Mining for Privacy: How to Bootstrap a Snarky Blockchain

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Abstract. Non-interactive zero-knowledge proofs, and more specifically succinct non-interactive zero-knowledge arguments (zk-SNARKS), have been proven to be the "swiss army knife" of the blockchain and distributed ledger space, with a variety of applications in privacy, interoperability and scalability. Many commonly used SNARK systems rely on a *structured reference string*, the secure generation of which turns out to be their Achilles heel: If the randomness used for the generation is known, the soundness of the proof system can be broken with devastating consequences for the underlying blockchain system that utilises them. In this work we describe and analyze, for the first time, a blockchain mechanism that produces a secure SRS with the characteristic that security is shown for the exact same conditions under which the blockchain protocol is proven to be secure. Our mechanism makes use of the recent discovery of *updateable* structure reference strings to perform this secure generation in a fully distributed manner. In this way, the SRS emanates from the normal operation of the blockchain protocol itself without the need of additional security assumptions or off-chain computation and/or verification. We provide concrete guidelines for the parameterisation of this system which allows for the completion of a secure setup in a reasonable period of time. We also provide an incentive scheme that, when paired with the update mechanism, properly incentivises participants into contributing to secure reference string generation.

1 Introduction

In the domain of distributed ledgers, non-interactive zero-knowledge proofs have many interesting applications. In particular, they have been successfully used to introduce privacy into these inherently public peer-to-peer systems. Most notably, Zerocash [1] demonstrates their usefulness in the creation of private currencies. Beyond this, there are numerous suggestions [21, 18, 26] to apply the same technology to smart contracts for increased privacy. Beyond privacy, other applications of such systems include blockchain interoperability, e.g., [13], and scalability, e.g., [4].

For the practical efficiency of these designs, two things are paramount: The succinctness of proofs, and the speed of verifying these proofs. The distributed

nature of these ledgers mandates that a large number of users store and verify each proof made, rendering many zero-knowledge proof systems not fit for purpose.

Research into so-called zk-SNARKs [23, 15, 17, 16, 22] aims at optimizing exactly these features, with proof sizes typically under a kilobyte, and verification times in the milliseconds. It is a well-known fact that non-interactive zero-knowledge requires some shared randomness, or a *common reference string*. For many succinct systems [23, 15, 17, 16, 22], a stronger property is necessary: Not only is a shared random value needed, but it must adhere to a specific *structure*. Such structured reference strings (or SRS) typically consist of related group elements: g^{x^i} for all $i \in \mathbb{Z}_n$, for instance.

The obvious way of sampling such a reference string from public randomness reveals the exponents used – and knowledge of these values breaks the soundness of the proof system itself. To make matters worse, the security of these systems typically relies (among others) on *knowledge of exponent* assumptions, which state that to create group elements related in such a way requires knowing the underlying exponents and hence any SRS sampler will have to "know" the exponents used and be trusted to erase them becoming effectively a single point of failure for the underlying system. While secure multi-party computation can be, and has been, used to reduce the trust placed on such a setup process (cf. [28]), the selection of the participants for the secure computation and the verification of the generation of the SRS by the MPC protocol retain an element of centralization. Using an MPC setup remains a controversial element in the setup of a decentralised system that requires SNARKs.

Recent work has found succinct zero-knowledge proof systems with *updateable* reference strings [16, 22]. In these systems, given a reference string, it is possible to produce an updated reference string, such that knowing the trapdoor of the new string requires both knowing the trapdoor of the old string, *and* knowing the randomness used in the update. [16] conjectured that a blockchain protocol may be used to securely generate such a reference string. Nevertheless, the exact blockchain mechanism that produces the SRS and the description of the security guarantees it can offer has, so far, remained elusive.

1.1 Our Contributions

In this work we describe and analyze, for the first time, a blockchain mechanism that produces a secure SRS with the characteristic that security is shown for the exact same conditions under which the blockchain protocol is proven to be secure. In this way, the SRS emanates from the normal operation of the blockchain protocol itself without the need of additional security assumptions or off-chain computation and/or verification.

We rely primarily on the *chain quality* property of "Nakamoto-style" ledgers [10] – distributed ledgers in which a randomised process selects which user may append a block to an already established chain. Such ledgers rely on honest hashing power majority – and can be shown to guarantee a chain quality property

which suggests that any sufficiently long chain segment will have some blocks created by an honest user, cf. [10, 24, 11].

Our construction integrates reference string updates into the block creation process, but we face additional difficulties due to the fact that update calculation is a computationally heavy and, contrary to brute-force hashing, useful operation. The issues arising from this are two fold. Firstly, an adversarial party can take shortcuts by supplying a low amount of entropy in their updates, and try to utilize this additional mining power to subvert the reference string which potentially has a large benefit for the adversary. Secondly, even non-colluding rational block creators may be incentivised to use bad randomness which would reduce or remove any security benefits of the updates. Our work addresses both of these issues.

We prove formally that our mechanism produces a secure reference string by providing an analysis in the universal composition framework [5]. Furthermore, we demonstrate via experimental analysis how to concretely parameterise a proof-of-work ledger to ensure that an adversary which takes shortcuts (while honest users do not) will still fail in subverting the reference string. The concrete results provided in our experimental section can be used to inform the selection of parameters in order to run our reference string generation mechanism in live blockchain systems.

We further introduce an incentives scheme which ensures that rational participants in the protocol, who intend to maximise their profits, will avoid low-entropy attacks. In short, the incentive mechanism mandates that a random fraction of update contributors in the final chain will be asked to reveal their trapdoor, which will be verified to be the output of a random oracle by the underlying ledger rules. Only if a user can demonstrate that their update is indeed random do they receive a suitably determined reward for their effort. Careful choice of the reward assignment enables us to demonstrate that rational participants will utilise high entropy exponents, thus contributing to the SRS computation.

1.2 Related Work

Beyond the obvious relation to the works introducing updateable reference strings in [16, 22] (most notably Sonic [22], which we follow closely), there have been attempts of practically answering the question of how to securely generate reference strings. These have been in a setting where the string is *not* updateable.

Notably [2] describes the mechanism used by Sprout, the first version of Zcash, during the initial setup of the cryptocurrency's SRS. It uses multi-party computation to generate a reference string, with a root of trust on the initial group of people participating. Due to performance constraints on the MPC protocol, the set of parties participating is relatively small, although only the honesty of a single participating party is required.

For the Sapling version of Zcash, a different approach was used when their reference string was replaced (due to an upgrade of the zero-knowledge statement, and proof system used). Their second CRS generation mechanism, described

in [3] uses a multiple-phase round-robin mechanism to generate a reference string for Groth's zk-SNARK [15]. They utilise a random beacon to ensure the uniform distribution of the result, and a coordinator to perform deterministic auxiliary computations.

2 Updateable Structured Reference Strings

While updateable structured reference strings (uSRSs) are modelled in the works we are building on [22, Section 3.2], we model their security in the setting of universal composability (UC) [5]. Here, a uSRS is a reference string with an underlying trapdoor τ, which has had a structure function S imposed on it. $S(\tau)$ is the reference string itself, while τ is not revealed to the adversary. We will primarily focus on the Sonic uSRS, whose update proofs we modify slightly to facilitate straight-line extraction of the trapdoor. We demonstrate that given these changes it fits within our model.

A uSRS scheme consists of a trapdoor domain T, an initial trapdoor τ_0, a set P of permissible permutations over T (i.e. bijective functions whose domain and codomain is T), a structure function S, and a lifting operation $\dagger$, which transforms a permutation in P to a corresponding operation over the structured reference string, i.e. $\forall q \in P, \tau \in T : q^\dagger(S(\tau)) = S(q(\tau))$. The latter allows applying a permutation to a reference string directly instead of applying it to its trapdoor. Finally, there must exist algorithms $a \leftarrow \mathsf{GenSRS}(S(\tau), q)$ and $q^\dagger(S(\tau)) \leftarrow \mathsf{ApplySRS}(S(\tau), a)$ for generating and applying updates respectively. The format of these updates is not specified, however the following constraints must be met:

1. Applying an honestly generated update is equivalent to applying the input permutation in P to the trapdoor:
   $\forall q \in P, \tau \in T : q^\dagger(S(\tau)) = \mathsf{ApplySRS}(S(\tau), \mathsf{GenSRS}(S(\tau), q))$.
2. Applying *any* update is equivalent to either applying *some* permissible permutation on the trapdoor, *or* doing nothing:
   $\forall a, \tau : \exists q \in P \cup \{\mathsf{id}\} : \mathsf{ApplySRS}(S(\tau), a) = q^\dagger(S(\tau))$.
3. Applying a random permutation is equivalent to selecting a new random trapdoor: Let D be the uniform distribution over T, and for all $\tau \in T$, let D_τ be the uniform distribution over $\{ \; q(\tau) \mid q \in P \; \}$. Then $\forall \tau \in T : D = D_\tau$.

We define a corresponding UC functionality $\mathcal{F}_{\mathsf{uSRS}}$, which provides a reference string $S(\tau_{\mathcal{H}})$ which the adversary can influence by applying any permutation in P on $\tau_{\mathcal{H}}$, before it is first queried by an honest party. For $\mathcal{F}_{\mathsf{uSRS}}$ to be useful, it is typical for inverting S to be hard – specifically that given $S(\tau_{\mathcal{H}})$ for a random trapdoor $\tau_{\mathcal{H}}$, no PPT adversary can determine $\tau_{\mathcal{H}}$ with non-negligible probability. Given such a hardness assumption, $\mathcal{F}_{\mathsf{uSRS}}$ ensures that the adversary can neither retrieve the trapdoor τ of the final SRS, nor significantly constrain the resulting reference string's domain beyond rejection sampling.

Functionality $\mathcal{F}_{\mathsf{uSRS}}$

The updateable structured reference string functionality $\mathcal{F}_{\mathsf{uSRS}}$ allows the adversary to update a reference string by applying a permutation from a set of permissible permutations P.

The functionality is parameterised by a trapdoor domain T, a structure function S, and a set of permissible permutations P over T.

State variables and initialisation values:

Variable	Description
$\tau_{\mathcal{H}} := \bot$	The honest part of the trapdoor
$\tau := \bot$	The trapdoor

When receiving a message HONEST-SRS *from $\mathcal{A}$:*

if $\tau_{\mathcal{H}} = \bot$ **then**
  let $\tau_{\mathcal{H}} \xleftarrow{R} T$
return $S(\tau_{\mathcal{H}})$

When receiving a message SRS *from a party $\mathcal{P}$:*

query $\mathcal{A}$ **with** (PERMUTE, $\mathcal{P}$) and **receive the reply** q
if $\tau = \bot$ **then**
  assert $q \in P \wedge \tau_{\mathcal{H}} \neq \bot$
  let $\tau \leftarrow q(\tau_{\mathcal{H}})$
return $S(\tau)$

2.1 The Sonic uSRS

Sonic's uSRS [22, Section 4.3] consists of a series of exponentiations of group elements in pairing groups $\mathbb{G}_1$ and $\mathbb{G}_2$, where a bilinear pairing $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ exists. Specifically, given generators $g \in \mathbb{G}_1, h \in \mathbb{G}_2$ (where $\mathsf{ord}(\mathbb{G}_1) = \mathsf{ord}(\mathbb{G}_2) = p$) and a depth parameter $d \in \mathbb{Z}_p$, the SRS has a trapdoor of $(\alpha, x) \in \mathbb{F}_p^{*2}$ (with $\tau_0 = (1, 1)$).

The corresponding structure function is defined through:

$$S((\alpha, x)) = \left(\left\{ g^{x^i}, h^{x^i}, h^{\alpha x^i} \right\}_{i=-d}^{d}, \left\{ g^{\alpha x^i} \right\}_{i=-d, i\neq 0}^{d} \right)$$

Observe that field *multiplications* over α or x can efficiently be applied to the corresponding structure through exponentiation: $g^{(\alpha x^i)\beta y^i} = (g^{\alpha x^i})^{\beta y^i}$.

