_{\text{right}}$ , and  $\Pi_{\text{enc}} \circ \Pi_{\text{ddh}} \circ \Lambda_{\text{bullet}}$  is a NIZKPoK for  $L_{\text{solvent}}$ . The first observation is that the first  $\Pi_{\text{enc}}$  can be removed since  $\Pi_{\text{equal}}$  already proves knowledge of  $C_s$ . Moreover, benefit from the nice feature of twisted ElGamal, two Bulletproofs can be generated and verified in aggregated mode, rather than being invoked twice independently. This trick roughly reduces the size of range proof part by half.

# 7.3 Eliminate Explicit Signature

At a high level, our DCP construction utilizes signature to provide authenticity and uses NIZK to provide soundness. The celebrated Fiat-Shamir transform [FS86] squashes an interactive public-coin proof into a non-interactive one by setting the verifier's challenges as the hash of the prover's history messages. Particularly, the Fiat-Shamir transform can convert a three round public-coin proof of knowledge into a signature scheme [AABN02]. To generate a signature of message m, the signer runs the PoK itself (with sk as the witness) by setting the challenge e as H(I,m), where H is a hash function modeled as a random oracle, I is the prover's first round message. The signature is the proof, i.e., the entire transcript.

Based on this connection between signature and ZKPs, Bünz et al. [BAZB20] suggested that one can leverage ZKPs to provide signature functionality instead of employing a separate signature scheme. However, their construction signs both the memo part and the NIZK proof part (cf. specification of CreateTransferTx (line 9) in Figure 3 [BAZB20]). Thus, the signing operation and signature are still explicit.

In PGC, when building NIZK for  $L_{\rm solvent}$  we use  $\Sigma_{\rm ddh}$  for  $L_{\rm ddh}$  as a sub-protocol, which is exactly a proof of knowledge of sender's secret key  $sk_s$ . Therefore, the signature component can be subsumed into the NIZK proof. More precisely, by appending all the information except  $\pi_{\rm ddh}$  to the prover's first round message in  $\Sigma_{\rm ddh}$ , the obtained zero-knowledge proof  $\pi_{\rm ddh}$  for  $L_{\rm ddh}$  also acts as sender's signature of (sn, memo,  $\pi_{\rm legal} \backslash \pi_{\rm ddh}$ ).<sup>13</sup> Moreover, [FKMV12] shows that NIZK proof obtained via the Fiat-Shamir transform from any Sigma protocol (with quasi unique response) for a hard relation is simulation sound and thus also non-malleable. We conclude that the signature scheme derived from  $\Sigma_{\rm ddh}$  is sEUF-CMA secure. Therefore, authenticity and confidentiality still hold. In this way, PGC obtains the signature functionality for free.

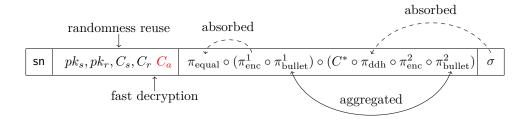


Figure 6: CTx structure of PGC and optimizations.

# 8 Comparison and Performance

# 8.1 Comparison to Related Works

We briefly compare PGC to other known account-based DCP in terms of functionalities.

<sup>&</sup>lt;sup>13</sup>A subtlety here is that our approach does not enable us to sign the whole information of (sn, memo,  $\pi_{legal}$ ), since there seems no way to append  $\pi_{legal}$  (containing  $\pi_{ddh}$ ) to the first round message of  $\Sigma_{ddh}$  before generating it.

Table 1: Comparison to other account-based DCP

Scheme	transparent setup	scalability	confidentiality	anonymity	regulation	supervision
zkLedger	$\checkmark$ + DL	O(n)	?	✓	O(m, f )	Х
Zether	$\checkmark$ + DL	O(1)	✓	Х	O( f )	Х
PGC	$\checkmark$ + DL	O(1)	✓	Х	O( f )	✓

n is the number of system users, m is the number of all transactions on the ledger. In zk-Ledger [NVV18], ctx size is linear of n, which is fixed at the very beginning. The regulation efficiency is linear of both m and |f| due to anonymity. Besides, its confidentiality is questionable due to the use of correlated randomness. Zether [BAZB20] potentially supports regulation, although not explicitly discussed in the paper. When PGC supports global supervision, it inherently relies on semi-transparent setup.

