

ABDKS: Attribute-Based Encryption with Dynamic Keyword Search in Fog Computing

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Abstract. Fog computing, as an extension of cloud computing, enables end user with limited resources to outsource computational and storage overheads to fog nodes. Attribute-based encryption with keyword search (ABKS) achieves both fine-grained access control and keyword search. However, the search algorithm in the previous ABKS scheme requires that each keyword between the target keyword set and the ciphertext keyword set be the same, otherwise the algorithm doesn't output any search result, which is not conducive to use. In this paper, we provide a new system in fog computing, the ciphertext-policy attribute-based encryption with dynamic keyword search (ABDKS). In ABDKS, the search algorithm requires only one keyword to be identical between the two keyword sets and outputs the corresponding correlation.

Considering the efficiency issue, we propose an improved version of ABDKS called ABDKS-E. In ABDKS-E, all user attribute secret keys are randomized and stored in the cloud rather than locally, and if someone needs to update attributes, any others don't need to update theirs' together. Moreover, we propose a heuristic concept called reverse outsourcing, i.e., the cloud is allowed to outsource computing tasks to idle users reversely. With the help of reverse outsourcing, the computational overheads of cloud in ABDKS-E can be further reduced.

Keywords: Fog computing · Outsourcing · Access control · Attribute-based encryption · Keyword search.

1 Introduction

Cloud computing is an Internet-based computing method, through which data can be stored, shared and processed. With the development of 5G, IoT, and the emergence of countless intelligent devices, tremendous data needs storage and procession in the cloud, which could give rise to huge network congestion and latency. Cloud computing is unable to meet the requirements of the contemporary era, so fog computing [3, 21, 22, 27] is proposed. As shown in Fig. 1, fog nodes are closer to the end user and process data at the edge of the network, which infiltrates into factories, automobiles, electrical appliances, street lamps

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and various articles in people’s daily life. Compared with cloud computing, fog computing can reduce request response time, save energy, and reduce network bandwidth, although it’s overall computing ability is not as powerful as cloud’s. While enjoying the convenience brought by fog computing services, data security is still a critical issue to be considered.

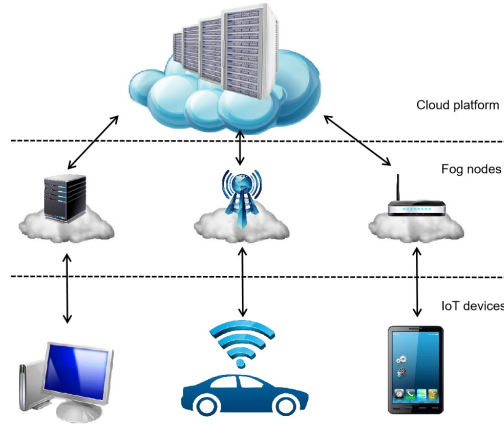


Fig. 1. The instruction of fog computing.

Sensitive information is usually encrypted before being uploaded to the cloud and the encrypted data should be amenable to access control. Ciphertext-policy attribute-based encryption (CP-ABE) [1] achieves fine-grained access control on encrypted data. For finding a ciphertext containing a specific keyword among all encrypted data, ciphertext-policy attribute-based encryption with keyword search (CP-ABKS) [23, 30] supports both fine-grained access control and keyword search simultaneously, which has a wide range of applications in industrial, academic and medical fields. In CP-ABKS, the user’s computational overheads increase with the complexity of the access structure. Outsourcing technology [8] is considered as a promising solution. CP-ABKS schemes in fog computing environment [18, 26] reduce user’s computational overheads by outsourcing computing tasks to fog nodes. Therefore, user only needs to perform few operations on resource-limited devices, such as smartphone or ipad.

1.1 Motivation

The search algorithms in previous CP-ABKS schemes [13, 18, 23, 26, 30] require that each keyword between the target keyword set and the ciphertext keyword set be the same when searching for a ciphertext. As long as the two keyword sets are not completely identical, their algorithms can’t output any search result.

Moreover, we find that schemes in [18, 23, 30] are vulnerable to what we call *peer-decryption attack*. In such attack, the ciphertext may be eavesdropped and decrypted by an adversary who has sufficient authorities but noting about the keywords. For example, while downloading a ciphertext, an employee maybe

eavesdropped and peeped by colleagues or bosses with the same or higher access authorities. In this case, the ciphertext can be decrypted by his peers or superiors, and his privacy will be revealed.

The CP-ABKS scheme for fog computing [18] adopted the mechanism of attribute update proposed by Zhang et al. [28]. In this mechanism, when updating an attribute for a user, the key authority center must update each user and each ciphertext associated with the attribute, regardless of whether or not those users have applied for attribute updates. Obviously, this mechanism no longer works if there are millions of users in the system.

Although, the cloud in CP-ABKS schemes [13, 18, 23, 26, 30] is supposed to have powerful computing power, little attention has been paid to reduce the computational burden of the cloud. There are countless intelligent devices connected to the Internet all over the world. They have certain computing power and are not in use most of the time. Is there any way to aggregate this computing resources to provide computing services for the cloud?

1.2 Our Contributions

Motivated by the observations in previous CP-ABKS schemes as above, we first present a new system on fog computing, the ABDKS, which achieves dynamic keyword search and peer-decryption resistance.

- **Dynamic keyword search:** The search algorithm of ABDKS returns the search result as long as *one keyword* is identical between the target keyword sets and the ciphertext keyword set. In addition, it outputs the correlation of the two keyword set for result ranking.
- **Peer-decryption resistance:** In ABDKS, anyone who wants to decrypt a ciphertext if and only if he has sufficient authority and knows *at least one element* in the ciphertext keyword set. Thus, the ABDKS can resist peer-decryption attack.

For efficiency issues, we further propose an improved version of ABDKS called ABDKS-E, which supports efficient attribute update and can be adapted to reverse outsourcing.

- **Efficient attribute update:** In ABDKS-E, all users' attribute secret keys are randomized and stored in the cloud. To update an attribute for a user, the key authority center only needs to regenerate a new secret key to replace the original one for him, instead of updating the corresponding attribute key of each user and updating the related ciphertext as in [18, 28].
- **Reverse outsourcing:** We first propose a heuristic concept called *reverse outsourcing*, i.e., the cloud is allowed to outsource computing tasks to *idle users* (with online devices that have some computing power but are not used) to reduce its workload. We assume that the cloud will reward idle users after they complete the corresponding computing tasks correctly, and propose the *rational idle user model* [9] in which users are more willing to earn rewards than saving computing resources. To demonstrate how reverse outsourcing works, we apply reverse outsourcing to ABDKS-E, and use game theory [11]

and Nash equilibrium [16] to analyze each user’s strategy: when and only when all users execute the scheme honestly, each user can get the maximum benefit.

1.3 Organization

This paper is organized as follows. Section 2 discusses several previous works. Section 3 describes the necessary preliminaries. Section 4 presents the system and security model. We give a concrete construction and explicit analysis of ABDKS in section 5 and section 6 respectively. In section 7, we introduce the construction of ABDKS-E. In the end, section 8 summarizes the paper and prospects for the future research.

2 Related Works

Sahai et al. [19] initially introduced the concept of attribute-based encryption (ABE). Generally, there are two types of ABE schemes, i.e., key-policy ABE (KP-ABE) [7] and ciphertext-policy ABE (CP-ABE) [1]. Bethencourt et al. [1] proposed the first CP-ABE scheme which realizes fine-grained access control based on the tree-based structure. From then on, large numbers of CP-ABE schemes have been proposed to achieve various functions [5,10,25]. However, with the increase of the number of attributes and the complexity of access structure, general CP-ABE schemes are computationally expensive.

Green et al. [8] provided a method to outsource the decryption of ABE ciphertexts. Li et al. [14] outsources both key-issuing and decryption. Zhang et al. [29] fully outsources key generation, encryption and decryption. In the wake of 5G and IoT techniques, fog computing is considered as a new data resource, which can provide many high-quality outsourcing services. Zuo et al. [31] proposed a practical CP-ABE scheme in fog computing environment and Zhang et al. [28] initially supports fog computing as well as attribute update.

Searching over encrypted data, the keyword can not be revealed because it may reflect sensitive information of ciphertext. In 2000, Song et al. [20] initially introduced a searchable encryption (SE) technique. Boneh et al. [2] proposed the first public key encryption with keyword search. After that, various SE schemes, such as single keyword search [24], multi-keyword search [4] and fuzzy keyword search [15] have emerged. To support both fine-grained access control and keyword search simultaneously, plenty of ciphertext-policy attribute-based encryption (CP-ABKS) schemes [6,12,13,17,18,23,26,30] have been proposed. Among them, [18,26] are constructed in fog computing environment.

