# Presentation and publication: General Congestion

<sup>2</sup> Attack on HTLC-Based Payment Channel

# Networks

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#### <sup>10</sup> — Abstract -

Payment Channel Networks (PCNs) have been a promising approach to scale blockchains. However, 11 PCNs have limited liquidity: large-amount or multi-hop payments may fail. The major threat of 12 PCNs liquidity is *payment griefing*, where the adversary who acts as the payee keeps withholding the 13 payment, so that coins involved in the payment cannot be used for routing other payments before 14 the payment expires. Payment griefing gives adversaries a chance to launch the congestion attack, 15 where the adversary griefs a large number of payments and paralyses the entire PCN. Understanding 16 congestion attacks, including their strategies and impact, is crucial for designing PCNs with better 17 liquidity guarantees. However, existing research has only focused on the specific attacking strategies 18 19 and specific aspects of their impact on PCNs.

We fill this gap by studying the *general congestion attack*. Compared to existing attack strategies, 20 in our framework each step serves an orthogonal purpose and is customisable, allowing the adversary 21 to focus on different aspects of the liquidity. To evaluate the attack's impact, we propose a generic 22 method of quantifying PCNs' liquidity and effectiveness of the congestion attacks. We evaluate our 23 general congestion attacks on Bitcoin's Lightning Network, and show that with direct channels to 24 1.5% richest nodes, and  $\sim 0.0096$  BTC of cost, the adversary can launch a congestion attack that 25 locks 47% ( $\sim$ 280 BTC) coins in the network; reduces success rate of payments by 16.0%  $\sim$ 60.0%; 26 increases fee of payments by  $4.5\% \sim 16.0\%$ ; increases average attempts of payments by  $42.0\% \sim 115.3\%$ ; 27 and increase the number of bankruptcy nodes (i.e., nodes with insufficient balance for making 28 normal-size payments) by  $26.6\% \sim 109.4\%$ , where the amounts of payments range from 0.001 to 0.019 29 BTC. 30

 $_{31}$  2012 ACM Subject Classification  $\, {\rm Security} \, {\rm and} \, {\rm privacy} \rightarrow {\rm Distributed} \, {\rm systems} \, {\rm security}$ 

32 Keywords and phrases Blockchain, PCN, Congestion

<sup>33</sup> Digital Object Identifier 10.4230/OASIcs...

# <sup>34</sup> **1** Introduction

Public blockchains suffer from limited throughput. Payment Channel Network (PCN) -35 introduced by the Lightning Network (LN) [16] – is one of the promising ways to scale 36 blockchains. Payment channels enable off-chain payments, i.e. payments that do not need to 37 be recorded on the blockchain. To open a payment channel, two nodes collateralise some coins 38 in a joint address. They can make a payment by signing a new transaction that updates their 39 balances. To close the channel, one of the two nodes commits the transaction recording the 40 latest balance allocation to the blockchain. If two nodes do not have a direct channel, they 41 can make payments to each other using *multi-hop payments*, i.e., payments going through 42 one or more intermediate channels. In a multi-hop payment, the payer has to find a path 43 that directs him to the payee. The payment is made by updating balances of these channels 44



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<sup>45</sup> in an atomic way. The atomic update can be achieved by Hash Time Locked Contracts <sup>46</sup> (HTLCs): the payment in each hop is locked by a hash value chosen by the payee, and all <sup>47</sup> payments proceed if the payee reveal a hash value's preimage before a timeout to redeem the <sup>48</sup> payment from the payer, otherwise the payment will expire. In a HTLC-based multi-hop <sup>49</sup> payment, the payee chooses a preimage, and nodes make HTLC payments on all involved <sup>50</sup> channels with this preimage's hash value. Revealing this preimage activates these HTLC <sup>51</sup> payments simultaneously.

**Payment griefing.** A well-known attack on HTLC-based multi-hop payments is *payment* 52 griefing [18], where the adversary makes a payment and withholds the preimage, so that 53 coins involved in this payment are locked and cannot be used in other payments before the 54 griefing payment expires. Thus, payment griefing can reduce the PCN's liquidity, i.e. the 55 ability of routing payments. In addition, payment griefing is free, as the payer does not need 56 to pay anything for failed payments. Moreover, payment griefing is also unaccountable, as 1) 57 the victim cannot distinguish between a normal failed payment and a griefing payment, and 58 2) the intermediate nodes can not know the payer and payee's identity. 59

Congestion attacks. Griefing opens an important attack vector on HTLC-based PCN's liquidity, namely the *congestion attack* [1,13]. In a congestion attack, the adversary initiates a large number of concurrent payments and griefs them. Consequently, some channels hit the limit of *max\_concurrent\_htlcs*, i.e., the number of concurrent unsettled payments allowed in the channel, and therefore cannot route payments before the adversary's payments expire. By launching a large-scale congestion attack, the entire PCN can be paralysed, i.e., the PCN cannot route further payments.

PCNs' liquidity: what is the real limit? Understanding congestion attacks is important 67 for understanding PCNs' liquidity and therefore future PCN design. However, congestion 68 attacks are still a new concept and haven't been well-studied yet. While existing research [1,13]69 only considers max concurrent htlcs as an exhaustible resource, it's unclear whether there 70 exists other resources that can be exhausted to create congestion. In addition, existing 71 congestion attacks apply a rather straightforward attack strategy, which will be analysed in 72 detail in §7. Moreover, we also observe that liquidity – the congestion attack's target – is not 73 well-defined yet. Besides the amount of locked balance and the number of locked channels 74 mentioned in Mizrahi et al. [13], some other metrics such as success rate of payments, fee 75 of payments, and number of attempts for making a payment have direct indications on the 76 PCN's liquidity. Congestion attacks over these metrics are not explored before. 77