#### 8.2 Benchmark of PGC

We first give a prototype implementation of PGC as a standalone cryptocurrency in C++ based on OpenSSL, and collect the benchmarks on a MacBook Pro with an Intel i7-4870HQ CPU (2.5GHz) and 16GB of RAM. The source code of PGC is publicly available at Github [libb]. For demo purpose only, we only focus on the transaction layer. The experimental results are described in the table below.

Unfortunately, we cannot find implementation of any existing cryptocurrencies in the account-based model with transaction confidentiality and auditing capability for a concrete comparison. As a closely related work, Zether [BAZB20] provides transaction confidentiality but there is no reported implementation as a standalone cryptocurrency. We note that Zether can be instantiated on Ethereum smart contract platform. Thus, we also consider PGC on top of the Ethereum smart contract platform for comparison with Zether.

Table 2: The computation and communication complexity of PGC.

PGC -	ctx size	transaction cost (ms)		
r GC –	$ ext{big-}\mathcal{O}$	bytes	generation ver	
confidential transaction	$(2\log_2(\ell) + 22) \mathbb{G}  + 11 \mathbb{Z}_p $	1408	27.10 7.5	
auditing policies –	proof size	auditing cost (ms)		
auditing policies –	$ ext{big-}\mathcal{O}$	bytes	generation	verify
limit policy	$(2\log_2(\ell) + 4) \mathbb{G}  + 5 \mathbb{Z}_p $	622	24.85	7.15
rate policy	$2 \mathbb{G} +1 \mathbb{Z}_p $	98	0.39	0.52
open policy	$2 \mathbb{G} +1 \mathbb{Z}_p $	98	0.21	0.31

Here we set the maximum number of coins as  $v_{\text{max}} = 2^{\ell} - 1$ , where  $\ell = 32$ . PGC operates elliptic curve P-256 [Ope], which has 128 bit security. The elliptic points are expressed in their compressed form. Each  $\mathbb{G}$  element can be stored as 33 bytes, and  $\mathbb{Z}_p$  element can be stored as 32 bytes. PGC also supports other elliptic curves. The experimental result of auditing cost for limit policy is obtained by considering two transactions. We emphasize that the proof size is independent of the number of transactions, while the efficiency of proof generation and verification will not degrade much as the number of transactions grows, since homomorphic add of twisted ElGamal is pretty fast.

Similar to Zether, PGC can provide privacy-preserving service for Ethereum and other smart contract platforms. In order to properly compare PGC mechanism with Zether mechanism [BAZB20], we further deploy PGC as an Ethereum smart contract. The experimental results and comparison to Zether [BAZB20] are described in the table below.

Table 3: PGC vs. Zether as smart contracts.

scheme	operation	gas Cost	EC Cost	transaction size (bytes)
	Burn	384k	329k	320
Zether	Fund	260k	41k	64
	Transfer	7188k	6455k	1472
	Burn	310k	161k	288
PGC	Fund	172k	42k	64
	Transfer	8282k	6695k	2240

We measure the total gas cost which includes the basic cost for sending a transaction, the storage cost, the proof/signature verification, and transaction size. In order to be consistent with Zether's result, we do not include the basic Ethereum transaction data.

The comparison results demonstrate that the performance of PGC is similar to Zether. Besides, PGC can bring confidential payment ability to ETH and other ERC-20 tokens on Ethereum, while Zether does not support ERC-20 tokens.