3 Preliminaries

In this section, we introduce some background knowledge, which includes access structure, access tree, bilinear maps, Diffie-Hellman assumption and its variants.

3.1 Access Structures

Definition 1 (Access structure [1]). Let $\{P_1, P_2, \dots, P_n\}$ be a set of parties. A collection $\mathbb{A} \subseteq 2^{\{P_1, P_2, \dots, P_n\}}$ is monotone if $\forall B, C: \text{if } B \in \mathbb{A} \text{ and } B \subseteq C \text{ then } C \in \mathbb{A}$. An access structure (respectively, monotone access structure) is a collection (respectively, monotone collection) \mathbb{A} of non-empty subsets of $\{P_1, P_2, \dots, P_n\}$, i.e., $\mathbb{A} \subseteq 2^{\{P_1, P_2, \dots, P_n\}} \setminus \{\emptyset\}$. The sets in \mathbb{A} are called the authorized sets, and the sets not in \mathbb{A} are called the unauthorized sets.

In this paper, attributes take the role of the parties and we only focus on the monotone access structure \mathbb{A} , which consists of the authorized sets of attributes. Obviously, attributes can directly reflect a user's authority.

Definition 2 (Access tree [1]). Let \mathcal{T} be a tree representing an access structure. Each non-leaf node of the tree represents a threshold gate, described by its children and a threshold value. If num_x is the number of children of a node x and k_x is its threshold value, then $0 \leq k_x \leq \text{num}_x$. When $k_x = 1$, the threshold gate is an OR gate and when $k_x = \text{num}_x$, it is an AND gate. Each leaf node x of the tree is describe by an attribute and a threshold value $k_x = 1$.

We introduce a few functions defined in [1] as follows. $\text{parent}(x)$ denotes the parent of the node x in the tree. The function $\text{att}(x)$ is defined only if x is a leaf node and denotes the attribute associated with the leaf node x in the tree. The access tree \mathcal{T} also defines an ordering between the children of every node, that is, the children of a node are numbered from 1 to num . The function $\text{index}(x)$ returns such a number associated with the node x , where the index values are uniquely assigned to nodes in the access structure for a given key in an arbitrary manner.

Definition 3 (Satisfying an access tree [1]). Let \mathcal{T} be an access tree with root r . Denote by \mathcal{T}_x the subtree of \mathcal{T} rooted at the node x . Hence \mathcal{T} is the same as \mathcal{T}_r . If a set of attributes γ satisfies the access tree \mathcal{T}_x , we denote it as $\mathcal{T}_x(\gamma) = 1$. We compute $\mathcal{T}_x(\gamma)$ recursively as follows. If x is a non-leaf node, evaluate $\mathcal{T}_{x'}(\gamma) = 1$ for all children x' of node x . $\mathcal{T}_x(\gamma)$ returns 1 if and only if at least k_x children return 1. If x is a leaf node, then $\mathcal{T}_x(\gamma)$ returns 1 if and only if $\text{att}(x) \in \gamma$.

3.2 Bilinear Map and DBDH Assumption

We briefly recall the definitions of the bilinear map and the decisional bilinear Diffie-Hellman (DBDH) assumption. Let \mathbb{G}_0 and \mathbb{G}_T be two multiplicative cyclic groups of prime order p . Let g be a generator of \mathbb{G}_0 and e be a efficient computable bilinear map, $e : \mathbb{G}_0 \times \mathbb{G}_0 \rightarrow \mathbb{G}_T$. The bilinear map e has a few properties: (1) Bilinearity: for all $u, v \in \mathbb{G}_0$ and $a, b \in \mathbb{Z}_p$, we have $e(u^a, v^b) = e(u, v)^{ab}$. (2) Non-degeneracy: $e(g, g) \neq 1$. We say that \mathbb{G}_0 is a bilinear group if the group operation in \mathbb{G}_0 and the bilinear map $e : \mathbb{G}_0 \times \mathbb{G}_0 \rightarrow \mathbb{G}_T$ are both efficiently computable. Notice that the map e is symmetric since $e(g^a, g^b) = e(g, g)^{ab} = e(g^b, g^a)$.

Given the bilinear map parameter $(\mathbb{G}_0, \mathbb{G}_T, p, e, g)$ and three random elements $(x, y, z) \in \mathbb{Z}_p^3$, if there is no probabilistic polynomial time (PPT) adversary algorithm \mathcal{B} can distinguish between $(g, g^x, g^y, g^z, e(g, g)^{xyz})$ and $(g, g^x, g^y, g^z, \vartheta)$, we can say that the DBDH assumption holds, where ϑ is randomly selected from \mathbb{G}_T . More specifically, the advantage ϵ of \mathcal{B} in solving the DBDH problem is defined as

$$\left| \Pr[\mathcal{A}(g, g^x, g^y, g^z, Z = e(g, g)^{xyz}) = 1] - \Pr[\mathcal{A}(g, g^x, g^y, g^z, Z = R) = 1] \right|. \quad (1)$$

Definition 4 (DBDH). *We say that the DBDH assumption holds if no PPT algorithm has a non-negligible advantage ϵ in solving DBDH problem.*

4 System and Security Model

In this section, we introduce the system description, system overview, threat model and security model of ABDKS.

4.1 System Description

As shown in Fig. 2, we consider a ciphertext retrieval scenario in fog computing. It consists of five parties: Key Authority Center (KAC), Data Owner (DO), Cloud Server (CS), End User (EU), and Fog Nodes. The specific role of each party is given as follows:

- **Key Authority Center (KAC):** The KAC is a fully trusted third party which is in charge of generating public parameters, secret keys, and handling attribute update.
- **Data Owner (DO):** The DO defines an access structure, chooses a set of keywords to generate a ciphertext CT with the help of fog nodes, then uploads CT to the CS.
- **Cloud Server (CS):** The CS has unlimited computing power and storage capacity, it can provide computing and storage services to users.
- **End User (EU):** Resource-constrained user can generate trapdoor with the help of fog nodes and issue search queries based on their authority. Moreover, it can take the advantage of fog nodes to decrypt ciphertext.
- **Fog Nodes:** The fog node can help reduce computational overheads during the encryption process of DO or the trapdoor generation and decryption processes of EU.

4.2 System Model

The ABDKS includes the following six algorithms:

- **Setup** $(1^\lambda, \mathcal{L}) \rightarrow (PK, MSK)$: Given security parameter λ and a set of all possible attributes \mathcal{L} , the KAC generates public key PK and master secret key MSK .

- **KeyGen**(PK, MSK, S) $\rightarrow SK$: On input the public key PK , the master secret key MSK and an attribute set S , the KAC generates a secret key SK for the EU.
- **Enc**(PK, \mathbb{A}, M, KW) $\rightarrow CT$: On input an access structure \mathbb{A} , a keyword set KW and the message M , the DO generates the ciphertext CT with the help of fog nodes, and uploads the ciphertext to the CS.
- **Trap**(SK, KW') $\rightarrow (Ta, Tk)$: To issue a search query, the EU generates a trapdoor (Ta, Tk) by his own secret key SK and a set of target keyword KW' with the help of fog nodes.
- **Search**($\{CT\}, (Ta, Tk)$) $\rightarrow \{(CT_{(i)}, \vec{mr}_{(i)}, l_{(i)})\}$ or \perp : On input a trapdoor (Ta, Tk) , the CS conducts searching operations among all ciphertexts $\{CT\}$. For each ciphertext $CT \in \{CT\}$, if the following two conditions satisfied, we call CT an accessible ciphertext:
 - there is at least one keyword kw such that $kw \in KW' \cap KW$,
 - the user is authorized to obtain CT ,
 where KW' is the target keyword set of (Ta, Tk) , KW is the ciphertext keyword set of CT . The algorithm finally returns the accessible ciphertext set $\{(CT_{(i)}, \vec{mr}_{(i)}, l_{(i)})\}$, where $CT_{(i)} \in \{CT\}$ denotes the i -th accessible ciphertext; matching result vector $\vec{mr}_{(i)}$ implies the relationship between KW' of (Ta, Tk) and $KW_{(i)}$ of the accessible ciphertext $CT_{(i)}$; correlation $l_{(i)}$ is the number of non-zero integers in $\vec{mr}_{(i)}$, indicating the correlation of KW' and $KW_{(i)}$. If no accessible ciphertext exists, the algorithm outputs \perp .
- **Dec**($CT_{(i)}, \vec{mr}_{(i)}, SK, KW'$) $\rightarrow M$: On input $CT_{(i)}, \vec{mr}_{(i)}, SK, KW'$, the EU can decrypt the accessible ciphertext $CT_{(i)}$ with the help of fog nodes and outputs M .

