

# Non-Malleable Codes for Bounded Polynomial Depth Tampering

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## Abstract

Non-malleable codes allow one to encode data in such a way that, after tampering, the modified codeword is guaranteed to decode to either the original message, or a completely unrelated one. Since the introduction of the notion by Dziembowski, Pietrzak, and Wichs (ICS '10 and J. ACM '18), a large body of work has focused on realizing such coding schemes secure against various classes of tampering functions. It is well known that there is no efficient non-malleable code secure against all polynomial size tampering functions. Nevertheless, non-malleable codes in the plain model (i.e., no trusted setup) secure against *bounded* polynomial size tampering are not known and obtaining such a code has been a major open problem.

We present the first construction of a non-malleable code secure against *all* polynomial size tampering functions that have *bounded polynomial depth*. This is an even larger class than all bounded polynomial *size* functions and, in particular, we capture all functions in non-uniform **NC** (and much more). Our construction is in the plain model (i.e., no trusted setup) and relies on several cryptographic assumptions such as keyless hash functions, time-lock puzzles, as well as other standard assumptions. Additionally, our construction has several appealing properties: the complexity of encoding is independent of the class of tampering functions and we obtain sub-exponentially small error.

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# 1 Introduction

A non-malleable code is a fascinating concept that (informally) allows one to encode messages such that it is impossible to modify the underlying message of a given codeword without decoding it first. More precisely, the operation applied to the codeword is called the *tampering function*, and the guarantee is that, with “high probability”, decoding a tampered codeword results in either the original message or an unrelated one. We refer to the probability that the attacker succeeds in coming up with a tampered codeword of a related messages as its *distinguishing advantage*, and we typically require this advantage to be negligible (i.e., smaller than the inverse of any polynomial). Note that in contrast to standard error-correcting (or detecting) codes, non-malleable codes can achieve security against tampering functions that modify *every* part of a codeword.

Non-malleable codes have proven to be a fundamental concept, giving rise to many beautiful connections and results, both in complexity theory (e.g., two-source extractors [Li12, Li13, CGL15, CZ16] and additive combinatorics [ADL18, ADKO15]) as well as in cryptography (e.g., non-malleable encryption and commitments [CMTV15, CDTV16, GPR16]).

In the paper that introduced non-malleable codes, Dziembowski, Pietrzak, and Wichs [DPW10, DPW18], observed that it is impossible to construct a non-malleable code secure against arbitrary tampering functions, since a tampering function which first decodes the codeword and then re-encodes a related message breaks security. By the same principle, it is impossible to construct a code with polynomial time decoding which is secure against all polynomial time tampering functions.<sup>1</sup> Therefore, the class of tampering functions has to be limited in some way—either in terms of computational power or in the way the functions can access the codeword. One natural limitation is by restricting the available computational complexity resources (e.g., running time, space, etc).

Already in the original work of Dziembowski et al. [DPW18] (see also [CG16] for a followup), it was shown that (with high probability) a random function is a non-malleable code secure against all circuits of size (say)  $2^{n/2}$ , where  $n$  is the size of a codeword. However, the code is clearly inefficient. Faust et al. [FMVW16] gave an efficient version of that result, but it is still not an explicit construction: For any polynomial bound  $S$ , there is an efficiently samplable *family* of codes such that (with high probability) a random member of the family is a non-malleable code secure against all functions computable by a circuit of size  $S$ . Stated differently, the result can be seen as an explicit construction (i.e., a single code) assuming an untamperable *common reference string* (CRS) which is longer than the running time of the tampering function. In the random oracle model (which can be thought of as an exponential size common random string), Faust et al. [FHMV17] constructed non-malleable codes secure against space-bounded tampering. Ball et al. [BDKM16] constructed a non-malleable code secure against bounded depth circuits with constant fan-in (which includes  $\mathbf{NC}^0$ ). Several works were able to get non-malleable codes secure against  $\mathbf{AC}^0$  tampering functions [CL17, BDKM18, BDG<sup>+</sup>18, BGW19] (actually even circuits of depth  $O(\log n / \log \log n)$ ).

Arguably, the holy grail in this line of works is to construct an explicit non-malleable code which is secure against all tampering functions from the class of bounded polynomial size circuits. Specifically, for a size bound  $S$ , we would like to get an efficient code which is non-malleable for all tampering functions that can be described by an arbitrary circuit of size  $S$ . Ideally, only decoding should require running-time greater than  $S$  and encoding should run in some a-priori

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<sup>1</sup>Here is the attack: the tampering function can decode the codeword and if it contains some pre-defined message (say all 0s), then it replaces it with garbage (which might not even correspond to a valid codeword), and otherwise it does not change the input.

fixed polynomial time, independent of  $S$ .

*Does there exist an explicit construction (in the plain model) of an efficient non-malleable code which is secure against all bounded polynomial size attackers?*

Ball et al. [BDKM18] made an important step towards this goal by using computational assumptions. Concretely, using public-key encryption and non-interactive zero-knowledge (NIZK), they gave a generic way to construct non-malleable codes secure against tampering classes  $\mathcal{F}$  using sufficiently strong average-case hardness for  $\mathcal{F}$ . This construction, however, still requires a CRS (for the public key of the encryption scheme and the CRS of the NIZK) albeit it is short (polynomial in the the security parameter and independent of the class  $\mathcal{F}$ ).

In a recent follow-up work, Ball et al. [BDK<sup>+</sup>19] managed to get rid of the CRS, but at the cost of (a) using non-standard assumptions, and (b) limiting the class of attacks and the level of security. In more detail, they showed a construction of an efficient non-malleable codes secure against all (uniform) tampering functions computable in an a-priori fixed polynomial time. But:

- Their construction relies (amongst other assumptions) on sub-exponentially sound **P**-certificates<sup>2</sup> which is a very strong and non-standard assumption. In particular, the *only* known instantiation requires *assuming* soundness of a non-trivial argument system (Micali’s CS proofs [Mic00]), which is true in the Random Oracle model.
- Their scheme is non-malleable only with respect to *uniform* polynomial time tampering (as opposed to the standard model of polynomial *size* tampering. In other words, the tampering attacker is restricted to being a *uniform* polynomial time algorithm, in contrast to the standard model of non-uniform polynomial time attackers.
- Their scheme achieves only *a-priori bounded* inverse polynomial distinguishing advantage, as opposed to achieving “full” security (i.e., negligible distinguishing advantage).
- Finally, both their encoding procedure, as well as the decoding procedure, run *longer* than the allowed tampering functions (i.e., the adversary can neither encode nor decode). In contrast, as mentioned, in principle encoding could be “efficient” in the sense that it is independent of the size/running-time of the tampering attacker.

To summarize, despite several beautiful steps towards resolving the above question, the answer is still largely unknown. Known partial solutions either require a CRS or strong and non-standard cryptographic assumptions that are only known how to be instantiated in the Random Oracle model (and even then only achieve a weaker form of non-malleability).

## 1.1 Our Results

We give the first full affirmative answer to the aforementioned question. Specifically, we construct an efficient non-malleable code that is (computationally) secure against tampering functions computable by any bounded polynomial size circuit. Our construction is in the plain model and relies on several generic and well-studied cryptographic building blocks: a time-lock puzzle [RSW96], a non-interactive non-malleable commitment [KS17, LPS17, BL18, KK19], and a non-interactive

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<sup>2</sup>These are “succinct” one-message arguments for languages in **P**, with proof length which is a fixed polynomial, independent of the time it takes to decide the language [CLP13].

SPS (super-polynomial-time simulatable) zero-knowledge protocol [BP04, BL18] (all in the plain model). While we cannot use the aforementioned primitives in their most general form, we identify certain additional properties from them that will be needed in our construction; additionally, we note that particular known constructions of them satisfy the additional desired properties; see below and in Section 2 for more details.

Our construction actually captures an even larger class of tampering functions. Specifically, we give a non-malleable code secure against all tampering functions that can be computed by *arbitrary* (unbounded) polynomial size circuit of *bounded polynomial depth*. We emphasize that while the circuit depth of the tampering function is bounded a priori by some fixed polynomial in the security parameter, the size of the circuit is *unbounded* and can be any polynomial in the security parameter.

**Theorem 1.1** (Informal Meta Theorem). *Assume the existence of a “special-purpose” time-lock puzzle, one-message non-malleable commitment, and one-message SPS zero-knowledge protocol. For any  $\bar{T} \in \text{poly}(\lambda)$ , there exists an explicit code where encoding takes time  $\text{poly}(\lambda)$ , decoding takes time  $\text{poly}(\bar{T}, \lambda)$ , and it is non-malleable against all tampering functions computable by a non-uniform arbitrary polynomial size (in  $\lambda$ ) circuit of depth  $\bar{T}$ .*

Our result is the first to handle all bounded polynomial size tampering functions (and in fact much more). In particular, as a special case, we capture all tampering functions in non-uniform  $\mathbf{NC}$  (while previously there was no construction even for  $\mathbf{NC}^1$ ). We emphasize that our scheme is *efficiently encodable*, namely, encoding time depends only on the security parameter and not on the (depth) complexity of the decoder. Furthermore, our construction readily extends to withstand (sub-)exponential size tampering functions (of depth  $\bar{T}$ ) without affecting the complexity of neither encoding nor decoding. Lastly, we note that the distinguishing advantage of any tampering function in our scheme can be made sub-exponentially small in  $\lambda$  at essentially no cost (since in any case we need to rely on sub-exponential hardness of the underlying building blocks).

In comparison, as mentioned, prior to this work, even dealing with just bounded polynomial size tampering was not known. The only approach towards polynomial size tampering [BDK<sup>+</sup>19] captured only *uniform* polynomial time tampering, but as mentioned above, even for this restricted class of tampering, their result has additional drawbacks: (1) it relies on a strong non-standard assumption ( $\mathbf{P}$ -certificates) that we only know how to satisfy in the random oracle model, and (2) it only gives inverse polynomial distinguishing advantage (as opposed to negligible distinguishing advantage).

We instantiate the time-lock puzzle using the construction of Rivest et al. [RSW96] and we show how to further use results of Bitansky and Lin [BL18] and Lin et al. [LPS17] to instantiate the required non-malleable commitment and zero-knowledge protocol. Thus, assuming the repeated squaring assumption [RSW96] (i.e., there is no way to significantly speed-up repeated squarings in a hidden-order group), a keyless multi-collision resistant hash function [BKP18] (i.e., a single function for which any PPT attacker with  $\ell(\lambda)$  bits of non-uniform advice cannot find more than  $\ell(\lambda)^c$  collisions for a constant  $c \in \mathbb{N}$ ),<sup>3</sup> as well as other standard assumptions, we obtain the following theorem.

**Theorem 1.2** (Informal). *Assume a keyless multi-collision resistant hash function, the repeated squaring assumption, an injective one-way function, and non-interactive witness-indistinguishable*

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<sup>3</sup>While keyless multi-collision resistance is a relatively new assumption, it is a natural and simple security property for keyless cryptographic hash functions, which in particular is satisfied by a random function.

proofs,<sup>4</sup> all being sub-exponentially secure. Then, for any  $\bar{T} \in \text{poly}(\lambda)$ , there exists an explicit code where encoding takes time  $\text{poly}(\lambda)$ , decoding takes time  $\text{poly}(\bar{T}, \lambda)$ , and it is non-malleable against all tampering functions computable by a non-uniform arbitrary polynomial size (in  $\lambda$ ) circuit of depth  $\bar{T}$ .

We refer to Section 1.2 for more details about the above assumptions.

**Non-malleable time-lock puzzle.** Our non-malleable code construction is secure for all bounded polynomial depth tampering functions and additionally it is efficiently encodable, meaning that encoding time is fixed as a function of the security parameter, but is otherwise independent of the time it takes to decode. We observe that the combination of these two properties actually implies a *time-lock puzzle* which is additionally *non-malleable*.<sup>5</sup> In other words, under the same assumptions as in Theorems 1.1 and 1.2, we get a *non-malleable* time-lock puzzle. We emphasize that the non-malleable time-lock puzzle that we obtain here is in the plain model, i.e., does not require any trusted setup.

In a followup work [EFKP20], the notion of non-malleable time-lock puzzles is formally defined and further studied, giving additional constructions and various applications. In particular, in [EFKP20] we give a more efficient construction than the one given in this work, which is proven secure in the (auxiliary-input) random oracle model. Additionally, in [EFKP20] we further construct time-lock puzzles which are concurrently non-malleability and show that they are useful in several desirable cryptographic protocols.

## 1.2 Related Work

Since the work of Dziembowski, Pietrzak, and Wichs [DPW10, DPW18] which introduced non-malleable codes, there has been a quite a significant amount of works on this subject in various different directions (for example, [AAG<sup>+</sup>16, KLT16, ADL18, ADKO15, CGL15, DLSZ15, CGM<sup>+</sup>16, CL17, Li17, Li18] to mention only a few in addition to the ones we mentioned earlier). Notably, various different classes of tampering functions were considered. The original work of [DPW10] presented a construction of non-malleable codes against bit-wise tampering functions. Also, Liu and Lysyanskaya [LL12] were the first to consider the class of split state tampering functions, where left and right halves of a codeword may be tampered arbitrarily, but independently. There has been a very long line of works on getting optimal constructions against such tampering functions (see the references above).

Next, we give more information about the building blocks used in our constructions and mention relevant related work.

**Time-lock puzzles.** These are puzzles that can be solved by “brute-force” in time  $\bar{T}$ , but cannot be solved significantly faster even using parallel processors. This concept was proposed by Rivest, Shamir, and Wagner [RSW96] (following Mays work [May92] on timed-release cryptography), and they have been used quite extensively studied since. The most popular instantiation relies on the

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<sup>4</sup>Non-interactive witness-indistinguishable proofs are known to exist based on various assumptions: trapdoor permutations and a particular derandomization-type assumption [BOV07], cryptographic bilinear maps [GOS06], or indistinguishability obfuscation and one-way permutations [BP15].

<sup>5</sup>Recall that time-lock puzzles are a cryptographic mechanism for sending messages “to the future”, by allowing a sender to quickly generate a puzzle with an underlying message that remains hidden until a receiver spends a moderately large amount of time solving it. Non-malleability guarantees that not only the puzzle hides the underlying message, but actually it is hard to “maul” it into a puzzle with a different “related” message.

*repeated squaring assumption* that postulates that  $\bar{T}$  repeated squarings mod  $N$ , where  $N = pq$  is a product of two secret primes, require “roughly”  $\bar{T}$  parallel time/depth. Bitansky et al. [BGJ<sup>+</sup>16] gave a construction of a time-lock puzzle from (strong) assumptions related to program obfuscation.

