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Information Conservational Security with "Black Hole" Keypad Compression and Scalable One-Time Pad—An Analytical Quantum Intelligence Approach to Pre and Post-Quantum Cryptography

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ABSTRACT— Although it is widely deemed impossible to overcome the information theoretic optimality of the one-time pad (OTP) cipher in pre and post-quantum cryptography, this work shows that the optimality of information theoretic security (ITS) of OTP is paradoxical from the perspective of information conservational computing and cryptography. To prove this point, ITS of OTP is extended to information conservational security (ICS) of scalable OTP (S-OTP) with percentage-based key extension where total key length can be reduced to a condensed tiny minimum through "black hole" keypad compression coupled with "big bang" data recovery. The cost is a limited increase in total data length and network traffic; the gain is making the transmission of long messages possible without weakening information theoretic security. It is proven that if ITS/OTP were optimal, ICS/S-OTP would be impossible; on the other hand, if ICS/S-OTP were not information theoretically secure, ITS/OTP would not be secure either. Thus, we have a proof by contradiction on the paradoxical nature of OTP optimality. It is further proven that a summation with percentage distribution is a special case of equilibrium-based bipolar quantum cellular automata. This proof bridges a classical world with a quantum world and makes it possible to combine the advantages of both approaches for pre and post-quantum cryptography. It is suggested that the findings of this work form an analytical paradigm of quantum intelligence machinery toward perfect information conservational security. Some mysteries in Nature and science are identified. In particular, the question is posted: Could modern science have been like a well-founded building with a floor of observable beings, truths, and entropy but missing its roof for equilibrium, harmony, information conservation, and logically definable causality?

KEYWORDS: Pre and Post-Quantum Cryptography; Scalable One-Time Pad; Information Conservational Security (ICS); "Black Hole" Keypad Compression; "Big Bang" Data Recovery; Percentage-Based Key Extension; Percentage-Base Fuzzy Entropy; Quantum-Fuzzy Collective Precision; Bipolar Quantum Entanglement; Quantum Intelligence; Classical World; Quantum World; Floor-Roof Mysteries

1 INTRODUCTION

Cryptography is essential for the security of digital communication. Many commonly used cryptosystems could, however, be completely broken by a *quantum* algorithm for integer *factorization* [1] once large quantum computers are commercially applicable. Post-quantum cryptography is to counter such quantum attacks and to keep digital communication secure [2]. A key for success is believed to identify mathematical operations for which quantum algorithms offer little advantage in speed, and then to build systems around them for both pre and post-quantum cryptography. It is commonly believed an impossible task to use a small keypad for transmitting long messages without weakening information theoretic security (ITS) [3].

One-Time Pad (OTP) is often regarded the only cipher with proven ITS for symmetric key cryptography. ITS derives security from information theory [4]. It was introduced in

1949 by American mathematician Claude Shannon—the inventor of information theory, who used it to prove the optimality of OTP in security [3]. Since then, OTP has been used for the most sensitive communications. It can now be used together with quantum key distribution (QKD) — a well-developed application of quantum cryptography applied in pre-quantum cryptography.

Since OTP is quantum proof to quantum factorization [1], it is often regarded the only candidate for post-quantum cryptography [2]. Unfortunately, the key requirement of equal or greater length than the original message hinders the general application of OTP. As a result, OTP is so far limited to transmitting relatively short messages with high security requirement. This limitation has been an impasse for both pre and post-quantum cryptography research. A major challenge is therefore to reduce the OTP key length without weakening ITS where the gain of key length reduction significantly overweighs its cost in data traffic increase.

This work extends ITS of OTP to information conservational security (ICS) of Scalable One-Time Pad (S-OTP) with an analytical quantum intelligence approach to the impassee. In this approach, ICS/S-OTP does not attempt to falsify Shannon's theorem on ITS/OTP directly. Instead, it gets around the problem by proving the paradoxical nature of the ITS optimality of OTP through a proof by contradiction. Then, with "black hole" keypad compression it reduces the key length to a tiny minimum through percentage-based key extension and makes the transmission of long messages practical without weakening ITS in its new context.

It is further proven that a summation with percentage distribution is a special or minimum case of equilibrium-based bipolar quantum cellular automata. This proof bridges a classical world with a quantum world and makes it possible to combine the advantages of both worlds for pre and post-quantum cryptography, one for efficient communication and another for collective computation.

It is suggested that the findings of this work form an analytical paradigm of quantum intelligence toward perfect information conservational security. Some mysteries in Nature and science are identified. In particular, the question is posted: *Could modern science have been like a well-founded building with a floor of observable beings and truths but missing its roof for equilibrium, harmony, information conservation, and logically definable causality?*

This work is organized into six sections with illustrations. Following this introduction, we have the sections of an axiomatic formulation, a basic methodological formulation, an advanced methodological formulation, a quantum architecture, and some conclusions. (Note: Some similar concepts of ICS are proposed in an unpublished technical report on a failed attempt [5]. This work is built upon the failure. The failed attempt is further compared in next section.)

2 AXIOMATIZATION

An OTP cipher is proven information theoretically secure and unbreakable [3,4] provided that the cipher key meets the four conditions of OTP:

- (a) truly random;
- (b) never reused;
- (c) kept secret from all possible attackers;
- (d) of equal or greater length than the message.

History shows that, however, when Shannon invented information theory and ITS [3,4], the first computer was not out yet. Since then, computing theory and technology have advanced beyond anyone's imagination. Although it was proven [3] that any cipher with the perfect secrecy property must use keys with effectively the same requirements as OTP keys, these proofs did not take later computing theories and technological development into consideration that can conceal the meaning of a message. For instance, when entropy became a key concept in information theory, information conservational computing/cryptography (ICC) as an analytical model for quantum intelligence (QI) [6,7] was not incepted yet. Furthermore, before the first computer was put on drawing board, double-

precision floating-point format for a wide dynamic range of numeric values was unimaginable. The wide range makes equilibrium-based ICC practical where a unit value 1.0 can be virtually composed of an infinite number of different fractions to form a basis for information conservation. Thus, ICC with QI is different from big prime factorization with a finite space of solutions [1], different from ITS, and different from classical information theory.

Entropy as a measure of disorder of a system provides a basis for classical information theory. Based on entropy unicity distance [3] is defined as the minimum amount of ciphertext required to permit a computationally unlimited adversary to recover the unique encryption key in a brute force attack.

The ICC extension, however, incorporates holistic set-theoretic keypad compression into S-OTP. As an analytical quantum-digital compatible extension to ITS, QI is brought into the picture, including bipolar equilibrium-based rebalancing, quantum-fuzzy collective precision, "black hole" keypad compression, "big bang" data recovery, logically definable causality, bipolar quantum entanglement, quantum teleportation, classical world, and quantum world.

Axiom 1. Within computational precision limitation, the plaintext L of any OTP message in binary format can be divided into a set of N integers $\{x_i\}$, $1 \le i \le N$, such that the pair $(X, \{x_i/X\})$ conserves the information of L, where $X = \sum_i x_i$ is referred to as a *virtual summation*, and $\{x_i/X\}$ a *virtual percentage distribution*.

Axiom 2. The semantics of a semantically meaningless message cannot be revealed from its ciphertext without the key even if it is only partially enciphered with a shorter OTP keypad or enciphered with a keypad that does not have sufficient entropy because such a message does not leak semantic information.

Axiom 3. While the OTP cipher conceals the bit pattern of a plaintext message for ITS based on entropy, an information conservational extension as described in Axiom 1 does not have to be strictly based on entropy because the pair $(X, \{x_i/X\})$ in Axiom 1 does not show semantics of the plaintext regardless of the entropy measure of its keypad.

Hypothesis. The pair $(X, \{x_i/X\})$ can be secured without using a greater or same length keypad as required by the OTP cipher for ITS.

Definition 1. Information conservational transformation (ICT) is referred to as a set of set-theoretic mathematical functions that form a transformation T to transform the bit pattern of a long message in form F_1 to a pair of (F_2,F_3) such that there is a reverse transformation T' that recovers F_1 from the pair (F_2,F_3) . Formally we have: $T(F_1)$: $F_1 \rightarrow (F_2,F_3)$, such that $\exists T'$ and $T'(F_2,F_3)$: $(F_2,F_3) \rightarrow F_1$. F_2 is assumed an integer summation $X = \sum_i x_i$, and F_3 a percentage distribution $\{x_i/X\}$. Thus, F_2 is significantly shorter than F_1 , and F_3 can be longer than F_1 with a limited data length increase.

