Traceable Constant-Size Multi-Authority Credentials

Chloé Hébant[1,2] and David Pointcheval[1,2]

[1] DIENS, École normale supérieure, CNRS, PSL University, Paris, France
[2] INRIA, Paris, France

Abstract Many attribute-based anonymous credential (ABC) schemes have been proposed allowing a user to prove the possession of some attributes, anonymously. They became more and more practical with, for the most recent papers, a constant-size credential to show a subset of attributes issued by a unique credential issuer. However, proving possession of attributes coming from K different credential issuers usually requires K independent credentials to be shown. Only attribute-based credential schemes from aggregatable signatures can overcome this issue.
In this paper, we propose new ABC schemes from aggregatable signatures with randomizable tags. We consider malicious credential issuers, with adaptive corruptions and collusions with malicious users. Whereas our constructions only support selective disclosures of attributes, to remain compact, our approach significantly improves the complexity in both time and memory of the showing of multiple attributes: for the first time, the cost for the prover is (almost) independent of the number of attributes *and* the number of credential issuers.
Whereas anonymous credentials require privacy of the user, we propose the first schemes allowing traceability.
We formally define an aggregatable signature scheme with (traceable) randomizable tags, which is of independent interest. We build concrete schemes from the recent linearly homomorphic signature scheme of PKC 20. As all the recent ABC schemes, our construction relies on signatures which unforgeability is proven in the bilinear generic group model.

Keywords: Anonymous credentials, aggregatable signatures, traceability

1 Introduction

In an anonymous credential scheme, a user asks to an organization (a credential issuer) a credential on an attribute, so that he can later claim its possession, even multiple times, but in an anonymous and unlinkable way.

Usually, a credential on one attribute is not enough and the user needs credentials on multiple attributes. Hence, the interest of an attribute-based anonymous credential scheme (ABC in short): depending on the construction, the user receives one credential per attribute or directly for a set of attributes. One goal is to be able to express relations between attributes (or at least selective disclosure), with *one showing*. As different attributes may have different meanings (e.g. a university delivers diploma while a city hall delivers a birth certification), there should be several credential issuers. Besides multi credential issuers, it can be useful to have a multi-show credential system to allow a user to prove an arbitrary number of times one credential still without breaking anonymity. For that, the showings are required to be unlinkable to each other.

Classically, a credential is a signature by the credential issuer of the attribute with the public key of the user. The latter is thus the only one able to prove the ownership with an interactive zero-knowledge proof of knowledge of the secret key. Anonymity is provided by the probabilistic encryption of the signature As many signature schemes with various interesting properties have been proposed, many ABC schemes have been designed with quite different approaches. We can gather them into two families: the ABC schemes where a credential is obtained on a set of attributes and then, according to the properties of the signature, it is possible either to prove the knowledge of a subset of the attributes (CL-signatures [CL03, CL04], blind signatures [BL13, FHS15]), or to modify some of the attributes to default values (sanitizable signatures [CL13]), or simply to remove them (unlinkable redactable signatures [CDHK15, San20], SPS-EQ with set commitments [FHS19]); and the ABC schemes where the user receives one

credential per attribute and then combines them (aggregatable signatures [CL11]). In the former family, whereas it is possible to efficiently show a subset of attributes issued in a unique credential, showing attributes coming from K different credential issuers requires K independent credentials to be proven. On the other hand, with aggregatable signatures, credentials on different attributes can be combined together even if they have been issued by different credential issuers. This leads to more compact schemes and this paper follows this latter approach.

Moreover, except some constructions based on blind signatures where the credentials can be shown only once, all ABC schemes allow multi-shows, exploiting randomizability properties of the signatures for anonymity and unlinkability of the showings. This avoids the need of encryption and heavy zero-knowledge proofs.

1.1 Our Contributions

Following the path of aggregatable signatures [CL11], our first contribution is the formalization of an aggregatable signature scheme with randomizable tags (ART-Sign) for which we propose a practical construction. With such a primitive, two signatures of different messages under different keys can be aggregated only if they are associated to the same tag. In our case, tags will eventually be like pseudonyms, but with some properties for being ephemeral (hence EphemerId scheme) and randomizable, even when they are associated to the same user.

However our goal is a compact ABC system, which is our second contribution: the EphemerId scheme generates keys for users, they will use for authentication. Public keys being randomizable, still for a same secret key, multiple authentications will remain unlinkable. In addition, these public keys will be used as (randomizable) tags with the above ART-Sign scheme when the credential issuer signs an attribute. Thanks to aggregation, multiple credentials for multiple attributes and from multiple credential issuers but under the same tag, and thus the same user, can be combined into a unique compact (constant-size) credential.

We achieve the optimal goal of constant-size multi-show credentials even for multiple attributes from multiple credential issuers and we stress that aggregation can be done on-the-fly, for any selection of attributes issued by multiple credential issuers: our scheme allows multi-show of any selective disclosure of attributes.

About security, whereas there exists a scheme proven in the universal composability (UC) framework [CDHK15], for our constructions, we consider a game-based security model for ABC inspired from [FHS19]. As we support different credentials issuers, we additionally consider malicious credential issuers, with adaptive corruptions, and collusion with malicious users. However, the keys need to be honestly generated, thus our proofs hold in the certified key setting. This is quite realistic, as this is enough to wait for a valid proof of knowledge of the secret key before certifying the public key. As all the recent ABC schemes, our constructions will rely on signature schemes proven in the bilinear generic group model.

Our last contribution is traceability, in the same vein as group signatures: whereas showings are anonymous, a tracing authority owns tracing keys for being able to link a credential to its owner. In such a case, we also consider malicious tracing authorities, with the non-frameability guarantee. As in [CL13] we thus define trace and judge algorithms to trace the defrauder and prove its identity to a judge. This excludes malicious behavior of the tracing authority.

1.2 Related Work

The most recent papers on attribute-based anonymous credential schemes are [FHS19, San20]. The former proposes the first constant-size credential to prove k-of-N attributes, with computational complexity in $O(N-k)$ for the prover and in $O(k)$ for the verifier. However, it only works for one credential issuer ($K=1$). The latter one improves this result enabling multiple showings of relations (r) of attributes. All the other known constructions allow, at best, selective (s) disclosures of attributes.

In [CL11], Canard and Lescuyer use aggregatable signatures to construct an ABC system. It is thus the closest to our approach. Instead of having *tags*, their signatures take *indices* as input. We follow a similar path but, we completely formalize this notion of tag/index with an EphemerId scheme. To our knowledge, aggregatable signatures are the only way to deal with multiple credential issuers but still showing a unique compact credential for the proof of possession of attributes coming from different credential issuers. However, the time-complexity of a prover during a verification depends on the number k of shown attributes. We solve this issue at the cost of a larger key for the credential issuers (but still in the same order as [FHS19, San20]) and a significantly better showing cost for the prover (also better than [FHS19, San20]). We can also note their tags/indices are 3 elements of $\mathbb{G}_1$, plus 2 elements of $\mathbb{G}_2$ and one element of $\mathbb{Z}_p$ which is much larger than our tags: only 3 elements in $\mathbb{G}_1$.

Scheme	P	T	k-of-N attributes from $K = 1$ credential issuer			
			|CI key| $\mathbb{G}_1, \mathbb{G}_2$	|Show| $\mathbb{G}_1, \mathbb{G}_2, (\mathbb{G}_T), \mathbb{Z}_p$	Prover exp., pairings	Verifier exp., pairings
[CL11]	s	✗	$\mathbf{1}, \mathbf{1}$	$16, 2, (4), 7$	$16\mathbb{G}_1 + 2\mathbb{G}_2 + 10\mathbb{G}_T$, $18 + k$	$12\mathbb{G}_1 + 20\mathbb{G}_T$, $18 + k$
[FHS19]	s	✗	$0, N$	$8, 1, 2$	$9\mathbb{G}_1 + 1\mathbb{G}_2, 0$	$4\mathbb{G}_1, k + 4$
[San20]	**r**	✗	$0, 2N + 1$	$2, 2, (1), 2$	$(2(N-k)+2)\mathbb{G}_1 + 2\mathbb{G}_2, 1$	$(k+1)\mathbb{G}_1 + 1\mathbb{G}_T, 5$
Sec. 6.1	s	✓	$0, 2k + 3$	$\mathbf{3}, \mathbf{0}, \mathbf{1}$	$\mathbf{6\mathbb{G}_1}, \mathbf{0}$	$\mathbf{4\mathbb{G}_1 + k\mathbb{G}_2}, \mathbf{3}$
Sec. 6.2	s	✓	$0, 2N + 2$	$\mathbf{3}, \mathbf{0}, \mathbf{1}$	$\mathbf{6\mathbb{G}_1}, \mathbf{0}$	$\mathbf{4\mathbb{G}_1 + 2N\mathbb{G}_2}, \mathbf{3}$

Scheme	$k = 1$-of-N attribute from K credential issuers			
	|CI key| $\mathbb{G}_1, \mathbb{G}_2$	|Show| $\mathbb{G}_1, \mathbb{G}_2, (\mathbb{G}_T), \mathbb{Z}_p$	Prover exp., pairings	Verifier exp., pairings
[CL11]	$\mathbf{K} \times (\mathbf{1}, \mathbf{1})$	$16, 2, (4), 7$	$16\mathbb{G}_1 + 2\mathbb{G}_2 + 10\mathbb{G}_T$, $18 + k$	$12\mathbb{G}_1 + 20\mathbb{G}_T$, $18 + k$
[FHS19]	$K \times (0, N)$	$K \times (8, 1, 2)$	$K \times (9\mathbb{G}_1 + 1\mathbb{G}_2, 0)$	$K \times (4\mathbb{G}_1, k + 4)$
[San20]	$K \times (0, 2N + 1)$	$K \times (2, 2, (1), 2)$	$K \times ((2(N-k)+2)\mathbb{G}_1 + 2\mathbb{G}_2, 1)$	$K \times ((k+1)\mathbb{G}_1 + 1\mathbb{G}_T, 5)$
Sec. 6.1	$K \times (0, 2k + 3)$	$\mathbf{3}, \mathbf{0}, \mathbf{1}$	$\mathbf{6\mathbb{G}_1}, \mathbf{0}$	$\mathbf{4\mathbb{G}_1 + k\mathbb{G}_2}, \mathbf{3}$
Sec. 6.2	$K \times (0, 2N + 2)$	$\mathbf{3}, \mathbf{0}, \mathbf{1}$	$\mathbf{6\mathbb{G}_1}, \mathbf{0}$	$\mathbf{4\mathbb{G}_1 + 2KN\mathbb{G}_2}, \mathbf{3}$

Figure 1. Comparison of different ABC systems.

On Figure 1, we provide some comparisons with the most efficient ABC schemes, where the column "P" (for policy) precises whether the scheme just allows selective disclosure of attributes (s) or relations between attributes (r). The column "T" (for traceability) checks whether traceability is possible or not. Then, "|CI key|" gives the size of the keys (public keys of the credential issuers) required to verify the credentials, "|Show|" is the communication bandwidth during a show, while "Prover" and "Verifier" are the computational cost during a show, for the prover and the verifier respectively. Bandwidths are in number of elements $\mathbb{G}_1$, $\mathbb{G}_2$, $\mathbb{G}_T$ and $\mathbb{Z}_p$. Computations are in number of exponentiations in $\mathbb{G}_1$, $\mathbb{G}_2$ and $\mathbb{G}_T$, and of pairings. We ignore multiplications. We denote N the global number of attributes owned by a user, k the number of attributes he wants to show and K the number of credential issuers involved in the issuing of the credentials. In the first table, we focus on the particular case of proving a credential with k attributes, among N attributes issued from 1 credential issuer. Our first scheme, from Section 6.1, is already the most efficient, but this is even better for a larger K, as shown in the second table. But this is for a limited number of attributes. Our second scheme, from Section 6.2 has similar efficiency, but with less limitations on the attributes. Note that both schemes have a constant-size communication for the showing of any number of attributes, and the computation cost for the prover is almost constant too (as we ignore multiplications). Our two instantiations are derived from the second linearly homomorphic signature scheme of [HPP20].

Very few papers deal with traceability: the first one [CL13] exploits sanitizable signatures, where the sanitizer can be traced back, but a closer look shows privacy weaknesses (see the Appendix A) and a more recent one [KL16] that has thereafter been broken [Ver17]. As a consequence, our scheme is the first traceable attribute-based anonymous credential scheme, hence the only one in the tables.

1.3 Organization

After precising some notations and reviewing classical definitions in Section 2, we informally describe, in Section 3, the two important primitives that we will use in our construction of anonymous credentials: the EphemerId and ART-Sign schemes. In Section 4, we provide the full definitions and a concrete instantiation. From that, we will be able to define and construct, in Section 5, our ABC scheme from EphemerId and ART-Sign schemes. The full instantiation is given in Section 6. Finally, traceability is defined and instantiated in Section 7.

2 Preliminaries

In this section, we recall the asymmetric pairing setting and some classical computational assumptions.

2.1 Notations

All along this paper, κ is the security parameter. We will consider an asymmetric bilinear setting $(\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, p, g, \mathfrak{g}, e)$, where $\mathbb{G}_1$, $\mathbb{G}_2$ and $\mathbb{G}_T$ are cyclic groups of prime order p (of length 2κ). The elements g and $\mathfrak{g}$ are generators of $\mathbb{G}_1$ and $\mathbb{G}_2$, respectively and e is a bilinear map from $\mathbb{G}_1 \times \mathbb{G}_2$ into $\mathbb{G}_T$, that is non-degenerated and efficiently computable. This is usually named a *pairing*.

For the sake of clarity, elements of $\mathbb{G}_2$ will be in Fraktur font. In addition, in all the public-key cryptographic primitives, keys will implicitly include the global parameters and secret keys will include the public keys.

Vectors will be denoted between brackets $[\ldots]$ and unions will be concatenations: $[a, b] \cup [a, c] = [a, b, a, c]$, keeping the ordering. On the other hand, sets will be denoted between parentheses $\{\ldots\}$, with possible repetitions: $\{a, b\} \cup \{a, c\} = \{a, a, b, c\}$ as in [San20], but without ordering.

2.2 Classical Assumptions and Useful Result

In an asymmetric bilinear setting $(\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, p, g, \mathfrak{g}, e)$, or just in a simple group $\mathbb{G}$, we can define the following assumptions.

Definition 1 (Discrete Logarithm (DL) Assumption). In a group $\mathbb{G}$ of prime order p, it states that for any generator g, given $y = g^x$, it is computationally hard to recover x.

Definition 2 (Decisional Diffie-Hellman (DDH) Assumption). In a group $\mathbb{G}$ of prime order p, it states that for any generator g, the two following distributions are computationally indistinguishable:

$$\mathcal{D}_{\mathsf{dh}}(g) = \{(g, g^x, h, h^x); h \xleftarrow{\$} \mathbb{G}, x, \xleftarrow{\$} \mathbb{Z}_p\}$$
$$\mathcal{D}^4_{\$}(g) = \{(g, g^x, h, h^y); h \xleftarrow{\$} \mathbb{G}, x, y, \xleftarrow{\$} \mathbb{Z}_p\}.$$

Definition 3 (Square Discrete Logarithm (SDL) Assumption). In a group $\mathbb{G}$ of prime order p, it states that for any generator g, given $y = g^x$ and $z = g^{x^2}$, it is computationally hard to recover x.