Our construction requires a “weak” notion of (sub-exponential) security that guarantees that the puzzle cannot be solved by sub-exponential size attackers that have depth  $\bar{T}^\epsilon$ . Therefore, using the instantiation that relies on repeated squarings, we only need to assume that there are no huge improvements in the parallel complexity of repeated squaring algorithms even for very large attackers.

**Non-malleable commitments.** Non-malleable commitments, introduced by Dolev, Dwork and Naor [DDN91], guarantee *hiding* (the committed value is kept secret from the receiver), *binding* (“opening” can yield only a single value determined in the commit phase), and *non-malleability* (guaranteeing that it is hard to “maul” a commitment to a given value into a commitment to a related value). Non-malleable commitments are extremely well studied with huge body of works trying to pin down the exact round complexity and minimal assumptions needed to obtain them [Bar02, PR05b, PR05a, LPV08, PPV08, LP09, PW10, Wee10, Goy11, LP11, GLOV12, GPR16, COSV16, COSV17, Khu17, LPS17, KS17, KK19].

We need a *non-interactive* (i.e., one-message) non-malleable commitment, of which relatively few constructions are known. Pandey et al. [PPV08] formulated a concrete property of a random oracle and showed that it suffices for non-interactive non-malleable commitments. This is a non-standard and non-falsifiable (Naor [Nao03]) assumption. Lin et al. [LPS17] showed a construction that satisfies non-malleability against uniform attackers assuming a keyless collision resistant hash function, time-lock puzzles, non-interactive commitments, and NIWI proofs, all with sub-exponential hardness. Bitansky and Lin [BL18] were able to get non-malleability against all attackers (i.e., even non-uniform ones) by either replacing the keyless collision resistant hash function with a keyless *multi-collision* resistant hash function,<sup>6</sup> or using a new assumption regarding sub-exponentially secure one-way functions admitting some strong form of hardness amplification. Most recently, Kalai and Khurana [KK19] gave a construction of a non-interactive non-malleable commitment from sub-exponential hardness of factoring or discrete log, and sub-exponential *quantum* hardness of Learning With Errors (LWE) or Learning Parity with Noise (LPN).

We will use the construction of Bitansky and Lin [BL18] and Lin et al. [LPS17] both of which rely on time-lock puzzles. Various properties of their non-malleable commitments will be crucial for our construction.

**One-message SPS zero-knowledge.** This is a one-message proof system for every language in **NP** in the plain model and without any setup assumptions that satisfies a relaxed notion of zero-knowledge referred to a super-polynomial time simulation (SPS) zero-knowledge [Pas03]. This concept was introduced by Barak and Pass [BP04] who also gave a construction assuming a keyless collision resistance hash function,<sup>7</sup> non-interactive commitments, and NIWI proofs, all with sub-exponential hardness. Their construction however satisfies soundness only against uniform attackers. Bitansky and Lin [BL18] showed how to overcome this limitation using keyless *multi-*

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<sup>6</sup>Actually, Bitansky and Lin [BL18] formulate an assumption about incompressible functions which is implied by keyless multi-collision resistant hash functions.

<sup>7</sup>Actually, Barak and Pass [BP04] formulate an assumption regarding the existence of a language in **P** which is hard to sample in slightly super-polynomial time but easy to sample in a slightly larger super-polynomial time. The existence of a keyless collision resistance hash function with sub-exponential hardness implies such a language.

collision resistant hash functions,<sup>8</sup> at the cost of obtaining a weaker soundness (allowing any attacker to output some bounded number of convincing proofs for false statements).

**Non-malleable codes vs. commitments.** (Non-interactive) non-malleable commitments and codes seem very similar. The only difference is that in the latter decoding should be efficient, while in the former it should be hard. There has been some evidence that the objects are not only syntactically related. For instance, non-malleable codes were used to construct non-malleable commitments [GPR16, CGM<sup>+</sup>16]. In the reverse direction, some works used ideas from the (vast) literature on non-malleable commitments to get new non-malleable codes [CGL16, OPVV18, BDK<sup>+</sup>19]. Our work continues the latter line of works and shows yet again that the notions are intimately related.

**Lower bounds for non-malleability.** We mentioned that there cannot be a non-malleable code secure against a class of tampering functions that includes the decoding procedures. In a very recent work, Ball et al. [BDKM20] gave various new lower bounds. The most related lower bound to this work is the one regarding (in)existence of non-malleable codes for  $\mathbf{NC}^1$  ( $\subseteq \mathbf{NC}$ ) in the standard model (a class that our construction captures). Their result introduces a notion of black-box reductions tailored for the setting of non-malleable codes and rules out such reductions for certain classes of tampering functions  $\mathcal{F}$ . Importantly, their impossibility results hold for constructions that rely only on the *minimal assumption* that there exists a distributional problem that is hard for the tampering class  $\mathcal{F}$ , but easy for  $\mathbf{P}$ . Our result bypasses the impossibility since we—in addition to an assumption of the above type (i.e. time-lock puzzles)—rely on standard cryptographic assumptions such as keyless multi-collision resistant hash functions, injective one-way functions, and non-interactive witness-indistinguishable proofs.

**The magic of the repeated squaring assumption.** In the past several years the repeated squaring assumption has played an important role in many works. In addition to the work about non-malleable commitments [LPS17] that we have already mentioned and the current work, this assumption was also used in several constructions of verifiable delay functions [Pie19, Wes19, EFKP19]. These functions are, roughly speaking, a publicly verifiable version of time-lock puzzles. The reason why this assumption has been so successful is that it brings a new dimension of hardness to the table, i.e., parallel-time, which is different from the type of hardness that standard cryptographic assumptions give.

**(Multi-)collisions resistance.** Collision resistant hash functions are (a family of) compressing functions where no efficient attacker can find a colliding pair in a random function from the family. The existence of such a family is a standard cryptographic assumption which is implied by many of the most classical assumptions such as factoring, discrete log, and more. A *keyless* collision resistant hash function is a *single* function where the above is hard for *uniform* attackers. Such functions exist in the random oracle model and may be heuristically instantiated using common constructions of cryptographic hash functions, such as SHA-3, where collisions are simply not known.

Multi-collision resistance [KNY19, BDRV18, BKP18, KNY18] is a relaxation of collision resistance where the goal of the attacker is to find a collection of *many* inputs (rather than just two) to a random function in the family that collide. The keyless version, introduced by [BKP18], is again a single function but now the security guarantee can be formulated so that it holds for all efficient attackers, even non-uniform ones. Concretely, the security guarantee is that while an attacker of size  $s$  can find about  $s$  inputs that collide, it cannot find many more, say  $s^5$  (i.e., multi-collisions cannot be compressed). Again, such functions exist in the random oracle model and may

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<sup>8</sup>See Footnote 6.

be heuristically instantiated using common constructions of cryptographic hash functions, such as SHA-3.

## 2 Technical Overview

At a very high level, as in several previous related works (e.g., [BDKM18, BDK<sup>+</sup>19]), we follow the Naor-Yung [NY90] paradigm that achieves CCA security of encryption by concatenating two instances of a CPA secure public key encryption scheme, followed by a (non-interactive) zero-knowledge proof of the equality of encrypted values. The novelty in this work stems from the way we instantiate and prove soundness of this approach in the context of non-malleable codes.

Concretely, the three main components in our construction are: a time-lock puzzle, a non-malleable commitment, and a one-message SPS zero-knowledge proof of consistency. As we will see later, these building blocks need to be instantiated in a very careful way to guarantee security. The construction  $\text{NMCode} = (\text{NMCode.E}, \text{NMCode.D})$  for a message space  $\{0, 1\}^\lambda$  and depth bound  $\bar{T}$  is informally described in Algorithm 1.

<u>NMCode.E(<math>m</math>):</u>	<u>NMCode.D(<math>Z, c, \pi</math>):</u>
1. Let $Z$ be a time-lock puzzle with hardness $\bar{T}$ and underlying message $m$ .	1. Verify the proof $\pi$ .
2. Let $c$ be a non-malleable commitment to $m$ .	2. If verifies, solve the puzzle $Z$ and output the underlying message. Otherwise, output 0.
3. Let $\pi$ be a zero-knowledge proof of consistency between $Z$ and $c$ .	
4. Output $\hat{Z} := (Z, c, \pi)$ .	

**Algorithm 1:** Our non-malleable code (*Informal*).

Let us provide some intuition and state some simple observations. Recall that a time-lock puzzle can be solved by “brute-force” in depth  $\bar{T}$ , but cannot be solved in depth  $\ll \bar{T}$ . However, time-lock puzzles may be malleable (in fact, the construction based on repeated squaring [RSW96] is easily malleable). Non-malleable commitments are, by definition, non-malleable but as opposed to time-lock puzzles, cannot be “brute-force” opened in polynomial time. Intuitively, adding the zero-knowledge proof of consistency in the above construction ties the hands of the attacker and achieves the desired properties of each of the primitives. The scheme inherits non-malleability from the non-malleable commitment while preserving the ability of solving the time-lock puzzle in polynomial time, which allows extraction of the underlying message and thereby decoding in polynomial time.

For efficiency, time-lock puzzles have a built-in trapdoor that allows one to generate puzzles *very fast* (while solving them requires many sequential resources). Thus, the running time of step 3 (generation of the zero-knowledge proof) takes fixed polynomial time (in the security parameter), independent of the depth bound  $\bar{T}$ . This is why  $\text{NMCode.E}$  has a fixed running time, polynomial in the security parameter, independent of  $\bar{T}$ . Negligible soundness of our construction, at a high level, is inherited from the security of the underlying primitives. Lastly, as we will explain shortly, we use



the non-interactive non-malleable commitments of Lin et al. [LPS17] and Bitansky and Lin [BL18] both of which are based on time-lock puzzles (and keyless collision resistant hash functions or keyless multi-collision resistant hash functions, respectively) and so this will work nicely with our usage of the time-lock puzzle in our construction.

While the intuition described above is rather solid, proving that the above construction satisfies non-malleability turns out to be challenging. We explain the high-level approach next.

## 2.1 Overview of the Proof

We will first explain the high-level approach when considering only uniform tampering functions and later explain how to handle non-uniform ones.

Since we only handle uniform tampering functions (for now), it will suffice to rely (in addition to time-lock puzzles) on a non-malleable commitment for uniform tampering functions and a one-message SPS zero-knowledge proof which satisfies uniform soundness. For the commitment scheme we will use the one of Lin, Pass, and Soni [LPS17] and for the zero-knowledge we will use the one of Barak and Pass [BP04]. We remark again that while the scheme of Lin et al. [LPS17] is also based on a time-lock puzzle, it will be convenient to use it not only in terms of assumptions, but to actually use specific properties of the scheme that will help us carry out the proof.

The proof is, naturally, by a hybrid argument where we start with the standard non-malleability game with a message  $m_0$  and in the last hybrid we will play the non-malleability game with a message  $m_1$ . Recall that the non-malleability game (a.k.a. Man-In-Middle game) consists of two stages. In the first stage, the adversary gets a codeword and it tries to maul it into a code with a related message. Then, roughly, the distribution of the underlying message in that tampered codeword should be simulatable without knowing the message itself.

In a high level, here are the sequence of hybrids that we consider. We describe the changes incrementally, namely, each hybrid starts with the scheme from the previous hybrid and makes a modification.

- Hybrid 0: The original scheme.
- Hybrid 1: Instead of using the zero-knowledge prover, we use the simulator.
- Hybrid 2: Instead of committing to  $m$ , we commit to 0.
- Hybrid 3: Instead of decoding by solving the time-lock puzzle, we decode by extracting from the commitment.
- Hybrid 4: Instead of using  $m$  as the underlying message in the time-lock puzzle, use 0.

Showing that hybrids 0, 1, and 2 are indistinguishable is simple. Hybrids 0 and 1 are indistinguishable due to the zero-knowledge property, and hybrids 1 and 2 are indistinguishable due to the hiding of the commitment scheme. The most challenging part is showing that hybrids 2 and 3 and hybrids 3 and 4 are indistinguishable.

**Hybrids 2 and 3.** The modification in this transition is from decoding via breaking the time-lock puzzle, to decoding via extraction from the non-malleable commitment. To prove indistinguishability, we show that the distribution of the underlying value in the right commitment does not change throughout the hybrids when considering both methods of decoding.

A careful inspection of the schemes in each hybrid reveals that in order for the proof to go through, we need to satisfy two conditions simultaneously:

1. The extractor of the commitment scheme (whose size is  $S_{\text{Ext}}$ ) cannot break zero-knowledge (which holds for all attackers of size at most  $S_{\text{ZK}}$ ). That is,

$$S_{\text{Ext}} \ll S_{\text{ZK}}.$$

2. The simulator of the zero-knowledge scheme (whose size is  $S_{\text{Sim}}$ ) cannot break non-malleability of the commitment (which holds for all attackers of size at most  $S_{\text{NMCom}}$ ). That is,

$$S_{\text{Sim}} \ll S_{\text{NMCom}}.$$

It also holds that  $S_{\text{NMCom}} \ll S_{\text{Ext}}$  since the commitment extractor can definitely break non-malleability (by extracting and re-committing to a related value). Therefore, the only way to satisfy the above two inequalities is if  $S_{\text{Sim}} \ll S_{\text{ZK}}$ , namely, a one-message zero-knowledge scheme where the simulator runs faster than the distinguisher!<sup>9</sup> Unfortunately, no such scheme is known as in all known schemes the simulator needs to “break” the underlying cryptographic primitives and so it has to have more resources than the distinguishers.

Our idea to make this go through is to introduce another axis of hardness which will allow us to satisfy both “inequalities” simultaneously—the axes of total size and depth. Namely, we think of algorithms as parallelizable machines and measure their complexity by counting their total size and parallel time (size and time/depth, in short). We will set the complexities of the above procedures as follows, where  $\lambda$  denotes the security parameter and where  $0 < c_1 < c_2 < c_3 < 1$ :

- $S_{\text{Ext}}$  (extraction from the non-malleable commitment): in quasi-polynomial size and depth.
- $S_{\text{ZK}}$  (zero-knowledge security): for all  $2^{\lambda^{c_1}}$  size (and depth) attackers.
- $S_{\text{Sim}}$  (ZK simulator complexity): in  $2^{\lambda^{c_2}}$  size but fixed polynomial depth.
- $S_{\text{NMCom}}$  (non-malleability): for all  $2^{\lambda^{c_3}}$  size attackers with arbitrary polynomial depth.

