

# RingCT 3.0 for Blockchain Confidential Transaction: Shorter Size and Stronger Security

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**Abstract.** In this paper, we propose the most competent blockchain ring confidential transaction protocol (RingCT3.0) for protecting the privacy of the sender’s identity, the recipient’s identity and the confidentiality of the transaction amount. For a typical 2-input transaction with a ring size of 1024, the ring signature size of our RingCT3.0 protocol is 97% less than the ring signature size of the original RingCT1.0 protocol used in Monero. Taking the advantage of our compact RingCT3.0 transcript size, privacy-preserving cryptocurrencies can enjoy a much lower transaction fee which will have a significant impact to the crypto-economy. Our implementation result shows that our protocol outperforms existing solutions, in terms of efficiency and security.

In addition to the significant improvement in terms of efficiency, our scheme is proven secure in a stronger security model. We remove the trusted setup assumption used in RingCT2.0. Our scheme is anonymous against ring insider (non-signing users who are included in the ring), while we show that the RingCT1.0 is not secure in this strong model.

Our RingCT3.0 protocol relies on our brand new designed ring signature scheme as an underlying primitive, which is believed to be the most efficient ring signature scheme up-to-date (in terms of signature size) without trusted setup. Our ring signature scheme is derived from our novel design of an efficient set membership proof of  $n$  public keys, with the proof size of  $O(\log n)$ . It is the first set membership proof without trusted setup for public keys in the base group, instead of in the exponent. These two primitives are of independent interest.

## 1 Introduction

Blockchain is a distributed ledger of transaction records between nodes, without relying on a trusted authority. Transaction records are synchronized to all nodes by a consensus algorithm, in order to provide a globally agreed, immutable history. However, all transaction data, including the sender’s public key, the recipient’s public key and the transaction amount, are publicly available. It may not be desirable for sensitive transaction in the area of FinTech. As a result, privacy-preserving blockchain received a lot of attention.

Monero, Dash and Zcash are three popular privacy-preserving cryptocurrencies having total market capitalization of USD 4 billion. They are ranked at 10, 13 and 21 of all cryptocurrencies as of October 2018. They used different cryptographic techniques: Monero [31] used linkable ring signatures, Pedersen commitment and Diffie-Hellman key agreement; Dash used coin shuffling; Zcash [5] used general zero-knowledge proof (zk-SNARK). These cryptographic techniques mainly suffer from two drawbacks: inefficient signature generation/verification, or large signature size. The latter is more concerned in public blockchains, since it is directly related to the transaction fee.

**Transaction Fee.** In public blockchain, the distributed ledger is maintained by *miners*. They are motivated for bookkeeping because they earn rewards in terms of new cryptocurrency mined

and the transaction fee from each transaction they recorded. The transaction fee is determined by the size of the transaction data. Different cryptocurrencies have their own *fee rate*, i.e., the price per kB of transaction data. During Nov 2017 to Feb 2018, Bitcoin’s fee rate reaches 0.002-0.008 BTC/kB (which is over USD 100/kB at that time); since then, Bitcoin’s fee rate is relatively stable at 0.0001 BTC/kB (which is about USD 1/kB). Monero has a relatively stable fee rate of about 0.0008 XMR/kB (which is about USD 0.2/kB).

Transaction fee depends on the length of the transaction data, which is dominated by the signature length of the senders. In Bitcoin, a *typical* transaction of 2-input-2-output contains 2 ECDSA signatures, the length of which is 1kB. As of April 2018, the average transaction fee of Bitcoin is USD 1 and the daily transaction fee of the whole Bitcoin system is USD 160,000. In Monero, the total signature size for a typical confidential transaction is 13.2kB. Therefore, any effort to reduce the signature size will have a significant impact to the crypto-economy. The improvement in signature size is relatively more important than the improvement in computation efficiency for public blockchains.

**Ring Signatures in Blockchain.** In this paper, we will focus on the ring signature, which is widely used in privacy-preserving blockchain applications. Ring signature [33] allows a user to dynamically choose a set of public keys (including his own) and to sign messages on behalf of the set, without revealing his identity. Ring signature provides perfect anonymity, such that it is impossible to decide whether two signatures are issued by the same user. In anonymous e-cash or cryptocurrency system, linkable-anonymity is more suitable than perfect anonymity since a double-spent payment can be detected. In a linkable ring signature [26], given any two signatures, the verifier knows that whether they are generated by the same signer (even though the verifier still does not know who the actual signer is).

**RingCT.** The first blockchain Confidential Transaction (CT) [27] is a proposed enhancement to the Bitcoin protocol for hiding payment values in the blockchain. To further achieve sender anonymity, a number of coin mixing protocols are proposed with CT.

In cryptocurrency Monero, linkable ring signature is used with CT to give a *Ring Confidential Transaction* (RingCT) protocol [31]. For a set of  $M$  transaction inputs, the  $M$  transaction inputs correspond to  $M$  ring signatures of ring size  $O(n)$  each, where  $n$  is the number of possible signer. In addition the net transaction amount (which should be equal to a commitment of zero) also corresponds to a ring signature of ring size  $O(n)$ . Therefore, Monero’s RingCT1.0 [31] has  $(M + 1)$  signatures of size  $O(n)$  each. Since the large signature size limits the the number  $n$  of possible signers, the value of  $n$  in Monero’s official wallet software ranges from 5 to 10 only. As a result, the sender anonymity for RingCT1.0 is at most 1-out-of-10. Due to the small ring size, there are some attacks to the anonymity of Monero users such as [22,36,29]. By launching an analysis of the Monero blockchain data, the signer anonymity can be revoked with a significant non-negligible probability.

The RingCT1.0 paper [31] does not give any notion and security model of RingCT, which are then later formalized in [35] and they give a RingCT2.0 protocol with  $(M + 1)$  signatures of size  $O(1)$  by using trusted public parameters. However, the use of trusted public parameters is not desirable in the setting of public blockchain. Note that the above comparison ignores the computation of range proof (e.g., an efficient range proof can be adopted from [9]), the  $M$  key images for linking double-spending transactions, and the computation of  $N$  committed output transaction amount.<sup>5</sup>

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<sup>5</sup> We note that a scheme called RuffCT of signature size  $O(\log n)$  is mentioned in [34]. Due to the lack of security analysis or security proof, we do not compare our scheme with RuffCT. The RuffCT is frequently discussed within the Monero development community and they plan to implement the RuffCT in the future Monero update.

RingCT	Communication		Prover (# $\mathbb{G}$ exponentiation)	Verifier (# $\mathbb{G}$ exponentiation)
	$\mathbb{G}$	$\mathbb{Z}_p$		
RingCT1.0 [31]	$M + 1$	$(M + 1)(n + 1)$	$2(M + 1)(n - 1)e_2 + 2(M + 1)e_1$	$2(M + 1)n \cdot e_2$
This paper	$(2 \log n + 9)M$	$8M + 2$	$e_{2n+1} + e_{n+1} + 3e_2$ $+(e_{2n} + e_n + \dots + e_1)$ $+ne_{M+1} + (n + 2)e_1$	$e_{2n+2 \log n+4}$ $+e_4 + e_3$ $+e_{M+2} + ne_1$

Table 1: Summary of RingCT without trusted setup for a set of  $M$  transaction inputs and each input generates a ring signature of ring size  $n$  (excluding the range proof, the key images and the input/output accounts).

Ring Signatures	Signature Size		Prover (# $\mathbb{G}$ exponentiation)	Verifier (# $\mathbb{G}$ exponentiation)
	$\mathbb{G}$	$\mathbb{Z}_p$		
[21]	$4 \log n$	$3 \log n + 1$	$\log n \cdot e_{n+1} + \frac{3}{2} \log n (e_2 + e_1)$	$e_{n+\log n+1} + n(e_3 + e_2)$
[7]	$\log n + 12$	$\frac{3}{2} \log n + 6$	$\frac{1}{2} \log n \cdot e_{n+1} + 2e_{2 \log n+1}$ $+(\frac{1}{2} \log n + 3)e_2 + 4e_1$	$2e_{n+\frac{1}{2} \log n+1}$ $+2e_{2 \log n+2} + 4e_2$
This paper	$2 \log n + 7$	$7$	$e_{2n+1} + e_{n+1} + 3e_2$ $+(e_{2n} + e_n + \dots + e_1)$	$e_{2n+2 \log n+4}$ $+e_4 + e_3$

Table 2: Summary of  $O(\log n)$ -size DL-based ring signatures for  $n$  public keys.

### 1.1 Our Contributions

Our goal is to construct a cost-efficient blockchain RingCT protocol by using an efficient ring signature scheme without trusted setup, and to prove the security in a stronger security model. Specifically, the contributions of this paper include:

1. We build a novel efficient ring signature scheme to construct RingCT3.0 protocol with the shortest RingCT transcript size, without using trusted setup. As shown in Table 1, our RingCT3.0 has ring signature size of  $O(M \log n)$  and the original RingCT1.0 [31] has ring signature size of  $O(Mn)$ . Consider a typical transaction (i.e., number of inputs  $M = 2$ ) with a ring size of 1024, our ring signature size (2kB) is 97% less than the ring signature size of [31] (98kB). In blockchain applications, the transaction fee is proportional to the transaction size. This improvement in the size provides significant savings of the transaction fee for privacy-preserving cryptocurrencies such as Monero. In addition, it becomes practical to include a larger ring size (e.g., to include  $10^5$  users with less than 4000 bytes for the signature size) and therefore (having a large ring size) makes it extremely difficult to launch an anonymity attack based on blockchain data analysis.
2. We give a strong security model for RingCT. In particular, we give a clearer security model for the balance property, and give a stronger security model for anonymity by considering insider attack. We will show that the original RingCT1.0 protocol in [31] is not secure in this anonymity model for insider attack. Then we will show that our RingCT3.0 is secure in this strong model.
3. Our significant improvements in the RingCT3.0 protocol rely on our proposed brand new ring signature scheme. It is the shortest ring signature without trusted setup in the literature (refer to Table 2). The idea comes from an innovative technique to construct an efficient set membership proof of  $n$  public keys in the base group, instead of in the exponent without trusted setup with the proof size of  $O(\log n)$ . We believe these two primitives are of independent interest and contributions.

## 2 Background

We give some background for the paper.

## 2.1 System Model

Public blockchain is a distributed system where users can join the system freely at anytime. User can have different roles:

- Sender/Recipient: The sender acts as the *signer* of a signature scheme to confirm that he wants to spend some money to the recipient in a transaction.
- Miner: The miner acts as the *verifier* of the signature scheme and checks if the signature (and the transaction) is valid. If it is true, all miners run the consensus protocol of the blockchain to agree on the order of all transactions.

In public blockchain, we assume that system parameters are simply agreed by all users and they do not need to be generated by a trusted party. For example, common hash function and elliptic curve can be used.

For the data model in the system, we use the model of Unspent Transaction Outputs (UTXO) as used in many blockchain systems. If a user wants to spend his digital assets in a transaction, he has to refer to the specific assets that he wants to spend. If he spends the same asset twice, the verifier will notice it and will reject the transaction.

## 2.2 Notations and Intractability Assumptions

**Vector Notations.** For a scalar  $c \in \mathbb{Z}_p$  and a vector  $\mathbf{a} = (a_1, \dots, a_n) \in \mathbb{Z}_p^n$ , we denote by  $\mathbf{b} = \mathbf{c}\mathbf{a}$  the vector of  $b_i = c \cdot a_i$  for  $i \in [1, n]$ . Let  $\langle \mathbf{a}, \mathbf{b} \rangle = \sum_{i=1}^n a_i b_i$  denote the inner product between two vectors  $\mathbf{a}, \mathbf{b}$ , and  $\mathbf{a} \circ \mathbf{b} = (a_1 \cdot b_1, \dots, a_n \cdot b_n)$  denote the Hadamard product.

We use  $\mathbf{k}^n$  to denote the vector containing the first  $n$ -th powers of  $k \in \mathbb{Z}_p$ . That is  $\mathbf{k}^n = (1, k, k^2, \dots, k^{n-1}) \in \mathbb{Z}_p^n$ . We use the vector notation to Pedersen vector commitment. Let  $\mathbf{g} = (g_1, \dots, g_n) \in \mathbb{G}^n$  be a vector of generators and  $\mathbf{a} = (a_1, \dots, a_n) \in \mathbb{Z}_p^n$ , then  $C = \mathbf{g}^{\mathbf{a}} = \prod_{i=1}^n g_i^{a_i}$ .

**Intractability Assumptions.** Denote  $\mathbb{G}$  as a cyclic group of order  $p$  generated by a security parameter  $1^\lambda$ . This paper uses the following intractability assumptions.

**Discrete Logarithm (DL) Assumption.** Given  $(g, g^a)$  where  $g \in \mathbb{G}, a \in \mathbb{Z}_p$ , no probabilistic polynomial time (PPT) adversary can output  $a$  with non-negligible probability.

**$q$ -Decisional Diffie-Hellman Inversion (DDHI) Assumption.** Given  $(g, g^a, \dots, g^{a^q}, T)$  where  $g \in \mathbb{G}, a \in \mathbb{Z}_p$ , no PPT adversary can decide if  $T = g^{1/a}$  with non-negligible probability.

## 3 Overview of RingCT3.0

We give a brief overview on how to improve the efficiency and the security of the RingCT protocol using a step-by-step approach.

### 3.1 Efficient RingCT3.0 Protocol

We give a new design of ring signature scheme to build an efficient RingCT without using trusted setup. This construction is composed of a number of new primitives and techniques.

**Set Membership Proof.** Our basic idea is to start with a set membership proof of a set of public keys, without trusted setup. We give the first set membership proof without trusted setup for public keys in the base group. The intuition is that we can have a set of public keys  $\mathbf{Y} = (Y_1, \dots, Y_n)$  and a binary vector  $\mathbf{b}_L = (b_1, \dots, b_n)$ . Denote  $\mathbf{Y}^{\mathbf{b}_L} = \prod_{i=1}^n Y_i^{b_i}$ . For a public key  $Y_i \in \mathbf{Y}$ , we set  $C = h^\beta Y_i$  for some public group element  $h$  and randomness  $\beta$ . We observe that  $C$  is a Pedersen commitment of the secret key  $x_i = \log_g Y_i$ . Also, when  $\mathbf{b}_L$  only has the bit at position  $i$  equal to 1, we have:

$$C = h^\beta Y_i = h^\beta \mathbf{Y}^{\mathbf{b}_L}.$$

Define  $\mathbf{b}_R$  as  $\mathbf{b}_R = \mathbf{b}_L - \mathbf{1}^n$ , where  $\mathbf{1}^n = (1, \dots, 1)$  of length  $n$ . We give a zero-knowledge proof for the above condition of  $\mathbf{b}_L$  by showing that:

$$\mathbf{b}_L \circ \mathbf{b}_R = \mathbf{0}^n, \quad \mathbf{b}_L - \mathbf{b}_R = \mathbf{1}^n, \quad \langle \mathbf{b}_L, \mathbf{1}^n \rangle = 1,$$

where  $\circ$  denotes the Hadamard product. Since the zero-knowledge proof hides the knowledge of  $\mathbf{b}_L$ , the position index  $i$  of the committed public key is hidden.

In order to give an efficient set membership proof of  $\mathbf{b}_L$  with length  $n$ , we use the inner product argument in [9] to reduce the proof size to  $\log n$ . One non-trivial tweak of our construction is to ensure the security of the Pedersen commitment  $C$  on the public key  $Y$ . We have to set  $h = \text{Hash}(\mathbf{Y})$  (Hash denotes a cryptographic hash function), such that the discrete logarithm (DL) between the public key  $Y$  and  $h$  is not known.

Our set membership proof is fundamentally different from the existing approaches. [21,7,8] proved that for a set of commitments, one of them is committed to  $g^0$ . They used  $n$  polynomials of degree  $\log n$  to hide the prover index and ran a zero-knowledge proof for the polynomials. Our scheme uses a zero-knowledge proof to prove that  $\mathbf{b}_L$  is a binary vector with Hamming weight 1 and uses the inner product argument in [9] to reduce the proof size to  $\log n$ . Details of the set membership proof is given in Appendix B. A comparison with the state-of-the-art  $\log n$ -size set membership proofs is shown Table 5. Our scheme is the most efficient in terms of communication size for ECC group, where  $|\mathbb{G}| \approx |\mathbb{Z}_p|$ . The computation of prover and verifier are slightly less efficient than the existing schemes, but the difference is smaller if we consider the acceleration of computing multi-exponentiation.

**Linkable Ring Signatures for RingCT3.0.** We propose the use of set membership proof for constructing ring signatures directly. The signer can directly give a zero-knowledge proof of knowing: (1) a committed public key ( $C = h^\beta Y_i$ ) which is in the set of  $n$  public keys, and (2) the secret key which corresponds to the committed public key. Details of the ring signature is given in Appendix C. However, turning it into linkable ring signature is a non-trivial task. Simply adding a key image to the ring signature does not give a secure linkable ring signature. It is because in one of the security models of linkable ring signature, the adversary is allowed to know all the secret keys in the ring. This is not compatible with the proof of ring signature, where the adversary does not know the pairwise discrete logarithm between public keys in the ring. As a result, the representation of public keys in the ring has to be changed for linkable ring signature.

Firstly, we convert our ring signature into a linkable ring signature by giving an extra linkability tag (also called key image in Monero) for each signer. The security proof of our ring signature scheme requires that the pairwise DL between different users' public key should be unknown to the adversary. However, in the security model of balance and non-slanderability for RingCT, the adversary is allowed to have more than one secret key. If we simply use the users' public keys as the representation of users in the ring, the scheme is not secure since the adversary knows the pairwise DL. The classical representation of user  $i$  in DL-setting is the public key  $Y_i = g^{x_i}$  where  $x_i$  is the user's secret key.

Therefore, we give a new proposal of using  $Y_i g_0^d$  as the *user representation* in the set  $\mathbf{Y}$ , where  $Y_i$  is the public key,  $g_0$  is the system parameter and  $d$  is the hash of all public keys in the ring. For each user representation  $Y_i g_0^d$ , the  $g_0$  component cannot be canceled out by  $Y_i$  due to the exponent  $d$  added. Now consider the pairwise DL between user representations. Even though the adversary knows the secret keys  $x_i$  of other users (which is allowed in the security model), the pairwise DL between different users' representation is still unknown guaranteed by the DL between  $g$  and  $g_0$ . (Refer to lemma 1).

### 3.2 Strong Security Model for RingCT3.0

As compared to the formal security model of RingCT proposed in [35], we propose a few improvements:

- We remove the use of trapdoor in system parameters. Therefore, this model is more suitable for public blockchain, as compared with RingCT2.0 [35].
- We give a clearer definition of the balance property. We observe that the balance property requires that any malicious user cannot (1) spend any account of an honest user, (2) spend her own accounts with the sum of input amount being different from that of output amount, and (3) double spend any of her accounts. Therefore, we break down the balance property into unforgeability, equivalency and linkability. As a result, the security of each property can be evaluated easily.
- We give a stronger security model of anonymity than the model in [35]. We consider the anonymity against insider attacks. Note that the original RingCT [31] is not secure in this strong model.