Definition 4 (Decisional Square Diffie-Hellman (DSqDH) Assumption). In a group $\mathbb{G}$ of prime order p, it states that for any generator g, the two following distributions are computationally indistinguishable:

$$\mathcal{D}_{\mathsf{sqdh}}(g) = \{(g, g^x, g^{x^2}), x \xleftarrow{\$} \mathbb{Z}_p\} \qquad \mathcal{D}^3_{\$}(g) = \{(g, g^x, g^y), x, y \xleftarrow{\$} \mathbb{Z}_p\}.$$

It is worth noticing that the DSqDH Assumption implies the SDL Assumption: if one can break SDL, from g, g^x, g^{x^2}, one can compute x and thus break DSqDH. A fortiori, this implies indistinguishability between the two distributions

$$\mathcal{D}_{\mathsf{sqdh}}(\mathbb{G}) = \{(g, g^x, g^{x^2}), g \xleftarrow{\$} \mathbb{G}, x \xleftarrow{\$} \mathbb{Z}_p\} \qquad \mathcal{D}^3_{\$}(\mathbb{G}) = \{(g_1, g_2, g_3) \xleftarrow{\$} \mathbb{G}^3\}.$$

In our construction we will use the following theorem on Square Diffie-Hellman tuples, stated and proven in [HPP20]:

Theorem 5. *Given n valid Square Diffie-Hellman tuples $(g_i, a_i = g_i^{w_i}, b_i = a_i^{w_i})$, together with w_i, for random $g_i \xleftarrow{\$} \mathbb{G}^*$ and $w_i \xleftarrow{\$} \mathbb{Z}_p^*$, outputting $(\alpha_i)_{i=1,\ldots,n}$ such that $(G = \prod g_i^{\alpha_i}, A = \prod a_i^{\alpha_i}, B = \prod b_i^{\alpha_i})$ is a valid Square Diffie-Hellman, with at least two non-zero coefficients α_i, is computationally hard under the DL assumption.*

Intuitively, from Square Diffie-Hellman tuples where the exponents are known but random and the bases are also known and random, it is impossible to construct a new Square Diffie-Hellman tuple melting the exponents. We refer to [HPP20] for the proof.

3 Overview of our New Primitives

The usual way to perform authentication is by presenting a certified public key and proving ownership, with a zero-knowledge proof of knowledge of the associated private key. The certified public key is essentially the signature by a Certification Authority (CA) on a public key and an identity pair, with a *standard* signature scheme. In case of attribute-based authentication, the attribute is signed together with the public key in the certificate. The latter thus signs two objects, with different goals, the public key associated to a private key, and the identity or an attribute.

In the same vein as labelled encryption schemes, we define tag-based signatures to dissociate the user-key which will be a provable tag and $\mathcal{A}\mathsf{ttr}$ which will be the signed message (attribute or identity). This flexibility will allow randomizability of one without affecting the other, leading to anonymous credentials.

3.1 Tag-based Signatures

For a pair $(\tilde{\tau}, \tau)$, where τ is a tag and $\tilde{\tau}$ corresponds to the secret part of the tag, one can define a new primitive called *tag-based signature*, where we assume all the used tags τ to be *valid* (either because they are all valid, or their validity can be checked):

- $\mathsf{Setup}(1^\kappa)$: Given a security parameter κ, it outputs the global parameter param, which includes the message space $\mathcal{M}$ and the tag space $\mathcal{T}$;
- $\mathsf{Keygen}(\mathsf{param})$: Given a public parameter param, it outputs a key pair $(\mathsf{sk}, \mathsf{vk})$;
- $\mathsf{GenTag}(\mathsf{param})$: Given a public parameter param, it generates a witness-word pair $(\tilde{\tau}, \tau)$;
- $\mathsf{Sign}(\mathsf{sk}, \tau, m)$: Given a signing key sk, a tag τ, and a message m, it outputs the signature σ under the tag τ;
- $\mathsf{VerifSign}(\mathsf{vk}, \tau, m, \sigma)$: Given a verification key vk, a tag τ, a message m and a signature σ, it outputs 1 if σ is valid relative to vk and τ, and 0 otherwise.

The security notion would expect no adversary able to forge, for any honest pair $(\mathsf{sk}, \mathsf{vk})$, a new signature for a pair (τ, m), for a valid tag τ, if the signature has not been generated using sk and the tag τ on the message m. Generically, $\tilde{\tau}$ can be sk and τ can be vk, then this is just a classical signature of m. Another case is when $\tilde{\tau} = \tau$, and then this can just be a classical signature of the message-pair (τ, m).

However more subtil situations can be handled: in our use-cases, τ will be a word for some language $\mathcal{L}$ representing the authorized users and $\tilde{\tau}$ a witness (for $\tau \in \mathcal{L}$). According to the language $\mathcal{L}$, which can be a strict subset of the whole set $\mathcal{T}$, one may have to prove the actual membership $\tau \in \mathcal{L}$ (the validity of the tag) for the validity of the signature. It might be important in the unforgeability security notion. On the other hand, one may also have to prove the knowledge of the witness $\tilde{\tau}$, in an interactive and zero-knowledge way for authentication.

The latter can be performed, using the interactive protocol $(\mathsf{ProveKTag}(\tilde{\tau}), \mathsf{VerifKTag}(\tau))$. This will be useful for the freshness in the authentication process. The former can also be proven using an interactive protocol $(\mathsf{ProveVTag}(\tilde{\tau}), \mathsf{VerifVTag}(\tau))$. However this verification can also be non-interactive or even public, without needing any private witness. The only requirement is that this proof or verification of membership should not reveal the private witness involved in the proof of knowledge, as the witness will be used for authentication.

Now the tag and the message are two distinct elements in the signature, we will introduce new properties for each of them:

- randomizable tags: if τ can be randomized, but still with an appropriate zero-knowledge proof of knowledge of $\tilde{\tau}$, one can get anonymous credentials, where τ is a randomizable public key and an attribute is signed;
- aggregatable signatures: one can aggregate signatures generated for different messages (attributes), even different keys (multi-authority) but all on the same tag τ.

By combining both properties, we will provide a compact scheme of attribute-based anonymous credentials. When a trapdoor allows to link randomized tags, one gets traceability.

3.2 Signatures with Randomizable Tags

As tags are seen as words in some language $\mathcal{L}$, randomizable tags will make sense for random-self reducible languages [TW87]: the word τ defined by a witness $\tilde{\tau}$ and some additional randomness r can be derived into another word $\tilde{\tau}'$ associated to $\tilde{\tau}'$ and r' (either r' only or both $\tilde{\tau}'$ and r' are uniformly random). When randomizing τ into τ', one must be able to keep track of the change from to update $\tilde{\tau}$ to $\tilde{\tau}'$ and the signatures. Formally, we will require to have the three algorithms:

$\mathsf{RandTag}(\tau)$: Given a tag τ as input, it outputs a new tag τ' and the randomization link $\rho_{\tau\to\tau'}$;

$\mathsf{DerivWitness}(\tilde{\tau}, \rho_{\tau\to\tau'})$: Given a witness $\tilde{\tau}$ (associated to the tag τ) and a randomization link between τ and a tag τ' as input, it outputs a witness $\tilde{\tau}'$ for the tag τ';

$\mathsf{DerivSign}(\mathsf{vk}, \tau, m, \sigma, \rho_{\tau\to\tau'})$: Given a valid signature σ on tag τ and message m, and $\rho_{\tau\to\tau'}$ the randomization link between τ and another tag τ', it outputs a new signature σ' on the message m and the new tag τ'. Both signatures are under the same key vk.

From a valid witness-word pair $(\tilde{\tau}, \tau) \leftarrow \mathsf{GenTag}(\mathsf{param})$, if $(\tau', \rho) \leftarrow \mathsf{RandTag}(\tau)$ and $\tilde{\tau}' \leftarrow \mathsf{DerivWitness}(\tilde{\tau}, \rho)$ then $(\tilde{\tau}', \tau')$ should also be a valid witness-word pair.

In addition, for compatibility with the tag and correctness of the signature scheme, we require that for all honestly generated keys $(\mathsf{sk}, \mathsf{vk}) \leftarrow \mathsf{Keygen}(\mathsf{param})$, all tags $(\tilde{\tau}, \tau) \leftarrow \mathsf{GenTag}(\mathsf{param})$, and all messages m, if $\sigma \leftarrow \mathsf{Sign}(\mathsf{sk}, \tau, m)$, $(\tau', \rho) \leftarrow \mathsf{RandTag}(\tau)$ and $\sigma' \leftarrow \mathsf{DerivSign}(\mathsf{vk}, \tau, m, \sigma, \rho)$, then the algorithm $\mathsf{VerifSign}(\mathsf{vk}, \tau', m, \sigma')$ should output 1.

For privacy reasons, in case of probabilistic signatures, it will not be enough to just randomize the tag, but the random coins too:

$\mathsf{RandSign}(\mathsf{vk}, \tau, m, \sigma)$: Given a valid signature σ on tag τ and message m, it outputs a new signature σ' on the same message m and tag τ.

Correctness extends the above one, where the algorithm $\mathsf{VerifSign}(\mathsf{vk}, \tau', m, \sigma'')$ should output 1 with $\sigma'' \leftarrow \mathsf{RandSign}(\mathsf{vk}, \tau', m, \sigma')$. One additionally expects unlinkability: the following distributions are (computationally) indistinguishable, for any vk and m (possibly chosen by the adversary), where for $i = 0, 1$, $(\tilde{\tau}_i, \tau_i) \leftarrow \mathsf{GenTag}(1^\kappa)$, $\sigma_i \leftarrow \mathsf{Sign}(\mathsf{sk}, \tau_i, m)$, $(\tau'_i, \rho_i) \leftarrow \mathsf{RandTag}(\tau_i)$, $\sigma'_i \leftarrow \mathsf{DerivSign}(\mathsf{vk}, \tau_i, m, \sigma_i, \rho_i)$ and $\sigma''_i \leftarrow \mathsf{RandSign}(\mathsf{vk}, \tau'_i, m, \sigma'_i)$:

$$\mathcal{D}_0 = \{(m, \mathsf{vk}, \tau_0, \sigma_0, \tau'_0, \sigma''_0, \tau_1, \sigma_1, \tau'_1, \sigma''_1)\} \qquad \mathcal{D}_1 = \{(m, \mathsf{vk}, \tau_0, \sigma_0, \tau'_1, \sigma''_1, \tau_1, \sigma_1, \tau'_0, \sigma''_0)\}.$$

3.3 Aggregatable Signatures

Boneh et al. [BGLS03] remarked it was possible to aggregate the BLS signature [BLS01], we will follow this path, but for tag-based signatures, with possible aggregation only between signatures with the same tag, in a similar way as the indexed aggregated signatures [CL11]. We will even consider aggregation of public keys, which can either be a simple concatenation or a more evolved combination as in [BDN18]. Hence, an aggregatable (tag-based) signature scheme ($\mathsf{Aggr\text{-}Sign}$) is a signature scheme with the algorithms:

$\mathsf{AggrKey}(\{\mathsf{vk}_j\}_{j=1}^{\ell})$: Given ℓ verification keys vk_j, it outputs an aggregated verification key avk;

$\mathsf{AggrSign}(\tau, (\mathsf{vk}_j, m_j, \sigma_j)_{j=1}^{\ell})$: Given ℓ signed messages m_j in σ_j under vk_j and the same tag τ, it outputs a signature σ on the message-set $\vec{M} = \{m_j\}_{j=1}^{\ell}$ under the tag τ and aggregated verification key avk.

We remark that keys can evolve (either in a simple concatenation or a more compact way) but messages also become sets. While we will still focus on signing algorithm of a single message with a single key, we have to consider verification algorithms on message-sets and for aggregated verification keys. In the next section, we combine aggregation with randomizable tags, and we will handle verification for message-sets.

Correctness of an aggregatable (tag-based) signature scheme requires that for any valid tag-pair $(\tilde{\tau}, \tau)$ and honestly generated keys $(\mathsf{sk}_j, \mathsf{vk}_j) \leftarrow \mathsf{Keygen}(\mathsf{param})$, if $\sigma_j = \mathsf{Sign}(\mathsf{sk}_j, \tau, m_j)$ are valid signatures for $j = 1, \cdots, \ell$, then for both key $\mathsf{avk} \leftarrow \mathsf{AggrKey}(\{\mathsf{vk}_j\}_{j=1}^{\ell})$ and signature $\sigma = \mathsf{AggrSign}(\tau, (\mathsf{vk}_j, m_j, \sigma_j)_{j=1}^{\ell})$, the verification $\mathsf{VerifSign}(\mathsf{avk}, \tau, \{m_j\}_{j=1}^{\ell}, \sigma)$ should output 1.

4 Aggregatable Signatures with Randomizable Tags

After the informal presentation of our new primitive, we describe the full definition of aggregatable signature scheme with randomizable tags. We will then provide a concrete construction that we will extend to attribute-based anonymous credentials. While the compactness of the credentials will exploit the aggregation of signature, as in [CL11], privacy will rely on the randomizability of the tags. But their specific format will allow more compact anonymous credentials.

4.1 Anonymous Ephemeral Identities

As our randomizable tags will be used as ephemeral identities (ephemeral key pairs), we denote them $\mathsf{EphemerId}$:

Definition 6 ($\mathsf{EphemerId}$). An $\mathsf{EphemerId}$ scheme consists of the algorithms:

$\mathsf{Setup}(1^\kappa)$: Given a security parameter κ, it outputs the global parameter param, which includes the tag space $\mathcal{T}$;

$\mathsf{GenTag}(\mathsf{param})$: Given a public parameter param, it outputs a tag τ and its secret part $\tilde{\tau}$;
$(\mathsf{ProveVTag}(\tilde{\tau}), \mathsf{VerifVTag}(\tau))$: This (possibly interactive) protocol corresponds to the verification of the tag τ. At the end of the protocol, the verifier outputs 1 if it accepts τ as a valid tag and 0 otherwise;
$\mathsf{RandTag}(\tau)$: Given a tag τ as input, it outputs a new tag τ' and the randomization link $\rho_{\tau\to\tau'}$ between τ and τ';
$\mathsf{DerivWitness}(\tilde{\tau}, \rho_{\tau\to\tau'})$: Given a witness $\tilde{\tau}$ (associated to the tag τ) and a link between the tags τ and τ' as input, it outputs a witness $\tilde{\tau}'$ for the tag τ';
$(\mathsf{ProveKTag}(\tilde{\tau}), \mathsf{VerifKTag}(\tau))$: This optional interactive protocol corresponds to the proof of knowledge of $\tilde{\tau}$. At the end of the protocol, the verifier outputs 1 if it accepts the proof and 0 otherwise.

The security notions are the usual properties of zero-knowledge proofs for the two protocols $(\mathsf{ProveKTag}(\tilde{\tau}), \mathsf{VerifKTag}(\tau))$ and $(\mathsf{ProveVTag}(\tilde{\tau}), \mathsf{VerifVTag}(\tau))$, with zero-knowledge and soundness. But the $\mathsf{RandTag}$ must also randomize the tag τ within an equivalence class, in an unlinkable way:

- Correctness: the language $\mathcal{L} \subset \mathcal{T}$ might be split in equivalence classes (denoted $\sim$, with possibly a unique huge class), then for any τ issued from GenTag and $\tau' \leftarrow \mathsf{RandTag}(\tau)$, we must have $\tau' \sim \tau$;
- Soundness: the verification process for the validity of the tag should not accept an invalid tag (not in the language);
- Knowledge Soundness: in case of the optional proof of knowledge, extraction of the witness should be possible when the verifier accepts the proof with non-negligible probability;
- Zero-knowledge: the proof of validity and the proof of knowledge should not reveal any information about the witness;
- Unlinkability: for any pair (τ_1, τ_2) issued from GenTag, the two distributions $\{(\tau_1, \tau_2, \tau_1', \tau_2')\}$ and $\{(\tau_1, \tau_2, \tau_2', \tau_1')\}$, where $\tau_1' \leftarrow \mathsf{RandTag}(\tau_1)$ and $\tau_2' \leftarrow \mathsf{RandTag}(\tau_2)$, must be (computationally) indistinguishable.

In the case of unique equivalence class for τ, one can expect perfect unlinkability. In case of multiple equivalence classes for τ, these classes should be computationally indistinguishable to provide unlinkability.