With this choice of parameters, it is evident that the commitment extractor cannot break zero-knowledge and also the zero-knowledge simulator cannot break non-malleability. It is also not too hard to instantiate the primitives with the above properties. The zero-knowledge scheme of Barak and Pass [BP04] readily satisfies the above properties if it is sub-exponentially hard. To get the required non-malleable commitment, we need to slightly adjust the scheme of Lin et al. [LPS17] (as they did not consider such tampering functions), but the changes are relatively minor.

**Hybrids 3 and 4.** In this hybrid, we change the time-lock puzzle’s underlying value and we want to use its hiding property. While seemingly being a relatively simple hybrid, it turns out that some complications arise. Specifically, to reduce to the underlying security of the time-lock puzzle, we need to come up with a bounded time attacker while there are two procedures that we need to run which seem to be of *arbitrary* depth. Specifically, in the reduction we need to simulate the whole experiment and use the distinguisher to break the security of the time-lock puzzle. Two procedures seem to require arbitrary large depth:

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<sup>9</sup>This kind of zero-knowledge simulation is known as *strong* super-polynomial simulation. Recently, Khurana and Sahai [KS17] managed to obtain it in two rounds, but we need a non-interactive scheme.

- The distinguisher itself, denoted  $D$  from now on.
- The extraction procedure of the non-malleable commitment (which we should execute as part of decoding).

We have *no* control over the depth (or size) of the distinguisher  $D$ , except that it is of arbitrary polynomial size and depth. However, we do know that its input, the message underlying the tampered code, is of bounded length. So, we *modify* the distinguisher and write it as a *truth table* which has hardcoded all of  $D$ 's outputs on every possible input. Call this distinguisher  $\tilde{D}$ . Observe that  $\tilde{D}$  (1) has the same input-output functionality as that of  $D$  (and so it serves as a good distinguisher), and (2) while  $\tilde{D}$ 's size is now exponential in the security parameter, its depth is some fixed polynomial in the security parameter!

For the extraction procedure, we intuitively make a similar modification. We rely on the fact that there is another brute-force extractor that requires exponential size but only fixed polynomial time. Note that for this to go through, the size of the extraction procedure has to be smaller than the hardness of the time-lock puzzle (and this can be achieved by making the time-lock puzzle sufficiently long and using sub-exponential security). So, we switch to this alternate extractor. Now, we can simulate the whole experiment in fixed polynomial depth and reduce to the security game of the underlying time-lock puzzle.

**The non-uniform case.** Extending to handle non-uniform tampering functions is challenging in the fully non-interactive setting and in the plain model. While it is relatively straight-forward to replace the non-malleable commitment scheme of Lin et al. [LPS17] (which is uniformly non-malleable) with the one of Bitansky and Lin [BL18], the challenge stems from finding an appropriate non-uniform analogue for the uniformly sound one-message zero-knowledge scheme of Barak and Pass [BP04]. Indeed, in the plain model and allowing only one message *there is no* non-uniformly sound zero-knowledge scheme (as accepting proofs for false statements just exist).

The closest candidate is the one of Bitansky and Lin [BL18] who constructed a non-uniformly *weakly* sound one-message zero-knowledge scheme. This notion captures all non-uniform attackers but the soundness guarantee is weak: every attacker *can* output some number of accepting proofs for false statements but not too many of those. Unfortunately, if we use this scheme directly in our construction instead of the current zero-knowledge scheme, the above proof outline fails. Specifically, when we switch to alternate decoding (which extracts from the commitment rather than breaks the time-lock puzzle), if the adversary uses such a maliciously crafted proof (which verifies), it can easily distinguish the two hybrids (as their outputs will be different). Another thing that makes the situation even harder is that the bad set of proofs is not global but actually attacker-dependent so we cannot just “black-list” some set of proofs in the decoding procedure.

To this end, we observe that in the security reduction, the attacker *is* fixed and so the set of “bad” proofs is *non-uniformly* known. Therefore, we can modify the alternate decoding procedure to check whether the tampered proof is one of some (polynomial size) non-uniformly hardcoded set of bad proofs—the ones that the given attacker can find. If it is one of these bad proofs, we output a fixed message, the one underlying the time-lock puzzle that corresponds to the false statement. In this way, we are guaranteed that even when switch to alternate decoding, for those maliciously crafted proofs, the attacker will not see any difference between the two hybrids.

### 3 Preliminaries

Unless stated otherwise, the logarithms in this paper are base 2. For a distribution  $\mathcal{D}$  we denote by  $x \leftarrow \mathcal{D}$  an element chosen from  $\mathcal{D}$  uniformly at random. For an integer  $n \in \mathbb{N}$  we denote by  $[n]$  the set  $\{1, \dots, n\}$ . We denote by  $U_n$  the uniform distribution over  $n$ -bit strings. A function  $\text{negl}: \mathbb{N} \rightarrow \mathbb{R}^+$  is *negligible* if for every constant  $c > 0$ , there exists an integer  $N_c$  such that  $\text{negl}(n) < n^{-c}$  for all  $n > N_c$ .

**Model of computation.** We consider uniform and non-uniform algorithms and we distinguish between their size and parallel time. The amount of non-uniformity is usually denoted by  $\kappa$ , the parallel time by  $T$ , and the size by  $S$ . We think of those algorithms as (possibly probabilistic) Turing machines with multiple heads that can operate in parallel. A non-uniform algorithm  $\mathcal{A}$  is described by a family of algorithms  $\{\mathcal{A}_\lambda\}_{\lambda \in \mathbb{N}}$ , one per security parameter  $\lambda$ . Each  $\mathcal{A}_\lambda$  corresponds to an algorithm that has input size  $n(\lambda)$  for some function  $n: \mathbb{N} \rightarrow \mathbb{N}$ . We say that  $\mathcal{A}$  is  $T$ -time, denoted  $\text{Time}[\mathcal{A}] = T(\lambda)$ , if for every  $\lambda \in \mathbb{N}$ , the parallel running time of  $\mathcal{A}_\lambda$  is at most  $T(\lambda)$ . We say that  $\mathcal{A}$  is  $S$ -size, denoted  $\text{Size}[\mathcal{A}] = S(\lambda)$ , if for every  $\lambda \in \mathbb{N}$ , the total work that the algorithm  $\mathcal{A}_\lambda$  does is at most  $S(\lambda)$ . Lastly, the amount of non-uniformity  $\kappa$  is chosen such that  $\kappa(\lambda)$  is an upper bound on the size of advice used per  $\lambda$ .

**Indistinguishability.** We recall the standard definition of computational indistinguishability.

**Definition 3.1** (Computational indistinguishability). Let  $\mathcal{X} = \{X_\lambda\}_{\lambda \in \mathbb{N}}$  and  $\mathcal{Y} = \{Y_\lambda\}_{\lambda \in \mathbb{N}}$  be two ensembles of random variables. Let  $T = T(\cdot)$ ,  $S = S(\cdot)$ , and  $\kappa = \kappa(\cdot)$  be functions. We say that  $\mathcal{X}$  and  $\mathcal{Y}$  are  $(S, T, \kappa)$ -indistinguishable if for all probabilistic  $S$ -size  $T$ -time  $\mathcal{A} = \{\mathcal{A}_\lambda\}_{\lambda \in \mathbb{N}}$  with non-uniformity  $\kappa$ , there is a negligible function  $\text{negl}(\cdot)$  such that for all  $\lambda \in \mathbb{N}$

$$|\Pr[\mathcal{A}(\mathcal{X}_\lambda) = 1] - \Pr[\mathcal{A}(\mathcal{Y}_\lambda) = 1]| \leq \text{negl}(\lambda).$$

We will use the notation  $\mathcal{X} \approx_S \mathcal{Y}$  for  $(S, S, S)$ -indistinguishability and  $\mathcal{X} \approx \mathcal{Y}$  for  $(S, S, S)$ -indistinguishability for all polynomial functions  $S$ .

#### 3.1 Non-Malleable Commitments

A tag-based non-interactive commitment scheme  $\text{NMCom} = (\text{NMCom.C}, \text{NMCom.O})$  has the following syntax:

1. For all  $m, \text{tag} \in \{0, 1\}^*$ ,  $(c, r) \leftarrow \text{NMCom.C}(1^\lambda, m, \text{tag})$  is the commitment/opening pair for the message  $m$  with tag  $\text{tag}$ .
2.  $\text{NMCom.O}(c, m, r, \text{tag}) \rightarrow \{0, 1\}$ , where 1 indicates that  $r$  is a valid opening of  $c$  to  $m$  under tag  $\text{tag}$  and 0 is returned otherwise

A commitments scheme must satisfy the following properties:

- Correctness: For every  $\lambda \in \mathbb{N}$  and  $m, \text{tag} \in \{0, 1\}^*$ ,

$$\Pr[\text{NMCom.O}(c, m, r, \text{tag}) = 1 ; c, r \leftarrow \text{NMCom.C}(1^\lambda, m, \text{tag})] = 1,$$

where the probability is taken over the randomness of  $\text{NMCom.C}$ .

- (Perfect) binding: For any string  $c$ , strings  $m, m' \in \{0, 1\}^*$  of the same length, strings  $\text{tag}, \text{tag}' \in \{0, 1\}^*$  of the same length, and opening strings  $r, r'$ ,

$$\text{if } \text{NMCom.O}(c, m, r, \text{tag}) = \text{NMCom.O}(c, m', r', \text{tag}') = 1, \text{ then } m = m'.$$

- $S$ -hiding: For all polynomials  $n = n(\lambda)$  and  $n' = n'(\lambda)$ , and all  $\text{tag} \in \{0, 1\}^{n'}$  and  $m, m' \in \{0, 1\}^n$ , it holds that

$$\{\text{NMCom.C}(1^\lambda, m, \text{tag})\}_{\lambda \in \mathbb{N}} \approx_S \{\text{NMCom.C}(1^\lambda, m', \text{tag})\}_{\lambda \in \mathbb{N}}.$$

A non-tag-based commitment scheme is the same as above but the tag is fixed to  $\text{tag} = \perp$ . In such cases, we omit the last input to  $\text{NMCom.C}$  and  $\text{NMCom.O}$ .

**Definition 3.2** (Underlying committed value,  $\text{val}$ ). Define  $\text{val}: \{0, 1\}^* \rightarrow \{0, 1\}^*$  to be the function that gets as input a commitment and outputs the underlying committed value. That is,  $\text{val}(c) = m$  if there exists  $r \in \{0, 1\}^*$  such that  $\text{NMCom.O}(c, m, r) = 1$ , and  $\perp$  otherwise.

**Definition 3.3** ((Over-)extractability). We say a commitment scheme is  $(S, T)$ -extractable if there exists a (uniform)  $S$ -size and  $T$ -time procedure  $\text{NMCom.E}$  and a negligible function  $\text{negl}(\cdot)$ , such that for all  $c \in \{0, 1\}^*$  it holds that  $\Pr[\text{NMCom.E}(c) \neq \text{val}(c)] \leq \text{negl}(|c|)$ . The scheme is  $(S, T)$ -over-extractable if the above holds only for strings  $c \in \{0, 1\}^*$  for which  $\text{val}(c) \neq \perp$ .

**Man In The Middle Execution (MIM).** Let  $\text{NMCom} = (\text{NMCom.C}, \text{NMCom.O})$  be a tag-based commitment scheme, and  $\mathcal{A}$  an arbitrary adversary. For security parameter  $\lambda \in \mathbb{N}$ , consider the following interactions by  $\mathcal{A}(1^\lambda)$ :

- Left interaction:  $\mathcal{A}(1^\lambda)$  interacts with the sender and receives commitment to a message  $m$  of length  $\lambda$  using identity tag as  $c \leftarrow \text{NMCom.C}(1^\lambda, m, \text{tag})$ .
- Right interaction:  $\mathcal{A}(1^\lambda)$  interacts with the receiver and tries to commit to related message  $\tilde{m}$  using identity  $\tilde{\text{tag}}$  of its choice. Specifically, for the commitment  $\tilde{c}$  sent to the receiver, let  $\tilde{m} = \text{val}(\tilde{c})$ . Furthermore, if  $\tilde{\text{tag}} = \text{tag}$ , then we set  $\tilde{m} = \perp$ .

Let  $\text{MIM}_{\text{NMCom}}^{\mathcal{A}}(\lambda, m)$  denote the random variable that describes  $\tilde{m}$  that  $\mathcal{A}$  commits to in the right interaction along with its output in the MIM execution  $\text{MIM}_{\text{NMCom}}^{\mathcal{A}}(\lambda, m)$  as described above.

**Definition 3.4** ( $(S, T, \kappa)$ -non-malleability [LPS17]). A tag-based commitment scheme  $\text{NMCom} = (\text{NMCom.C}, \text{NMCom.O})$  is said to be  $(S, T, \kappa)$ -non-malleable if for all  $S$ -size  $T$ -time algorithms  $\mathcal{A} = \{\mathcal{A}_\lambda\}_{\lambda \in \mathbb{N}}$  with non-uniform advice of size  $\kappa$ , it holds that

$$\{\text{MIM}_{\text{NMCom}}^{\mathcal{A}}(\lambda, m_0)\}_{\lambda \in \mathbb{N}, m_0 \in \{0, 1\}^m} \approx \{\text{MIM}_{\text{NMCom}}^{\mathcal{A}}(\lambda, m_1)\}_{\lambda \in \mathbb{N}, m_1 \in \{0, 1\}^m}.$$

### 3.2 One-Message SPS Zero-Knowledge Proofs

We recall the definitions of a one-message SPS zero-knowledge (1ZK) systems, following Barak and Pass [BP04]. In a high-level, this is a non-interactive protocol (just one message from the prover to the verifier) which provides soundness for the verifier and zero-knowledge for the prover. We emphasize that we want a protocol in the plain model, namely, without any setup assumptions (such as a trusted common reference string).

More precisely, a one-message SPS zero-knowledge argument system  $(P, V)$  for an NP relation  $R(x, w)$  with associated language  $L$  consists of two polynomial-time algorithms:

- $\pi \leftarrow P(x, w, 1^\lambda)$ : Given an instance  $x$ , witness  $w$ , and security parameter  $1^\lambda$ ,  $P$  produces a proof  $\pi$ .
- $b \leftarrow V(x, \pi)$ : Given a proof  $\pi$  for instance  $x$ ,  $V$  outputs a bit  $b$ , where  $b = 1$  indicates acceptance.

We require the following standard three properties from such a zero-knowledge system.