# 8.3 Benchmark of Twisted ElGamal

Twisted ElGamal is an important building block of PGC. It is additively homomorphic over  $\mathbb{Z}_p$ , and enjoys nice interoperability with signature and zero-knowledge proofs. Moreover, its practical efficiency is competitive to other homomorphic PKE schemes based on number-theoretic assumptions, for example, the Paillier PKE [Pai99]. These features makes twisted ElGamal extremely useful in numerous privacy-preserving scenarios beyond cryptocurrencies.

**Efficiency.** We provide the benchmark of twisted ElGamal, and compare it with Paillier PKE implemented in open-source libraillier [liba]. The experimental results are collected from the same platform as PGC.

Table 4: Benchmarks of twisted ElGamal and Paillier PKE (average of 1000 runs)

timing (ms)	Setup	KeyGen	Enc	Dec	ReRand	HomoAdd	HomoSub	Scalar
Paillier		1644.53	32.211	31.367	_	0.0128		_
twisted ElGamal	52.7s + 5.1s	0.009	0.089	0.584	0.100	0.004	0.004	0.074

size (bytes)	public parameters	public key	secret key	ciphertext
Paillier	_	384	384	768
twisted ElGamal	66	33	32	66

For a fair comparison, we choose 32 bits message space and 128 bit security level [key] (twisted ElGamal operates on elliptic curve prime256v1 and Paillier operates on 3072 bit modulus). In twisted ElGamal, the setup algorithm runs "one-and-for-all", it first generates and serializes a key-value hash map (roughly 64MB) to accelerate decryption (this step takes roughly 52.7s), then loads it to the memory (this step takes roughly 5.1s). The hash map could be included as a part of public parameters, which can be safely shared among users. The decryption time decreases linearly as the hash map size grows. In Paillier PKE, global setup is not supported, and libpaillier does not provide APIs for re-randomization, homomorphic subtraction and scalar operations.

**ZKP** friendliness. Next, we demonstrate the advantages of twisted ElGamal over standard ElGamal in terms of zero-knowledge proof friendliness. We compare the efficiency of two useful gadgets built from standard ElGamal and twisted ElGamal respectively. Gadget-1 allows a sender to prove the message m encrypted under  $C = \mathsf{Enc}(pk, m; r)$  lies in range  $[0, 2^\ell)$ . In this setting, the prover knows m and r. Gadget-2 allows a receiver to prove the message m encrypted under  $C = \mathsf{Enc}(pk, m)$  lies in range  $[0, 2^\ell)$ . In this setting, the prover knows sk and thus is able to learn m by decrypting C.

In either cases, Gadget-2 can be built from Gadget-1 by re-randomizing the ciphertext and appending a NIZK for consistency of re-randomization. Note that twisted ElGamal and its accompanying Sigma protocols are as efficient as their standard version, so the efficiency difference of gadgets lies in the Bulletproof's bridging overhead. To see the comparison results more clearly, we fix baselines of two gadgets and only focus on the invoking cost. We set the baseline of Gadget-1 as a (twisted) ElGamal ciphertext, a NIZKPoK for plaintext and randomness knowledge, as well as a Bulletproof; set the baseline of Gadget-2 additionally includes a re-randomized (twisted) ElGamal ciphertext and a NIZK for consistency of re-randomization. The Bulletproof's bridging overhead is computed as the difference with the baseline of Gardet-1/2. When implementing Gadget-1/2 with standard ElGamal, the Bulletproof's bridging overhead includes generating a Pedersen commitment  $h^r g^m$  and a NIZKPoK for its consistency to the ElGamal ciphertext ( $g^r, pk^r g^m$ ). In contrast, when implementing Gadget-1/2 with twisted ElGamal, the Bulletproof's bridging overhead is zero.

Table 5: Bulletproof's bridging overheads of standard ElGamal and twisted ElGamal.

bridging overhead	building block	size	generation	verify
Gadgets-1/2	standard ElGamal	$n(2 \mathbb{G} + \mathbb{Z}_p )$	n(2Exp)	n(2Exp+1Add)
Gaugets-1/2	twisted ElGamal	0	0	0

n is the number of ciphertexts that need to be processed. In many of the range proof applications like secure machine learning, a single prover needs to conduct range proofs via gadgets-1/2 for million of ciphertexts. In this case, the saving is significant.