4.2 Aggregatable Signatures with Randomizable Tags

We can now provide the formal definition of an aggregatable signature scheme with randomizable tags, where some algorithms exploit compatibility between the $\mathsf{EphemerId}$ scheme and the signature scheme:

Definition 7 (Aggregatable Signatures with randomizable tags (ART-Sign)). An ART-Sign scheme, associated to an $\mathsf{EphemerId}$ scheme $\mathcal{E} = (\mathsf{Setup}, \mathsf{GenTag}, (\mathsf{ProveVTag}, \mathsf{VerifVTag}), \mathsf{RandTag}, \mathsf{DerivWitness})$ consists of the algorithms $(\mathsf{Setup}, \mathsf{Keygen}, \mathsf{Sign}, \mathsf{AggrKey}, \mathsf{AggrSign}, \mathsf{DerivSign}, \mathsf{RandSign}, \mathsf{VerifSign})$:

$\mathsf{Setup}(1^\kappa)$: Given a security parameter κ, it runs $\mathcal{E}.\mathsf{Setup}$ and outputs the global parameter param, which includes $\mathcal{E}.\mathsf{param}$ with the tag space $\mathcal{T}$, and extends it with the message space $\mathcal{M}$;
$\mathsf{Keygen}(\mathsf{param})$: Given a public parameter param, it outputs a key-pair $(\mathsf{sk},\mathsf{vk})$;
$\mathsf{Sign}(\mathsf{sk}, \tau, m)$: Given a signing key, a valid tag τ, and a message $m \in \mathcal{M}$, it outputs the signature σ;
$\mathsf{AggrKey}(\{\mathsf{vk}_j\}_{j=1}^{\ell})$: Given ℓ verification keys vk_j, it outputs an aggregated verification key avk;

$\mathsf{AggrSign}(\tau, (\mathsf{vk}_j, m_j, \sigma_j)_{j=1}^{\ell})$: Given ℓ signed messages m_j in σ_j under vk_j and the same valid tag τ, it outputs a signature σ on the message-set $\vec{M} = \{m_j\}_{j=1}^{\ell}$ under the tag τ and aggregated verification key avk;

$\mathsf{VerifSign}(\mathsf{avk}, \tau, \vec{M}, \sigma)$: Given a verification key avk, a valid tag τ, a message-set $\vec{M}$ and a signature σ, it outputs 1 if σ is valid relative to avk and τ, and 0 otherwise;

$\mathsf{DerivSign}(\mathsf{avk}, \tau, \vec{M}, \sigma, \rho_{\tau\to\tau'})$: Given a signature σ on a message-set $\vec{M}$ under a valid tag τ and aggregated verification key avk, and the randomization link $\rho_{\tau\to\tau'}$ between τ and another tag τ', it outputs a signature σ' on the message-set $\vec{M}$ under the new tag τ' and the same key avk;

$\mathsf{RandSign}(\mathsf{avk}, \tau, \vec{M}, \sigma)$: Given a signature σ on a message-set $\vec{M}$ under a valid tag τ and aggregated verification key avk, it outputs a new signature σ' on the message-set $\vec{M}$ and the same tag τ.

We stress that all the tags must be valid.

Note that using algorithms from $\mathcal{E}$, tags are randomizable at any time, and signatures adapted and randomized, even after an aggregation: avk and $\vec{M}$ can either be single key and message or aggregations of keys and messages. One can remark that only protocol $(\mathsf{ProveVTag}, \mathsf{VerifVTag})$ from $\mathcal{E}$ is involved in the ART-Sign scheme, as one just needs to check the validity of the tag, not the ownership. The latter will be useful in anonymous credentials with fresh proof of ownership.

Unforgeability. In the Chosen-Message Unforgeability security game, the adversary has unlimited access to the following oracles, with lists KList and TList initially empty:

- $\mathcal{O}\mathsf{GenTag}()$ outputs the tag τ and keeps track of the associated witness $\tilde{\tau}$, with $(\tilde{\tau}, \tau)$ appended to TList;
- $\mathcal{O}\mathsf{Keygen}()$ outputs the verification key vk and keeps track of the associated signing key sk, with $(\mathsf{sk}, \mathsf{vk})$ appended to KList;
- $\mathcal{O}\mathsf{Sign}(\tau, \mathsf{vk}, m)$, for $(\tilde{\tau}, \tau) \in \mathsf{TList}$ and $(\mathsf{sk}, \mathsf{vk}) \in \mathsf{KList}$, outputs $\mathsf{Sign}(\mathsf{sk}, \tau, m)$.

It should not be possible to generate a signature that falls outside the range of $\mathsf{DerivSign}$, $\mathsf{RandSign}$, or $\mathsf{AggrSign}$:

Definition 8 (Unforgeability for ART-Sign). An ART-Sign scheme is said unforgeable if, for any adversary $\mathcal{A}$ that, given signatures σ_i for tuples $(\tau_i, \mathsf{vk}_i, m_i)$ of its choice but for τ_i and vk_i issued from the GenTag and Keygen algorithms respectively (for Chosen-Message Attacks), outputs a tuple $(\mathsf{avk}, \tau, \vec{M}, \sigma)$ where both τ is a valid tag and σ is a valid signature w.r.t. $(\mathsf{avk}, \tau, \vec{M})$, there exists a subset J of the signing queries with a common tag $\tau' \in \{\tau_i\}_i$ such that $\tau \sim \tau'$, $\forall j \in J, \tau_j = \tau'$, avk is an aggregated key of $\{\mathsf{vk}_j\}_{j\in J}$, and $\vec{M} = \{m_j\}_{j\in J}$, with overwhelming probability.

Since there are multiple secrets, we can consider corruptions of some of them:

- $\mathcal{O}\mathsf{CorruptTag}(\tau)$, for $(\tilde{\tau}, \tau) \in \mathsf{TList}$, outputs $\tilde{\tau}$;
- $\mathcal{O}\mathsf{Corrupt}(\mathsf{vk})$, for $(\mathsf{sk}, \mathsf{vk}) \in \mathsf{KList}$, outputs sk.

The forgery should not involve a corrupted key (but corrupted tags are allowed). Note again that all the tags are valid (either issued from GenTag or verified). In the unforgeability security notion, some limitations might be applied to the signing queries: one-time queries (for a given tag-key pair) or a bounded number of queries.

Unlinkability. Randomizability of both the tag and the signature are expected to provide anonymity, with some unlinkability property:

Definition 9 (Unlinkability for ART-Sign). An ART-Sign scheme is said unlinkable if, for any avk and $\vec{M}$, no adversary $\mathcal{A}$ can distinguish the distributions $\mathcal{D}_0$ and $\mathcal{D}_1$, where for $i = 0, 1$, we have $(\tilde{\tau}_i, \tau_i) \leftarrow \mathsf{GenTag}(1^\kappa)$, $(\tau'_i, \rho_i) \leftarrow \mathsf{RandTag}(\tau_i)$, σ_i is any valid signature of $\vec{M}$ under τ_i and vk, $\sigma'_i \leftarrow \mathsf{DerivSign}(\mathsf{avk}, \tau_i, \vec{M}, \sigma_i, \rho_i)$ and $\sigma''_i \leftarrow \mathsf{RandSign}(\mathsf{avk}, \tau'_i, \vec{M}, \sigma'_i)$:

$$\mathcal{D}_0 = \{(\vec{M}, \mathsf{avk}, \tau_0, \sigma_0, \tau'_0, \sigma''_0, \tau_1, \sigma_1, \tau'_1, \sigma''_1)\} \quad \mathcal{D}_1 = \{(\vec{M}, \mathsf{avk}, \tau_0, \sigma_0, \tau'_1, \sigma''_1, \tau_1, \sigma_1, \tau'_0, \sigma''_0)\}.$$

4.3 One-Time ART-Sign Scheme with Square Diffie-Hellman Tags (SqDH)

Our construction will provide an aggregatable signature with randomizable tags based on the second linearly homomorphic signature scheme of [HPP20].

Description of the EphemerId Scheme. With tags in $\mathcal{T} = \mathbb{G}_1^3$, in an asymmetric bilinear setting $(\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, p, g, \mathfrak{g}, e)$, and τ is a Square Diffie-Hellman tuple $(h, h^{\tilde{\tau}}, h^{\tilde{\tau}^2})$, one can define the SqDH EphemerId scheme:

- $\mathsf{Setup}(1^\kappa)$: Given a security parameter κ, let $(\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, p, g, \mathfrak{g}, e)$ be an asymmetric bilinear setting, where g and $\mathfrak{g}$ are random generators of $\mathbb{G}_1$ and $\mathbb{G}_2$ respectively. The set of tags is $\mathcal{T} = \mathbb{G}_1^3$. We then define $\mathsf{param} = (\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, p, g, \mathfrak{g}, e; \mathcal{T})$;
- $\mathsf{GenTag}(\mathsf{param})$: Given a public parameter param, it randomly chooses a generator $h \xleftarrow{\$} \mathbb{G}_1^*$ and outputs $\tilde{\tau} \xleftarrow{\$} \mathbb{Z}_p^*$ and $\tau = (h, h^{\tilde{\tau}}, h^{\tilde{\tau}^2}) \in \mathbb{G}_1^3$.
- $\mathsf{ProveVTag}(\tilde{\tau}), \mathsf{VerifVTag}(\tau)$: The prover constructs the proof $\pi = \mathsf{proof}(\tilde{\tau} : \tau = (h, h^{\tilde{\tau}}, h^{\tilde{\tau}^2}))$ (see the Appendix C.2 for the Groth-Sahai [GS08] proof). The verifier outputs 1 if it accepts the proof and 0 otherwise.
- $\mathsf{RandTag}(\tau)$: Given a tag τ as input, it chooses $\rho_{\tau \to \tau'} \xleftarrow{\$} \mathbb{Z}_p$ and constructs $\tau' = \tau^{\rho_{\tau \to \tau'}}$ the derived tag. It outputs $(\tau', \rho_{\tau \to \tau'})$.
- $\mathsf{DerivWitness}(\tilde{\tau}, \rho_{\tau \to \tau'})$: The derived witness remains unchanged: $\tilde{\tau}' = \tilde{\tau}$.

Valid tags are Square Diffie-Hellman pairs in $\mathbb{G}_1$:

$$\mathcal{L} = \{(h, h^x, h^{x^2}), h \in \mathbb{G}_1^*, x \in \mathbb{Z}_p^*\} = \cup_{x \in \mathbb{Z}_p^*} \mathcal{L}_x \qquad \mathcal{L}_x = \{(h, h^x, h^{x^2}), h \in \mathbb{G}_1^*\}$$

The randomization does not affect the exponents, hence there are $p - 1$ different equivalence classes $\mathcal{L}_x$, for all the non-zero exponents $x \in \mathbb{Z}_p^*$, and correctness is clearly satisfied within equivalence classes. The validity check (see the Appendix C.2) is sound as the Groth-Sahai commitment is in the perfectly binding setting. Such tags also admit an interactive Schnorr-like zero-knowledge proof of knowledge of the exponent $\tilde{\tau}$ for $(\mathsf{ProveKTag}(\tilde{\tau}), \mathsf{VerifKTag}(\tau))$ which also provides extractability (knowledge soundness). Under the DSqDH and DL assumptions, given the tag τ, it is hard to recover the exponent $\tilde{\tau} = x$. The tags, after randomization, are uniformly distributed in the equivalence class, and under the DSqDH-assumption, each class is indistinguishable from $\mathbb{G}_1^3$, and thus one has unlinkability.

Description of the One-Time SqDH-based ART-Sign Scheme. The above EphemerId scheme can be extended into an ART-Sign scheme where implicit vector messages are signed. As the aggregation can be made on signatures of messages under the same tag but from various signers, the description is given for signers indexed by j and one-component messages indexed by (j, i). However, the scheme needs to be state-full as there is the limitation for a signer j not to sign more than one message by index (j, i) for a given tag: a signer must use two different indices to sign two messages for one tag.

- $\mathsf{Setup}(1^\kappa)$: It extends the above setup with the set of messages $\mathcal{M} = \mathbb{Z}_p$;

$\mathsf{Keygen}(\mathsf{param})$: Given the public parameters param, it outputs the signing and verification keys

$$\mathsf{sk}_{j,i} = (\ \mathsf{SK}_j = [\ t,\ u,\ v\ ],\ \mathsf{SK}'_{j,i} = [\ r_i,\ s_i\ ]\ ) \xleftarrow{\$} \mathbb{Z}_p^5,$$
$$\mathsf{vk}_{j,i} = (\ \mathsf{VK}_j = [\ \mathfrak{g}^t, \mathfrak{g}^u, \mathfrak{g}^v\ ],\ \mathsf{VK}'_{j,i} = [\ \mathfrak{g}^{r_i}, \mathfrak{g}^{s_i}\ ]\ ) \in \mathbb{G}_2^5.$$

Note that one could dynamically add new $\mathsf{SK}'_{j,i}$ and $\mathsf{VK}'_{j,i}$ to sign implicit vector messages: $\mathsf{sk}_j = \mathsf{SK}_j \cup [\mathsf{SK}'_{j,i}]_i$, $\mathsf{vk}_j = \mathsf{VK}_j \cup [\mathsf{VK}'_{j,i}]_i$;

$\mathsf{Sign}(\mathsf{sk}_{j,i}, \tau, m)$: Given a signing key $\mathsf{sk}_{j,i} = [t, u, v, r, s]$, a message $m \in \mathbb{Z}_p$ and a public tag $\tau = (\tau_1, \tau_2, \tau_3)$, it outputs the signature

$$\sigma = \tau_1^{t+r+ms} \times \tau_2^u \times \tau_3^v.$$

$\mathsf{AggrKey}(\{\mathsf{vk}_{j,i}\}_{j,i})$: Given verification keys $\mathsf{vk}_{j,i}$, it outputs the aggregated verification key $\mathsf{avk} = [\mathsf{avk}_j]_j$, with $\mathsf{avk}_j = \mathsf{VK}_j \cup [\mathsf{VK}'_{j,i}]_i$ for each j;

$\mathsf{AggrSign}(\tau, (\mathsf{vk}_{j,i}, m_{j,i}, \sigma_{j,i})_{j,i})$: Given tuples of verification key $\mathsf{vk}_{j,i}$, message $m_{j,i}$ and signature $\sigma_{j,i}$ all under the same tag τ, it outputs the signature $\sigma = \prod_{j,i} \sigma_{j,i}$ of the concatenation of the messages verifiable with $\mathsf{avk} \leftarrow \mathsf{AggrKey}(\{\mathsf{vk}_{j,i}\}_{j,i})$;

$\mathsf{DerivSign}(\mathsf{avk}, \tau, \vec{M}, \sigma, \rho_{\tau\to\tau'})$: Given a signature σ on tag τ and a message-set $\vec{M}$, and $\rho_{\tau\to\tau'}$ the randomization link between τ and another tag τ', it outputs $\sigma' = \sigma^{\rho_{\tau\to\tau'}}$;

$\mathsf{RandSign}(\mathsf{avk}, \tau, \vec{M}, \sigma)$: The scheme being deterministic, it returns σ;

$\mathsf{VerifSign}(\mathsf{avk}, \tau, \vec{M}, \sigma)$: Given a valid tag $\tau = (\tau_1, \tau_2, \tau_3)$, an aggregated verification key $\mathsf{avk} = [\mathsf{avk}_j]$ and a message-set $\vec{M} = [\mathsf{m}_j]$, with both for each j, $\mathsf{avk}_j = \mathsf{VK}_j \cup [\mathsf{VK}'_{j,i}]_i$ and $\mathsf{m}_j = [m_{j,i}]_i$, and a signature σ, one checks if the following equality holds or not, where $n_j = \#\{\mathsf{VK}'_{j,i}\}$:

$$e(\sigma, \mathfrak{g}) = e\left(\tau_1, \prod_j \mathsf{VK}_{j,1}{}^{n_j} \times \prod_i \mathsf{VK}'_{j,i,1} \cdot \mathsf{VK}'_{j,i,2}{}^{m_{j,i}}\right) \times e\left(\tau_2, \prod_j \mathsf{VK}_{j,2}{}^{n_j}\right) \times e\left(\tau_3, \prod_j \mathsf{VK}_{j,3}{}^{n_j}\right).$$

In case of similar public keys in the aggregation (a unique index j), $\mathsf{avk} = \mathsf{VK} \cup [\mathsf{VK}'_i]_i$ and verification becomes, where $n = \#\{\mathsf{VK}'_i\}$,

$$e(\sigma, \mathfrak{g}) = e\left(\tau_1, \mathsf{VK}_1{}^n \times \prod_{i=1}^{n} \mathsf{VK}'_{i,1} \cdot \mathsf{VK}'_{i,2}{}^{\vec{M}_i}\right) \times e\left(\tau_2, \mathsf{VK}_2{}^n\right) \times e\left(\tau_3, \mathsf{VK}_3{}^n\right).$$

Recall that the validity of the tag has to be verified, either with a proof of knowledge of the witness (as it will be the case in the ABC scheme, or with the proof $\pi = \mathsf{proof}(\tilde{\tau} : \tau = (h, h^{\tilde{\tau}}, h^{\tilde{\tau}^2}))$ (see the Appendix C.2 for the Groth-Sahai [GS08] proof).