This work: general congestion attacks. In this paper, we fill this gap by introducing 78 general congestion attack, which generalises the existing congestion attack in terms of attack 79 strategies and targeted metrics. We introduce a framework for launching congestion attacks, 80 where the adversary generates Sybil nodes connecting to a carefully chosen set of nodes, 81 establishes channels with them, initiates numerous multi-hop payments between its nodes, 82 and griefs these payments simultaneously. Compared to existing studies that put less effort 83 on the order of payments to be griefed [13, 21], we provide five strategies for ranking these 84 payments, and each strategy focuses on some specific aspects of liquidity. To quantify the 85 effectiveness of congestion attacks, we introduce a generic method of quantifying PCNs' 86 liquidity. We evaluate the congestion attack on Bitcoin's LN – the first and most well-known 87 PCN. Our results show that congestion attacks can significantly damage the liquidity of 88 PCNs. In particular, with direct channels to 1.5% richest nodes, the adversary can launch a 89 congestion attack that locks 47% (~280 BTC) coins in the network; reduces success rate 90 of payments by  $16.0\% \sim 60.0\%$ ; increases fee of payments by  $4.5\% \sim 16.0\%$ ; increases average 91 attempts of payments by  $42.0\% \sim 115.3\%$ ; and increase the number of bankruptcy nodes by 92



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 $_{93}$  26.6%~109.4%, where the amounts of payments range from 0.001 to 0.019 BTC.

<sup>94</sup> While being effective, our general congestion attacks are cheap to launch. The only cost

<sup>95</sup> of general congestion attacks is the fee for establishing channels. Our evaluation shows that,

<sup>96</sup> a successful attack on LN requires channel fee of approximately 0.0096 BTC. The adversary
 <sup>97</sup> does not lose its custody (i.e., coins in the channel) during the attack, as payments for

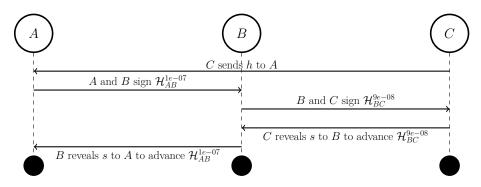
<sup>98</sup> griefing will expire.

Roadmap. Section 2 provides the background of PCNs and griefing. Section 3 describes the security model and the congestion attack. Section 4 describes the method of quantifying PCNs' liquidity. Section 5 evaluates congestion attacks on LN. Section 6 discusses the cost of congestion and strategy to utilise it for making a profit. Section 7 reviews relevant literature, and provides a quantitative comparison between the general congestion attack and the existing ones. Section 8 concludes this paper. Appendix A outlines detailed evaluation results.

#### <sup>106</sup> 2 Background

#### <sup>107</sup> 2.1 Payment Channel Networks

Lightning Network (LN) [16] introduces the idea of Payment Channel Networks. A payment 108 channel allows two parties to pay each other without the need to publish every payment to 109 the blockchain. Instead, they collateralise their coins into a 2-of-2 multi-signature address. 110 They can make payments by mutually signing new transactions with updated balances. They 111 can make payments with each other by mutually signing new transactions with updated 112 amounts of their collateralised coins. To close the channel, one party commits the latest 113 state of channel balance to the blockchain, and coins in the channel will be allocated to both 114 parties accordingly. 115



**Figure 1** A multi-hop payment from A to C via an intermediate node B.

The system can be further extended to support multi-hop payments. Most multi-hop payment protocols are based on Hash Time Locked Contracts (HTLCs). HTLC is a contract between two parties which guarantees that a payment will be made if the payee shows the preimage of a hash value before a negotiated block height on the blockchain. If the payee does not show the preimage and the timeout expires, the payment is deemed invalid.

Figure 1 describes a multi-hop payment where A pays 9e-08 BTC to C via an intermediate node B in Bitcoin's LN. First, C chooses a random string s as preimage and sends its hash value h = H(s) to A, where  $H(\cdot)$  is a cryptographic hash function. A then signs a HTLC contract  $\mathcal{H}_{AB}^{1e-07}$  with B stating "A will pay 1e-07 BTC to B if B can show the value of s within (e.g.) 144 blocks". B also signs a HTLC contract  $\mathcal{H}_{BC}^{9e-08}$  with C saying that "B will



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pay 9e-08 BTC to C if C can show the value of s within (e.g.) 138 blocks". Then C shows 126 s to B to redeem 9e-08 BTC in  $\mathcal{H}_{BC}^{9e-08}$  from B. Meanwhile, B can redeem 1e-07 BTC in  $\mathcal{H}_{AB}^{1e-07}$  from A by revealing s to A. B is incentivised to reveal s, as B does not want to lose 127 128 money. The timelock of AB is set to be longer than BC, so B always has sufficient time to 129 reveal s to A. 130

By routing this payment, B gets 1e-08 BTC from A. This is known as "fee", which is 131 paid by the payer and is used for encouraging nodes to route multi-hop payments. In LN, fee 132 consists of a fixed base fee and proportional fee that fluctuates according to the congestion 133 level of the network. To minimise the cost, payers usually search for a path with the least 134 fee when making payments. 135

#### 2.2 Payment Griefing 136

If the payee C reveals the preimage on time and the intermediate node B is rational, the 137 multi-hop payment will happen. However, as we mentioned before, there exists an attack 138 called *payment griefing* [4], where the payee withholds the preimage until HTLCs expire. 139 Before HTLC expires, coins involved in all channels of this payment are locked and cannot 140 route other payments. 141

Payment griefing is a threat to PCNs' liquidity. If a big portion of coins in a PCN are 142 locked, the PCN will no longer be able to route payments. Payment griefing is cheap, as 143 the payment does not really happen and the payer does not pay for the fee to intermediate 144 nodes. Identifying payment griefing can be hard, as nodes cannot distinguish whether the 145 withholding is due to network delay, on purpose, or by accident. If the PCN's routing protocol 146 is privacy-preserving, payment griefing can even be launched anonymously. For example, 147 Bitcoin's LN adopts Sphinx [7], where each intermediate node only has the knowledge of 148 nodes who directly connect with him. 149

#### 2.3 Congestion attack 150

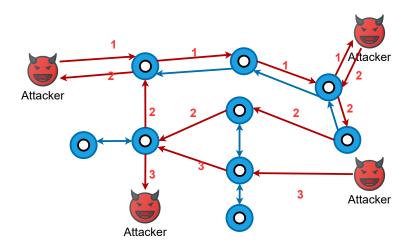


Figure 2 Congestion attack.