Very recently, exponential ElGamal and Paillier are accepted as ISO standard [ISO]. Based on better ZKP friendliness over exponential ElGamal and better efficiency over Paillier, we believe twisted ElGamal has the potential to be a candidate of standardized homomorphic PKE in the future.

# Acknowledgments

We thank Benny Pinkas and Jonathan Bootle for clarifications on Sigma protocols and Bulletproofs in the early stages of this research. We thank Yi Deng and Shunli Ma for helpful comments. We particularly thank Shuai Han for many enlightening discussions. Yu Chen is supported by National Natural Science Foundation of China (Grant No. 61772522, No. 61932019). Man Ho Au is supported by National Natural Science Foundation of China (Grant No. 61972332).

# References

- [AABN02] Michel Abdalla, Jee Hea An, Mihir Bellare, and Chanathip Namprempre. From identification to signatures via the fiat-shamir transform: Minimizing assumptions for security and forward-security. In Advances in Cryptology EUROCRYPT 2002, pages 418–433, 2002.
- [BAZB20] Benedikt Bünz, Shashank Agrawal, Mahdi Zamani, and Dan Boneh. Zether: Towards privacy in a smart contract world. In *Financial Cryptography and Data Security FC 2020*, volume 12059, pages 423–443. Springer, 2020.
- [BBB<sup>+</sup>18] Benedikt Bünz, Jonathan Bootle, Dan Boneh, Andrew Poelstra, Pieter Wuille, and Gregory Maxwell. Bulletproofs: Short proofs for confidential transactions and more. In 2018 IEEE Symposium on Security and Privacy, SP 2018, pages 315–334, 2018.
- [BBM00] Mihir Bellare, Alexandra Boldyreva, and Silvio Micali. Public-key encryption in a multi-user setting: Security proofs and improvements. In *Advances in Cryptology EUROCRYPT 2000*, volume 1807 of *LNCS*, pages 259–274. Springer, 2000.
- [BCC<sup>+</sup>16] Jonathan Bootle, Andrea Cerulli, Pyrros Chaidos, Jens Groth, and Christophe Petit. Efficient zero-knowledge arguments for arithmetic circuits in the discrete log setting. In *Advances in Cryptology EUROCRYPT 2016*, pages 327–357, 2016.
- [BDN18] Dan Boneh, Manu Drijvers, and Gregory Neven. Compact multi-signatures for smaller blockchains. In Advances in Cryptology ASIACRYPT 2018, pages 435–464, 2018.