Security of the One-Time SqDH-based ART-Sign Scheme. As argued in [HPP20], the signature scheme defined above is unforgeable in the generic group model [Sho97], if signing queries are asked at most once per tag-index pair:

Theorem 10. *The One-Time SqDH-based ART-Sign is unforgeable with one signature only per index, for a given tag, even with adaptive corruptions of keys and tags, in the generic group model.*

Proof. As argued in [HPP20], when the bases of the tags are *random*, even if the exponents are known, the signature that would have signed messages $\vec{M} = (g, g^{m_1}, \ldots, g, g^{m_n})$ is an unforgeable linearly-homomorphic signature. This means it is only possible to linearly combine signatures with the same tag. As issued signatures are on pairs (g, g^{m_i}), under a different pair

of keys for each such signed pair (whether they are from the same global signing key SK or not, as we exclude repetitions for an index), which can be seen as tuples $(1, 1, \ldots, g, g^{m_i}, \ldots, 1, 1)$, completed with 1's, the invariant generators g imply coefficients 0 and 1 in the linear combination: all the pairs (g, g^{m_i}) have been signed under the same tag. This proves unforgeability, even with corruptions of the tags, but without repetitions of tag-index. One can also consider corruptions of the signing keys, as they are all independent: one just needs to guess under which key will be generated the forgery.

About unlinkability, it relies on the DSqDH assumption, but between credentials that contain the same messages at the same shown indices (the same message-vector $\vec{M}$):

Theorem 11. *The One-Time SqDH-based ART-Sign, with message-vectors, is unlinkable under the* DSqDH *assumption.*

Proof. As already noticed, the tags are randomizable among all the square Diffie-Hellman triples with the same exponent, which are indistinguishable from random triples in $\mathbb{G}_1^3$, so for any pair of tags $(\tilde{\tau}_i, \tau_i) \leftarrow \mathsf{GenTag}(1^\kappa)$, for $i = 0, 1$, when randomized into τ_i' respectively, the distributions $(\tau_0, \tau_1, \tau_0', \tau_1')$ and $(\tau_0, \tau_1, \tau_1', \tau_0')$ are indistinguishable under the DSqDH assumption. For any avk and $\vec{M}$, the signatures are deterministic and unique for a tag τ, so they are functions (even if not efficiently computable) of $(\mathsf{avk}, \tau, \vec{M})$, so the distributions $(\vec{M}, \mathsf{avk}, \tau_0, \sigma_0, \tau_1, \sigma_1, \tau_0', \sigma_0', \tau_1', \sigma_1)$ and $(\vec{M}, \mathsf{avk}, \tau_0, \sigma_0, \tau_1, \sigma_1, \tau_1', \sigma_1', \tau_0', \sigma_0)$ are also indistinguishable under the DSqDH assumption. No need of randomization of the signatures.

4.4 Bounded ART-Sign Scheme with Square Diffie-Hellman Tags (SqDH)

The above signature scheme limits to one-time signatures: only one signature can be generated for a given tag-index, otherwise signatures can be later forged on any message for this index, by linearity: the vector space spanned by (g, g^m) (in case of just one signature issued for one index) is just $(g^\alpha, g^{\alpha m})$ and the constraint of g for the first component implies $\alpha = 1$; on the other hand, the vector space spanned by (g, g^m) and $(g, g^{m'})$ (in case of two signatures issued for one index) is $\mathbb{G} \times \mathbb{G}$, and even the constraint of g for the first component does not limit anything for the second component.

This will be enough for our ABC application, as one usually has one attribute value for a specific kind of information (age, city, diploma, etc), but in practice this implies the signer to either keep track of all the indices already signed for one tag or to sign all the messages at once. We provide another kind of combinations, that could be applied on our SqDH signature scheme that will have interesting application to an ABC scheme.

Description of the Bounded SqDH-based ART-Sign Scheme. We propose here an alternative where the limitation is on the total number n of messages signed for each tag by each signer:

$\mathsf{Setup}(1^\kappa)$: It extends the above $\mathsf{Ephemerld}$-setup with the set of messages $\mathcal{M} = \mathbb{Z}_p$;

$\mathsf{Keygen}(\mathsf{param}, n)$: Given the public parameters param and a length n, it outputs the signing and verification keys

$$\mathsf{sk}_j = [\ t,\ u,\ v,\ s_1,\ \ldots,\ s_{2n-1}\ ] \xleftarrow{\$} \mathbb{Z}_p^{2n+2},$$
$$\mathsf{vk}_j = \mathfrak{g}^{\mathsf{sk}_j} = [\ T, U, V, S_1, \ldots, S_{2n-1}\ ] \in \mathbb{G}_2^{2n+2}.$$

$\mathsf{Sign}(\mathsf{sk}_j, \tau, m)$: Given a signing key $\mathsf{sk}_j = [t, u, v, s_1, \ldots, s_{2n-1}]$, a message $m \in \mathbb{Z}_p$ and a public tag $\tau = (\tau_1, \tau_2, \tau_3)$, it outputs the signature

$$\sigma = \tau_1^{t + \sum_1^{2n-1} s_\ell m^\ell} \times \tau_2^u \times \tau_3^v.$$

$\mathsf{AggrKey}(\{\mathsf{vk}_j\}_j)$: Given verification keys vk_j, it outputs the aggregated verification key $\mathsf{avk} = [\mathsf{vk}_j]_j$;

$\mathsf{AggrSign}(\tau, (\mathsf{vk}_j, m_{j,i}, \sigma_{j,i})_{j,i})$: Given tuples of verification key vk_j, message $m_{j,i}$ and signature $\sigma_{j,i}$ all under the same tag τ, it outputs the signature $\sigma = \prod_{j,i} \sigma_{j,i}$ of the concatenation of the messages verifiable with $\mathsf{avk} \leftarrow \mathsf{AggrKey}(\{\mathsf{vk}_j\}_j)$;

$\mathsf{DerivSign}(\mathsf{avk}, \tau, \vec{M}, \sigma, \rho_{\tau\to\tau'})$: Given a signature σ on tag τ and a message-set $\vec{M}$, and $\rho_{\tau\to\tau'}$ the randomization link between τ and another tag τ', it outputs $\sigma' = \sigma^{\rho_{\tau\to\tau'}}$;

$\mathsf{RandSign}(\mathsf{avk}, \tau, \vec{M}, \sigma)$: The scheme being deterministic, it returns σ;

$\mathsf{VerifSign}(\mathsf{avk}, \tau, \vec{M}, \sigma)$: Given a valid tag $\tau = (\tau_1, \tau_2, \tau_3)$, an aggregated verification key $\mathsf{avk} = [\mathsf{vk}_j]_j$ and a message-set $\vec{M} = [\mathsf{m}_j]_j$, with for each j, $\mathsf{m}_j = [m_{j,i}]_i$, and a signature σ, one checks if the following equality holds or not, where $n_j = \#\{m_{j,i}\}$:

$$e(\sigma, \mathfrak{g}) = e\left(\tau_1, \prod_j T_j^{n_j} \times \prod_{\ell=1}^{2n-1} S_{j,\ell}^{\sum_i m_{j,i}^\ell}\right) \times e\left(\tau_2, \prod_j U_j^{n_j}\right) \times e\left(\tau_3, \prod_j V_j^{n_j}\right)$$

Recall that the validity of the tag has to be verified, as for the other version.

Security of the Bounded SqDH-based ART-Sign Scheme. The linear homomorphism of the signature from [HPP20] still allows combinations. But when the number of signing queries is at most $2n$ per tag, the verification of the signature implies 0/1 coefficients only:

Theorem 12. *The bounded SqDH-based ART-Sign is unforgeable with a bounded number of signing queries per tag, even with adaptive corruptions of keys and tags, in both the generic group model and the random oracle model.*

Proof. As argued in [HPP20] and recalled in Theorem 5, when the bases of the tags are random, even if the exponents are known, the signature that would have signed messages $\vec{M} = (g^m, \ldots, g^{m^{2n-1}})$, for $m \in \mathbb{Z}_p$, is an unforgeable linearly-homomorphic signature. This means it is only possible to linearly combine signatures with the same tag. We fix the limit to n signatures σ_i queried on distinct messages m_i, for $i = 1, \ldots, n$ under vk_j: one can derive the signature $\sigma = \prod \sigma_i^{\alpha_i}$ on $\left(g^{\sum_i \alpha_i m_i^1}, \ldots, g^{\sum_i \alpha_i m_i^{2n-1}}\right)$. Whereas the forger claims this is a signature on $\left(g^{\sum_i a_i^1}, \ldots, g^{\sum_i \alpha_i a_i^n}\right)$, on $n_j \leq n$ values $a_1, \ldots, a_{n_j}$, as one cannot combine more than n attributes. Because of the constraint on τ_2, we additionally have $\sum \alpha_i = n_j \bmod p$:

$$\sum_{i=1}^{n} \alpha_i m_i^\ell = \sum_{i=1}^{n_j} a_i^\ell \bmod p \qquad \text{for } \ell = 0, \ldots, 2n-1$$

Let us first move on the left hand side the elements $a_k \in \{m_i\}$, with only $n' \leq n_j$ new elements, we assume to be the first ones, and we note $\beta_i = \alpha_i$ if $m_i \notin \{a_k\}$ and $\beta_i = \alpha_i - 1$ if $m_i \in \{a_k\}$:

$$\sum_{i=1}^{n} \beta_i m_i^\ell = \sum_{i=1}^{n'} a_i^\ell \bmod p \qquad \text{for } \ell = 0, \ldots, 2n-1$$

We thus have the system

$$\sum_{i=1}^{n} \beta_i m_i^\ell + \sum_{i=1}^{n'} \gamma_i a_i^\ell = 0 \bmod p \qquad \text{for } \ell = 0, \ldots, 2n-1, \text{ with } \gamma_i = -1$$

This is a system of $2n$ equations with at most $n + n' \leq 2n$ unknown values β_i's and γ_i's, and the Vandermonde matrix is invertible: $\beta_i = 0$ and $\gamma_i = 0$ for all index i. As a consequence, the vector $(\alpha_i)_i$ only contains 0 or 1 components.

This proves unforgeability, even with corruptions of the tags, but with a number of signed messages bounded by n. One can also consider corruptions of the signing keys, as they are all independent: one just needs to guess under which key will be generated the forgery.

About unlinkability, it relies on the DSqDH assumption, with the same proof as the previous one-time scheme, except we can consider un-ordered message-sets $\vec{M}$:

Theorem 13. *The bounded SqDH-based ART-Sign, with message-sets, is unlinkable.*

A slightly more compact scheme is described in the Appendix B.

5 Multi-Authority Anonymous Crendentials

In this section, we first define an anonymous attribute-based credential scheme, in the certified key setting (we assume a Certification Authority that first checks the knowledge of the secret keys before certifying public keys. The latter are then always checked before used by any players in the system). We assume that an identity id is associated (and included) to any vk, which is in turn included in sk. Then, we will show how to construct such a scheme based on EphemerId and ART-Sign schemes.

5.1 Definition

Our general definition supports multiple users $(\mathcal{U}_i)_i$ and multiple credential issuers $(\mathsf{CI}_j)_j$:

Definition 14 (Anonymous Credential). An anonymous credential system is defined by the following algorithms:

$\mathsf{Setup}(1^\kappa)$: It takes as input a security parameter and outputs the public parameters param;
$\mathsf{CIKeyGen}(\mathsf{ID})$: It generates the key pair $(\mathsf{sk}, \mathsf{vk})$ for the credential issuer with identity ID;
$\mathsf{UKeyGen}(\mathsf{id})$: It generates the key pair $(\mathsf{usk}, \mathsf{uvk})$ for the user with identity id;
$(\mathsf{CredObtain}(\mathsf{usk}, \mathsf{vk}, a), \mathsf{CredIssue}(\mathsf{uvk}, \mathsf{sk}, a))$: A user with identity id (associated to $(\mathsf{usk}, \mathsf{uvk})$) runs CredObtain to obtain a credential on the attribute a from the credential issuer ID (associated to $(\mathsf{sk}, \mathsf{vk})$) running CredIssue. At the end of the protocol, the user receives a credential σ;
$\mathsf{CredAggr}(\mathsf{usk}, \{(\mathsf{vk}_j, a_j, \sigma_j)\}_j)$: It takes as input a secret key usk of a user and a list of credentials $(\mathsf{vk}_j, a_j, \sigma_j)$ and outputs a credential σ of the aggregation of the attributes;
$(\mathsf{CredShow}(\mathsf{usk}, \{(\mathsf{vk}_j, a_j)\}_j, \sigma), \mathsf{CredVerify}(\{(\mathsf{vk}_j, a_j)\}_j))$: In this two-party protocol, a user with identity id (associated to $(\mathsf{usk}, \mathsf{uvk})$) runs CredShow and interacts with a verifier running CredVerify to prove that he owns a valid credential σ on $\{a_j\}_j$ issued respectively by credential issuers ID_j (associated to $(\mathsf{sk}_j, \mathsf{vk}_j)$).

5.2 Security Model

The security model of anonymous credentials was already defined in various papers. We follow [FHS19, San20], with multi-show unlinkable credentials, but considering multiple credential issuers. Informally, the scheme needs to have the three properties:

- Correctness: the verifier must accept any credential obtained by an aggregation of honestly issued credentials on attributes;
- Unforgeability: the verifier should not accept a credential on a set of attributes for which the user did not obtain all the individual credentials for himself;
- Anonymity: credentials shown multiple times by a user should be unlinkable, even for the credential issuers. This furthermore implies that credentials cannot be linked to their owners.