In order to permit UC-extraction, we use a low-level NIZK functionality $\mathcal{F}_{\mathsf{NIZK}}^{\mathcal{R}}$ (formally defined in Appendix B.2) to prove the knowledge of two group exponents for each update. The NP relation $\mathcal{R}_{\mathsf{srs}} \subset \mathbb{G}_1^2 \times \mathbb{F}_p^{*2}$ for this NIZK is:

$$\mathcal{R}_{\mathsf{srs}}((A, B), (a, b)) \iff A = g^a \wedge B = g^b$$

We omit the $e(g, h^\alpha)$ term presented in Sonic, as this can be computed from the rest of the SRS, and is therefore immaterial to the update procedure. The

permitted trapdoor permutations are field multiplications: $P := \{ (\alpha, x) \mapsto (\alpha\beta, xy) \mid (\beta, y) \in \mathbb{F}_p^{*2} \}$. Correspondingly, $\dagger$ exponentiates group elements: If $q = (\alpha, x) \mapsto (\alpha\beta, xy)$, then $q^\dagger = (\{G_i, H_i, H'_i\}_{i=-d}^d, \{G'_i\}_{i=-d, i\neq 0}^d) \mapsto \{G_i^{y^i}, H_i^{y^i}, H_i'^{\beta y^i}\}_{i=-d}^d, \{G_i'^{\beta y^i}\}_{i=-d,i\neq 0}^d\}$. The corresponding update procedure is as follows:

procedure $\mathsf{GenSRS}(\mathsf{srs}, q)$
  let $(\beta, y) \leftarrow q((1, 1))$
  send $(\text{PROVE}, (g^y, g^{\beta y}), (y, \beta y))$ **to** $\mathcal{F}_{\mathsf{NIZK}}^{\mathcal{R}_{\mathsf{srs}}}$ and
    receive the reply π
  let $\rho \leftarrow (g^y, g^{\beta y}, \pi)$
  return $(q^\dagger(\mathsf{srs}), \rho)$

The verification/application procedure ensures correct computation by checking the consistency of various pairing computations:

procedure $\mathsf{ApplySRS}(\mathsf{srs}, (\mathsf{srs}', \rho))$
  let $(\{G_i, H_i, H'_i\}_{i=-d}^d, \{G'_i\}_{i=-d,i\neq 0}^d) \leftarrow \mathsf{srs}$
  let $((\{I_i, J_i, J'_i\}_{i=-d}^d, \{I'_i\}_{i=-d,i\neq 0}) \leftarrow \mathsf{srs}'$
  let $(A, B, \pi) \leftarrow \rho$
  let $b \leftarrow 1$
  if $e(I'_1, h) \neq e(B, H'_1) \vee e(g, J'_1) \neq e(B, H'_1) \vee e(I_1, h) \neq e(A, H_1) \vee e(g, J_1) \neq e(A, H_1) \vee I_0 \neq g \vee J_0 \neq h$ **then**
    let $b \leftarrow 0$
  for $i = -d$ **to** d **do**
    if $\neg(i = d \vee e(I_i, J_1) = e(I_1, J_i) = e(I_{i+1}, h) = e(g, J_{i+1})) \vee \neg(e(I_i, J'_0) = e(g, J'_i)) \vee (i \neq 0 \wedge \neg e(I_i, J'_0) = e(I'_i, h))$ **then**
      let $b \leftarrow 0$
  send $(\text{VERIFY}, (A, B), \pi)$ **to** $\mathcal{F}_{\mathsf{NIZK}}^{\mathcal{R}_{\mathsf{srs}}}$ and **receive the reply** b_π
  if $b \wedge b_\pi$ **then return** srs'
  else return srs

3 Building SRS from Chain Quality

This section shows how to securely initialise a uSRS using a distributed ledger by requiring block creators to perform updates on an evolving uSRS during an initial setup period. After waiting for agreement on the final uSRS, it can be safely used. To formally model this approach, we discuss the ideal and real worlds used in our simulation proof. Both worlds have access to a ledger, however, the ideal world's ledger is independent of the reference string (which is instead provided by the independent $\mathcal{F}_{\mathsf{uSRS}}$ functionality), while the real world's ledger is programmed to generate it using updates.

3.1 Our Ledger Abstraction

Our construction of the updateable structured reference string functionality relies heavily on the properties of *common prefix*, *chain quality*, and *chain growth* defined in the "Bitcoin backbone" analysis by Garay et al. [10], for Nakamoto-style consensus algorithms. Despite our use in the section title, we make use of

all three properties, not just that of chain quality. We emphasise chain quality, as it is the property central to ensuring an honest update has occurred. We briefly and informally restate the three properties:

- Common prefix: Given the current chains Π_1 and Π_2 of two parties, and removing k blocks from the first, it is a prefix of the second: $\Pi_1^{\lceil k} \prec \Pi_2$.
- Chain quality: For any party's current chain Π, any consecutive l blocks in this chain will include μ blocks created by an honest party.
- Chain growth: If at any point in time, a party's chain is of length c, then s time slots later, it will be at least of length $c + \gamma$.

Chain growth + quality give liveness. However, that assumes that honest blocks pick up all transactions they can. The UC formalisation we give doesn't require this, and instead requires the (weaker) condition that transaction liveness holds, i.e. that transactions will be eventually included. I think this is a reasonable compromise. These parameters determine the length of the two phases of our protocol. In the first phase, we construct the reference string itself from the liveness parameter (assuming $\mu \geq 1$), and in the second phase, we wait until this reference string has propagated to all users. The length of the first phase is at least $\delta_1 \geq \lceil l\gamma^{-1} \rceil s$, and that of the second at least $\delta_2 \geq \lceil k\gamma^{-1} \rceil s$. Combined, they make up the total uSRS generation delay $\delta \geq (\lceil l\gamma^{-1} \rceil + \lceil k\gamma^{-1} \rceil)s$.

We assume a ledger which guarantees the backbone properties, formally described in Appendix A.1. While we do not demonstrate that any specific existing proof-of-work ledger (or those based on a different leader-selection mechanism) formally UC-realise this specific formalisation, we conjecture that all ledgers with "Nakamoto-style" (as opposed to BFT-style) consensus do so. Our functionality further depends on a *global clock* $\mathcal{G}_{\mathsf{clock}}$, defined in Appendix B.1. For the purposes of this paper, it is sufficient that this is a beacon providing monotonically increasing values representing the current time to any party requesting them.

In addition to this, we assume that each block created can contain additional information, provided by its creator (the "miner"), which can be aggregated to construct a "leader state". In the real world, each created block is associated with an *update* a, and the ledger is parameterised by two procedures, Gen, and Apply, which describe the honest selection of updates, and the semantics of updates respectively. Looking forward, these correspond closely to GenSRS and $\mathsf{ApplySRS}$, with added time-sensitivity: They do nothing after the setup period. Gen is randomized, takes a leader state σ and the current time t as inputs, and produces an update a. Apply takes a leader state σ, an update a, and an update time t, and returns a successor state σ': $\sigma' = \mathsf{Apply}(\sigma, (a, t))$. For a chain, the leader state may be computed by sequentially applying all updates in the chain, starting from an initial state $\varnothing$.

The adversary controls when and which party creates a new block, as well as the transactions each new block contains (provided it does not violate the backbone properties). For transactions created by a corrupted party, the adversary can further control the block's timestamp (within the reasonable limits of not being in the future, and being after the previous block), and the desired update a itself. For honest parties updates, Gen is used instead.

The UC interfaces our ledger provides are:

- SUBMIT – Submitting new transactions for the ledger.
- READ – Reading the confirmed sequence of transactions.
- PROJECTION – Reading the currently chain's sequence of (potentially unconfirmed) transactions.
- LEADER-STATE – Reading the confirmed leader state.
- ADVANCE – The adversary instructs a party to adopt a longer chain.
- EXTEND – The adversary instructs a party to create a new block.

3.2 The Ideal World

Our ideal world consists of two functionalities, composed in parallel (by which we mean: the environment may address either, and they do not interact). The first is a variant of $\mathcal{F}_{\mathsf{uSRS}}$, with the modification that it cannot be addressed by honest parties before δ time slots have passed. Formally, this modification is made with a wrapper functionality $\mathcal{W}_{\mathsf{delay}}(\mathcal{F}, \delta)$, described in Appendix B.4.

The second is the Nakamoto-style ledger functionality, parameterised with arbitrary leader-state generation and application procedures: $\mathsf{Gen} = \mathsf{GenIdeal}$ and $\mathsf{Apply} = \mathsf{ApplyIdeal}$, and the following ledger parameters:

1. A common prefix parameter k.
2. Chain quality parameters μ and l.
3. Chain growth parameters γ and s.

Formally then, our ideal world consists of the pair $(\mathcal{W}_{\mathsf{delay}}(\delta, \mathcal{F}_{\mathsf{uSRS}}), \mathcal{F}^{\mathsf{ideal}}_{\mathsf{nakLedger}})$, as well as the global functionality $\mathcal{G}_{\mathsf{clock}}$.

3.3 The Hybrid World

We operate in a hybrid world consisting of a separate Nakamoto-style ledger $\mathcal{F}^{\mathsf{real}}_{\mathsf{nakLedger}}$, a NIZK functionality $\mathcal{F}^{\mathcal{R}_{\mathsf{srs}}}_{\mathsf{NIZK}}$, and the global clock $\mathcal{G}_{\mathsf{clock}}$. Further, a uSRS scheme is used, with algorithms GenSRS and $\mathsf{ApplySRS}$, the structure function S, permissible permutations P, and an initial trapdoor τ_0. The ledger is then parameterised by the same chain parameters as those in the ideal world, and the following leader-state procedures:

function $\mathsf{Apply}((\mathsf{srs}, \sigma^{\mathsf{ideal}}), ((a^{\mathsf{srs}}, a^{\mathsf{ideal}}), t))$
  if $\mathsf{srs} = \varnothing$ **then let** $\mathsf{srs} \leftarrow S(\tau_0)$
  if $t \leq \delta_1$ **then**
    let $\mathsf{srs} \leftarrow \mathsf{ApplySRS}(\mathsf{srs}, \mathsf{a}^{\mathsf{srs}})$
  return $(\mathsf{srs}, \mathsf{ApplyIdeal}(\sigma^{\mathsf{ideal}}, a^{\mathsf{ideal}}, t))$

function $\mathsf{Gen}((\mathsf{srs}, \sigma^{\mathsf{ideal}}), t)$
  if $t > \delta_1$ **then**
    return $(\epsilon, \mathsf{GenIdeal}(\sigma^{\mathsf{ideal}}, t))$
  else
    let $q \xleftarrow{R} P$

$\textbf{return}\ (\mathsf{GenSRS}(\mathsf{srs}, q), \mathsf{GenIdeal}(\sigma^{\mathsf{ideal}}, t))$

Key here is that once a block is received after the initial chain quality period, any reference string update it may declare is no longer carried out – at this point the uSRS is not necessarily stable, as the chain may still be reorganised, but should not change for this particular chain. Further, these procedures always mimic the ideal-world behaviour, extending it rather than replacing it. This demonstrates the composability of allowing block leaders to produce updates: One system using updates for security does not impact other parallel uses.

There is little additional work to be done to UC-emulate the ideal-world behaviour, besides ensuring that queries are routed appropriately, especially on how the reference string is queried in the hybrid world. We describe this with a small "adaptor" protocol in Appendix A.2, which we refer to as LEDGER-ADAPTOR. This forwards most queries, and treats uSRS queries as querying the appropriate part of the leader state after time δ, and by ignoring them before.
Formally then, our real (or more precisely hybrid) world consists of the system LEDGER-ADAPTOR$(\delta, \mathcal{F}^{\mathsf{real}}_{\mathsf{nakLedger}}(\mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}}))$, as well as the global functionality $\mathcal{G}_{\mathsf{clock}}$.