- [BDZ03] Feng Bao, Robert H. Deng, and Huafei Zhu. Variations of diffie-hellman problem. In *Information and Communications Security, 5th International Conference, ICICS 2003*, pages 301–312, 2003.
- [BFM88] Manuel Blum, Paul Feldman, and Silvio Micali. Non-interactive zero-knowledge and its applications (extended abstract). In *Proceedings of the 20th Annual ACM Symposium on Theory of Computing, STOC 1988*, pages 103–112, 1988.
- [BKP14] Alex Biryukov, Dmitry Khovratovich, and Ivan Pustogarov. Deanonymisation of clients in bitcoin P2P network. In *Proceedings of the 2014 ACM SIGSAC Conference on Computer and Communications Security, CCS 2014*, pages 15–29, 2014.
- [BL12] Daniel J. Bernstein and Tanja Lange. Computing small discrete logarithms faster. In *Progress in Cryptology INDOCRYPT 2012*, pages 317–338, 2012.
- [BN06] Mihir Bellare and Gregory Neven. Multi-signatures in the plain public-key model and a general forking lemma. In *Proceedings of the 13th ACM Conference on Computer and Communications Security, CCS 2006*, pages 390–399. ACM, 2006.
- [BNM+14] Joseph Bonneau, Arvind Narayanan, Andrew Miller, Jeremy Clark, Joshua A. Kroll, and Edward W. Felten. Mixcoin: Anonymity for bitcoin with accountable mixes. In Financial Cryptography and Data Security 18th International Conference, FC 2014, pages 486–504, 2014.
- [BPS00] Olivier Baudron, David Pointcheval, and Jacques Stern. Extended notions of security for multicast public key cryptosystems. In *Automata, Languages and Programming, 27th International Colloquium, ICALP 2000*, pages 499–511, 2000.
- [BR93] Mihir Bellare and Phillip Rogaway. Random oracles are practical: A paradigm for designing efficient protocols. In 1st ACM Conference on Computer and Communications Security, pages 62–73, 1993.
- [CB21] Panagiotis Chatzigiannis and Foteini Baldimtsi. Miniledger: Compact-sized anonymous and auditable distributed payments. In *Computer Security ESORICS 2021 26th European Symposium on Research in Computer Security*, volume 12972 of *Lecture Notes in Computer Science*, pages 407–429. Springer, 2021.
- [CCN21] Panagiotis Chatzigiannis, Konstantinos Chalkias, and Valeria Nikolaenko. Homomorphic decryption in blockchains via compressed discrete-log lookup tables. In 5th International Workshop on Cryptocurrencies and Blockchain Technology CBT 2021, page 899, 2021.
- [CGS97] Ronald Cramer, Rosario Gennaro, and Berry Schoenmakers. A secure and optimally efficient multiauthority election scheme. In Advances in Cryptology - EUROCRYPT 1997, pages 103–118, 1997.
- [CMTA20] Yu Chen, Xuecheng Ma, Cong Tang, and Man Ho Au. PGC: Pretty Good Confidential Transaction System with Auditability. In The 25th European Symposium on Research in Computer Security, ESORICS 2020, 2020.
- [CP92] David Chaum and Torben P. Pedersen. Wallet databases with observers. In *Advances in Cryptology CRYPTO 1992*, pages 89–105, 1992.
- [CTW21] Yu Chen, Qiang Tang, and Yuyu Wang. Hierarchical integrated signature and encryption (or key separation vs. key reuse: Enjoy the best of both worlds). In *Advances in Cryptology ASIACRYPT* 2021, pages 575–606, 2021.
- [CZ14] Yu Chen and Zongyang Zhang. Publicly evaluable pseudorandom functions and their applications. In 9th International Conference on Security and Cryptography for Networks, SCN 2014, pages 115–134, 2014.
- [Dam] Ivan Damgård. On sigma protocols. http://www.cs.au.dk/~ivan/Sigma.pdf.
- [Das] Dash. https://www.dash.org.
- [Dia21] Benjamin E. Diamond. Many-out-of-many proofs and applications to anonymous zether. 2021.
- [Fin] https://cst.qd.sdu.edu.cn/info/1037/1001.htm.
- [FKMV12] Sebastian Faust, Markulf Kohlweiss, Giorgia Azzurra Marson, and Daniele Venturi. On the non-malleability of the fiat-shamir transform. In *Progress in Cryptology INDOCRYPT 2012*, pages 60–79, 2012.
- [FLS90] Uriel Feige, Dror Lapidot, and Adi Shamir. Multiple non-interactive zero knowledge proofs based on a single random string (extended abstract). In 31st Annual Symposium on Foundations of Computer Science, STOC 1990, pages 308–317, 1990.
- [FMMO19] Prastudy Fauzi, Sarah Meiklejohn, Rebekah Mercer, and Claudio Orlandi. Quisquis: A new design for anonymous cryptocurrencies. In Advances in Cryptology - ASIACRYPT 2019, volume 11921 of Lecture Notes in Computer Science, pages 649–678. Springer, 2019.
- [FS86] Amos Fiat and Adi Shamir. How to prove yourself: practical solutions to identification and signature problems. In *Advances in Cryptology CRYPTO 1986*, pages 186–194, 1986.