**Anonymity against Insider Attacks.** We observe that anonymity for RingCT protocol is more complicated than the anonymity of linkable ring signatures. Given the knowledge of the input and output amount, the level of anonymity of RingCT protocol may be lowered. For a transaction with multiple input accounts, multiple linkable ring signatures are generated. Yet, they are correlated when validating the balance of input and output amount. This extra relationship may lower the level of anonymity.

The previous model of anonymity [35] only considered outsider security (i.e., not against the recipient and other members of the ring). In this paper, we define two stronger models for anonymity: anonymity against recipient (who knows all the output account secret keys and their amounts) and anonymity against ring insider (who knows some input account secret keys and their amounts). The collusion of recipient and ring insider will inevitably lower the level of anonymity. Therefore, we do not capture it in our security model.

**Anonymity of RingCT1.0.** We first review the original RingCT1.0 [31]. Denote  $\mathbb{A}_{\text{in}}$  as the set of all input accounts and  $\mathbb{A}_S$  as the set of real signers. Arrange  $\mathbb{A}_{\text{in}}$  as an  $M \times n$  matrix with each row containing only one account in  $\mathbb{A}_S$ . The original RingCT1.0 [31] requires that all signing accounts are in the same column in  $\mathbb{A}_{\text{in}}$ . One ring signature is generated for each row. The graphical representation is as follows:

$$\mathbb{A}_{\text{in}} = \begin{array}{|c|c|c|c|c|} \hline \text{act}_1^{(1)} & \cdot & \cdot & \cdot & \text{act}_1^{(n)} \\ \hline \vdots & & & & \vdots \\ \hline \text{act}_k^{(1)} & & & & \text{act}_k^{(n)} \\ \hline \vdots & & & & \vdots \\ \hline \text{act}_M^{(1)} & & & & \text{act}_M^{(n)} \\ \hline \end{array}, \mathbb{A}_S = \begin{array}{|c|} \hline \text{act}_1^{(\text{ind})} \\ \hline \vdots \\ \hline \text{act}_k^{(\text{ind})} \\ \hline \vdots \\ \hline \text{act}_M^{(\text{ind})} \\ \hline \end{array}$$

The real signers must be located in the same column. It is because RingCT1.0 [31] includes an extra ring signature, where each “ring public key” is computed by the product of all coins in each column, divided by all output coins. It is used to guarantee the balance of the input and output amount.

We observe that if the adversary knows one of the secret key in the first column, he can check if any of the key images is generated from this secret key. If not, the adversary can rule out the possibility that the real signer is from the first column. The level of anonymity is already lowered. By knowing  $n - 1$  secret keys in different columns, the adversary can find out which  $M$  input accounts are the real signer. Therefore, the original RingCT1.0 [31] is not secure against our model of anonymity against ring insider.

**Anonymity of RingCT3.0.** In order to provide anonymity against ring insider, we have to break the distribution of real signer in  $\mathbb{A}_{\text{in}}$  in RingCT1.0 [31]. We achieve this by three steps:

(1) generate one re-randomized input coin  $C_{in,k}^*$  for each input coin of the real signer (denoted as  $C_{in,k}^{(i)}$ ); (2) prove in zero-knowledge that  $C_{in,k}^*$  and  $C_{in,k}^{(i)}$  are the Pedersen commitments on the same amount; (3) prove the sum of amounts of all re-randomized input coins is equal to that of all the output coins  $C_{out,j}$ . The first step is done by simply re-randomizing the Pedersen commitment  $C_{in,k}^{(i)}$ . The third step is done via showing the  $\prod_k C_{in,k}^* / \prod_j C_{out,j}$  is a commitment to zero. It can be done without the need of using ring signatures. The most complicated part is the second step. We can prove the validity of  $C_{in,k}^*$  by further changing the user representation of our RingCT3.0 as:

$$Y_i \left( \frac{C_{in,k}^{(i)}}{C_{in,k}^*} \right)^{d_1} g_0^{d_2},$$

where  $Y_i$  is the input public key corresponding to  $C_{in,k}^{(i)}$ ,  $g_0$  is the system parameter, and  $d_1, d_2$  are the hash of all public keys in the ring. Recall that the coin  $C_{in,k}^* = h_c^a g_c^\kappa$  is a commitment to input amount  $a$ . If  $Y_i$  is the real signer's public key, then  $\frac{C_{in,k}^{(i)}}{C_{in,k}^*} = g_c^{\kappa'}$  for some  $\kappa'$ . Then the RingCT3.0 includes an extra proof of knowledge of  $\kappa'$ . The re-randomized input coins together with the new user representation provide anonymity against insider for RingCT3.0.

## 4 Security Model for RingCT

We give the security definitions and models for RingCT which is modified from [35]. A RingCT protocol consists of a tuple of polynomial time algorithms (**Setup**, **KeyGen**, **Mint**, **AccountGen**, **Spend**, **Verify**), the syntax of which are described as follows:

- $pp \leftarrow \mathbf{Setup}(1^\lambda)$ : it takes a security parameter  $\lambda \in \mathbb{N}$ , and outputs the system parameters  $pp$ . All algorithms below have implicitly  $pp$  as part of their inputs.
- $(sk, pk) \leftarrow \mathbf{KeyGen}()$ : In order to provide anonymity to the recipient, the concept of *stealth address* was used in RingCT1.0 [31]. It can be viewed as dividing the algorithm into two parts: generating a long-term key pairs for each user, and generating a one-time key pairs for each transaction.
  - **LongTermKeyGen**. It outputs a long term secret key  $ltsk$  and a long term public key.
  - **OneTimePKGen**. On input a long term public key  $ltpk$ , it outputs  $pk$  and the auxiliary information  $R_{ot}$ .
  - **OneTimeSKGen**. On input a one-time public key  $pk$ , an auxiliary information  $R_{ot}$  and a long term secret key  $ltsk$ , it outputs the one-time secret key  $sk$ .
- $(cn, ck) \leftarrow \mathbf{Mint}(pk, a)$ : it takes as input a public key  $pk$  and an amount  $a$ , outputs a coin  $cn$  for  $pk$  as well as the associated coin key  $ck$ .
- $(act, ask)/\perp \leftarrow \mathbf{AccountGen}(sk, pk, cn, ck, a)$ : it takes as input a user key pair  $(sk, pk)$ , a coin  $cn$ , a coin key  $ck$  and an amount  $a$ . It returns  $\perp$  if  $ck$  is not the coin key of  $cn$  with amount  $a$ . Otherwise, it outputs the account  $act \doteq (pk, cn)$  and the corresponding account secret key is  $ask \doteq (sk, ck, a)$ .
- $(\mathbb{A}_{out}, \pi, \mathbb{S}, \mathbb{C}k_{out})/\perp \leftarrow \mathbf{Spend}(m, \mathbb{K}_S, \mathbb{A}_S, \mathbb{A}_{in}, \mathbb{O})$ : it takes as input a group  $\mathbb{A}_S$  of accounts together with the corresponding account secret keys  $\mathbb{K}_S$ , an arbitrary set  $\mathbb{A}_{in}$  of groups of input accounts containing  $\mathbb{A}_S$ , a set  $\mathbb{O}$  of output public keys with the corresponding output amounts, and some transaction string  $m \in \{0, 1\}^*$ , it outputs  $\perp$  if the sum of output amount in  $\mathbb{O}$  is different from the sum of input amount in  $\mathbb{K}_S$ . Otherwise, it outputs a set of output accounts  $\mathbb{A}_{out}$ , a proof  $\pi$ , a set  $\mathbb{S}$  of serial numbers and a set of output coin keys  $\mathbb{C}k_{out}$ . Each serial number  $S_i \in \mathbb{S}$  corresponds to one account secret key  $ask_i \in \mathbb{K}_S$ .
- $1/0/-1 \leftarrow \mathbf{Verify}(m, \mathbb{A}_{in}, \mathbb{A}_{out}, \pi, \mathbb{S})$ : it takes as input a message  $m$ , a set of input accounts  $\mathbb{A}_{in}$ , a set of output accounts  $\mathbb{A}_{out}$ , a proof  $\pi$  and a set of serial numbers  $\mathbb{S}$ , the algorithm outputs  $-1$  if the serial numbers in  $\mathbb{S}$  is spent previously. Otherwise, it checks if the proof  $\pi$  is valid for the transaction  $tx$ , and outputs 1 or 0, meaning a **valid** or **invalid** spending respectively.

**Perfect Correctness.** The perfect correctness property requires that a user can spend any group of her accounts w.r.t. an arbitrary set of groups of input accounts, each group containing the same number of accounts as the group she intends to spend.

A RingCT protocol is called perfectly correct if for all PPT adversaries  $\mathcal{A}$ , it holds that

$$\Pr \left[ \begin{array}{l} pp \leftarrow \mathbf{Setup}(1^\lambda); \\ \forall j \in [1, M], a_j \leftarrow_R \mathbb{Z}_p : \\ \mathbf{Verify}(m, \mathbb{A}_{in}, \mathbb{A}_{out}, \pi, \mathbb{S}) = 1 \\ \mathbb{A}_S \doteq \{act_j\}_{j \in [1, M]}, \mathbb{K}_S \doteq \{ask_j\}_{j \in [1, M]}; \\ (m, \mathbb{A}_{in}, \mathbb{O}) \leftarrow \mathcal{A}(pp, \mathbb{A}_S, \mathbb{K}_S); \\ (\mathbb{A}_{out}, \pi, \mathbb{S}, \mathbb{C}_{k_{out}}) \leftarrow \mathbf{Spend}(m, \mathbb{K}_S, \mathbb{A}_S, \mathbb{A}_{in}, \mathbb{O}). \end{array} \right] = 1.$$

We give the definition of some oracles and lists that will be used in the security models in Table 3 and 4.

**Anonymity.** The anonymity of RingCT is more complicated than the anonymity of linkable ring signatures, due to the extra knowledge of transaction amount. The previous model of anonymity in RingCT1.0 only considered outsider security only (i.e., not against the recipient and other members of the ring). As introduced in section 3.2, we define two stronger models for anonymity: anonymity against recipient (who knows all the output amounts) and anonymity against ring insider (who knows some input account secret keys and their amounts).

*Anonymity against recipient.* The anonymity against recipient property requires that without the knowledge of any input account secret key and input amount (which are within a valid *Range*), the spender's accounts are successfully hidden among all the honestly generated accounts, even when the output accounts and the output amounts are known.

**Definition 1.** A RingCT protocol is called *anonymous against recipient* if for all PPT adversaries  $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$  in the following experiment  $\text{Expr}_{AR}$ :

$$\begin{array}{l} pp \leftarrow \mathbf{Setup}(1^\lambda); \\ (m, \{\text{pk}_{k,i}\}_{k \in [1, M], i \in [1, n]}, \{\text{pk}_{out,j}\}_{j \in [1, N]}) \leftarrow \mathcal{A}_1^{orc}(pp), \\ \text{where } (\text{sk}_{k,i}, \text{pk}_{k,i}) \in \mathcal{U} \text{ and } ((\text{pk}_{k,i}, \cdot), \cdot) \notin \mathcal{L}; \\ \text{for } k \in [1, M], i_k^* \leftarrow_R [1, n], \mathbf{a}_{k,i}, \mathbf{a}_{out,j} \leftarrow_R \text{Range}, \text{where } \sum_k \mathbf{a}_{k,i_k^*} = \sum_j \mathbf{a}_{out,j}. \\ (cn_{k,i}, ck_{k,i}) \leftarrow \mathbf{Mint}(\text{pk}_{k,i}, \mathbf{a}_{k,i}), \\ (act_{k,i}, ask_{k,i}) \leftarrow \mathbf{AccountGen}(\text{sk}_{k,i}, \text{pk}_{k,i}, cn_{k,i}, ck_{k,i}, \mathbf{a}_{k,i}); \\ \mathbb{A}_{in} \doteq \{act_{k,i}\}, \mathbb{A}_S \doteq \{act_{k,i_k^*}\}, \mathbb{K}_S \doteq \{ask_{k,i_k^*}\}, \mathbb{O} \doteq \{\text{pk}_{out,j}, \mathbf{a}_{out,j}\}; \\ (\mathbb{A}_{out}^*, \pi^*, \mathbb{S}^*, \mathbb{C}_{k_{out}}^*) \leftarrow \mathbf{Spend}(m, \mathbb{K}_S, \mathbb{A}_S, \mathbb{A}_{in}, \mathbb{O}); \\ (k^*, ind^*) \leftarrow \mathcal{A}_2^{orc}(pp, (\mathbb{A}_{out}^*, \pi^*, \mathbb{S}^*, \mathbb{O}, \mathbb{C}_{k_{out}}^*, \mathbb{A}_{in})), \end{array}$$

it holds that:  $|\Pr[\mathcal{A} \text{ wins in Expr}_{AR}] - \frac{1}{n}| \leq \text{negl}(\lambda)$ .  $\mathcal{A}$  wins if  $ind^* = i_k^*$  and  $\mathbb{A}_{in} \cap \mathcal{C} = \emptyset$ .

*Anonymity against ring insider.* The anonymity against ring insider property requires that without the knowledge of output account secret key and output amount (which are within a valid *Range*), the spender's accounts are successfully hidden among all uncorrupted accounts.

**Definition 2.** A RingCT protocol is called *anonymous against ring insider* if for all PPT adversaries  $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$  in the following experiment  $\text{Expr}_{AI}$ :

$$\begin{array}{l} pp \leftarrow \mathbf{Setup}(1^\lambda); \\ (m, \{\text{act}_{k,i}\}_{k \in [1, M], i \in [1, n]}, \{\text{pk}_{out,j}\}_{j \in [1, N]}) \leftarrow \mathcal{A}_1^{orc}(pp), \\ \text{where } (\cdot, \text{pk}_{out,j}) \in \mathcal{U}, \text{ and } (\text{act}_{k,i}, \mathbf{a}_{k,i}, ck_{k,i}) \in \mathcal{G}; \\ \text{for } j \in [1, N], i_k^* \leftarrow_R [1, n], \mathbf{a}_{out,j} \leftarrow_R \text{Range}, \text{where } \sum_k \mathbf{a}_{k,i_k^*} = \sum_j \mathbf{a}_{out,j}. \\ \mathbb{A}_{in} \doteq \{act_{k,i}\}, \mathbb{A}_S \doteq \{act_{k,i_k^*}\}, \mathbb{K}_S \doteq \{ask_{k,i_k^*}\}, \mathbb{O} \doteq \{\text{pk}_{out,j}, \mathbf{a}_{out,j}\}; \\ (\mathbb{A}_{out}^*, \pi^*, \mathbb{S}^*, \mathbb{C}_{k_{out}}^*) \leftarrow \mathbf{Spend}(m, \mathbb{K}_S, \mathbb{A}_S, \mathbb{A}_{in}, \mathbb{O}); \\ (k^*, ind^*, n^*) \leftarrow \mathcal{A}_2^{orc}(pp, (\mathbb{A}_{out}^*, \pi^*, \mathbb{S}^*, \mathbb{A}_{in})), \end{array}$$



List	Description
$\mathcal{U}$	The list of user key pairs generated by the challenger.
$\mathcal{G}$	The list of accounts, their balance and coin keys generated by Mint or Spend.
$\mathcal{I}$	The list of accounts with public keys generated by the challenger.
$\mathcal{L}$	The list of accounts and account secret keys with public keys generated by the challenger.
$\mathcal{T}$	The list of output generated by the Spend oracle with the corresponding input accounts.
$\mathcal{C}$	The list of corrupted accounts.

Table 3: Definition of Lists

Oracle	Description
$\text{PkGen}(i, b, j)$	on input a query number $i$ , a bit $b$ and an index $j$ , if $b = 0$ , it runs algorithm $(\text{ltsk}_i, \text{ltpk}_i) \leftarrow \text{LongTermKeyGen}()$ , adds $(i, \text{ltsk}_i, \text{ltpk}_i)$ to an initially empty list $\mathcal{U}'$ and returns the long term public key $\text{pk}_i$ . If $b = 1$ , it retrieves $(j, \text{ltsk}_j, \text{ltpk}_j)$ from $\mathcal{U}'$ and runs $(\text{pk}_i, R_{\text{ot}}) \leftarrow \text{OneTimePKGen}(\text{ltpk}_j)$ , $\text{sk}_i \leftarrow \text{OneTimeSKGen}(\text{pk}_i, R_{\text{ot}}, \text{ltsk}_j)$ . It adds $(\text{sk}_i, \text{pk}_i)$ to an initially empty list $\mathcal{U}$ and returns $\text{pk}_i$ .
$\text{ActGen}(\text{pk}, \mathbf{a})$	on input a public key $\text{pk}$ and an amount $\mathbf{a}$ , it runs algorithm $(\text{cn}, \text{ck}) \leftarrow \text{Mint}(\text{pk}, \mathbf{a})$ , and outputs $(\text{act}, \text{ck})$ for address $\text{pk}$ . It adds $(\text{act}, \mathbf{a}, \text{ck})$ to an initially empty list $\mathcal{G}$ . If $(\text{sk}, \text{pk}) \in \mathcal{U}$ , then the associated secret key with account $\text{act}$ is $\text{ask} \doteq (\text{sk}, \text{ck}, \mathbf{a})$ . It adds $\text{act}$ and $(\text{act}, \text{ask})$ to an initially empty list $\mathcal{I}$ and $\mathcal{L}$ respectively.
$\text{Corrupt}(\text{act})$	on input an account $\text{act}$ , it retrieves $(\text{act}, \text{ask}) \in \mathcal{L}$ . It adds $\text{act}$ to an initially empty list $\mathcal{C}$ , and finally returns $\text{ask}$ .
$\text{Spend}(\mathbf{m}, \mathbb{A}_S, \mathbb{A}_{\text{in}}, \mathbb{O})$	takes in a string $\mathbf{m}$ , input accounts $\mathbb{A}_{\text{in}}$ containing $\mathbb{A}_S$ and the output set $\mathbb{O}$ , it retrieves $(\text{act}_i, \text{ask}_i) \in \mathcal{L}$ for all $\text{act}_i \in \mathbb{A}_S$ . Denote $\mathbb{K}_S$ as the set of these $\text{ask}_i$ . It runs $(\mathbb{A}_{\text{out}}, \pi, \mathbb{S}, \mathbb{Ck}_{\text{out}}) \leftarrow \text{Spend}(\mathbf{m}, \mathbb{K}_S, \mathbb{A}_S, \mathbb{A}_{\text{in}}, \mathbb{O})$ and returns $(\mathbb{A}_{\text{out}}, \pi, \mathbb{S}, \mathbb{Ck}_{\text{out}})$ after adding $(\mathbb{A}_{\text{out}}, \pi, \mathbb{S}, \mathbb{A}_S)$ to an initially empty list $\mathcal{T}$ . For all $\text{act}_j = (\text{pk}_j, \text{cn}_j) \in \mathbb{A}_{\text{out}}$ , with $\mathbf{a}_j$ as the corresponding amount in $\mathbb{O}$ and $\text{ck}_j$ as the corresponding coin key in $\mathbb{Ck}_{\text{out}}$ , it adds $(\text{act}_j, \mathbf{a}_j, \text{ck}_j)$ to the list $\mathcal{G}$ . If $(\text{sk}_j, \text{pk}_j) \in \mathcal{U}$ , it sets $\text{ask}_j \doteq (\text{sk}_j, \text{ck}_j, \mathbf{a}_j)$ , where $\text{ck}_j$ is the corresponding coin key in $\mathbb{Ck}_{\text{out}}$ . It adds $\text{act}_j$ to the list $\mathcal{I}$ and adds $(\text{act}_j, \text{ask}_j)$ to the list $\mathcal{L}$ .