For the two above security notions of unforgeability and anonymity, one can consider malicious adversaries able to corrupt some parties. We thus define the following lists: HU the list of honest user identities, CU the list of corrupted user identities, similarly we define HCI and CCI for the honest/corrupted credential issuers. For a user identity id, we define $\mathsf{Att}[\mathsf{id}]$ the list of the attributes of id and $\mathsf{Cred}[\mathsf{id}]$ the list of his individual credentials obtained from the credential issuers. All these lists are initialized to the empty set. For both unforgeability and anonymity, the adversary has unlimited access to the oracles:

- $\mathcal{O}\mathsf{HCI}(\mathsf{ID})$ corresponds to the creation of an honest credential issuer with identity ID. If he already exists (i.e. $\mathsf{ID} \in \mathsf{HCI} \cup \mathsf{CCI}$), it outputs $\perp$. Otherwise, it adds $\mathsf{ID} \in \mathsf{HCI}$ and runs $(\mathsf{sk}, \mathsf{vk}) \leftarrow \mathsf{CIKeyGen}(\mathsf{ID})$ and returns vk;
- $\mathcal{O}\mathsf{CCI}(\mathsf{ID}, \mathsf{vk})$ corresponds to the corruption of a credential issuer with identity ID and optionally public key vk. If he does not exist yet (i.e. $\mathsf{ID} \notin \mathsf{HCI} \cup \mathsf{CCI}$), it creates a new corrupted credential issuer with public key vk by adding ID to CCI. Otherwise, if $\mathsf{ID} \in \mathsf{HCI}$, it removes ID from HCI and adds it to CCI and outputs sk;
- $\mathcal{O}\mathsf{HU}(\mathsf{id})$ corresponds to the creation of an honest user with identity id. If the user already exists (i.e. $\mathsf{id} \in \mathsf{HU} \cup \mathsf{CU}$), it outputs $\perp$. Otherwise, it creates a new user by adding $\mathsf{id} \in \mathsf{HU}$ and running $(\mathsf{usk}, \mathsf{uvk}) \leftarrow \mathsf{UKeyGen}(\mathsf{id})$. It initializes $\mathsf{Att}[\mathsf{id}] = \{\}$ and $\mathsf{Cred}[\mathsf{id}] = \{\}$ and returns uvk;
- $\mathcal{O}\mathsf{CU}(\mathsf{id}, \mathsf{uvk})$ corresponds to the corruption of a user with identity id and optionally public key uvk. If the user does not exist yet (i.e. $\mathsf{id} \notin \mathsf{HU} \cup \mathsf{CU}$), it creates a new corrupted user with public key uvk by adding id to CU. Otherwise, if $\mathsf{id} \in \mathsf{HU}$, it removes id from HU and adds it to CU and outputs usk and all the associated credentials $\mathsf{Cred}[\mathsf{id}]$;
- $\mathcal{O}\mathsf{ObtIss}(\mathsf{id}, \mathsf{ID}, a)$ corresponds to the issuing of a credential from a credential issuer with identity ID (associated to $(\mathsf{sk}, \mathsf{vk})$) to a user with identity id (associated to $(\mathsf{usk}, \mathsf{uvk})$) on the attribute a. If $\mathsf{id} \notin \mathsf{HU}$ or $\mathsf{ID} \notin \mathsf{HCI}$, it outputs $\perp$. Otherwise, it runs $\sigma \leftarrow (\mathsf{CredObtain}(\mathsf{usk}, \mathsf{id}), \mathsf{CredIssue}(\mathsf{uvk}, \mathsf{sk}, a))$ and adds (ID, a) to $\mathsf{Att}[\mathsf{id}]$ and (ID, a, σ) to $\mathsf{Cred}[\mathsf{id}]$;
- $\mathcal{O}\mathsf{Obtain}(\mathsf{id}, \mathsf{ID}, a)$ corresponds to the issuing of a credential from the adversary playing the role of a malicious credential issuer with identity ID (associated to vk) to an honest user with identity id (associated to $(\mathsf{usk}, \mathsf{uvk})$) on the attribute a. If $\mathsf{id} \notin \mathsf{HU}$ or $\mathsf{ID} \notin \mathsf{CCI}$, it outputs $\perp$. Otherwise, it runs $\mathsf{CredObtain}(\mathsf{usk}, a)$ and adds (ID, a) to $\mathsf{Att}[\mathsf{id}]$ and (ID, a, σ) to $\mathsf{Cred}[\mathsf{id}]$;
- $\mathcal{O}\mathsf{Issue}(\mathsf{id}, \mathsf{ID}, a)$ corresponds to the issuing of a credential from an honest credential issuer with identity ID (associated to $(\mathsf{sk}, \mathsf{vk})$) to the adversary playing the role of a malicious user with identity id (associated to uvk) on the attribute a. If $\mathsf{id} \notin \mathsf{CU}$ or $\mathsf{ID} \notin \mathsf{HCI}$, it outputs $\perp$. Otherwise, it runs $\mathsf{CredIssue}(\mathsf{uvk}, \mathsf{sk}, a)$ and adds (ID, a) to $\mathsf{Att}[\mathsf{id}]$ and (ID, a, σ) to $\mathsf{Cred}[\mathsf{id}]$;
- $\mathcal{O}\mathsf{Show}(\mathsf{id}, \{(\mathsf{ID}_j, a_j)\}_j)$ corresponds to the showing by an honest user with identity id (associated to $(\mathsf{usk},\mathsf{uvk})$) of a credential on the set $\{(\mathsf{ID}_j, a_j)\}_j \subset \mathsf{Att}[\mathsf{id}]$. If $\mathsf{id} \notin \mathsf{HU}$, it outputs $\perp$. Otherwise, it runs $\mathsf{CredShow}(\mathsf{usk}, \{(\mathsf{vk}_j, a_j)\}_j, \sigma)$ with the adversary playing the role of a malicious verifier.

Definition 15 (Unforgeability). An anonymous credential scheme is said unforgeable if, for any polynomial time adversary adversary $\mathcal{A}$ having access to $\mathcal{O} = \{\mathcal{O}\mathsf{HCI}, \mathcal{O}\mathsf{CCI}, \mathcal{O}\mathsf{HU}, \mathcal{O}\mathsf{CU}, \mathcal{O}\mathsf{ObtIss}, \mathcal{O}\mathsf{Issue}, \mathcal{O}\mathsf{Show}\}$, $\mathsf{Adv}^{\mathrm{unf}}(\mathcal{A}) = |\Pr[\mathsf{Exp}^{\mathrm{unf}}_{\mathcal{A}}(1^\kappa) = 1]|$ is negligible where

$\mathsf{Exp}^{\mathrm{unf}}_{\mathcal{A}}(1^\kappa)$:
  $\mathsf{param} \leftarrow \mathsf{Setup}(1^\kappa)$
  $\{(\mathsf{ID}_j, a_j)\}_j \leftarrow \mathcal{A}^{\mathcal{O}}(\mathsf{param})$
  $b \leftarrow (\mathcal{A}(), \mathsf{CredVerify}(\{(\mathsf{vk}_j, a_j)\}_j))$
  If $\exists \mathsf{id} \in \mathsf{CU}$, $\forall j$, either $\mathsf{ID}_j \in \mathsf{CCI}$, or $\mathsf{ID}_j \in \mathsf{HCI}$ and $(\mathsf{ID}_j, a_j) \in \mathsf{Att}[\mathsf{id}]$,
    then return 0
  Return b

Intuitively, the adversary wins the security game if it manages to prove its ownership of a credential, on behalf of a corrupted user $\mathsf{id} \in \mathsf{CU}$ whereas this user did not ask the attributes to the honest credential issuers. Note that attributes from the corrupted credential issuers can be generated by the adversary itself, using the secret keys.

Definition 16 (Anonymity). An anonymous credential scheme is said anonymous if, for any polynomial time adversary $\mathcal{A}$ having access to $\mathcal{O} = \{\mathcal{O}\mathsf{HCI}, \mathcal{O}\mathsf{CCI}, \mathcal{O}\mathsf{HU}, \mathcal{O}\mathsf{CU}, \mathcal{O}\mathsf{Obtain}, \mathcal{O}\mathsf{Show}\}$, $\mathsf{Adv}^{\mathrm{ano}}(\mathcal{A}) = |\Pr[\mathsf{Exp}^{\mathrm{ano}-1}_{\mathcal{A}}(1^\kappa) = 1] - \Pr[\mathsf{Exp}^{\mathrm{ano}-0}_{\mathcal{A}}(1^\kappa) = 1]|$ is negligible where

$\mathsf{Exp}^{\mathrm{ano}-b}_{\mathcal{A}}(1^\kappa)$:
$\mathsf{param} \leftarrow \mathsf{Setup}(1^\kappa)$
$(\mathsf{id}_0, \mathsf{id}_1, \{(\mathsf{ID}_j, a_j)\}_j) \leftarrow \mathcal{A}^{\mathcal{O}}(\mathsf{param})$
If for some ID_j, $(\mathsf{ID}_j, a_j) \notin \mathsf{Att}[\mathsf{id}_0] \cap \mathsf{Att}[\mathsf{id}_1]$, then return 0
$(\mathsf{CredShow}(\mathsf{usk}_b, \{a_j\}_j, \sigma), \mathcal{A}())$
$b^* \leftarrow \mathcal{A}^{\mathcal{O}}()$
If $\mathsf{id}_0 \in \mathsf{CU}$ or $\mathsf{id}_1 \in \mathsf{CU}$, then return 0
Return b^*

First, note that we do not hide the attributes nor the issuers during the showing, but just the user, as we want to prove their ownership by the anonymous user. Intuitively, the adversary wins the security game if it can distinguish showings from users id_0 and id_1 of its choice, on the same set of attributes $\{(\mathsf{ID}_j, a_j)\}_j$, even after having verified credentials from the two identities, as it has access to the oracle $\mathcal{O}\mathsf{Show}$. Note that contrarily to [San20], unless the attributes contain explicit ordering (as it will be the case with our first construction), we are dealing with unlinkability as soon as the sets of attributes are the same for the two players (with the second construction).

5.3 Anonymous Credential from EphemerId and ART-Sign Scheme

Let $\mathcal{E}$ be an EphemerId scheme and $\mathsf{S}^{\mathrm{art}}$ an ART-Sign scheme, one can construct an anonymous attribute-based credential scheme. The user's keys will be tag pairs and the credentials will be ART-Sign signatures on both the tags and the attributes. Since the signature is aggregatable and the tag is randomizable, the user can anonymously show any aggregation of credentials:

- $\mathsf{Setup}(1^\kappa)$: Given a security parameter κ, it runs $\mathsf{S}^{\mathrm{art}}.\mathsf{Setup}$ and outputs the public parameters param which includes all the parameters;
- $\mathsf{CIKeyGen}(\mathsf{ID})$: Credential issuer CI with identity ID, runs $\mathsf{S}^{\mathrm{art}}.\mathsf{Keygen}(\mathsf{param})$ to obtain his key pair $(\mathsf{sk}, \mathsf{vk})$;
- $\mathsf{UKeyGen}(\mathsf{id})$: User $\mathcal{U}$ with identity id, runs $\mathcal{E}.\mathsf{GenTag}(\mathsf{param})$ to obtain his key pair $(\mathsf{usk}, \mathsf{uvk})$. In the case witnesses are required for the signatures, $(\mathsf{usk}, \mathsf{uvk})$ are provided to the credential issuers;
- $(\mathsf{CredObtain}(\mathsf{usk}, a), \mathsf{CredIssue}(\mathsf{uvk}, \mathsf{sk}, a))$: User $\mathcal{U}$ with identity id and key-pair $(\mathsf{usk}, \mathsf{uvk})$ asks the credential issuer CI for a credential on attribute a: $\sigma = \mathsf{S}^{\mathrm{art}}.\mathsf{Sign}(\mathsf{sk}, \mathsf{uvk}, a)$;
- $\mathsf{CredAggr}(\mathsf{usk}, \{(\mathsf{vk}_j, a_j, \sigma_j)\}_j)$: Given credentials σ_j on attributes (ID_j, a_j) under the same user key uvk, it outputs the signature $\sigma = \mathsf{S}^{\mathrm{art}}.\mathsf{AggrSign}(\mathsf{uvk}, \{(\mathsf{vk}_j, a_j, \sigma_j)\}_j)$ on the set of attributes $\{a_j\}_j$ under uvk and the aggregated verification key avk of all the vk_j;
- $(\mathsf{CredShow}(\mathsf{usk}, \{(\mathsf{vk}_j, a_j)\}_j, \sigma), \mathsf{CredVerify}(\{(\mathsf{vk}_j, a_j)\}_j)$: User $\mathcal{U}$ randomizes his public key $(\mathsf{uvk}', \rho) = \mathcal{E}.\mathsf{RandTag}(\mathsf{uvk})$ and computes the aggregated key $\mathsf{avk} = \mathsf{S}^{\mathrm{art}}.\mathsf{AggrKey}(\{\mathsf{vk}_j\}_j)$. Then, it adapts the secret key $\mathsf{usk}' = \mathcal{E}.\mathsf{DerivWitness}(\mathsf{usk}, \rho)$ as well as the aggregated signature $\sigma' = \mathsf{S}^{\mathrm{art}}.\mathsf{DerivSign}(\mathsf{avk}, \mathsf{uvk}, \{a_j\}_j, \sigma, \rho)$ and randomizes it: $\sigma'' = \mathsf{S}^{\mathrm{art}}.\mathsf{RandSign}(\mathsf{avk}, \mathsf{uvk}', \{a_j\}_j, \sigma')$. Finally, it sends to the verifier $\mathcal{V}$ the anonymous credential $(\mathsf{avk}, \{a_j\}_j, \mathsf{uvk}', \sigma'')$. The verifier first checks the freshness of the credential with a proof of ownership of uvk' using the interactive protocol $(\mathcal{E}.\mathsf{ProveKTag}(\mathsf{usk}'),$

$\mathcal{E}.\mathsf{VerifKTag}(\mathsf{uvk}')$) and then verifies the validity of the credential with $\mathsf{S}^{\mathrm{art}}.\mathsf{VerifSign}(\mathsf{avk}, \mathsf{uvk}', \{a_j\}_j, \sigma'')$.

If one considers corruptions, when one corrupts a user, his secret key is provided, when one corrupts a credential issuer, his secret key is provided.

By replacing all the algorithms by their instantiations for the proposed constructions of EphemerId and ART-Sign schemes, we obtain our constructions of anonymous attribute-based credential schemes. The SqDH construction uses an aggregatable signature with (public) randomizable tag, and unforgeability holds even if the witnesses are known. As a consequence, this construction allows corruption of the Credential Issuers and of the users.

Theorem 17. *Assuming EphemerId achieves knowledge soundness and ART-Sign is unforgeable, the generic construction is an unforgeable attribute-based credential scheme, in the certified key model.*

Proof. Let $\mathcal{A}$ be an adversary against the unforgeability of our anonymous credential scheme. We build an adversary $\mathcal{B}$ against the unforgeability of the ART-Sign. As we are in the certified key model, even for the corrupted players, the simulator knows the secret keys, as they can be extracted at the certification time. Our adversary $\mathcal{B}$ runs the unforgeability security game of the ART-Sign, and answers the oracle queries asked by $\mathcal{A}$ as follows:

- $\mathcal{O}\mathsf{HCI}(\mathsf{ID})$: If $\mathsf{ID} \in \mathsf{HCI} \cup \mathsf{CCI}$, $\mathcal{B}$ outputs $\perp$. Otherwise, it adds $\mathsf{ID} \in \mathsf{HCI}$, asks the query $\mathcal{O}\mathsf{Keygen}()$ and forwards the answer to $\mathcal{A}$;
- $\mathcal{O}\mathsf{CCI}(\mathsf{ID}, \mathsf{vk})$: If $\mathsf{ID} \notin \mathsf{HCI} \cup \mathsf{CCI}$, $\mathcal{B}$ adds $\mathsf{ID} \in \mathsf{CCI}$. Otherwise, if $\mathsf{ID} \in \mathsf{HCI}$ with keys $(\mathsf{sk}, \mathsf{vk})$, it moves ID from HCI to CCI. It then asks the query $\mathcal{O}\mathsf{Corrupt}(\mathsf{vk})$ and forwards the answer to $\mathcal{A}$;
- $\mathcal{O}\mathsf{HU}(\mathsf{id})$: If $\mathsf{id} \in \mathsf{HU} \cup \mathsf{CU}$, $\mathcal{B}$ outputs $\perp$. Otherwise, it adds $\mathsf{id} \in \mathsf{HU}$, asks the query $\mathcal{O}\mathsf{GenTag}()$ and forwards the answer to $\mathcal{A}$;
- $\mathcal{O}\mathsf{CU}(\mathsf{id}, \mathsf{uvk})$: If $\mathsf{id} \notin \mathsf{HU} \cup \mathsf{CU}$, $\mathcal{B}$ adds $\mathsf{id} \in \mathsf{CU}$. Otherwise, if $\mathsf{id} \in \mathsf{HU}$ with keys $(\mathsf{usk}, \mathsf{uvk})$, it moves id from HU to CU, asks the query $\mathcal{O}\mathsf{CorruptTag}(\mathsf{uvk})$ and forwards the answer to $\mathcal{A}$;
- $\mathcal{O}\mathsf{ObtIss}(\mathsf{id}, \mathsf{ID}, a)$: If $\mathsf{id} \notin \mathsf{HU}$ or $\mathsf{ID} \notin \mathsf{HCI}$, $\mathcal{B}$ outputs $\perp$. Otherwise, id is associated to $(\mathsf{usk}, \mathsf{uvk})$ and ID is associated to $(\mathsf{sk}, \mathsf{vk})$. Then $\mathcal{B}$ asks the query $\mathcal{O}\mathsf{Sign}(\mathsf{vk}, \mathsf{uvk}, a)$, adds (ID, a) to $\mathsf{Att}[\mathsf{id}]$ and (ID, a, σ) to $\mathsf{Cred}[\mathsf{id}]$ and outputs σ.
- $\mathcal{O}\mathsf{Obtain}(\mathsf{id}, \mathsf{ID}, a)$: If $\mathsf{id} \notin \mathsf{HU}$ or $\mathsf{ID} \notin \mathsf{CCI}$, $\mathcal{B}$ outputs $\perp$. Otherwise, id is associated to $(\mathsf{usk}, \mathsf{uvk})$ and ID is associated to $(\mathsf{sk}, \mathsf{vk})$. Then $\mathcal{B}$ runs $\sigma = \mathsf{Sign}(\mathsf{sk}, \mathsf{uvk}, a)$ and adds (ID, a) to $\mathsf{Att}[\mathsf{id}]$ and (ID, a, σ) to $\mathsf{Cred}[\mathsf{id}]$;
- $\mathcal{O}\mathsf{Issue}(\mathsf{id}, \mathsf{ID}, a)$: If $\mathsf{id} \notin \mathsf{CU}$ or $\mathsf{ID} \notin \mathsf{HCI}$, $\mathcal{B}$ outputs $\perp$. Otherwise, id is associated to $(\mathsf{usk}, \mathsf{uvk})$ and ID is associated to $(\mathsf{sk}, \mathsf{vk})$. Then $\mathcal{B}$ runs $\sigma = \mathsf{Sign}(\mathsf{sk}, \mathsf{uvk}, a)$ and adds (ID, a) to $\mathsf{Att}[\mathsf{id}]$ and (ID, a, σ) to $\mathsf{Cred}[\mathsf{id}]$;
- $\mathcal{O}\mathsf{Show}(\mathsf{id}, \{(\mathsf{ID}_j, a_j)\}_j)$: If $\mathsf{id} \notin \mathsf{HU}$ or $\{(\mathsf{ID}_j, a_j)\}_j) \not\subset \mathsf{Att}[\mathsf{id}]$, $\mathcal{B}$ outputs $\perp$. Otherwise, id is associated to $(\mathsf{usk}, \mathsf{uvk})$ and each ID_j is associated to $(\mathsf{sk}_j, \mathsf{vk}_j)$. Furthermore, for each (ID_j, a_j), there is σ_j such that $(\mathsf{ID}_j, a_j, \sigma_j) \in \mathsf{Cred}[\mathsf{id}]$. Then $\mathcal{B}$ first randomizes the key uvk with $(\mathsf{uvk}', \rho) = \mathcal{E}.\mathsf{RandTag}(\mathsf{uvk})$, computes the aggregated key $\mathsf{avk} = \mathsf{S}^{\mathrm{art}}.\mathsf{AggrKey}(\{\mathsf{vk}_j\}_j)$ and adapts the secret key $\mathsf{usk}' = \mathcal{E}.\mathsf{DerivWitness}(\mathsf{usk}, \rho)$. From the obtained credentials σ_j, it computes the aggregated signature $\sigma = \mathsf{S}^{\mathrm{art}}.\mathsf{AggrSign}(\mathsf{uvk}, \{(\mathsf{vk}_j, a_j, \sigma_j)\}_j)$, adapts it: $\sigma' = \mathsf{S}^{\mathrm{art}}.\mathsf{DerivSign}(\mathsf{avk}, \mathsf{uvk}, \{a_j\}_j, \sigma, \rho)$, and randomizes it: $\sigma'' = \mathsf{S}^{\mathrm{art}}.\mathsf{RandSign}(\mathsf{avk}, \mathsf{uvk}', \{a_j\}_j, \sigma')$. $\mathcal{B}$ outputs $(\mathsf{avk}, \{a_j\}_j, \mathsf{uvk}', \sigma'')$ and makes the $\mathcal{E}.\mathsf{ProveKTag}(\mathsf{usk}')$ part of the interactive proof of ownership.

Eventually, the adversary $\mathcal{A}$ runs a showing for $\{(\mathsf{vk}_j, a_j)\}_j$, with a credential $(\mathsf{avk}, \{a_j\}_j, \mathsf{uvk}^*, \sigma^*)$ and a proof of knowledge of usk^* associated to uvk^*: in case of success, $\mathcal{B}$ outputs the signature $(\mathsf{avk}, \{a_j\}_j, \mathsf{uvk}^*, \sigma^*)$.

In case of validity of the showing, except with negligible probability,

– from the knowledge soundness of the EphemerId scheme, this means there is $\mathsf{id} \in \mathsf{CU}$, associated to $(\mathsf{usk}, \mathsf{uvk})$, with $\mathsf{uvk} \sim \mathsf{uvk}^*$;
– from the unforgeability of the aggregatable signature with randomizable tags, all the tags a_j's have been signed for uvk and vk. These individual credentials have thus been issued either by the adversary on behalf of a corrupted credential issuer $\mathsf{ID}_j \in \mathsf{CCI}$ or from an oracle query to ID_j for id.

This is thus a legitimate showing with overwhelming probability: $\mathcal{B}$ win with negligible probability. Hence, the adversary $\mathcal{A}$ can only win with negligible probability.

As explained above, the security relies on both the soundness of the EphemerId scheme and the unforgeability of the aggregatable signature with randomizable tags. In our construction, the witness is not needed for signing, and unforgeability of the ART-Sign holds even if the witnesses are all known to the adversary. Hence, corruption of users would just help to run the proof of knowledge of the witnesses, and corruption of credential issuers for the issuing of credentials, which would not help for forgeries (in the above security model). Of course, we also have to take care of the way keys are generated and the number of signatures that will be issued to guarantee the unforgeability.

Theorem 18. *Assuming EphemerId is zero-knowledge and ART-Sign is unlinkable, the generic construction is an anonymous attribute-based credential scheme, in the certified key model.*

Proof. From the unlinkability of the ART-Sign, the tuple $(\mathsf{avk}, \vec{M}, \tau', \sigma'')$ does not leak any information about the initial tag τ. Hence, a credential does not leak any information about uvk_b. In addition, if the proof of knowledge of the witness is zero-knowledge, it does not leak any information about uvk_b either.

6 SqDH-based Anonymous Credentials

Thanks to our aggregate signatures that tolerate corruptions of users and signers, we will be able to consider corruptions of users and credential issuers, and even possible collusions. In the first construction, we consider attributes where the index i determines the topic (age, city, diploma) and the exact value is encoded in $a_i \in \mathbb{Z}_p^*$ (possibly $H(m) \in \mathbb{Z}_p^*$ if the value is a large bitstring), or 0 when empty. The second construction will not require any such ordering on the attributes. Free text will be possible.

6.1 The Basic SqDH-based Anonymous Credential Scheme

The basic construction directly follows the instantiation of the above construction with the SqDH-based ART-Sign:

Setup(1^κ): Given a security parameter κ, let $(\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, p, g, \mathfrak{g}, e)$ be an asymmetric bilinear setting, where g and $\mathfrak{g}$ are random generators of $\mathbb{G}_1$ and $\mathbb{G}_2$ respectively. We then define $\mathsf{param} = (\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, p, g, \mathfrak{g}, e, \mathcal{H})$, where $\mathcal{H}$ is an hash function in $\mathbb{G}_1$;

CIKeyGen(ID): Credential issuer CI with identity ID, generates its keys for n kinds of attributes

$$\mathsf{sk}_j = (\ \mathsf{SK}_j = [\ t,\ \ u,\ \ v\ ],\ \mathsf{SK}'_{j,i} = [\ r_i,\ \ s_i\ ]_i\ ) \xleftarrow{\$} \mathbb{Z}_p^{3+2n},$$
$$\mathsf{vk}_j = (\ \mathsf{VK}_j = [\ \mathfrak{g}^t, \mathfrak{g}^u, \mathfrak{g}^v\ ],\ \mathsf{VK}'_{j,i} = [\ \mathfrak{g}^{r_i}, \mathfrak{g}^{s_i}\ ]_i\ ) \in \mathbb{G}_2^{3+2n}.$$

More keys for new attributes can be generated on-demand: by adding the pair $[r, s] \xleftarrow{\$} \mathbb{Z}_p^2$ to the secret key and $[\mathfrak{g}^r, \mathfrak{g}^s]$ to the verification key, the keys can works on $n+1$ kinds of attributes;

UKeyGen(id): User $\mathcal{U}$ with identity id, sets $h = \mathcal{H}(\mathsf{id}) \in \mathbb{G}_1^*$, generates its secret tag $\tilde{\tau} \xleftarrow{\$} \mathbb{Z}_p^*$ and computes $\tau = (h, h^{\tilde{\tau}}, h^{\tilde{\tau}^2}) \in \mathbb{G}_1^3$: $\mathsf{usk} = \tilde{\tau}$ and $\mathsf{uvk} = \tau = (h, h^{\tilde{\tau}}, h^{\tilde{\tau}^2})$;

$(\mathsf{CredObtain}(\mathsf{usk}, a_i), \mathsf{CredIssue}(\mathsf{uvk}, \mathsf{sk}, a_i))$: User $\mathcal{U}$ with identity id and $\mathsf{uvk} = (\tau_1, \tau_2, \tau_3)$ asks to the credential issuer CI for a credential on the attribute a_i: $\sigma = \tau_1^{t+r_i+a_i s_i} \times \tau_2^u \times \tau_3^v$. The credential issuer uses the appropriate index i, making sure this is the first signature for this index;

$\mathsf{CredAggr}(\mathsf{usk}, \{(\mathsf{VK}_j, \mathsf{VK}'_{j,i}, a_{j,i}, \sigma_{j,i})\}_{j,i})$: Given credentials $\sigma_{j,i}$ on attributes $(\mathsf{ID}_j, a_{j,i})$ under the same user key uvk, it outputs the signature $\sigma = \prod_{j,i} \sigma_{j,i}$;

$(\mathsf{CredShow}(\mathsf{usk}, \{(\mathsf{VK}_j, \mathsf{VK}'_{j,i}, a_{j,i})\}_{j,i}, \sigma)$, $\mathsf{CredVerify}(\{(\mathsf{VK}_j, \mathsf{VK}'_{j,i}, a_{j,i})\}_{j,i})$:
First, user $\mathcal{U}$ randomizes his public key with a random $\rho \xleftarrow{\$} \mathbb{Z}_p^*$ into $\mathsf{uvk}' = (\tau_1^\rho, \tau_2^\rho, \tau_3^\rho)$, concatenates the keys $\mathsf{avk} = \cup_j([\mathsf{VK}_j] \cup [\mathsf{VK}'_{j,i}]_i)$, and adapts the signature $\sigma' = \sigma^\rho$. Then it sends the anonymous credential $(\mathsf{avk}, \{a_{j,i}\}_{j,i}, \mathsf{uvk}', \sigma')$ to the verifier. The latter first checks the freshness of the credential with a proof of ownership and validity of uvk' using a Schnorr-like interactive proof and then verifies the validity of the credential: with $n_j = \#\{\mathsf{VK}'_{j,i}\}$:

$$e(\sigma, \mathfrak{g}) = e\left(\tau_1, \prod_j {\mathsf{VK}_{j,1}}^{n_j} \times \prod_i {\mathsf{VK}'_{j,i,1} \cdot \mathsf{VK}'_{j,i,2}}^{a_{j,i}}\right) \times e\left(\tau_2, \prod_j {\mathsf{VK}_{j,2}}^{n_j}\right) \times e\left(\tau_3, \prod_j {\mathsf{VK}_{j,3}}^{n_j}\right).$$

We stress that for the unforgeability of the signature, generator h for each tag must be random, and so it is generated as $\mathcal{H}(\mathsf{id})$, with a hash function $\mathcal{H}$ in $\mathbb{G}_1$. This way, the credential issuers will automatically know the basis for each user. There is not privacy issue as this basis is randomized when used in an anonymous credential. On the other hand, the user can choose his secret key $\tilde{\tau}$, and has to prove the knowledge of the witness for the validity of the tag. This is thus an interactive protocol. In this construction, we can consider a polynomial number n of attributes per credential issuer, where a_i is associated to key $\mathsf{vk}_{j,i}$ of the Credential Issuer CI_j. Again, to keep the unforgeability of the signature, the credential issuer should provide at most one attribute per key $\mathsf{vk}_{j,i}$ for a given tag. At the showing time, for proving the ownership of k attributes (possibly from K different credential issuers), the users has to perform $k-1$ multiplications in $\mathbb{G}_1$ to aggregate the credentials into one, and 4 exponentiations in $\mathbb{G}_1$ for randomization, but just one element from $\mathbb{G}_1$ is sent, as anonymous credential, plus an interactive Schnorr-like proof of SqDH-tuple with knowledge of usk (see the Appendix C.1: 2 exponentiations in $\mathbb{G}_1$, 2 group elements from $\mathbb{G}_1$, and a scalar in $\mathbb{Z}_p$); whereas the verifier first has to perform 4 exponentiations and 2 multiplications in $\mathbb{G}_1$ for the proof of validity/knowledge of usk, and less than $3k$ multiplications and k exponentiations in $\mathbb{G}_2$, and 3 pairings to check the credential. While this is already better than [CL11], we can get a better construction.

6.2 A Compact SqDH-based Anonymous Credential Scheme

Instead of having a specific key $\mathsf{VK}'_{j,i}$ for each family of attributes $a_{j,i}$, and thus limiting to one issuing per family of attributes for each user, we can use the bounded SqDH-based ART-Sign, with free-text attributes: we consider $2n-1$ keys, where n is the maximum number of attributes issued for one user by a credential issuer, whatever the attributes are:

$\mathsf{Setup}(1^\kappa)$: Given a security parameter κ, let $(\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, p, g, \mathfrak{g}, e)$ be an asymmetric bilinear setting, where g and $\mathfrak{g}$ are random generators of $\mathbb{G}_1$ and $\mathbb{G}_2$ respectively. We then define $\mathsf{param} = (\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, p, g, \mathfrak{g}, e, \mathcal{H})$, where $\mathcal{H}$ is an hash function in $\mathbb{G}_1$;

$\mathsf{CIKeyGen}(\mathsf{ID})$: Credential issuer CI with identity ID, generates its keys for n maximum attributes per user

$$\mathsf{sk}_j = [\ t,\ u,\ v,\ s_1, \ldots, s_{2n-1}\ ] \xleftarrow{\$} \mathbb{Z}_p^{2n+2},$$
$$\mathsf{vk}_j = \mathfrak{g}^{\mathsf{sk}_j} = [\ T, U, V, S_1, \ldots, S_{2n-1}\ ] \in \mathbb{G}_2^{2n+2}.$$

$\mathsf{UKeyGen}(\mathsf{id})$: User $\mathcal{U}$ with identity id, sets $h = \mathcal{H}(\mathsf{id}) \in \mathbb{G}_1^*$, is given a randomly chosen a generator $h \xleftarrow{\$} \mathbb{G}_1^*$, generates its secret tag $\tilde{\tau} \xleftarrow{\$} \mathbb{Z}_p^*$ and computes $\tau = (h, h^{\tilde{\tau}}, h^{\tilde{\tau}^2}) \in \mathbb{G}_1^3$: $\mathsf{usk} = \tilde{\tau}$ and $\mathsf{uvk} = \tau = (h, h^{\tilde{\tau}}, h^{\tilde{\tau}^2})$;