4 Security Analysis

As for any UC proof, we require a simulator which ensures the ideal world behaves indistinguishably from the real world. Our simulator, $\mathcal{S}_{\text{LEDGER-ADAPTOR}}$, is formally described in Appendix A.3. Intuitively, this simulator ensures that the real and ideal world's ledgers are equivalent, and that the real world uSRS is equal to the uSRS produced in the ideal world.

In order to achieve this, the simulator ensures that the initial "honest" reference string provided by $\mathcal{F}_{\mathsf{uSRS}}$ is the basis of the uSRS of a simulated execution of the real-world protocol. Doing so relies primarily on three things: First, the simulator's ability to extract the trapdoor from any adversarial reference string update. Second, the simulator's ability to, given the adversarial trapdoors, then produce a valid "honest" update which ensures the reference string is a permutation of the ideal-world "honest" string. And finally, the simulator's knowledge that the final reference string in its simulation will have at least one honest update, in which it can apply this trick.

The simulator observes each of the competing chains in the ledger, and when the first honest update occurs in each, instead coerces the chain into a permutation of the ideal honest reference string. For each subsequent honest update, the simulator instead performs the update, remembering the randomness used. The result is that for each chain, the simulator either knows the *entire* trapdoor of the reference string (if there was no honest update), or all except for the first honest update. By the backbone properties, the simulator knows that the first case will not apply, and that only one prefix of valid updates will exist, after δ time has passed. As a result, the simulator knows exactly which permutation to apply to the honest ideal reference string to match the simulation's result.

As $\mathcal{F}_{\mathsf{uSRS}}$ only provides a single honest SRS, the simulator applies a random permutation to this for each initial honest update, ensuring that the updates of different chains remain unlinkable.

We will prove UC-emulation, and will therefore refer to the ideal and real worlds frequently throughout the proof. Beyond this, the simulator locally simulates the NIZK functionality and the ledger functionality. To be clear which functionality we are talking about at any point, we will use $\mathcal{F}^{\mathsf{ideal}}_{\mathsf{nakLedger}}$, $\mathcal{F}^{\mathsf{simul}}_{\mathsf{nakLedger}}$, and $\mathcal{F}^{\mathsf{real}}_{\mathsf{nakLedger}}$ to refer to the ideal, simulated, and real ledgers respectively. We refer to the real-world NIZK functionality as $\mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}}$, and the simulted NIZK as $\mathcal{S}_{\textsc{ledger-adaptor}}.\mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}}$. The notation $\mathcal{F}.x$ is used to mean "the variable x within the functionality $\mathcal{F}$" – it is also used to refer to the ideal trapdoor $\mathcal{F}_{\mathsf{uSRS}}.\tau_{\mathcal{H}}$.

The simulator needs to compute two separate items related to the SRS:

- Simulated honest updates a, given an honest reference string, and the combined prior trapdoors.
- The permutation applied to either $\tau_{\mathcal{H}}$ or τ_0, given a reference string.

For Sonic, we provide the following algorithms $\mathcal{X}_a$, and $\mathcal{X}_q$ below. $\mathcal{X}_a$ receives as inputs the trapdoor τ, the honest reference string srs, and access to the simulated NIZK functionality $\mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}}$. The latter is used to record the simulated proof necessary for a valid update.

procedure $\mathcal{X}_a((\alpha, x), \mathsf{srs}, \mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}})$
  $(\{G_i, H_i, H'_i\}^d_{i=-d}, \{G'_i\}^d_{i=-d,i\neq 0}) \leftarrow \mathsf{srs}$
  let $X \leftarrow (G_1^{x^{-1}}, G_1'^{x^{-1}\alpha^{-1}})$
  query $\mathcal{A}$ **with** $(\textsc{prove}, X)$ and
    receive the reply π,
    satisfying $\pi \neq \bot \wedge (X, \pi) \notin \mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}}.\overline{\Pi} \wedge (\cdot, \pi) \notin \mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}}.\Pi$, else
    sampling from $\{0,1\}^\kappa$
  let $\mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}}.\Pi \leftarrow \mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}}.\Pi \cup \{(X, \pi)\}$
  let $\rho \leftarrow (G_1^{x^{-1}}, G_1'^{x^{-1}\alpha^{-1}}, \pi)$
  return (srs, ρ)

Given a series of SRS updates, $\mathcal{X}_q$ computes the permutation applied to the reference string's trapdoor as far back as possible. It receives as inputs the sequence of updates $\vec{a}$, a mapping W from NIZK statements to corresponding witnesses (as far as the simulator knows them), and a mapping A from honest updates to the permutation applied to the honest SRS. It returns a permutation in P, which can be applied either to the initial trapdoor τ_0, or the initial honest trapdoor $\tau_{\mathcal{H}}$, to create the same SRS as the sequence of updates. When performing lookups in this map, we use equality modulo zero-knowledge proofs: Two updates with different proofs are considered the same in this instance. This ensures adversarially mauled proofs can be traced back to their honest counterpart.

procedure $\mathcal{X}_q(\vec{a}, W, A)$
  let $(\gamma, z) \leftarrow \tau_0$
  let $\mathsf{srs} = S(\tau_0)$
  for $a \in \vec{a}$ **do**

```
        let srs′ ← ApplySRS(srs, a)
        // Skip invalid updates
        if srs = srs′ then continue
        let srs ← srs′
        let (·, (A, B, π)) ← a
        if ((A, B), π) ∈ W then
            let (β, y) ← W(((A, B), π))
            let (γ, z) ← (γβ, zy)
        else
            // The update is honest.
            // Start with its permutation.
            assert a ∈ A
            let (γ, z) ← A(a)(τ0)
    return λ(β, y) : (βγ, yz)
```

Before we prove correctness of the simulation for Sonic, we validate that its updates satisfy the necessary constraints laid out in Section 2. We restate these properties in Lemmas 1-3, and prove them.

Lemma 1. *Applying an honestly generated update to a well-formed SRS is equivalent to applying the input permutation in P to the trapdoor:*
$\forall q \in P, \tau \in T : q^\dagger(S(\tau)) = \mathsf{ApplySRS}(S(\tau), \mathsf{GenSRS}(S(\tau), q))$.

Proof (for Sonic).
This follows from each of the checks being satisfied, and the correctness of $\mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}}$. □

Lemma 2. *Applying* any *update is equivalent to either applying* some *permissible permutation on the trapdoor,* or *doing nothing:*
$\forall a, \tau : \exists q \in P \cup \{\mathsf{id}\} : \mathsf{ApplySRS}(S(\tau), a) = q^\dagger(S(\tau))$.

Proof. Suppose a structured input $S(\tau) = (\{g^{x^i}, h^{x^i}, h^{\alpha x^i}\}_{i=-d}^{d}, \{g^{\alpha x^i}\}_{i=-d, i\neq 0}^{d})$, and an update $a = (\mathsf{srs}', \rho)$, where $\mathsf{srs}' = (\{g^{k_i}, h^{m_i}, h^{n_i}\}_{i=-d}^{d}, \{g^{l_i}\}_{i=-d,i\neq 0}^{d})$, and $\rho = (g^y, g^{\beta y}, \cdot)$.

$\mathsf{ApplySRS}$ will return either srs', or $S(\tau)$. In the latter case, structure is trivially preserved. In the former case, we know all of the following hold, due to the conditions checked in $\mathsf{ApplySRS}$:

- $e(g^{l_1}, h) = e(g, h^{n_1}) = e(g^{\beta y}, h^{\alpha x})$
- $e(g^{k_1}, h) = e(g, h^{m_1}) = e(g^y, h^x)$
- $\forall i \in [-d, d) : e(g^{k_i}, h^{m_1}) = e(g^{k_1}, h^{m_i}) = e(g^{k_{i+1}}, h) = e(g, h^{m_{i+1}})$
- $\forall i \in [-d, d] : e(g^{k_i}, h^{n_0}) = e(g, h^{n_i})$
- $\forall i \in [-d, d] \setminus \{0\} : e(g^{k_i}, h^{n_0}) = e(g^{l_i}, h)$

As $e(g, h)$ is a generator over $\mathbb{G}_T$, and each of the above can be expressed as an equality of exponentiations of the form $e(g, h)^a = e(g, h)^b$, this we simplify these to equalities within $\mathbb{F}_p^*$ of their exponents:

- $l_1 = n_1 = \alpha\beta xy$
- $k_1 = m_1 = xy$
- $\forall i \in [-d, d) : k_i m_1 = k_1 m_i = k_{i+1} = m_{i+1}$
- $\forall i \in [-d, d] : k_i n_0 = n_i$
- $\forall i \in [-d, d] \setminus \{0\} : k_i n_0 = l_i$

From the above follows directly that $n_0 = \alpha\beta$, $k_i = m_i = (xy)^i$, and $l_i = n_i = \alpha\beta(xy)^i$. As a result, srs' matches exactly the structured reference string $S((\alpha\beta, xy)) = q^\dagger(S(\tau))$. The update has preserved structure. □

Lemma 3. *Applying a random permutation is equivalent to selecting a new random trapdoor: Let D be the uniform distribution over T, and for all $\tau \in T$, let D_τ be the uniform distribution over $\{ q(\tau) \mid q \in P \}$. Then $\forall \tau \in T : D = D_\tau$.*

Proof (for Sonic). As each well formed SRS has a trapdoor, if the trapdoor of srs is (α, x), GenSRS will sample randomly from $\mathbb{F}_p^*$ two values β and y, such that the outputted SRS is the structure function applied to $(\alpha\beta, xy)$. Due to multiplication in prime fields with a fixed element (here α and x) being a bijective functions the result $(\alpha\beta, xy)$ is also distributed uniformly at random in $\mathbb{F}_p^{*2}$, therefore being indistinguishable from a new, randomly sampled trapdoor. □

We additionally prove the following auxiliary lemma that will be used in the proof of our main theorem.

Lemma 4. *In the ideal-world execution of $\mathcal{S}_{\text{LEDGER-ADAPTOR}}$, $\mathcal{X}_q(\vec{a}, W, A)$ does not abort, and outputs a permutation $q \in P$, such that its inverse, applied to the underlying trapdoor of the SRS generated from the given sequence of updates $\vec{a}$, is either the initial trapdoor τ_0, or the honest trapdoor $\tau_{\mathcal{H}}$.*

Proof (for Sonic). $\mathcal{X}_q$ has no abort conditions. Furthermore, the output is a pair of field multiplications over $\mathbb{F}_p^*$, which is a permutation in P. The permutation applied corresponds directly to how the underlying trapdoor of the uSRS is updated by longest suffix of updates in $\vec{a}$ for which the trapdoor is known – i.e. the NIZK proofs contained in the updates have a witness in W or a permutation of the honest trapdoor is recorded in A. When this isn't the case, the update is skipped, and the trapdoor reset, ensuring that any trapdoors preceeding a non-extractable value are ignored. The case that the trapdoors are known for *all* of the updates is trivial; as by definition inverting this permutation will result in the initial trapdoor τ_0.

If, however, at any point the witness for an update is not known, at this point the trapdoor must be honestly generated. The reasoning for this is straightforward: As this update was not skipped, the NIZK proofs embedded in it must verify. The only way for the proofs to verify, and the NIZK functionality *not* to have recorded the corresponding witnesses, however, it that the simulator added the proof manually to the NIZK's set of valid proofs. This only happens at one point – in the update algorithm $\mathcal{X}_a$, which is used *only* for random permutations

applied to the honest reference strings. Further, only one such instance may occur in each chain – this stems from the fact that witness-less SRS updates are only carried out by the simulator if it can fully extract the trapdoor of a series of updates. As this happening by accident – or the adversary re-applying an honest update elsewhere – implies breaking the hardness of the structure function (specifically, breaking DLOG), we can say with overwhelming probability that at most one witness-less update is in each sequence of updates.