Definition 2. Scalability is referred to as using ICT once or multiple times systematically to transform a long message or large data set in form F_1 into (F_2, F_3) such that the total OTP cipher key length is reduced to a condensed minimum for secure transmission of the (F_2, F_3) pair. In this case, a cipher is referred to as Scalable OTP (S-OTP), and classical OTP is referred to as Virtually Scalable OTP or VS-OTP.

Definition 3. S-OTP is said having *information conservational security (ICS)* provided that: (a) The key length required is significantly shorter than the original message; (b) The shorter key does not weaken the ITS of OTP in terms of OTP being VS-OTP; (c) The gain significantly overweighs the cost.

Definition 4. Information conservational percentage-based key extension (PBKE) is referred to as using an S-OTP keypad to secure a summation $X = \sum_i x_i$ plus a partial percentage distribution and using the secured percentages as extended keys to encrypt the remaining percentages. "Black Hole" Keypad Compression (BHKC) is referred to as key length reduction of a lengthy message to a condensed tiny minimum through a hierarchy of ICTs and PBKEs; "Big Bang" Data Recovery (BBDR) is referred to as the reverse process of BHKC.

It could be argued that S-OTP is just OTP plus data compression, and there is nothing new. The reality is:

- Energy/Information conservation or preservation as a paramount law in modern science and ICC as a long sought goal in physics and information theory has been computationally unreachable;
- The keypad length requirement of ITS/OTP has been a longstanding impasse;
- ICS/S-OTP only reduces keypad length but not total data length. It actually costs a limited data traffic increase—a guarantee of no information or entropy loss.
- ICS could, if proved, contradict the ITS optimality of OTP for the first time ever;
- ICS could be proven a holistic extension to ITS with a higher level of abstraction;
- ICS could be extendable to quantum-digital compatible computing and cryptography machinery.

The inception of ICS accounts for the new development in computing technology. Double precision floating-point format of IEEE binary128 is used as a basis in this work that was not available when information theory was initially developed based on Shannon entropy [4].

Basic Cost-Gain Analysis. Let each 1K-bit data divided into ten 96-bit sections plus one 64-bit section, total $1K=960+64=1024=2^{10}$ bits. L=32K bits would be divided into 320 of 96-bit sections plus 32 of 64-bit sections, a set of 352 integers $\{x_i\}$, $1 \le i \le 352$. The summation of all integers is smaller than 96+9=105 bits because $352 < 2^9$. Based on IEEE binary128 standard with 112 significant bits precision and 16-bits exponent, we would have one integer summation $X = \sum_i x_i$ with no more than 105 bits (<112) and a set of 352 percentages $\{x_i/X\}$ in double precision floating-point format, each has 128-bits. While an OTP key requires the minimum length of 32K bits that is the message length. Without computational precision problem, an S-OTP key could potentially be reduced to the length of the 105-bit summation in 128-bit double precision floating-point format, a $32K/128 = 2^{15}/2^7 = 256$ -fold key length reduction. Assuming more room is given to the S-OTP key for partial concealment of the percentage distribution after they are used to encrypt other percentages, we could have:

- (1) For a 1K-bit key, the saving would be 32K/1K=32-fold;
- (2) For a 2K-bit key, the saving would be 32K/2K=16-fold;
- (3) For a 4K-bit key, the saving would be 32K/4K=8-fold;
- (4) For an 8K-bit key, the saving would be 32K/8K=4-fold.

The summation and the percentage distribution total data length approximates to $128 + 352 \times 128$ bits ≈ 44 K bits for the 32K-bit message. The cost is roughly a (44-32)/32 = 12/32 = 3/8 = 37.5% increase in data length and network traffic. The gain could include (i) to make the transmission of long messages possible; (ii) to reduce the cost of QKD by 32K/2K = 16 folds. Evidently, the gain significantly overweighs the cost.

It should be remarked that the failed approach reported in [5] also attempted to secure a summation and a percentage distribution. While both are aimed at information conservation, the former attempted to compress the whole plaintext to a condensed tiny minimum and was proven impossible due to the loss of information or entropy. The latter is focused on compressing the keypad only with a limited network traffic increase as a tradeoff.

Theorem 1 (Security Theorem). ICS of S-OTP does not weaken the ITS of OTP provided that (a) summation $X = \sum_i x_i$ is enciphered with a same length or longer private S-OTP key equivalent to an OTP key, and (b) the percentage distribution $\{x_i/X\}$ is made completely misleading and uncorrectable without the S-OTP key.

Proof. If the percentage distribution is completely misleading and uncorrectable by an attacker in a brute force attack, the plaintext concealed in the ciphertext of a summation is safe. On the other hand, without the key to the summation, to find a correct percentage through trial and error for revealing the plaintext would be as difficult as finding a virtual percentage to reveal a section of the plaintext of an OTP ciphertext in a virtual summation per Axioms 1-3. This leads to a proof by contradiction. Under the two conditions, if an attacker could still find a percentage to reveal the plaintext of a data section in S-OTP ciphertext, it would be possible to do it similarly to reveal the plaintext of a data section of an OTP ciphertext and its virtual summation. If this could really happen, OTP as VS-OTP would be made irrelevant by S-OTP because entropy and unicity distance would be bypassed. That would contradict not only the optimality of OTP but also the ITS of OTP.

Theorem 2 (Optimality Theorem). If OTP is information theoretically secure, S-OTP must be information theoretically secure under the two conditions of Theorem 1. However, if S-OTP is information conservationally secure, OTP is not information theoretically optimal as long as information conservation belongs to information theory.

Proof. S-OTP is information theoretically secure per Theorem 1 under two conditions. However, if S-OTP were information-theoretically secure, OTP would not be information theoretically optimal because its key length could be scaled down using ICS with a limited data overhead for a benefit that significantly overweighs its cost.

Theorem 3 (Security Level Theorem). ICS is a holistic top-down approach to information security at a higher level of abstraction than ITS—a bottom-up approach strictly based on observability of entropy and unicity distance. This difference makes it possible for key extension in S-OTP that does not have to be strictly based on entropy.

Proof. Since an S-OTP cipher is to conceal a summation and a percentage distribution without plaintext semantics, it is different from an OTP cipher but more general. While plaintext can be revealed based on semantics due to low entropy of an OTP key, summation and percentage distribution do not show such semantics. A guessed percentage in a trial cannot be confirmed without its key or a confirmed distribution; a guessed summation cannot be confirmed without its key or a confirmed distribution plus deterministic semantic analysis. Thus, ICS of S-OTP is a fundamentally different approach. It does not have to be strictly based on entropy as for ITS of OTP. After all, OTP is VS-OTP, and information conservation is a paramount law of modern science. Entropy, on the other hand, is information but not a law.

3 METHODOLOGY

3.1 Basic ICC Algorithm with Key Extension

The rationale of S-OTP is that, given an unsigned big integer L representing large data item D to be transmitted, L can be divided into a set of short integers $\{x_i\} = \{x_1, x_2, ..., x_b, ..., x_N\}$ representing sections of D. The math summation can be obtained as $X = \sum_i x_i$, $1 \le i \le N$, which is concealed in the ciphertext of key K_I in the same way as using an OTP key. The ciphertext of X is much shorter than L to transmit. The percentage distribution $\{x_i/X\}$ can be longer than L but it can be made completely misleading with K_I indirectly. While part of the distribution can be concealed together with the summation using K_I , the concealed percentages can be used as encryption keys for the other part of the percentage distribution, and the other part of distribution can be similarly used as encryption keys. For instance, assume the data sections are random without repetitions, a section of the whole distribution can be used to encrypt another section in the same length on a one on one basis without reuse. Such encryption can be done section by section until the last section of P terms of the percentages can be partially concealed with K_I together with the summation. Thus, keys distribution through QKD are made unnecessary. The percentage-based key extension (PBKE) encryption technique can be defined in Eq. 1.

$$\forall i, P \le i \le N, \{x_i/X\}' = \{w_1(x_i/X) + w_2(x_{i-P}/X)\},$$
 (1) where w_i is a private decimal coefficient derived from concealed percentages.

Assumption. Eq. 1 assumes: (a) $\{(x_i/X)\}$ is a subset of the percentage distribution nonlinearly correlated with each other; (b) Integers P and N are private until the receiver count the number of concealed percentages by K_I ; (c) If the remaining percentages are not enough for two sections but longer than one section, an adjustment is done to P terms. With these conditions there is no common factor involved. Thus, Eq. 1 is quantum proof to factorization.