4.3 Threat Model

In this paper, we assume that the KAC is a fully trusted third party, while the CS and fog nodes are honest-but-curious entities, which exactly follow the protocol specifications but also are curious about the sensitive information of ciphertexts and trapdoors. Users are not allowed to collude with CS or fog nodes. Nevertheless, malicious users may collude with each other to access some unauthorized ciphertexts. During transmitted, the ciphertext may be eavesdropped and decrypted by peer-decryption adversary who has sufficient authorities but noting about the keywords.

4.4 Security Model

The ABDKS achieves chosen plaintext security, and the security game between a PPT adversary \mathcal{A} and the challenger \mathcal{C} is as follows.

- *Initialization*: \mathcal{A} chooses and submits a challenge access structure \mathbb{A}^* to \mathcal{C} .
- *Setup*: \mathcal{C} runs **Setup** algorithm and returns the public key PK to \mathcal{A} .
- *Phase 1*: \mathcal{A} adaptively submits any attribute set S to \mathcal{C} with the restriction that S doesn't satisfy \mathbb{A}^* . In response, \mathcal{C} runs **KeyGen** algorithm and answers \mathcal{A} with the corresponding SK .

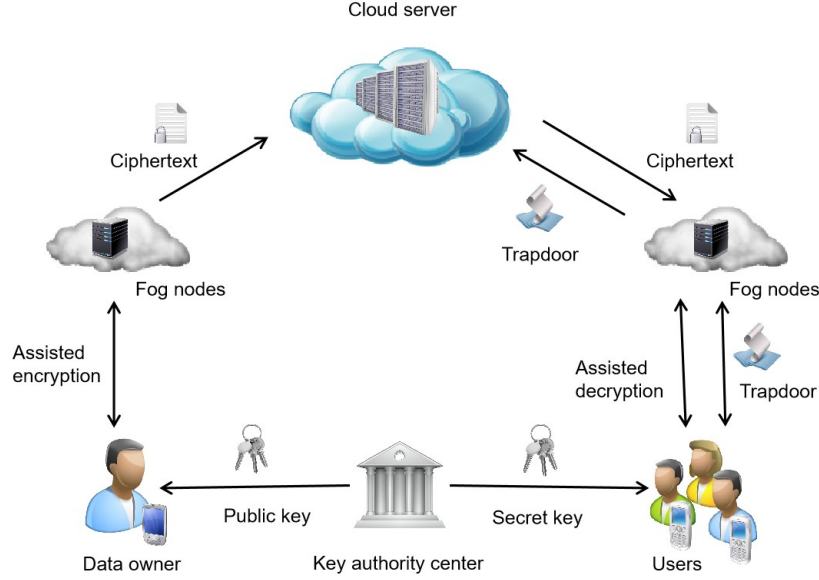


Fig. 2. System description of fog computing.

- *Challenge*: \mathcal{A} chooses two equal-length challenge messages (m_0, m_1) , a set of keywords KW^* and submits them to \mathcal{C} . Then \mathcal{C} picks a random bit $\vartheta \in \{0, 1\}$, runs **Enc** algorithm to encrypt (m_{ϑ}, KW^*) , and returns the challenge ciphertext CT^* to \mathcal{A} .
- *Phase 2*: This phase is the same as Phase 1.
- *Guess*: \mathcal{A} outputs a guess bit ϑ' of ϑ . We say that \mathcal{A} wins the game if and only if $\vartheta' = \vartheta$. The advantage of \mathcal{A} to win this security game is defined as $Adv(\mathcal{A}) = \left| \Pr[\vartheta' = \vartheta] - \frac{1}{2} \right|$.

Definition 5. The ABDKS achieves IND-CPA security if there exist no PPT adversary winning the above security game with a non-negligible advantage ϵ under the DBDH assumption.

In addition, the ABDKS also achieves chosen keyword security, and the security game between \mathcal{A} and \mathcal{C} is as follows.

- *Initialization*: \mathcal{A} chooses and submits a challenge access structure \mathbb{A}^* to \mathcal{C} .
- *Setup*: \mathcal{C} runs **Setup** algorithm and gives PK to \mathcal{A} .
- *Phase 1*: \mathcal{A} adaptively submits any attribute set S and keyword set KW to \mathcal{C} with the restriction that S doesn't satisfy \mathbb{A}^* . In response, \mathcal{C} runs **Trap** algorithm and responds \mathcal{A} with the corresponding trapdoor (Ta, Tk) .
- *Challenge*: \mathcal{A} submits two challenge keyword sets KW^{0*} and KW^{1*} with equal number of keywords. Then, \mathcal{C} picks a random bit $\vartheta \in \{0, 1\}$, and returns the challenge ciphertext CT^* .
- *Phase 2*: This phase is the same as Phase 1.

- *Guess*: \mathcal{A} outputs a guess bit ϑ' of ϑ . We say that \mathcal{A} wins the game if and only if $\vartheta' = \vartheta$. The advantage of \mathcal{A} to win this security game is defined as $Adv(\mathcal{A}) = \left| \Pr[\vartheta' = \vartheta] - \frac{1}{2} \right|$.

Definition 6. *The ABDKS achieves IND-CKA security if there exist no PPT adversary winning the above security game with a non-negligible advantage ϵ under the DBDH assumption.*

5 Construction of ABDKS

Here, we present the concrete construction of ABDKS scheme with dynamic keyword search and peer-decryption resistance.

Without loss of generality, we suppose that there are n possible attributes in total and $\mathcal{L} = \{a_1, a_2, \dots, a_n\}$ is the set of all possible attributes. Assume $\mathbb{G}_0, \mathbb{G}_T$ are multiplicative cyclic groups with prime order p and the generator of \mathbb{G}_0 is g . Let λ be the security parameter which determines the size of groups. Let $e : \mathbb{G}_0 \times \mathbb{G}_0 \rightarrow \mathbb{G}_T$ be a bilinear map and $H : \{0, 1\}^* \rightarrow \mathbb{Z}_p$ be a hash function which maps any string to a random element of \mathbb{Z}_p . We also define the Lagrange coefficient $\Delta_{i,L}(x) = \prod_{j \in L, j \neq i} \frac{x-j}{i-j}$, where $i \in \mathbb{Z}_p$ and a set L , of elements in \mathbb{Z}_p . The details of our scheme are as follows.

- **Setup**($1^\lambda, \mathcal{L}$) $\rightarrow (PK, MSK)$: Given a security parameter λ and all possible attributes \mathcal{L} , the KAC chooses a bilinear group \mathbb{G}_0 with prime order p and generator g . Next, it randomly picks $\alpha, \beta \in \mathbb{Z}_p$ and $h \in \mathbb{G}_0$. For each attribute $a_j \in \mathcal{L}$, it randomly selects a $v_j \in \mathbb{Z}_p$ and sets $PK_j = g^{v_j}$. Finally, it generates the master secret key MSK and the public key PK as

$$PK = \{\mathbb{G}_0, g, h, g^\alpha, e(g, g)^\beta, e(g, h)^\beta, \{PK_j = g^{v_j} \mid \forall a_j \in \mathcal{L}\}\}; \quad (2)$$

$$MSK = \{\alpha, \beta, \{v_j \mid \forall a_j \in \mathcal{L}\}\}. \quad (3)$$

- **KeyGen**(MSK, S) $\rightarrow SK$: While receiving an attribute set S from the EU, the KAC randomly selects $r, r' \in \mathbb{Z}_p$ and returns the secret key SK as

$$SK = \left\{ g^{\beta+\alpha r}, g^{\alpha r} h^{r'}, h^{\alpha r} h^{r'}, g^{r'}, \left\{ g^{\frac{\alpha r}{v_j}}, h^{\frac{\alpha r}{v_j}} \mid \forall a_j \in S \right\} \right\}. \quad (4)$$

- **Enc**(PK, \mathbb{A}, M, KW) $\rightarrow CT$: The DO randomly chooses $ck \in \mathbb{Z}_p$ as a symmetric encryption key and encrypts message M with ck , $E_{ck}(M)$, by using symmetric encryption (AES). Then, it encrypts ck as follows:

1. The DO sends an access structure \mathbb{A} to fog nodes, which in turn represent \mathbb{A} with an access tree \mathcal{T} . Then the fog nodes randomly choose a polynomial q_x for each node x of \mathcal{T} from the root node R in a top-down manner: for each node x of \mathcal{T} , the degree of q_x is $d_x = k_x - 1$, where k_x is the threshold value of x ; beginning with root node R , fog nodes pick a random $s_1 \in \mathbb{Z}_p$, set $q_R(0) = s_1$ and randomly choose $d_R = k_R - 1$ other points of q_R to define the polynomial completely; for any other node x , fog nodes set $q_x(0) = q_{parent(x)}(index(x))$ and choose d_x other points to define q_x completely. Fog nodes send the **attribute ciphertext** CT'_1 ,

$$CT'_1 = \left\{ \mathcal{T}, g^{s_1}, h^{s_1}, \{C_{j,1} = g^{v_j q_x(0)}, C_{j,2} = h^{v_j q_x(0)} \mid \forall a_j = \text{att}(x) \in \mathcal{X} \} \right\}, \quad (5)$$

to DO, where \mathcal{X} is a set of attributes corresponding with all leaf nodes in \mathcal{T} . Given CT'_1 , the DO randomly picks $s_2 \in \mathbb{Z}_p$ and generates CT_1 as

$$CT_1 = \{g^{s_2}, g^{s_1} g^{s_2}, h^{s_1}, h^{s_2}, CT'_1\}. \quad (6)$$

2. The DO chooses the ciphertext keyword set $KW = \{kw_1, kw_2, \dots, kw_t\}$, where kw_i means the i -th keyword. Then, it randomly selects $r_1, r_2 \in \mathbb{Z}_p$, computes $g^{\frac{1}{r_1}}$, and sends $h^{s_2 r_1}, KW_1, KW_2$ to fog nodes, where

$$KW_1 = \{H(kw_1)r_1, H(kw_2)r_1, \dots, H(kw_t)r_1\}; \quad (7)$$

$$KW_2 = \{H(kw_1)r_2, H(kw_2)r_2, \dots, H(kw_t)r_2\}. \quad (8)$$

The fog nodes return DO the **keyword ciphertext**

$$CT'_2 = \{C_1^i = h^{\frac{s_2 r_1}{H(kw_i) r_1}}, C_2^i = g^{\frac{1}{H(kw_i) r_2}} \mid \forall i \in [1, t]\}. \quad (9)$$

3. The DO generates $CT_2 = \{e(g, h)^{\beta s_2}, g^{\frac{1}{r_1}}, CT'_2\}$ and outputs the final ciphertext CT as

$$CT = \{\mathcal{T}, E_{ck}(M), C = ck \cdot e(g, g)^{\beta s_2} \cdot e(g, g)^{\frac{1}{r_2}}, CT_1, CT_2\}. \quad (10)$$

- **Trap**(SK, KW') $\rightarrow (Ta, Tk)$: To issue a search query of the target keyword set $KW' = \{kw'_1, kw'_2, \dots, kw'_t\}$, the **Trap** algorithm proceeds as follows.
 1. The EU randomly chooses $x', y', z', r_3 \in \mathbb{Z}_p$, and uses SK to generate the **attribute trapdoor** Ta as

$$Ta = (Ta_0, Ta_1, Ta_2, Ta_3, Ta_4) = \left(x' + y', h^{(\alpha r + r')(x' + y') + z'}, \right. \\ \left. g^{r'(x' + y') + z'}, \{Ta_3^j = g^{\frac{\alpha r x'}{v_j}} \mid \forall a_j \in S\}, \{Ta_4^j = h^{\frac{\alpha r y'}{v_j}} \mid \forall a_j \in S\} \right). \quad (11)$$

2. The EU computes g^{r_3} and sends $\{H(kw'_1)r_3, \dots, H(kw'_t)r_3\}$ to fog nodes, which return $\{g^{H(kw'_j)r_3} \mid \forall j \in [1, t]\}$ to the former. Finally, the EU generates the **keyword trapdoor** Tk as

$$Tk = (Tk_0, Tk_1) = (g^{(\beta + \alpha r) + r_3}, \{Tk_1^j = g^{H(kw'_j)r_3} \mid \forall j \in [1, t]\}). \quad (12)$$

- **Search**($\{CT\}, (Ta, Tk)$) $\rightarrow \{(CT_{(i)}, \overrightarrow{m}_{(i)}, l_{(i)})\}$ or \perp : For each $CT \in \{CT\}$, the algorithm conducts the following two steps: access precomputation and keyword matching.
 1. **Access Precomputation**: Due to the access precomputation process is a recursive procedure, we define the recursive algorithm $F'_x(C_{j,1}, C_{j,2}, Ta_3^j, Ta_4^j, x)$ intaking $C_{j,1}, C_{j,2}, x$ in CT'_1 of CT and Ta_3^j, Ta_4^j in (Ta, Tk) . For each node x of \mathcal{T} in CT , the CS runs a recursive algorithm as follows:

- (a) If x is a leaf node of \mathcal{T} . Let $a_j = att(x)$. If $a_j \in S$, the CS computes

$$\begin{aligned} F'_x(C_{j,1}, C_{j,2}, Ta_3^j, Ta_4^j, x) &= e(C_{j,2}, Ta_3^j) \cdot e(C_{j,1}, Ta_4^j) \\ &= e(h^{v_j q_x(0)}, g^{\frac{\alpha r x'}{v_j}}) \cdot e(g^{v_j q_x(0)}, h^{\frac{\alpha r y'}{v_j}}) = e(g, h)^{\alpha r q_x(0)(x'+y')}. \end{aligned} \quad (13)$$

If $a_j \notin S$, set $F'_x(C_{j,1}, C_{j,2}, Ta_3^j, Ta_4^j, x) = null$.

- (b) If x is a non-leaf node, the recursive algorithm is defined as: for all child nodes z of x , where $a_i = att(z)$, the CS computes $F'_z = F'_z(C_{i,1}, C_{i,2}, Ta_3^i, Ta_4^i, z)$ recursively. Let S_x be an arbitrary k_x -sized set of child nodes z satisfying $F'_z \neq null$. If S_x doesn't exist, $F'_x = null$. Otherwise, the CS calculates

$$\begin{aligned} F'_x &= \prod_{z \in S_x} F'_z{}^{\Delta_{i, S'_x}(0)} \\ &= \prod_{z \in S_x} (e(g, h)^{\alpha r(x'+y')q_{parent(z)}(index(z))})^{\Delta_{i, S'_x}(0)} \\ &= e(g, h)^{\alpha r q_x(0)(x'+y')}, \end{aligned} \quad (14)$$

where $i = index(z)$ and $S'_x = \{index(z) : z \in S_x\}$.

By calling the above algorithm on the root node R of \mathcal{T} , the CS gets $F'_R = e(g, h)^{\alpha r s_1(x'+y')}$. Then, the CS computes D' as

$$D' = e(g, h)^{\beta s_2} \cdot \left[\frac{F'_R \cdot e(Ta_1, g^{s_1+s_2})}{e(Ta_2, h^{s_1+s_2})} \right]^{\frac{1}{T_{a_0}}} = e(g, h)^{(\beta+\alpha r)s_2}. \quad (15)$$

2. **Keyword Matching:** Given TK and CT , the CS interacts $Tk_1 = \{Tk_1^j = g^{H(kw'_j)r_3} \mid \forall j \in [1, t]\}$ of TK with $\{C_1^i = h^{\frac{s_2 r_1}{H(kw_i)r_1}} \mid \forall i \in [1, t]\}$ of CT : for each $j \in [1, t]$, the CS checks whether there exists an $i \in [1, t]$ such that

$$D' \cdot e(Tk_1^j, C_1^i) = e(Tk_0, h^{s_2}). \quad (16)$$

- (a) If no i makes the above formula hold, the CS outputs a matching result $mr_j = 0$, which means that kw'_j in KW' of Tk is not in KW of CT .
(b) If there is an i such that the above formula holds, which means that there exists a kw_i in KW such that $kw'_j = kw_i$, the CS sets $mr_j = i$.

After that, CS sets the **matching result vector** $\vec{mr} = (mr_1, mr_2, \dots, mr_t)$, the **correlation** $l \in [0, t]$ be the number of non-zero elements in \vec{mr} , which represents the number of identical keywords between KW' and KW . If $l > 0$, the CS outputs the accessible ciphertext tuple (CT, \vec{mr}, l) ; otherwise, it turns to the next ciphertext.

If there is no ciphertext satisfying the above conditions, the algorithm outputs \perp . Otherwise, the CS ranks the accessible ciphertext tuples as $\{CT_{(i)}, \vec{mr}_{(i)}, l_{(i)}\}$ based on the correlation, as shown in Table 1, which is allowed to be obtained by the EU.

- **Dec** $((CT, \vec{mr}), SK, KW') \rightarrow M$: The EU accesses (CT, \vec{mr}) in Table 1 according to l , and decrypt CT with the help of fog nodes in the following steps:

Table 1. Accessible ciphertext for the EU generated by the CS.