In a congestion attack, the adversary establishes payment channels with existing nodes 151 in the PCN, and make numerous multi-hop payments between its nodes simultaneously. 152 Then, the adversary withholds preimages until these payments expire. Before that, coins 153 locked in these payments cannot be used in other payments. If the adversary has sufficient 154



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<sup>155</sup> custody, it can lock a great portion of coins in the PCN so that the PCN may be paralysed.
<sup>156</sup> Figure 2 shows the intuition of the congestion attack, where the adversary generates Sybil
<sup>157</sup> nodes connecting to a carefully chosen set of nodes, establishes channels with them, initiates
<sup>158</sup> numerous multi-hop payments between its nodes, and griefs these payments simultaneously.

To reduce the custody required for an attack, Mizrahi et al. [13] proposed to lock the channel by initiating numerous payments with small amounts to occupy all *max\_concurrent\_htlcs*, the maximum number of concurrent payments in a channel. In addition, they propose three strategies for enumerating payment paths. Tikhomirov et al. [21] provided another strategy, where the adversary attacks a single channel rather than a path for each step.

We compare the strategies of our attack with those of Mizrahi et al. [13] and Tikhomirov et al. [21] in §3.2, and compare the cost and effectiveness in §7. The result show that. If the attacker is fee-sensitive, then our attack is preferred because our fees are 16% of and 5% of other two. Whereas, if the attacker has a restricted custody in hand, then the attack by person Mizrahi et al. is more preferred, as the custody required is only 1.5% of our attack (in case of locking 41% network's capacity).

#### **General congestion attack**

#### 171 **3.1 Model**

We consider a HTLC-based PCN that is identical to Bitcoin's Lightning Network as described 172 in §2.1. We model the PCN as a weighted directed graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ .  $\mathcal{V}$  is the set of nodes 173 in the network, and all  $v_*$  in the remainder of the paper refer to a specific element (i.e. a 174 node) in  $\mathcal{V}$ .  $\mathcal{E}$  is a set of tuples  $(v_s, v_t, capacity)$ , which represents a channel with a capacity 175 of capacity from  $v_s$  to  $v_t$ . Since channels in LN is bi-directional, a channel is represented in 176 the graph as two opposite edges  $(v_s, v_t, capacity)$  and  $(v_t, v_s, capacity)$ . A payment P is a 177 dictionary  $\{amount : \alpha, path : path\}$ , where  $\alpha$  is the amount of coins that the payer wants 178 to pay to the payee and *path* is the path of payment (a list of edges). Then,  $\mathcal{P}$  denotes list 179 of payments enumerated by the attacker. Meanwhile, we use l to refer the length of path. 180 For simplicity, we do not consider the impact of timelocks on our attack. The payer also 181 needs to pay some fee  $f_i$  for each intermediate node i on the path. We can now get the size 182 of a multi-hop payment  $\theta$  as 183

$$\theta = \alpha \cdot l + \sum_{i=2}^{l} f_i$$

In addiciton, we use  $\Gamma_v$  to refer the total capacity of all channels connecting to v. Therefore, the richest node we defined earlier is the node with the largest  $\Gamma_v$ .

We consider nodes in the PCN are rational. Each honest node in a multi-hop payment 186 will reveal the preimage of the hashlock to the upstream node once the node knows it. We 187 assume a malicious adversary, who has sufficient coins and aims at paralysing the entire 188 PCN with minimal cost. The adversary does not control any node in the beginning, but 189 has the knowledge of the network topology and each channel's capacity and fee policy. The 190 information can be retrieved from PCN's P2P protocol, evidenced by existing studies [5,10]. 191 When the adversary establishes a channel with a node, the node is willing to provide sufficient 192 capacity. According to liquidity providers such as Bitrefill [3], purchasing capacity from 193 nodes is easy and costs negligible coins. Moreover, if an adversary just wants to attack PCN 194 for a period of time, it can use the channel lease marketplace like lightning pool [14] to get 195 incoming liquidity at a much lower cost. 196



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# 197 3.2 Attack framework

We generalise the congestion attack in terms of attack strategies and liquidity metrics. We
 propose a framework for launching congestion attacks. The framework consists of four steps
 as follows.

Node selection: the adversary chooses a set of nodes and establishes channels with
 them.

- 203 2. Payment enumeration: the adversary enumerates all payments between its nodes.
- <sup>204</sup> **3. Path ranking:** the adversary orders these payments.
- <sup>205</sup> 4. Launching attack: the adversary starts to make and abort payments in this order.

Comparison with existing attack strategies. While existing congestion attacks [13,21] start by enumerating griefing paths or griefing channels, our attack chooses nodes in the beginning and enumerates paths within the given set of nodes. Such design allows us to divide the attack into multiple steps with orthogonal purposes, revealing the complete design space for congestion attacks. Specifically, the adversary decides resources (e.g., channel capacity or the max\_concurrent\_htlc parameter) to congest when enumerating payments, and decides its focus of liquidity metrics when ranking payments.

In addition, in existing congestion attacks [13, 21], the adversary has to create new channels when attacking a new channel or path. Therefore, the adversary has to establish a large number of channels, which incurs additional transaction fees on the underlying blockchain. In contrast, our attack chooses nodes in the beginning and enumerates paths within the given set of nodes, and therefore requires much fewer channels.

### 218 3.3 Node selection

The adversary's first step is to join the PCN by establishing channels with existing nodes. We analyse the adversary's strategy on the type of nodes and the number of nodes to establish channels with.

Type of nodes to establish channels with. We suggest establishing channels with the richest (w.r.t. total capacity of involved channels) nodes (which we call *hubs*) in the network, as they are likely to route more griefing payments than a normal node. If establishing channels with nodes with little capacity, the adversary has to establish channels with more nodes, leading to more fee on establishing channels.