Preprocessing. This work assumes that the plaintext in binary format to be enciphered is random without zero and full sections/subsections (all 1s but no 0s) and no repeated

sections. The assumption is reasonable because full, zero, and repeated sections can be categorically compressed and concealed in the P terms.

Theorem 4 (Possibility Theorem). With information conservational PBKE, a partial percentage distribution $\{(x_t/X)\}$ can be made completely misleading and uncorrectable to meet the two conditions of Theorem 1.

Proof. PBKE as defined by Eq. 1 is to make a partial percentage distribution misleading and uncorrectable by an attacker without key K_1 where the distribution does not have to remain rational. The theorem follows from six conditions. First, Eq. 1 has no common factor. Second, with QKD, K_1 is random. Third, with complete concealment of the summation and partial percentage distribution by K_1 , the remaining part of the distribution does not sum to 1.0 and no longer correlated. Fourth, private coefficients conceal any reuse and makes an encryption key sufficiently large. Fifth, data preprocessing guarantees no extreme integer values of zero or full data sections/subsections and no repeated sections. Sixth, different pieces of encrypted data are isolated and interlocked with information conservation such that a guessed key for a percentage p_i in a trial cannot be confirmed until an encrypted percentage is confirmed; a guessed percentage distribution cannot be confirmed without being tested against a confirmed summation; a guessed summation cannot be confirmed without its key or a percentage distribution plus semantic analysis which is generally non-deterministic.

Definition 5. Based on theorem 4, Eq 1. converts Shannon entropy to a new kind of entropy which is referred to as *percentage-based fuzzy entropy (PBFE)*. The former is a measure of disorder of binary bit patterns; the later is a measure of misleading and uncorrectability of percentages in decimal values. One is bit-based and another is value-based. One leads to unicity distance and another leads to *value distance*.

Example: Fuzzy values 0.8 and 0.795327596312 are equally misleading to the correct decimal percentage p = 0.67853291 even though 0.8 has much fewer digits and a lower Shannon entropy than 0.795327596312.

The S-OTP key K_1 for enciphering the summation plus P terms of the percentages could have (128+128P)=128(1+P) bits. Assuming a 32K-bit data length we have the comparison between S-OTP and OTP: (i) Let P=7, the key length 128(1+P) approximates to 1K bits with 32-fold key reduction. (ii) Let P=15, the key is about 2K bits with 16-fold key reduction. (iii) Let P=31, the key is about 4K bits with 8-fold key reduction. (iv) Let P=63, the key is about 4K bits with 4-fold key reduction. (v) Let 4K bits with 2-fold key reduction.

S-OTP-Method1

Assume IEEE binary128 is used within the limit of computational precision, and sender Alice and receiver Bob share private key K_1 distributed through QKD. $\forall i, x_i > 0$.

Part I. Encryption

- **Step 1**. Let $\{x_i\}$ represent preprocessed data sections;
- **Step 2**. Let math summation $X = \sum_{i} x_{i}$, for all i (not XOR);
- *Step 3.* Calculate percentage distribution $\{x_i/X\}$;
- **Step 4.** Apply Eq. 1 to encrypt the N-P terms $\{x_i/X\}$ to $\{x_i/X\}$, and encrypt summation X and P terms of $\{x_i/X\}$ with key K_1 to $X' = [K_1 \oplus (X \text{ concatenates P terms of } x_i/X)]$ where \oplus is XOR (not math summation);
- **Step 5**. Alice Transmits $E = (X', \{x_i/X\}')$ to Bob as a pair.

Part II. Decryption

- **Step 1.** Use K_1 and Eq. 1 to decipher or decrypt E to the pair $(X, \{x_i/X\})$;
- **Step 2.** Use $\{x_i/X\}$ to decrypt the summation X to $\{x_i\}$;
- **Step 3**. Recover the message from $\{x_i\}$ with concatenation.

Example. Assuming the plaintext data D to be transmitted is represented by L = 1048549998213983988 divided into the three sections 1048549, 998213, and 983988. Assume sender Alice and receiver Bob share key K_1 .

Part I – Encryption (some steps are omitted)

- Let $x_1 = 1048549$, $x_2 = 998213$, $x_3 = 983988$; $X = x_1 + x_2 + x_3 = 3030750$;
- Calculate percentage distribution $\{x_i/X\} = \{34.5970\%, 32.9362\%, 32.4668\%\};$
- Encrypt and transmit;

Part II – Decryption (some steps are omitted):

- Use K_1 and Eq. 1 to decrypt the message;
- *U*=(3030750, {34.5970%, 32.9362%, 32.4668%});
- Recover $x_1 = 1048549$, $x_2 = 998213$, and $x_3 = 983988$;
- $L = concatenate(x_1, x_2, x_3) = 1048549998213983988;$
- Recover D from L.

Theorem 5. S-OTP-Method1 is information conservationally secure provided that Theorem 4 holds.

Proof. The conditions of ICS are met: (a) The key length required is significantly shorter than the message due to ICT and PBKE. (b) The shorter key does not weaken the ITS of OTP being VS-OTP because the summation is enciphered with an S-OTP key equivalent to an OTP key, percentage distribution is self-enciphered on a one by one basis without key reuse are made completely misleading and uncorrectable without the key per Theorem 4. (c) The gain of multi-fold key reduction significantly overweighs the cost—a limited data traffic increase of 37.5% countered by a 4- to 32-fold reduction of key length and key distribution cost. After all, safe transmission of long messages is made possible without weakening ITS/OTP. ■

It is undoubtedly a big step to advance from ITS/OTP to ICS/S-OTP. While ITS/OTP takes a bottom-up approach to cryptography, ICS/S-OTP takes a top-down approach. One is probabilistic and another holistic. One is observational and another information conservational. One is statistical and another logical. Notably, the plaintext in S-OTP is holistically transformed into isolated but interlocked and meaningless little pieces—one summation plus many percentages—whose complete correlation cannot be revealed until the plaintexts of all pieces are revealed with direct and indirect keys through key extension. It is understandable that this leads to doubts and confusions.

Clarification 1. It can be argued that, since S-OTP exposes the approximate number of data sections, it cannot be compared with OTP where the plaintext is enciphered with a same length or longer keypad. This is an invalid argument. A same length OTP key exposes the exact data length while a longer keypad exposes the approximate data length.

Clarification 2. It can be argued that the S-OTP idea is quite similar to the already well-researched key stretching technology [8]. This is an interesting observation where an enhanced or stretched key should be of sufficient size such as 128 bits to make it infeasible to break by a brute force attacks. Notably, a 2K-bit or longer integer key and 128-bit decimal keys are used in S-OTP as well. It seems similar. However, a fundamental difference is information conservation. With ICS/S-OTP, key K_I without reuse conceals a summation and the whole percentage distribution directly or indirectly in the similar way as an OTP key does with ITS. With PBKE the percentage distribution can be used as keys for themselves. Since the summation and percentage distribution are resulted from ICT, no semantics of bare plaintext is involved. While a guessed password can be readily tested in a trial, a decimal key for a percentage cannot be confirmed until the encrypted percentage and the entire distribution is confirmed. A percentage distribution cannot be confirmed without being tested against a confirmed summation. A guessed summation cannot be confirmed without its key or a confirmed percentage distribution. Thus, the isolated and meaningless little pieces are self-interlocked that have to be revealed holistically with either the key K_I or through impossible exhaustive semantic analysis. Thus, S-OTP looks similar but fundamentally different from key stretching that must have a deterministic space such as 2¹²⁸.

It should be pointed out, however, since the summation and percentage distribution can be deemed attributes, PBKE can be deemed an information conservational extension to attributed-based privacy [9,10,11]. It extends attribute-based asymmetric privacy for data sharing to a symmetric key protocol for general-purpose cryptography.

Clarification 3. Yet another major argument is that using an S-OTP key to encrypt a long message to reach ITS requires the min-entropy of the S-OTP key to be the same as OTP key, and the length of S-OTP key must be equal or larger than the length of the OTP key. Thus, it is impossible for the ICS of S-OTP not to weaken the ITS of OTP for long messages, and Definition 3 has to be wrong.

A counter argument to the critique seems difficult but readily lies in the context of OTP being VS-OTP. Based on Axioms 1-3 and Definitions 1-4, OTP is virtually scalable where any plaintext message can have a virtual summation with a virtual percentage distribution. This leads to the paradoxical nature of ITS optimality of OTP (see Theorems 1-4) because, for the first time, a computationally unlimited adversary can guess a virtual percentage and a virtual summation with a trial and error method regardless of any entropy measure of the key. Now, if an adversary can be forced to bypass an S-OTP key to guess a percentage and a summation, the adversary can also bypass an OTP key to guess a virtual percentage and a virtual summation. Thus, ITS/OTP faces a newly discovered reality where some conventional concept is no longer classical from an ICS perspective.