- [GGM16] Christina Garman, Matthew Green, and Ian Miers. Accountable privacy for decentralized anonymous payments. In Financial Cryptography and Data Security - 20th International Conference, FC 2016, pages 81–98, 2016.
- [Gol06] Oded Goldreich. Foundations of Cryptography: Volume 1. Cambridge University Press, New York, NY, USA, 2006.
- [GR10] Steven D. Galbraith and Raminder S. Ruprai. Using equivalence classes to accelerate solving the discrete logarithm problem in a short interval. In 13th International Conference on Practice and Theory in Public Key Cryptography, PKC 2010, pages 368–383, 2010.
- [Gri] Grin. https://grin-tech.org/.
- [HAB<sup>+</sup>17] Ethan Heilman, Leen Alshenibr, Foteini Baldimtsi, Alessandra Scafuro, and Sharon Goldberg. Tumblebit: An untrusted bitcoin-compatible anonymous payment hub. In 24th Annual Network and Distributed System Security Symposium, NDSS 2017, 2017.
- [HP01] Stuart Haber and Benny Pinkas. Securely combining public-key cryptosystems. In *Proceedings of the 8th ACM Conference on Computer and Communications Security, CCS 2001*, pages 215–224. ACM, 2001.
- [ISO] Homomorphic standard. ISO/IEC 18033-6:2019.
- [key] https://www.keylength.com/.
- [Kur02] Kaoru Kurosawa. Multi-recipient public-key encryption with shortened ciphertext. In Public Key Cryptography - PKC 2002, pages 48–63, 2002.
- [liba] libPaillier. https://github.com/snipsco/paillier-libraries-benchmarks.
- [libb] libPGC. https://github.com/yuchen1024/Kunlun/adcp.
- [Max] Greg Maxwell. Confidential transactions.
- [Max13] Gregory Maxwell. Coinjoin: Bitcoin privacy for the real world, 2013. https://people.xiph.org/~greg/confidential\_values.txt.
- [MM] Sarah Meiklejohn and Rebekah Mercer. Möbius: Trustless tumbling for transaction privacy. PoPETs, 2018(2):105-121.
- [Nak08] Satoshi Nakamoto. Bitcoin: A peer-to-peer electronic cash system, 2008. https://bitcoin.org/bitcoin.pdf.
- [Noe15] Shen Noether. Ring signature confidential transactions for monero, 2015. https://eprint.iacr.org/2015/1098.
- [NVV18] Neha Narula, Willy Vasquez, and Madars Virza. zkledger: Privacy-preserving auditing for distributed ledgers. In 15th USENIX Symposium on Networked Systems Design and Implementation, NSDI 2018, pages 65–80, 2018.
- [Ope] https://github.com/openssl/openssl/blob/4e6647506331fc3b3ef5b23e5dbe188279ddd575/include/openssl/obj\_mac.h.
- [Pai99] Pascal Paillier. Public-key cryptosystems based on composite degree residuosity classes. In *Advances in Cryptology EUROCRYPT 1999*, pages 223–238, 1999.
- [Ped91] Torben P. Pedersen. Non-interactive and information-theoretic secure verifiable secret sharing. In Advances in Cryptology - CRYPTO 1991, pages 129–140, 1991.
- [Poe] Andrew Poelstra. Mimblewimble. https://download.wpsoftware.net/bitcoin/wizardry/mimblewimble.pdf.
- [Pol78] John M. Pollard. Monte carlo methods for index computation (mod p). *Mathematics of Computation*, 32, 1978.
- [PS00] David Pointcheval and Jacques Stern. Security arguments for digital signatures and blind signatures.  $J.\ Cryptology,\ 13(3):361-396,\ 2000.$
- [PSST11] Kenneth G. Paterson, Jacob C. N. Schuldt, Martijn Stam, and Susan Thomson. On the joint security of encryption and signature, revisited. In *Advances in Cryptology ASIACRYPT 2011*, pages 161–178, 2011.
- [RMK14] Tim Ruffing, Pedro Moreno-Sanchez, and Aniket Kate. Coinshuffle: Practical decentralized coin mixing for bitcoin. In 19th European Symposium on Research in Computer Security, ESORICS 2014, pages 345–364, 2014.
- [RS13] Dorit Ron and Adi Shamir. Quantitative analysis of the full bitcoin transaction graph. In Financial Cryptography and Data Security 17th International Conference, FC 2013, pages 6–24, 2013.
- [Sch91] Claus-Peter Schnorr. Efficient signature generation by smart cards. *Journal of Cryptology*, 4(3):161–174, 1991.