Number of nodes to establish channels with. The number of nodes to establish channels with depends on how the adversary enumerates griefing payments (i.e., the second step). By establishing channels with sufficient nodes, the total size of enumerated payments will take the majority of the network capacity, and therefore the congestion attack will take effect. Later in §5.1, we will show that for Bitcoin's Lightning Network, by establishing channels with the top 1.5% richest nodes the enumerated payments can occupy 47% of the network capacity.

# 234 3.4 Enumerating payments

After establishing payment channels, the adversary enumerates all possible payments between its nodes. To this end, the adversary has to find all paths between each pair of its nodes, and calculate the maximum amount that each path can afford. Our payment enumerating algorithm builds upon the Ford–Fulkerson algorithm [9] - a maximum flow algorithm in graph theory. Maximum flow is a classic problem in graph theory, which aims at finding the



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**Algorithm 1** Enumerating payments.

# Input:

Input:					
1: The entire network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$					
2: The adversary's node list $\mathcal{N}$					
Output:					
3: The list of payments $\mathcal{P}$					
$4: \mathcal{P} \leftarrow []$					
5: for $(v_1, v_2)$ in $\mathcal{N}$ .combination() do					
6: $path\_list \leftarrow BFS(\mathcal{G}, v_1, v_2)$					
7: <b>for</b> $path \in path\_list$ <b>do</b>					
8: $P \leftarrow \{path : [], amount : 0\}$					
9: $P.path \leftarrow path$					
10: $capacity\_list \leftarrow [edge.capacity for edge in path]$					
11: $\alpha \leftarrow min(capacity\_list)$	$\triangleright$ Max viable amount				
12: <b>if</b> $\alpha = 0$ <b>then continue</b>	$\triangleright$ This path is not viable				
13: $P.amount \leftarrow \alpha$	$\triangleright P$ is a viable payment				
14: Append $P$ to $\mathcal{P}$					
15: Consume $P$ in $\mathcal{G}$					
16: <b>end for</b>					
17: end for					
18: return $\mathcal{P}$					

maximum amount of flow that the network allows from a source to a sink. Ford–Fulkerson 240 algorithm is one of the most effective algorithms to solve the maximum flow problem. Given 241 a weighted directed graph and two vertices. The Ford-Fulkerson algorithm uses Breadth-242 First Search (BFS) [12] to find all paths between these two vertices. For each path, the 243 maximum viable amount is the minimum weight of edges. Algorithm 1 describes the process 244 of enumerating payments in Python syntax. First, it enumerates the binary combinations 245 of nodes who the adversary establishes channels as the starting and ending points of the 246 path. Second, it executes BFS to find all paths between each two adversary nodes similar 247 Ford–Fulkerson. Third, we derive the most viable amount using the least channel capacity 248 for each path. Last, it consumes this payment from the graph and adds this payment to our 249 payment list, i.e., we subtract the amount of this payment from the capacity of all channels 250 on the path. 251

#### 252 3.5 Ranking payments

Different griefing payments have different impacts on the PCNs' liquidity. With limited
balance, the adversary has to start from griefing important payments for maximising the
attack's effect.

**Rank-by-length, -amount and -size.** We first consider three ranking criteria, namely the length, amount, and size  $\theta$  of a payment. *Rank-by-length* aims at maximising the effect while minimising the cost, as long payments lock most capacity with the least amount. *Rank-by-amount* aims at attacking the channel with large capacity. Since real-time balance in LN cannot be seen, payer tends to prefer to go through channels with large capacities to reduce the number of attempts. *Rank-by-size* aims at maximising the attack effect without considering the custody, as payments with large sizes cost most collateral.



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**Rank-by-fee.** Inspired by B'eres et al. [5], we consider *rank-by-fee*, where the adversary starts from attacking channels with lower fees. This aims at maximising the average channel fee of normal payments after the attack. The ranking criterium Score(P) for payment P is calculated as

$$Score(P) = -\frac{\sum_{i=2}^{l} f_i}{l}$$

**Rank-by-bankrupt.** In addition, we consider the bankruptcy rates presented by Dandekar 267 et al. [6, 17, 20] as a ranking criterium. The adversary first attacks channels that make most 268 nodes "bankrupt". Dandekar et al. introduced credit networks [6], where nodes are connected 269 by edges with a limited resource called *credit*. We model the PCN as a credit network 270 following Ramsever et al. [17], where each channel's credit is its capacity. Liquidity in a credit 271 network is quantified as the probability that nodes become *bankrupt*, i.e., loss of all credit. 272 Dandekar et al. [6] proved that the probability that a node v goes bankrupt is upper-bounded 273 by  $\frac{1}{\Gamma_v+1}$ , where  $\Gamma_v$  is the total capacity of all channels connecting to v. Griefing can be 274 seen as "removing" the capacity of nodes, and therefore increases the probability of nodes 275 becoming bankrupt. In rank-by-bankrupt, the adversary first attacks payments that reduce 276 the most mathematical expectations of the probability of nodes becoming bankrupt. 277

The bankruptcy criterium is quantified as the total increased probability Score(P) of nodes in P becoming bankruptcy

$$Score(P) = Score_t(v_1, \alpha) + \sum_{i=1}^{l} Score_i(v_i, \alpha) + Score_t(v_{l+1}, \alpha)$$

where  $Score_t(v_1, \alpha)$  and  $Score_t(v_{l+1}, \alpha)$  are the increased probability of node  $v_1$  and  $v_{l+1}$ , respectively, and  $Score_i(v_i, \alpha)$  is the increased probability of intermediate nodes  $v_i$  where  $i \in [2, l]$ . For node  $v = v_1$  or  $v_{l+1}$ , the capacity is reduced by  $\alpha$ , so

$$Score_t(v, \alpha) = \frac{1}{\Gamma_v - \alpha + 1} - \frac{1}{\Gamma_v + 1}$$

Meanwhile, for intermediate nodes  $v = v_i$  where  $i \in [1, l-1]$ , the capacity is reduced by  $2\alpha$ , so

$$Score_i(v, \alpha) = \frac{1}{\Gamma_v - 2\alpha + 1} - \frac{1}{\Gamma_v + 1}$$