$(\mathsf{CredObtain}(\mathsf{usk}, a), \mathsf{CredIssue}(\mathsf{uvk}, \mathsf{sk}, a))$: User $\mathcal{U}$ with identity id and $\mathsf{uvk} = (\tau_1, \tau_2, \tau_3)$ asks to the credential issuer CI for a credential on the attribute a: $\sigma = \tau_1^{t+\sum_{\ell=1}^{2n-1} s_\ell a^\ell} \times \tau_2^u \times \tau_3^v$. Note that $a \in \mathbb{Z}_p^*$, so it can be a hash value of the actual free-text attribute;

$\mathsf{CredAggr}(\mathsf{usk}, \{(\mathsf{vk}_j, a_{j,i}, \sigma_{j,i})\}_{j,i})$: Given credentials $\sigma_{j,i}$ on attributes $(\mathsf{ID}_j,\ a_{j,i})$ under the same user key uvk, it outputs the signature $\sigma = \prod_{j,i} \sigma_{j,i}$;

$(\mathsf{CredShow}(\mathsf{usk}, \{(\mathsf{vk}_j, a_{j,i})\}_{j,i}, \sigma), \mathsf{CredVerify}(\{(\mathsf{vk}_j, a_{j,i})\}_{j,i})$: First, a user $\mathcal{U}$ randomizes his public key with a random $\rho \xleftarrow{\$} \mathbb{Z}_p^*$, $\mathsf{uvk}' = (\tau_1^\rho, \tau_2^\rho, \tau_3^\rho)$, concatenates the keys $\mathsf{avk} = \cup_j[\mathsf{vk}_j]$, and adapts the signature $\sigma' = \sigma^\rho$. Then it sends the anonymous credential $(\mathsf{avk}, \{a_{j,i}\}_{j,i}, \mathsf{uvk}', \sigma')$ to the verifier. The latter first checks the freshness of the credential with a proof of ownership and validity of uvk' using a Schnorr-like interactive proof and then verifies the validity of the credential: with $n_j = \#\{a_{j,i}\}$:

$$e(\sigma, \mathfrak{g}) = e\left(\tau_1, \prod_j T_j^{n_j} \prod_{\ell=1}^{2n-1} S_{j,\ell}^{\sum_i a_{j,i}^\ell}\right) \times e\left(\tau_2, \prod_j U_j^{n_j}\right) \times e\left(\tau_3, \prod_j V_j^{n_j}\right)$$

Again, we stress that for the unforgeability of the signature, generator h for each tag must be random. And the credential issuer should provide at most n attributes per user, even if in this construction, we can consider an exponential number N of attributes per credential issuer, as $a_{j,i}$ is any scalar in $\mathbb{Z}_p^*$. More concretely, $a_{j,i}$ can be given as the output of a hash function into $\mathbb{Z}_p$ from any bitstring. At the showing time, for proving the ownership of k attributes (possibly from K different credential issuers), the users has to perform $k-1$ multiplications in $\mathbb{G}_1$ to aggregate the credentials into one, and 4 exponentiations in $\mathbb{G}_1$ for randomization, but just one group element for $\mathbb{G}_1$ is sent, as anonymous credential, plus an interactive Schnorr-like proof of SqDH-tuple with knowledge of usk (see the Appendix C.1: 2 exponentiations in $\mathbb{G}_1$, 2 group elements from $\mathbb{G}_1$, and a scalar in $\mathbb{Z}_p$); whereas the verifier first has to perform 4 exponentiations and 2 multiplications in $\mathbb{G}_1$ for the proof of validity/knowledge of usk, and less than $2n \cdot (K+3k)$ multiplications in $\mathbb{G}_2$, $2n \cdot k$ exponentiations in $\mathbb{G}_2$ and 3 pairings to check the credential.

In the particular case of just one credential issuer with verification key $\mathsf{vk} = (T, U, V, [S_i]_{i=1}^{2n-1})$, the verification of the credential σ on the k attributes $\{a_i\}$ just consists of

$$e(\sigma, \mathfrak{g}) = e\left(\tau_1, T^k \prod_{\ell=1}^{2n-1} S_\ell^{\sum_i a_i^\ell}\right) \times e\left(\tau_2, U^k\right) \times e\left(\tau_3, V^k\right).$$

The communication is of constant size (one group element in $\mathbb{G}_1$). We stress that n is just a limit of the maximal number of attributes issued by the credential issuer for one user but the universe of the possible attributes is exponentially large, and there is no distinction between the families of attributes.

7 Traceable Anonymous Credentials

As the SqDH-based $\mathsf{ART\text{-}Sign}$ schemes provide computational unlinkability only, it opens the door of possible traceability in case of abuse, with anonymous but traceable tags:

Definition 19 (Traceable EphemerId). This is an extension of an $\mathsf{EphemerId}$ scheme with a modified GenTag algorithm and an additional $\mathsf{TraceId}$ one:

$\mathsf{GenTag}(1^\kappa)$**:** Given a security parameter 1^κ, it outputs the user-key pair (usk,uvk) and the tracing key utk;

$\mathsf{TraceId}(\mathsf{utk}, \mathsf{uvk}')$**:** Given the tracing key utk associated to uvk and a public key uvk', it outputs a proof π of whether $\mathsf{uvk} \sim \mathsf{uvk}'$ or not.

$\mathsf{JudgeId}(\mathsf{uvk}, \mathsf{uvk}', \pi)$**:** two public keys and a proof, the judge checks the proof π and outputs 1 if it is correct.

Providing the tracing keys to a tracing authority at the key generation time for the users will allow traceability.

7.1 Traceable Anonymous Credentials

For traceability, we need an additional player: the *tracing authority*. During the user's key generation, this tracing authority will either be the certification authority, or a second authority, that also has to certify user's key uvk once it has received the tracing key utk.

In case of abuse of a credential σ under anonymous key uvk', a tracing algorithm outputs the initial uvk and id, with a proof a correct tracing. A new security notion is quite important: *non-frameability*, which means that the tracing authority should not be able to declare guilty a wrong user: only correct proofs are accepted by the judge. We consider a non-interactive proof of tracing, produced by the $\mathsf{TraceId}$ algorithm and verified by anybody using the $\mathsf{JudgeId}$ algorithm. This proof could be interactive.

7.2 Traceable SqDH-based Anonymous Credentials

With our Square Diffie-Hellman based $\mathsf{EphemerId}$ scheme where $\mathsf{uvk} = \tau = (h, h^{\tilde{\tau}}, h^{\tilde{\tau}^2})$ in an asymmetric bilinear setting $(\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, p, g, \mathfrak{g}, e)$ where g and $\mathfrak{g}$ are random generators of $\mathbb{G}_1$ and $\mathbb{G}_2$ respectively, $\mathsf{usk} = \tilde{\tau}$ and $\mathsf{utk} = \mathfrak{g}^{\tilde{\tau}}$. The latter tracing key indeed allows to check whether $\tau' \sim \tau$ or not: $e(\tau'_1, \mathsf{utk}) = e(\tau'_2, \mathfrak{g})$ and $e(\tau'_2, \mathsf{utk}) = e(\tau'_3, \mathfrak{g})$. If one already knows the tags are valid (SqDH tuples), this is enough to verify whether $e(\tau'_1, \mathsf{utk}) = e(\tau'_2, \mathfrak{g})$ holds or not. But we provide the complete proof, as it is already quite efficient: in order to prove it, the $\mathsf{TraceId}$ algorithm can use a Groth-Sahai proof as shown in the Appendix C.3 that proves, in a zero-knowledge way, the existence of utk such that

$$e(\tau_1, \mathsf{utk}) = e(\tau_2, \mathfrak{g}) \qquad e(\tau_2, \mathsf{utk}) = e(\tau_3, \mathfrak{g})$$
$$e(\tau'_1, \mathsf{utk}) = e(\tau'_2, \mathfrak{g}) \qquad e(\tau'_2, \mathsf{utk}) = e(\tau'_3, \mathfrak{g}).$$

The first line proves that utk is the good tracing key for $\mathsf{uvk} = \tau$, and the second line shows it applies to $\mathsf{uvk}' = \tau'$ too. These are the equations verified by $\mathsf{JudgeId}$ algorithm. This can also be a proof of innocence of id with key uvk if the first line is satisfied while the second one is not.

With such a proof, the tracing authority cannot frame a user. We thus have a secure traceable anonymous credential scheme. Note however that, since we let the user choose the secret key $\tilde{\tau}$ in GenTag, one user could decide to use the same as another user. Either the tracing authority first checks that, using the new tracing key on all the previous tags, and reject, or this is considered a collusion of users, and at the tracing time, both users will be accused.

Acknowledgments

We warmly thank Olivier Sanders for fruitful discussions. This work was supported in part by the European Community's Seventh Framework Programme (FP7/2007-2013 Grant Agreement no. 339563 – CryptoCloud).

References

BDN18. Dan Boneh, Manu Drijvers, and Gregory Neven. Compact multi-signatures for smaller blockchains. In Thomas Peyrin and Steven Galbraith, editors, *ASIACRYPT 2018, Part II*, volume 11273 of *LNCS*, pages 435–464. Springer, Heidelberg, December 2018.

BFI+10. Olivier Blazy, Georg Fuchsbauer, Malika Izabachène, Amandine Jambert, Hervé Sibert, and Damien Vergnaud. Batch Groth-Sahai. In Jianying Zhou and Moti Yung, editors, *ACNS 10*, volume 6123 of *LNCS*, pages 218–235. Springer, Heidelberg, June 2010.

BGLS03. Dan Boneh, Craig Gentry, Ben Lynn, and Hovav Shacham. Aggregate and verifiably encrypted signatures from bilinear maps. In Eli Biham, editor, *EUROCRYPT 2003*, volume 2656 of *LNCS*, pages 416–432. Springer, Heidelberg, May 2003.

BL13. Foteini Baldimtsi and Anna Lysyanskaya. Anonymous credentials light. In Ahmad-Reza Sadeghi, Virgil D. Gligor, and Moti Yung, editors, *ACM CCS 2013*, pages 1087–1098. ACM Press, November 2013.

BLS01. Dan Boneh, Ben Lynn, and Hovav Shacham. Short signatures from the Weil pairing. In Colin Boyd, editor, *ASIACRYPT 2001*, volume 2248 of *LNCS*, pages 514–532. Springer, Heidelberg, December 2001.

CDHK15. Jan Camenisch, Maria Dubovitskaya, Kristiyan Haralambiev, and Markulf Kohlweiss. Composable and modular anonymous credentials: Definitions and practical constructions. In Tetsu Iwata and Jung Hee Cheon, editors, *ASIACRYPT 2015, Part II*, volume 9453 of *LNCS*, pages 262–288. Springer, Heidelberg, November / December 2015.

CL03. Jan Camenisch and Anna Lysyanskaya. A signature scheme with efficient protocols. In Stelvio Cimato, Clemente Galdi, and Giuseppe Persiano, editors, *SCN 02*, volume 2576 of *LNCS*, pages 268–289. Springer, Heidelberg, September 2003.

CL04. Jan Camenisch and Anna Lysyanskaya. Signature schemes and anonymous credentials from bilinear maps. In Matthew Franklin, editor, *CRYPTO 2004*, volume 3152 of *LNCS*, pages 56–72. Springer, Heidelberg, August 2004.

CL11. Sébastien Canard and Roch Lescuyer. Anonymous credentials from (indexed) aggregate signatures. In Abhilasha Bhargav-Spantzel and Thomas Groß, editors, *DIM'11, Proceedings of the 2013 ACM Workshop on Digital Identity*, pages 53–62. ACM, 2011.

CL13. Sébastien Canard and Roch Lescuyer. Protecting privacy by sanitizing personal data: a new approach to anonymous credentials. In Kefei Chen, Qi Xie, Weidong Qiu, Ninghui Li, and Wen-Guey Tzeng, editors, *ASIACCS 13*, pages 381–392. ACM Press, May 2013.

FHS15. Georg Fuchsbauer, Christian Hanser, and Daniel Slamanig. Practical round-optimal blind signatures in the standard model. In Rosario Gennaro and Matthew J. B. Robshaw, editors, *CRYPTO 2015, Part II*, volume 9216 of *LNCS*, pages 233–253. Springer, Heidelberg, August 2015.

FHS19. Georg Fuchsbauer, Christian Hanser, and Daniel Slamanig. Structure-preserving signatures on equivalence classes and constant-size anonymous credentials. *Journal of Cryptology*, 32(2):498–546, April 2019.

GS08. Jens Groth and Amit Sahai. Efficient non-interactive proof systems for bilinear groups. In Nigel P. Smart, editor, *EUROCRYPT 2008*, volume 4965 of *LNCS*, pages 415–432. Springer, Heidelberg, April 2008.

HPP20. Chloé Hébant, Duong Hieu Phan, and David Pointcheval. Linearly-homomorphic signatures and scalable mix-nets. In Aggelos Kiayias, Markulf Kohlweiss, Petros Wallden, and Vassilis Zikas, editors, *PKC 2020*, volume 12111 of *LNCS*, pages 597–627. Springer, 2020. https://ia.cr/2019/547.

KL16. Nesrine Kaaniche and Maryline Laurent. Attribute-based signatures for supporting anonymous certification. In Ioannis G. Askoxylakis, Sotiris Ioannidis, Sokratis K. Katsikas, and Catherine A. Meadows, editors, *ESORICS 2016, Part I*, volume 9878 of *LNCS*, pages 279–300. Springer, Heidelberg, September 2016.

San20. Olivier Sanders. Efficient redactable signature and application to anonymous credentials. In Aggelos Kiayias, Markulf Kohlweiss, Petros Wallden, and Vassilis Zikas, editors, *PKC 2020*, volume 12111 of *LNCS*, pages 628–656. Springer, 2020. https://ia.cr/2019/1201.

Sho97. Victor Shoup. Lower bounds for discrete logarithms and related problems. In Walter Fumy, editor, *EUROCRYPT'97*, volume 1233 of *LNCS*, pages 256–266. Springer, Heidelberg, May 1997.

TW87. Martin Tompa and Heather Woll. Random self-reducibility and zero knowledge interactive proofs of possession of information. In *28th FOCS*, pages 472–482. IEEE Computer Society Press, October 1987.

Ver17. Damien Vergnaud. Comment on 'Attribute-Based Signatures for Supporting Anonymous Certification' by N. Kaaniche and M. Laurent (ESORICS 2016). *The Computer Journal*, 60(12):1801–1808, 2017.

A Canard-Lescuyer Scheme

In 2013, Canard and Lescuyer proposed a traceable attribute-based anonymous credential scheme [CL13], based on sanitizable signatures: "Protecting privacy by sanitizing personal data: a new approach to anonymous credentials".

The intuition consists in allowing the user to "sanitize" the global credentials issued by the credential issuer, in order to keep visible only the required attributes. Then for unlinkability, the signatures are encrypted under an ElGamal encryption scheme.

Unfortunately, in their scheme, the public key contains $g \xleftarrow{\$} \mathbb{G}_1$ and $\mathfrak{g} \xleftarrow{\$} \mathbb{G}_2$, and the ElGamal secret key is $\alpha \xleftarrow{\$} \mathbb{Z}_p$, the tracing key. The public encryption key is $h = g^\alpha$, but they also need $\mathfrak{h} = \mathfrak{g}^\alpha$ to be published for some verifications.

With this value $\mathfrak{h}$, anybody can break the semantic security of the ElGamal encryption, and then break the privacy of the anonymous credential.

B Another Bounded SqDH-Based ART-Sign

We can slightly reduce the parameters of the bounded SqDH-based ART-Sign, but with some limitations on the number of attributed to be signed. It relies on a hash function, modelled as a random oracle in the security analysis.