Subsequently, we can conclude that S-OTP does not weaken the ITS of OTP as long as the two security conditions of Theorem 1 are met no matter whether ITS in its new context is stronger, weaker, or same as before (a topic left for future study). Our focus then should be on the two security conditions with: (a) holistic information conservation, (b) semantic information hiding, (c) summation isolation from percentage distribution, (d) partitioning and concealing the percentage distribution, and (e) keypad compression with collective precision. While Shannon entropy is used as a basis, other entropy measures are left for future research effort for possible enhancement.

3.2 ICC with Quantum-Fuzzy Collective Precision

S-OTP-Method1 with percentage distribution has its limitation in computational precision and network traffic overhead. When the math summation gets huge—a usual case, the precision of a single percentage is problematic. The 37.5% data traffic increase is also a significant factor. These problems can be better solved with the massive parallel quantum-fuzzy collective precision property of ICC [6,7]. In ICC, a big integer total can be divided into many subtotals of different data sections. If each subtotal is further divided into bipolar import and export, each can be normalized by its corresponding column subtotal. An information conservational matrix can then be derived through column-major normalization for parallel collective precision without using a grand total.

Without dealing with the grand total, a section subtotal can be increased to reduce the number of sections as well as percentages and data traffic overhead. For instance, section size can be increased from 96 bits (12 bytes) to a maximum 112 bits (14 bytes) for IEEE binary128 standard. 1K-bits would result in 10 sections. L=32K bits would be divided into 320 sections instead of 352 sections. The summation would be less than 128 bits. The summation and the percentage distribution total data length approximates to $128 + 320 \times 128$ bits ≈ 40 K vs. 44K as for Method1. The cost would be roughly (40-32)/32=12/32=25% network traffic increase vs. 37.5% as for Method1.

ICC with collective precision is made achievable with bipolar fuzzy sets defined in the quantum lattices or Cartesian products including $B_1 = \{-1,0\} \times \{0,+1\}$, $B_F = [-1,0] \times [0,+1]$, and $B_{\infty} = [-1,0] \times [0,+1]$, respectively, as shown in **Fig. 1**.

Bipolar fuzzy sets forms an equilibrium-based mathematical abstraction—a set theoretic or information theoretic generalization of classical sets and fuzzy sets. Bipolar fuzzy sets are quantum sets that provide a basis for logically definable causality and quantum intelligence [15]. Within the generalization, truth-based computing can be used freely as long as certain equilibrium conditions are not violated.

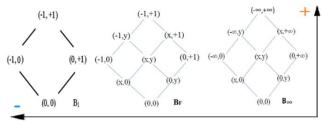


Figure 1. Hasse diagrams of YinYang bipolar quantum-fuzzy lattices B_1 , B_F , B_{∞} (adapted from [12,13,14])

In this subsection, we show an ICC example with collective precision. A key concept is an information conservational bipolar matrix M. With a percentage distribution built into M, an energy or information total or summation can be decrypted through equilibrium-based rebalancing to result in all the subtotals in parallel. This makes it possible to develop digital or quantum machinery with massive parallelism and collective precision that is not achievable with a linearly normalized percentage distribution.

M consists of bipolar elements. A bipolar element is an energy/information importexport or negative-positive variable $x = (a^{-}, b^{+})$ defined as the length of the bipolar interval from a to b. For instance, |(-2.5, 3.5)| = 3.5 - 2.5 = 2.5 + 3.5 = 6. In general,, let $|\varepsilon|$ denote the energy of, we have

$$|\mathcal{E}|(a^{-}, b^{+}) = |(a^{-}, b^{+})| = b^{+} - a^{-} = |a^{-}| + |b^{+}|.$$
 (2)

A 3-partner US-China-EU trade example is used to illustrate the basic idea of ICC with collective precision. First, the 3-parners' bipolar import-export data for 2014 are shown in **Fig. 2a** as a cognitive map (CM) in million Euros. The total energy/information in the trade scenario is characterized by the total import-export

$$|\varepsilon|(-3030750, +0) = |\varepsilon|(-0, +3030750) = 3030750.$$

Using collective bipolar interaction in ICC, accurate calculation can be carried out with the bipolar quantum cellular automaton (BQCA) $E(t+1) = M \times E(t)$ based on a column-major normalized bipolar matrix M that does not need sequential calculation of a percentage distribution [6,7]. (Note: Illustrations in this paper are in fixed-point format for readability. In real computing, they are in floating-point format.)

In this ICC example E(1) is the transpose of the initial bipolar column vector with certain total energy/information. A cognitive map (CM) C is referred to as a bipolar or unipolar conceptual graph or an import/export network. M is obtained with column-major normalization of an I/O-consistent and interactive CM in which all elements are directly or indirectly interrelated [6,7].

$$C(t) = \begin{bmatrix} (0,0) & (-420,079,+111,308) & (-311,035,+206,127) \\ (-111,308,+420,079) & (0,0) & (-164,777,+302,049) \\ (-206,127,+311,035) & (-302,049,+164,777) & (0,0) \end{bmatrix}$$

$$M = normalize(C^{T}(t)) = \begin{bmatrix} (0.000\,0.000) & (-0.112\,0.421) & (-0.209\,0.316) \\ (-0.401\,0.106) & (0.000\,0.000) & (-0.307\,0.167) \\ (-0.297\,0.197) & (-0.165\,0.303) & (0.000\,0.000) \end{bmatrix}$$

Equilibrium-based rebalancing is illustrated in **Fig. 2b** and curved in **Fig. 2c**. **Fig. 2d** verifies such rebalancing with sequential computing. **Fig. 2e** shows 200% is rebalanced to a perfect percentage distribution built in *M*. Thus, matrix *M* can be deemed a quantum encryption of a percentage distribution. Here both of a quantum world and a classical world are logically unified into a formal analytical paradigm without using bra-ket notation.

Equilibrium-based rebalancing can balance a total or a set of subtotals to a perfect equilibrium state with percentage distribution coded in *M* in an iterative and massively parallel process. Although a perfect equilibrium-state may be neither practical nor desirable in business, equilibrium-based rebalancing provides a new approach to pre and post-quantum cryptography. It finds a mathematical abstraction for collective precision and bridges the classical and quantum worlds.

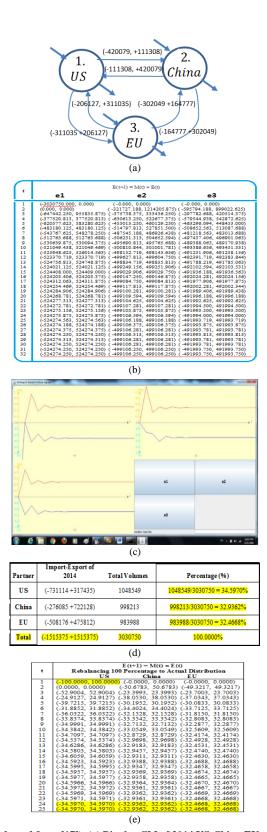


Figure 2 Illustration (Adapted from [17]). (a) Bipolar CM of 2014 US-China-EU trade (in Million Euros); (b) Rebalancing of total import/export to an equilibrium state; (c) Curves of the rebalancing; (d) Digital computing; (e) Quantum-fuzzy rebalancing of 200%

S-OTP-Method2

Assume IEEE binary128 is used with the precision of 112 significant bits and assume sender Alice and receiver Bob share private key K₁ distributed through QKD.

Part I. Encryption

- Step 1. Data Transformation. Given binary data D to be transmitted, let the unsigned integer number set $\{x_i\} = \{x_1, x_2, ..., x_i, ..., x_n\}$ represent preprocessed data sections of D and let math summation $X = \sum_i x_i$, $1 \le i \le N$.
- Step 2. Bipolar Cognitive Mapping. Construct an I/O-consistent bipolar cognitive map C based on $\{x_i\}$ such that $\{x_i\}$ is decomposed into an unbalanced relational data set $\{e_{ij}\}$ = $\{(e_{ij}, e_{ij}^+)\}$ where each bipolar link weight $e_{ij} = (e_{ij}, e_{ij}^+)$ and $|d_i| \equiv \sum_j |\varepsilon| e_{ij}$ (energy/information of row i) with ratio $|e_{ij}|/|e_{ij}^+| > l$, a threshold for non-zero bipolar elements. Thus, the set $\{e_{ij}\}$ forms a bipolar cognitive map C with total information $X = \sum_i |x_i|$. (Note: C is not unique an area of further research.)
- Step 3. Bipolar Energy/Information Normalization. Normalize the transpose C^T to an information conservational matrix M (a bipolar quantum-fuzzy logic gate (BQFLG) or a bipolar quantum-fuzzy cognitive map (BQFCM)) such that the BQCA $E(t+1)=M\times E(t)$ is asymptotic to an equilibrium state [6,7] (further discussed in next section with Eqs. 3.4).
- **Step 4. Data Encryption.** Apply Eq. 1 to change the remaining terms of M to a misleading M'; and use K_1 to encipher X and part of M to X'.
- Step 5. Transmit the pair E = (X', M').