Ciphertext	Matching result vector	Correlation
$CT_{(1)}$	$\vec{mr}_{(1)}$	$l_{(1)}$
$CT_{(2)}$	$\vec{mr}_{(2)}$	$l_{(2)}$
\vdots	\vdots	\vdots

1. The EU selects a random $d \in \mathbb{Z}_p$, keeps it secret, and sends a randomized secret key SK' to fog nodes, where

$$\begin{aligned} SK' &= \{SK'_1, SK'_2, SK'_3, SK'_4 = \{SK'_4{}^{ij}\}\} \\ &= \left\{ g^{\frac{\beta+\alpha r}{d}}, g^{\frac{\alpha r}{d}} h^{\frac{r'}{d}}, g^{\frac{r'}{d}}, \{g^{\frac{\alpha r}{v_j d}} \mid \forall a_j \in S\} \right\}. \end{aligned} \quad (17)$$

2. The fog nodes interact SK' and CT to perform some precomputation, which greatly reduces the computational costs of user decryption. The interaction procedure is similar to the access precomputation of **Search** algorithm described above. Due to the precomputation process is a recursive procedure, we define the recursive algorithm $F_x(C_{j,1}, SK'_4{}^{ij}, x)$ intaking $C_{j,1}$, x in CT_1^i of CT and $SK'_4{}^{ij}$ in SK' . For each node x of \mathcal{T} in CT , the fog nodes perform the following recursive algorithm:

- (a) If x is a leaf node of \mathcal{T} . Let $a_j = att(x)$. If $a_j \notin S$, $F_x(C_{j,1}, SK'_4{}^{ij}, x) = null$. If $a_j \in S$, then fog nodes compute

$$F_x(C_{j,1}, SK'_4{}^{ij}, x) = e(g^{v_j q_x(0)}, g^{\frac{\alpha r}{v_j d}}) = e(g, g)^{\frac{\alpha r q_x(0)}{d}}. \quad (18)$$

- (b) If x is a non-leaf node, the recursive algorithm is defined as: for all child nodes z of x , where $a_i = att(z)$, the fog nodes calculate $F_z = F_z(C_{i,1}, SK'_4{}^i, z)$ recursively. Let S_x be an arbitrary k_x -sized set of z satisfying $F_z \neq null$. If S_x doesn't exist, $F_x = null$. Otherwise, the fog nodes compute

$$F_x = \prod_{z \in S_x} F_z^{\Delta_{i, S'_x}(0)} = e(g, g)^{\frac{\alpha r q_x(0)}{d}}. \quad (19)$$

By running the above algorithm recursively, the fog nodes obtain $F_R = e(g, g)^{\frac{\alpha r s_1}{d}}$ for the root node R of \mathcal{T} and return D to the EU, where

$$D = \frac{e(SK'_1, g^{s_2}) \cdot F_R \cdot e(SK'_3, h^{s_1} h^{s_2})}{e(SK'_2, g^{s_1} g^{s_2})} = e(g, g)^{\frac{\beta s_2}{d}}. \quad (20)$$

3. For the \vec{mr} , there is at least one element mr_j in \vec{mr} such that $mr_j = i \neq 0$, i.e., the j^{th} keyword kw'_j in KW' is identical to the i^{th} keyword kw_i in KW . Then, the EU derives ck as

$$\frac{C}{D^d \cdot e(g^{H(kw'_j)}, C_2^i)} = \frac{ck \cdot e(g, g)^{\beta s_2} \cdot e(g, g)^{\frac{1}{r_2}}}{(e(g, g)^{\frac{\beta s_2}{d}})^d \cdot e(g^{H(kw'_j)}, g^{\frac{1}{H(kw_i)r_2}})} = ck. \quad (21)$$

4. Finally, the EU decrypts $E_{ck}(M)$ with ck by symmetric decryption.

Remark 1. Attribute update is very important to keep the system dynamic and protect data from eavesdropping and sniffing by revoked users. we briefly describe how to apply the basic idea of attribute update in [28] to the ABDKS as follows:

1. To update $a_j \rightarrow a_w$, the KAC randomly selects $v'_j \neq v_j \in \mathbb{Z}_p$ and computes $uk_{j \rightarrow w} = \frac{v_j}{v_w}$, $uk_{j \rightarrow j} = \frac{v_j}{v'_j}$, $cu_{j \rightarrow j} = \frac{v'_j}{v_j}$. The KAC updates the public attribute

key of a_j as $PK'_j = PK_j^{\frac{1}{uk_{j \rightarrow j}}} = g^{v'_j}$ and sends $uk_{j \rightarrow w}$, $uk_{j \rightarrow j}$, $cu_{j \rightarrow j}$ to the updated user, non-updated users, the CS respectively.

2. The updated user updates its secret key as

$$SK_u = \left\{ g^{\beta+\alpha r}, g^{\alpha r} h^{r'}, h^{\alpha r} h^{r'}, g^{r'}, \{g^{\frac{\alpha r}{v_i}}, h^{\frac{\alpha r}{v_i}} \mid \forall a_i \in S \setminus \{a_j\}\}, g^{\frac{\alpha r}{v_j} \cdot uk_{j \rightarrow w}}, h^{\frac{\alpha r}{v_w} \cdot uk_{j \rightarrow w}} \right\}. \quad (22)$$

Each non-updated user updates its secret keys as

$$SK_{nu} = \left\{ g^{\beta+\alpha r}, g^{\alpha r} h^{r'}, h^{\alpha r} h^{r'}, g^{r'}, \{g^{\frac{\alpha r}{v_i}}, h^{\frac{\alpha r}{v_i}} \mid \forall a_i \in S \setminus \{a_j\}\}, g^{\frac{\alpha r}{v_j} \cdot uk_{j \rightarrow j}}, h^{\frac{\alpha r}{v_j} \cdot uk_{j \rightarrow j}} \right\}. \quad (23)$$

The CS updates the attribute ciphertext CT'_1 of CT as

$$CT'_1 = \left\{ \mathcal{T}, g^{s_1}, h^{s_1}, C_{j,1} = g^{v_j q_x(0) \cdot cu_{j \rightarrow j}}, C_{j,2} = h^{v_j q_x(0) \cdot cu_{j \rightarrow j}}, \left\{ C_{i,1} = g^{v_i q_x(0)}, C_{i,2} = h^{v_i q_x(0)} \mid \forall a_i \in att(x) \in \mathcal{X} \setminus \{a_j\} \right\} \right\}. \quad (24)$$

6 Analysis of ABDKS

In this section, we provide a security analysis of ABDKS and demonstrate its performance from in a theoretical point of view.

6.1 Security Analysis

Theorem 1. *Supposed that a PPT adversary \mathcal{A} can break the IND-CPA security of ABDKS with a non-negligible advantage $\epsilon > 0$, then there exists a PPT simulator \mathcal{B} that can distinguish a DBDH tuple from a random tuple with an advantage $\frac{\epsilon}{2}$.*

Proof. Given the bilinear map parameter $(\mathbb{G}_0, \mathbb{G}_T, p, e, g)$. The DBDH challenger \mathcal{C} selects $a', b', c' \in \mathbb{Z}_p$, $\theta \in \{0, 1\}$, $\mathcal{R} \in \mathbb{G}_T$ at random. Let $\mathcal{Z} = e(g, g)^{a'b'c'}$, if $\theta = 0$, \mathcal{R} else. Next, \mathcal{C} sends \mathcal{B} the tuple $\langle g, g^{a'}, g^{b'}, g^{c'}, \mathcal{Z} \rangle$. Then, \mathcal{B} plays the role of challenger in the following security game.

- *Initialization:* \mathcal{A} chooses and submits a challenge access structure \mathbb{A}^* to \mathcal{B} .
- *Setup:* \mathcal{B} chooses $\beta', x \in \mathbb{Z}_p$ at random and sets $h = g^x$, $g^\alpha = g^{a'}$, $e(g, g)^\beta = e(g, g)^{\beta'+a'b'}$ = $e(g, g)^{\beta'} e(g^{a'}, g^{b'})$, $e(g, h)^\beta = (e(g, g)^\beta)^x$. For each attribute $a_j \in \mathcal{L}$, \mathcal{B} picks a random $s_j \in \mathbb{Z}_p$. If $a_j \in \mathbb{A}^*$, set $PK_j = g^{v_j} = g^{\frac{a'}{s_j}}$; otherwise, $PK_j = g^{v_j} = g^{s_j}$. Then, \mathcal{B} sends $PK = \{\mathbb{G}_0, g, h, g^\alpha, e(g, g)^\beta, e(g, h)^\beta, \{PK_j \mid \forall a_j \in \mathcal{L}\}\}$ to \mathcal{A} .

