# Bent Boolean functions: A better procedure to generate non-crypto 4-bit S-boxes. 

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#### Abstract

S-boxes are used in block ciphers for nonlinear substitution very frequently. If the 16 elements of a 4-bit S-box are unique, distinct and vary between 0 and $f$ in hex then the said 4-bit S-box is called as a crypto 4-bit S-box. There are 16! crypto 4-bit S-boxes available in crypto literature. Other than crypto 4-bit S-boxes there are another type of 4-bit S-boxes exist. In such 4-bit S-boxes 16 elements of the 4 -bit S-box are not unique and distinct i.e. at least one element must repeat more than one time. They are called as non-crypto 4-bit S-boxes. There are $16^{16}-16$ ! Numbers of non-crypto 4-bit S-boxes can be found in crypto-literature. The non-crypto 4-bit S-boxes can be generated from 4-bit Boolean Functions (BFs) in the same manner as crypto 4-bit S-boxes are generated in [1]. But to generate crypto 4-bit S-boxes the security of the generated 4-bit S-boxes is sacrificed into some extend. Since 128704 -bit balanced BFs are responsible for 16! crypto 4-bit S-boxes and the nonlinearity of the balanced 4-bit BFs are at most 4. So the 4-bit BFs with highest nonlinearity 6 are left abandoned. These 4-bit BFs are called as 4-bit Bent BFs. Here in this paper we generate non-crypto 4-bit S-boxes from 4-bit Bent BFs. The generated non-crypto 4-bit S-boxes are analyzed with the existing cryptanalysis techniques to prove them much secure 4-bit S-boxes from crypto angle.


1. Introduction and Scope: 4-bit S-boxes are made of 16 elements. The hex values of 16 elements of a 4-bit S-box are may or may not be unique and distinct. If the 16 elements have 16 unique hex values varies from 0 to F in hex then the 4-bit S-box is termed as crypto 4-bit S-box [2]. Left alone 4-bit S-boxes are non-crypto 4-bit S-boxes. In non-crypto 4-bit S-boxes 16 elements must not have 16 unique hex values and the hex values must be within 0 to $F$ in hex. For a brief example a crypto 4-bit S-box is given in row 1 of table.1.1 and a non-crypto 4-bit S-box is given in row 2 of table.1.1. Here Index 0 is termed as the position of Most Significant Element (MSE) and Index F is termed as position of Least Significant Element (LSE) respectively.

| Row | Index | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{D}$ | $\mathbf{E}$ | $\mathbf{F}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Crypto | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |
| 2 | Non-crypto | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | A | B |

Table.1.1: A brief example of a crypto 4-bit S-box (identity 4-bit S-box) and a non-crypto 4-bit S-box.
Each of 16 elements from MSE to LSE of a crypto 4-bit S-box are converted into 16-bit long binary numbers. The first MSBs (as given in table.1.2) of each of 16 elements from MSE to LSE constitutes $4^{\text {th }}$ output BF or $4^{\text {th }}$ OPBF. In the same manner $2^{\text {nd }}$ bits from MSB, $3^{\text {rd }}$ bit from MSB and LSBs constitute $3^{\text {rd }}$ output BF or $3^{\text {rd }} \mathrm{OPBF}, 2^{\text {nd }}$ output BF or $2^{\text {nd }}$ OPBF and $1^{\text {st }}$ output BF or $1^{\text {st }}$ OPBF respectively [3][4]. The non-crypto 4-bit S-boxes are decomposed into four OPBFs in the same manner. The four OPBFs of the crypto 4-bit S-box are given in table.1.1 and are shown in row $2,3,4$, 5 of table.1.2 with the four OPBFs of the non-crypto 4-bit S-box are given in table.1.1 and are shown in row $8,9,10$ and 11 of table.1.2 respectively. The table.1.2 is given as follows,

| Row | Index | MSB | Crypto 4-bit S-box |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |
| 1 | Crypto |  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |
| 2 | $4^{\text {th }}$ OPBF | Bal | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | $3^{\text {rd }}$ OPBF | Bal | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 4 | $2^{\text {nd }}$ OPBF | Bal | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 5 | $1{ }^{\text {st }}$ OPBF | Bal | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 6 |  | LSB | Non crypto 4-bit S-box |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 7 | Non-crypto | MSB | 0 | 0 | 1 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | A | B |
| 8 | $4^{\text {th }}$ OPBF | Unbal | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 9 | $3^{\text {rd }}$ OPBF | Unbal | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 10 | $2^{\text {nd }}$ OPBF | Unbal | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |
| 11 | $1^{\text {st }}$ OPBF | Unbal | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 12 |  | LSB | Table.1.2: crypto and non-crypto 4-bit S-box to 4-bit BFs |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

In this example as well as in related relevant crypto literature it is found that the four OPBFs of a crypto 4-bit S-box must be balanced but the four OPBFs of a non-crypto 4-bit S-box may or may not be balanced [3][4]. So the maximum nonlinearity of any OPBF of a crypto 4-bit S-box is at most 4 [2]. So the security of a crypto 4-bit S-box has certain limitations far below maximum. But the non-crypto 4-bit S-boxes can be generated from unbalanced OPBFs. So the maximum nonlinearity of any OPBF of a non-crypto 4-bit S-box is at most 6 .i.e. the maximum value of the maximum nonlinearity. The BFs with maximum nonlinearity 6 are called as 4 -bit Bent Boolean functions or 4-bit Bent BFs. The non-crypto S-boxes generated from Bent BFs must have good security.