Part II. Decryption

- **Step 1.** Use K_1 and Eq. 1 to recover the pair (X,M);
- **Step 2**. Use M to decipher and depolarize X to recover $\{x_i\}$;
- **Step 3.** Recover D from $\{x_i\}$ with concatenation.

Illustration. Applying S-OTP-Method2 we have the decryption example in **Fig. 2**. The total information of the last row of **Fig. 2b** approximate to exactly the same result as that of S-OTP-Mehrod1:

```
d_1 = |\mathcal{E}|(-731114, +317435) = 1048549;

d_2 = |\mathcal{E}|(-276085, +722128) = 998213;

d_3 = |\mathcal{E}|(-508176, +475812) = 983988

D = Concatenate(d_1, d_2, d_3) = 1048549998213983988.
```

Theorem 6. S-OTP-Method2 is information conservationally secure provided that the summation is enciphered with a same length or longer S-OTP key and matrix M is made completely misleading and uncorrectable by an attacker without the key.

Proof. It follows the proof for S-OTP-Method1.

Remarks: (a) Theorem 6 closes a loophole in Theorem 8 of Ref. [6]. (b) The value of S-OTP-Method2 lies in its collective precision, not in key compression and data traffic.

3.3 The Quantum Intelligence Nature of ICC

The bipolar approach is based on a formal logic named bipolar dynamic logic and an algebra named bipolar quantum linear algebra, which are proven equilibrium-based bipolar dynamic generalizations of Boolean logic and linear algebra respectively. This development leads to an analytical quantum intelligence paradigm of ICC [6,7].

Given an $n \times n$ square bipolar interactive matrix M and an $n \times 1$ column bipolar vector E(t) such that $E(t+1)=M\times E(t)$, if the absolute energy/information subtotal $|\varepsilon_{col}/M_{*j}(t)|$ of each column j of M equals 1.0 or $\forall j$, $|\varepsilon_{col}/M_{*j}(t)| \equiv 1.0$, M is referred to as an information

conservational bipolar quantum logic gate (BQLG) matrix or a bipolar quantum-fuzzy cognitive map (BQFCM) [6]. With the BQLG matrix M we have an information conservational bipolar quantum cellular automaton (BQCA) (Eq. 3).

$$|\varepsilon|E(t+1) = |\varepsilon|(M \times E(t)) \equiv |\varepsilon|E(t).$$
 (3)

Eq. 3 defines a general-purpose quantum cellular automata theory—an equilibrium-based unification of matter and antimatter atoms. Computationally, a BQCA can be regulated to achieve information conservation, regeneration, degeneration, and/or oscillation. It provides a basis for equilibrium-based quantum intelligence (QI) (**Fig. 3**) [6,7].

QI leads to the theory of ICC and S-OTP-Method2. In ICC, an I/O-consistent CM can always be designed and normalized to a BQLG matrix *M* for a BQCA to be asymptotic to a bipolar equilibrium state even though some link weights are weaker and need more iterations (*t*) to be balanced. This property provides a basis for pre and post-quantum cryptography.

The transpose $C^T(t)$ is used to obtain its column-major normalized BQLG matrix M for ICC. The normalization follows Eq. 4. In Eq. 4, the denominator $|\varepsilon_{col}/(C^T_{*j})|$ denotes the absolute energy/information subtotal of column j in C^T . The notation $|\varepsilon_{col}/(M_{*j})|$ denotes the normalized absolute energy/information subtotal of column j of matrix M.

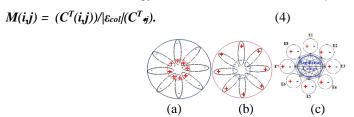


Figure 3. BQCA unification of matter and antimatter atoms (Adapted from [16,17,18]): (a) Matter; (b) Antimatter; (c) Unification

3.4 The Digital Nature of S-OTP-Method2

Notably, S-OTP-Method2 is based on bipolar equilibrium-based rebalancing. Bipolarity is a quantum feature that form the bipolar reality of negative-positive particles. The bipolar property, however, can be depolarized for digital cryptography. Thus, a unipolar CM can be revealed from a bipolar one with depolarization—a move from quantum world to classical world. Since a bipolar representation is a generalization of unipolar representation and subsumes unipolar cases, all the elements of a polarized map can simply have zero negative energy/information, it leads to the simplified CM as in **Fig. 4** coded as a unipolar positive matrix C(t) that does not distinguish import-export bipolarity. This leads to a unipolar cipher that is basically the same as S-OTP-Method2 except using a positive CM and a positive matrix M. **Fig. 4** shows such a decryption example where in the last row we have the same result.

```
d_1 = |\varepsilon|(-0, +1048549) = 1048549;

d_2 = |\varepsilon|(-0, +998213) = 998213;

d_3 = |\varepsilon|(-0, +983988) = 983988;

D = Concatenate(d_1, d_2, d_3) = 1048549998213983988.
```

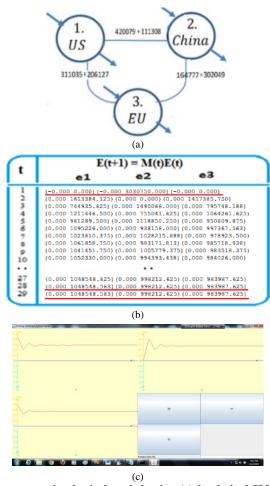


Figure 4. Information-conservational unipolar rebalancing: (a) depolarized CM; (b) Positive distribution; (c) Positive curve (scaled)

3.5 Two Puzzles Explained

(A) How can matrix C(t) in symmetry (C(t)(i,j)=C(t)(j,i)) be used in cryptography?

The answer is that, although matrix C(t) is symmetrical, a column-major normalized M can be non-linear and asymmetrical because the normalization is by dividing its column subtotal of $C^{T}(t)$, but not by the global total (corresponding to the overall summation). For instance,

$$C(t) = \begin{bmatrix} 0 & 531587 & 517162 \\ 531587 & 0 & 466826 \\ 517162 & 466826 & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0.000 & 0.532 & 0.526 \\ 0.507 & 0.000 & 0.474 \\ 2 & 0.493 & 0.468 & 0.000 \end{bmatrix};$$

where C is symmetrical, but M is not. The non-linear asymmetrical property of M can be characterized with a set of linear equations. Let the three subtotals (or data sections) be x, y, and z, respectively, for the 3×3 matrix M we have $m_{10}\times x - m_{01}\times y = 0$; $m_{20}\times x - m_{02}\times z = 0$; and $m_{21}\times y - m_{12}\times z = 0$; and $m_{ij}\neq m_{ji}$. The set of equations have infinite number of solutions because all column coefficients of M correlate non-linearly with each other due to non-linear normalization with different local column subtotals. This is fundamentally different from percentage distribution where all percentages are normalized with a global total and linearly correlated.

(B) If a unipolar positive matrix is sufficient, why do we need a bipolar equilibrium-based matrix for cryptography?

There are three top answers to this question:

- (i) The universe consists of negative-positive particles. Without bipolarity, there would be no information conservation, logically definable causality, bipolar quantum computing, and analytical quantum intelligence [6,7]. Thus, bipolarity leads to an analytical paradigm of ICC compatible to digital computing (further discussed later).
- (ii) Bipolarity is set-theoretically different from bilinearity or bijection, one defines a 2-to-2 mapping of equilibrium-based non-linear bipolar dynamic entanglement with logically definable causality and information conservation, another defines a 1-to-1 mapping without entanglement and definable causality. Of course, a negative sign can be eliminated with depolarization, but the bipolar semantics remains intact. **Fig. 5** shows the necessity of quantum bipolarity.
- (iii) On the sender side, a bipolar matrix doubles the number of elements in a unipolar matrix, doubles the parallelism, and doubles collective precision with equilibrium-based rebalancing even though a full N by N matrix is unnecessary. On the receiver side, a bipolar matrix avoids decrypting a large total because, given a big integer E in 100%, we have the *equivalence law of energy/information distribution/conservation*:

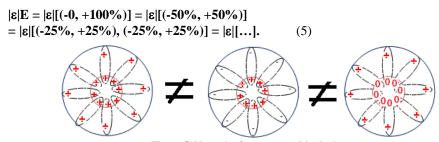


Figure 5. Necessity for quantum bipolarity

4 ADVANCED METHODOLOGY

4.1 Minimal BQCA

Theorem 7 (Minimal BQCA Theorem). Summation and percentage distribution is a one-step bipolar quantum cellular automaton with all zero values for the negative poles. Thus, Mehtod1 can be the minimal case of Method2.