- *Phase 1*: \mathcal{A} adaptively submits any attribute set $S \in \mathcal{L}$ to \mathcal{B} with the restriction that $S \not\subseteq \mathbb{A}^*$. In response, \mathcal{B} picks $\hat{r}, \tilde{r} \in \mathbb{Z}_p$ at random, computes $g^{\hat{r}} = \frac{g^{\hat{r}}}{g^{b'}}$, $g^{\beta+\alpha\hat{r}} = g^{\beta'+a'b'+a'(\hat{r}-b')} = g^{\beta'+a'\hat{r}}$, $g^{\alpha\hat{r}}h^{\tilde{r}}$, $h^{\alpha\hat{r}}h^{\tilde{r}}$, $g^{\tilde{r}}$. For each $a_j \in S$, if $a_j \in \mathbb{A}^*$, \mathcal{B} computes $g^{\frac{\alpha\hat{r}}{v_j}} = g^{s_j\hat{r}}$ and $h^{\frac{\alpha\hat{r}}{v_j}} = h^{s_j\hat{r}}$; otherwise, $g^{\frac{\alpha\hat{r}}{v_j}} = g^{\frac{a'\hat{r}}{s_j}}$ and $h^{\frac{\alpha\hat{r}}{v_j}} = h^{\frac{a'\hat{r}}{s_j}}$. Afterwards, \mathcal{B} answers \mathcal{A} with the corresponding secret key $SK = \{g^{\beta'+\alpha\hat{r}}, g^{\alpha\hat{r}}h^{\tilde{r}}, h^{\alpha\hat{r}}h^{\tilde{r}}, g^{\tilde{r}}, \{g^{\frac{\alpha\hat{r}}{v_j}}, h^{\frac{\alpha\hat{r}}{v_j}} \mid \forall a_j \in S\}\}$.
- *Challenge*: \mathcal{A} chooses two equal-length challenge messages (m_0, m_1) , a set of keywords $KW^* = \{kw_1^*, kw_2^*, \dots, kw_t^*\}$ and submits them to \mathcal{B} . Then, \mathcal{B} randomly chooses $r_1, r_2, s_1 \in \mathbb{Z}_p$, sets $g^{s_2} = g^{c'}$, $h^{s_2} = g^{c'x}$, $e(g, h)^{\beta s_2} = \mathcal{Z} \cdot e(g, g)^{\beta'c'x}$ and generates

$$CT_1^* = \left\{ \mathcal{T}^*, g^{s_1}, h^{s_1}, \{C_{j,1} = g^{v_j q_{x^*}^{(0)}}, C_{j,2} = h^{v_j q_{x^*}^{(0)}} \mid \forall a_j = \text{att}(x^*) \in \mathcal{X}^* \} \right\},$$

where \mathcal{X}^* is a set of attributes corresponding with all leaf nodes in \mathcal{T}^* ; $CT_1^* = \{g^{s_2}, g^{s_1}g^{s_2}, h^{s_1}, h^{s_2}, CT_1^*\}$; $CT_2^* = \left\{ \{C_1^{i*} = h^{\frac{s_2}{H(kw_i^*)r_1}}, C_2^{i*} = g^{\frac{1}{H(kw_i^*)r_2}} \mid \forall i \in [1, t]\} \right\}$; $CT_2^* = \left\{ e(g, h)^{\beta s_2}, g^{\frac{1}{r_1}}, CT_2^* \right\}$. After that, \mathcal{B} randomly picks $\theta' \in \{0, 1\}$, sets $C^* = m_{\theta'} \cdot e(g, g)^{\beta s_2} \cdot e(g, g)^{\frac{1}{r_2}}$ where $e(g, g)^{\beta s_2} = \mathcal{Z} \cdot e(g, g)^{\beta'c'}$, and returns \mathcal{A} the final challenge ciphertext $CT^* = \{\mathcal{T}^*, C^*, CT_1^*, CT_2^*\}$.

- *Phase 2*: This phase is the same as Phase 1.
- *Guess*: \mathcal{A} outputs a guess bit θ'' of θ' . If $\theta'' = \theta'$, \mathcal{B} guesses $\theta = 0$ which indicates that $\mathcal{Z} = e(g, g)^{a'b'c'}$ in the above game. Otherwise, \mathcal{B} guesses $\theta = 1$ i.e., $\mathcal{Z} = \mathcal{R}$.

If $\mathcal{Z} = e(g, g)^{a'b'c'}$, then CT^* is available and \mathcal{A} 's advantage of guessing θ' is ϵ . Therefore, \mathcal{B} 's probability to guess θ correctly is

$$\Pr \left[\mathcal{B} \left(g, g^{a'}, g^{b'}, g^{c'}, \mathcal{Z} = e(g, g)^{a'b'c'} \right) = 0 \right] = \frac{1}{2} + \epsilon. \quad (25)$$

Else $\mathcal{Z} = \mathcal{R}$, then CT^* is random from the view of \mathcal{A} . Hence, \mathcal{B} 's probability to guess θ correctly is

$$\Pr \left[\mathcal{B} \left(g, g^{a'}, g^{b'}, g^{c'}, \mathcal{Z} = \mathcal{R} \right) = 1 \right] = \frac{1}{2}. \quad (26)$$

In conclusion, \mathcal{B} 's advantage to win the above security game is

$$\begin{aligned} Adv(\mathcal{B}) &= \frac{1}{2} \left(\Pr \left[\mathcal{B} \left(g, g^{a'}, g^{b'}, g^{c'}, \mathcal{Z} = e(g, g)^{a'b'c'} \right) = 0 \right] \right. \\ &\quad \left. + \Pr \left[\mathcal{B} \left(g, g^{a'}, g^{b'}, g^{c'}, \mathcal{Z} = \mathcal{R} \right) = 1 \right] \right) - \frac{1}{2} = \frac{1}{2}\epsilon. \end{aligned} \quad (27)$$

□

Theorem 2. *Supposed that a PPT adversary \mathcal{A} can break the IND-CKA security of ABDKS with a non-negligible advantage $\epsilon > 0$, then there exists a PPT simulator \mathcal{B} that can distinguish a DBDH tuple from a random tuple with an advantage $\frac{\epsilon}{2}$.*

Proof. The proof process of this theorem is similar to that of Theorem 1. The DBDH challenger \mathcal{C} sends \mathcal{B} the tuple $\langle g, g^{a'}, g^{b'}, g^{c'}, \mathcal{Z} \rangle$, in which $\mathcal{Z} = e(g, g)^{a'b'c'}$ or \mathcal{R} . \mathcal{A} chooses a challenge access structure \mathbb{A}^* initially. \mathcal{B} returns public key in the same way as in Theorem 1. Then \mathcal{A} adaptively submits any attribute set S and keyword set KW to \mathcal{B} , where $S \not\models \mathbb{A}^*$. Since \mathcal{B} can generate secret keys as in Theorem 1, it can naturally answer \mathcal{A} with the corresponding trapdoor (Ta, Tk) . In the challenge phase, \mathcal{A} submits two challenge keyword sets KW^{0*} and KW^{1*} with equal number of keywords. \mathcal{B} randomly picks $\theta' \in \{0, 1\}$, generates CT_2^* with $KW^{\theta'*}$ and returns the challenge ciphertext (CT_1^*, CT_2^*) . If \mathcal{A} 's advantage of guessing θ' is ϵ , then \mathcal{B} 's advantage to distinguish a DBDH tuple from a random tuple is $\frac{\epsilon}{2}$. \square

Remark 2. Previous ABKS schemes [18,23,30] are vulnerable to peer-decryption attack, in which ciphertext may be eavesdropped and decrypted by an adversary who has sufficient authority but noting about the keywords. In those schemes, the symmetric secret key (or message) is encrypted as $ck \cdot e(g, g)^{\beta s_2}$ by an access structure \mathbb{A} . Any adversary \mathcal{A} with attribute set $S_{\mathcal{A}} \models \mathbb{A}$ can calculate $e(g, g)^{\beta s_2}$ so to get ck . While in ABDKS, the symmetric secret key (or message) is encrypted as $ck \cdot e(g, g)^{\beta s_2} \cdot e(g, g)^{\frac{1}{r_2}}$. As shown in Eq.(21), \mathcal{A} without any information of keywords $KW = \{kw_i\}$ cannot compute $e(g, g)^{\frac{1}{r_2}}$ even if he has sufficient authority $S_{\mathcal{A}} \models \mathbb{A}$ to compute $e(g, g)^{\beta s_2}$. Thus, the ABDKS resists peer-decryption attack, since adversary \mathcal{A} can't get ck .

6.2 Comparison with Other Schemes

From a theoretical point of view, we compared our ABDKS with a few up-to-the-minute CP-ABE schemes [18,26,28] in fog computing environment as shown in Table 2. Besides fine-grained access control, keyword search and attribute update, the ABDKS has richer functions such as dynamic keyword search and peer-decryption resistance.

Table 2. Functional comparison among previous ABKS schemes and ABDKS.

Schemes	Fine-grained access control	Keyword search	Attribute update	Dynamic keyword search	Peer-decryption resistance
[28]	✓		✓		
[26]	✓	✓			
[18]	✓	✓	✓		
ABDKS	✓	✓	✓	✓	✓

The ABDKS achieves dynamic keyword search rather than single keyword search in [18], even though, we compared the computational overheads of ABDKS with [18] from the perspective of user, fog nodes, cloud as shown in Table 3.