Table 4: Definition of Oracles  $\text{Orc}$ 

it holds that:  $\left| \Pr[\mathcal{A} \text{ wins in Expr}_{AI}] - \frac{1}{n-n^*} \right| \leq \text{negl}(\lambda)$ .  $\mathcal{A}$  wins if  $\text{ind}^* = i_k^*$  and there are  $n^*$  distinct values of  $i$  such that  $\text{act}_{k^*, i} \in \mathcal{C}$ .

**Balance.** The balance property requires that any malicious user cannot (1) spend any account of an honest user, (2) spend her own accounts with the sum of input amount being different from that of output amount, and (3) double spend any of her accounts. Therefore, the balance property can be modeled by three security models: unforgeability, equivalence and linkability.

The following definition of unforgeability captures the property that no PPT adversary can forge a signature where all input accounts are uncorrupted.

**Definition 3.** A RingCT protocol is called unforgeable if for all PPT adversaries  $\mathcal{A}$ , it holds that

$$\Pr[\mathcal{A} \text{ Wins} : \text{pp} \leftarrow \text{Setup}(1^\lambda); (\mathbf{m}, \mathbb{A}_{\text{in}}, \mathbb{A}_{\text{out}}, \pi, \mathbb{S}) \leftarrow \mathcal{A}^{\text{Orc}}(\text{pp})] \leq \text{negl}(\lambda),$$

where  $\text{Orc}$  denotes all oracles  $\text{PkGen}$ ,  $\text{ActGen}$ ,  $\text{Spend}$  and  $\text{Corrupt}$  defined in Table 4. Finally,  $\mathcal{A}$  wins in the experiment if  $\text{Verify}(\mathbf{m}, \mathbb{A}_{\text{in}}, \mathbb{A}_{\text{out}}, \pi, \mathbb{S}) = 1$ ,  $(\mathbb{A}_{\text{out}}, \pi, \mathbb{S}, \cdot) \notin \mathcal{T}$ , and for all  $\text{act}_i \in \mathbb{A}_{\text{in}}$ ,  $\text{act}_i \in \mathcal{I} \setminus \mathcal{C}$ .

The following definition of equivalence captures the property that no PPT adversary can output a valid signature and coin keys where the sum of input amount is different from the sum of output amount. It holds even if these accounts are corrupted.

**Definition 4.** A RingCT protocol is called equivalent w.r.t. insider corruption if for all PPT adversaries  $\mathcal{A}$ , it holds that

$$\Pr [\mathcal{A} \text{ Wins} : pp \leftarrow \text{Setup}(1^\lambda); (\mathbf{m}, \mathbb{A}_{\text{in}}, \mathbb{A}_{\text{out}}, \pi, \mathbb{S}, \mathbb{Ck}_{\text{out}}, \mathbb{Amt}_{\text{out}}) \leftarrow \mathcal{A}^{\text{orc}}(pp)] \leq \text{negl}(\lambda),$$

where  $\mathbb{Amt}_{\text{out}}$  is a set of output amount.  $\mathcal{A}$  wins in the experiment if it satisfies the following conditions: (1)  $\text{Verify}(\mathbf{m}, \mathbb{A}_{\text{in}}, \mathbb{A}_{\text{out}}, \pi, \mathbb{S}) = 1$ , (2) for all coin key  $ck_i \in \mathbb{Ck}_{\text{out}}$  and amount  $\mathbf{a}_{\text{out},i} \in \mathbb{Amt}_{\text{out}}$ ,  $\text{AccountGen}(\text{sk}_i, \text{pk}_i, \text{cn}_i, ck_i, \mathbf{a}_{\text{out},i}) \neq \perp$ , (3) for all account  $act_j \in \mathbb{A}_{\text{in}}$ ,  $(act_j, \mathbf{a}_{\text{in},j}) \in \mathcal{G}$ ,  $\sum_j \mathbf{a}_{\text{in},j} \neq \sum_i \mathbf{a}_{\text{out},i}$ .

The following definition of linkability captures the property that no PPT adversary can output two valid signatures with distinct serial numbers  $\mathbb{S}_1$  and  $\mathbb{S}_2$ , where there is at most  $|\mathbb{S}_1| + |\mathbb{S}_2| - 1$  input account that is corrupted or not generated by the challenger.

**Definition 5.** A RingCT protocol is called linkable w.r.t. insider corruption if for all PPT adversaries  $\mathcal{A}$ , it holds that

$$\Pr [\mathcal{A} \text{ Wins} : pp \leftarrow \text{Setup}(1^\lambda); \{\mathbf{m}_i, \mathbb{A}_{\text{in},i}, \mathbb{A}_{\text{out},i}, \pi_i, \mathbb{S}_i\}_{i=1,2} \leftarrow \mathcal{A}^{\text{orc}}(pp)] \leq \text{negl}(\lambda).$$

$\mathcal{A}$  wins in the experiment if  $\text{Verify}(\mathbf{m}_i, \mathbb{A}_{\text{in},i}, \mathbb{A}_{\text{out},i}, \pi_i, \mathbb{S}_i) = 1$  for  $i = 1, 2$ ;  $\mathbb{S}_1 \cap \mathbb{S}_2 = \emptyset$ ; and  $(\mathbb{A}_{\text{in},1} \cup \mathbb{A}_{\text{in},2}) \cap \mathcal{C} + (\mathbb{A}_{\text{in},1} \cup \mathbb{A}_{\text{in},2}) \setminus \mathcal{I} \leq |\mathbb{S}_1| + |\mathbb{S}_2| - 1$ .

**Non-Slanderability.** The non-slanderability property requires that a malicious user cannot slander any honest user after observing an honestly generated spending. It is infeasible for any malicious user to produce a valid spending that shares at least one serial number with a previously generated honest spending.

**Definition 6.** A RingCT protocol is called non-slanderable if for all PPT adversaries  $\mathcal{A}$ , it holds that

$$\Pr [\mathcal{A} \text{ Wins} : pp \leftarrow \text{Setup}(1^\lambda); (\hat{\mathbb{S}}, \mathbf{m}, \mathbb{A}_{\text{in}}, \mathbb{A}_{\text{out}}, \pi, \mathbb{S}) \leftarrow \mathcal{A}^{\text{orc}}(pp)] \leq \text{negl}(\lambda).$$

$\mathcal{A}$  wins if  $\hat{\mathbb{S}} \cap \mathbb{S}^* \neq \emptyset$ ;  $\text{Verify}(\mathbf{m}, \mathbb{A}_{\text{in}}, \mathbb{A}_{\text{out}}, \pi, \mathbb{S}) = 1$ ;  $(\mathbb{A}_{\text{out}}, \pi, \mathbb{S}, \cdot) \notin \mathcal{T}$ ; and  $(\cdot, \cdot, \hat{\mathbb{S}}, \mathbb{A}_S) \in \mathcal{T}$  and for all  $act \in \mathbb{A}_S$ ,  $act \notin \mathcal{C}$ .

## 5 RingCT 3.0

For the ease of presentation, we first present a basic construction of RingCT3.0 with linkable ring signature. Then we optimize our construction to  $\log(n)$ -size by using the inner-product argument in [9], where  $n$  is the size of the ring.

### 5.1 Our Basic Construction

We give our basic construction in this section. Our scheme uses a zero-knowledge range proof of a value committed in a Pederson commitment. Denote  $\text{RP} = (\text{RSetup}, \text{RProof}, \text{RVerify})$  as a zero-knowledge range proof for the statement:

$$\text{PoK} : \{(\mathbf{a}, \kappa) : C = h_c^2 g_c^\kappa \wedge \mathbf{a} \in [R_{\min}, R_{\max}]\}.$$

The range proof can be instantiated by the Bulletproof [9].

Our basic construction is as follows.

**Setup.** On input security parameter  $1^\lambda$  and the maximum size of ring  $n_{\max}$ , it picks a group  $\mathbb{G}$  of prime order  $p$  and some generators  $g_c, h_c, g, u \in \mathbb{G}$ ,  $\mathbf{h} = (h_1, \dots, h_{n_{\max}}) \in \mathbb{G}^{n_{\max}}$ . Suppose that

$H_j : \{0, 1\}^* \rightarrow \mathbb{Z}_p$  for  $j = 1, 2, 4, 5$ ,  $H_3 : \{0, 1\}^* \rightarrow \mathbb{G}$  and  $H_6 : \mathbb{G} \rightarrow \mathbb{Z}_p$  are collision resistant hash functions. It also runs RSetup of the range proof. Assume these parameters are known in the system.

**KeyGen.** The **KeyGen** algorithm is divided as follows.

- **LongTermKeyGen.** The user picks his long term secret key  $\text{ltsk} \doteq (x_1, x_2) \in \mathbb{Z}_p^2$  and computes his long term public key  $\text{ltpk} \doteq (g^{x_1}, g^{x_2})$ .
- **OneTimePKGen.** On input a long term public key  $\text{ltpk} = (g^{x_1}, g^{x_2})$ , it picks a random  $r_{\text{ot}} \in \mathbb{Z}_p$  and computes a one-time public key  $\text{pk} = g^{x_1} \cdot g^{H_6((g^{x_2})^{r_{\text{ot}}})}$ . It outputs  $\text{pk}$  and the auxiliary information  $R_{\text{ot}} \doteq g^{r_{\text{ot}}}$ .
- **OneTimeSKGen.** On input a one-time public key  $\text{pk}$ , an auxiliary information  $R_{\text{ot}}$  and a long term secret key  $\text{ltsk} = (x_1, x_2)$ , it checks if  $\text{pk} = g^{x_1} \cdot g^{H_6(R_{\text{ot}}^{x_2})}$ . If it is correct, then it outputs the one-time secret key  $\text{sk} = x_1 + H_6(R_{\text{ot}}^{x_2})$ .

**Mint.** On input a public key  $\text{pk}$ , an amount  $\mathbf{a} \in \mathbb{Z}_p$ , the algorithm chooses  $\kappa \in \mathbb{Z}_p$  uniformly at random and computes the coin  $C = g_c^\kappa h_c^{\mathbf{a}}$ . It returns the coin  $C$  and the coin key  $\text{ck} = \kappa$ .

**AccountGen.** On input a user key pair  $(\text{sk}, \text{pk})$ , a coin  $C$  and a coin key  $\text{ck} = \kappa$  (where the pair  $(\text{pk}, C)$  is listed as the output of a transaction) and an amount  $\mathbf{a}$ , it checks if  $C = g_c^\kappa h_c^{\mathbf{a}}$ . If it is true, then it outputs the account  $\text{act} \doteq (\text{pk}, C)$  and the corresponding account secret key is  $\text{ask} \doteq (\text{sk}, \text{ck}, \mathbf{a})$ .

**Spend.** On input a set of  $M$  signer's input accounts  $\mathbb{A}_S$  with a set of account secret keys  $\{\text{ask}_k = (\text{sk}_k, \kappa_{\text{in},k}, \mathbf{a}_{\text{in},k})\}_{k \in [1, M]}$ , a set of  $nM$  input accounts  $\mathbb{A}_{\text{in}}$  which contains  $\mathbb{A}_S$  (where  $n < n_{\text{max}}$  is the size of the ring), a set of  $N$  output amount  $\{\mathbf{a}_{\text{out},j}\}_{j \in [1, N]}$  corresponding to  $N$  recipient's public keys  $\{\text{pk}_{\text{out},j}\}_{j \in [1, N]}$ , and a transaction message  $\mathbf{m}$ , it first checks the amount balance. If  $\sum_{k=1}^M \mathbf{a}_{\text{in},k} \neq \sum_{j=1}^N \mathbf{a}_{\text{out},j}$ , the transaction amount is not correct and it returns  $\perp$ .

Arrange  $\mathbb{A}_{\text{in}}$  as an  $M \times n$  matrix with each row containing only one account in  $\mathbb{A}_S$ . Denote the column index  $\text{ind}_k$  as the position of the  $k$ -th element in  $\mathbb{A}_S$  appearing in row  $k$ , column  $\text{ind}_k$  of  $\mathbb{A}_{\text{in}}$ . The graphical representation is as follows:

$$\mathbb{A}_{\text{in}} = \begin{array}{|c|c|c|c|c|} \hline \text{act}_1^{(1)} & \cdot & \cdot & \cdot & \text{act}_1^{(n)} \\ \hline \vdots & & & & \vdots \\ \hline \text{act}_k^{(1)} & & & & \text{act}_k^{(n)} \\ \hline \vdots & & & & \vdots \\ \hline \text{act}_M^{(1)} & & & & \text{act}_M^{(n)} \\ \hline \end{array}, \mathbb{A}_S = \begin{array}{|c|} \hline \text{act}_1^{(\text{ind}_1)} \\ \hline \vdots \\ \hline \text{act}_k^{(\text{ind}_k)} \\ \hline \vdots \\ \hline \text{act}_M^{(\text{ind}_M)} \\ \hline \end{array}$$

The spend protocol can be roughly separated into two parts. The first part is mainly about the balance of the input and output amount. The second part is mainly about the ring signature providing anonymity of the sender.

We first give some sub-protocols for the *balance* property as follows.

1. **Generate One-Time Public Key:** The sender converts all recipient's long term public keys to one-time public keys by **OneTimePKGen**. The auxiliary information is appended to the transaction message  $\mathbf{m}$ .
2. **Generate Output Coins.** It first runs  $(C_{\text{out},j}, \kappa_{\text{out},j}) \leftarrow \text{Mint}(\mathbf{a}_{\text{out},j})$ , for all  $j \in [1, N]$ . It sets  $\mathbb{A}_{\text{out}} = \{(\text{pk}_{\text{out},j}, C_{\text{out},j})\}_{j \in [1, N]}$  as the set of  $N$  output accounts. The sender can later privately send the amount  $\mathbf{a}_{\text{out},j}$  and coin key  $\kappa_{\text{out},j}$  to each secret key owner of  $\text{pk}_{\text{out},j}$ . Denote  $\mathbb{Ck}_{\text{out}}$  as the set of all coin keys.
3. **Generate Range Proof.** It runs the RProof of the range proof for all  $\mathbf{a}_{\text{out},j}$  where  $j \in [1, N]$ . Denote  $\pi_{\text{range}}$  as the set of output of RProof for all  $j$ .

4. **Generate New Input Coin.** Denote the coin for  $act_k^{(\text{ind}_k)}$  as  $C_{\text{in},k}^{(\text{ind}_k)}$ . The sender picks a random  $\delta_k \in \mathbb{Z}_p$  and computes the randomized input coin:  $C_{\text{in},k}^* \doteq C_{\text{in},k}^{(\text{ind}_k)} g_c^{-\delta_k}$ . Recall that the coin key of  $C_{\text{in},k}^{(\text{ind}_k)}$  is  $(\mathbf{a}_{\text{in},k}, \kappa_{\text{in},k})$ . Therefore,  $C_{\text{in},k}^* = g_c^{\kappa_{\text{in},k} - \delta_k} h_c^{\mathbf{a}_{\text{in},k}}$ . Denote the set of new input coins as  $\mathbb{C}_{\text{in}} = \{C_{\text{in},k}^*\}_{k \in [1, M]}$ .
5. **Generate Balance Proof.** If sum of input amount is equal to the sum of output amount, we have  $\prod_{j=1}^N C_{\text{out},j} / \prod_{k=1}^M C_{\text{in},k}^* = g_c^{\sum_{j=1}^N \kappa_{\text{out},j} - \sum_{k=1}^M (\kappa_{\text{in},k} - \delta_k)}$ . Denote  $\Delta \doteq \sum_{j=1}^N \kappa_{\text{out},j} - \sum_{k=1}^M (\kappa_{\text{in},k} - \delta_k)$ . The sender picks a random  $r_\Delta \in \mathbb{Z}_p$  and computes:

$$R_\Delta = g_c^{r_\Delta}, \quad c_\Delta = H_1(R_\Delta, \mathbb{A}_{\text{out}}, \mathbb{C}_{\text{in}}), \quad z_\Delta = r_\Delta + \Delta \cdot c_\Delta.$$

Denote the proof of balance as  $\pi_{\text{bal}} = (c_\Delta, z_\Delta)$ .

Next, we give some sub-protocols for the *ring signature* part. The sender generates a ring signature for each row  $k$  of  $\mathbb{A}_{\text{in}}$ . Denote  $act_k^{(i)} = (\text{pk}_{\text{in},k}^{(i)}, C_{\text{in},k}^{(i)})$  for  $i \in [1, n]$  and the signer index is  $\text{ind}_k$ . The sender runs as follows.

1. **Generate One-Time Secret Key:** The sender converts his long term secret key to one-time secret keys by **OneTimeSKGen**.
2. **Generate Key Images.** Denote  $(\text{sk}, \cdot, \cdot)$  as the account secret key for  $act_k^{(\text{ind}_k)}$ . It computes the key image  $U = u^{\frac{1}{\text{sk}}}$ .
3. **Ring Formation.** Denote the concatenated string  $\text{str} = act_k^{(1)} || \dots || act_k^{(n)} || C_{\text{in},k}^*$ . The prover computes  $d_1 = H_2(1, \text{str})$  and  $d_2 = H_2(2, \text{str})$ . The prover sets

$$\mathbf{Y} = (\text{pk}_{\text{in},k}^{(1)} \left( \frac{C_{\text{in},k}^{(1)}}{C_{\text{in},k}^*} \right)^{d_1} g_0^{d_2}, \dots, \text{pk}_{\text{in},k}^{(n)} \left( \frac{C_{\text{in},k}^{(n)}}{C_{\text{in},k}^*} \right)^{d_1} g_0^{d_2}).$$

It is easy to see that the  $\text{ind}_k$ -th element of  $\mathbf{Y}$  is  $g^{\text{sk}} g_c^{d_1 \delta_k} g_0^{d_2}$ .

4. **Prepare Signer Index.** The sender generates a binary vector  $\mathbf{b}_L = (b_1, \dots, b_n)$ , where  $b_i = 1$  when  $i = \text{ind}_k$  and  $b_i = 0$  otherwise. Define  $\mathbf{b}_R = \mathbf{b}_L - \mathbf{1}^n$ . We will prove in zero knowledge that  $\mathbf{b}_L$  is a binary vector with only one bit equal to 1. It is equivalent to showing:  $\mathbf{b}_L \circ \mathbf{b}_R = \mathbf{0}^n$ ,  $\mathbf{b}_L - \mathbf{b}_R = \mathbf{1}^n$ ,  $\langle \mathbf{b}_L, \mathbf{1}^n \rangle = 1$ .
5. **Signature Generation.** It consists of the following steps.