- [Sha71] D. Shanks. Class number, a theory of factorization and genera. *Proc. Symposium in Pure Mathematics*, 20, 1971.
- [Sol] Solana. https://github.com/solana-labs/spl-zk-token.
- [TMZC17] Raymond K. H. Tai, Jack P. K. Ma, Yongjun Zhao, and Sherman S. M. Chow. Privacy-preserving decision trees evaluation via linear functions. In 22nd European Symposium on Research in Computer Security, ESORICS 2017, pages 494–512, 2017.
- [Woo14] Gavin Wood. Ethereum: A secure decentralized transaction ledger. http://gavwood.com/paper.pdf, 2014. https://www.ethereum.org/.
- [ZCa] Zcash: Privacy-protecting digital currency. https://z.cash/.

# A Missing Proofs and Protocols

# A.1 Security Proof of Twisted ElGamal

**Theorem A.1.** Twisted ElGamal is IND-CPA secure based on the divisible DDH assumption.

*Proof.* We proceed via two games. Let  $S_i$  be the probability that  $\mathcal{A}$  wins in Game i.

Game 0. The real IND-CPA security experiment. Challenger  $\mathcal{CH}$  interacts with  $\mathcal{A}$  as below:

- 1. Setup:  $\mathcal{CH}$  generates pp and keypair according to the definition, then sends pp and  $pk = g^{sk}$  to  $\mathcal{A}$ .
- 2. Challenge:  $\mathcal{A}$  submits  $m_0, m_1$  to  $\mathcal{CH}$  as the target messages.  $\mathcal{CH}$  picks a random bit  $\beta$  and randomness r, computes  $X = pk^r$ ,  $Y = g^r h^{m_\beta}$ , sends C = (X, Y) to  $\mathcal{A}$ .
- 3. Guess:  $\mathcal{A}$  outputs its guess  $\beta'$  for  $\beta$  and wins if  $\beta' = \beta$ .

According to the definition of Game 0, we have:

$$\mathsf{Adv}_{\mathcal{A}}(\lambda) = \Pr[S_0] - 1/2$$

Game 1. Same as Game 0 except  $\mathcal{CH}$  generates the challenge ciphertext in a different way.

3. Challenge:  $\mathcal{A}$  submits  $m_0, m_1$  to  $\mathcal{CH}$  as the target messages.  $\mathcal{CH}$  picks a random bit  $\beta$  and two independent randomness r and s, computes  $X = pk^r$ ,  $Y = g^s h^{m_\beta}$ , sends C = (X, Y) to  $\mathcal{A}$ .

In Game 1, the distribution of C is independent of  $\beta$ , thus we have:

$$\Pr[S_1] = 1/2$$

It remains to prove  $\Pr[S_1]$  and  $\Pr[S_0]$  are negligibly close. We prove this by showing if not so, one can build an adversary  $\mathcal{B}$  breaks the divisible DDH assumption with the same advantage. Given  $(g, g^a, g^b, g^c)$ ,  $\mathcal{B}$  is asked to decide if it is a divisible DDH tuple or a random tuple. To do so,  $\mathcal{B}$  interacts with  $\mathcal{A}$  by simulating  $\mathcal{A}$ 's challenger in the following IND-CPA experiment.