For user, compared with [18], the user workload of ABDKS is significantly lower in the Enc phase, and slightly higher in other phases.

For fog nodes, the ABDKS has more computational costs in the Enc and Trap phases than those of [18]. Specifically, in the Enc phase of ABDKS, $\{C_{j,2} |$

Table 3. Comparison of computational overhead between ABDKS and [18].

Algorithm	ABDKS		[18]	
	Fog nodes	User	Fog nodes	User
Enc	$2(1+n+t)g$	$5g+2e$	$(n+2)g$	$(n+4)g+e$
Trap	tg	$(2S+3)g$	$2(S+1)g$	$(2S+1)g$
Search	$(2Se+e+g)+(t^2e+e)$		$(S+1)e+2g$	
Dec	$(n+3)e$	$e+g$	$(n+2)e$	e

¹ e : Bilinear pairing. g : Exponentiation in group. S : Number of user attributes. n : Number of attributes in \mathcal{T} . t : Number of keywords.

$j \leq n\}$ in CT' is generated by fog nodes, which can be regarded as the index like $\{I_{l,1} \mid l \leq n\}$ in [18]. While in [18], $\{I_{l,1} \mid l \leq n\}$ is generated by the DO. This means that, compared with [18], the ABDKS outsources part of the index generation from user to fog nodes. In the Trap phase, if only single keyword considered as in [18], the computational costs of fog nodes in ABDKS will be reduced to g rather than tg which is greatly lower than that of [18].

For cloud, in order to achieve dynamic keyword search, the ABDKS also has more computational costs in the Search phase than that of [18]. In fact, [18] can also achieve dynamic search, but if so, their computational costs of EU in the Enc phase will grow to $(nt+4)g+e$, and the costs of CS in the Search phase will grow to $(t^2S+1)e+2g$, which are both greatly larger than those of ABDKS. Therefore, the ABDKS is more efficient than [18] with the same functionality.

7 ABDKS-E

As shown in Remark 1, we applied the method of [28] to ABDKS to support attribute update. In this case, the attribute secret key $\{g^{\frac{ax}{v_j}}, h^{\frac{ax}{v_j}} \mid \forall a_j \in S\}$ of secret key SK in Eq.(4) is stored on the user side. Updating an attribute from a_j to a_w for updated user requires the joint efforts of KAC, all users with a_j and the CS. Obviously, this method is complicated and not applicable when there are millions of users in the system. Therefore, we provide an improved version of ABDKS called ABDKS-E, i.e., ABDKS with efficient attribute update. In ABDKS-E, all users' attribute secret keys are randomized and stored in the cloud rather than user side. To update an attribute for a user, we simply need the KAC to generate a new secret key to replace the original one for the updated user. No changes are required to any non-updated user or ciphertext associated with the updated attribute.

7.1 Construction of ABDKS-E

At first, we define a notion called **lagrange-route product**. Actually in traditional CP-ABE schemes, the calculation of the lagrange-route product is implied in the decryption process of ciphertexts.

Definition 7 (Lagrange-Route Product). *If the access tree \mathcal{T} is exposed to the CS, the CS can compute the lagrange coefficient of each node in \mathcal{T} . For each leaf node z of \mathcal{T} , there is only one route from z to the root node R . We define the route as a set $S_{z \rightarrow R} = (x_0, x_1, x_2, \dots, x_{R-1})$, where $x_0 = z$ and x_{R-1} is R 's child node. Then, the lagrange-route product is defined as:*

$$\pi_z = \prod_{x \in S_{z \rightarrow R}} \Delta_{i, q_x}(0). \quad (28)$$

Following the notations in Section 5, the details of ABDKS-E are shown as follows.

- **Setup**($1^\lambda, \mathcal{L}$) \rightarrow (PK, MSK): This algorithm is the same as that of ABDKS, it generates the master secret key MSK and the public key PK as

$$\begin{aligned} PK &= \{\mathbb{G}_0, g, h, g^\alpha, e(g, g)^\beta, e(g, h)^\beta, \{PK_j = g^{v_j} \mid \forall a_j \in \mathcal{L}\}\}; \\ MSK &= \{\alpha, \beta, \{v_j \mid \forall a_j \in \mathcal{L}\}\}. \end{aligned}$$

- **KeyGen**(MSK, S, U_{id}) \rightarrow ($SK_{EU}, SK_{CS}, SK_{Fog}$): While receiving an attribute set S and user's ID U_{id} from the EU, the KAC randomly selects $r, r', z' \in \mathbb{Z}_p$, computes $x' = H(U_{id} \parallel r \parallel r')$, $y' = H(r \parallel U_{id} \parallel r')$, and generates the user secret key SK_{EU} , the auxiliary decryption secret key SK_{Fog} , and the randomized attribute secret key SK_{CS} as

$$\begin{aligned} SK_{EU} &= \{x', y', g^{(\beta+\alpha r)}\}; \quad SK_{Fog} = \{g^{(\beta+\alpha r)x'}, g^{\alpha r x'} h^{r' x'}, g^{r' x'}\}; \\ SK_{CS} &= \left\{ \begin{array}{l} x' + y', h^{(\alpha r + r')(x' + y') + z'}, g^{r'(x' + y') + z'}, \\ \{SK_{j,1} = g^{\frac{\alpha r x'}{v_j}}, SK_{j,2} = h^{\frac{\alpha r y'}{v_j}} \mid \forall a_j \in S\} \end{array} \right\}. \end{aligned}$$

The KAC sends SK_{EU} to EU, SK_{Fog} to Fog nodes, and (U_{id}, SK_{CS}) to the CS respectively. The CS stores (U_{id}, SK_{CS}) for all users.

- **Enc**(PK, \mathbb{A}, M, KW) \rightarrow CT : The encryption process is the same as that of ABDKS. The DO submits

$$CT = \{\mathcal{T}, E_{ck}(M), C = ck \cdot e(g, g)^{\beta s_2} \cdot e(g, g)^{\frac{1}{r_2}}, CT_1, CT_2\}$$

to the CS, where $CT_1 = \{g^{s_2}, g^{s_1} g^{s_2}, h^{s_1}, h^{s_2}, CT'_1\}$, $CT'_1 = \{\mathcal{T}, g^{s_1}, h^{s_1}, \{C_{j,1} = g^{v_j q_x(0)}, C_{j,2} = h^{v_j q_x(0)} \mid \forall a_j = att(x) \in \mathcal{X}\}\}$, $CT_2 = \{e(g, h)^{\beta s_2}, g^{\frac{1}{r_1}}, CT'_2\}$, $CT'_2 = \{C_1^i = h^{\frac{s_2}{H(kw_i)r_1}}, C_2^i = g^{\frac{1}{H(kw_i)r_2}} \mid \forall i \in [1, t]\}$.

- **Trap**(SK_{EU}, SK_{CS}, KW') \rightarrow (Ta, Tk): The CS takes SK_{CS} as Ta , and the EU generates Tk in the same way of ABDKS, where

$$Tk = (Tk_0, Tk_1) = (g^{(\beta+\alpha r)+r_3}, \{Tk_1^j = g^{H(kw'_j)r_3} \mid \forall j \in [1, t]\}).$$

- **Search**($\{CT\}, Ta, Tk$) $\rightarrow \{CT_{(i)}, \vec{mr}_{(i)}, l_{(i)}\}$ **or** \perp : This algorithm is the same as that of ABDKS, it outputs \perp or Table 1.
- **PreDec**(CT_1, SK_{CS}) $\rightarrow \vec{F}$: In ABDKS-E, the CS can do some precomputation for users to reduce their local computational overheads. For each leaf node x of \mathcal{T} , supposed $a_i = att(x)$. The CS computes $F_x = F_x(C_{i,1}, SK_{i,1}, x) = e(g^{v_j q_x(0)}, g^{\frac{\alpha r x'}{v_j}}) = e(g, g)^{\alpha r q_x(0) x'}$. Then the CS constructs the precomputation vector \vec{F} , where

$$\vec{F} = \{(F_{x_1})^{\pi_{x_1}}, (F_{x_2})^{\pi_{x_2}}, \dots, (F_{x_n})^{\pi_{x_n}}\}. \quad (29)$$

Finally, the CS constructs Table 4 for EU as follows.

Table 4. Accessible ciphertext for the user generated by the CS.