*Commit 1.* It sets  $h = H_3(\mathbf{Y})$ , picks random  $\alpha, \beta, \rho, r_\alpha, r_{\text{sk}}, r_\delta \in \mathbb{Z}_p$ ,  $\mathbf{s}_L, \mathbf{s}_R \in \mathbb{Z}_p^n$  and computes:

$$\begin{aligned} B &= h^\alpha \mathbf{Y}^{\mathbf{b}_L} = h^\alpha \text{pk}_{\text{in},k}^{(\text{ind}_k)} (g_c^{\delta_k})^{d_1} g_0^{d_2}, & A &= h^\beta \mathbf{h}^{\mathbf{b}_R}, \\ S_1 &= h^{r_\alpha} g^{r_{\text{sk}}} g_c^{d_1 r_\delta}, & S_2 &= h^\rho \mathbf{Y}^{\mathbf{s}_L} \mathbf{h}^{\mathbf{s}_R}, & S_3 &= U^{r_{\text{sk}}}. \end{aligned}$$

*Challenge 1.* Denote the concatenated string  $\text{str}' = \mathbf{Y} || B || A || S_1 || S_2 || S_3 || U$ . It computes  $y = H_4(1, \text{str})$ ,  $z = H_4(2, \text{str})$  and  $w = H_4(3, \text{str})$ .

*Commit 2.* It can construct two degree 1 polynomials of variable  $X$ :

$$l(X) = \mathbf{b}_L - z \cdot \mathbf{1}^n + \mathbf{s}_L \cdot X, \quad r(X) = \mathbf{y}^n \circ (w \cdot \mathbf{b}_R + wz \cdot \mathbf{1}^n + \mathbf{s}_R \cdot X) + z^2 \cdot \mathbf{1}^n.$$

Denote  $t(X) = \langle l(X), r(X) \rangle$ , which is a degree 2 polynomial. We can write  $t(X) = t_0 + t_1 X + t_2 X^2$ , and  $t_0, t_1, t_2$  can be computed by using  $(\mathbf{b}_L, \mathbf{b}_R, \mathbf{s}_L, \mathbf{s}_R, w, y, z)$ . In particular, observe that

$$\begin{aligned} t_0 &= w \langle \mathbf{b}_L, \mathbf{b}_R \circ \mathbf{y}^n \rangle + zw \langle \mathbf{b}_L - \mathbf{b}_R, \mathbf{y}^n \rangle + z^2 \langle \mathbf{b}_L, \mathbf{1}^n \rangle - wz^2 \langle \mathbf{1}^n, \mathbf{y}^n \rangle - z^3 \langle \mathbf{1}^n, \mathbf{1}^n \rangle, \\ &= z^2 + w(z - z^2) \langle \mathbf{1}^n, \mathbf{y}^n \rangle - z^3 \langle \mathbf{1}^n, \mathbf{1}^n \rangle. \end{aligned}$$

It picks random  $\tau_1, \tau_2 \in \mathbb{Z}_p$ , and computes:  $T_1 = g^{\tau_1} h^{\tau_1}$ ,  $T_2 = g^{\tau_2} h^{\tau_2}$ .

*Challenge 2.* It computes  $x = H_5(v, w, y, z, T_1, T_2, \mathbf{m})$ .

*Response.* It computes:

$$\begin{aligned} \tau_x &= \tau_1 \cdot x + \tau_2 \cdot x^2, & \mu &= \alpha + \beta \cdot w + \rho \cdot x, & z_\alpha &= r_\alpha + \alpha \cdot x, \\ z_{sk} &= r_{sk} + sk \cdot x, & z_\delta &= r_\delta + \delta_k \cdot x, & \mathbf{l} = l(x) &= \mathbf{b}_L - z \cdot \mathbf{1}^n + \mathbf{s}_L \cdot x, \\ \mathbf{r} &= r(x) = \mathbf{y}^n \circ (w \cdot \mathbf{b}_R + wz \cdot \mathbf{1}^n + \mathbf{s}_R \cdot x) + z^2 \cdot \mathbf{1}^n, & t &= \langle \mathbf{l}, \mathbf{r} \rangle. \end{aligned}$$

It outputs  $\sigma_{\text{ring}} = (B, A, S_1, S_2, T_1, T_2, \tau_x, \mu, z_\alpha, z_{sk}, \mathbf{l}, \mathbf{r}, t)$  and the key image  $U$ . Note that  $\mathbf{l}, \mathbf{r}$  can be reduced to logarithm size.

**Output.** Suppose we have  $M$  ring signatures  $\sigma_{\text{ring},k}$  and key images  $U_k$  for  $k \in [1, M]$  representing each row of input account. Denote  $\mathbb{S}$  as a set of serial number  $\{U_1, \dots, U_M\}$ . Then the output of the spend algorithm is  $(\mathbb{A}_{\text{out}}, \pi = (\pi_{\text{range}}, \mathbb{C}_{\text{in}}^*, \pi_{\text{bal}}, \{\sigma_{\text{ring},k}\}_{k \in [1, M]}), \mathbb{S}, \mathbb{C}k_{\text{out}})$ .

**Verify.** On input a message  $\mathbf{m}$ , a set of input accounts  $\mathbb{A}_{\text{in}}$ , a set of output accounts  $\mathbb{A}_{\text{out}}$ , a proof  $\pi$  and a set  $\mathbb{S}$  of serial numbers and a set  $\mathbb{U}$  of serial numbers in the past, then it checks:

1. If there exists any  $U^{(k)}$  in both  $\mathbb{S}$  and  $\mathbb{U}$ , returns 0 and exits since it is a double spending of the previous transaction. We can use Bloom filter on  $\mathbb{U}$  to speed up the detection of double spending.
2. It runs the RVerify algorithm of the range proof with input from  $\pi_{\text{range}}$  and the output coins in  $\mathbb{A}_{\text{out}}$ .
3. It checks the proof of balance as follows. It parses  $\pi_{\text{bal}} = (c_\Delta, z_\Delta)$  and computes  $R'_\Delta = g_c^{z_\Delta} \left( \frac{\prod_{j=1}^N C_{\text{out},j}}{\prod_{k=1}^M C_{\text{in},k}^*} \right)^{-c_\Delta}$ . It returns 0 if  $c_\Delta \neq H_1(R_\Delta, \mathbb{A}_{\text{out}}, \mathbb{C}_{\text{in}})$ .
4. It checks each ring signature  $\sigma_{\text{ring},k}$  and key images  $U_k \in \mathbb{S}$  for  $k \in [1, M]$  as follows. For simplicity, we drop the index  $k$  when the context is clear. It computes  $d_1, d_2$  and  $\mathbf{Y}$  as in the Ring Formation of the **Spend** protocol, using  $\mathbb{A}_{\text{in}}$  and  $\mathbb{C}_{\text{in}}^*$ . Denote the concatenated string  $\text{str} = \mathbf{Y} || B || A || S_1 || S_2 || S_3 || U$ . It computes  $h = H_3(\mathbf{Y})$ ,  $y = H_4(1, \text{str})$ ,  $z = H_4(2, \text{str})$ ,  $w = H_4(3, \text{str})$  and  $x = H_5(v, w, y, z, T_1, T_2, \mathbf{m})$ . Define  $\mathbf{h}' = (h'_1, \dots, h'_n) \in \mathbb{G}^n$  such that  $h'_i = h_i^{y^{-i+1}}$  for  $i \in [1, n]$ . It returns 1 if all of the following hold and returns 0 otherwise:

$$t = \langle \mathbf{l}, \mathbf{r} \rangle, \tag{1}$$

$$g^t h^{\tau_x} = g^{z^2 + w(z-z^2)\langle \mathbf{1}^n, \mathbf{y}^n \rangle - z^3 \langle \mathbf{1}^n, \mathbf{1}^n \rangle} \cdot T_1^x \cdot T_2^{x^2}, \tag{2}$$

$$h^\mu \mathbf{Y}^l \mathbf{h}'^r = B \cdot A^w \cdot S_2^x \cdot \mathbf{Y}^{-z \cdot \mathbf{1}^n} \cdot \mathbf{h}'^{wz \cdot \mathbf{y}^n + z^2 \cdot \mathbf{1}^n}, \tag{3}$$

$$h^{z_\alpha} g^{z_{sk}} g_c^{d_1 z_\delta} = S_1 (B g_0^{-d_2})^x, \tag{4}$$

$$U^{z_{sk}} = S_3 u^x. \tag{5}$$

## 5.2 Security

We give the security analysis of our construction with stealth address. The security proofs are given in the appendix.

**Theorem 1 (Balance).** *Our scheme is unforgeable if the DL assumption holds in  $\mathbb{G}$  in the random oracle model (ROM). Our scheme is equivalent w.r.t. insider corruption if the DL assumption holds in  $\mathbb{G}$  in the ROM and RP is a secure zero-knowledge range proof. Our scheme is linkable w.r.t. insider corruption if the DL assumption holds in  $\mathbb{G}$  in the ROM.*

**Theorem 2 (Anonymity).** *Our scheme is anonymous against recipient if the  $q$ -DDHI assumption holds in  $\mathbb{G}$  in the ROM, where  $q$  is the number of Spend oracle query. Our scheme is anonymous against ring insider if the  $q$ -DDHI assumption holds in  $\mathbb{G}$  in the ROM and RP is a secure zero-knowledge range proof.*

**Theorem 3 (Non-slanderability).** *Our scheme is non-slanderable w.r.t. insider corruption if the DL assumption holds in  $\mathbb{G}$  in the ROM.*

### 5.3 Our Efficient Construction

The last step towards our final construction is to use the improved inner product argument in [9] to compress the  $O(n)$ -size vector  $\mathbf{l}, \mathbf{r}$  in the ring signature part to a  $O(\log n)$ -size proof. Denote **IPProve**, **IPVerify** as the inner product argument. Details of the algorithm can be found in [9]. We give the modified **Spend'** and **Verify'** algorithms as follows.

- **Spend'**. On input  $(\mathfrak{m}, \mathbb{K}_S, \mathbb{A}_S, \mathbb{A}_{in}, \mathbb{O})$ , it runs  $(\mathbb{A}_{out}, \pi = (\pi_{range}, \mathbb{C}_{in}^*, \pi_{bal}, \{\sigma_{ring,k}\}_{k \in [1,M]}), \mathbb{S}, \mathbb{C}k_{out}) \leftarrow \mathbf{Spend}(\mathfrak{m}, \mathbb{K}_S, \mathbb{A}_S, \mathbb{A}_{in}, \mathbb{O})$ . For each  $\sigma_{ring,k} = (B, A, S_1, S_2, S_3, T_1, T_2, \tau_x, \mu, z_\alpha, z_{sk}, z_\delta, \mathbf{l}, \mathbf{r}, t)$ , it computes  $P = \mathbf{Y}' \mathbf{h}' \mathbf{r}'$ , where  $\mathbf{Y}$  and  $\mathbf{h}'$  are defined in **Spend**. it runs  $(\mathbf{L}, \mathbf{R}, a, b) \leftarrow \mathbf{IPProve}(\mathbf{Y}, \mathbf{h}', t, P, \mathbf{l}, \mathbf{r})$ . Note that  $\mathbf{L}, \mathbf{R}$  are vectors of  $\mathbb{G}$  with size  $\log n$ . It sets  $\sigma'_{ring,k} = (B, A, S_1, S_2, S_3, T_1, T_2, \tau_x, \mu, z_\alpha, z_{sk}, z_\delta, t, P, \mathbf{L}, \mathbf{R}, a, b)$ . The algorithm outputs  $(\mathbb{A}_{out}, \pi = (\pi_{range}, \mathbb{C}_{in}^*, \pi_{bal}, \{\sigma'_{ring,k}\}_{k \in [1,M]}), \mathbb{S}, \mathbb{C}k_{out})$ .
- **Verify'**. On input  $(\mathfrak{m}, \mathbb{A}_{in}, \mathbb{A}_{out}, \pi = (\pi_{range}, \mathbb{C}_{in}^*, \pi_{bal}, \{\sigma'_{ring,k}\}_{k \in [1,M]}), \mathbb{S})$ , denote  $\sigma'_{ring,k} = (B, A, S_1, S_2, S_3, T_1, T_2, \tau_x, \mu, z_\alpha, z_{sk}, z_\delta, t, P, \mathbf{L}, \mathbf{R}, a, b)$ . It runs  $0/1 \leftarrow \mathbf{IPVerify}(\mathbf{Y}, \mathbf{h}', t, P, (\mathbf{L}, \mathbf{R}, a, b))$ , where  $\mathbf{Y}$  and  $\mathbf{h}'$  are defined in **Verify**. It outputs 0 if **IPVerify** outputs 0. Otherwise, it runs as the **Verify** algorithm, except that equation (3) is modified to:

$$h^\mu P = B \cdot A^w \cdot S_2^x \cdot \mathbf{Y}^{-z \cdot \mathbf{1}^n} \cdot \mathbf{h}'^{wz \cdot \mathbf{y}^n + z^2 \cdot \mathbf{1}^n}.$$

The security of our final construction follows from the security of the improved inner product argument in [9], which is based on the DL assumption in the random oracle model.

## 6 Analysis

**Proof Size of RingCT.** RingCT3.0 has size of  $O(M \log n)$  excluding the key images and committed outputs, where  $n$  is the size of the ring and  $M$  is the number of transaction input. As shown in Figure 1a, RingCT3.0 is significantly shorter than Monero's RingCT1.0 even for small ring size (ring size  $\geq 16$ ) and hence RingCT3.0 can reduce the transaction fee by more than 90%. Since the proof size increases logarithmically, the sender can increase the anonymity level by increasing the ring size without increasing the cost drastically. Increasing the ring size of 1000 only increases the transaction fee by 45%.

Consider a typical transaction (i.e., number of inputs  $M = 2$ ) with a ring size of 1024, our ring signature size (2kB) is 97% less than the ring signature size of [31] (98kB). Monero has a relatively stable fee rate of about 0.0008 XMR/kB (which is about USD 0.2/kB). For the ring size of 1024, the cost of the ring signature part for RingCT1.0 is already about USD 20, which is not practical. On the other hand, the cost of the ring signature part for RingCT3.0 is only about USD 0.4.

**Running Time of RingCT.** We implemented the RingCT3.0 in Ubuntu 16.04, Intel Core i5-6200U 2.3GHz, 8GB RAM. We used the BouncyCastle's Java library for Curve 25519 in our implementation. Each element in  $\mathbb{G}$  is represented by 33 bytes and each element in  $\mathbb{Z}_p$  is represented by 32 bytes.

We compare the running time of the **Spend** protocol of RingCT3.0 in Figure 2a and RingCT1.0 in Figure 2c. Our RingCT3.0 outperforms RingCT1.0 when the ring size exceeds 64. Our RingCT3.0 is better for larger ring size and more user input. When the ring size is 1024 and the input size is 20, RingCT3.0 is about 2 times faster than RingCT1.0.

We compare the running time of the **Verify** protocol of RingCT3.0 in Figure 2b and RingCT1.0 in Figure 2d. Our RingCT3.0 outperforms RingCT1.0 when the ring size exceeds 32. When the ring size reaches 1024 and the input size is 20, RingCT3.0 is more than 2 times faster than RingCT1.0.

In general, the running time of RingCT3.0 is comparatively shorter than the time of generating a block of transactions, which is 2 minutes in Monero and 10 minutes in Bitcoin. Therefore, RingCT3.0 will not be the bottleneck of the blockchain system.

**Tradeoff for Strong Anonymity** One interesting analysis is about the tradeoff in having a stronger anonymity. We compare the proof size of using different levels of anonymity (we exclude the common components, such as key images, range proof, etc.). In the lower anonymity setting (RingCT 1.0), the proof size is  $(M + 1)$  ring signatures for  $M$  transaction inputs. In the higher anonymity setting (RingCT 3.0), the proof size is  $M$  ring signatures,  $M$  re-randomized input coins (due to  $\mathbb{C}_{in}^*$ ), and the balance proof size. Therefore, the higher anonymity setting has 1 ring signature less, and has the extra  $M$  re-randomized input coins (which consists of  $M$  group elements) and the balance proof size (which consists of 2 group elements). The ring signature has the size of  $O(\log n)$ , where  $n$  is the size of the ring. For a typical transaction of  $M = 2$ , our higher anonymity setting is even more efficient.

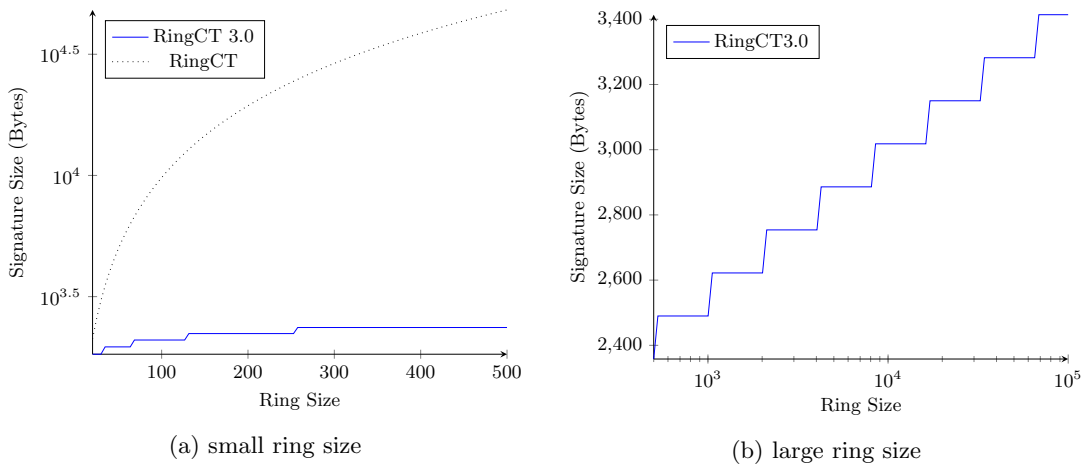


Fig. 1: Comparison of RingCT and RingCT3.0 for a transaction with 2 inputs.