Finally, we note that for this witness-less update, the remaining trapdoor defines a permutation of $\mathcal{F}_{\mathsf{uSRS}}.\tau_{\mathcal{H}}$. Algorithm $\mathcal{X}_q$ extracts the trapdoors from all subsequent updates to compute the permutation applied to this honest trapdoor – ensuring precisely that inverting this permutation results in $\mathcal{F}_{\mathsf{uSRS}}.\tau_{\mathcal{H}}$. $\square$

Theorem 1. *For any updateable reference string scheme* $\mathsf{GenSRS}, \mathsf{ApplySRS}, S, T, \tau_0$*, with algorithms* $\mathcal{X}_q$ *(satisfying Lemma 4) and* $\mathcal{X}_a$ *(which simulates honest updates),* LEDGER-ADAPTOR *(in the* $(\mathcal{F}^{\mathsf{real}}_{\mathsf{nakLedger}}, \mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}})$*-hybrid world, parameterised as in Subsection 3.3) UC-emulates the pair of functionalities* $(\mathcal{F}^{\mathsf{ideal}}_{\mathsf{nakLedger}}, \mathcal{W}_{\mathsf{delay}}(\delta, \mathcal{F}_{\mathsf{uSRS}}))$*, parameterised as in Subsection 3.3, with the simulator* $\mathcal{S}_{\text{LEDGER-ADAPTOR}}$*, in the presence of the global clock functionality* $\mathcal{G}_{\mathsf{clock}}$*.*

Proof. If the environment can distinguish between these worlds, there must exist a minimal series of interactions the environment, combined with its adversary, can make to cause the other UC ITMs to behave sufficiently differently to allow distinguishing. We will show that for any interaction the environment makes, it will not learn enough information to distinguish the two worlds, and therefore that across *all* (polynomially many) interactions it also cannot distinguish. First, we consider what actions the adversary/environment pair can take. The interactions fall into the following categories:

1. Honest or adversarial SUBMIT, READ, LEADER-STATE, or PROJECTION queries.
2. Interactions with $\mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}}$, or $\mathcal{G}_{\mathsf{clock}}$.
3. ADVANCE queries.
4. EXTEND queries.
5. SRS queries.

We will establish the following invariants throughout the execution of the UC security game:

- $\mathcal{G}_{\mathsf{clock}}$ has the same internal state in both the real and ideal worlds.
- $\mathcal{S}_{\text{LEDGER-ADAPTOR}}.\mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}}$ has the same internal state as the real-world $\mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}}$, except that it does not know the witnesses for honestly generated proofs or their mauled variants.
- $\mathcal{S}_{\text{LEDGER-ADAPTOR}}.\mathcal{F}^{\mathsf{simul}}_{\mathsf{nakLedger}}$ has the same internal state as the real-world ledger $\mathcal{F}^{\mathsf{real}}_{\mathsf{nakLedger}}$, and differs from the ideal-world ledger $\mathcal{F}^{\mathsf{ideal}}_{\mathsf{nakLedger}}$ only in that all state updates contain an addition SRS update term.

Ledger reads and submissions. Given these invariants, it is clear that the environment cannot distinguish between the results of READ and PROJECTION queries – they must return the same value! Further, as the adaptor protocol strips the SRS component from the leader state, and the ideal world's leader state is precisely defined as being without this component, it is clear that also LEADER-STATE queries will be indistinguishable (even if made directly by the adversary, since these are answered by $\mathcal{F}^{\mathsf{simul}}_{\mathsf{nakLedger}}$). For transaction submissions by either the environment or the adversary, both worlds will add the transaction, with the current timestamp, to their ledger's submitted transactions, and will notify the adversary *once*, and return the transaction together with the timestamp. This does not reveal any information to the environment which could be used to distinguish.

Queries to other functionalities. Likewise, $\mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}}$ queries clearly will not permit the environment to distinguish, or invalidate the above mentioned invariants – they do not go beyond the NIZK functionality, and this does not read – only update – the witness map. Similarly for $\mathcal{G}_{\mathsf{clock}}$, as this exists in both worlds, and is not manipulated by the simulator (or any other entity), beyond read-only operations, it will behave identically.

ADVANCE *queries.* The simulator first simulates advancing a specified party's ledger state on $\mathcal{F}^{\mathsf{simul}}_{\mathsf{nakLedger}}$. If this succeeds, the simulator knows that the advancement will succeed in the ideal world as well, where the ledger state is less constrained. It removes the SRS updates from the ledger state being switched to, and issues a corresponding advance query to $\mathcal{F}^{\mathsf{ideal}}_{\mathsf{nakLedger}}$. If the simulated ADVANCE does not succeed, it will also have failed in the real world execution, both of which will abort. If the update succeeds, the invariant between the various ledger states is preserved – up to the lack of SRS updates in the ideal world, they are the same. If the update fails, both worlds terminate execution.

$(\text{EXTEND}, \mathcal{P}, B, t, a)$ *queries.* Let us first detail what EXTEND queries do. Called by the adversary, if the party parameter $\mathcal{P}$ represents an honest party, the query runs Gen to generate a new update a to apply to this party's view of the leadership state. If the party is adversarial on the other hand, an adversary-supplied update parameter a is used instead. With the timestamp t (or the accurate time for honestly created blocks), block content B, state update a, and a randomly sampled ID, a new block is created, and appended to $\mathcal{P}$'s projected chain. Finally, it is asserted that the common prefix property still holds.

Once the simulator intercepts such a query, it needs to ensure not only that the same EXTENDs are carried out in the simulated and ideal ledgers, but also that honest SRS updates are (when necessary) sourced from the $\mathcal{F}_{\mathsf{uSRS}}$ functionality. In the case that the party extending the chain is adversarial, this is simple – split the adversarial real world update a into an SRS update and an ideal-world update (it is worth noting that these need not be valid), and forward only the ideal-world update in an EXTEND query to $\mathcal{F}^{\mathsf{ideal}}_{\mathsf{nakLedger}}$. This already results in the real and ideal ledgers satisfying the invariant, leaving the simulated ledger. For this, the simulator manually inserts the ID returned from the ideal-world ledger,

inserts the new block, and asserts the same common prefix condition as the real world does, ensuring these two ledgers are in the same state and – crucially – abort under the same conditions. The returned value is identical to that returned in the real world.

For honest updates, things are more complex. If current time is after when honest SRS updates are performed, the honest SRS update is set to ϵ, as in the real world. Otherwise, the SRS is reconstructed from the party's current projected ledger view, and the simulator attempts to extract the trapdoor permutation from this SRS. If it succeeds in extracting the entire trapdoor, the simulator ensures it is updated such that it can no longer do so: It updates the uSRS to a permutation of the honest uSRS supplied by $\mathcal{F}_{\mathsf{uSRS}}$, by first applying a fresh permutation to it, recording this in a map of such permutations A, and extracting the corresponding update using $\mathcal{X}_a$.

By Lemma 3, this is indistinguishable from the result of Gen, which the environment expects. In case the full trapdoor cannot be extracted, Gen is used to generate the "honest" SRS update, ensuring the simulator knows the trapdoor for this update as well (as it retains the NIZK witness used). Finally, the ideal ledger is sent an EXTEND query, with a^{ideal} set to $\bot$. Execution proceeds as in the adversarial case, with the SRS part of the update being distributed equally in the real and simulated ledgers, and the ideal-world component being generated directly by the ideal world functionality (and therefore also being distributed the same as in the real world, which sampled from the same distribution).

SRS *queries.* Finally, a user may query the SRS. If this happens before time δ, both worlds return $\bot$ – the delay wrapper does so in the ideal world, and the adaptor protocol does so in the real world. Otherwise, the real world reconstructs the leadership state, and returns only the SRS component, while the ideal world queries the simulator for a trapdoor permutation, and, if the SRS is not yet finalised, applies it to the honest SRS.

Recall that after every extension, $\mathcal{F}_{\mathsf{nakLedger}}$ ensures that the common prefix property holds. Further, once a party's projected ledger state has some common prefix, this is only ever extended – either by extending the whole projection (in EXTEND), or by switching to a different one with the same prefix (in ADVANCE). After time δ, if chain quality and liveness hold, we can split each party's projected chain into two parts: Blocks with a timestamp at or before the time δ_1, and those with a timestamp after it. As EXTEND enforces timestamps to be monotonically increasing, these concatenate to form the entire chain. By the liveness property, and as it is at least time δ, we know that the first part contains at least l blocks, and the second at least k blocks. Chain quality, ensures that the first part contains at least μl honest blocks, while Apply ignores updates with a timestamps after δ_1. Combined, these facts imply that, for any party, the valid SRS updates, taken from their stable chain, are identical.

After the first SRS query, both the ideal and real worlds will not change what value they return, the former because it has then recorded the final trapdoor, and the latter because the common prefix containing valid reference string updates cannot change. The first query is therefore the most interesting.

From Lemma 4, we know that the permutation q extracted by the simulator when it is queried for the SRS permutation will, inverted and applied to the SRS' underlying trapdoor, either result in τ_0, or $\mathcal{F}_{\mathsf{uSRS}}.\tau_{\mathcal{H}}$. From the above we know that the SRS the simulator is extracting from matches that which honest parties generate – containing at least one honest update (by chain quality). As the first honest update in any chain is extracted from a $\mathcal{F}_{\mathsf{uSRS}}$-provided reference string, (and, by Lemma 1, it *is* valid) it cannot be τ_0. Therefore, the simulator, by providing q to $\mathcal{F}_{\mathsf{uSRS}}$, satisfies its requirements of a permissible permutation in P, and ensures that once the permutation is applied, the same SRS is returned: $S(q(\tau_{\mathcal{H}})) = S(q(q^{-1}(\tau))) = S(\tau)$.

In the above we have brushed aside the issue of aborts, however these are also simple to deal with. $\mathcal{F}_{\mathsf{uSRS}}$ aborts if given an invalid permutation, which the simulator does not do. In the real world, if liveness or chain quality are violated, $\mathcal{F}_{\mathsf{nakLedger}}$ aborts. In each query, the simulator ensures that the same query is run against the simulated ledger, ensuring that both will abort under the same conditions. This is the primary purpose for which $\mathcal{F}_{\mathsf{uSRS}}$ asks for a permutation on each invocation, despite only using it on the first, as well as why it supplies the identity of the calling party. □

As it is possible to construct (non-succinct) non-interactive zero-knowledge from a random oracle, we can remove the requirement on $\mathcal{F}_{\mathsf{NIZK}}^{\mathcal{R}}$ and instead rely on a random oracle $\mathcal{F}_{\mathsf{RO}}$ (formally described in Appendix B.3).

Corollary 1. *For any updateable reference string scheme* GenSRS*,* ApplySRS*, with algorithm $\mathcal{X}_q$ (satisfying Lemma 4) and $\mathcal{X}_a$ (which extracts valid updates), it is possible to realise the pair of functionalities* $(\mathcal{F}_{\mathsf{nakLedger}}^{\mathsf{ideal}}, \mathcal{W}_{\mathsf{delay}}(\delta, \mathcal{F}_{\mathsf{uSRS}})$ *in the* $(\mathcal{F}_{\mathsf{nakLedger}}^{\mathsf{real}}, \mathcal{F}_{\mathsf{RO}})$*-hybrid world, and in the presence of* $\mathcal{G}_{\mathsf{clock}}$*.*

Proof. By Theorem 1, we already know this realisation is possible in the $\mathcal{F}_{\mathsf{NIZK}}^{\mathcal{R}}$ hybrid world. We can employ Fischlin's transform [8] in combination with a simple sigma protocol to prove knowledge of pairs of exponents. A simple option for this proof is a parallel composition of two Schnorr proofs of knowledge of exponent [25]. It is important that these are a single proof, and not two separate proofs of knowledge of exponent, as the latter would enable the adversary to create proofs which are only partially extractable. We posit that these would still allow for simulation, however the simulator would be tasked with a more difficult, and implementation specific book-keeping. □

5 Implementation and Parameter Selection

We have implemented [6] the Sonic update mechanism, and using this provide performance estimates for SRS generation in a live blockchain network. Further, we simulate the optimal adversarial attack strategy, and demonstrate how this may be used to select optimal parameters for the secure generation of reference strings. We demonstrate that for currently typical applications, these parameters are practical for real-world usage.