#### 280 3.6 Launching attack

To obtain a list of griefing payments before launching the attack, we use channels' capacities 281 rather than their balances for determining the amounts of payments. As balances are 282 fluctuating in real-time, some of the enumerated payments may not succeed during the attack. 283 In real-world PCNs, if a node cannot route a payment, the node will reply to the payer with 284 an error message, e.g., LN calls this error *InsufficientFunds*. Thus, we introduce a retry 285 mechanism that, when a griefing payment is rejected, the adversary reduces its amount by a 286 parameter *step*, and retries the same path until it is successful or the amount reaches zero. 287 Algorithm 2 describes the attack process. To avoid being detected due to the retry pattern, 288 the adversary can obfuscate the payment pattern, e.g., by dividing payments into multiple 289 ones with random amounts. 290



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**Algorithm 2** Launching attack.

#### Input:

- 1: The list of ranked payments  $\mathcal{P}_{ranked}$
- 2: The dropping step ratio  $step\_ratio$
- 3: The custody of the adversary  ${\cal B}$

```
4: for P in \mathcal{P}_{ranked} do
       if B < 0 then
5:
           return;
6:
7:
       end if
8:
       step\_amount \leftarrow step\_ratio * P.amount
       while True do
9:
           response \leftarrow make \ payment(P)
10:
           if response = InsufficientFunds then
11:
               P.amount = P.amount - step\_amount
12:
              if P.amount \leq 0 then break
13:
              continue
14:
           end if
15:
16:
           B = B - P.amount
           break
17:
18:
       end while
19: end for
```

### <sup>291</sup> **4** Quantifying PCNs' liquidity and congestion attacks' impact

We propose a generic method of quantifying PCNs' liquidity and congestion attacks' impact. In our method, we generate a batch of payments, simulate them on the PCN, and calculate liquidity metrics. The liquidity metrics include the success rate, the average cost and the number of attempts of payments, and the number of bankruptcy nodes. A congestion attack's impact is quantified as the liquidity difference before and after the attack.

# <sup>297</sup> 4.1 Generating payments for simulation

We follow the approach of Béres et al. [5] to test PCNs' liquidity. Specifically, we generate a batch of n payments, of which payers and payees are random and the amount  $x_t$  is fixed. We test multiple batches of payments with different amounts to cover regular payment scenarios, which will be discussed in §5.

We simulate these payments in the PCN. Each payment is allowed to try r times to find 302 a viable path. If it finds a path within r tries, we consider it successful, otherwise failed. 303 Then the payments can be categorized into three states according to the status before and 304 after the attack, namely added, survived, or removed. Added means the payment fails before 305 the attack but is successful after the attack. Survived means the payment is successful both 306 before and after the attack. *Removed* means the payment is successful before the attack but 307 fails after the attack. Since our analysis is based on successful payments, payments that fail 308 both before and after the attack are ignored here. 309

Interestingly, some payments are *added*. Assuming a payments  $P_1 = A \rightarrow B \rightarrow C \rightarrow D$ fails before the attack since *BC* has insufficient balance. For example, if some successful payments that would have gone through *BC* failed after the attack, then channel *BC* would



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<sup>313</sup> become available to  $P_1$ . Another example is that the attack may cause some payments to <sup>314</sup> change their paths. Suppose a successful payment  $P_2$  originally went through EF and was <sup>315</sup> forced to go through CB after the attack, then the balance of BC will increase and make it <sup>316</sup> have enough balance to route P1.

#### 317 4.2 Calculating liquidity metrics

We derive the PCNs' liquidity from the execution results of the simulated payments. We consider the following five metrics: 1) amount of locked funds, 2) success rate of payments, 3) fee of payments, 4) average attempt times of payments and 5) the number of bankruptcy nodes. A congestion attack's impact is quantified by the difference of liquidity metrics before and after the attack.

#### **5** Evaluation of congestion attacks

In this section, we evaluate congestion attacks on Bitcoin's Lightning Network (LN), the 324 first and most well-known PCN. We analyse the impact of congestion attacks with different 325 strategies in terms of the defined five liquidity metrics. Our results show that the adversary 326 who adopts congestion can severely limit the functionality of the entire PCN. Specifically, 327 the adversary can launch a congestion attack that locks 47% (~280 BTC) coins in the 328 network; reduces success rate of payments by  $16.0\% \sim 60.0\%$ ; increases fee of payments by 329  $4.5\% \sim 16.0\%$ ; increases average attempts of payments by  $42.0\% \sim 115.3\%$ ; and increase the 330 number of bankruptcy nodes by  $26.6\% \sim 109.4\%$ , where the amounts of payments range from 331 0.001 to 0.019 BTC. 332

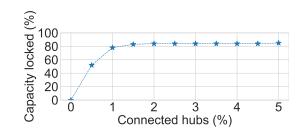
#### **5.1** Experimental setting

Implementation and simulation setting. We simulate and implement our attack 334 using Python 3.7.4 and NetworkX. For simplicity, we implement all algorithms sequentially. 335 Adversaries can use multi-threaded programming to speed up the algorithm if they prefer 336 efficiency. The topology provided by B'eres et al. [5] is the snapshot of the LN in 2019 (we 337 checked the network snapshot for 2021 and found the topology to be similar to 2019, so 338 we believe the results are similar of simulation on the 2021 snapshot). The snapshot also 339 includes the fee policy for each channel as well as the capacity. Since the balance distribution 340 characteristics of LN are not publicly available, we apply the random uniform distribution 341 for initialising the channels' balances similar to existing studies [5]. To amortise the bias 342 from randomness, we run each group of simulations with a certain strategy and custody level 343 for ten times. Our results show that the coefficient of variation for the quantitative impact 344 of the different balance distributions is only 1%. 345