- Completeness: For every  $\lambda \in \mathbb{N}$  and every  $(x, w) \in R$ ,

$$\Pr_P[V(x, \pi) = 1 : \pi \leftarrow P(x, w, 1^\lambda)] = 1$$

- $(S_P, \kappa)$ -soundness: For all  $S_P$ -size adversaries  $\mathcal{A} = \{\mathcal{A}_\lambda\}_{\lambda \in \mathbb{N}}$  with non-uniform advice of size  $\kappa$ , there exists a negligible function  $\text{negl}(\cdot)$  such that for any  $\lambda \in \mathbb{N}$ ,

$$\Pr_{(x, \pi) \leftarrow \mathcal{A}_\lambda} \left[ \begin{array}{c} x \notin L \\ V(x, \pi, 1^\lambda) = 1 \end{array} \right] \leq \text{negl}(\lambda).$$

- $(S_D, S_{\text{Sim}}, T_{\text{Sim}})$ -zero-knowledge: There exists a  $S_{\text{Sim}}$ -size  $T_{\text{Sim}}$ -time (uniform) algorithm  $\text{Sim} = \{\text{Sim}_\lambda\}_{\lambda \in \mathbb{N}}$ , such that for every  $(x, w) \in R$ ,

$$\{\pi \leftarrow P(x, w, 1^\lambda)\}_{\lambda \in \mathbb{N}} \approx_{S_D} \{\pi \leftarrow \text{Sim}_\lambda(x)\}_{\lambda \in \mathbb{N}}.$$

We say the system is  $(S_D, S_{\text{Sim}})$ -zero-knowledge if it is  $(S_D, S_{\text{Sim}}, S_{\text{Sim}})$ -zero-knowledge.

**(Non-uniform) weak-soundness.** In order to capture non-uniform attackers, Bitansky and Lin [BL18] introduced a relaxation of the soundness requirement and showed that one can obtain it for all non-uniform attackers. Their notion of soundness is called *weak soundness*. It allows each adversary to find some (small, bounded) number of accepting proofs for false statements, somewhat proportional to the size of its own description (and not much more than that). The precise definition is given next.

**Definition 3.5** ( $(S_P, K)$ -weak-soundness [BL18]). For any  $S_P$ -time probabilistic adversary  $\mathcal{A} = \{\mathcal{A}_\lambda\}_{\lambda \in \mathbb{N}}$ , there exists a negligible function  $\text{negl}(\cdot)$  and a collection of sets  $\mathcal{Z} = \{\mathcal{Z}_\lambda\}_{\lambda \in \mathbb{N}}$ , where  $|\mathcal{Z}_\lambda| = K(|\mathcal{A}_\lambda|)$ , such that for any  $\lambda \in \mathbb{N}$ ,

$$\Pr_{\mathcal{A}_\lambda} \left[ \begin{array}{c} x \notin L(R) \cup \mathcal{Z}_\lambda \\ V(x, \pi, 1^\lambda) = 1 \end{array} : (x, \pi) \leftarrow \mathcal{A}_\lambda \right] \leq \text{negl}(\lambda).$$

### 3.3 Time-Lock Puzzles

A time-lock puzzle [RSW96] is a puzzle that can be generated given a solution, but given just the puzzle, it is moderately-hard to find the solution. A time-lock puzzle is initialized with some difficult parameter  $t$  and its security holds against a class of algorithms which we parametrize by their respective size and parallel time. The following definition follows Bitansky et al. [BGJ<sup>+</sup>16].

**Definition 3.6** (Time-lock puzzles). A time-lock puzzle (TL) is a tuple  $\text{TL} = (\text{TL.Gen}, \text{TL.Sol})$  with the following syntax

- $Z \leftarrow \text{TL.Gen}(1^\lambda, t, s)$ : A probabilistic algorithm that takes as input a security parameter, a difficulty parameter  $t$  and a solution  $s \in \{0, 1\}^\lambda$ , where  $\lambda$  is a security parameter, and outputs a puzzle  $Z$ .
- $s \leftarrow \text{TL.Sol}(Z)$ : A deterministic algorithm that takes as input a puzzle  $Z$  and outputs a solution  $s$ .

We require the following three properties:

- Completeness: For every security parameter  $\lambda$ , difficulty parameter  $t$ , solution  $s \in \{0, 1\}^\lambda$ , and puzzle  $Z$  in the support of  $\text{TL.Gen}(1^\lambda, t, s)$ ,  $\text{TL.Sol}(Z)$  outputs  $s$ .
- Efficiency:
  - $Z \leftarrow \text{TL.Gen}(1^\lambda, t, s)$  runs in time  $\text{poly}(\log t, \lambda)$ .
  - $s \leftarrow \text{TL.Sol}(Z)$  can be computed in time  $t \cdot \text{poly}(\lambda)$  for  $Z$  in the support of  $\text{TL.Gen}(1^\lambda, t, \cdot)$
- $(S, \epsilon)$ -hardness: There exists a polynomial  $\underline{t}(\cdot)$ , such that for every  $t(\cdot) \geq \underline{t}(\cdot)$ , and every  $S$ -size  $t^\epsilon$ -time adversary  $\mathcal{A} = \{\mathcal{A}_\lambda\}_{\lambda \in \mathbb{N}}$ , there exists a negligible function  $\text{negl}(\cdot)$  such that for every  $\lambda \in \mathbb{N}$  and every pair  $s_0, s_1 \in \{0, 1\}^\lambda$ , it holds that

$$\Pr \left[ \begin{array}{l} b \leftarrow \{0, 1\}, Z \leftarrow \text{TL.Gen}(1^\lambda, t, s_b) \\ b' \leftarrow \mathcal{A}_\lambda(Z) \end{array} : b' = b \right] \leq \frac{1}{2} + \text{negl}(\lambda).$$

## 4 Definition of Non-Malleable Codes

In this section we give our definition of non-malleable codes. Our definition follows closely the definition of [BDKM18]. One difference though is that, rather than defining non-malleability for an abstract class of tampering functions, we define non-malleability directly for the class of tampering functions that we consider in this work.

Let  $\Sigma$  and  $\Sigma'$  be sets of strings. A *coding scheme* consists of two algorithms  $\text{NMCode} = (\text{NMCode.E}, \text{NMCode.D})$  such that  $\text{NMCode.E}: \Sigma \rightarrow \Sigma'$  and  $\text{NMCode.D}: \Sigma' \rightarrow \Sigma$ . In words,  $\text{NMCode.E}$  (“encoding”) maps messages to codewords and  $\text{NMCode.D}$  (“decoding”) maps codewords to messages. The algorithm  $\text{NMCode.E}$  can be randomized and  $\text{NMCode.D}$  is assumed to be deterministic. For *correctness*, we require that for every message  $m \in \Sigma$ , it holds that

$$\Pr_{\text{NMCode.E}} [\text{NMCode.D}(\text{NMCode.E}(m)) = m] = 1.$$

$\text{NMCode.E}$  may also accept as an explicit input a security parameter in unary (in which case the syntax is  $\text{NMCode.E}(1^\lambda, m)$ ).

**Non-malleability.** Intuitively, this notion requires that given a codeword, as long as one cannot decode it, it is hard to generate a codeword with a different related underlying message. A function that takes a codeword and tries to generate a codeword for a related message out of it is called a *tampering function*. As mentioned, we have to limit the possible tampering functions in some way. Otherwise, a tampering function could decode a codeword and re-encode a related message.

**Definition 4.1** (Tampering experiment). For an algorithm  $\mathcal{A} = \{\mathcal{A}_\lambda\}_{\lambda \in \mathbb{N}}$ , a security parameter  $\lambda \in \mathbb{N}$ , and a string  $s \in \{0, 1\}^\lambda$ , define the tampering experiment:

$$\text{Tamper}_{\mathcal{A}, s}^{\text{NMCode}}(\lambda) = \left\{ \begin{array}{l} Z \leftarrow \text{NMCode.E}(1^\lambda, s); \tilde{Z} = \mathcal{A}_\lambda(Z); \tilde{s} = \text{NMCode.D}(\tilde{Z}) \\ \text{Output: } \tilde{s} \end{array} \right\},$$

where the randomness of the above experiment comes from the randomness of  $\text{NMCode.E}$ .

**Definition 4.2** ( $(S, T, \kappa)$ -non-malleability). We say that a code  $\text{NMCode}$  is  $(S, T, \kappa)$ -non-malleable if for every  $S$ -size  $T$ -time algorithm  $\mathcal{A} = \{\mathcal{A}_\lambda\}_{\lambda \in \mathbb{N}}$  with  $\kappa$  bits of non-uniformity, there exists a (uniform) probabilistic polynomial-time simulator  $\text{Sim}$  such that

$$\{\text{Tamper}_{\mathcal{A}, s}^{\text{NMCode}}(\lambda)\}_\lambda \approx \{\text{Ideal}_{\text{Sim}, s}(\lambda)\}_\lambda,$$

where

$$\text{Ideal}_{\text{Sim}, s}(\lambda) = \left\{ \begin{array}{l} \tilde{s} \cup \{\text{same}\} \leftarrow \text{Sim}^{\mathcal{A}_\lambda}(1^\lambda) \\ \text{Output: } s \text{ if output of Sim is same and otherwise } \tilde{s} \end{array} \right\}.$$

**Medium non-malleability.** We next define a different notion of non-malleability, referred to as *medium* non-malleability, which implies the one above (Definition 4.2) but is slightly easier to work with.

**Definition 4.3** ( $(S, T, \kappa)$ -medium non-malleability). We say that a code  $\text{NMCode}$  is  $(S, T, \kappa)$ -medium non-malleable if there exists a function  $g$  such that for every  $s_0, s_1 \in \{0, 1\}^\lambda$  and every  $S$ -size  $T$ -time algorithm  $\mathcal{A} = \{\mathcal{A}_\lambda\}_{\lambda \in \mathbb{N}}$  with  $\kappa$  bits of non-uniformity, it holds that

$$\{\text{MedTamper}_{\mathcal{A}, s_0}^{\text{NMCode}}(\lambda)\}_{\lambda \in \mathbb{N}} \approx \{\text{MedTamper}_{\mathcal{A}, s_1}^{\text{NMCode}}(\lambda)\}_{\lambda \in \mathbb{N}},$$

where the tampering experiment (whose randomness comes from the randomness of  $\text{NMCode.E}$ ) is defined as follows:

$$\text{MedTamper}_{\mathcal{A}, s}^{\text{NMCode}}(\lambda) = \left\{ \begin{array}{l} Z \leftarrow \text{NMCode}(1^\lambda, s); \tilde{Z} = \mathcal{A}_\lambda(Z); \tilde{s} = \text{NMCode.D}(\tilde{Z}) \\ \text{Output: same}^* \text{ if } g(Z, \tilde{Z}) = 1, \text{ and } \tilde{s} \text{ otherwise} \end{array} \right\},$$

and where  $g(\cdot, \cdot)$  is a predicate such that for every  $\mathcal{A}$  as above,  $\lambda \in \mathbb{N}$ , and  $s \in \{0, 1\}^\lambda$ ,

$$\Pr_{Z \leftarrow \text{NMCode.D}(1^\lambda, s)} [g(Z, \mathcal{A}_\lambda(Z)) = 1 \wedge \text{NMCode.D}(\mathcal{A}_\lambda(Z)) \neq s] \leq \text{negl}(\lambda).$$

## 5 The Building Blocks

### 5.1 Time-Lock Puzzle

**Theorem 5.1.** *Assuming the sub-exponential hardness of the repeated squaring assumption, there exists a time-lock puzzle which is  $(S^{\text{TL}}, \epsilon)$ -hard for a fixed  $\epsilon \in (0, 1)$  and where  $\xi^{\text{TL}} = 2^{3\lambda}$ .*



We need a time-lock puzzle which, when instantiated with difficulty parameter  $t$ , is hard for machines that have parallel time at most  $t^\epsilon$  for some fixed  $\epsilon \in (0, 1)$ , even if their total size is  $2^{3\lambda}$ . We instantiate this primitive by relying on the repeated squaring assumption with sub-exponential hardness. The latter says that for some  $\epsilon, \epsilon' \in (0, 1)$  and any large enough  $t$  the following holds: any  $2^{\lambda^{\epsilon'}}$ -size  $t^\epsilon$ -time algorithm cannot distinguish  $(g, N, t, g^{2^t} \bmod N)$  from  $(g, N, t, g')$  for uniform  $g, g' \in Z_{p \cdot q}^*$ , where  $p$  and  $q$  are two random  $\lambda$ -bit primes. Note that it is common to assume the above assumption even for  $((1 - \epsilon) \cdot t)$ -time algorithms our assumption is much weaker.

To generate a puzzle  $Z$  with difficulty  $t$  and a message  $m$ , one does the following (we assume here for simplicity that  $m$  is short enough but it is easy to extend this): Sample an RSA modulus  $N = pq$  to be a product of two random  $\text{poly}(\lambda)$ -bit primes (with some large enough polynomial; see below), and computes  $Z = (g, N, t, m + g^{2^t} \bmod N)$ , where  $g$  is a randomly chosen element in  $Z_N^*$ . Note that using  $p$  and  $q$  it is possible to compute  $g^{2^t} \bmod N$  in fixed polynomial time in  $\lambda$  (and  $\log t$  which is absorbed by the  $\text{poly}(\lambda)$  term) by first computing  $a = 2^t \bmod \phi(N)$  (where  $\phi(N) = (p - 1)(q - 1)$ ) and then computing  $Z = g^a \bmod N$ .

Assuming the sub-exponential hardness of the repeated squaring assumption, we want a time-lock puzzle whose guarantee is that the underlying value is completely hidden as long as the attacker has size less than  $2^{3\lambda}$  size and  $t^\epsilon$  time. To achieve this, the bit-length of  $p$  and  $q$  needs to be large enough. That is, we need to instantiate our primes with say  $\tilde{\lambda} = (3\lambda)^{1/\epsilon}$  bits which would give security for attackers of size  $2^{3\lambda}$  and  $t^\epsilon$  time.