- 1. Setup:  $\mathcal{B}$  sets  $pp = (\mathbb{G}, g, h, p)$  as public parameters, sets  $g^b$  as the public key, where b is interpreted as the corresponding secret key  $b \in \mathbb{Z}_p$  and unknown to  $\mathcal{B}$ .  $\mathcal{B}$  sends pp and  $pk = g^b$  to  $\mathcal{A}$ .
- 2. Challenge:  $\mathcal{A}$  submits  $m_0, m_1$  to  $\mathcal{B}$ .  $\mathcal{B}$  picks a random bit  $\beta$  and sets  $X = g^a, Y = g^c h^{m_\beta}$ , sends C = (X, Y) to  $\mathcal{A}$ .
- 3. Guess:  $\mathcal{A}$  outputs a guess  $\beta'$ .  $\mathcal{B}$  outputs "1" if  $\beta' = \beta$  and "0" otherwise.

If  $(g, g^a, g^b, g^c)$  is a divisible DDH tuple,  $\mathcal{B}$  simulates Game 0 perfectly (with randomness c = a/b). Else,  $\mathcal{B}$  simulates Game 1 perfectly (with two independent randomness a/b and c). Thereby, we have  $\mathsf{Adv}_{\mathcal{B}} = |\Pr[S_0] - \Pr[S_1]|$ , which is negligible in  $\lambda$  by the divisible DDH assumption.

Putting all the above together, Theorem A.1 follows.

Remark A.1. We stress that twisted ElGamal does not require trusted setup. It remains secure even the adversary knows the discrete relation between (g, h).

#### A.2 NIZK for 1-message/3-recipient Twisted ElGamal Encryption

To enable global supervision, the sender need to encrypt the transfer amount using 1-message/3-recipient twisted ElGamal, then proves the correctness of encryption. According to our ADCP construction and definition of twisted ElGamal,  $L_{\rm equal}$  can be written as:

$$\{(pk_1, X_1, Y_1, pk_2, X_2, Y_2, pk_3, X_3, Y_3) \mid \exists r_1, r_2, r_3, v \text{ s.t. } X_i = pk_i^{r_i} \land Y_i = g^{r_i}h^v \text{ for } i = 1, 2, 3\}.$$

As analyzed before, for twisted ElGamal randomness can be safely reused in the 1-plaintext/2-recipient setting.  $L_{\text{equal}}$  can thus be simplified to:

$$\{(pk_1, pk_2, pk_3, X_1, X_2, X_3, Y) \mid \exists r, v \text{ s.t. } Y = g^r h^v \land X_i = pk_i^r \text{ for } i = 1, 2, 3\}.$$

Sigma protocol for  $L_{\rm equal}$ . To obtain a NIZK for  $L_{\rm equal}$ , we first design a Sigma protocol  $\Sigma_{\rm equal} = ({\sf Setup}, P, V)$  for  $L_{\rm equal}$ . The Setup algorithm of  $\Sigma_{\rm equal}$  is same as that of the twisted ElGamal. On statement  $(pk_1, pk_2, pk_3, X_1, X_2, X_3, Y)$ , P and V interact as below:

- 1. P picks  $a, b \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$ , sends  $A_1 = pk_1^a$ ,  $A_2 = pk_2^a$ ,  $A_3 = pk_3^a$ ,  $B = g^a h^b$  to V.
- 2. V picks  $e \stackrel{\mathbb{R}}{\leftarrow} \mathbb{Z}_p$  and sends it to P as the challenge.
- 3. P computes z = a + er, t = b + ev using witness w = (r, v), then sends (z, t) to V. V accepts iff: (i)  $pk_i^z = A_i X_i^e$  for  $i = \{1, 2, 3\}$ ; (ii)  $g^z h^t = BY^e$ .

Figure 7:  $\Sigma_{\text{equal}}$  for  $L_{\text{equal}}$ : proof of knowledge of three twisted ElGamal ciphertexts encrypting the same value under different public keys

Lemma A.2.  $\Sigma_{\text{equal}}$  is a public-coin SHVZK proof of knowledge for  $L_{\text{equal}}$ .

We omit the proof here since it is similar to the 1-message/2-recipient case.