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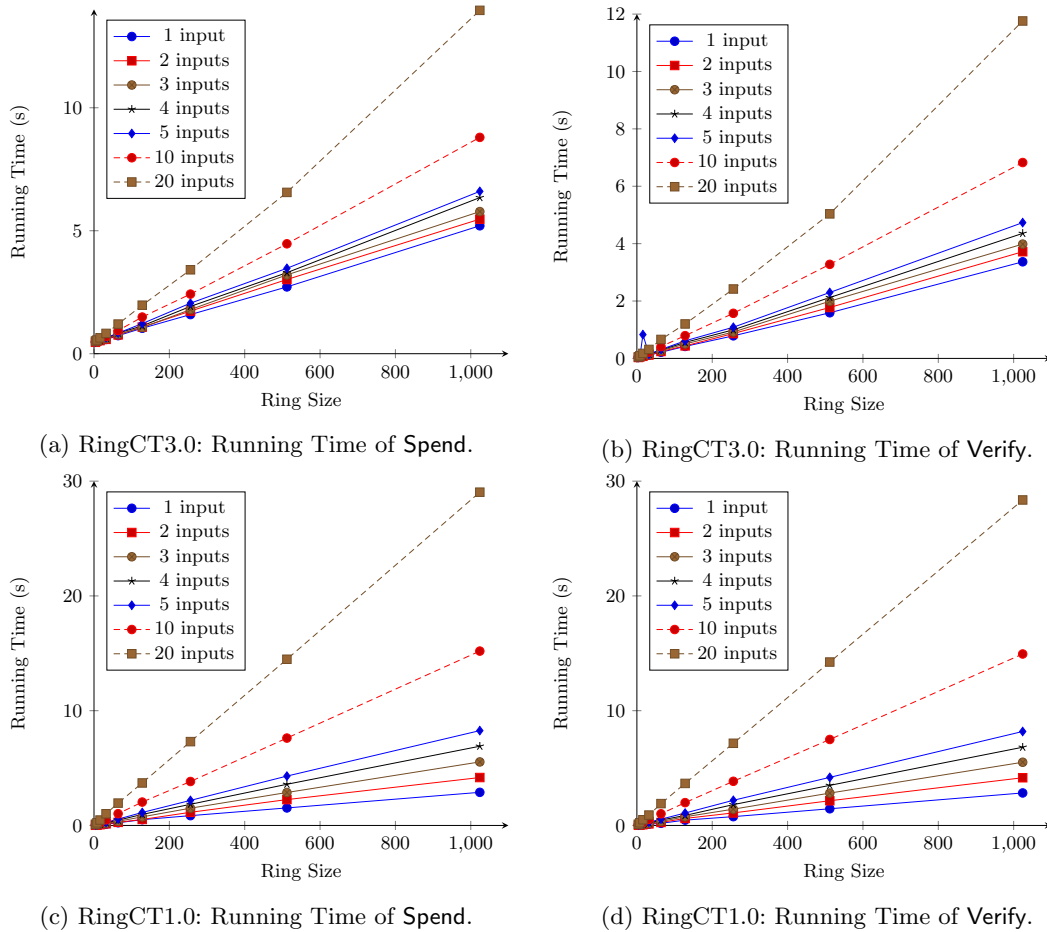


Fig. 2: Performance of RingCT3.0 and RingCT1.0.

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## A Security Proofs

**Theorem 4.** *Our scheme is anonymous against recipient if the  $q$ -DDHI assumption holds in  $\mathbb{G}$  in the ROM, where  $q$  is the number of Spend oracle query.*

*Proof.* Observe that the adversary is given  $\mathbb{A}_{\text{out}}^*, \pi^* = (\pi_{\text{range}}, \mathbb{C}_{\text{in}}^*, \pi_{\text{bal}}, \{\sigma_{\text{ring},k}\}_{k \in [1,M]})$ ,  $\mathbb{S}^*, \mathbb{O}$  and  $\mathbb{C}k_{\text{out}}^*$ . Note that  $\mathbb{A}_{\text{out}}^*, \mathbb{O}, \mathbb{C}k_{\text{out}}^*$  only contain information about the output account (including the output amount). Therefore, we only have to check if  $\pi^*, \mathbb{S}^*$  reveal the information of the input account or amount.

Suppose the simulator is given the DDHI problem instance  $(g, g^a, \dots, g^{a^q}, T)$  and wants to decide if  $T = g^{1/a}$ . The simulator picks some distinct random  $\zeta, h^*, h_1, \dots, h_q \in \mathbb{Z}_p$  and  $u = g^{\zeta \cdot \prod_{i=1}^q (a - h^* + h_i)}$  as part of the system parameters.

For the PkGen query, the simulator picks some random  $\text{ltsk}$  and computes the corresponding  $\text{ltpk}$  and one-time  $\text{pk}$ . Except for one time, the simulator returns  $\text{ltpk}^* = (g^{a-h^*}, g^{x_2})$  for some random  $x_2 \in \mathbb{Z}_p^*$ . The simulator honestly runs the ActGen oracle, except that the  $\text{ask}^*$  corresponding to  $\text{ltpk}^*$  is unknown. For the Corrupt oracle, the simulator declares failure and exits if the  $\text{ask}^*$  is requested.

If the adversary queries the  $H_6$  oracle, it either returns  $\zeta, h^*, h_1, \dots, h_q$ , or a random element in  $\mathbb{Z}_p^*$ .

If the adversary queries the Spend oracle with input  $(\mathbf{m}, \mathbb{A}_S, \mathbb{A}_{\text{in}}, \mathbb{O})$  with with known  $\text{ltsk}$ s correspond to some public keys in  $\mathbb{A}_S$ , then the simulator can generate valid signatures for that part. Otherwise  $\text{pk} = (g^{a-h^*} g^{\hat{h}})$  corresponds to a public key in  $\mathbb{A}_S$ , where  $\hat{h}$  is the oracle output of  $H_6$ . If  $\hat{h} \neq h_j$  for some  $j$ , the simulator declares failure and exits. If  $\hat{h} = h_j$ , then the simulator set  $U = g^{\zeta \cdot \prod_{i=1, i \neq j}^q (a - h^* + h_i)}$ . The simulator chooses most of the proof elements  $(B, S_2, T_2, T_3, \tau_x, \mu, z_\alpha, z_{\text{sk}}, z_\delta, \mathbf{l}, \mathbf{r})$  and challenges  $(x, y, z, w)$  uniformly at random from their respective domains, and then computes  $t$  from equation (1),  $T_1$  from equation (2),  $A$  from equation (3),  $S_1$  from equation (4) and  $S_3$  from equation (5). Then the simulator sets the value  $x, y, z, w$  in the random oracle  $H_4, H_5$ . Then the output for  $\sigma_{\text{ring}}$  is complete. The rest of the Spend oracle can be simulated easily.

In the challenge phase, the adversary gives  $(\mathbf{m}, \{\text{pk}_{k,i}\}_{k \in [1, M], i \in [1, n]}, \{\text{pk}_{\text{out},j}\}_{j \in [1, N]})$  to the simulator. If  $\text{pk}^* = g^{a-h^*} g^{h^*} = g^a$  does not corresponds to some  $\text{pk}_{k,i}$ , the simulator declares failure and exits. Without loss of generality, assume  $\text{pk}_{k',i_k^*} = \text{pk}^*$  for some  $k', i_k^*$ . According to the security model, the simulator retrieves  $(\text{sk}_{k,i}, \text{pk}_{k,i}) \in \mathcal{U}$ , picks  $\mathbf{a}_{k,i}, \mathbf{a}_{\text{out},j} \leftarrow_R \text{Range}$  where  $\sum_k \mathbf{a}_{k,i_k^*} = \sum_j \mathbf{a}_{\text{out},j}$ . With the knowledge of all input, output amount and input, output coin keys, the simulator can honestly generate  $\pi_{\text{range}}, \mathbb{C}_{\text{in}}^*, \pi_{\text{bal}}$ . If the DL assumption holds, the Pederson commitments in  $\mathbb{C}_{\text{in}}^*$  hides all the input amounts. For the generation of the ring signature part, for  $k \neq k'$ ,  $\sigma_{\text{ring},k}$  can be honestly generated using the account secret key. For  $k = k'$ , the generation of  $\sigma_{\text{ring},k'}$  is described as follows. The simulator sets  $U = g^{\zeta \cdot \prod_{i=1}^q (a - h^* + h_i)/a} := g^{\zeta \cdot \sum_{i=0}^{q-1} A_i a^i T^{\zeta A_{-1}}}$ , for some constant  $A_{q-1}, \dots, A_0, A_{-1} \in \mathbb{Z}_p$ . Note that  $A_{-1} \neq 0$  since  $h^* \neq h_i$  for all  $i$ . The rest of  $\sigma_{\text{ring},k'}$  is simulated as in the Spend oracle. According to the security model, the adversary is given  $\mathbb{A}_{\text{out}}^*, \pi^* = (\pi_{\text{range}}, \mathbb{C}_{\text{in}}^*, \pi_{\text{bal}}, \{\sigma_{\text{ring},k}\}_{k \in [1, M]}), \mathbb{S}^*, \mathbb{O}, \mathbb{Ck}_{\text{out}}^*$  and  $\mathbb{A}_{\text{in}}$ , where  $\mathbb{S}^*$  is the set of serial numbers,  $\mathbb{O}$  is the set of output public keys and output amounts and  $\mathbb{A}_{\text{in}}$  is the set of input accounts.

Finally the adversary outputs  $(k^*, \text{ind}_k^*)$ . With probability  $1/M$ ,  $k' = k^*$ . If the adversary guesses  $\text{ind}_k^* = i_k^*$  correctly, then the simulator outputs  $T = g^{1/a}$  as the solution to the DDHI problem. Therefore, our scheme is anonymous against outsider if the DDHI assumption holds.  $\square$

Before giving the formal proof of balance, we first give a lemma.

**Lemma 1.** *The mutual discrete logarithm between each element in  $\mathbf{Y}$  (formed by the Ring Formation step) is not known if the discrete logarithm assumption holds in  $\mathbb{G}$ .*

*Proof.* Suppose the simulator is given a DL problem instance  $(g_0, X)$  and wants to compute  $\log_{g_0} X$ . The simulator picks some random  $\zeta_1, \zeta_2 \in \mathbb{Z}_p$  and sets  $g = X^{\zeta_1}$  and  $g_c = X^{\zeta_2}$ . Consider two elements  $Y_1 \doteq g^{\text{sk}_1} g_c^{d_1 \delta_1} g_0^{d_2}$  and  $Y_2 \doteq g^{\text{sk}_2} g_c^{d_1 \delta_2} g_0^{d_2}$  in  $\mathbf{Y}$ . Suppose that the discrete logarithm value  $V = \log_{Y_2} Y_1$  is known. Then

$$V = \frac{\text{sk}_1 \log_{g_0} g + d_1 \delta_1 \log_{g_0} g_c + d_2}{\text{sk}_2 \log_{g_0} g + d_1 \delta_2 \log_{g_0} g_c + d_2} = \frac{\log_{g_0} X(\zeta_1 \text{sk}_1 + \zeta_2 d_1 \delta_1) + d_2}{\log_{g_0} X(\zeta_1 \text{sk}_2 + \zeta_2 d_1 \delta_2) + d_2}.$$

It implies that the simulator can answer the DL problem as

$$\log_{g_0} X = \frac{d_2(1 - V)}{\zeta_1(V \text{sk}_2 - \text{sk}_1) + \zeta_2 d_1(V \delta_2 - \delta_1)}.$$

if  $\zeta_1(V\text{sk}_2 - \text{sk}_1) + \zeta_2 d_1(V\delta_2 - \delta_1) \neq 0$ . By the random choice of  $\zeta_1$  and  $\zeta_2$ ,  $\zeta_1(V\text{sk}_2 - \text{sk}_1) + \zeta_2 d_1(V\delta_2 - \delta_1) = 0$  with negligible probability, except  $V\text{sk}_2 = \text{sk}_1$  and  $V\delta_2 = \delta_1$ . If  $V\text{sk}_2 = \text{sk}_1$  and  $V\delta_2 = \delta_1$ , then:

$$Y_1 = g^{\text{sk}_1} g_c^{d_1 \delta_1} g_0^{d_2} = g^{V\text{sk}_2} g_c^{V d_1 \delta_2} g_0^{d_2} = (Y_2 g_0^{-d_2})^V g_0^d = Y_1 g_0^{d_2(1-V)},$$

which leads to a contradiction since  $d_2 \neq 0$  and  $V \neq 1$ .

**Theorem 5.** *Our scheme is unforgeable if the DL assumption holds in  $\mathbb{G}$  in the random oracle model.*

*Proof.* Suppose the simulator is given a discrete logarithm problem instance  $g, g^a$ . It picks random group elements for  $\mathbf{h}, u, h_c$  and  $g_c$  such that the discrete logarithm with respect to  $g$  is unknown. It picks a random  $\beta \in \mathbb{Z}_p$  and sets  $u = g^{a\beta}$ .

For the PkGen query, the simulator picks some random  $\text{sk} = \iota \in \mathbb{Z}_p$  and returns  $\text{pk} = g^\iota$ . Except for one-time, the simulator returns  $\text{pk}^* = g^a$ . The simulator honestly runs the ActGen oracle, except that the  $\text{ask}^*$  for input  $\text{pk}^*$  is unknown. For the Corrupt oracle, the simulator declares failure and exits if the  $\text{ask}^*$  is requested. For the  $H_6$  oracle, it returns a random  $h \in \mathbb{G}$  (therefore the discrete logarithm with respect to  $g$  is unknown).

If the adversary queries the Spend oracle, the simulator runs as the Spend algorithm if  $\text{pk}^*$  is not the signing account's public key. If  $\text{pk}^*$  is the signing account's public key, the simulator can first honestly generate  $\pi_{\text{range}}, \mathbb{C}_{\text{in}}^*, \pi_{\text{bal}}$  using the knowledge of all input, output amount and input, output coin keys. It generates the set of output accounts  $\mathbb{A}_{\text{out}}^*$  and coin keys  $\mathbb{C}_{\text{out}}^*$  using Mint. The generation of the ring signature part  $\sigma_{\text{ring}}$  for  $\text{pk}^*$  is described as follows. The simulator sets  $U = g^\beta$ . The simulator chooses most proof elements  $(B, S_2, T_2, T_3, \tau_x, \mu, z_\alpha, z_{\text{sk}}, z_\delta, \mathbf{l}, \mathbf{r})$  and challenges  $(x, y, z, w)$  uniformly at random from their respective domains, and then computes  $t$  from equation (1),  $T_1$  from equation (2),  $A$  from equation (3),  $S_1$  from equation (4) and  $S_3$  from equation (5). Then the simulator sets the value  $x, y, z, w$  in the random oracle  $H_4, H_5$ . Then the output for  $\sigma_{\text{ring}}$  is complete. Hence the simulator can answer the Spend oracle query.

Finally, the adversary  $\mathcal{A}$  outputs  $(\mathbf{m}, \mathbb{A}_{\text{in}}, \mathbb{A}_{\text{out}}, \pi, \mathbb{S})$  such that for all  $\text{act}_i \in \mathbb{A}_{\text{in}}, \text{act}_i \in \mathcal{I} \setminus \mathcal{C}$ , and  $(\mathbb{A}_{\text{out}}, \pi, \mathbb{S}, \cdot) \notin \mathcal{T}$ . If  $\mathcal{A}$  wins,  $\text{Verify}(\mathbf{m}, \mathbb{A}_{\text{in}}, \mathbb{A}_{\text{out}}, \pi, \mathbb{S}) = 1$ . Denote the output  $\pi = (\pi_{\text{range}}, \mathbb{C}_{\text{in}}^*, \pi_{\text{bal}}, \{\sigma_{\text{ring}, k}\}_{k \in [1, M]})$ . The simulator rewinds  $H_5$  (in step challenge 2) for three times. For each transcript, denote the challenge as  $x_i$  and the responses as  $(\tau_{x,i}, \mu_i, z_{\alpha,i}, z_{\text{sk},i}, z_{\delta,i}, \mathbf{l}_i, \mathbf{r}_i, t_i)$  for  $i \in [1, 3]$ . Denote  $\mathbf{l}_i = (l_{i,1}, \dots, l_{i,n})$  and  $\mathbf{r}_i = (r_{i,1}, \dots, r_{i,n})$ .

- To extract  $BA^w$ , it picks some  $\eta_i \in \mathbb{Z}_p$  such that  $\sum_{i=1}^2 \eta_i = 1, \sum_{i=1}^2 \eta_i x_i = 0$ . By equation (3), we have:

$$\begin{aligned} BA^w &= h^{\sum_{i=1}^2 \eta_i \mu_i} \mathbf{Y}^{\sum_{i=1}^2 \eta_i \cdot \mathbf{l}_i + z \cdot \mathbf{1}^n} \mathbf{h}^{\sum_{i=1}^2 \eta_i \mathbf{r}_i - w z \cdot \mathbf{y}^n - z^2 \cdot \mathbf{1}^n} \\ &:= h^{\gamma'} \mathbf{Y}^{\mathbf{b}'_{\mathbf{L}}} \mathbf{h}^{w \cdot \mathbf{b}'_{\mathbf{R}}}, \end{aligned} \quad (6)$$

for some  $\gamma', \mathbf{b}'_{\mathbf{L}}, \mathbf{b}'_{\mathbf{R}}$ .

- To extract  $S_2$ , it picks some  $\eta'_i \in \mathbb{Z}_p$  such that  $\sum_{i=1}^2 \eta'_i = 0, \sum_{i=1}^2 \eta'_i x_i = 1$ . By equation (3), we have:

$$S_2 = h^{\sum_{i=1}^2 \eta'_i \mu_i} \mathbf{Y}^{\sum_{i=1}^2 \eta'_i \mathbf{l}_i} \cdot \mathbf{h}^{\sum_{i=1}^2 \eta'_i \mathbf{r}_i} := h^{\rho'} \mathbf{Y}^{\mathbf{s}'_{\mathbf{L}}} \mathbf{h}^{\mathbf{s}'_{\mathbf{R}}},$$

for some  $\rho', \mathbf{s}'_{\mathbf{L}}, \mathbf{s}'_{\mathbf{R}}$ .

Putting back the extracted values  $BA^w$  and  $S_2$  into equation (3), we have:

$$h^\mu \mathbf{Y}^{\mathbf{l}} \mathbf{h}^{\mathbf{r}} = (h^{\gamma'} \mathbf{Y}^{\mathbf{b}'_{\mathbf{L}}} \mathbf{h}^{w \cdot \mathbf{b}'_{\mathbf{R}}}) \cdot (h^{\rho'} \mathbf{Y}^{\mathbf{s}'_{\mathbf{L}}} \mathbf{h}^{\mathbf{s}'_{\mathbf{R}}})^x \cdot \mathbf{Y}^{-z \cdot \mathbf{1}^n} \cdot \mathbf{h}^{w z \cdot \mathbf{y}^n + z^2 \cdot \mathbf{1}^n}.$$

The mutual discrete logarithm between  $\mathbf{Y}$  is not known if the discrete logarithm assumption holds by lemma 1. Since  $h$  and the elements in  $\mathbf{h}$  are randomly chosen from the group, the mutual discrete logarithm between  $h$ , the elements in  $\mathbf{h}$  and  $\mathbf{Y}$  is not known.

Then we have:

$$\begin{aligned} \mathbf{l} &= \mathbf{b}'_{\mathbf{L}} - z \cdot \mathbf{1}^n + \mathbf{s}'_{\mathbf{L}} \cdot x, \\ \mathbf{r} &= \mathbf{y}^n \circ (w \cdot \mathbf{b}'_{\mathbf{R}} + wz \cdot \mathbf{1}^n + \mathbf{s}'_{\mathbf{R}} \cdot x) + z^2 \cdot \mathbf{1}^n. \end{aligned}$$

By the same set of 3 rewinding transcripts, we can also extract the commitments  $T_1, T_2$  as follows.

- To extract  $T_1$ , it picks some  $\delta_i \in \mathbb{Z}_p$  such that  $\sum_{i=1}^3 \delta_i = 0$ ,  $\sum_{i=1}^3 \delta_i x_i = 1$ ,  $\sum_{i=1}^3 \delta_i x_i^2 = 0$ . By equation (2), we have:

$$T_1 = g^{\sum_{i=1}^3 \delta_i t_i} h^{\sum_{i=1}^3 \delta_i \tau_{x,i}} := g^{t'_1} h^{r'_1}$$

for some  $t'_1, r'_1$ .