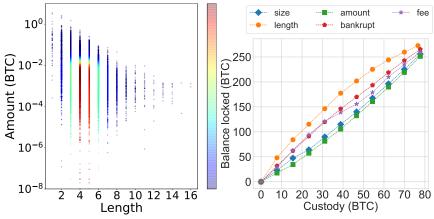
Payment routing mechanism. In the real-world scenario, payments may sometimes fail, 346 as nodes cannot know the real-time balances of channels they do not involve. LN introduces 347 a success probability mechanism to optimise the routing. Specifically, if intermediate node 348 A is unable to forward a payment because of insufficient balance, then it will return an 349 error to the sender. The sender will temporarily reduce the success probability of this node. 350 The path finding mechanism of LN is finding the shortest path on a weighted graph. For 351 simplicity, we set the weight as channel fee. The routing algorithm is the plain Dijkstra [8] 352 algorithm. When an attempt fails, we temporarily remove the first node on the current path 353 with insufficient balance and try again. A payment is allowed to try r times for finding a 354 viable path. If it finds a path within r tries, it is successful, otherwise we consider it fails. 355



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**Figure 3** The percentage of connected hubs v.s. locked capacity.



(a) Distribution of enumerated payments. (b) Locked balance.

**Figure 4** Characterisation of enumerated payments and the amount of locked balance.

We test attacks with different levels of custody of the adversary, i.e., Parameters. 356  $\{7.7, 15.4, \ldots, 77\}$  BTC, all ranking criteria in Section 3.5, and step ratio = 0.1 in Al-357 gorithm 2. When testing LN's liquidity, we pick batch size n = 7000 for testing liquidity(which 358 is identical to to existing works [5]), payment amount  $x_t \in \{0.001, 0.007, 0.013, 0.019\}$  BTC, 359 and payment retry times r = 10. In total, we ran 10 \* 4 \* 10 \* 5 = 2000 (retry times \* # of 360 payment amounts \* # of different custody levels \* # of strategies) simulations. We consider 361 the threshold of bankruptcy as 0.006 BTC, which is the average amount of payments in 362 LN [5]. 363

<sup>364</sup> **Choosing entry nodes.** We test the percentage of the capacity that the adversary can lock <sup>365</sup> by establishing channels with different numbers of richest nodes in LN. Figure 3 shows that, <sup>366</sup> by establishing channels with the top 1.5% (42) richest nodes, the enumerated payments take <sup>367</sup> ~ 83% of the capacity of the entire network. In addition, the total amount of enumerated <sup>368</sup> payments converges with the percentage of hubs increasing.

### **5.2** Impact of congestion attacks

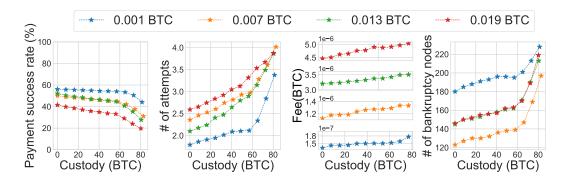
We simulate the general congestion attack with five strategies in §3.5, and evaluate their impact in terms of the five metrics defined in §4. Figure 4 summarises the results, and Figure 7 in Appendix A provides the results in more detail. For Figure 4(b) and 4, the baseline (when y = 0) is the scenario without any attack.

**Characterisation of enumerated payments.** As mentioned before, we establish channels with the 42 richest nodes in the network. Algorithm 1 enumerates 35,402 payments in total. Figure 4(a) visualises the distribution of these payments w.r.t. their amounts and



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**Figure 5** Overview of impacts of rank-by-fee.

<sup>377</sup> lengths. Red indicates there are many griefing paths under that path length and payment <sup>378</sup> amount, while blue means the opposite. The amount ranges from zero to 0.1 BTC, while <sup>379</sup> the length ranges from 1 to 13. On average, most payments have a length of  $3 \sim 6$  and an <sup>380</sup> amount of  $1e - 05 \sim 0.01$  BTC.

Locked balance. With a custody of 80 BTC (13% of the total capacity), an adversary can lock 280 BTC (47% of the total capacity) in LN, where rank-by-length is the most efficient strategy for locking balance. The average length of griefing payments is 3.8, which implies that there is room for optimisation of our path enumeration algorithm, since LN allows a maximum payment length of 20 hops.

Impact on different liquidity metrics. While Appendix A outlines the evaluation results of all attack strategies on all liquidity metrics, Figure 5 shows the result of the rank-by-fee strategy on LN as an example. With the rank-by-fee strategy and  $7 \sim 80$  BTC as custody, the attack can reduce the payments' success rates by  $21.4\% \sim 52.3\%$ , increase the fee by  $9.3\% \sim 27\%$ , increase the number of attempts by  $50\% \sim 88.7\%$ , and increase the number of bankrupt nodes by  $26.7\% \sim 60\%$ .

### 392 6 Discussion

Budget analysis. The budget of launching congestion attacks is twofold: 1) channel fee for establishing channels and 2) custody deposited into channels. When preparing for a congestion attack, the adversary needs to pay the transaction fee for opening channels. Transaction fee is negligible as analysed in §5.1. After the congestion attack, the custody is refunded as payments are expired. For LN, the required custody is 77 BTC (13% of the network capacity). In Bitcoin, there are more than 10,000 addresses with more than 157 BTC [2], making them having sufficient capacity to launch a congestion attack.

Profit from congestion attacks. The adversary can apply griefing on other nodes' 400 channels, so that more payments go through its controlled channels. To receive most fees 401 following this approach, the adversary redirects as many payments to its channels as possible. 402 Existing research [22] shows that the probability is proportional to the adversarial node's 403 betweenness centrality, while maximising the betweenness centrality by removing channels 404 can be formalised as the *destructive betweenness improvement* problem that is NP-hard [11]. 405 To our knowledge, there exists no approximation algorithm to solve this problem, and we 406 consider designing such algorism as future work. 407



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# **7** Related work and comparison

#### 409 7.1 Attacks on PCNs

Congestion attacks. Congestion attack was informally discussed by Lightning Network
community [1]. Mizrahi et al. [13] first systematically studied the congestion attack on PCNs.
In their proposed attack, the adversary makes a large number of small payments, in order to
make channels hit max\_concurrent\_htlcs, the maximum number of concurrent payments.
Tikhomirov et al. [21] used the same idea to lock the balance of the channel, but they only
grief a single channel at a time.