Proof. Mehtod2 entails an $N \times N$ square matrix multiplied by a column vector in an information conservational BQCA. When $N \times N$ is reduced to $N \times 1$, the matrix becomes a column vector of percentage distributions $\{w_i\} = \{x_i/X\}$ summing up to 1.0, the single number must be the summation X of N sections, such that the column vector multiplied by a single element matrix results in a column vector energy/information distribution $\{x_i\}$. The Matrix multiplication can be deemed the minimal BQCA which requires a final equilib-

rium state be reached in a single step such as
$$\begin{pmatrix} w_0 \\ w_1 \\ w_{i+1} \end{pmatrix} [X] = \begin{pmatrix} x_0 \\ x_1 \\ x_{i+1} \end{pmatrix}$$
. Thus, Mehtod1 would

be the minimal case of Method2 if it is optimized to have a minimum number of percentages with collective precision.

The above theorem provides a unifying bridge between two mathematical abstractions, one for a bipolar quantum world and another for a classical unipolar world. Since Method1 is suitable for reducing network traffic and Method2 for collective precision, the two can now be combined in an optimization.

4.2 Optimization

Method1 uses percentage distribution; Method2 uses collective precision. In Method1 each data section depends on a single percentage resulted from linear normalization by a grand total. When the data length is long, Method 1 will have a precision problem. In Method2, each data section depends on column-major normalization by much smaller subtotals where percentage distribution is not directly calculated using a grant total as the denominator. If each column has an average of n > 2 non-zero numbers as weights, the precision requirement on the weights is n-times smaller. The larger the number n the more parallelism in high precision decryption. When n equals N, Method2 reaches maximum parallelism with N-fold precision enforcement for a positive matrix M and 2N-fold for a bipolar matrix M. This observation leads to the inception of information conservational quantum-fuzzy collective precision [6].

Observation 1: Asymptoticity. If M is information conservational, a BQCA E(t+1) = $M \times E(t)$ is asymptotic to an equilibrium state as determined by M per references [6,7].

Observation 2: Information Conservation. If an original message D is converted to an energy/information total E through a BQCA transformation, the information conservational matrix M of the BQCA can serve as a key to decode the total information to the original message D on the receiver side per ref. [6]. However, to transmit matrix M will cost much more than to transmit a percentage distribution. Thus, Method1 is more efficient than Method2 for network traffic but only Method2 can enable collective precision, optimization, and efficient decryption.

Theorem 8. If M is information conservational, a BQCA $E(t+1) = M \times E(t)$ can be used to derive the minimum number of percentages in a distribution embedded in M.

Proof. Given $|\varepsilon|E(t) = 100$ (%), the theorem follows from the asymptoticity law [6] directly (e.g. Fig. 2e). \blacksquare

Theorem 9. A percentage distribution of N divisions can be converted to an $N\times N$ (unipolar or bipolar) information conservational matrix M with collective precision and enhanced parallelism for decryption where M is information conservational and BQCA E(t+1)= $M \times E(t)$ is asymptotic to an equilibrium state.

ized from different matrices. The theorem follows from
$$\begin{pmatrix} w_0 \\ w_1 \\ w_1 \end{pmatrix} [E] = \begin{pmatrix} w_0 E \\ w_1 E \\ w_1 E \\ w_1 E \end{pmatrix}$$
 where

Proof. Notice that
$$M$$
 is information conservational but not unique that can be normalized from different matrices. The theorem follows from $\begin{pmatrix} w_0 \\ w_1 \\ w_{i+1} \end{pmatrix} [E] = \begin{pmatrix} w_0 E \\ w_1 E \\ w_0 E \end{pmatrix}$ where $\begin{pmatrix} w_0 \\ w_1 \\ w_{i+1} \end{pmatrix}$ is strictly proportional to $\begin{pmatrix} w_0 E \\ w_1 E \\ w_{i+1} E \\ w_n E \end{pmatrix}$. That is, M can be derived from either of the two mutually proportional column vectors.

Based on the above findings we can conclude that, on the sender side, matrix M can be used for determining the minimum percentage distribution with collective precision that can achieve N to 2N fold reduction of precision requirement due to column-major normalization (Re. Eq. 4). On the receiver side, M can be used to decrypt a big total to subtotals (or data sections) with collective precision in a reverse way (Fig. 2b and Fig. 4b). Thus, Method1 and Method2 can be used in a combination. Method2 focuses on ICC with collective precision; Method1 focuses on secure and efficient data transmission, this lead to the block diagram design in Fig. 6 that combines the advantages of Method1 and Method2 while eliminating drawbacks of each of them.

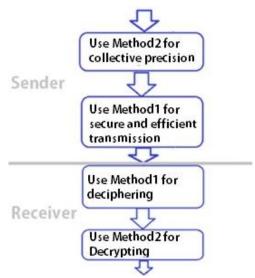


Figure 6. Method1 and Method2 in combination

The combination of two methods together leads to an optimized algorithm named S-OTP-Method₁₊₂.

S-OTP-Method1+2

For 32K-bit binary data (or less) divided into 320 (or fewer) sections $\{x_i\}$, assume sender Alice and receiver Bob share private key K_1 distributed through QKD. An integer P is private until the sender found the number of concealed percentages. Assume no zero-value or full data section. Assume IEEE binary128 standard with 112 significant bits precision

Part I -Sender Side

- **Step 1**. Let integer set $\{x_i\}$ represent preprocessed data sections;
- **Step 2**. Let math summation $X = \sum_i x_i$ for all i
- **Step 3**. Use the collective precision advantage of Method2 to calculate percentage distribution $\{x_i/X\}$ (See Fig. 2e);
- **Step 4.** Use Method₁ and K_1 to encrypt X and P terms of $\{x_i/X\}$ to X' and use Eq. 1 to encrypt the remaining (N P) terms of the percentage distribution to $\{x_i/X\}'$;
- **Step 5**. Alice Transmits the pair $E=(X', \{x_i/X\}')$ to Bob use Method₁.

Part II. Receiver Side

- **Step 1**. Use K_1 and Eq. 1 to decipher E to recover the pair $(X, \{x_i/X\})$;
- **Step 2.** Use $\{x_i/X\}$ to derive an information conservational matrix M so that Method2 can be used to decrypt the summation X to $\{x_i\}$;
- **Step 3**. Recover the message from $\{x_i\}$ with concatenation.

Example. Assuming the plaintext data D to be transmitted is represented by the big integer L = 1048549998213983988 divided into the three sections 1048549, 998213, and 983988. Assume sender Alice and receiver Bob share K_1 distributed through QKD.

Part I – Encryption (some steps are omitted)

- Let $x_1 = 1048549$, $x_2 = 998213$, $x_3 = 983988$;
- $X = x_1 + x_2 + x_3 = 3030750;$
- Use Method2 to calculate percentage distribution (see Fig. 2(e)) and {x/X} = {34.5970%, 32.9362%, 32.4668%}:
- Use Method1 to encrypt $(X, \{x_i/X\})$ to $(X, \{x_i/X\})$ ' and transmit;

Part II – Decryption (some steps are omitted):

- Use K_1 and Eq. 1 to decrypt the message;
- *U*=(3030750, {34.5970%, 32.9362%, 32.4668%});
- Derive an information conservational BQCA based on U using Method2;
- Use the BQCA and Method2 to decrypt U (see Method2 and Fig. 2(b));

- $x_1 = 1048549$, $x_2 = 998213$, and $x_3 = 983988$;
- $L = concatenate(x_1, x_2, x_3) = 1048549998213983988;$
- Recover D from L.

It should be noted that the above example illustrates the basic concept but may be too small to be fully illustrative to the collective precision property of Method2. With a large number of sections, collective precision would be crucial. In that case, Method1 would face a critical precision problem due to a huge summation denominator. Method2, however, does not have to deal with the big summation. Thus, collective precision could spark an analytical paradigm of ICC for supercomputing machinery with analytical quantum intelligence.