## 5.2 Non-Malleable Commitment

**Theorem 5.2.** *Assume that there is a keyless multi-collision resistant hash function, the repeated squaring assumption, NIWI proof for all  $\mathbf{NP}$ , and injective one-way functions, all with sub-exponential hardness. Then, there exists a non-interactive commitment which is  $S^{\text{NMCom}}$ -hiding,  $(S_{\text{Ext}_1}^{\text{NMCom}}, T_{\text{Ext}_1}^{\text{NMCom}})$ -extractable via  $\text{NMCom.Ext}_1$  and  $(S_{\text{Ext}_2}^{\text{NMCom}}, T_{\text{Ext}_2}^{\text{NMCom}})$ -extractable via  $\text{NMCom.Ext}_2$ , and  $(S_{\text{NM}}^{\text{NMCom}}, T_{\text{NM}}^{\text{NMCom}})$ -non-malleable for all polynomial functions  $S^{\text{NMCom}}$  and  $T_{\text{NM}}^{\text{NMCom}}$ , and where  $S_{\text{Ext}_1}^{\text{NMCom}}(\lambda) = T_{\text{Ext}_1}^{\text{NMCom}}(\lambda) = 2^{\log^2 \lambda}$ ,  $S_{\text{Ext}_2}^{\text{NMCom}}(\lambda) = 2^{2\lambda}$ ,  $T_{\text{Ext}_2}^{\text{NMCom}}(\lambda) = \lambda^3$ , and  $S_{\text{NM}}^{\text{NMCom}} = 2^\lambda$ .*

**Theorem 5.3.** *Assume that there is a keyless collision resistant hash function, the repeated squaring assumption, NIWI proof for all  $\mathbf{NP}$ , and injective one-way functions, all with sub-exponential hardness. Then, there exists a non-interactive commitment which is  $S^{\text{NMCom}}$ -hiding,  $(S_{\text{Ext}_1}^{\text{NMCom}}, T_{\text{Ext}_1}^{\text{NMCom}})$ -extractable via  $\text{NMCom.Ext}_1$  and  $(S_{\text{Ext}_2}^{\text{NMCom}}, T_{\text{Ext}_2}^{\text{NMCom}})$ -extractable via  $\text{NMCom.Ext}_2$ , and  $(S_{\text{NM}}^{\text{NMCom}}, T_{\text{NM}}^{\text{NMCom}}, \kappa_{\text{NM}}^{\text{NMCom}})$ -non-malleable for all polynomial functions  $S^{\text{NMCom}}$  and  $T_{\text{NM}}^{\text{NMCom}}$ , and where  $S_{\text{Ext}_1}^{\text{NMCom}}(\lambda) = T_{\text{Ext}_1}^{\text{NMCom}}(\lambda) = 2^{\log^2 \lambda}$ ,  $S_{\text{Ext}_2}^{\text{NMCom}}(\lambda) = 2^{2\lambda}$ ,  $T_{\text{Ext}_2}^{\text{NMCom}}(\lambda) = \lambda^3$ ,  $S_{\text{NM}}^{\text{NMCom}} = 2^\lambda$ , and  $\kappa_{\text{NM}}^{\text{NMCom}} = 0$ .*

The difference between the two theorems are that in the former we obtain non-malleability for non-uniform attackers but using a keyless multi-collision resistant hash, while in the latter we obtain non-malleability only for uniform attackers but we are using a keyless (plain) collision resistant hash.

We need a one-message non-malleable tag-based commitment scheme (Definition 3.1) which is hiding for all (non-uniform) polynomial-size distinguishers, extractable either in size and time  $2^{\log^2 \lambda}$  or in  $2^\lambda$ -size and  $\lambda^3$ -time, and non-malleable for all exponential size and polynomial time tampering functions.

**The uniform scheme.** To get the scheme satisfying the properties listed in Theorem 5.3 we use the scheme of Lin et al. [LPS17]. Let us review their scheme and explain why and how it satisfies the above properties. In a high-level, they use two types of commitment scheme, each with a different “axis” of hardness. From sub-exponentially secure injective one-way functions, they obtain a sub-exponentially secure commitment scheme  $\text{Com}^s$ . By instantiating  $\text{Com}^s$  with different security parameters, one can obtain a family of  $\gamma$  commitment schemes  $\{\text{Com}_i^s\}_{i \in [\gamma]}$  such that  $\text{Com}_{i+1}^s$  is harder than  $\text{Com}_i^s$  for all  $1 \leq i \leq \gamma - 1$  in the *axis of size*. Namely, using size which is sufficient to extract from  $\text{Com}_i^s$  it is still hard to break  $\text{Com}_{i+1}^s$ . Also, the extraction procedure is essentially a brute force algorithm that “tries all option” and so it is highly parallelizable and requires fixed parallel time (depth).

A similar trick is performed using time-lock puzzles. They are used to obtain a family of  $\gamma$  commitment schemes  $\{\text{Com}_i^t\}_{i \in [\gamma]}$  such that  $\text{Com}_{i+1}^t$  is harder than  $\text{Com}_i^t$  for all  $1 \leq i \leq \gamma - 1$  in the *axis of time*. Namely, in time which is sufficient to extract from  $\text{Com}_i^t$  it is still hard to break  $\text{Com}_{i+1}^t$ . The extraction procedure is highly sequential and requires very small total size. In particular, in size which is sufficient to extract from any  $\text{Com}^t$  it is still hard to break any  $\text{Com}^s$ .

To construct a non-malleable commitment scheme  $\text{NMCom}$ , their key idea is to combine a  $\text{Com}^s$  and  $\text{Com}^t$  scheme with opposite strength. That is,

$$\text{NMCom}(1^\lambda, m, \text{tag}) = \text{Com}_{\text{tag}}^s(1^\lambda, s) \parallel \text{Com}_{\gamma - \text{tag}}^t(1^\lambda, s \oplus m), \text{ where } s \leftarrow \{0, 1\}^{|m|}.$$

The hiding and non-malleability proofs are the same as in [LPS17]. Hiding is immediate from hiding of the two underlying commitments, and we sketch the main idea behind the proof of non-malleability next. Non-malleability holds by considering two cases. First, if the left tag  $i$  is smaller than the right tag  $j$ , the  $\text{Com}_j^t$  commitment on the right remains hiding for attackers of size and time enough for extracting from both  $\text{Com}_i^t$  and  $\text{Com}_j^s$ . Therefore the right committed value remains hidden, while the right is extracted. Otherwise, if the left tag  $i$  is larger than the right commitment  $j$ , then the  $\text{Com}_i^s$  commitment on the left remains hiding for attackers of size and time enough for extracting from both  $\text{Com}_j^s$  and  $\text{Com}_{\gamma - j}^t$ . Thus, the left committed value remains hidden, while the right is extracted.

Lastly, we explain how to implement the two extraction procedures that we need. Recall that we need one extraction procedure that works in quasi-linear size and time and another procedure that works in exponential size and fixed polynomial time. The former is implemented by extracting from the right commitment (by breaking the time-lock puzzles) and the latter is implemented by breaking the left commitment (by checking in parallel all possible openings).

Of course, the above construction is not the final construction of [LPS17] as it supports only a small number of tags (while our goal is to support an exponential number of tags). To get around this they present a tag-amplification technique that is based on a tree-like structure and the way they avoid blow-up in the commitment size is by using a (keyless) collision resistant hash function (which causes the final construction to be non-malleable only with respect to uniform attackers). We refer to [LPS17] for the precise details.

**The non-uniform scheme.** To get the scheme satisfying the properties listed in Theorem 5.2 we use the scheme of Bitansky and Lin [BL18] (which in turns is based on the scheme of [LPS17]). Here, they present a new tag-amplification technique, inspired by a interactive tag-amplification technique of Khurana and Sahai [KS17], where they make it non-interactive using their one-message zero-knowledge protocol (which is based on keyless multi-collision resistant hash functions).

For our purposes, the details of this transformation are not very relevant—the only thing that is important is the structure of their final commitment. Indeed, it consists of the same time or space hard commitments of [LPS17] (along with various proofs). These are extractable in the same manner, either in quasi-linear time or in exponential size and polynomial time.

### 5.3 One-Message Zero-Knowledge

**Theorem 5.4.** *Assume the existence of a one-way permutation, a NIWI proof systems for all NP, a keyless multi-collision resistant hash function, all sub-exponentially secure. Then, there exists a one-message SPS zero-knowledge argument system satisfying  $(S_P, K)$ -weak-soundness and  $(S_D, S_{\text{Sim}}, T_{\text{Sim}})$ -zero-knowledge for all polynomials  $S_P(\lambda)$ , and where  $K \in \text{poly}(\lambda)$  is a fixed polynomial,  $S_D(\lambda) = 2^{\lambda^\eta}$  and  $S_{\text{Sim}}(\lambda) = 2^{\lambda^{\eta'}}$  for some constants  $\eta, \eta' \in (0, 1)$ , and  $T_{\text{Sim}}(\lambda) = \lambda^2$ .*

**Theorem 5.5.** *Assume the existence of a one-way permutation, a NIWI proof systems for all NP, a collision resistant hash function secure against uniform polynomial-time algorithms, all sub-exponentially secure. Then, there exists a one-message SPS zero-knowledge argument system satisfying  $(S_P, \kappa)$ -soundness and  $(S_D, S_{\text{Sim}}, T_{\text{Sim}})$ -zero-knowledge for all polynomial  $S_P(\lambda)$ , and where  $\kappa(\lambda) = 0$ ,  $S_D(\lambda) = 2^{\lambda^\eta}$  and  $S_{\text{Sim}}(\lambda) = 2^{\lambda^{\eta'}}$  for some constants  $\eta, \eta' \in (0, 1)$ , and  $T_{\text{Sim}}(\lambda) = \lambda^2$ .*

The difference between the two theorems are that in the former we obtain weak-soundness for non-uniform attackers but using a keyless multi-collision resistant hash, while in the latter we obtain (plain) soundness only for uniform attackers but we are using a keyless (plain) collision resistant hash.

Barak and Pass [BP04] showed that a one-message zero-knowledge system exists assuming a collection of sub-exponentially hard primitives: a one-way permutation, a NIWI for all NP, and a keyless collision resistant hash function. Intuitively, their construction follows the Feige-Lapidot-Shamir paradigm [FLS90] where the protocol consists of a commitment to 0 and a WI argument for the statement that either the prover knows a witness for the given instance, or it used a commitment to a special (hard to guess) value. The special value which is hard to guess is, intuitively, a collision in an appropriately chosen hash function and this is why soundness only applies to uniform malicious provers. The simulator is a parallel machine that can find such a collision by brute force using a super-polynomial size procedure which has only fixed polynomial time (it tries all possibilities in parallel). Their construction gives Theorem 5.5.

Bitansky and Lin [BL18] constructed a one-message zero-knowledge argument system by replacing the uniform hash function used by Barak and Pass with a *keyless* multi collision resistant hash function [BKP18]. Their construction gives Theorem 5.4.

## 6 The Non-Malleable Code

In this section, we present a construction of a non-malleable code that satisfies non-malleability against all (non-uniform) polynomial size attackers that have bounded polynomial depth. In other words, the only way to maul a codeword is by having high depth.

Our construction relies on several building blocks on which we elaborate next.

1. A time-lock puzzle (Definition 3.6 and Theorem 5.1)  $\text{TL} = (\text{TL.Gen}, \text{TL.Sol})$  which, for all large enough difficulty parameters  $t$ , allows to generate puzzles which are hard for any (non-uniform) machine whose parallel time/depth is at most  $t^\epsilon$ , even it has size  $2^{3\lambda}$ .

More precisely, for a difficulty parameter  $t$ , it is  $(S^{\text{TL}}, \epsilon)$ -hard for a fixed  $\epsilon \in (0, 1)$  and for  $S^{\text{TL}}(\lambda) = 2^{3\lambda}$ .

2. A one-message SPS zero-knowledge argument system (Definition 3.2 and Theorem 5.4)  $\text{ZK} = (\text{ZK.P}, \text{ZK.V})$  which is weakly sound w.r.t. all (non-uniform) polynomial-size attackers, there is a (uniform) simulator that requires sub-exponential size and fixed polynomial time, and zero-knowledge holds w.r.t. sub-exponential size adversaries.

More precisely, it is  $(S_P^{\text{ZK}}, K^{\text{ZK}})$ -sound and  $(S_D^{\text{ZK}}, S_{\text{Sim}}^{\text{ZK}}, T_{\text{Sim}}^{\text{ZK}})$ -zero-knowledge for all polynomial functions  $S_P^{\text{ZK}}$  and where  $K^{\text{ZK}} \in \text{poly}(\lambda)$  is a fixed polynomial,  $S_D^{\text{ZK}}(\lambda) = 2^{\lambda^\eta}$ ,  $S_{\text{Sim}}^{\text{ZK}}(\lambda) = 2^{\lambda^{\eta'}}$ , and  $T_{\text{Sim}}^{\text{ZK}}(\lambda) = \lambda^2$ .

3. A one-message non-malleable tag-based commitment scheme (Definition 3.1 and Theorem 5.2)  $\text{NMCom} = (\text{NMCom.C}, \text{NMCom.O})$  which is hiding for all (non-uniform) polynomial-size distinguishers, extractable either in size and time  $2^{\log^2 \lambda}$  or in  $2^\lambda$  size and  $\lambda^3$  time, and non-malleable for all exponential size and polynomial time tampering functions.

More precisely, it is  $S^{\text{NMCom}}$ -hiding,  $(S_{\text{Ext}_1}^{\text{NMCom}}, T_{\text{Ext}_1}^{\text{NMCom}})$ -extractable via  $\text{NMCom.Ext}_1$  and  $(S_{\text{Ext}_2}^{\text{NMCom}}, T_{\text{Ext}_2}^{\text{NMCom}})$ -extractable via  $\text{NMCom.Ext}_2$ , and  $(S_{\text{NM}}^{\text{NMCom}}, T_{\text{NM}}^{\text{NMCom}})$ -non-malleable for all polynomial functions  $S^{\text{NMCom}}$  and  $T_{\text{NM}}^{\text{NMCom}}$ , and where  $S_{\text{Ext}_1}^{\text{NMCom}}(\lambda) = T_{\text{Ext}_1}^{\text{NMCom}}(\lambda) = 2^{\log^2 \lambda}$ ,  $S_{\text{Ext}_2}^{\text{NMCom}}(\lambda) = 2^{2\lambda}$ ,  $T_{\text{Ext}_2}^{\text{NMCom}}(\lambda) = \lambda^3$ , and  $S_{\text{NM}}^{\text{NMCom}} = 2^\lambda$ .

4.  $\text{Sig} = (\text{Sig.G}, \text{Sig.S}, \text{Sig.V})$ . A one-time signature scheme, unforgeable for polynomial-size attackers.

We show that assuming the existence of the above primitives, there is a code which is non-malleable for *all* polynomial-size attackers that run in bounded polynomial depth. We denote the latter  $\bar{T}$ . Our main result is summarized in the following theorem.