While we have not modified a full blockchain client to utilise this extended consensus, we discuss the impact it would have on each of the following points:

- block verification
- block generation
- chain reorganisation
- network usage
- local storage

While the bitcoin backbone paper [10] provides bounds on chain parameters in given situations, these have three main drawbacks in the context of this paper:

1. The bounds are not tight.
2. The criteria for security is stricter than required: It asserts liveness and persistence are never violated, while this paper only requires them in a few select cases.
3. The analysis is in the synchronous model – while the generation and verification of reference strings can take a significant amount of time.

To obtain parameters to practically generate reference strings, we measure the time taken for computing and verifying updates, and factor this processing overhead into a simulation of the optimal adversarial strategy to subvert the reference string generation procedure.

The implementation and numbers provided for execution time and storage use the commonly used BLS12-381 curve pair. Circuits which have been practically deployed tend to require a depth in the hundreds of thousands; less than half a million typically, so we will often assume a uSRS depth of 500,000. All data points shown are available at [6], and may be reproduced with the provided source code.

5.1 Execution Time of uSRS Operations

We tested our implementation of the uSRS generation mechanism on a AMD Ryzen 7 2700X 8-core processor with hyper-threading enabled. This processor is a standard consumer-grade CPU – in proof-of-work mining it is likely that miners will have access to better hardware. All operations have been parallelised, and the verification operation has been additionally optimised to use less pairing operations. The workload, especially for uSRS generation, is also highly parallelisable (consisting of primarily a large number of group exponentiations), suggesting further improvements by utilising GPUs and clusters of machines are possible. If such improvements are applied, the total time *delay* required for the secure generation procedure, as well as the optimal intended block time could be reduced proportionally to the increase in parallelisation; assuming paralellisation across 10 machines could reduce both by an order of magnitude, for instance.

We measured the time taken for create and verify a uSRS update in relation to the uSRS depth in Figure 1. We measure the overhead of the Fischlin

proofs discussed in Corollary 1, amounting to 23.956ms for proving and 1.567ms for verifying (a Fiat-Shamir proof of the same type was measured to 0.921ms and 0.870ms respectively), using SHA-3 in place of a random oracle. For larger dimensions of reference strings, neither have much impact on the total runtime.

Finally, we implemented *aggregate updates*: The bulk of Sonic's update verification procedure is concerned with verifying the structure of the reference string, while a few parts of it verify that it is an exponentiation of the previous string. By retaining only the latter parts, a series of updates can be verified almost as quickly as a single update. The verification of aggregate proofs has an overhead of 1.634ms per update included in the aggregate. The bulk of this cost arises from the verification of the Fischlin proof. This allows for even large chain reorganisations to be quickly verified.

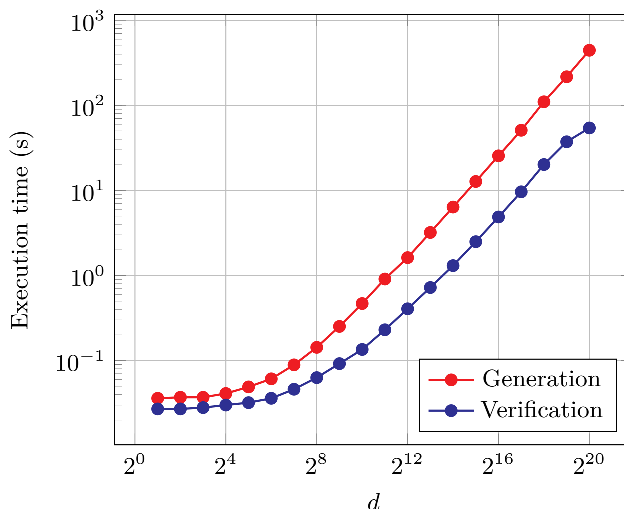

Fig. 1. The time taken to produce and verify uSRS updates.

5.2 Simulating the Optimal Attack Strategy

The mechanism we have presented in this paper operates in two phases. In the first phase, the adversary has the chance to *subvert* the reference string, while in the second phase it can carry out a denial of service attack, potentially convincing users that an incorrect (but not subverted) reference string is the canonical one.

For the first phase, the adversary's optimal strategy is to mine entirely independently from any honest activity. The reason for this is straight-forward: the adversary cannot adopt any honest block – doing so would break the subversion of its reference string. Further, the adversary has no reason to share any of its own blocks except if it reached the threshold of having a fully valid

subverted reference string – it only gives the honest network a chance to catch up, in the case that the adversary is ahead. This allows for a straightforward simulation of the consensus protocol: The probability of either honest parties, or the adversary creating an individual block is exponentially distributed. In addition to this, honest parties have a fixed processing overhead before they may start mining: This may include a networking delay, but more crucially it includes the time taken to verify a newly received block's uSRS update, and to produce the subsequent update. We assume that the adversary can bypass large parts of this overhead, by virtue of network dominance, by skipping verification, and by producing reference string updates with small (and therefore insecure) exponents.

The overhead manifests as shifting the honest party's exponential distribution for block generation by a constant factor. More precisely, we parameterise each experiment by:

- The intended time between blocks b
- The combined networking, verification, and update overhead d
- The fraction of adversarial mining power α

Of these three, d can be seen as fixed, depending on the depth of the uSRS being generated, and the corresponding speed of verification and update generation. For simplicity, we assume a uSRS depth of 500,000, which corresponds to d being approximately 250 seconds on our single-CPU setup.

We draw the time of the next adversarial block from the exponential distribution with $\lambda = \alpha/b$, and the next honest block from the exponential distribution with $\lambda = (1-\alpha)/b$, shifted to the right by d. The simulation is then advanced to the lesser of the two times, which is resampled from the same distribution. The number of times the adversary or the honest parties have extended their chain is counted, and the honest parties win at any point if and only if the honest chain is longer than the adversarial chain.

We run one million experiments in parallel, either up to a fixed end time, or until a large enough fraction of the experiments end in honest victory. We refer to the probability of an adversarial success as the probability of subversion ϵ. In Figure 2, we demonstrate that for a fixed d, a tradeoff exists between the target time between block b, and the time until any given subversion threshold ϵ is met.

A practical limit of this simulation approach is that it cannot by itself determine the length of time needed to wait until ϵ is negligible for most typical security parameters. We can however observe that for fixed parameters, ϵ decreases approximately exponentially as time passes, as seen in Figure 3, outside of a brief initial window.

While the second phase – that where the adversary attempts to create disagreement as to which reference string is the canonical one – may initially seem different, its optimal strategy is identical, as it essentially wishes to create as long as possible a fork, starting one block prior to the end of the first phase (to select a different reference string). As creating the longest fork *forking at this point* does not allow the adversary to accept honest blocks after it, nor gives

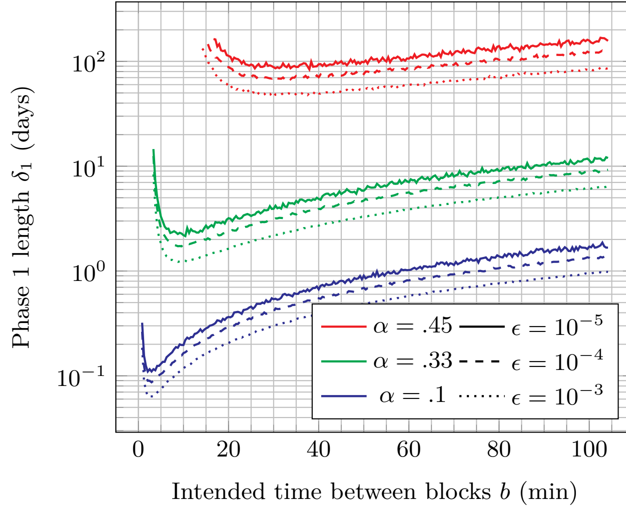

Fig. 2. The time required to generate a secure uSRS, as a function of the intended time between blocks. This depends on the proportion of adversarial mining power a, and the bound ϵ on the probablity of subversion. Each data point represents the time until at most a fraction of ϵ of one million parallel experiments ended in adversarial victory. Values are given assuming $d = 250s$, and both axes scale linearly to d.

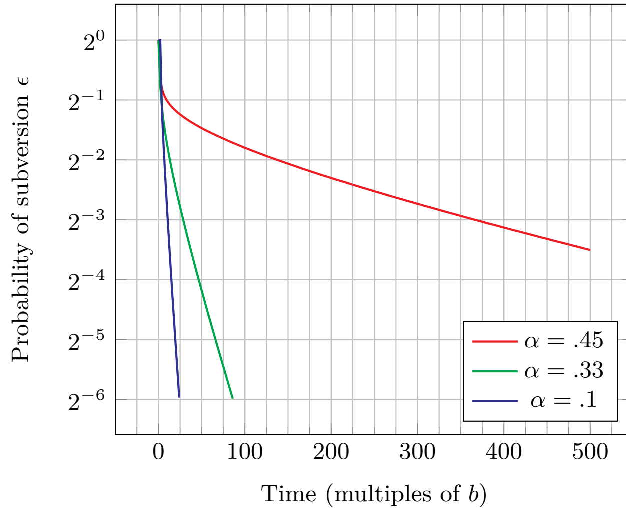

Fig. 3. The probability of the reference string being subverted ϵ, as a function of the time passed, in multiples of the intended time between blocks b. This depends on the proportion of adversarial mining power α, and the compound overhead d. b is selected to be approximately at the minimum seen in Figure 2, with $d = .15b$, $d = .4b$, and $d = 2b$ for the $\alpha = .45$, .33, and .1 respectively.

the adversary a reason to share its blocks, the adversarial strategy – and this analysis – is the same.

5.3 Storage and Network Usage

A reference string consists of $4d + 1$ elements in $\mathbb{G}_1$ and $4d + 2$ elements in $\mathbb{G}_2$. For the commonly used BLS12-381 curve pair, $\mathbb{G}_1$ elements have a storage requirement of 48 bytes each, and $\mathbb{G}_2$ elements of 96 bytes each. An update by itself includes an additional two $\mathbb{G}_1$ elements, and a Fischlin proof, which itself consists of twelve iterations, each with 2 elements in $\mathbb{F}_p^*$ (each of which requires 32 bytes to store), two elements of $\mathbb{G}_1$, and a 16-bit nonce. Each part of an aggregate update has an additional two $\mathbb{G}_2$ elements.

As it is not necessary to retain intermediate reference strings, and aggregate updates are sufficient, for a chain of length l, and with an uSRS depth of d, this is a storage requirement of $576d + 288$ bytes for the uSRS itself, and $l \cdot (2 \cdot 48 + 2 \cdot 96 + 12 \cdot (2 \cdot 32 + 2 \cdot 48 + 2)) = 2,232l$ bytes for storing updates.

For 500,000 gates and chains of length 20,000, this corresponds to a total storage requirement of 318MiB, with the reference string itself being the largest part, at 275MiB.

Although this is quite manageable as a storage requirement, it must be considered that the SRS itself (and a single update of around 2KiB) has to be re-transmitted with each block. While at the common home-internet upload speed of 10Mb/s, a block would take slightly under 4 minutes to transmit, it is reasonable to assume that miners would invest in high-grade connections to offset the change of their block being replaced with a competitors. Speeds up to 10Gb/s are commercially available, which would reduce the transmission time to under a second.

One remaining issue is that of denial-of-service. The receipt and verification of a reference string is costly, and should therefore only be done *after* a blocks proof-of-work has been received, which should depend on a commitment to the subsequently sent reference string – such as the update proof itself. An attacker can still perform a limited denial of service attack with blocks they legitimately mined – however this uses no more resources in verification than a legitimate block would.