Theorem 10. S-OTP-Method1+2 is information conservationally secure provided that S-OTP-Method1 is secure.

Proof. Since Method2 is only used for optimization with collective precision but not for communication, the theorem follows the proof for S-OTP-Method1 directly.

4.3 Hierarchical Extension with BHKC and BBDR

S-OTP-Method1+2 can be extended to a hierarchical algorithm. With BHKC for a long message, a hierarchical process of ICTs can scale down or reduce an OTP keypad length to a condensed tiny minimum. Taking QKD cost saving into consideration the benefit of BHKC/BBDR significantly overweighs the cost.

S-OTP-Hierarchical

Assume IEEE binary128 is used with the precision of 112 significant bits, and assume the sender and receiver share private key K_1 . Assume preprocessed data sections.

Part I -Sender Side BHKC

- Step 1. Level L = 0; OTP key length required equals to data length, that is not practical for long messages of multiple mega or gaga bits; let block index b = 0; let total number of blocks B = (total number of bits)/32K;
- Step 2. Level L = L+1, for each 32K-bit (longer or shorter) data block, apply S-OTP-Method₁₊₂ to reduce the key length to 2K, 4K, or 8K compressed data bits, respectively, as designated, to conceal a 128-bit summation and P terms of percentages. Total key length is reduced after this step by 32-, 16-, or 8-folds, respectively, as designated. The unconcealed percentages are made completely misleading with Eq. 1 and uncorrectable by an attacker without private key. The compressed data for each 32K are left for further compression without enciphering;
- **Step 3.** If total key length for compressed data is greater than 2K, 4K, or 8K bits as designated, go to Step 2;
- Step 4. Encrypt the remaining percentages with Eq. 1, and encipher the compressed data with K_1
- *Step 5*. Transmit the ciphertext and all the misleading percentages for each 32K-bit data block at each level to the receiver.

Part II - Receiver Side BBDR

- **Step 1**. Use S-OTP-Method₁₊₂ to decipher with K_1 and Eq. 1.
- **Step 2**. L = L 1; if L > 0, go to Step (1);
- Step 3. Recover the original message or data set D by concatenating the data sections.

Figures 7a and 7b show the sketches of BHKC and BBDR, respectively.

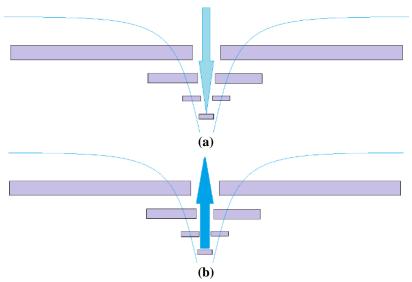


Figure 7. Sketches of Information Conservational Encryption and Decryption: (a) "Black Hole" Keypad Compression; (b) "Big Bang" Data Recovery

BHKC Example 1. Given 64G-bit message (or shorter), assuming 32-fold key reduction with IEEE binary128 for each round of ICT with key extension,

- L = 0, total key length = 64G bits, too long;
- L = 1, total key length = 64G/32 = 2G, too long (data length increase $64G \times 25\% = 16G$);
- L=2, total key length = $2G/32 = 2^{11}M/2^5 = 2^6M = 64M$, still too long (data length increase $2G \times 25\% = 512M$);
- L=3, total key length = 64M/32=2M, too long (data length increase $64M\times25\%=16M$), impractical and not applicable;
- L = 4, total key length = $2M/32 = 2^{11}K/2^5 = 2^6K = 64K$, too long (data length increase $2M \times 25\% = 512K$);
- L = 5, total key length = 64K/32 = 2K, practical key applied (data length increase $64K \times 25\% = 16K$).
- Total network traffic overhead: (16G + 512M + 16M + 512K + 16K) < 17G > (25% of 64G)

BHKC Example 2. For the same problem as in Example 1 but assuming 16-fold key reduction for each round of ICT instead of 32-fold, we have

- L = 0, total key length = 64G bits, too long;
- L = 1, total key length = 64G/16 = 4G, too long (data length increase $64G \times 25\% = 16G$);
- $L=2, \ total \ key \ length=4G/16=2^{12}M/2^4=2^8M=256M \ too \ long \ (data \ length \ increase \ 4G\times 25\%=1G);$
- L = 3, total key length = 256M/16 = 16M, too long (data length increase $256M \times 25\% = 64M$);
- L = 4, total key length = 16M/16 = 1M, too long (data length increase $16M \times 25\% = 4M$);
- L = 5, total key length = 1M/16 = 64K, too long (data length increase $1M \times 25\% = 256K$);
- L = 6, total key length = 64K/16 = 4K, practical and applied (data length increase $64K \times 25\% = 16K$).
- Total network traffic overhead: 16G + 1G + 64M + 4M + 1M + 256K + 16K < 18G > (25% of 64G)

Theorem 11. S-OTP-Hierarchical is information conservationally secure provided that S-OTP-Method1+2 is secure.

Proof. Since the section percentages in different data blocks at different layers are not correlated in anyway, the theorem follows Theorems 10 and 11 directly. ■

With S-OTP-Hierarchical, sender-receiver collusion on collective precision could be important. On the sender side, collective precision can be used for testing intermediate results efficiently and precisely to guarantee that the receiver side will get the correct message. On the receiver side, it can be used to decrypt the total to many subtotals precisely and efficiently in massive parallelism. The two sides can be colluded through public protocols based on the ranges of data length and number of sections.

4.4 "Event Horizon" or "Firewall"

The large number of little pieces, mostly encrypted percentages, form the metaphorical "event horizon" or the "firewall" of a "black hole." The "black hole" compresses intermediate keypads (summations and partial percentage distributions) of many data blocks layer by layer to a condensed final summation which is concealed together with some percentages for key extension by a short S-OTP keypad. Without the key, however, the large number of encrypted percentages remain isolated for different blocks of data. Since the "event horizon" is larger than the plaintext in data size, a mild network traffic increase incurs as a tradeoff. That is, more binary bits are transmitted for scaling an OTP key to an S-OTP key without information loss and information leaking. The gain is, however, much more significant than the cost because S-OTP makes the transmission of long messages possible with a short key distributed through QKD. Otherwise, 64G-bit data would need a same length OTP keypad that is impossible.

4.5 PBKE in BHKC

Information conservational PBKE is critical for BHKC in ICS/S-OTP. Notably, three short keys in Eq. 1 are replaced with three partially concealed random percentages p_1 , p_2 , p_3 and p_4 . If the three keys had been distributed through QKD all data blocks would have to use the same keys. Otherwise, each data block would require three different keys from QKD that is impractical. With three percentages p_1 , p_2 , p_3 and p_4 as extended keys, each data block has its own set of keys for the encryption of the remaining percentages. These keys are indeed random and there are plenty of them.

Notably the keypads at the intermediate levels of a hierarchy are further segmented into sections concealed layer by layer until the last summation is concealed together with some percentages with a short S-OTP key K_1 distributed through QKD. In this way, the short key conceals the last 128-bit summation directly and the big "event horizon" indirectly. The S-OTP cipher, however, is impossible to break because the enciphered data are semantically meaningless, and the higher the hierarchy the final summation and its percentage distribution are farther away from plaintext semantics.

Theorem 12 (Law on BHKC). A same length OTP keypad for a long plaintext message can always be compressed through BHKC to a condensed tiny minimum S-OTP keypad with the same level of information theoretic security.

Proof. Based on Axiom 2, the semantics of a semantically meaningless message enciphered with an OTP key cannot be recovered without the key. With S-OTP-Hierarchical the higher the hierarchy, the deeper the "black hole", the farther away is the final summation from plaintext semantics and the more difficult/impossible to perform semantic analysis even for a computationally unlimited adversary. Thus, the longer the message, the higher a hierarchy, the deeper the "black hole", and the smaller the S-OTP/OTP ratio can be. That means any lengthy OTP cipher keypad can be reduced to a tiny minimum ICS/S-OTP key through BHKC and PBKE without weakening ITS/OTP as proven in Theorems 3.

Clarification 4. As we know information (message) is ultimately presented as binary (0/1) bits in modern computing and communication, this leads to the question: When we scale an OTP keypad to a tiny S-OTP key through BHKC, is there really no information (entropy) loss during the process? The answer to the above question is that, within the limitation of computational precision, there is no information loss because the final summation is not really a grand total but a total of segmented intermediate summations and partial percentages. Since ICS/S-OTP reduces the keypad length with a mild network traf-

fic increase at each layer of a hierarchy, BHKC does not suffer information loss. In addition, since collective precision can be colluded based on certain public protocols between senders and receivers, BBDR is guaranteed full data recovery.