**Theorem 6.1.** *Assume a time-lock puzzle  $\text{TL}$ , a one-message SPS zero knowledge system  $\text{ZK}$ , a one-message non-malleable commitment scheme  $\text{NMCom}$ , and a one-time signature scheme  $\text{Sig}$ , as above. Then, there exist constants  $\alpha, \beta, \gamma \in \mathbb{N}$  such that for any large enough polynomial  $\bar{T}$ , there is a code  $\text{NMCode} = (\text{NMCode.E}, \text{NMCode.D})$  (described below in Algorithms 2, 3, and 4) with the following properties:*

1. *The input of  $\text{NMCode.E}$  is a message from  $\{0, 1\}^\lambda$  and it outputs a codeword in  $\{0, 1\}^{\lambda^\alpha}$ .*
2. *The running time of  $\text{NMCode.E}$  is  $\lambda^\beta$  and the running time of  $\text{NMCode.D}$  is  $(\bar{T} \cdot \lambda)^\gamma$ .*
3. *It is  $(S, \bar{T})$ -non-malleable for all polynomials  $S(\lambda)$ .*

**The construction.** Fix  $\bar{T}$ , the upper bound on the depth of the tampering function. The high level idea of the construction is to combine the hardness for parallel machines that comes from a time-lock puzzle together with non-malleability that comes from a non-malleable commitment. Specifically, the way we combine them is so that an encoding of a message  $m$  consists of a time-lock puzzle for  $m$ , a non-malleable commitment for  $m$ , and a zero-knowledge proof that ties them together and asserts that they have the same underlying message. The construction is described formally in Algorithms 2, 3, and 4

Algorithm  $\text{NMCode.E}(1^\lambda, m)$  for  $m \in \{0, 1\}^\lambda$ :

1.  $(\text{vk}, \text{sk}) \leftarrow \text{Sig.G}(1^\lambda)$ .
2.  $Z \leftarrow \text{TL.Gen}(1^\lambda, \bar{T}^{2/\epsilon}, m; r_{\text{TL}})$  with uniformly random  $r_{\text{TL}}$ .
3.  $(c, r_{\text{NMCom}}) \leftarrow \text{NMCom.C}(1^\lambda, m, \text{tag} = \text{vk})$ .
4. Compute a ZK proof  $\pi \leftarrow \text{ZK.P}(\cdot, \cdot, 1^\lambda)$  for the relation  $\mathcal{R}_u$  from Algorithm 4 using  $(\text{vk}, Z, c)$  as the instance and  $(r_{\text{TL}}, m, r_{\text{NMCom}})$  as the witness.
5.  $\sigma \leftarrow \text{Sig.S}(\text{sk}, (Z, c, \pi))$ .
6. Output  $\hat{Z} = (\text{vk}, Z, c, \pi, \sigma)$ .

**Algorithm 2:** The encoding procedure  $\text{NMCode.E}$ .

Algorithm  $\text{NMCode.D}(\text{vk}, Z, c, \pi, \sigma)$ :

1. Verify the signature  $\sigma$ :  

$$\text{Sig.V}(\text{vk}, (Z, c, \pi), \sigma) \stackrel{?}{=} 1.$$
2. Verify the proof  $\pi$ :  

$$\text{ZK.V}((\text{vk}, Z, c), \pi) \stackrel{?}{=} 1.$$
3. If both accept, output  $\text{TL.Sol}(Z)$ . Otherwise, output  $0^\lambda$ .

**Algorithm 3:** The decoding procedure  $\text{NMCode.D}$ .

**Sub-exponential security.** The theorem extends to show that the resulting non-malleable code cannot be mauled in depth better than  $\bar{T}$  even if the total size of the solver is exponential in  $\lambda$ . For that, we need to make all of our underlying building blocks sub-exponentially secure (in particular, they have to remain secure in the presence of an exponential size adversary). We focus on the polynomial regime for simplicity.

**Organization.** The proof of Theorem 6.1 consists of two parts: (1) efficiency analysis showing that the encoding and decoding procedures can be implemented with the required complexities and (2) showing that the code is non-malleable. Part (1) is proven in Section 6.1 and Part (2) is proven in Section 6.2.

## 6.1 Efficiency Analysis

Fix a security parameter  $\lambda \in \mathbb{N}$  and a message  $m \in \{0, 1\}^\lambda$ . The encoding (i.e., the output of  $\text{NMCode.E}(1^\lambda, m)$ ) consists of a verification key of a signature scheme, a time-lock puzzle, a non-malleable commitment scheme, a zero-knowledge proof, and a signature. All of these are of fixed polynomial size in  $\lambda$ .

The procedure  $\text{NMCode.E}$ , on input  $(1^\lambda, s)$ , runs in time  $\text{poly}(\log \bar{T}, \lambda)$ . Indeed, steps 1, 3, and 5 are independent of  $\bar{T}$  and take  $\text{poly}(\lambda)$  time. Step 2, by definition of time-lock puzzles,

Relation  $\mathcal{R}_u((vk, Z, c), (r_{\text{TL}}, m, r_{\text{NMCom}}))$ :

- Instance: a verification key  $vk$ , a puzzle generated by  $\text{TL.Gen}(1^\lambda, \bar{T}^{2/\epsilon}, m)$ , and a commitment  $c$ .
- Witness: a string  $t_{\text{TL}} \in \{0, 1\}^*$ , a string  $m \in \{0, 1\}^\lambda$ , and a string  $r_{\text{NMCom}} \in \{0, 1\}^*$ .
- Statement:  $\text{TL.Gen}(1^\lambda, \bar{T}^{2/\epsilon}, m; r_{\text{TL}}) = Z$  and  $\text{NMCom.O}(c, m, r_{\text{NMCom}}, \text{tag} = vk) = 1$ .

**Algorithm 4:** The Relation  $\mathcal{R}_u$ .

takes time  $\text{poly}(\log \bar{T}, \lambda)$ . Finally, step 4 takes time  $\text{poly}(\log \bar{T}, \lambda)$  due to the running time of the verification procedure of the underlying language. The procedure  $\text{NMCode.D}$  can be computed in time  $\bar{T}^{2/\epsilon} \cdot \text{poly}(\lambda)$ . Indeed, verifying the proof and the signature both take fixed polynomial time  $\text{poly}(\lambda)$  and the last step takes time  $\bar{T}^{2/\epsilon} \cdot \text{poly}(\lambda)$ , by definition.

## 6.2 Proof of Non-Malleability

In what follows, we prove that the coding scheme from Algorithms 2 and 3 is medium-non-malleable for all polynomial-size  $S$  and bounded polynomial-time  $\bar{T}$  tampering functions. Let  $g(Z, Z')$  be the procedure defined in Algorithm 5.

$g(Z, Z')$ :

1. Parse  $Z$  as  $(vk, Z, c, \pi, \sigma)$  and  $Z'$  as  $(vk', Z', c', \pi', \sigma')$ .
2. If  $vk = vk'$  and  $\sigma'$  verifies (that is,  $\text{Sig.V}(vk', (Z, c', \pi'), \sigma')=1$ ), output 1. Otherwise output 0.

*Running time:* The procedure  $g$  has fixed polynomial size (and time) in its input size.

**Algorithm 5:** The procedure  $g$ .

**Claim 6.2.** *For every non-uniform polynomial-size tampering function  $\mathcal{A} = \{\mathcal{A}_\lambda\}_{\lambda \in \mathbb{N}}$ , every difficulty parameter  $t$ , and every  $m \in \{0, 1\}^\lambda$ , it holds that*

$$\Pr_{\hat{Z} \leftarrow \text{NMCode.E}(1^\lambda, m)} \left[ g(\hat{Z}, \mathcal{A}_\lambda(\hat{Z})) = 1 \wedge \text{NMCode.D}(\mathcal{A}_\lambda(\hat{Z})) \neq m \right] \leq \text{negl}(\lambda).$$

**Proof.** Let  $\hat{Z} = (vk, Z, c, \pi, \sigma)$  and  $\mathcal{A}_\lambda(\hat{Z}) = \hat{Z}' = (vk', Z', c', \pi', \sigma')$ . If  $g(\hat{Z}, \hat{Z}') = 1$ , then  $vk = vk'$  and  $\text{Sig.V}(vk', (Z, c', \pi'), \sigma')=1$ . Also, recall that  $Z$  is a puzzle with underlying message  $m$ . Thus, if  $\text{NMCode.D}(\hat{Z}') \neq m$ , it means that  $(Z, c, \pi) \neq (Z', c', \pi')$ . Thus,  $\mathcal{A}_\lambda$  can be used to create (in polynomial-time) a valid signature  $\sigma'$  w.r.t. verification key  $vk$  for a new statement which is a contradiction to the security of the one-time signature. ■

We next show that w.r.t. the above  $g$  (Algorithm 5), for any polynomial-size algorithm  $\mathcal{A} = \{\mathcal{A}_\lambda\}_{\lambda \in \mathbb{N}}$  such that  $\text{Time}[\mathcal{A}] \leq \bar{T}$  and any  $m_0, m_1 \in \{0, 1\}^\lambda$ , it holds that

$$\{\text{MedTamer}_{\mathcal{A}, m_0}^{\text{NMCode}}(\lambda)\}_{\lambda \in \mathbb{N}} \approx \{\text{MedTamer}_{\mathcal{A}, m_1}^{\text{NMCode}}(\lambda)\}_{\lambda \in \mathbb{N}},$$

where

$$\text{MedTamer}_{\mathcal{A},m}^{\text{NMCode}}(\lambda) = \left\{ \begin{array}{l} \hat{Z} \leftarrow \text{NMCode.E}(1^\lambda, m); \tilde{m} = \text{NMCode.D}(\mathcal{A}_\lambda(\hat{Z})) \\ \text{Output: same* if } g(Z, \mathcal{A}_\lambda(Z)) = 1, \text{ and } \tilde{m} \text{ otherwise} \end{array} \right\}.$$

We do so by defining a sequence of hybrid experiments where we slowly change how  $\text{NMCode.E}$  and  $\text{NMCode.D}$  work and showing that every two consecutive hybrids are indistinguishable. For consistency of notation with what follows, we denote the non-malleable code from Algorithms 2 and 3 used in the original scheme by  $\text{NMCode}_0 = (\text{NMCode}_0.\text{E}, \text{NMCode}_0.\text{D})$ , where  $\text{NMCode}_0.\text{E} \equiv \text{NMCode.E}$  and  $\text{NMCode}_0.\text{D} \equiv \text{NMCode.D}$ . The first experiment that we define corresponds to the experiment  $\{\text{MedTamer}_{\mathcal{A},m_0}^{\text{NMCode}_0}(\lambda)\}_{\lambda \in \mathbb{N}}$  and the last one corresponds to an experiment where we encode  $m_1$ . From that point, one can “reverse” the sequence of experiment to reach the experiment  $\{\text{MedTamer}_{\mathcal{A},m_1}^{\text{NMCode}_0}(\lambda)\}_{\lambda \in \mathbb{N}}$ . We omit this part to avoid repetition.

Throughout the following sequence of hybrids, we treat  $\mathcal{A}$  and  $m_0, m_1$  as fixed.

**Experiment  $\mathcal{H}_0(\lambda)$ .** This is the original experiment, where we encode  $m_0$  under  $\text{NMCode}_0$  (see Algorithms 2 and 3) and execute the experiment  $\{\text{MedTamer}_{\mathcal{A},m_0}^{\text{NMCode}_0}(\lambda)\}_{\lambda \in \mathbb{N}}$ .

**Experiment  $\mathcal{H}_1(\lambda)$ .** This experiment is the same as Experiment  $\mathcal{H}_0(\lambda)$  except that we use the simulator of the ZK proof to generate  $\pi$ . This gives rise to the scheme  $\text{NMCode}_1 = (\text{NMCode}_1.\text{E}, \text{NMCode}_0.\text{D})$ , where  $\text{NMCode}_1.\text{E}$  is describer in Algorithm 6. Using this scheme we execute the experiment  $\{\text{MedTamer}_{\mathcal{A},m_0}^{\text{NMCode}_1}(\lambda)\}_{\lambda \in \mathbb{N}}$ . By the zero-knowledge property of ZK, this hybrid is indistinguishable from  $\mathcal{H}_0(\lambda)$ .

Algorithm  $\text{NMCode}_1.\text{E}(m)$  for  $m \in \{0, 1\}^\lambda$ :

1.  $(\text{vk}, \text{sk}) \leftarrow \text{Sig.G}(1^\lambda)$ .
2.  $Z \leftarrow \text{TL.Gen}(1^\lambda, \overline{T}^{2/\epsilon}, m)$ .
3.  $(c, r) \leftarrow \text{NMCom.C}(1^\lambda, m, \text{tag} = \text{vk})$ .
4. Use the simulator  $\text{Sim}$  to simulate a proof for the relation  $\mathcal{R}_u$  using  $(\text{vk}, Z, c)$  as the instance.
5.  $\sigma \leftarrow \text{Sig.S}(\text{sk}, (Z, c, \pi))$ .
6. Output  $\hat{Z} = (\text{vk}, Z, c, \pi, \sigma)$ .

**Algorithm 6:** The encoding procedure  $\text{NMCode}_1.\text{E}$  used in  $\mathcal{H}_1(\lambda)$ .

**Claim 6.3.** *It holds that*

$$\{\text{MedTamer}_{\mathcal{A},m_0}^{\text{NMCode}_0}(\lambda)\}_{\lambda \in \mathbb{N}} \approx \{\text{MedTamer}_{\mathcal{A},m_0}^{\text{NMCode}_1}(\lambda)\}_{\lambda \in \mathbb{N}}.$$

**Proof.** Let  $\hat{Z}_0 \leftarrow \text{NMCode}_0.\text{E}(1^\lambda, m_0)$  and  $\hat{Z}_1 \leftarrow \text{NMCode}_1.\text{E}(1^\lambda, m_0)$ . Next, since  $\mathcal{A}$ ,  $g$ , and sampling  $\hat{Z}_0$  and  $\hat{Z}_1$  are efficient (i.e., polynomial size), the zero-knowledge property of ZK (see Section 3.2) guarantees that

$$\left| \Pr[g(\hat{Z}_0, \mathcal{A}_\lambda(\hat{Z}_0)) = 1] - \Pr[g(\hat{Z}_1, \mathcal{A}_\lambda(\hat{Z}_1)) = 1] \right| \leq \text{negl}(\lambda). \quad (1)$$

Second, conditioning on the event that  $g(\hat{Z}_0, \mathcal{A}_\lambda(\hat{Z}_0)) = 0$  together with the fact that  $\text{NMCode}_0.\text{D}$  is efficient (i.e., polynomial size), by the zero-knowledge property,

$$\{\text{NMCode}_0.\text{D}(\mathcal{A}_\lambda(\hat{Z}_0))\}_{\lambda \in \mathbb{N}} \approx \{\text{NMCode}_1.\text{D}(\mathcal{A}_\lambda(\hat{Z}_1))\}_{\lambda \in \mathbb{N}}. \quad (2)$$

Combining Equations (1) and (2), the claim follows. ■

**Experiment  $\mathcal{H}_2(\lambda)$ .** This experiment is the same as Experiment  $\mathcal{H}_1(\lambda)$  except that instead of committing to  $m_0$  with a non-malleable commitment, we commit to  $0^\lambda$ . This gives rise to the scheme  $\text{NMCode}_2 = (\text{NMCode}_2.\text{E}, \text{NMCode}_0.\text{D})$  which is described in Algorithm 7. Using this scheme we execute the experiment  $\{\text{MedTammer}_{\mathcal{A}, m_0}^{\text{NMCode}_2}(\lambda)\}_{\lambda \in \mathbb{N}}$ . By the hiding property of  $\text{NMCom}$ , this hybrid is indistinguishable from  $\mathcal{H}_1(\lambda)$ .