Ciphertext	Matching result vector	Correlation	Precomputation
$CT_{(1)}$	$\vec{mr}_{(1)}$	$l_{(1)}$	$\vec{F}_{(1)}$
$CT_{(2)}$	$\vec{mr}_{(2)}$	$l_{(2)}$	$\vec{F}_{(1)}$
\vdots	\vdots	\vdots	\vdots

- **Dec**($CT, SK_{EU}, SK_{Fog}, KW', \vec{F}$) $\rightarrow ck$: The EU is allowed to download the tuple (CT, \vec{F}, \vec{mr}) in Table 4 according to l , and decrypt CT with the help of fog nodes in the following steps:
 1. Given \vec{F} , the EU picks out $F_{x_i} \in \vec{F}$ such that $att(x_i) \in S \cap \mathcal{X}$. Then the EU computes

$$P = \prod_{att(x) \in S \cap \mathcal{X}} (F_x)^{\pi_x} = e(g, g)^{\alpha r x' \sum_{att(x) \in S \cap \mathcal{X}} q_x(0) \prod_{x \in S_z \rightarrow R} \Delta_{i, q_x(0)}} = e(g, g)^{\alpha r s_1 x'}.$$

2. The fog nodes interact SK_{Fog} and CT to compute Q as

$$Q = \frac{e(g^{(\beta + \alpha r)x'}, g^{s_2}) \cdot e(g^{r'x'}, h^{s_1} h^{s_2})}{e(g^{\alpha r x'} h^{r'x'}, g^{s_1} g^{s_2})} = e(g, g)^{\beta s_2 x' - \alpha r s_1 x'} \quad (30)$$

and return Q to the EU.

3. There is at least one mr_j in \vec{mr} such that $mr_j = i \neq 0$, i.e., $kw'_j = kw_i$. Thus, the EU can make use of the j^{th} keyword in KW' and i^{th} keyword in KW to derives ck as

$$ck = \frac{C}{(P \cdot Q)^{\frac{1}{x'}} \cdot e(g^{H(kw'_j)}, C_2^i)} = \frac{ck \cdot e(g, g)^{\beta s_2} \cdot e(g, g)^{\frac{1}{r_2}}}{(e(g, g)^{\beta s_2 x'})^{\frac{1}{x'}} \cdot e(g^{H(kw'_j)}, g^{\frac{1}{H(kw_i) r_2}})}.$$

4. Eventually, the EU can decrypt $E_{ck}(M)$ with ck by symmetric decryption.
- **Attribute update:** When the attribute set of EU changes from S to S_{new} , the KAC only needs to run **KeyGen**(MsK, S_{new}, U_{id}) to generate a tuple of new secret keys ($SK_{EU}, SK_{CS}, SK_{Fog}$) to replace the original ones.

7.2 Analysis of ABDKS-E

The ABDKS-E can also achieve IND-CPA and IND-CKA security and the security proofs are similar to those of ABDKS, so we omit them here. Compared with ABDKS, ABDKS-E is more efficient for attribute update. When a user requests to update an attribute, the KAC only needs to generate a set of new keys $(SK_{EU}, SK_{CS}, SK_{Fog})$ for him, instead of updating everyone's corresponding attribute secret key in previous schemes [18, 28].

7.3 Reverse Outsourcing

We find that some computing tasks of the CS in ABDKS-E can be reversely outsourced to idle users. Idle users refer to those with smart devices that are not in use and connected to the Internet. Each idle user can provide a small amount of computational resource for the cloud, but the total is very considerable. Here, we formally introduce the concept of reverse outsourcing.

Definition 8 (Reverse Outsourcing). *As is known to all, the cloud service provider can provide outsourcing services for end users to reduce their local computational burden. The reverse outsourcing is on the contrary i.e., the cloud can divide a computing task into several parts and outsource them to different idle users.*

When the CS outsources a computing task to idle users, they must follow the protocol specification. If idle users complete the task and pass the inspection, they will be rewarded by the CS. In this paper, the reverse outsourcing is applied to rational idle user model, which is defined as follows .

Definition 9 (Rational Idle User Model). *Rational idle user are selfish and lazy, and always attempt to maximize their profits, which means that they prefer to get rewards from the CS, rather than save the computational resource of their smart devices. Therefore, for each rational idle user U_i , it holds that $ut_i^{++} > ut_i^+ > ut_i^- > ut_i^{--}$, where*

- ut_i^{++} is the utility of U_i when he can get rewards without following the protocol specification.
- ut_i^+ is the utility of U_i when he follows the protocol specification and gets rewards.
- ut_i^- is the utility of U_i when he doesn't get rewards without following the protocol specification.
- ut_i^{--} is the utility of U_i when he follows the protocol specification but doesn't get rewards.

In rational idle user model, any system user is independent from each other. Since the performance of U_i satisfies $ut_i^{++} > ut_i^+ > ut_i^- > ut_i^{--}$, he may try to defraud rewards without following the protocol honestly. So each U_i has two strategies: follow the protocol or not. In order to analyze the best strategy for each U_i , we formalize the reverse outsourcing game by means of game theory and introduce the notion of Nash equilibrium.

Definition 10 (Reverse Outsourcing Game). *The reverse outsourcing game is a tuple $G_{RO} = \{U, T, ST, R, V\}$, where*

- $U = \{U_1, U_2, \dots, U_n\}$ is the set of n rational idle users, where $n \geq 1$. Each of them needs to complete a computing task in order to get rewards from the CS.
- $T = \{T_1, T_2, \dots, T_n\}$ is the set of computing tasks, where T_i is assigned to U_i .
- $ST = \{st_1, st_2, \dots, st_n\}$ is the set of rational idle users' strategies in G_{RO} . In particular, $st_i \in \{st_i^0, st_i^1\}$ is the set of U_i 's strategies. st_i^0 denotes that U_i wants to be rewarded without following the protocol specification; st_i^1 denotes that U_i follows the protocol honestly.
- $R = \{R_1, R_2, \dots, R_n\}$ is the set of computational results, where R_i is the result of T_i .
- V is a verification algorithm to check whether R is valid or not. If R is valid, i.e., each R_i is valid, then every rational idle user will get the same reward. Otherwise, none of them will get anything.

Definition 11 (Nash Equilibrium of G_{RO}). *For a given strategy $ST^* = (st_1^*, st_2^*, \dots, st_n^*)$, ST^* is Nash equilibrium for G_{RO} , if and only if for any rational idle user $U_i \in U$, when the game G_{RO} is finished, for any $st_j \in \{st_j^0, st_j^1\}$, it holds that*

$$ut_i(st_i^* | ST^* \setminus \{st_i^*\}) \geq ut_i(st_i | ST^* \setminus \{st_i^*\}), \quad (31)$$

where $st_i^* \in \{st_i^0, st_i^1\}$.

In ABDKS-E, algorithm **PreDec** can be reversely outsourced to a set of rational idle users U . For instance, for each leaf node x of \mathcal{T} , the CS sends a tuple $\{C_{i,1}, SK_{i,1}, x\}$ to a rational idle user U_i and asks him to compute the function $F_x(C_{i,1}, SK_{i,1}, x) = e(g, g)^{\alpha r q_x(0)x'}$. The probability that U_i does not follow the protocol but obtains the correct result $e(g, g)^{\alpha r q_x(0)x'}$ is negligible. If U_i cheats, for example, by randomly generating an incorrect result, the cheating behavior can be detected by a small change to the **Enc** algorithm: the DO adds $H(ck)$ into CT . In this case, when the EU obtains CT from Table 4, which means his attribute set $S \models \mathcal{T}$, he can report an error to the CS. This is because he can't calculate a ck' such that $H(ck') \neq H(ck)$ since the cheated user U_i gave him an incorrect result of $e(g, g)^{\alpha r q_x(0)x'}$. Then, any participant U_i will not get rewarded. Thus, the utility of U_i for choosing a strategy $st_i \in \{st_i^0, st_i^1\}$ satisfies

$$ut_i(st_i | ST \setminus \{st_i\}) = \begin{cases} ut_i^+, & st_i = st_i^1 \text{ and } \forall st_j = st_j^1 \text{ for } st_j \in ST \setminus \{st_i\}; \\ ut_i^-, & st_i = st_i^0 \text{ or } \exists st_j = st_j^0 \text{ for } st_j \in ST \setminus \{st_i\}. \end{cases}$$

The utility of U_i reaches the maximum when all users follow the protocol. According to Nash equilibrium theory, each user follows the protocol honestly and returns the right result, which is the best strategy to maximize its own profit. Not following the protocol and generating wrong results will not increase the utility of U_i but consume his computational resources without rewards. Therefore, if and only if all rational idle users implement the protocol honestly, the profit of each user can be maximized.

8 Conclusion

In this paper, we propose an attribute-based encryption with dynamic keyword search (ABDKS) scheme in fog computing environment at first. The ABDKS initially achieves dynamic keyword search and peer-decryption resistance, which makes it more practical and secure. The strict security proof has shown that it is selective CPA and CKA security. Then, we improve ABDKS to ABDKS-E for efficient attribute update. We also propose a heuristic concept the reverse outsourcing to reduce the workload of cloud. In the future, we will continue to focus on privacy issues of cloud computing, such as how to protect the privacy of user identity and data access policy.

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