The two congestion attacks focus on a single attack liquidity metric, or put limitations 416 on the attack strategy, and therefore can be seen as special cases of our general congestion 417 attack. In addition, as their attacks focus on a single path or channel at a time, the adversary 418 has to establish new channels when attacking a new path or channel. Establishing a large 419 number of channels makes the adversary easier to be identified, and existing nodes may 420 not be willing to establish too many channels in a short time period. Moreover, to occupy 421 max\_concurrent\_htlcs, the adversary in their two attacks has to make a large number of 422 concurrent payments compared to our attack. This also makes the adversary's behaviour 423 easier to be identified. 424

Other attacks on PCNs. There have been attacks on PCNs with different goals. In the lockdown attack [15], the adversary griefs the victim's channels to isolate it from the network. In the hijacking attack [22], the adversary publishes channels with small fee to attract payments, and withhold all payments through its channels. Rohrer et al. [19] discussed two attacks, namely channel exhaustion and node isolation. While congestion attacks aim at paralysing the entire PCN, these three attacks aim at exhausting individual channels or isolating individual nodes.

### 432 7.2 Quantitative comparison with existing congestion attacks

We quantitatively compare existing congestion attacks [13, 21] with ours w.r.t. different 433 budget level of custody and channel fee and different max concurrent htlcs value distribution. 434 For both attacks, we simulate the capacity-first strategy. The strategy iterates the following 435 process: when a path is enumerated, calculate the total capacity of involved channels whose 436 max concurrent htlcs values have been filled, then remove these channels from the network. 437 Locking a channel by using max concurrent htlcs takes max concurrent htlcs \* 2 payments 438 (as a channel has two directions). The smallest payment amount is 5.46e-06 BTC (i.e. the 439 dust limit). Thus, the custody required for griefing a path is 2 \* max\_concurrent\_htlcs \* 440 5.46e-06 BTC. When enumerating a path, we check whether both ends have channels with 441 the adversary. If not, the adversary has to establish channels with them, leading to a fee of 442 0.0002 BTC ( $\sim 18.89$  USD at the time of writing). 443

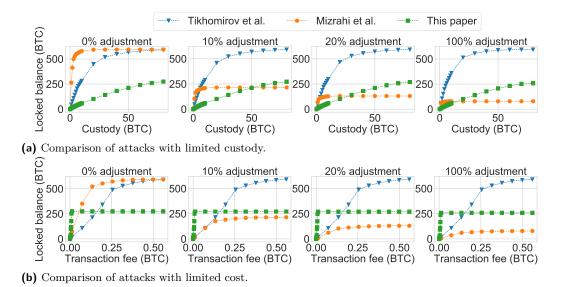
To quantify the impact of max\_concurrent\_htlcs, we test the locked capacity when different portions of channels adjust max\_concurrent\_htlcs. Given the size limit of Bitcoin transactions, the maximum value of max\_concurrent\_htlcs in Bitcoin's LN is 483. Thus, we assume the adjusted value of max\_concurrent\_htlcs is uniformly distributed in interval [1,483]. When the custody is limited, we assume the fee is unlimited, and vice versa.

Figure 6 shows the experimental results. Each experiment is repeated 10 times, and the variation of experimental results is about 2.4%. As the results are similar after the 20% channel adjustment, we skipped the simulation in 30%-90% for brevity. Figure 6(a) shows the performance of the three attacks under different custody. When all channels share



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**Figure 6** Comparison with congestion attack. x% adjustment means x% channels adjust their  $max\_concurrent\_htlcs$ .

the same max\_concurrent\_htlcs, Mizrahi et al.'s attack locks most capacity. When more 453 channels adjust max\_concurrent\_htlcs, the locked capacity becomes less. This is because 454 when channels in a path have different max\_concurrent\_htlcs values, the adversary can only 455 congest the channel with the smallest max concurrent htlcs in this path by spamming this 456 path only, making the strategy of Mizrahi et al. less effective. Meanwhile, Tikhomirov et 457 al.'s attack and our attack are not affected by max concurrent htlcs. This is because our 458 attack does not rely on max\_concurrent\_htlcs and the number of concurrent htlcs occupied 459 by our attack averaged only 3.8 per channel, and Tikhomirov et al.'s attack focuses on a 460 channel at a time. Figure 6(b) shows that, both Tikhomirov's and Mizrahi's attacks require 461 more transaction fee compared to our attack. This is because, in their attacks, the adversary 462 has to open a new channel when attacking a new path. With sufficient transaction fee, 463 Tikhomirov's locks more money compared to our attack. 464

To lock in 250 BTC of liquidity. The attack by Mizrahi et al et al. requires 1 BTC of custody and pays a transaction fee of 0.05 BTC, the attack by person Tikhomirov et al. requires 8 BTC of custody and a fee of 0.15 BTC, while our attack requires 65 BTC of custody and a fee of 0.008 BTC. Therefore, if the attacker is fee-sensitive, then our attack is preferred because our fees are 16% of and 5% of other two. Whereas, if the attacker has a restricted custody in hand, then the attack by person Mizrahi et al. is more preferred, as the custody required is only 1.5% of our attack.