5.4 Conclusion

Figure 2 provides insight into the space of tradeoffs which can be made for the secure generation of reference string. While the secure generation of a reference string is possible even for a small honest majority, the time required to do so is much higher than for a more relaxed setting, with δ_1 being approximately three *months* for $\alpha = .45$, in contrast to around two *days* for $\alpha = .33$. The full setup is double this: six months for $\alpha = .45$, and four days for $\alpha = .3$. Perhaps surprisingly, the probability of subversion ϵ has a more muted effect on the required setup time, increasing it approximately logarithmically.

The minima observed for δ_1 suggest that simply deploying this system on existing blockchain systems as they are currently parameterised is unwise: Most blockchains emphasise small values of b to enable transactions to settle quickly, with even notoriously slow chains such as Bitcoin having values on the lower end of our scale. This is directly linked to the compound overhead of verification and update generation – when b is small, the adversary can better use it's advantage of bypassing large parts of the verification and update procedure. As previously noted, there is a lot of room for speedup by assuming miners use greater computation power – if each miner used ten machines, even the $\alpha = .45$ case would be reduced to under a month in total.

6 Discussion

Our analysis in the previous section is based on the assumption that parties either behaving completely honest or completely adversarial. There is no gray area. While such an analysis for the generation of a new reference string from a ledger protocol is itself useful, real-world situations are likely to be more complex, and may evolve dynamically over time. In this section we discuss practical adjustments that may be made.

6.1 Dealing with Low-Entropy Exponent Attacks

While our analysis indicates that in a Byzantine, honest majority setting, our protocol produces a trustworthy reference string, it also asks participants to dedicate computational resources to updates. It follows that in a rational setting, players need to be properly incentivized to follow the protocol.

This is palpable since there is a protocol deviation that breaks the security of the reference string. Specifically, by choosing the exponent in a specific low-entropy fashion, (e.g., $y = 2^l$) the computation of the update, which primarily relies on repeated squaring, can be done significantly faster. In more detail, instead of using a random permutation q, a specific choice of (β, y) is made that eases the computation of srs' – in the most extreme cast, for $(\beta, y) = (1, 1)$, the update is trivial.

In order to facilitate a mitigation for this class of attacks, we will need to assume an additional property of the underlying ledger, in particular it must provide a "resettable" randomness beacon: With each ADVANCE operation (where adversary must be restricted in how often it may do such ADVANCE queries), a random beacon value is sampled in a variable bcn and is associated with the corresponding block. Prior work [7] demonstrates that such beacon values allow for the adversary to bias them only by "resetting" it at most a certain number of times, say t, before they are fixed by entering the ledger's confirmed state, with the exact value of t depending on the chain parameters.

We can then amend GenSRS to derive its random values from the random oracle, by sending the query $(\mathsf{bcn}, \mathsf{nonce})$ to $\mathcal{F}_{\mathsf{RO}}$, where nonce is a randomly selected nonce, and bcn is the previous block's beacon value. The response is

interpreted as a trapdoor permutation (β, y) to apply to the reference string, and the nonce is stored by miners locally, and kept private. We adapt the phase 1 period δ_1 so that at least $l' := l(1-\theta)^{-1} + c$ blocks will be produced, where θ and c are new security parameters (to be discussed below). Next, after phase 2 ends, we can be sure that the beacon value associated with the end of phase 1 has been reset at most t times.

We extract from bcn l' biased coins, each with probability θ (e.g., if a bcn is a random hash value of length 256 we can obtain as many coins with no bias). For each block, if the corresponding coin is 1, it is required to reveal its randomness within a period of time equal to the liveness parameter. Specifically, a party which created one of the selected blocks may reveal its nonce. If its update matches this nonce, the party receives an additional reward of value R times the standard block reward.

Choosing c to be sufficiently large, with overwhelming probability, at least l blocks will be left unopened – one of which will be honestly produced even against a minority Byzantine adversary. This does not reduce to the standard chain-quality property, but rather a stronger (but equally plausible) variant: for any subsequence of at least length l of some given chain, any randomly sampled subset of at least l blocks must contain at least one honest block with overwhelming probability. In a UC formalisation, this sampling process would be exposed, allowing sampling with a specific block's bcn value. The functionality would abort in case no honest parties' blocks were returned. The negation of the sampled set is the set that will be asked to be opened.

Consider now a rational miner with hashing power α. We know that, at best, using an underlying blockchain like Bitcoin, the relative rewards such a miner may expect are at most $\alpha/(1-\alpha)$ in expectation; this assumes a selfish mining strategy that wins all network races against the other honest participants. Now consider a miner who uses low entropy exponents to save on computational power on created blocks and, as a result, boosts their hashing power α to an increased relative hashing power of $\alpha' > \alpha$. The attacker can further try to influence the blockchain by forking and selectively disclosing blocks which has the effect of resetting the bcn value to a preferred one. To see that the impact of this is minimal, we prove the following easy lemma.

Lemma 5. *Consider a mapping $\rho \mapsto \{0,1\}^{l'}$ that generates l' independent biased coin flips, each with probability θ, when ρ is uniformly selected. Consider any fixed $n \leq l'$ positions and suppose an adversary gets to choose any one out of t independent draws of the mapping's random input with the intention to increase the number of successes in the n positions. The probability of obtaining more than $n(1+\epsilon)\theta$ successes is $\exp(-\Omega(\epsilon^2\theta n) + \ln t)$.*

Proof. In case $t = 1$, result follows from a Chernoff bound on the event E defined as obtaining more than $n(1+\epsilon)\theta$ successes, and has probability $\exp(-\Omega(\epsilon^2\theta n))$. Given that each reset is an independent draw of the same experiment, by applying a union bound we obtain lemma's statement. □

The optimal strategy of a miner utilising low-entropy attacks is to minimise the number of blocks of other miners are chosen, to increase its relative reward. Lemma 5 demonstrates that at most a factor of $(1-\epsilon)$ damage can be done in this way. Regardless of whether a miner utilises low-entropy attacks or not, their optimal strategy beyond this is selfish mining, in the low-entropy attack mining in expectation $l'\alpha'/(1-\alpha')$ blocks [10]. A rational miner utilising low-entropy attacks will not gain any additional rewards, while a miner not doing so will gain at least $l'\alpha/(1-\alpha)(1-\epsilon)\theta R$ rewards from revealing their randomness, by Lemma 5. It follows that for a rational miner, this strategy can be advantageous to plain selfish mining only in case:

$$\frac{\alpha'}{1-\alpha'} > (1+\theta(1-\epsilon)R)\frac{\alpha}{1-\alpha}$$

If we assume a miner can increase their effective hash rate by a factor of c, using low-entropy exponents, then their advantage in the low entropy case is $\alpha' = \alpha c/(\alpha c + \beta)$, where $\beta = 1 - \alpha$ is the relative mining power of all other miners. If follows that the miner benefits if and only if:

$$\begin{aligned} \frac{\alpha c}{\alpha c + \beta} \cdot \frac{\alpha c + \beta}{\beta} &> (1+\theta(1-\epsilon)R)\frac{\alpha}{\beta} \\ \implies c &> (1+\theta(1-\epsilon)R) \end{aligned}$$

If we adopt a sufficiently large intended time interval between blocks it is possible to bound the relative savings of a selfish miner using low-entropy exponents; following the parameterisation of Subsection 5.2, if a selfish miner using such exponents can improve their hashing power by at most a multiplicative factor $\chi \in (0,1)$ then we can mitigate such attack by setting R to $(\chi\theta(1-\epsilon))^{-1}$.

6.2 Bootstrapping Private Ledgers

In creating a new ledger which requires a secure SRS – for instance to use zk-SNARKs for privacy-preserving features, it is crucial to note that **the reference string should not be used until time δ.**

Before this time, it is not secure, and may allow the proving of false statements. While the cryptographic security parameters of ledgers are long when considering transaction confirmation times, in this case they are used for an initial setup period. Bearing this in mind, the initial setup of such a ledger must be with reduced functionality – however as discussed in Subsection 5.4, the time may well be acceptable for reasonable parameters.

In many cases, bootstrapped ledgers are not given entirely onto themselves to rely on honest majority, but instead rely on checkpointing [19] to ensure that even an adversarial majority does not break the chain's security properties. Even though this scenario explicitly allows for dishonest majority, it is important to note that there are still assumptions of liveness – in other words, chain quality and chain growth are still observed, although with worse parameters.

Furthermore, if the checkpointing authority itself also provides an SRS update (for instance, by computing the initial SRS), the security of the SRS would also reduce to the honesty of *either* the checkpointing authority *or* the majority of mining power. It is worth noting that this is not a perfect situation, however – the checkpointing authority may *appear* honest, but subvert the SRS. By contrast, it is possible to detect a malicious checkpointing authority for chain properties.

To assuage such fears, we suggest employing the traditional SRS initialisation procedure, of permitting a sufficiently diverse selection of people to contribute to the initial reference string, as in [3, 2], in addition to this SRS generation scheme.

6.3 Cleaner Reference Strings from Random Beacons

While Sonic functions with limited adversarial influence on its reference string, it may be desirable to remove this influence entirely. Following the approach of [3], it is possible to use a random beacon to create a "pure" reference string from the "impure" one presented so far. To sketch the design: The random beacon would be queried after time δ, and the randomness used to select a trapdoor permutation over the reference string. This would then be applied by each party independently, arriving at the same – randomly distributed – reference string.

As this is not required for Sonic, we did not perform this analysis in depth. However broadly the approach to the simulation would be to perform the SRS generation identically, and then program the random beacon to invert all permutations applied to the honest reference string. Since this include the one honest permutation applied on every honest update, this is indistinguishable from a random value to the adversary.

It is worth noting that the requirement of a random beacon is on the stronger side of requirements, especially as it should itself not allow adversarial influence to provide the desired advantage. Approaches using block hashes for randomness introduce exactly the kind of limited influence which we are attempting to remove!

6.4 On NIZK Proofs and Composability

Since the primary purpose of our structured reference string is to construct a NIZK, the usage of a NIZK in the reference string generation procedure is circular. Nevertheless, we believe there is a practical benefit to this approach, as the NIZK used in reference string generation need be neither succinct, nor particularly efficient – as seen in Subsection 5.1, the proving and verification speeds are not of great concern.

As demonstrated in Corollary 1 the requirement NIZK reduces to a more base form of reference string – a uniform reference string, i.e. one sampled with *public* randomness. This distinction may appear minor, however, as previously mentioned, such uniform reference strings are trivial to construct in the random oracle model, while structured reference strings cannot use the same approach –

using public randomness for the latter would reveal the trapdoor of a structured reference string.

Nevertheless, it is worth noting that this NIZK is only used to satisfy the properties of UC extractions – to implement the algorithms $\mathcal{X}_q$ and $\mathcal{X}_a$. In a property based setting, the algebraic group model (AGM) [9] would provide similar extraction properties; we therefore conjecture that similar results could be achieved without the use of a NIZK in this setting. Currently, the composability of the AGM, and other similar knowledge assumptions, is not well understood – it is for primarily this reason that we are relying on a "lower-level" NIZK.

6.5 Upgrading Reference Strings

As distributed ledgers are typically long-lived, and may well outlive any reference string used within it – or have been running before a reference string was needed. Indeed, the Zcash protocol has seen upgrades in its reference string. A reference string being replaced with a new one is innocuous without further context, however it is important to consider how they are usually used in zero-knowledge proofs. If the proof they are used in is stateless, upgrading from an insecure to a secure reference string behaves as one may naively expect: It ensures that after the upgrade, security properties hold.

In the example of Zcash, which runs a variant of the Zerocash [1] protocol, the situation is more muddy. Zerocash makes *stateful* zero-knowledge proofs. Suppose a user is sceptical of the security of the initial setup – and there is good reason to be [27] – but is convinced the second reference string is secure. Is such a user able to use Zcash with confidence in its security?