Algorithm  $\text{NMCode}_2.\text{E}(m)$  for  $m \in \{0, 1\}^\lambda$ :

1.  $(\text{vk}, \text{sk}) \leftarrow \text{Sig.G}(1^\lambda)$ .
2.  $Z \leftarrow \text{TL.Gen}(1^\lambda, \overline{T}^{2/\epsilon}, m)$ .
3.  $(c, r) \leftarrow \text{NMCom.C}(1^\lambda, 0^\lambda, \text{tag} = \text{vk})$ .
4. Use the simulator  $\text{Sim}$  to simulate a proof for the relation  $\mathcal{R}_u$  using  $(\text{vk}, Z, c)$  as the instance.
5.  $\sigma \leftarrow \text{Sig.S}(\text{sk}, (Z, c, \pi))$ .
6. Output  $\hat{Z} = (\text{vk}, Z, c, \pi, \sigma)$ .

**Algorithm 7:** The encoding procedure  $\text{NMCode}_2.\text{E}$  used in  $\mathcal{H}_2(\lambda)$ .

**Claim 6.4.** *It holds that*

$$\{\text{MedTammer}_{\mathcal{A}, m_0}^{\text{NMCode}_1}(\lambda)\}_{\lambda \in \mathbb{N}} \approx \{\text{MedTammer}_{\mathcal{A}, m_0}^{\text{NMCode}_2}(\lambda)\}_{\lambda \in \mathbb{N}}.$$

**Proof.** Let  $\hat{Z}_1 \leftarrow \text{NMCode}_1.\text{E}(1^\lambda, m_0)$  and  $\hat{Z}_2 \leftarrow \text{NMCode}_2.\text{E}(1^\lambda, m_0)$ . Again, as in the proof of Claim 6.3, since  $\mathcal{A}$ ,  $g$ , and sampling  $\hat{Z}_1$  and  $\hat{Z}_2$  are efficient (i.e., polynomial size), the hiding property of the commitment scheme  $\text{NMCom}$  guarantees that

$$\left| \Pr[g(\hat{Z}_1, \mathcal{A}_\lambda(\hat{Z}_1)) = 1] - \Pr[g(\hat{Z}_2, \mathcal{A}_\lambda(\hat{Z}_2)) = 1] \right| \leq \text{negl}(\lambda). \quad (3)$$

Second, conditioning on the event that  $g(Z_1, \mathcal{A}_\lambda(Z_1)) = 0$  together with the fact that  $\text{NMCode}_0.\text{D}$  is efficient (i.e., polynomial size), by the hiding property of the commitment scheme,

$$\text{NMCode}_0.\text{D}(\mathcal{A}_\lambda(\hat{Z}_1)) \approx \text{NMCode}_0.\text{D}(\mathcal{A}_\lambda(\hat{Z}_2)). \quad (4)$$

Combining Equations (3) and (4), the claim follows. ■



**Experiment  $\mathcal{H}_3(\lambda)$ .** This experiment is the same as Experiment  $\mathcal{H}_2(\lambda)$  except that we use an alternate decoding procedure. The alternate decoding procedure does not solve the time-lock puzzle in order to decode the secret  $m$ , but rather it “breaks” the commitment scheme and extracts  $m$  from it using  $\text{NMCom.Ext}_1$  *unless* the (tampered) proof is one of a fix set of “bad” proofs.

Concretely, recall that by weak-soundness of ZK, every algorithm (and in particular  $\mathcal{A}$ ) can come up with some small bounded number of proofs that are verified yet are for false statements. If the proof on the right size is one of those proofs, we will output a hard coded value instead of trying to extract the value from the commitment.

More precisely, the adversary  $\mathcal{A}$  can find a set  $\mathcal{Z}'$  (that depends on the adversary and the hybrid) of size at most  $K \triangleq K^{\text{ZK}}(|\mathcal{A}_\lambda| + O(1)) \in \text{poly}(\lambda)$  of proofs that are verified yet are for false statements. We denote by  $\mathcal{Z}$  the augmented set of proofs (for false statements) together with the instance and with the values underlying the time-lock puzzle in each such statements. Namely,  $\mathcal{Z}$  is a set that consists of tuples of the form  $(\pi, \text{vk}, Z, c, \tilde{m})$ , where  $\pi$  is a proof from  $\mathcal{Z}'$  for the instance  $(\text{vk}, Z, c)$  and  $\tilde{m}$  is the message underlying  $Z$ .

This gives rise to the scheme  $\text{NMCode}_3 = (\text{NMCode}_2.\text{E}, \text{NMCode}_1.\text{D})$  which is described in Algorithm 8. Using this scheme we execute the experiment  $\{\text{MedTamper}_{\mathcal{A}, m_0}^{\text{NMCode}_3}(\lambda)\}_{\lambda \in \mathbb{N}}$ .

Algorithm  $\text{NMCode}_1.\text{D}(\text{vk}, Z, c, \pi, \sigma)$ :

1. Verify the signature  $\sigma$ :

$$\text{Sig.V}(\text{vk}, (Z, c, \pi), \sigma) \stackrel{?}{=} 1.$$

2. Verify the ZK proof  $\pi$ :

$$\text{ZK.V}((\text{vk}, Z, c), \pi) \stackrel{?}{=} 1.$$

3. If either test from Steps 1 or 2 does not pass or  $c = \perp$ , output  $0^\lambda$  and terminate.
4. If  $\pi$  is a proof which is in  $\mathcal{Z}$ , output the corresponding message  $\tilde{m}$  and terminate.
5. Otherwise (both tests pass,  $c \neq \perp$ , and  $\pi \notin \mathcal{Z}'$ ), use the extractor  $\text{NMCom.Ext}_1(c)$  to get the underlying value  $\tilde{m}$ . Output  $\tilde{m}$  (if extraction fails,  $\tilde{m} = \perp$ ).

**Algorithm 8:** The decoding procedures  $\text{NMCode}_1.\text{D}$  used in  $\mathcal{H}_3(\lambda)$ .

**Claim 6.5.** *It holds that*

$$\{\text{MedTamper}_{\mathcal{A}, m}^{\text{NMCode}_2}(\lambda)\}_{\lambda \in \mathbb{N}} \approx \{\text{MedTamper}_{\mathcal{A}, m}^{\text{NMCode}_3}(\lambda)\}_{\lambda \in \mathbb{N}}.$$

**Proof.** By definition of the  $\text{MedTamper}$  experiment, letting  $Z_3 \leftarrow \text{NMCode}_3.\text{E}(1^\lambda, m)$ , it is enough to show that

$$P_3 \triangleq \Pr[\text{NMCode}_0.\text{D}(\mathcal{A}_\lambda(\hat{Z}_3)) \neq \text{NMCode}_1.\text{D}(\mathcal{A}_\lambda(\hat{Z}_3)) \wedge g(\hat{Z}_3, \mathcal{A}_\lambda(\hat{Z}_3)) = 0] \leq \text{negl}(\lambda). \quad (5)$$

Define the following three quantities where  $\hat{Z}_0 \leftarrow \text{NMCode}_0.\text{E}(1^\lambda, m)$ ,  $\hat{Z}_1 \leftarrow \text{NMCode}_1.\text{E}(1^\lambda, m)$ ,

and  $\hat{Z}_2 \leftarrow \text{NMCode}_2.\text{E}(1^\lambda, m)$ :

$$\begin{aligned} P_0 &\triangleq \Pr[\text{NMCode}_0.\text{D}(\mathcal{A}_\lambda(\hat{Z}_0)) \neq \text{NMCode}_1.\text{D}(\mathcal{A}_\lambda(\hat{Z}_0)) \wedge g(\hat{Z}_0, \mathcal{A}_\lambda(\hat{Z}_0)) = 0], \\ P_1 &\triangleq \Pr[\text{NMCode}_0.\text{D}(\mathcal{A}_\lambda(\hat{Z}_1)) \neq \text{NMCode}_1.\text{D}(\mathcal{A}_\lambda(\hat{Z}_1)) \wedge g(\hat{Z}_1, \mathcal{A}_\lambda(\hat{Z}_1)) = 0], \\ P_2 &\triangleq \Pr[\text{NMCode}_0.\text{D}(\mathcal{A}_\lambda(\hat{Z}_2)) \neq \text{NMCode}_1.\text{D}(\mathcal{A}_\lambda(\hat{Z}_2)) \wedge g(\hat{Z}_2, \mathcal{A}_\lambda(\hat{Z}_2)) = 0]. \end{aligned}$$

Note that  $\text{NMCode}_3.\text{E} \equiv \text{NMCode}_2.\text{E}$  and so  $P_2 = P_3$ . Thus, to prove Equation (5), it is enough to show that  $P_2 \leq \text{negl}(\lambda)$ . For this, we would like to show that  $P_0$ ,  $|P_0 - P_1|$  and  $|P_1 - P_2|$  are all bounded by a negligible function in  $\lambda$ .

**Claim 6.6.**  $P_0 \leq \text{negl}(\lambda)$ .

**Proof.** If  $\text{NMCode}_0.\text{D}(\hat{Z}_0) \neq \text{NMCode}_1.\text{D}(\hat{Z}_0)$ , then it must be that the ZK proof  $\pi$  (1) verifies yet the statement being proven is false and (2)  $\pi \notin \mathcal{Z}'$ . If this happens with noticeable probability in  $\lambda$ , we can use it to break the *weak* soundness of the ZK by outputting an accepting proof for a false statement which is not in the set  $\mathcal{Z}'$ . To do this, we need to run the tampering function  $\mathcal{A}_\lambda$  on an honestly generated codeword ( $\text{NMCode}_0.\text{E}$ ) which is a uniform polynomial-size procedure. This is a contradiction to weak-soundness. ■

**Claim 6.7.**  $|P_0 - P_1| \leq \text{negl}(\lambda)$ .

**Proof.** We argue that if the difference is noticeable, then we can break the zero-knowledge property of ZK. Indeed, we can verify that the statement proven is true or false without faster than the security of zero-knowledge. Verifying that  $\text{TL.Sol}(Z) = m$  can be done in polynomial-time (using the randomness used to generate  $Z$ ). Extracting the committed value by running  $\text{NMCom.E}$  takes size  $S_{\text{Ext}_1}^{\text{NMCom}}(\lambda) = 2^{\log^2 \lambda}$  which is much smaller than the size required to break the zero-knowledge property (the latter is  $S_D^{\text{ZK}}(\lambda) = 2^{\lambda^\eta}$ ). ■

**Claim 6.8.**  $|P_1 - P_2| \leq \text{negl}(\lambda)$ .

**Proof.** Assume towards contradiction that  $|P_1 - P_2| \geq 1/p(\lambda)$  for a polynomial  $p(\cdot)$ . We show that we can break the non-malleability of the commitment scheme. To this end, we need to show an attacker-distinguisher pair  $(D, \mathcal{B})$  for the Man-In-the-Middle execution such that  $\mathcal{B}$  is an algorithm whose size is  $\ll 2^\lambda$  and whose depth is some fixed polynomial and  $D$  can distinguish (in polynomial-time)  $\text{MIM}_{\text{NMCom}}^A(\lambda, m)$  from  $\text{MIM}_{\text{NMCom}}^A(\lambda, 0^\lambda)$  for a fixed message  $m \in \{0, 1\}^\lambda \setminus \{0^\lambda\}$ . The algorithm  $\mathcal{B} = (\mathcal{B}_0, \mathcal{B}_1)$  is defined as follows.

On input  $1^\lambda$ , algorithm  $\mathcal{B}_0$  does:

1. Generate  $(\text{vk}, \text{sk}) \leftarrow \text{Sig.G}(1^\lambda)$ .
2. Send  $\text{tag} = \text{vk}$  to the challenger as the desired tag and outputs  $\text{sk}$  to  $\mathcal{B}_1$ .

On input  $(\text{sk}, \text{vk}, c)$ , algorithm  $\mathcal{B}_1$  does:

1. Sample  $Z \leftarrow \text{TL.Gen}(1^\lambda, \bar{T}^{2/\epsilon}, m)$ .
2. Use the simulator  $\text{Sim}$  (which has size  $2^{\lambda^{\eta'}}$  and depth  $\lambda^2$ ) to simulate a proof  $\pi$ , and sign  $\sigma \leftarrow \text{Sig.S}(\text{sk}, (Z, c, \pi))$ . Let  $\hat{Z} = (\text{vk}, Z, c, \pi, \sigma)$ .

3. Compute  $\mathcal{A}_\lambda(\hat{Z})$  which we interpret as  $\hat{Z}' = (\mathbf{vk}', Z', c', \pi', \sigma')$ .
4. If either  $g(\hat{Z}, \hat{Z}') = 1$ ,  $\sigma'$  does not verify,  $\pi'$  does not verify, or  $\pi' \in Z'$ , output  $\perp$ . Otherwise, output  $(c', Z')$ .

The distinguisher  $D$  receives either the message  $m'$  underlying  $c'$  or  $\perp$ , as well as  $\text{out} = (c', Z')$ . The distinguisher  $D$  outputs 0 if either its input  $c'$  is  $\perp$  or if  $\text{TL.Sol}(Z') = m'$ . It outputs 1 otherwise. It holds that

$$\Pr_{c \leftarrow \text{NMCom.C}(m, \text{tag}=\mathbf{vk})} [D(m', \text{out}) = 1] = P_1(\lambda)$$

and

$$\Pr_{c \leftarrow \text{NMCom.C}(0^\lambda, \text{tag}=\mathbf{vk})} [D(m', \text{out}) = 1] = P_2(\lambda).$$

Thus,

$$\left| \Pr_{c \leftarrow \text{NMCom.C}(m, \text{tag}=\mathbf{vk})} [D(m', \text{out}) = 1] - \Pr_{c \leftarrow \text{NMCom.C}(0^\lambda, \text{tag}=\mathbf{vk})} [D(m', \text{out}) = 1] \right| \geq \frac{1}{p(\lambda)}$$

which is a contradiction to the non-malleability of  $\text{NMCom}$ . Indeed, the dominant part in  $\mathcal{B}$ 's implementation is running the simulator of  $\text{ZK}$  (the rest of the steps require fixed polynomial size). The simulator runs in sub-exponential time and a fixed polynomial depth (in  $\lambda$ ), while non-malleability of the commitment scheme holds for all exponential size attackers with any polynomial depth. Lastly,  $D$  runs in (uniform) polynomial time directly by definition.  $\blacksquare$

**Experiment  $\mathcal{H}_4(\lambda)$ .** This experiment is the same as Experiment  $\mathcal{H}_3(\lambda)$  except that we modify the alternate decoding procedure to use the extractor  $\text{NMCom.Ext}_2$  instead of  $\text{NMCom.Ext}_1$ . Namely, we execute the experiment  $\{\text{MedTamper}_{\mathcal{A}, m_1}^{\text{NMCode}_4}(\lambda)\}_{\lambda \in \mathbb{N}}$ . This gives rise to the scheme  $\text{NMCode}_4 = (\text{NMCode}_2.\text{E}, \text{NMCode}_2.\text{D})$  which is described in Algorithm 9. Using this scheme we execute the experiment  $\{\text{MedTamper}_{\mathcal{A}, m_0}^{\text{NMCode}_4}(\lambda)\}_{\lambda \in \mathbb{N}}$ .