- To extract  $T_2$ , it picks some  $\delta'_i \in \mathbb{Z}_p$  such that  $\sum_{i=1}^3 \delta'_i = 0$ ,  $\sum_{i=1}^3 \delta'_i x_i = 0$ ,  $\sum_{i=1}^3 \delta'_i x_i^2 = 1$ . By equation (2), we have:

$$T_2 = g^{\sum_{i=1}^3 \delta'_i t_i} h^{\sum_{i=1}^3 \delta'_i \tau_{x,i}} := g^{t'_2} h^{r'_2}$$

for some  $t'_2, r'_2$ .

Putting back the extracted values  $T_1$  and  $T_2$  into equation (2), we have:

$$g^t h^{\tau_x} = g^{z^2 + w(z-z^2)\langle \mathbf{1}^n, \mathbf{y}^n \rangle - z^3 \langle \mathbf{1}^n, \mathbf{1}^n \rangle} \cdot (g^{t'_1} h^{r'_1})^x \cdot (g^{t'_2} h^{r'_2})^{x^2}.$$

Since  $h$  is a random group element by the simulation of  $H_6$ , we have:

$$t = z^2 + w(z-z^2)\langle \mathbf{1}^n, \mathbf{y}^n \rangle - z^3 \langle \mathbf{1}^n, \mathbf{1}^n \rangle + t'_1 x + t'_2 x^2.$$

Denote  $t'_0 = z^2 + w(z-z^2)\langle \mathbf{1}^n, \mathbf{y}^n \rangle - z^3 \langle \mathbf{1}^n, \mathbf{1}^n \rangle$ .

By equation (1), we have  $t = \langle \mathbf{l}, \mathbf{r} \rangle$ . Observe that we already extracted  $\mathbf{l}, \mathbf{r}$  as:

$$\begin{aligned} \langle \mathbf{l}, \mathbf{r} \rangle &= (\mathbf{b}'_{\mathbf{L}} - z \cdot \mathbf{1}^n + \mathbf{s}'_{\mathbf{L}} \cdot x) \cdot (\mathbf{y}^n \circ (w \cdot \mathbf{b}'_{\mathbf{R}} + wz \cdot \mathbf{1}^n + \mathbf{s}'_{\mathbf{R}} \cdot x) + z^2 \cdot \mathbf{1}^n) \\ &= w \langle \mathbf{b}'_{\mathbf{L}}, \mathbf{b}'_{\mathbf{R}} \circ \mathbf{y}^n \rangle + wz \langle \mathbf{b}'_{\mathbf{L}} - \mathbf{b}'_{\mathbf{R}}, \mathbf{y}^n \rangle + z^2 \langle \mathbf{b}'_{\mathbf{L}}, \mathbf{1}^n \rangle \\ &\quad - wz^2 \langle \mathbf{1}^n, \mathbf{y}^n \rangle - z^3 \langle \mathbf{1}^n, \mathbf{1}^n \rangle + t'_1 x + t'_2 x^2, \end{aligned}$$

for some  $t''_1, t''_2 \in \mathbb{Z}_p$ . Since the above holds for all  $w, x, y, z$ , we have:

$$\mathbf{b}'_{\mathbf{L}} \circ \mathbf{b}'_{\mathbf{R}} = \mathbf{0}^n, \quad \mathbf{b}'_{\mathbf{L}} - \mathbf{b}'_{\mathbf{R}} = \mathbf{1}^n, \quad \langle \mathbf{b}'_{\mathbf{L}}, \mathbf{1}^n \rangle = 1.$$

Therefore, it implies that  $\mathbf{b}'_{\mathbf{L}}$  is a binary vector with one bit equal to 1. Putting back  $\mathbf{b}'_{\mathbf{L}}$  in equation (13), we have:

$$BA^w = h^{\gamma'} \cdot \text{pk}_{\text{in},k}^{(\text{ind}_k)} \left( \frac{C_{\text{in},k}^{(\text{ind}_k)}}{C_{\text{in},k}^*} \right)^{d_1} g_0^{d_2} \cdot \mathbf{h}^{w \cdot \mathbf{b}'_{\mathbf{R}}},$$

for some index  $\text{ind}_k$ . Since the above is true for all  $w$ , then we have  $B = h^{\alpha'} \text{pk}_{\text{in},k}^{(\text{ind}_k)} \left( \frac{C_{\text{in},k}^{(\text{ind}_k)}}{C_{\text{in},k}^*} \right)^{d_1} g_0^{d_2}$  for some  $\alpha' \in \mathbb{Z}_p$ .

By the same set of rewinding transcripts, we can also extract from equation (4):

$$Bg_0^{-d_2} = h^{\frac{z\alpha, 1-z\alpha, 2}{x_1-x_2}} g^{\frac{z\text{sk}, 1-z\text{sk}, 2}{x_1-x_2}} g_c^{\frac{d_1(z\delta, 1-z\delta, 2)}{x_1-x_2}} := h^{\alpha''} g^{\text{sk}'} g_c^{d_1 \delta'_k} \quad (7)$$

Since the above is true for all  $h$  and  $d_1$ , then we have  $\text{pk}_{\text{in},k}^{(\text{ind}_k)} = g^{\text{sk}'}$ . Hence if  $g^{\text{sk}'} = \text{pk}_*$ , then the simulator returns  $\text{sk}'$  as the solution to the DL problem. It happens with probability  $1/n$ .

**Theorem 6.** *Our scheme is equivalent w.r.t. insider corruption if the DL assumption holds in  $\mathbb{G}$  in the random oracle model and RP is a secure zero-knowledge range proof.*

*Proof.* Suppose the simulator is given a discrete logarithm problem instance  $g, g^a$ . It picks random group elements for  $\mathbf{h}$ ,  $u$ , and  $h_c$  such that the discrete logarithm with respect to  $g$  is unknown. It sets  $g_c = g^a$ .

For the **AddGen** query, the simulator picks some random  $\mathbf{sk} = \iota \in \mathbb{Z}_p$  and returns  $\mathbf{pk} = g^\iota$ . The simulator honestly runs the **ActGen** oracle, the **Corrupt** oracle and the **Spend** oracle.

In the challenge phase, the adversary outputs  $(\mathbf{m}, \mathbb{A}_{\text{in}}, \mathbb{A}_{\text{out}}, \pi, \mathbb{S}, \mathbb{C}k_{\text{out}}, \mathbb{A}mt_{\text{out}})$ . If  $\mathcal{A}$  wins, then for all coin key  $\kappa_i \in \mathbb{C}k_{\text{out}}$ , amount  $\mathbf{a}_{\text{out},i} \in \mathbb{A}mt_{\text{out}}$  and output coins  $C_{\text{out},i}$  in  $\mathbb{A}_{\text{out}}$ ,

$$C_{\text{out},i} = g_c^{\kappa_i} h_c^{\mathbf{a}_{\text{out},i}}.$$

Denote the output  $\pi = (\pi_{\text{range}}^*, \mathbb{C}_{\text{in}}^*, \pi_{\text{bal}}^*, \{\sigma_{\text{ring},k}\}_{k \in [1,M]})$ . Recall from the proof of unforgeability that we have  $B = h_c^{\alpha'} \mathbf{pk}_{\text{in},k}^{(\text{ind}_k)} \left( \frac{C_{\text{in},k}^{(\text{ind}_k)}}{C_{\text{in},k}^*} \right) d_1 g_0^{d_2}$ . Together with equation (7), we have

$$\frac{C_{\text{in},k}^{(\text{ind}_k)}}{C_{\text{in},k}^*} = g_c^{\delta'_k}, \quad (8)$$

by the random choice of  $d_1$ .

Denote  $\pi_{\text{bal}}^* = (c_\Delta, z_\Delta)$ . The simulator computes:

$$R_\Delta^* = g_c^{z_\Delta} \left( \frac{\prod_{j=1}^N C_{\text{out},j}}{\prod_{k=1}^M C_{\text{in},k}^*} \right)^{-c_\Delta}.$$

The simulator rewinds  $H_1$  to obtain another proof  $(c'_\Delta, z'_\Delta)$ . Therefore, we have:

$$g_c^{z_\Delta - z'_\Delta} = \left( \frac{\prod_{j=1}^N C_{\text{out},j}}{\prod_{k=1}^M C_{\text{in},k}^*} \right)^{c_\Delta - c'_\Delta}.$$

Denote  $\Delta^* = \frac{z_\Delta - z'_\Delta}{c_\Delta - c'_\Delta}$ . From equation (8), we have:

$$\prod_{j=1}^N C_{\text{out},j} = \prod_{k=1}^M C_{\text{in},k}^{(\text{ind}_k)} g_c^{-\delta'_k} \cdot g_c^{\Delta^*}.$$

Recall that if  $\mathcal{A}$  wins, then for all account  $act_j \in \mathbb{A}_{\text{in}}$ ,  $(act_j, \mathbf{a}_{\text{in},j}, \kappa_{\text{in},k}) \in \mathcal{G}$ . It implies that  $C_{\text{in},k}^{(\text{ind}_k)} = h_c^{\mathbf{a}_{\text{in},k}} g_c^{\kappa_{\text{in},k}}$  and  $\mathbf{a}_{\text{in},k}$  is within a valid range. Similarly, the range proof  $\pi_{\text{range}}^*$  of the challenge signature shows that  $C_{\text{out},j} = h_c^{\mathbf{a}_{\text{out},j}} g_c^{\kappa_{\text{out},j}}$  and  $\mathbf{a}_{\text{out},j}$  is within a valid range. Therefore, we have:

$$\prod_{j=1}^N h_c^{\mathbf{a}_{\text{out},j}} g_c^{\kappa_{\text{out},j}} = \prod_{k=1}^M h_c^{\mathbf{a}_{\text{in},k}} g_c^{\kappa_{\text{in},k} - \delta'_k} \cdot g_c^{\Delta^*}.$$

If  $Z \doteq \sum_{j=1}^N \mathbf{a}_{\text{out},j} - \sum_{k=1}^M \mathbf{a}_{\text{in},k} \neq 0$ , then:

$$h_c^Z = g_c^{\sum_{k=1}^M (\kappa_{\text{in},k} - \delta'_k) + \Delta^* - \sum_{j=1}^N \kappa_{\text{out},j}}.$$

Then the simulator returns  $\frac{\sum_{k=1}^M (\kappa_{\text{in},k} - \delta'_k) + \Delta^* - \sum_{j=1}^N \kappa_{\text{out},j}}{Z}$  as the solution to the DL problem.

**Theorem 7.** *Our scheme is linkable w.r.t. insider corruption if the DL assumption holds in  $\mathbb{G}$  in the random oracle model.*

*Proof.* Suppose the simulator is given a DL problem instance  $g, g^a$ . It picks random elements in  $\mathbb{G}$  for  $\mathbf{h}$ ,  $u$ ,  $h_c$  and  $g_c$ .

For the **PkGen** query, the simulator picks some random  $\mathbf{sk} = \iota \in \mathbb{Z}_p$  and returns  $\mathbf{pk} = g^\iota$ . Except for one-time, the simulator returns  $\mathbf{pk}^* = g^a$ . The simulator honestly runs the **ActGen** oracle, except

that the  $ask^*$  for input  $pk^*$  is unknown. For the **Corrupt** oracle, the simulator declares failure and exits if the  $ask^*$  is requested. If the adversary queries the **Spend** oracle, the simulator runs as in the proof of unforgeability.

Finally, the adversary  $\mathcal{A}$  outputs  $(\mathbf{m}_i, \mathbb{A}_{in,i}, \mathbb{A}_{out,i}, \pi_i, \mathbb{S}_i)$  for  $i = 1, 2$ , such that all  $U$  in  $\mathbb{S}_1$  and  $\mathbb{S}_2$  are distinct. From the proof of unforgeability, the simulator can rewinds  $x$  and from equation (7):

$$h^{\alpha''} g^{\text{sk}'} g_c^{d_1 \delta'_k} = h^{\frac{z_{\alpha,1} - z_{\alpha,2}}{x_1 - x_2}} g^{\frac{z_{\text{sk},1} - z_{\text{sk},2}}{x_1 - x_2}} g_c^{\frac{d_1(z_{\delta,1} - z_{\delta,2})}{x_1 - x_2}},$$

which implies  $\text{sk}' = \frac{z_{\text{sk},1} - z_{\text{sk},2}}{x_1 - x_2}$ . From equation (5), we also have:

$$U^{z_{\text{sk},1} - z_{\text{sk},2}} = u^{x_1 - x_2}.$$

Therefore,  $U = u^{\frac{1}{\text{sk}'}}$  and  $g^{\text{sk}'}$  refers to one public key in  $\mathbb{A}_{in,1} \cup \mathbb{A}_{in,2}$ . There are  $|\mathbb{S}_1| + |\mathbb{S}_2|$  distinct values of  $U$ .

If  $\mathcal{A}$  wins, then  $(\mathbb{A}_{in,1} \cup \mathbb{A}_{in,2}) \cap \mathcal{C} + (\mathbb{A}_{in,1} \cup \mathbb{A}_{in,2}) \setminus \mathcal{I} \leq |\mathbb{S}_1| + |\mathbb{S}_2| - 1$ . It means that there exists at least one  $U$  corresponding to one public key  $g_c^{\text{sk}'}$  with account  $act^* \doteq (g_c^{\text{sk}'}, \cdot) \in \mathcal{I} \setminus \mathcal{C}$ . With probability at least  $\frac{1}{q_a + q_s - q_c}$ ,  $g^{\text{sk}'} = g^a$ , where  $q_a, q_s, q_c$  is the number of oracle queries to the **ActGen**, **Spend**, **Corrupt** oracles respectively. Then the simulator returns  $\text{sk}'$  as the solution to the DL problem.

**Theorem 8.** *Our scheme is non-slanderable w.r.t. insider corruption if the DL assumption holds in  $\mathbb{G}$  in the random oracle model.*

*Proof.* Suppose the simulator is given a DL problem instance  $g, g^a$ . It picks random elements in  $\mathbb{G}$  for  $\mathbf{h}, u, h_c$  and  $g_c$ .

For the **PkGen** query, the simulator picks some random  $\text{sk} = \iota \in \mathbb{Z}_p$  and returns  $pk = g^\iota$ . Except for one-time, the simulator returns  $pk^* = g^a$ . The simulator honestly runs the **ActGen** oracle, except that the  $ask^*$  for input  $pk^*$  is unknown. For the **Corrupt** oracle, the simulator declares failure and exits if the  $ask^*$  is requested. If the adversary queries the **Spend** oracle, the simulator runs as in the proof of unforgeability.

Finally, the adversary  $\mathcal{A}$  outputs  $(\hat{\mathbb{S}}, \mathbf{m}, \mathbb{A}_{in}, \mathbb{A}_{out}, \pi, \mathbb{S})$ , where  $(\cdot, \cdot, \hat{\mathbb{S}}, \mathbb{A}_S) \in \mathcal{T}$ .

If  $pk^* \notin \mathbb{A}_S$ , the simulator declares failure and exits (it happens with probability  $(1 - \frac{1}{q_p})^{|\mathbb{A}_S|}$ , where  $q_p$  is the number of **PkGen** oracle queries.). By the winning condition, there exists some  $U \in \hat{\mathbb{S}} \cap \mathbb{S}^*$ . Following the proof of linkability, the simulator can extract  $\text{sk}'$  such that  $U = u^{\frac{1}{\text{sk}'}}$  and  $g^{\text{sk}'} \in \mathbb{A}_S$ . With probability  $\frac{1}{|\mathbb{A}_S|}$ ,  $g^{\text{sk}'} = pk^*$ . Then the simulator return  $\text{sk}'$  as the solution to the DL problem.

## B Set Membership Proof without Trusted Setup

We first review the definition of set membership proof in [11] and then we give our new construction without using trusted setup.

**Definition 7.** [11] *Let  $C = (\text{Gen}, \text{Com}, \text{Open})$  be the generation, the commit and the open algorithm of a commitment scheme. For an instance  $c$ , a proof of set membership with respect to commitment scheme  $C$  and set  $\Phi$  is a proof of knowledge for the following statement:*

$$PK\{(\mu, \rho) : c \leftarrow \text{Com}(\mu; \rho) \wedge \mu \in \Phi\}.$$

The security model for set membership proof follows the standard definitions of zero-knowledge proof: perfect completeness, computational soundness and perfect zero-knowledge.

In this section, we consider the following modified set membership proof for a set  $\Phi$  of base group elements :

$$PK\{(\mu, \rho) : c = g^\mu h^\rho \wedge g^\mu \in \Phi\}.$$

**Related Works.** Generally speaking, accumulator with the property of zero-knowledge can be viewed as a special case of set membership proof. As summarized in [16,2], it includes the RSA-based [13,23,2], pairing-based [30,15,1,12,16], Merkle-tree-based [6] and lattice-based [24] constructions. However, all existing RSA-based and pairing-based accumulators, and [6] require a trusted setup. The scheme in [6] also requires a trusted setup for generating composite order group. Furthermore, the pairing-based approach can only accumulate values in the exponent. It means that if we want to accumulate public keys, eventually we have to prove the knowledge of a secret key as a proof of double discrete logarithm, which is inefficient. For the Merkle-tree-based accumulator [10], it can be combined with zk-SNARK [32] to achieve zero-knowledge. Nevertheless, zk-SNARK also needs trusted setup. Recently, zk-STARK [4] is proposed to remove the need of trusted setup. However, zk-STARK proofs is 1000 times longer than zk-SNARK proofs and hence it is not practical. For the lattice-based accumulator without trusted setup [24], the asymptotic proof size is  $O(\log(n))$ , but the actual size is impractical ( $> 30\text{MB}$ ).

## B.1 Our Basic Construction

Our construction is essentially a set membership proof for group elements which is the domain of public keys. It is the first set membership proof for public keys in the base group, instead of in the exponent. The intuition of our scheme is introduced in the previous section. Our construction is as follows.