Description of the Bounded SqDH-based ART-Sign Scheme 2. We thus propose here a second version, still with the limitation on the total number of messages signed for each tag, but the public keys are twice smaller:

$\mathsf{Setup}(1^\kappa)$: It extends the above Ephemerld-setup with the set of messages $\mathcal{M} = \{0,1\}^*$, but also a hash function $\mathcal{H}$ into $\mathbb{Z}_p$;

$\mathsf{Keygen}(\mathsf{param}, n)$: Given the public parameters param and a length n, it outputs the signing and verification keys

$$\mathsf{sk}_j = [\ t,\ u,\ v,\ s_1,\ \ldots,\ s_n\ ] \xleftarrow{\$} \mathbb{Z}_p^{n+3},$$
$$\mathsf{vk}_j = \mathfrak{g}^{\mathsf{sk}_j} = [\ T, U, V, S_1, \ldots, S_n\ ] \in \mathbb{G}_2^{n+3}.$$

$\mathsf{Sign}(\mathsf{sk}_j, \tau, m)$: Given a signing key $\mathsf{sk}_j = [t, u, v, s_1, \ldots, s_n]$, a message $m \in \mathbb{Z}_p$ and a public tag $\tau = (\tau_1, \tau_2, \tau_3)$, it outputs the signature

$$\sigma = \tau_1^{t+\sum_{\ell=1}^n s_\ell \mathcal{H}(m)^\ell} \times \tau_2^u \times \tau_3^v.$$

$\mathsf{AggrKey}(\{\mathsf{vk}_j\}_j)$: Given verification keys vk_j, it outputs the aggregated verification key $\mathsf{avk} = [\mathsf{vk}_j]_j$;

$\mathsf{AggrSign}(\tau, (\mathsf{vk}_j, m_{j,i}, \sigma_{j,i})_{j,i})$: Given tuples of verification key vk_j, message $m_{j,i}$ and signature $\sigma_{j,i}$ all under the same tag τ, it outputs the signature $\sigma = \prod_{j,i} \sigma_{j,i}$ of the concatenation of the messages verifiable with $\mathsf{avk} \leftarrow \mathsf{AggrKey}(\{\mathsf{vk}_j\}_j)$;

$\mathsf{DerivSign}(\mathsf{avk}, \tau, \vec{M}, \sigma, \rho_{\tau\to\tau'})$: Given a signature σ on tag τ and a message-set $\vec{M}$, and $\rho_{\tau\to\tau'}$ the randomization link between τ and another tag τ', it outputs $\sigma' = \sigma^{\rho_{\tau\to\tau'}}$;

$\mathsf{RandSign}(\mathsf{avk}, \tau, \vec{M}, \sigma)$: The scheme being deterministic, it returns σ;

$\mathsf{VerifSign}(\mathsf{avk}, \tau, \vec{M}, \sigma)$: Given a valid tag $\tau = (\tau_1, \tau_2, \tau_3)$, an aggregated verification key $\mathsf{avk} = [\mathsf{vk}_j]_j$ and a message-set $\vec{M} = [\mathsf{m}_j]_j$, with for each j, $\mathsf{m}_j = [m_{j,i}]_i$, and a signature σ, one checks if the following equality holds or not, where $n_j = \#\{m_{j,i}\}$:

$$e(\sigma, \mathfrak{g}) = e\left(\tau_1, \prod_j T_j^{n_j} \times \prod_{\ell=1}^n S_{j,\ell}^{\sum_i \mathcal{H}(m_{j,i})^\ell}\right) \times e\left(\tau_2, \prod_j U_{j,2}^{n_j}\right) \times e\left(\tau_3, \prod_j V_{j,3}^{n_j}\right)$$

We also recall that the validity of the tag has to be verified, as before, for the signature to be considered valid.

Security of the Bounded SqDH-based ART-Sign Scheme 2. The linear homomorphism of the signature from [HPP20] still allows combinations. But when the number of signing queries is at most n per tag, the verification of the signature implies 0/1 coefficients only, with overwhelming probability:

Theorem 20. *The bounded SqDH-based ART-Sign defined above is unforgeable with a bounded number of signing queries per tag, even with adaptive corruptions of keys and tags, in both the generic group model and the random oracle model, as soon as $q_{\mathcal{H}}^n \ll p$, where $q_{\mathcal{H}}$ is the number of hash queries and p the order of the group (the output of the hash function).*

Proof. As argued in [HPP20], when the bases of the tags are random, even if the exponents are known, the signature that would have signed messages $\vec{M} = (g^{m^1}, \ldots, g^{m^n})$, for $m \in \mathbb{Z}_p$, is an unforgeable linearly-homomorphic signature. This means it is only possible to linearly combine signatures with the same tag: from up to n signatures σ_i on distinct messages m_i, for $i = 1, \ldots, n$ under vk_j, one can derive the signature $\sigma = \prod \sigma_i^{\alpha_i}$ on $\left(g^{\sum_i \alpha_i m_i^1}, \ldots, g^{\sum_i \alpha_i m_i^n}\right)$. Whereas the forger claims this is a signature on $\left(g^{\sum_i a_i^1}, \ldots, g^{\sum_i \alpha_i a_i^n}\right)$, on n_j values $a_1, \ldots, a_{n_j}$. Because of the constraint on τ_2, we have $\sum \alpha_i = n_j \bmod p$:

$$\sum_{i=1}^{n} \alpha_i m_i^\ell = \sum_{i=1}^{n_c} a_i^\ell \bmod p \qquad \text{for } \ell = 0, \ldots, n$$

Let us first move on the left hand side the elements $a_k \in \{m_i\}$, with only $n' \leq n_j$ new elements, we assume to be the first ones, and we note $\beta_i = \alpha_i$ if $m_i \notin \{a_k\}$ and or $\beta_i = \alpha_i - 1$ if $m_i \in \{a_k\}$:

$$\sum_{i=1}^{n} \beta_i m_i^\ell = \sum_{i=1}^{n'} a_i^\ell \bmod p \qquad \text{for } \ell = 0, \ldots, n$$

Our goal is to prove that $n' = 0$ and the α_i's are only 0 or 1.

So, first, let us assume that $n' = 0$: there is no new element. The matrix $(m_i^\ell)_{i,\ell}$, for $i = 1, \ldots, n$ and $\ell = 0, \ldots, n-1$ is a Vandermonde matrix, that is invertible: hence the unique possible vector (β_i) is the zero-vector. As a consequence, the vector $(\alpha_i)_i$ only contains 0 or 1 components.

Now, we assume $n' = 1$: there is exactly one element $a_1 \notin \{m_i\}$. We can move it on the left side:

$$\beta_0 a_1^\ell + \sum_{i=1}^{n} \beta m_i^\ell = 0 \bmod p \qquad \text{for } \ell = 0, \ldots, n, \text{ with } \beta_0 = -1$$

Again, the matrix $(m_i^\ell)_{i,\ell}$, for $i = 0, \ldots, n$ where we denote $m_0 = a_1$, and $\ell = 0, \ldots, n$, is a Vandermonde matrix, that is invertible: hence the unique possible vector (β_i) is the zero-vector, which contradicts the fact that $\beta_0 = -1$.

Eventually, we assume $n' > 1$: there are at least two elements $a_k \notin \{m_i\}$. We can move a_1 on the left side:

$$\beta_0 a_1^\ell + \sum_{i=1}^{n} \beta m_i^\ell = \sum_{i=2}^{n'} a_i^\ell \bmod p \qquad \text{for } \ell = 0, \ldots, n, \text{ with } \beta_0 = -1$$

Again, because of the invertible matrix, for the $n' - 1$ elements on the right hand side, there is a unique possible vector (β_i), and the probability for $\beta_0 = -1$ is negligible, as the new elements a_k are random (if they are issued from a hash value): probability $1/p$ for each possible choice on the $n' - 1 < n$ attributes on the right hand side. Hence, as soon as $q_{\mathcal{H}}^n \ll p$, the probability for a combination to allow $\beta_0 = -1$ is negligible.

As a conclusion, one can only combine initial messages with a weight 1 (or 0). This proves unforgeability, even with corruptions of the tags, but with a number of signed messages bounded by n, and random messages (issued from a hash function). One can also consider corruptions of the signing keys, as they are all independent: one just needs to guess under which key will be generated the forgery.

Unlinkability remains unchanged.

C Zero-Knowledge Proofs

C.1 Zero-Knowledge Proof for Square Diffie-Hellman Tuples

During both the certification of the tag τ and the showing protocol, the user must provide a proof of validity of the SqDH tuple, in an extractable way, as this must also be a proof of knowledge.

As an SqDH-tuple $(\tau_1 = h, \tau_2 = h^{\tilde{\tau}}, \tau_3 = h^{\tilde{\tau}^2}) \in \mathbb{G}_1^3$ is a Diffie-Hellman tuple $(\tau_1, \tau_2, \tau_2, \tau_3)$, one can use a Schnorr-like proof:

- The prover chooses a random scalar $r \xleftarrow{\$} \mathbb{Z}_p$, and sets and sends $U \leftarrow \tau_1^r$, $V \leftarrow \tau_2^r$;
- The verifier chooses a random challenge $e \xleftarrow{\$} \{0,1\}^\kappa$;
- The prover sends back the response $s = e\tilde{\tau} + r \bmod p$;
- The verifier checks whether both $\tau_1^s = \tau_2^e \times U$ and $\tau_2^s = \tau_3^e \times V$.

This provides an interactive zero-knowledge proof of knowledge of the witness $\tilde{\tau}$ that (τ_1, τ_2, τ_3) is an SqDH-tuple.

C.2 Groth-Sahai Proof for Square Diffie-Hellman Tuples

If you just need a proof of validity of the tuple, this is possible, using the Groth-Sahai methodology [GS08], to provide a non-interactive proof of Square Diffie-Hellman tuple: in the asymmetric pairing setting, one sets a reference string $(\mathfrak{v}_{1,1}, \mathfrak{v}_{1,2}, \mathfrak{v}_{2,1}, \mathfrak{v}_{2,2}) \in \mathbb{G}_2^4$, such that $(\mathfrak{v}_{1,1}, \mathfrak{v}_{1,2}, \mathfrak{v}_{2,1}, \mathfrak{v}_{2,2})$ is a Diffie-Hellman tuple.

Given a Square Diffie-Hellman tuple $(\tau_1 = h, \tau_2 = h^{\tilde{\tau}}, \tau_3 = h^{\tilde{\tau}^2}) \in \mathbb{G}_1^3$, one first commits $\tilde{\tau}$: $\mathsf{Com} = (\mathfrak{c} = \mathfrak{v}_{2,1}^{\tilde{\tau}} \mathfrak{v}_{1,1}^{\mu}, \mathfrak{d} = \mathfrak{v}_{2,2}^{\tilde{\tau}} \mathfrak{g}^{\tilde{\tau}} \mathfrak{v}_{1,2}^{\mu})$, for a random $\mu \xleftarrow{\$} \mathbb{Z}_p$, and one sets $\pi_1 = \tau_1^\mu$ and $\pi_2 = \tau_2^\mu$, which satisfy

$$e(\tau_1, \mathfrak{c}) = e(\tau_2, \mathfrak{v}_{2,1}) \cdot e(\pi_1, \mathfrak{v}_{1,1}) \qquad e(\tau_1, \mathfrak{d}) = e(\tau_2, \mathfrak{v}_{2,2} \cdot \mathfrak{g}) \cdot e(\pi_1, \mathfrak{v}_{1,2})$$
$$e(\tau_2, \mathfrak{c}) = e(\tau_3, \mathfrak{v}_{2,1}) \cdot e(\pi_2, \mathfrak{v}_{1,1}) \qquad e(\tau_2, \mathfrak{d}) = e(\tau_3, \mathfrak{v}_{2,2} \cdot \mathfrak{g}) \cdot e(\pi_2, \mathfrak{v}_{1,2})$$

The proof $\mathsf{proof} = (\mathfrak{c}, \mathfrak{d}, \pi_1, \pi_2)$, when it satisfies the above relations, guarantees that (τ_1, τ_2, τ_3) is a Square Diffie-Hellman tuple. This proof is furthermore zero-knowledge, under the DDH assumption in $\mathbb{G}_2$: by switching $(\mathfrak{v}_{1,1}, \mathfrak{v}_{1,2}, \mathfrak{v}_{2,1}, \mathfrak{g} \times \mathfrak{v}_{2,2})$ into a Diffie-Hellman tuple, one can simulate the proof, as the commitment is perfectly hiding.

As explained in [HPP20], one can apply a batch verification [BFI+10], and pack them in a unique one with random scalars $x_{1,1}, x_{1,2}, x_{2,1}, x_{2,2} \xleftarrow{\$} \mathbb{Z}_p$:

$$e(\tau_1^{x_{2,1}} \tau_2^{x_{2,2}}, \mathfrak{c}^{x_{1,1}} \mathfrak{d}^{x_{1,2}}) = e(\tau_2^{x_{2,1}} \tau_3^{x_{2,2}}, \mathfrak{v}_{2,1}^{x_{1,1}} \mathfrak{v}_{2,2}^{x_{1,2}} \mathfrak{g}^{x_{1,2}}) \times e(\pi_1^{x_{2,1}} \pi_2^{x_{2,2}}, \mathfrak{v}_{1,1}^{x_{1,1}} \mathfrak{v}_{1,2}^{x_{1,2}})$$

One thus just has to compute 13 exponentiations and 3 pairing evaluations for the verification, instead of 12 pairing evaluations.

C.3 Groth-Sahai Proof for Square Diffie-Hellman Tracing

For the proof of tracing, one wants to show $\tau' \sim \tau$, where τ is the reference tag for a user (certified at the registration time). With the tracing key $\mathsf{utk} = \mathfrak{g}^{\tilde{\tau}}$, one needs to show

$$e(\tau_1, \mathsf{utk}) = e(\tau_2, \mathfrak{g}) \qquad e(\tau_2, \mathsf{utk}) = e(\tau_3, \mathfrak{g})$$
$$e(\tau'_1, \mathsf{utk}) = e(\tau'_2, \mathfrak{g}) \qquad e(\tau'_2, \mathsf{utk}) = e(\tau'_3, \mathfrak{g})$$

but without revealing $\mathsf{utk} \in \mathbb{G}_2$. This is equivalent, for random $\alpha_1, \alpha_2, \alpha'_1, \alpha'_2 \xleftarrow{\$} \mathbb{Z}_p$, to have:

$$e(T_1, \mathsf{utk}) = e(T_2, \mathfrak{g}) \qquad \text{with} \qquad T_1 = {\tau_1}^{\alpha_1} \cdot {\tau_2}^{\alpha_2} \cdot {\tau'_1}^{\alpha'_1} \cdot {\tau'_2}^{\alpha'_2}$$
$$T_2 = {\tau_2}^{\alpha_1} \cdot {\tau_3}^{\alpha_2} \cdot {\tau'_2}^{\alpha'_1} \cdot {\tau'_2}^{\alpha'_2}$$

One can commit utk: as above, with the reference string $(\mathfrak{v}_{1,1}, \mathfrak{v}_{1,2}, \mathfrak{v}_{2,1}, \mathfrak{v}_{2,2}) \in \mathbb{G}_2^4$, such that $(\mathfrak{v}_{1,1}, \mathfrak{v}_{1,2}, \mathfrak{v}_{2,1}, \mathfrak{v}_{2,2})$ is a Diffie-Hellman tuple, one computes $\mathsf{Com} = (\mathfrak{c} = \mathfrak{v}_{2,1}^{\lambda}\mathfrak{v}_{1,1}^{\mu}, \mathfrak{d} = \mathfrak{v}_{2,2}^{\lambda}\mathfrak{v}_{1,2}^{\mu} \times \mathsf{utk})$, for random $\lambda, \mu \xleftarrow{\$} \mathbb{Z}_p$, and one sets $\pi_1 = T_1^{\lambda}$ and $\pi_2 = T_1^{\mu}$, which should satisfy

$$e(T_1, \mathfrak{c}) = e(\pi_1, \mathfrak{v}_{2,1}) \cdot e(\pi_2, \mathfrak{v}_{1,1}) \qquad e(T_1, \mathfrak{d}) = e(T_2, \mathfrak{g}) \cdot e(\pi_1, \mathfrak{v}_{2,2}) \cdot e(\pi_2, \mathfrak{v}_{1,2})$$

The random values $\alpha_1, \alpha_2, \alpha_1', \alpha_2'$ can be either chosen by the verifier in case of interactive proof, or set from $H(\tau_1, \tau_2, \tau_3, \tau_1', \tau_2', \tau_3')$.