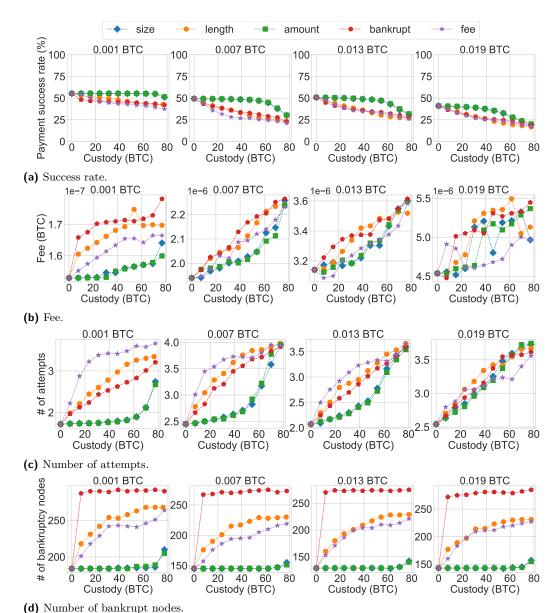
## 472 8 Conclusion

In this paper, we propose the general congestion attack on payment channel networks (PCNs). Our general congestion attack generalises the existing congestion attacks in terms of attack strategies, targeted metrics and optimisation techniques. We develop concrete steps for launching congestion attacks, and provide a generic method of quantifying PCNs' liquidity and effectiveness of congestion attacks. We evaluate our congestion attacks on Lightning Network – the first and most well-known PCN. Our evaluation results show that the congestion attack is cheap to launch and can greatly reduce the LN's liquidity.



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#### Detailed evaluation results and analysis Α 530

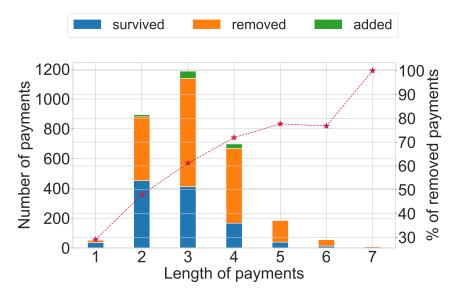
**Figure 7** Overview impact of all strategies.

Payment success rate (Figure 7(a)). Larger payments are less likely to succeed both 531 before and after the attack. This is because if a payment's amount is bigger, fewer channels 532 can route this payment. In addition, rank-by-fee is more effective than the other four 533 strategies in terms of the success rate. We suspect the reason to be that the rank-by-fee 534 strategy exhausts some channels that route a large number of payments by specifying a low 535 fee rate. 536

Rank-by-amount does not perform well when custody is limited and the amount of 537 payments is low. This is because the adversary starts from attacking channels with high 538



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**Figure 8** Status of payments. Red line indicates the percentage of removed payments. The bar shows the number of payments in different states at different lengths and the line chart shows the percentage of payments blocked at different lengths.

capacity, and therefore concerns less about payments with small amounts. As the amount of payment increases, the gap between rank-by-amount and other strategies begins to narrow, which confirms our thinking. In addition, rank-by-amount and rank-by-size have similar performance in success rate. This is because the distribution of payment lengths is centralised as shown in Figure 4(a), and the payment amount becomes the main factor determining the payment size.

Average fee of payments (Figure 7(b)). Note that we only count *survived* payments when calculating the fee. Payments with long paths are more likely to be blocked by the attack and removing them will decrease the average fee, offsetting the rise caused by the attack. Thus, the average fee of successful payments fail to reflect the increase in fees caused by the congestion.

We use the scenario where the payment amount is 0.013 BTC as an example. The result (in Figure 8) shows that, long payments are more likely to be influenced by our attack. Specifically, all 7-hop payments are blocked by the attack, while only about 30% of 1-hop payments are affected. This is because a long payment indicates that there are more channels in the path, so the payments are more likely to be attacked. These long path payments that fail as a result of the attack will result in a decrease in the average fee. Therefore, we only count *survived* payments to give a true evaluation of the effect of the attack.

The five strategies performed similarly in terms of fees, similar to payment success rates. The fee upward trend of the fee is because the attack will make channels with low fee unavailable, and payments will be re-directed to channels with high fee. However, sometimes the fee decreases as the custody increases. This is because the attack does not only block payments, but also add some payments, as we observe in §4.1. Both *blocked* payments and *added* payments may make some channels with low fee to be available again.

Congestion attacks can be launched together with the route hijacking attacks [22]. In a route hijacking attack, the adversary uses channels with zero fee to attract and hijack payments. Congestion attacks can increase payment fee, and therefore make the adversary's channels more attractive to payments.



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**Table 1** The proportion  $\mu$  of deceptive channels out of channels with capacities greater than the payment amount. Parameter  $\bar{a}$  is the number of attempts before the attack.

	0.001	0.007	0.013	0.019
$\mu$	15.0%	31.8%	31.2%	39.2%
$\bar{a}$	1.76	2.59	2.66	3.11

Average attempt times of payments (Figure 7(c)). When the amount of payments 567 increased from 0.007 BTC to 0.013 BTC, the number of attempts does not rise significantly. 568 In addition, the average number of attempts with 0.001 BTC is much lower than payments 569 with a bigger amount. We observe that the average number of attempts is proportional 570 to the number of "deceptive channels". We say a channel is deceptive for a payment with 571 amount x when the capacity of the channel is greater than x, but the balance is less than x. 572 The number of attempts depends on the ratio between the number of deceptive channels and 573 the number of channels with balance greater than the payment amount. As aforementioned, 574 we assume the balance of each channel is uniformly distributed. Under this assumption, 575 given the amount of simulated payments x, this proportion  $\mu$  is calculated as 576

$$\mu = \frac{\sum_{i=1}^{n} \frac{x}{c_i}}{n}$$

where *n* is the number of channels with capacities greater than *x*, and  $c_i$  is the *i*-th channel's capacity. Table 1 shows the trend of  $\mu$  and  $\bar{a}$  is consistent, which confirms our suspicions.

Number of bankruptcy nodes (Figure 7(d)). When it comes to bankruptcy rates, 580 the rank-by-bankrupt strategy is particularly effective in the number of bankruptcy nodes 581 and the success rate of small payments. Figure 7(d) compares their performance. The 582 results show that when the attack custody is limited, rank-by-bankrupt outperforms all other 583 strategies in terms of the number of bankruptcy nodes. The additional bankruptcy nodes 584 caused by rank-by-bankrupt have the characteristic of poor capacity. Specifically, the average 585 capacity of these victims is 0.02 BTC, while the average capacity of all nodes in the network 586 is 0.42 BTC, which is consistent with the theory we cited in §3.5. 587