- **Setup.** On input security parameter  $1^\lambda$  and the maximum size of the set of membership public key  $N$ , it picks a group  $\mathbb{G}$  of prime order  $p$  and some generators  $g \in \mathbb{G}, \mathbf{h} = (h_1, \dots, h_N) \in \mathbb{G}^N$ . Suppose that  $H_j : \{0, 1\}^* \rightarrow \mathbb{Z}_p$  for  $j = 1, 2, 3, 4$ ,  $H_6 : \{0, 1\}^* \rightarrow \mathbb{G}$  are collision resistant hash functions. Let  $C = (\text{Gen}, \text{Com}, \text{Open})$  be the Pedersen commitment scheme. Assume these parameters are known in the system.
- **PKGen.** It randomly picks  $x \in \mathbb{Z}_p$  and outputs a public key  $Y = g^x$ .
- **Prove.** On input the set of  $n \leq N$  public keys as  $\mathbf{Y} = (Y_1, Y_2, \dots, Y_n)$  and denote the set member  $\sigma = Y_{i^*} \in \mathbf{Y}$ , with corresponding secret key  $x_{sk, i^*}$ . The prover runs as follows.
  1. *Prepare Index.* The prover generates a binary vector  $\mathbf{b}_L = (b_1, \dots, b_n)$ , where  $b_i = 1$  when  $i = i^*$  and  $b_i = 0$  otherwise. Define  $\mathbf{b}_R = \mathbf{b}_L - \mathbf{1}^n$ . It proves in zero knowledge that  $\mathbf{b}_L$  is a binary vector with only one bit equal to 1. It is equivalent to showing:

$$\mathbf{b}_L \circ \mathbf{b}_R = \mathbf{0}^n, \quad \mathbf{b}_L - \mathbf{b}_R = \mathbf{1}^n, \quad \langle \mathbf{b}_L, \mathbf{1}^n \rangle = 1.$$

2. *Commit 1.* It computes  $h = H_6(\mathbf{Y})$ . It picks random  $\alpha, \beta, \rho, r_\alpha, r_{sk} \in \mathbb{Z}_p, \mathbf{s}_L, \mathbf{s}_R \in \mathbb{Z}_p^n$  and computes:

$$A_1 = h^\alpha \mathbf{Y}^{\mathbf{b}_L} = h^\alpha Y_{i^*}, \quad A_2 = h^\beta \mathbf{h}^{\mathbf{b}_R}, \quad S_1 = h^{r_\alpha} g^{r_{sk}}, \quad S_2 = h^\rho \mathbf{Y}^{\mathbf{s}_L} \mathbf{h}^{\mathbf{s}_R}.$$

Note that  $A_1$  is the Pedersen commitment of the secret key of  $Y_{i^*}$  for randomness  $\alpha$ .

3. *Challenge 1.* Denote the concatenated string  $\text{str} = \mathbf{Y} || A_1 || A_2 || S_1 || S_2$ . It computes  $y = H_2(\text{str})$ ,  $z = H_3(\text{str})$  and  $w = H_4(\text{str})$ .
4. *Commit 2.* It can construct two degree 1 polynomials of variable  $X$ :

$$\begin{aligned} l(X) &= \mathbf{b}_L - z \cdot \mathbf{1}^n + \mathbf{s}_L \cdot X, \\ r(X) &= \mathbf{y}^n \circ (w \cdot \mathbf{b}_R + wz \cdot \mathbf{1}^n + \mathbf{s}_R \cdot X) + z^2 \cdot \mathbf{1}^n. \end{aligned}$$

Denote  $t(X) = \langle l(X), r(X) \rangle$ , which is a degree 2 polynomial. We can write  $t(X) = t_0 + t_1 X + t_2 X^2$ , and  $t_0, t_1, t_2$  can be computed by using  $(\mathbf{b}_L, \mathbf{b}_R, \mathbf{s}_L, \mathbf{s}_R, w, y, z)$ . In particular, observe that

$$\begin{aligned} t_0 &= w \langle \mathbf{b}_L, \mathbf{b}_R \circ \mathbf{y}^n \rangle + zw \langle \mathbf{b}_L - \mathbf{b}_R, \mathbf{y}^n \rangle + z^2 \langle \mathbf{b}_L, \mathbf{1}^n \rangle - wz^2 \langle \mathbf{1}^n, \mathbf{y}^n \rangle - z^3 \langle \mathbf{1}^n, \mathbf{1}^n \rangle, \\ &= z^2 + w(z - z^2) \langle \mathbf{1}^n, \mathbf{y}^n \rangle - z^3 \langle \mathbf{1}^n, \mathbf{1}^n \rangle. \end{aligned}$$

It picks random  $\tau_1, \tau_2 \in \mathbb{Z}_p$ , and computes:

$$T_1 = g^{t_1} h^{\tau_1}, \quad T_2 = g^{t_2} h^{\tau_2}.$$

5. *Challenge 2.* It computes  $x = H_1(w, y, z, T_1, T_2)$ .

6. *Response.* It computes:

$$\begin{aligned}\tau_x &= \tau_1 \cdot x + \tau_2 \cdot x^2, \\ \mu &= \alpha + \beta \cdot w + \rho \cdot x, \\ z_\alpha &= r_\alpha + \alpha \cdot x, \\ z_{sk} &= r_{sk} + x_{sk, i^*} \cdot x, \\ \mathbf{l} &= l(x) = \mathbf{b}_L - z \cdot \mathbf{1}^n + \mathbf{s}_L \cdot x, \\ \mathbf{r} &= r(x) = \mathbf{y}^n \circ (w \cdot \mathbf{b}_R + wz \cdot \mathbf{1}^n + \mathbf{s}_R \cdot x) + z^2 \cdot \mathbf{1}^n, \\ t &= \langle \mathbf{l}, \mathbf{r} \rangle.\end{aligned}$$

It outputs  $A_1$  and  $\sigma = (A_2, S_1, S_2, T_1, T_2, \tau_x, \mu, z_\alpha, z_{sk}, \mathbf{l}, \mathbf{r}, t)$ .

- **Verify.** On input a set of public keys  $\mathbf{Y}$ ,  $A_1$  and the proof  $\sigma = (A_2, S_1, S_2, T_1, T_2, \tau_x, \mu, z_\alpha, z_{sk}, \mathbf{l}, \mathbf{r}, t)$ , denote the concatenated string  $\mathbf{str} = \mathbf{Y} \| A_1 \| A_2 \| S_1 \| S_2$ . It computes  $h = H_6(\mathbf{Y})$ ,  $y = H_2(\mathbf{str})$ ,  $z = H_3(\mathbf{str})$ ,  $w = H_4(\mathbf{str})$  and  $x = H_1(w, y, z, T_1, T_2)$ . Define  $\mathbf{h}' = (h'_1, \dots, h'_n) \in \mathbb{G}^n$  such that  $h'_i = h_i^{y^{-i+1}}$  for  $i \in [1, n]$ . It checks if all of the following hold:

$$t = \langle \mathbf{l}, \mathbf{r} \rangle, \quad (9)$$

$$g^t h^{\tau_x} = g^{z^2 + w(z-z^2)(\mathbf{1}^n, \mathbf{y}^n) - z^3(\mathbf{1}^n, \mathbf{1}^n)} \cdot T_1^x \cdot T_2^{x^2}, \quad (10)$$

$$h^\mu \mathbf{Y}^{\mathbf{l}} \mathbf{h}'^{\mathbf{r}} = A_1 \cdot A_2^w \cdot S_2^x \cdot \mathbf{Y}^{-z \cdot \mathbf{1}^n} \cdot \mathbf{h}'^{wz \cdot \mathbf{y}^n + z^2 \cdot \mathbf{1}^n}, \quad (11)$$

$$h^{z_\alpha} g^{z_{sk}} = S_1 A_1^x. \quad (12)$$

**Theorem 9.** *The set membership proof is secure if the discrete logarithm assumption holds in  $\mathbb{G}$  in the random oracle model.*

*Proof. Completeness.* The completeness of the protocol is straightforward.

**Zero-knowledge.** The simulator chooses most of the proof elements  $(A_1, S_2, T_2, T_3, \tau_x, \mu, z_\alpha, z_{sk}, \mathbf{l}, \mathbf{r})$  and challenges  $(x, y, z, w)$  uniformly at random from their respective domains, and then computes  $t$  from equation (9),  $T_1$  from equation (10),  $A_2$  from equation (11) and  $S_1$  from equation (12). Then the simulator sets the value  $x, y, z, w$  in the random oracle  $H_1, H_2, H_3, H_4$ . Hence it achieves the zero-knowledge property.

**Soundness.** Suppose the adversary returns a proof  $\sigma$  for a set of public keys  $\mathbf{Y}^* = \{Y_1, \dots, Y_n\}$ , the extractor rewinds challenge 2 for 3 times. For each transcript, denote the challenge as  $x_i$  and the responses as  $(\tau_{x,i}, \mu_i, z_{\alpha,i}, z_{sk,i}, \mathbf{l}_i, \mathbf{r}_i, t_i)$  for  $i \in [1, 3]$ . Denote  $\mathbf{l}_i = (l_{i,1}, \dots, l_{i,n})$  and  $\mathbf{r}_i = (r_{i,1}, \dots, r_{i,n})$ .

- To extract  $A_1 A_2^w$ , it picks some  $\eta_i \in \mathbb{Z}_p$  such that:  $\sum_{i=1}^2 \eta_i = 1, \sum_{i=1}^2 \eta_i x_i = 0$ . By equation (11), we have:

$$\begin{aligned}A_1 A_2^w &= h^{\sum_{i=1}^2 \eta_i \mu_i} \mathbf{Y}^{\sum_{i=1}^2 \eta_i \cdot \mathbf{l}_i + z \cdot \mathbf{1}^n} \mathbf{h}'^{\sum_{i=1}^2 \eta_i \mathbf{r}_i - wz \cdot \mathbf{y}^n - z^2 \cdot \mathbf{1}^n} \\ &:= h^{\alpha'} \mathbf{Y}^{\mathbf{b}'_L} \mathbf{h}^{w \cdot \mathbf{b}'_R},\end{aligned} \quad (13)$$

for some  $\alpha', \mathbf{b}'_L, \mathbf{b}'_R$ .

- To extract  $S_2$ , it picks some  $\eta'_i \in \mathbb{Z}_p$  such that  $\sum_{i=1}^2 \eta'_i = 0, \sum_{i=1}^2 \eta'_i x_i = 1$ . By equation (11), we have:

$$S_2 = h^{\sum_{i=1}^2 \eta'_i \mu_i} \mathbf{Y}^{\sum_{i=1}^2 \eta'_i \mathbf{l}_i} \cdot \mathbf{h}'^{\sum_{i=1}^2 \eta'_i \mathbf{r}_i} := h^{\rho'} \mathbf{Y}^{\mathbf{s}'_L} \mathbf{h}^{\mathbf{s}'_R},$$

for some  $\rho', \mathbf{s}'_L, \mathbf{s}'_R$ .

Putting back the extracted values into equation (11), we have:

$$h^\mu \mathbf{Y}^{\mathbf{l}} \mathbf{h}'^{\mathbf{r}} = (h^{\alpha'} \mathbf{Y}^{\mathbf{b}'_L} \mathbf{h}^{w \cdot \mathbf{b}'_R}) \cdot (h^{\rho'} \mathbf{Y}^{\mathbf{s}'_L} \mathbf{h}^{\mathbf{s}'_R})^x \cdot \mathbf{Y}^{-z \cdot \mathbf{1}^n} \cdot \mathbf{h}'^{wz \cdot \mathbf{y}^n + z^2 \cdot \mathbf{1}^n}.$$



Since the above holds for all  $x$ , and assuming the DL assumption holds between  $h$ ,  $g$  and  $\mathbf{h}$ , we have

$$\begin{aligned} \mathbf{l} &= \mathbf{b}'_{\mathbf{L}} - z \cdot \mathbf{1}^n + \mathbf{s}'_{\mathbf{L}} \cdot x, \\ \mathbf{r} &= \mathbf{y}^n \circ (w \cdot \mathbf{b}'_{\mathbf{R}} + wz \cdot \mathbf{1}^n + \mathbf{s}'_{\mathbf{R}} \cdot x) + z^2 \cdot \mathbf{1}^n. \end{aligned}$$

By the same set of 3 rewinding transcripts, we can also extract some other commitments as follows.

- To extract  $T_1$ , it picks some  $\delta_i \in \mathbb{Z}_p$  such that  $\sum_{i=1}^3 \delta_i = 0$ ,  $\sum_{i=1}^3 \delta_i x_i = 1$ ,  $\sum_{i=1}^3 \delta_i x_i^2 = 0$ . By equation (10), we have:

$$T_1 = g^{\sum_{i=1}^3 \delta_i t_i} h^{\sum_{i=1}^3 \delta_i \tau_{x,i}} := g^{t'_1} h^{r'_1},$$

for some  $t'_1, r'_1$ .

- To extract  $T_2$ , it picks some  $\delta'_i \in \mathbb{Z}_p$  such that:  $\sum_{i=1}^3 \delta'_i = 0$ ,  $\sum_{i=1}^3 \delta'_i x_i = 0$ ,  $\sum_{i=1}^3 \delta'_i x_i^2 = 1$ . By equation (10), we have:

$$T_2 = g^{\sum_{i=1}^3 \delta'_i t_i} h^{\sum_{i=1}^3 \delta'_i \tau_{x,i}} := g^{t'_2} h^{r'_2},$$

for some  $t'_2, r'_2$ .

Putting back the extracted values into equation (10), we have:

$$g^t h^{\tau_x} = g^{z^2 + w(z-z^2)\langle \mathbf{1}^n, \mathbf{y}^n \rangle - z^3 \langle \mathbf{1}^n, \mathbf{1}^n \rangle} \cdot (g^{t'_1} h^{r'_1})^x \cdot (g^{t'_2} h^{r'_2})^{x^2}.$$

By the random oracle model, the DL between  $h = H_6(\mathbf{Y})$  and  $g$  is not known by the adversary. So we have:

$$t = z^2 + w(z-z^2)\langle \mathbf{1}^n, \mathbf{y}^n \rangle - z^3 \langle \mathbf{1}^n, \mathbf{1}^n \rangle + t'_1 x + t'_2 x^2.$$

By equation (9), we have  $t = \langle \mathbf{l}, \mathbf{r} \rangle$ . Observe that:

$$\begin{aligned} \langle \mathbf{l}, \mathbf{r} \rangle &= (\mathbf{b}'_{\mathbf{L}} - z \cdot \mathbf{1}^n + \mathbf{s}'_{\mathbf{L}} \cdot x) (\mathbf{y}^n \circ (w \cdot \mathbf{b}'_{\mathbf{R}} + wz \cdot \mathbf{1}^n + \mathbf{s}'_{\mathbf{R}} \cdot x) + z^2 \cdot \mathbf{1}^n) \\ &= w \langle \mathbf{b}'_{\mathbf{L}}, \mathbf{b}'_{\mathbf{R}} \circ \mathbf{y}^n \rangle + wz \langle \mathbf{b}'_{\mathbf{L}} - \mathbf{b}'_{\mathbf{R}}, \mathbf{y}^n \rangle + z^2 \langle \mathbf{b}'_{\mathbf{L}}, \mathbf{1}^n \rangle \\ &\quad - wz^2 \langle \mathbf{1}^n, \mathbf{y}^n \rangle - z^3 \langle \mathbf{1}^n, \mathbf{1}^n \rangle + t''_1 x + t''_2 x^2, \end{aligned}$$

for some  $t''_1, t''_2 \in \mathbb{Z}_p$ . Since the above holds for all  $w, x, y, z$ , we have:

$$\mathbf{b}'_{\mathbf{L}} \circ \mathbf{b}'_{\mathbf{R}} = \mathbf{0}^n, \quad \mathbf{b}'_{\mathbf{L}} - \mathbf{b}'_{\mathbf{R}} = \mathbf{1}^n, \quad \langle \mathbf{b}'_{\mathbf{L}}, \mathbf{1}^n \rangle = 1.$$

Therefore, it implies that  $\mathbf{b}'_{\mathbf{L}}$  is a binary vector with one bit equal to 1. By equation (11), we have  $A_1 A_2^w = h^{\alpha'} Y_{i^*} \mathbf{h}^{w \cdot \mathbf{b}'_{\mathbf{R}}} = h^{\alpha'} g^{x_{sk,i^*}} \mathbf{h}^{w \cdot \mathbf{b}'_{\mathbf{R}}}$  for some index  $i^*$ . Since the above is true for all  $w$ , then we have  $A_1 = h^{\alpha'} g^{x_{sk,i^*}}$ .

By the same set of rewinding transcripts, from equation 12, we also have:

$$A_1 = h^{\frac{z_{\alpha,1} - z_{\alpha,2}}{x_1 - x_2}} g^{\frac{z_{sk,1} - z_{sk,2}}{x_1 - x_2}} := h^{\alpha'} g^{sk'}.$$

By the random oracle model, the DL between  $h$  and  $g$  is not known by the adversary. So we can extract  $x_{sk,i^*} = sk'$ , where  $A_1$  is a commitment to  $Y_{i^*} = g^{x_{sk,i^*}}$ . It completes the soundness proof.  $\square$

## B.2 Set Membership Proof with Logarithm Size

Our scheme in the last section is linear size of  $n$  for the part of  $\mathbf{l}$  and  $\mathbf{r}$ . Observe that the verifier can compute  $A_1 \cdot A_2^w \cdot S_2^x \cdot \mathbf{Y}^{-z \cdot \mathbf{1}^n} \cdot \mathbf{h}'^{wz \cdot \mathbf{y}^n + z^2 \cdot \mathbf{1}^n}$ . We note that verifying both equations (9) and (11) is equivalent to verifying the witness  $\mathbf{l}$  and  $\mathbf{r}$  satisfying the inner-product relation. Therefore, it can be fitted into the improved inner-product argument framework from [9] to give a zero knowledge proof  $\pi$  of  $\mathbf{l}, \mathbf{r}$  such that:

$$P = \mathbf{Y}'^l \mathbf{h}'^r \quad \wedge \quad t = \langle \mathbf{l}, \mathbf{r} \rangle.$$

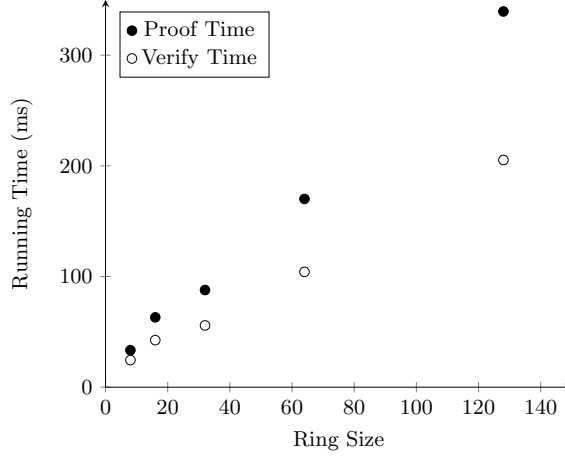


Fig. 3: Efficiency of Our Set Membership Proof.

Membership Proof	Communication		Prover	Verifier
	$\mathbb{G}$	$\mathbb{Z}_p$	(# $\mathbb{G}$ exponentiation)	(# $\mathbb{G}$ exponentiation)
[3]	$4 \log n + 5$	$3 \log n + 4$	$(4 \log n + 4)e_2$	$(2 \log n + 1)e_3 + e_{\log n+3} + e_2$
[21]	$4 \log n$	$3 \log n + 1$	$\frac{5}{2} \log n \cdot e_2 + \frac{3}{2} \log n \cdot e_1$	$e_{\log n+3} + n(e_3 + e_2)$
[7]	$\log n + 12$	$\frac{3}{2} \log n + 6$	$\frac{1}{2} \log n \cdot e_{n+1} + 2e_{2 \log n+1} + (\frac{1}{2} \log n + 3)e_2 + 4e_1$	$2e_{n+\frac{1}{2} \log n+1} + 2e_{2 \log n+2} + 4e_2$
[8] Scheme 1	7	$4 \log n + 4$	$5e_{\log n+1} + (\log n + 2)e_2$	$e_{3 \log n+1} + e_{\log n+2} + e_{\log n+1} + e_2$
[8] Scheme 2	$2.7 \sqrt{\log n} + 5$	$1.9 \log n + 2.7 \sqrt{\log n} + 4$	$5e_{3.2 \log n+1} + 2.7 \log n \cdot e_2$	$e_{9.5 \log n+1} + e_{3.2 \log n+1} + e_{1.6 \log n+2} + e_2$
This paper	$2 \log n + 6$	7	$e_{2n+1} + e_{n+1} + 3e_2 + (e_{2n} + e_n + \dots + e_1)$	$e_{2n+2 \log n+4} + e_4 + e_3$

Table 5: Summary of set membership proofs for  $n$  members with  $O(\log n)$  size. Denote  $e_i$  as the multi-exponentiation of length  $i$ .