**Claim 6.9.** *It holds that  $\{\text{MedTamper}_{\mathcal{A}, m_0}^{\text{NMCode}_3}(\lambda)\}_{\lambda \in \mathbb{N}}$  and  $\{\text{MedTamper}_{\mathcal{A}, m_0}^{\text{NMCode}_4}(\lambda)\}_{\lambda \in \mathbb{N}}$  are identically distributed.*

**Proof.** The difference between the two experiments is the way that decoding extracts the message underlying the commitment scheme. Specifically, in the experiment that corresponds to  $\text{NMCode}_3$  it works by running  $\text{NMCom.Ext}_1$  while in  $\text{NMCode}_4$  it works by running  $\text{NMCom.Ext}_2$ . Both extraction procedures output the same exact value on every input and so since the output of the experiment contains only the output of this procedure, the two experiments are identically distributed.  $\blacksquare$

**Experiment  $\mathcal{H}_5(\lambda)$ .** This experiment is the same as Experiment  $\mathcal{H}_4(\lambda)$  except that we use  $m_1$  as the underlying message for  $\text{TL.Gen}$  (rather than  $m_0$ ), namely, we execute the experiment  $\{\text{MedTamper}_{\mathcal{A}, m_1}^{\text{NMCode}_4}(\lambda)\}_{\lambda \in \mathbb{N}}$ .

**Claim 6.10.** *It holds that*

$$\{\text{MedTamper}_{\mathcal{A}, m_0}^{\text{NMCode}_4}(\lambda)\}_{\lambda \in \mathbb{N}} \approx \{\text{MedTamper}_{\mathcal{A}, m_1}^{\text{NMCode}_4}(\lambda)\}_{\lambda \in \mathbb{N}}.$$

Algorithm  $\text{NMCode}_2.\text{D}(\text{vk}, Z, c, \pi, \sigma)$ :

1. Verify the signature  $\sigma$ :

$$\text{Sig.V}(\text{vk}, (Z, c, \pi), \sigma) \stackrel{?}{=} 1.$$

2. Verify the ZK proof  $\pi$ :

$$\text{ZK.V}((\text{vk}, Z, c), \pi) \stackrel{?}{=} 1.$$

3. If either test from Steps 1 or 2 does not pass or  $c = \perp$ , output  $0^\lambda$  and terminate.

4. If  $\pi$  is a proof which is in  $\mathcal{Z}$ , output the corresponding message  $\tilde{m}$  and terminate.

5. Otherwise (both tests pass,  $c \neq \perp$ , and  $\pi \notin \mathcal{Z}'$ ), use the extractor  $\text{NMCom.Ext}_2(c)$  to get the underlying value  $\tilde{m}$ . Output  $\tilde{m}$  (if extraction fails,  $\tilde{m} = \perp$ ).

**Algorithm 9:** The decoding procedures  $\text{NMCode}_2.\text{D}$  used in  $\mathcal{H}_4(\lambda)$ .

**Proof.** Let  $Z_0 \leftarrow \text{NMCode}_3.\text{E}(m_0)$  and  $Z_1 \leftarrow \text{NMCode}_3.\text{E}(m_1)$ . We first argue that

$$|\Pr[g(Z_0, \mathcal{A}_\lambda(Z_0)) = 1] - \Pr[g(Z_1, \mathcal{A}_\lambda(Z_1)) = 1]| \leq \text{negl}(\lambda).$$

This proof uses the fact that  $\mathcal{A}$  has bounded depth. Towards contradiction, if the above difference is noticeable, then we can use  $\text{NMCode}_3.\text{E}$ ,  $g$ , and  $\mathcal{A}$  to create a polynomial size and depth  $\bar{T} + \text{poly}(\lambda)$  machine that gets as input a time-lock puzzle  $Z$  that corresponds to either  $m_0$  or to  $m_1$  (each with probability  $1/2$ ), computes the appropriate non-malleable encoding  $\hat{Z}$ , and then decide whether  $\text{TL.Sol}(Z)$  is  $m_0$  or  $m_1$  with noticeable probability. This is a contradiction to the assumption on the underlying time-lock puzzle which stays that no machine with the above size and depth  $(\bar{T}^{2/\epsilon})^\epsilon = \bar{T}^2$  (which is much larger than  $\bar{T} + \text{poly}(\lambda)$  for large enough  $\bar{T}$ ) can solve it.

Next, we complete the proof by showing that

$$\text{NMCode}_2.\text{D}(\mathcal{A}_\lambda(\hat{Z}_0)) \approx \text{NMCode}_2.\text{D}(\mathcal{A}_\lambda(\hat{Z}_1)).$$

Assume towards contradiction that there is a distinguisher  $D$  between the above two distributions which are the message underlying the tampered codeword encoded using  $\text{NMCode}_2.\text{E}$ . Let  $\tilde{D}$  be a distinguisher that has the same truth table as  $D$  but is implemented in (at most)  $2^\lambda$  size and  $\leq \lambda$  time. This can be done by a procedure that has hardwired the whole truth table and it uses the input, which is of size  $\lambda$ , to access an entry. As before, we can fix  $D$  and this distinguisher to create the non-malleable codeword, which we can then use by simulating  $\text{NMCode}_2.\text{E}$  and  $\text{NMCode}_2.\text{D}$  to simulate the codeword and its decoding, and then to decide what was the underlying message. This requires an algorithm of (at most) fixed polynomial size and fixed polynomial time in  $\lambda$ , except  $\text{NMCom.Ext}_2$  which is  $2^{2\lambda}$ -size and  $\lambda^3$ -time, and  $\tilde{D}$  which is  $2^\lambda$ -size and  $\lambda$ -time. Thus, we obtain a procedure of size  $S = 2^{2\lambda} + 2^\lambda + \text{poly}(\lambda)$  and time  $T \in \text{poly}(\lambda)$  (for some fixed polynomial  $\text{poly}$ ) that has noticeable correlation with  $\text{TL.Sol}(Z)$ . The latter, by assumption, cannot be done by a  $\bar{T}$ -time procedure, even if its size is  $2^{3\lambda}$ , which implies our contradiction by choosing  $\bar{T}$  large enough so that  $T < \bar{T}$ . ■

## 7 The Case of Uniform Tampering

In Section 6 we gave a construction of a non-malleable code secure against all tampering functions that can be described as non-uniform polynomial size algorithm with *bounded* polynomial depth. In this section we focus on the natural class of tampering functions that consists of *uniform* polynomial size algorithm with bounded polynomial parallel running time. This is the class that was considered in the work of Ball et al. [BDK<sup>+</sup>19].

The construction is essentially the same as the one for non-uniform tampering functions and the main differences are in how we instantiate the building blocks and how the security proof goes through. Let us precisely list the building blocks with which we use the scheme from Section 6 (Algorithms 2, 3, and 4). We note that the time-lock puzzle and the signature scheme that we use (Items 1 and 4 below) are the same as the one we used in Section 6.

1. A time-lock puzzle (Definition 3.6 and Theorem 5.1)  $\text{TL} = (\text{TL.Gen}, \text{TL.Sol})$  which, for all large enough difficulty parameters  $t$ , allows to generate puzzles which are hard for any (non-uniform) machine whose parallel time is at most  $t^\epsilon$ , even it has size  $2^{3\lambda}$ .

More precisely, for a difficulty parameter  $t$ , it is  $(S^{\text{TL}}, \epsilon)$ -hard for a fixed  $\epsilon \in (0, 1)$  and for  $S^{\text{TL}}(\lambda) = 2^{3\lambda}$ .

2. A one-message zero-knowledge argument system (Definition 3.2 and Theorem 5.5)  $\text{ZK} = (\text{ZK.P}, \text{ZK.V})$  which is sound w.r.t. all uniform polynomial-size attackers, there is a (uniform) simulator that requires sub-exponential size and fixed polynomial time, and zero-knowledge holds w.r.t. sub-exponential size adversaries.

More precisely, it is  $(S_P^{\text{ZK}}, \kappa^{\text{ZK}})$ -sound and  $(S_D^{\text{ZK}}, S_{\text{Sim}}^{\text{ZK}}, T_{\text{Sim}}^{\text{ZK}})$ -zero-knowledge for all polynomial functions  $S_P^{\text{ZK}}$  and where  $\kappa^{\text{ZK}} = 0$ ,  $S_D^{\text{ZK}}(\lambda) = 2^{\lambda^\eta}$ ,  $S_{\text{Sim}}^{\text{ZK}}(\lambda) = 2^{\lambda^{\eta'}}$ , and  $T_{\text{Sim}}^{\text{ZK}}(\lambda) = \lambda^2$ .

3. A one-message non-malleable tag-based commitment scheme (Definition 3.1 and Theorem 5.3)  $\text{NMCom} = (\text{NMCom.C}, \text{NMCom.O})$  which is hiding for all (non-uniform) polynomial-size distinguishers, extractable either in size and time  $2^{\log^2 \lambda}$  or in  $2^\lambda$  size and  $\lambda^3$  time, and non-malleable for all *uniform* exponential size and polynomial time tampering functions.

More precisely, it is  $S^{\text{NMCom}}$ -hiding,  $(S_{\text{Ext}_1}^{\text{NMCom}}, T_{\text{Ext}_1}^{\text{NMCom}})$ -extractable via  $\text{NMCom.Ext}_1$  and  $(S_{\text{Ext}_2}^{\text{NMCom}}, T_{\text{Ext}_2}^{\text{NMCom}})$ -extractable via  $\text{NMCom.Ext}_2$ , and  $(S_{\text{NM}}^{\text{NMCom}}, T_{\text{NM}}^{\text{NMCom}}, \kappa_{\text{NM}}^{\text{NMCom}})$ -non-malleable for all polynomial functions  $S^{\text{NMCom}}$  and  $T_{\text{NM}}^{\text{NMCom}}$ , and where  $S_{\text{Ext}_1}^{\text{NMCom}}(\lambda) = T_{\text{Ext}_1}^{\text{NMCom}}(\lambda) = 2^{\log^2 \lambda}$ ,  $S_{\text{Ext}_2}^{\text{NMCom}}(\lambda) = 2^{2\lambda}$ ,  $T_{\text{Ext}_2}^{\text{NMCom}}(\lambda) = \lambda^3$ ,  $S_{\text{NM}}^{\text{NMCom}} = 2^\lambda$ , and  $\kappa_{\text{NM}}^{\text{NMCom}} = 0$ .

4.  $\text{Sig} = (\text{Sig.G}, \text{Sig.S}, \text{Sig.V})$ . A one-time signature scheme, unforgeable for polynomial-size attackers.

**Overview of the proof.** The proof works by defining a sequence of hybrid experiments, where in the first the Man-In-the-Middle game is played with a message  $m_0$  and in the last with a message  $m_1$ . The sequence of experiments is analogous to the one described in Section 6 except that we do not need worry about “weak-soundness” of the ZK scheme and so some transitions follow due to slightly different reasons.

- $\mathcal{H}_0(\lambda)$ : This is the original experiment.

- $\mathcal{H}_1(\lambda)$ : This is the same experiment, except that we use the simulator to generate proofs. Experiments  $\mathcal{H}_0(\lambda)$  and  $\mathcal{H}_1(\lambda)$  are indistinguishable due to the zero-knowledge property of ZK.
- $\mathcal{H}_2(\lambda)$ : This is the same experiment, except that we commit to 0 and not to  $m_0$ . Experiments  $\mathcal{H}_1(\lambda)$  and  $\mathcal{H}_2(\lambda)$  are indistinguishable due to the hiding property of NMCom.
- $\mathcal{H}_3(\lambda)$ : This is the same experiment, except that we switch to alternate decoding where, rather than solving the time-lock puzzle, the commitment is being extracted via  $\text{NMCom.Ext}_1$ . Indistinguishability of  $\mathcal{H}_2(\lambda)$  and  $\mathcal{H}_3(\lambda)$  is proven by defining the same three quantities  $P_0$ ,  $P_1$ , and  $P_2$  such that  $P_2$  corresponds to  $|\Pr[\mathcal{H}_2(\lambda) = 1] - \Pr[\mathcal{H}_3(\lambda) = 1]|$ .
  - $|P_0| \leq \text{negl}(\lambda)$ : This follows by soundness of ZK. (Recall that soundness against uniform attacker suffices since our attacker is uniform.)
  - $|P_1 - P_0| \leq \text{negl}(\lambda)$ : This follows by observing that by the choice of the hardness parameters of the scheme, one can extract from the commitment (in size and time quasi-polynomial) without breaking zero-knowledge (which holds for sub-exponential attackers).
  - $|P_2 - P_1| \leq \text{negl}(\lambda)$ : This follows by observing that if this is false, then we can use the experiment to break non-malleability. Since the ZK simulator (which consumes sub-exponential size and fixed polynomial time) cannot break non-malleability (which holds for all exponential size polynomial time attackers) this implies a contradiction.
- $\mathcal{H}_4(\lambda)$ : This is the same experiment, except that we modify the alternate decoding to use the other extraction procedure  $\text{NMCom.Ext}_2$ . Since the extraction procedures have the same functionality, experiments  $\mathcal{H}_3(\lambda)$  and  $\mathcal{H}_4(\lambda)$  are identical.
- $\mathcal{H}_5(\lambda)$ : This is the same experiment, except that we use time-lock puzzle to encode the other message  $m_1$ . Experiments  $\mathcal{H}_5(\lambda)$  and  $\mathcal{H}_4(\lambda)$  are indistinguishable due to the security of the time-lock puzzle (and using the fact that the attacker has bounded polynomial time).

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