If Zcash had not had safeguards in place, the answer would be no. While the protocol may operate as intended currently, and the user can be convinced of that, due to the stateful nature of the proofs, the user cannot be convinced of the correctness of this state. The Zcash cryptocurrency did employ safeguards, similar to those we will outline below. We stress the importance of such here, as it may not be obvious to all developers intending to improve the security of reference strings they use.

Specifically, for a Zerocash-based system, an original reference string's backdoor could have been used to create mismatched transactions, and to effectively "mint" large coins illicitly. This process is undetectable at the time, and the minted coins would persist across a reference string upgrade. Our fictitious user may therefore be rightfully suspicious as to the value of any coins he is sold – they may be a part of an almost infinite pool!

Such an attack, once carried out (especially against a currency) is hard to recover from – it is impossible to identify "legitimate" owners of the currency, even if the private transaction history were deanonymised, and the culprit identified. The culprit may have traded whatever he created already. Simply invalidating the transaction would therefore harm those he traded with, not himself. In an extreme case, if he traded one-to-one with legitimate owners of the currency, he would succeed in effectively stealing the honest users funds. If such an attack

is identified, the community has two unfortunate options: Annul the funds of potentially legitimate users, or accept a potentially large amount of inflation.

We may assume a less grim scenario however: Suppose we are *reasonably confident* in the security of our old reference string, but we are *more confident* of the new one. Is it possible to convince users that we have genuinely upgraded our security? We suggest the usage of a type of *firewalling* property. Such properties are common in the domain of cross-chain transfers [14], and are designed to prevent a catastrophic failure on one chain damaging another.

For monetary transfers, the firewall would guarantee an upper-bound of funds was not exceeded. Proving that the firewall property is preserved is easily done if a small loss of privacy is accepted – each private coin being re-minted before it can be used after the upgrade, during which time its value must be declared. Assuming everything operates fine, and the firewall property is not violated, users interacting with the post-firewall state can be confident as to the upper bound of funds available. Further, attacks on the system can be identified: If an attacker mints too many coins, eventually the firewall property will be violated, indicating that too many coins were in circulation – bringing the complex problem of how to handle this situation with it. We believe that a firewall property does however give peace of mind to users of the system, and is a practical means to assuage concerns about the security of a system which had – at some point – a questionable reference string.

In Zcash, a soft form of such firewalling is available, in that funds are split across several "pools", each of which uses a different proving mechanism. The total value of each pool can be observed, and values under zero would be considered a cause for alarm, and rejected. Zcash use the terminology "turnstiles" [29], and no attacks have not been observed through them.

A further consideration for live systems is that as Subsection 5.2 shows, the time required for generation strongly depends on the frequency between blocks. This may be in conflict with other considerations for selecting the block time – potential solutions for this include only performing updates on "superblocks" – blocks which meet a higher proof-of-work (or other selection mechanism) criteria than usual.

6.6 The Root of Trust

An important question for all protocols in the distributed ledger setting is whether a user entering the system at some point during its runtime can be convinced to trust in its security. Early proof-of-stake protocols, such as [20], did poorly at this, and were subject to "stake-bleeding" attacks [12] for instance – effectively meaning new users joining could not safely join the network.

For reference strings, if a newly joining user is prepared to accept that the honest majority assumption holds, they may trust the security of the reference string, as per Theorem 1. There is a curious difference to the security of the consensus protocol however: to trust the consensus – at least for proof-of-work based protocols – it is most important to trust a *current* honest majority, as these protocols are assumed to be able to recover from dishonest majorities at

some point in their past. The security of the reference string on the other hand only relies on assuming honest majority during the initial δ time units. This may become an issue if a large period of time passes – why should someone trust the intentions of users during a different age? In practice, it may make sense to “refresh” a reference string regularly to renew faith in it.

Most subversion attacks are detectable – they require lengthy forks which are unlikely to occur during a legitimate execution. In an optimistic case, where no attack is attempted, this may provide an additional level of confirmation: if there are no widespread claims of large forks during the initial setup, then the reference string is likely secure (barring large-scale out-of-band censorship). A flip side to this is that it may be a lot easier to sow doubt, however, as there is no way to *prove* this: A malicious actor could create a fork long after the initial setup, and claim that it is evidence of an attack to undermine the credibility of the system.

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A Formal UC Specification

A.1 The Nakamoto Ledger

The basic functionality of this ledger allows the submission of transactions, and retrieving each of the following:

– A confirmed prefix of the ledger state.
– A "projection" of the ledger state – i.e. what the local state will approach, if there is no chain reorganisation.
– The confirmed "leader state", which models the mechanism used for the SRS generation.

When any of these values is queried, the functionality ensures that liveness and chain quality properties still hold. The adversary further has the power to instruct the creation of a new block, on behalf of any party, and to instruct any party to adopt a different chain. In both cases, the functionality ensures that the common prefix property is preserved. The adversary has full control over the contents of both honest and adversarial blocks, as well as their order.

Functionality $\mathcal{F}_{\mathsf{nakLedger}}$

A ledger following a Nakamoto-style consensus, with each party having a *projected* chain, a prefix of which is common to all parties. Common prefix, chain quality and chain growth are guaranteed.

State variables and initialisation values:

Variable	Description
$\Pi := \mathcal{P} \mapsto \epsilon$	Mapping of parties to projected ledger states
$T := \varnothing$	Multiset of submitted transactions
$\mathsf{hon} := \varnothing$	Mapping of block ids 1 if they are honest, or 0

When receiving a message $(\text{SUBMIT}, \mathsf{tx})$ *from a party* $\mathcal{P}$*:*

send READ **to** $\mathcal{G}_{\mathsf{clock}}$ and **receive the reply** t
let $T \leftarrow T \cup \{(\mathsf{tx}, t)\}$
query $\mathcal{A}$ **with** $(\text{TRANSACTION}, \mathsf{tx}, t)$

When receiving a message READ *from a party* $\mathcal{P}$*:*

assert $\mathsf{liveness}(\mathcal{P}) \wedge \mathsf{chainQuality}(\mathcal{P})$
return $\mathsf{map}(\mathsf{proj}_1, \mathsf{txs}(\Pi(\mathcal{P})^{\lceil k}))$

When receiving a message PROJECTION *from a party* $\mathcal{P}$*:*

assert $\mathsf{liveness}(\mathcal{P}) \wedge \mathsf{chainQuality}(\mathcal{P})$
return $\mathsf{map}(\mathsf{proj}_1, \mathsf{txs}(\Pi(\mathcal{P})))$

When receiving a message LEADER-STATE *from a party* $\mathcal{P}$*:*

assert $\mathsf{liveness}(\mathcal{P}) \wedge \mathsf{chainQuality}(\mathcal{P})$
let $\vec{a} \leftarrow \mathsf{map}(\lambda(\cdot, a, \cdot, t) : (a, t), \Pi(\mathcal{P})^{\lceil k})$
return $\mathsf{foldl}(\mathsf{Apply}, \varnothing, \vec{a})$

When receiving a message $(\text{EXTEND}, \mathcal{P}, B, t, a)$ *from* $\mathcal{A}$*:*

send READ **to** $\mathcal{G}_{\mathsf{clock}}$ and **receive the reply** t'
let $\mathsf{id} \xleftarrow{R} \{0,1\}^{\kappa}$
if $\mathcal{P} \in \mathcal{H}$ **then**
  let $\vec{a} \leftarrow \mathsf{map}(\lambda(\cdot, a, \cdot, t) : (a, t), \Pi(\mathcal{P}))$
  let $\sigma \leftarrow \mathsf{foldl}(\mathsf{Apply}, \varnothing, \vec{a})$

let $a \xleftarrow{R} \mathsf{Gen}(\sigma, t')$
let $t \leftarrow t'$ **let** $\mathsf{hon}(\mathsf{id}) \leftarrow 1$
else
let $\mathsf{hon}(\mathsf{id}) \leftarrow 0$
if $t' < t$ **then let** $t \leftarrow t'$
else if $\exists t'' : (\cdot, \cdot, \cdot, t'') = \mathsf{last}(\Pi(\mathcal{P})) \wedge t'' > t$ **then let** $t \leftarrow t''$
let $\Pi(\mathcal{P}) \leftarrow \Pi(\mathcal{P}) \parallel (B, a, \mathsf{id}, t)$
assert $\forall \mathcal{P}' \in \mathit{Parties} : \Pi(\mathcal{P})^{\lceil k} \prec \Pi(\mathcal{P}')$
return (B, a, id, t)

When receiving a message $(\textsc{advance}, \mathcal{P}, \Sigma')$ *from* $\mathcal{A}$*:*
assert $\exists \mathcal{P}' \in \mathit{Parties} : \Sigma' \prec \Pi(\mathcal{P}')$
assert $\forall \mathcal{P}' \in \mathit{Parties} : \Sigma'^{\lceil k} \prec \Pi(\mathcal{P}') \wedge \Pi(\mathcal{P}')^{\lceil k} \prec \Sigma'$
let $\Pi(\mathcal{P}) \leftarrow \Sigma'$

Helper procedures:
function $\mathsf{txs}(\Pi_{\mathcal{P}})$
let $\vec{B} \leftarrow \mathsf{map}(\mathsf{proj}_1, \Pi_{\mathcal{P}})$
return $\mathsf{concat}(\vec{B})$
procedure $\mathsf{liveness}(\mathcal{P})$
send READ **to** $\mathcal{G}_{\mathsf{clock}}$ and **receive the reply** t
if $\exists t_0 < t : |[\, t_b \mid (\cdot, \cdot, \cdot, t_b) \in \Pi(\mathcal{P}), t_0 - s \leq t_b < t_0 \,]|$
$< \gamma \wedge t_0 - s \geq 0$ **then**
return $\bot$
return $\forall (\mathsf{tx}, t') \in T : t' + \lceil (l + k)\gamma^{-1} \rceil s > t \ \vee (\mathsf{tx}, t') \in \mathsf{txs}(\Pi(\mathcal{P})^{\lceil k})$
procedure $\mathsf{chainQuality}(\mathcal{P})$
let $\vec{\mathsf{id}} \leftarrow \mathsf{map}(\mathsf{proj}_3, \Pi(\mathcal{P})^{\lceil k})$
return $\forall i \in \mathbb{Z}_{|\vec{a}|-l} : \left(\sum_{j \in \mathbb{Z}_l} \mathsf{ids}(\vec{\mathsf{id}}_{i+j})\right) \geq \mu l$

In order to judge chain growth, this functionality needs access to a simple global clock, given in Appendix B.1.

A.2 The Adaptor Protocol

We provide a small protocol which adapts the honest interface of the Nakamoto ledger to match that of the ideal world – specifically ensuring the leadership state seen matches the ideal world's, and that and SRS is read only if sufficient time has passed.

Protocol LEDGER-ADAPTOR

The protocol adaptor fits the interface of $\mathcal{F}^{\mathsf{real}}_{\mathsf{nakLedger}}$ to match those of $\mathcal{F}_{\mathsf{uSRS}}$ and $\mathcal{F}^{\delta}_{\mathsf{delayLedger}}{}^{\mathsf{ideal}}$. It operates in the $(\mathcal{F}^{\mathsf{real}}_{\mathsf{nakLedger}}, \mathcal{G}_{\mathsf{clock}})$-hybrid world.