The size of  $\pi$  is  $2 \cdot \lceil \log_2(n) \rceil$  elements in  $\mathbb{G}$  and 2 elements in  $\mathbb{Z}_p$ . The signer’s work is dominated by  $\log n + 1$  multi-exponentiations in  $\mathbb{G}$  of size  $2n, n, n/2, \dots, 1$  respectively. The verifier’s work is dominated by a single multi-exponentiations in  $\mathbb{G}$  of size  $2n + 2 \log_2 n + 1$ .

To sum up, the set membership proof output is  $\sigma = (A_1, A_2, S_1, S_2, T_1, T_2, \tau_x, \mu, z_\alpha, z_{sk}, t, \pi)$ , which has size  $2 \cdot \lceil \log_2(n) \rceil + 6$  elements in  $\mathbb{G}$  and 7 elements in  $\mathbb{Z}_p$ . The signer’s work is dominated by three multi-exponentiations in  $\mathbb{G}$  of size  $2n + 1, 2n$  and  $n + 1$  respectively. The verifier’s work is dominated by two multi-exponentiations in  $\mathbb{G}$  of size  $2n + 2 \log_2 n + 1$  and  $n + 4$  respectively.

We implemented our scheme in Intel Core i5 3.1GHz, 8Gb RAM, MacOS 10.13.4. We used the BouncyCastle’s library for Curve 25519 in our implementation. Each element in  $\mathbb{G}$  is represented by 33 bytes and each element in  $\mathbb{Z}_p$  is represented by 32 bytes. The results are shown in Figure 3. It can be seen that the proof and verification time is almost linear to the size of the set, which is consistent to our analysis.

### B.3 Comparison

As introduced in the previous section (§3.1), our set membership proof is fundamentally different from the existing approaches [21,7,8]. A comparison with the state-of-the-art  $\log n$ -size set membership proofs is shown Table 5. Our scheme is the most efficient in terms of communication size for ECC group, where  $|\mathbb{G}| \approx |\mathbb{Z}_p|$ .

#### B.4 Extensions for Range Proof

We can construct set membership proof for arbitrary set of integers, if we modify our scheme by using  $\mathbb{G}$  as a group of composite order  $N = pq$ . By considering the set as  $(x_1, x_2, \dots, x_n)$  and we use  $Y_i = g^{x_i}$  in the above construction.

As compared to the range proof in [9], it only allows the proof for a continuous range from zero to a maximum value. It is because the range proof is essentially performed in a bitwise manner. In our scheme, we can perform proof of arbitrary disjoint range, e.g. proving  $x \in [2, 5] \cup [8, 10] \cup [12, 14]$ . It may be useful in some privacy-preserving smart contract scenario. For example, it can be used to show that a transaction is performed between Monday to Friday in January 2018. It is equivalent to prove that the date variable  $x \in [1, 5] \cup [8, 12] \cup [19, 23] \cup [26, 30]$ .

There exists range proof in composite order group, with proof size independent of the size of the range [25]. To prove a number  $x \in [A, B]$ , Lipmaa [25] used the Lagrange theorem for showing  $x - A \geq 0$  and  $B - x \geq 0$ . If we want to use [25] to prove set membership for  $M$  disjoint ranges, then the prover needs to have  $M$  disjunction of the Lipmaa’s proof. On the other hand, the proof size of our scheme is  $O(\log N)$ , where  $N$  is the total size of the disjoint ranges. Therefore, our scheme is more efficient for multiple disjoint ranges.

#### B.5 Application to Zerocoin

Zerocoin [28] was the first anonymous cryptocurrency proposal which supports large anonymity sets. The original Zerocoin protocol is used by many cryptocurrencies (Zcoin, PIXV, SmartCash, Zoin, and HexxCoin) with a combined market capitalization of about USD 660 million at the time of writing. Each zerocoin is a commitment  $C$  to a so-called serial number  $S$ . When a zerocoin is spent, the spender reveals the serial number  $S$  and proves in zero-knowledge of  $C$  that she knows  $C$  is a commitment to the serial number  $S$  and  $C$  belongs to a large set of minted zerocoins  $\mathcal{C}$ .

[28] used RSA-based accumulator for zero-knowledge proof of  $C$ . Therefore, it suffers from the drawback of having trusted setup. We propose the use of our set membership proof of  $C$  for the set of minted zerocoins  $\mathcal{C}$ . Our set membership proof can be easily modified to fit the Zerocoin protocol, by replacing the RSA-based accumulator. The only modification needed is changing the proof of knowledge of secret key, to the proof of knowledge of randomness used in the zerocoin commitment  $C$ .

## C Efficient Ring Signatures without Trusted Setup

In this section, we give an efficient ring signature without trusted setup, whose signature size is logarithmic in the size of the ring.

**Backgrounds.** The classical ring signatures [33] for a set of  $n$  public keys are constructed by computing  $n$  “pseudo-signatures” and only one real secret key (out of all  $n$  secret keys) is used to sign. Many ring signatures use disjunctive proofs to demonstrate possession of one-out-of- $n$  secret keys. However, they have signature size of  $O(n)$ , with the exception of [14] having  $O(\sqrt{n})$  size.

Ring signatures can also be constructed by cryptographic accumulator [17]. Accumulator allows the signer to “compress”  $n$  public keys into a constant size *value* and there is a *witness* showing that the signer public key is in the set of public keys. Then, a ring signature can be constructed by a zero-knowledge proof of knowing (1) the *witness* and the signer public key that correspond to the accumulated *value*, and (2) the secret key that corresponds to the signer public key. The advantage of this approach is constant signature size. However, the existing RSA-based and pairing-based accumulators both require a trusted setup for generating system parameters. It is not desirable for system without mutually trusted party.<sup>6</sup> The accumulator without trusted setup by lattice [24] is impractical.

<sup>6</sup> Some may suggest the use of multi-party computation for generating such system parameters. However, we have to ensure that all participated parties did not collude and they performed secure erasure.

Some new ring signature schemes are proposed using zero-knowledge argument for low degree polynomials. In [21], the signer’s index  $b$  can be expressed as a binary string  $(b_1, \dots, b_m)$ , where  $m = \log n$ . For  $i \in [1, m]$ , each secret  $b_i$  is hidden in a polynomial  $p_i$  of degree  $m$ . The zero-knowledge proof is the evaluation of these polynomials using the challenge value as input. [7] used  $u$ -ary representation of  $b$  instead of binary. These ring signatures have size of  $O(\log(n))$ .

The logarithmic ring signature from lattice-based accumulator [24], one-out-of-many proof [19] and one-shot zero-knowledge proof [18] are still at least 100 times longer than our discrete logarithm-based construction. For a ring size of 1024, the signature size of [24] is 59.1MB for  $\lambda = 100$  and the signature size of [19] is 1.021MB for  $\lambda = 133$ . For a ring size of 4096, the signature size of [18] is 103KB for  $\lambda = 128$ . They are still far less efficient than the DL-based construction.

**Our Ring Signature by Set Membership Proof.** We propose the use of set membership proof for constructing ring signatures directly. The signer can directly give a zero-knowledge proof of knowing: (1) a committed public key which is in the set of  $n$  public keys, and (2) the secret key which corresponds to the committed public key. Compared to the accumulator-based two-step approach, we avoid the need to generate *witness* and accumulated *value* first and then to compute zero-knowledge proof on them. Theoretically, our proposal is a more direct approach of constructing ring signatures. In practice, we can also avoid the use of trusted setup for existing RSA-based and pairing-based accumulators. The major obstacle of our approach is how to construct an efficient set membership proof of “a committed public key is in the set of  $n$  public keys”.

We give a concrete instantiation of using set membership proof for constructing ring signatures. We use the set membership proof together with the zero-knowledge proof of the secret key corresponding to the committed public key. As a result, we obtain an efficient ring signature without trusted setup, whose signature size is logarithmic to the size of the ring.

Note that this approach is not efficient with the existing set membership proofs [11,21,8]. The non-interactive version of [11] does not allow dynamic set formation. The set membership proof of [21,8] is for the set of integers in the exponents. If the public keys are considered as the set, then a further zero-knowledge proof of knowledge of the corresponding secret key requires an inefficient zero-knowledge proof of double discrete logarithms. Our new set membership proof enables this new paradigm of ring signatures.<sup>7</sup>

Compared with existing accumulator-based ring signatures, the RSA-based or pairing-based constructions require trusted setup. It is not desirable for distributed systems without trusted authority. Our construction does not require such trusted setup. The lattice-based construction is not practical [37]. A number of ring signatures are proposed recently using zero-knowledge argument for polynomial evaluation [21,7]. They both achieve signature size of  $O(\log n)$ . Table 2 shows the comparison with these efficient ring signatures, except [37] due to its large impractical size. Our construction is the most efficient in terms of signature size (we consider ECC where  $|\mathbb{G}| \approx |\mathbb{Z}_p|$ ) and verifier’s computation. In terms of prover efficiency, our scheme is the fastest for larger ring size ( $n > 128$ ).

## C.1 Definitions

A ring signature scheme consists of some PPT algorithms (**Setup**, **KeyGen**, **Sign**, **Verify**) for generating a system parameter available to all users, generating keys for users, signing messages and verifying ring signatures. We follow the standard definition of ring signatures algorithms and its security model (correctness, unforgeability and perfect anonymity) in [21].

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In practice, it is not a simple task. Zcash performed some complicated procedures (<https://z.cash/technology/paramgen.html>) to convince the general public that the system parameters are properly generated.

<sup>7</sup> [7,21] use the property that “DL-based public key is equal to Pedersen commitment of zero” and then the ring signature includes the one-out-of-many proofs that one Pedersen commitment is committed to zero. Our scheme does not involve the commitment to zero.

## C.2 Ring Signature with Logarithm Size

- **Setup.** On input security parameter  $1^\lambda$  and the maximum size of ring  $N$ , it picks a group  $\mathbb{G}$  of prime order  $p$  and some generators  $g \in \mathbb{G}, \mathbf{h} = (h_1, \dots, h_N) \in \mathbb{G}^N$ . Suppose that  $H_j : \{0, 1\}^* \rightarrow \mathbb{Z}_p$  for  $j = 4, 5$  and  $H_3 : \{0, 1\}^* \rightarrow \mathbb{G}$  are collision resistant hash functions. Assume these parameters are known in the system.
- **KeyGen.** For each user, he picks a random  $sk \in \mathbb{Z}_p$  as his secret key and computes his public key as  $Y = g^{sk}$ .
- **Sign.** On input the set of  $n \leq N$  public keys as  $\mathbf{Y} = (Y_1, Y_2, \dots, Y_n)$ , the signer index  $i^* \in [1, n]$ , the signer secret key  $sk^*$  and a message  $\mathbf{m}$ , the signer runs as follows:
  1. *Prepare Signer Index.* The sender generates a binary vector  $\mathbf{b}_L = (b_1, \dots, b_n)$ , where  $b_i = 1$  when  $i = i^*$  and  $b_i = 0$  otherwise. Define  $\mathbf{b}_R = \mathbf{b}_L - \mathbf{1}^n$ . We will prove in zero knowledge that  $\mathbf{b}_L$  is a binary vector with only one bit equal to 1. It is equivalent to showing:

$$\mathbf{b}_L \circ \mathbf{b}_R = \mathbf{0}^n, \quad \mathbf{b}_L - \mathbf{b}_R = \mathbf{1}^n, \quad \langle \mathbf{b}_L, \mathbf{1}^n \rangle = 1.$$

2. *Signature Generation.* It consists of the following steps.

*Commit 1.* It sets  $h = H_3(\mathbf{Y})$ , picks random  $\alpha, \beta, \rho, r_\alpha, r_{sk}, \in \mathbb{Z}_p, \mathbf{s}_L, \mathbf{s}_R \in \mathbb{Z}_p^n$  and computes:

$$B = h^\alpha \mathbf{Y}^{\mathbf{b}_L} = h^\alpha Y_{i^*}, \quad A = h^\beta \mathbf{h}^{\mathbf{b}_R}, \quad S_1 = h^{r_\alpha} g^{r_{sk}}, \quad S_2 = h^\rho \mathbf{Y}^{\mathbf{s}_L} \mathbf{h}^{\mathbf{s}_R}.$$

*Challenge 1.* Denote the concatenated string  $\text{str}' = \mathbf{Y} \parallel B \parallel A \parallel S_1 \parallel S_2 \parallel U$ . It computes  $y = H_4(1, \text{str})$ ,  $z = H_4(2, \text{str})$  and  $w = H_4(3, \text{str})$ .

*Commit 2.* It can construct two degree 1 polynomials of variable  $X$ :

$$\begin{aligned} l(X) &= \mathbf{b}_L - z \cdot \mathbf{1}^n + \mathbf{s}_L \cdot X, \\ r(X) &= \mathbf{y}^n \circ (w \cdot \mathbf{b}_R + wz \cdot \mathbf{1}^n + \mathbf{s}_R \cdot X) + z^2 \cdot \mathbf{1}^n. \end{aligned}$$

Denote  $t(X) = \langle l(X), r(X) \rangle$ , which is a degree 2 polynomial. We can write  $t(X) = t_0 + t_1 X + t_2 X^2$ , and  $t_0, t_1, t_2$  can be computed by using  $(\mathbf{b}_L, \mathbf{b}_R, \mathbf{s}_L, \mathbf{s}_R, w, y, z)$ . In particular, observe that

$$\begin{aligned} t_0 &= w \langle \mathbf{b}_L, \mathbf{b}_R \circ \mathbf{y}^n \rangle + zw \langle \mathbf{b}_L - \mathbf{b}_R, \mathbf{y}^n \rangle + z^2 \langle \mathbf{b}_L, \mathbf{1}^n \rangle - wz^2 \langle \mathbf{1}^n, \mathbf{y}^n \rangle - z^3 \langle \mathbf{1}^n, \mathbf{1}^n \rangle, \\ &= z^2 + w(z - z^2) \langle \mathbf{1}^n, \mathbf{y}^n \rangle - z^3 \langle \mathbf{1}^n, \mathbf{1}^n \rangle. \end{aligned}$$

It picks random  $\tau_1, \tau_2 \in \mathbb{Z}_p$ , and computes:

$$T_1 = g^{t_1} h^{\tau_1}, \quad T_2 = g^{t_2} h^{\tau_2}.$$

*Challenge 2.* It computes  $x = H_5(v, w, y, z, T_1, T_2, \mathbf{m})$ .

*Response.* It computes:

$$\begin{aligned} \tau_x &= \tau_1 \cdot x + \tau_2 \cdot x^2, \\ \mu &= \alpha + \beta \cdot w + \rho \cdot x, \\ z_\alpha &= r_\alpha + \alpha \cdot x, \\ z_{sk} &= r_{sk} + sk \cdot x, \\ \mathbf{l} &= l(x) = \mathbf{b}_L - z \cdot \mathbf{1}^n + \mathbf{s}_L \cdot x, \\ \mathbf{r} &= r(x) = \mathbf{y}^n \circ (w \cdot \mathbf{b}_R + wz \cdot \mathbf{1}^n + \mathbf{s}_R \cdot x) + z^2 \cdot \mathbf{1}^n, \\ t &= \langle \mathbf{l}, \mathbf{r} \rangle. \end{aligned}$$

It outputs  $\sigma_{\text{ring}} = (B, A, S_1, S_2, T_1, T_2, \tau_x, \mu, z_\alpha, z_{sk}, z_\delta, \mathbf{l}, \mathbf{r}, t)$ .

Note that the two vectors  $\mathbf{l}, \mathbf{r}$  with length  $n$  can be reduced to  $\pi \doteq (P, \mathbf{L}, \mathbf{R}, a, b)$  (which consists of  $2 \log(n) + 1$  elements in  $\mathbb{G}$  and 2 elements in  $\mathbb{Z}_p$ ) by the inner product argument.

- **Verify.** On input a set of public keys  $\mathbf{Y}$ , the signature  $\sigma$ , the message  $\mathbf{m}$ , denote the concatenated string  $\mathbf{str} = \mathbf{Y}||B||A||S_1||S_2||U$ . It computes  $h = H_3(\mathbf{Y})$ ,  $y = H_4(1, \mathbf{str})$ ,  $z = H_4(2, \mathbf{str})$ ,  $w = H_4(3, \mathbf{str})$  and  $x = H_5(v, w, y, z, T_1, T_2, \mathbf{m})$ . Define  $\mathbf{h}' = (h'_1, \dots, h'_n) \in \mathbb{G}^n$  such that  $h'_i = h_i^{y^{-i+1}}$  for  $i \in [1, n]$ . It checks if all of the following hold:

$$\begin{aligned} t &= \langle \mathbf{l}, \mathbf{r} \rangle, \\ g^t h^{\tau_x} &= g^{z^2 + w(z-z^2)\langle \mathbf{1}^n, \mathbf{y}^n \rangle - z^3 \langle \mathbf{1}^n, \mathbf{1}^n \rangle} \cdot T_1^x \cdot T_2^{x^2}, \\ h^\mu \mathbf{Y}^{\mathbf{l}} \mathbf{h}'^{\mathbf{r}} &= B \cdot A^w \cdot S_2^x \cdot \mathbf{Y}^{-z \cdot \mathbf{1}^n} \cdot \mathbf{h}'^{wz \cdot \mathbf{y}^n + z^2 \cdot \mathbf{1}^n}, \\ h^{z_\alpha} g^{z_{\text{sk}}} &= S_1 B^x. \end{aligned}$$

It returns 1 if all of the above hold and returns 0 otherwise.

If the signature is compressed by the inner product argument, it also runs the verification for it.

**Security.** The security for unforgeability and anonymity can be reduced to the discrete logarithm assumption in the random oracle model. We do not repeat the security proof here as it is very similar to the proof of the security of RingCT3.0 and the set membership proof.

**Efficiency.** To sum up, the ring signature output is  $\sigma = (B, A, S_1, S_2, T_1, T_2, \tau_x, \mu, z_\alpha, z_{\text{sk}}, t, \pi)$ , which has size  $2 \cdot \lceil \log_2(n) \rceil + 7$  elements in  $\mathbb{G}$  and 7 elements in  $\mathbb{Z}_p$ . The signer's work is dominated by three multi-exponentiations in  $\mathbb{G}$  of size  $2n + 1$ ,  $2n$  and  $n + 1$ , respectively. The verifier's work is dominated by two multi-exponentiations in  $\mathbb{G}$  of size  $2n + 2 \log_2 n + 1$  and  $n + 4$ , respectively.