It should be remarked, however, that a short key for transmitting a long message without information loss does not suggest unsecure information leaking in anyway. S-OTP transforms a long message into a large number of isolated, semantically meaningless, and holistically interlocked little pieces using information conservational PBKE where an original message can only be recovered with the key collectively. Otherwise, no deterministic algorithm can break the cipher as axiomatically proven that ICS/S-OTP does not weaken ITS/OTP.

5. QUANTUM EXTENSION

While collective precision adds a number of new features to cryptography, its biggest potential could be its equilibrium-based extension of quantum mechanics and quantum computing to an equilibrium-based analytical paradigm. It further suggests that Method2 is suitable for developing bipolar quantum-digital compatible ICC machinery. While the illogical aspect of bra-ket quantum mechanics prevents quantum computing from becoming an analytical paradigm, ICC makes the quantum dream logically possible and quantum-digital compatible with a formal set-theoretic theory of quantum intelligence.

From a security perspective, quantum-digital compatible ICC machinery can be an ultimate solution to achieve perfect security. Although ITS/OTP is widely believed the only unbreakable crypto system even if the adversary has unlimited computing power, the optimality paradox as proven in Theorems 1-3 cast doubt on this claim from the perspective of ICS. Even though researchers are trying hard to find a solution for post-quantum cryptography, a perfect digital computing solution may not exist. If quantum-proof cryptography is not realizable through digital communication, the only option left for post-quantum cryptography could be quantum teleportation. Bipolar quantum entanglement provides such an equilibrium-based logical basis [6,7].

The nature of S-OTP-Mehtod2 makes it suitable for equilibrium-based bipolar quantum rebalancing with perfect information conservation. Encryption would be unnecessary for quantum communication as any attack to a quantum channel would jeopardize perfect information conservation and stop the communication. Such a quantum teleportation machine is drafted in **Figs. 8(a,b,c)**. **Fig. 8(a)** sketches a bipolar quantum-digital compatible ICC crypto machine. **Fig. 8(b)** sketches bipolar quantum teleportation. **Fig. 8(c)** portrays a bipolar qubit register. **Fig. 8(d)** tables YinYang bipolar universal modus ponens (BUMP) as a ubiquitous law for bipolar quantum entanglement with definable causality—a logical basis for bipolar quantum teleportation.

While the bipolar quantum dream may still seem to be "far-fetched" in terms of quantum-digital compatibility for ICC, a newly reported discovery of a class of subatomic particles (fermions) [19] injected new life into this line of research. The new discovery is a family of particles that are their own antiparticles. This family of particles strengthen the ontological basis of equilibrium-based bipolar quantum rebalancing and may make quantum computing more practical and powerful.

The equilibrium-based architecture in Fig. 8 forms the basis of analytical quantum intelligence machinery. The significance of this new paradigm is further supported by the realization of quantum teleportation between long distances [20,21] that gives hope for practical application of ICC with perfect quantum information conservation and security.

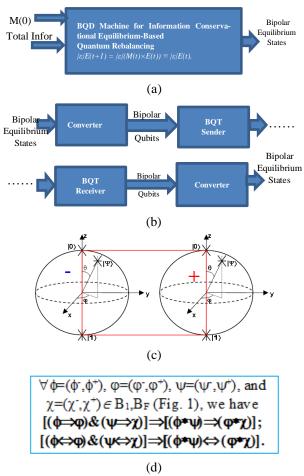


Figure 8. ICC—An Analytical Paradigm of Quantum Mechanics:
(a) Bipolar quantum-digital compatible machinery; (b) Bipolar quantum teleportation (BQT); (c) Bipolar qubit register [14,18]; (d) Bipolar universal modus ponens (BUMP) [14]

6. CONCLUSIONS

Based on an analytical theory of quantum intelligence [6,7,15,22,23,24], information theoretic security has been extended to information conservational security that has led OTP to S-OTP. PBKE has been proposed. Based on PBKE, "Black hole" keypad compression coupled with "big bang" data recovery has been introduced for S-OTP with gains significantly overweigh a mild cost in network traffic increase. It is proven that if the security of OTP were optimal, S-OTP would be impossible; on the other hand, if S-OTP were not information theoretically secure, OTP would not be secure either. Thus, we have a proof by contradiction on the paradoxical nature of OTP optimality from an information conservational perspective. The proof has further led to an analytical paradigm of quantum intelligence machinery toward perfect information conservational security. It has been shown that

- S-OTP makes it possible for transmitting long messages or large data sets with a condensed tiny keypad.
- Math summation with percentage distribution without using big primes makes S-OTP quantum proof to quantum factorization [1] for both pre and post-quantum cryptography.
- ICC with collective precision can be massively parallel, accurate, efficient, and suitable for developing supercomputers with digital technology.
- ICC is quantum-digital compatible and suitable for developing teleportation machinery with perfect information conservation and quantum security.
- It has been shown that a summation with percentage distribution is a special/minimal case of equilibrium-based bipolar quantum cellular automata. This finding has made it possible to combine the advantages of both approaches for pre and post-quantum cryptography, one for efficient and minimum data communication and another for effective and acurate computation with collective precision. On the other hand, the finding has unified a classical world with the quantum world in a bipolar equilibrium-based mathematical abstraction.

While Dirac bra-ket notation [25] is adopted as a standard for quantum mechanics and quantum computing, the standard limits quantum computing within Hilbert space—a complex spacetime geometry that stopped short of providing logically definable causality for the Copenhagen interpretation [26, 27]. With formal logically definable causality [6, 7, 14, 15, 22], the quantum intelligence approach extends Bohr's particle-wave complementarity principle to a bipolar interpretation and a formal system—an equilibrium-based analytical arm for research on quantum mechanics and quantum computing including but not limited to quantum cosmology (e.g. [28, 29, 30]), quantum life (e.g. [31, 32,33]), and quantum cryptography (e.g. [1, 2, 19, 20, 21, 34, 35, 36]).

Whereas OTP has been prevented from being widely used by its key length requirement, S-OTP has got around the problem through ICC without weakening ITS in its new context. Thus, S-OTP qualifies itself as a unique extension from ITS to ICS. While this work has been focused on information conservational security, the equivalency or non-equivalency between ICS and ITS with different entropy measures (ex. [37]) may deserve further investigation for possible improvement. Although the limited network traffic overhead with ICS/S-OTP may not be a problem for a high-speed network such as 5G or higher broadband, as a major cost it may be further reduced. On other hand, the size of large matrices used for analysis in this paper can be reduced if needed with limited cost. Thus, further improvements of ICS/S-OTP are possible.

Floor-Roof Mysteries. According to the floor-roof theory of science [15,24], ITS of OTP has been developed based on information theory rooted in probability and statistics—a floor or foundation of modern science focused on observability and entropy without logically definable causality and bipolar information conservation. ICS of S-OTP, on the other hand, has been a set-theoretic development rooted in bipolar quantum intelligence—a roof of modern science focused on equilibrium-based rebalancing and information conservation with logically definable causality [6,7]. Thus, this work has opened some major challenges to computing, cryptography, and science in general. Among them are the following floor-roof mysteries:

- Is ICS an information theoretic extension to ITS or just a new technological development?
- Is ICS a falsification of ITS in terms of optimality?
- Shannon concluded on the impossibility for perfect secrecy beyond OTP with key length greater than or equal to the message to be enciphered [3]. Although this paper did not attempt to falsify the theorem directly, however, if ICS/S-OTP does not weaken ITS/OTP, could Shannon's conclusion be wrong? On the other hand, if ICS of S-OTP were not secure, could ITS of OTP be secure?

 Could modern science, such as modern physics and information theory [3,4], have been like a well-founded building with a floor of observable beings, truths, and entropy but missing its roof for equilibrium, harmony, information conservation, and logically definable causality [6, 7, 5, 22, 2, 24]?

Floor-Roof Assertions. Despite the mysteries, we have the following floor-roof assertions:

- Can the floor perform some functions not performed by the roof? The answer is definitely YES.
- Can the roof perform some functions not performed by the floor? The answer is definitely YES.
- Can information conservational security solve some unsolved problems by information theoretic security? The
 answer should be logically YES.
- Does modern science need equilibrium, harmony, information conservation, and logically definable causality
 as its roof? The answer should be logically YES [38].

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