When receiving a message $(\textsc{submit}, \mathsf{tx})$ *from a party* $\mathcal{P}$*:*
send $(\textsc{submit}, \mathsf{tx})$ **to** $\mathcal{F}_{\mathsf{nakLedger}}$

When receiving a message READ *from a party* $\mathcal{P}$*:*

send READ **to** $\mathcal{F}_{\mathsf{nakLedger}}$ and **receive the reply** txs
return txs

When receiving a message PROJECTION *from a party* $\mathcal{P}$*:*

send PROJECTION **to** $\mathcal{F}_{\mathsf{nakLedger}}$ and **receive the reply** txs
return txs

When receiving a message LEADER-STATE *from a party* $\mathcal{P}$*:*

send LEADER-STATE **to** $\mathcal{F}_{\mathsf{nakLedger}}$ and **receive the reply** $(\cdot, \sigma^{\mathsf{ideal}})$
return σ^{ideal}

When receiving a message SRS *from a party* $\mathcal{P}$*:*

send READ **to** $\mathcal{G}_{\mathsf{clock}}$ and **receive the reply** t
if $t < \delta$ **then return** $\bot$
else
  send LEADER-STATE **to** $\mathcal{F}_{\mathsf{nakLedger}}$ and
    receive the reply $(\mathsf{srs}, \cdot)$
  return srs

Forward SUBMIT*,* READ*, and* PROJECTION *queries to* $\mathcal{F}^{\mathsf{real}}_{\mathsf{nakLedger}}$

A.3 The Simulator

Simulator $\mathcal{S}_{\text{LEDGER-ADAPTOR}}$

The simulator between the protocol adaptor over $\mathcal{F}^{\mathsf{real}}_{\mathsf{nakLedger}}$, and $\mathcal{F}^{\delta}_{\mathsf{delayLedger}}{}^{\mathsf{ideal}}$ and $\mathcal{F}_{\mathsf{uSRS}}$. It operates in the $\mathcal{G}_{\mathsf{clock}}$-hybrid world.

State variables and initialisation values:

Variable	Description
$\mathcal{F}^{\mathsf{simul}}_{\mathsf{nakLedger}}$	A simulation of the hybrid-world ledger
$\mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}}$	A simulation of the low-level NIZK functionality
$A := \varnothing$	Map from honest updates to the applied permutation

When receiving a message $(\text{TRANSACTION}, \mathsf{tx}, t)$ *from* $\mathcal{F}^{\mathsf{ideal}}_{\mathsf{nakLedger}}$*:*

simulate sending $(\text{SUBMIT}, \mathsf{tx})$ **to** $\mathcal{F}^{\mathsf{simul}}_{\mathsf{nakLedger}}$

When receiving a message $(\text{SUBMIT}, \mathsf{tx})$ *from* $\mathcal{A}$ *for* $\mathcal{F}^{\mathsf{real}}_{\mathsf{nakLedger}}$*:*

send $(\text{SUBMIT}, \mathsf{tx})$ **to** $\mathcal{F}^{\mathsf{ideal}}_{\mathsf{nakLedger}}$

When receiving a message $(\text{PERMUTE}, \mathcal{P})$ *from* $\mathcal{F}_{\mathsf{uSRS}}$*:*

simulate sending LEADER-STATE **to** $\mathcal{F}^{\mathsf{simul}}_{\mathsf{nakLedger}}$
  through $\mathcal{P}$ and
  receive the reply $(\mathsf{srs}, \cdot)$
let $q \leftarrow \mathcal{X}_q(\mathsf{map}(\mathsf{proj}_1, \vec{a}), \mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}}.W, A)$
return q

When receiving a message $(\text{EXTEND}, \mathcal{P}, B, t, a)$ *from* $\mathcal{A}$ *for* $\mathcal{F}^{\mathsf{real}}_{\mathsf{nakLedger}}$*:*

send READ **to** $\mathcal{G}_{\mathsf{clock}}$ and **receive the reply** t'

if $\mathcal{P} \in \mathcal{H} \land t' \leq \lceil l\gamma^{-1} \rceil s$ **then**
  let $\vec{a} \leftarrow \mathsf{map}(\mathsf{proj}_2, \mathcal{F}^{\mathsf{simul}}_{\mathsf{nakLedger}}.\Pi(\mathcal{P}))$
  let $(\mathsf{srs}, \cdot) \leftarrow \mathsf{foldl}(\mathsf{Apply}, \varnothing, \vec{a})$
  let $q \leftarrow \mathcal{X}_q(\mathsf{map}(\mathsf{proj}_1, \vec{a}), \mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}}.W, A)$
  if $q^{-1\dagger}(\mathsf{srs}) \neq S(\tau_0)$ **then**
    `// We cannot extract a trapdoor;`
    `// the SRS is already secure`
    let $q \xleftarrow{R} P; a^{\mathsf{srs}} \leftarrow \mathsf{GenSRS}(\mathsf{srs}, q)$
  else
    `// We produce an update to match a`
    `// random "initial" SRS`
    let $q \xleftarrow{R} P$
    send HONEST-SRS **to** $\mathcal{F}_{\mathsf{uSRS}}$ and
      receive the reply $\mathsf{srs}_{\mathcal{H}}$
    let $\tau \leftarrow q(\tau_0)$
    let $a^{\mathsf{srs}} \leftarrow \mathcal{X}_a(q(\tau), q^{\dagger}(\mathsf{srs}'))$
    let $A(a^{\mathsf{srs}}) \leftarrow q$
  let $a^{\mathsf{ideal}} \leftarrow \bot$
else if $\mathcal{P} \in \mathcal{H}$ **then**
  let $a^{\mathsf{srs}} \leftarrow \epsilon$
  let $a^{\mathsf{ideal}} \leftarrow \bot$
else let $(a^{\mathsf{srs}}, a^{\mathsf{ideal}}) \leftarrow a$
send $(\text{EXTEND}, \mathcal{P}, B, t, a^{\mathsf{ideal}})$ **to** $\mathcal{F}^{\mathsf{ideal}}_{\mathsf{nakLedger}}$ and
  receive the reply $(B, a^{\mathsf{ideal}}, \mathsf{id}, t)$
if $\mathcal{P} \in \mathcal{H}$ **then**
  let $\mathcal{F}^{\mathsf{simul}}_{\mathsf{nakLedger}}.\mathsf{hon}(\mathsf{id}) \leftarrow 1$
else
  let $\mathcal{F}^{\mathsf{simul}}_{\mathsf{nakLedger}}.\mathsf{hon}(\mathsf{id}) \leftarrow 0$
let $\mathcal{F}^{\mathsf{simul}}_{\mathsf{nakLedger}}.\Pi(\mathcal{P}) \leftarrow \mathcal{F}^{\mathsf{simul}}_{\mathsf{nakLedger}}.\Pi(\mathcal{P}) \parallel (B, (a^{\mathsf{srs}}, a^{\mathsf{ideal})}, \mathsf{id}, t)$
assert $\forall \mathcal{P}' \in \mathit{Parties} : \mathcal{F}^{\mathsf{simul}}_{\mathsf{nakLedger}}.\Pi(\mathcal{P})^{\lceil k} \prec \mathcal{F}^{\mathsf{simul}}_{\mathsf{nakLedger}}.\Pi(\mathcal{P}')$
return $(B, (a^{\mathsf{srs}}, a^{\mathsf{ideal}}), \mathsf{id}, t)$

When receiving a message $(\text{ADVANCE}, \mathcal{P}, \Sigma')$ *from* $\mathcal{A}$ *for* $\mathcal{F}^{\mathsf{real}}_{\mathsf{nakLedger}}$*:*
  simulate sending $(\text{ADVANCE}, \mathcal{P}, \Sigma')$ **to** $\mathcal{F}^{\mathsf{simul}}_{\mathsf{nakLedger}}$
  `// Remove SRS updates from` Σ'
  let $\Sigma' \leftarrow \mathsf{map}(\lambda(B, (a^{\mathsf{srs}}, a^{\mathsf{ideal}}), t) : (B, a^{\mathsf{ideal}}, t), \Sigma)$
  send $(\text{ADVANCE}, \mathcal{P}, \Sigma')$ **to** $\mathcal{F}^{\mathsf{ideal}}_{\mathsf{nakLedger}}$

Forward requests to $\mathcal{F}^{\mathcal{R}}_{\mathsf{NIZK}}$*, and all other adversarial messages for* $\mathcal{F}^{\mathsf{real}}_{\mathsf{nakLedger}}$ *to* $\mathcal{F}^{\mathsf{simul}}_{\mathsf{nakLedger}}$*.*

B Minor UC Functionalities

B.1 The Global Clock

Functionality $\mathcal{G}_{\mathsf{clock}}$

The global clock allows parties to agree on some discrete notion of time.

State variables and initialisation values:

Variable	Description
$t := 0$	Current time
$T := \varnothing$	Timekeepers
$A := \varnothing$	Agreements to advance

When receiving a message REGISTER *from a party* $\mathcal{P}$*:*

let $T \leftarrow T \cup \{\mathcal{P}\}$

When receiving a message DEREGISTER *from a party* $\mathcal{P}$*:*

let $T \leftarrow T \setminus \{\mathcal{P}\}$

When receiving a message UPDATE *from a party* $\mathcal{P}$*:*

let $A(\mathcal{P}) \leftarrow \top$
if $\forall \mathcal{P} \in T : A(\mathcal{P})$ **then**
  let $t \leftarrow t + 1; A \leftarrow \lambda\mathcal{P} : \bot$
  query $\mathcal{A}$ **with** TICK-TOCK

When receiving a message READ *from a party* $\mathcal{P}$*:*

return t

B.2 Non-Interactive Zero-Knowledge

Functionality $\mathcal{F}_{\mathsf{NIZK}}^{\mathcal{R}}$

The (malleable) non-interactive zero-knowledge functionality $\mathcal{F}_{\mathsf{NIZK}}^{\mathcal{R}}$ allows proving of statements in an NP relation $\mathcal{R}$.

State variables and initialisation values:

Variable	Description
$W := \varnothing$	Mapping of statement/proof pairs to witnesses
$\Pi := \varnothing$	Set of statement/proof pairs
$\overline{\Pi} := \varnothing$	Set of known invalid statement/proof pairs

When receiving a message (PROVE, x, w) *from a party* $\mathcal{P}$*:*

if $\neg x\mathcal{R}w$ **then**
  return $\bot$
query $\mathcal{A}$ **with** (PROVE, x) and **receive the reply** π,
  satisfying $\pi \neq \bot \land (x, \pi) \notin \overline{\Pi} \land (\cdot, \pi) \notin \Pi$, else **sampling from** $\{0, 1\}^{\kappa}$
let $\Pi \leftarrow \Pi \cup \{(x, \pi)\}; W(x, \pi) \leftarrow w$
return π

When receiving a message $(\text{MAUL}, x, \pi, \pi')$ *from* $\mathcal{A}$*:*

if $(x, \pi) \in \Pi$ **then**
  let $\Pi \leftarrow \Pi \cup \{(x, \pi')\}$
  let $W(x, \pi') \leftarrow W(x, \pi)$

When receiving a message (VERIFY, x, π) *from a party* $\mathcal{P}$*:*

```
if (x, π) ∉ Π ∪ Π̄ ∧ π ≠ ⊥ then
    query A with (VERIFY, x, π)
    // The adversary has been given a chance to
    // prove the statement. It didn't take it.
    if (x, π) ∉ Π ∪ Π̄ then
        let Π̄ ← Π̄ ∪ (x, π)
return (x, π) ∈ Π
```

B.3 Random Oracle

Functionality $\mathcal{F}_{\mathsf{RO}}$

The random oracle functionality $\mathcal{F}_{\mathsf{RO}}$ returns a uniform random value in $\{0,1\}^\kappa$ for each input.

State variables and initialisation values:

Variable	Description
$H := \varnothing$	A map from inputs to (fixed) outputs

When receiving a message (QUERY, x) *from a party* $\mathcal{P}$*:*

if $x \notin H$ **then let** $H(x) \xleftarrow{R} \{0,1\}^\kappa$
return $H(x)$

B.4 Delay Wrapper

Functionality $\mathcal{W}_{\mathsf{delay}}(\delta, \mathcal{F})$

The wrapper functionality $\mathcal{W}_{\mathsf{delay}}(\delta, \mathcal{F})$ of $\mathcal{F}$ accepts *honest* inputs only after δ time slots.

When receiving a message M *from a party* $\mathcal{P}$*:*

send READ **to** $\mathcal{G}_{\mathsf{clock}}$ and **receive the reply** t
if $t < \delta \wedge \mathcal{P} \in \mathcal{H}$ **then return** $\perp$
else
  send M **to** $\mathcal{F}$ and **receive the reply** y
  return y