In this paper 4-bit Bent Boolean function is defined with proper example in section 2. The generation of non-crypto 4-bit S-boxes from 4-bit Bent functions is explained with example in section 3. And the generated non-crypto 4-bit S-boxes are analyzed with latest cryptanalysis techniques to show the result in section 4 . The results and the utility of non-crypto S-boxes in encryption and decryption algorithms are also established in section 5. Section 6 is dedicated to the conclusion of this research article.
2. 4-bit Bent Boolean functions: The maximum nonlinearity described in [2] is one of the key features to select 4bit BFs for generation of 4-bit S-boxes. In Lucifer [5][6] and DES [7][8] crypto 4-bit S-boxes are used for nonlinear substitution and these crypto 4-bit S-boxes must be composed of balanced [2] 4-bit BFs. So the maximum nonlinearity of the 4-bit BFs used in generation of crypto 4-bit S-boxes are at most 4. So the security of the 4-bit Sboxes in generated crypto 4-bit S-boxes by balanced 4-bit BFs is neglected. Now it can be seen from file entitled "4bit Bent BFs" is that the maximum nonlinearity of 4-bit BFs is 6 . There are 448 nonlinear [8] and their complement 448 non-affine [2] 4-bit BFs exist with nonlinearity 6. These 996 4-bit BFs are termed as 4-bit Bent BFs. The First Order Strict Avalanche Criterion (FO-SAC) [2], Successive First Order Strict Avalanche Criterion (SFO-SAC) [2], and Multiple Higher Order Strict Avalanche Criterion (MHO-SAC) [2] shows the best results for these 996 4-bit Bent BFs. So these 996 4-bit Bent BFs are the best options to generate 4-bit S-boxes with good security. The only disadvantage is that these 996 4-bit Bent BFs generate only non-crypto S-boxes. Since S-boxes are used for nonlinear substitutions, the arrangement of the algorithm can be able to overcome this disadvantage. So the 4-bit non-crypto S-boxes generated from these 996 4-bit Bent BFs are better ones from the crypto security angle. All these properties of these 9964 -bit Bent BFs are noted in table 2.1 given in appendix which is a part of the c language program generated text file entitled "4-bit Bent BFs" enclosed with this paper as supplementary material. Description of the table and the file is given below.
3.

Description of the table 2.1 or the file "4-bit Bent BFs":

1. Here Sl.No. column shows serial numbers of the Bent 4-bit BFs in ascending order of their decimal equivalents and the corresponding complement Bent 4-bit BFs in descending order of their decimal equivalents.
2. Here BF(Dec) column shows the decimal equivalents of the Bent 4-bit BFs in ascending order.
3. Here $\mathbf{B F}$ (Binary) shows the 16 bit long binary output vectors of the Bent 4 -bit BFs in ascending order of their decimal equivalents.
4. Here in $\mathbf{1 0}$ column $\mathbf{1}$ shows the numbers of 1 s and $\mathbf{0}$ shows the numbers of 0 s in the 16 bit long binary output vectors of the Bent 4-bit BFs.
5. Here $\mathbf{L}$ column shows that the Bent 4-bit BFs are nonlinear. In this column 1 means Linear Bent 4-bit BFs, 2 means Nonlinear Bent 4-bit BFs, 3 means Affine Bent 4-bit BFs and 4 means Non-Affine Bent 4-bit BFs.
6. Here $\mathbf{C B F}$ (Dec) column shows the decimal equivalents of the Complement Bent 4-bit BFs in descending order.
7. Here CBF(Binary) shows the 16 bit long binary output vectors of the Complement Bent 4 -bit BFs in descending order of their decimal equivalents.
8. Here in $\mathbf{1 0}$ column $\mathbf{1}$ shows the numbers of 1 s and $\mathbf{0}$ shows the numbers of 0 s in the 16 bit long binary output vectors of the Complement Bent 4-bit BFs.
9. Here $\mathbf{L}$ column shows that the Complement Bent 4 -bit BFs are non-affine. In this column 1 means Linear Complement Bent 4-bit BFs, 2 means Nonlinear Complement Bent 4-bit BFs, 3 means Affine Complement Bent 4bit BFs and 4 means Non-Affine Complement Bent 4-bit BFs.
10. Here in $\mathbf{M m}$ column $\mathbf{M}$ means maximum hamming distance and $\mathbf{m}$ means minimum hamming distance among 32 hamming distances of the 16 bit long binary output vectors of the Bent 4-bit BFs to the 16 linear and 16 affine BFs.
11. Here in FO-SAC 8421 column 1 is under 8 means the $4^{\text {th }}$ OPBF satisfies FO-SAC individually. Similarly 1 is under 4 means the $3^{\text {rd }}$ OPBF satisfies FO-SAC individually. Again 1 is under 2 means the $2^{\text {nd }}$ OPBF satisfies FOSAC individually and 1 is under 1 means the 1 st OPBF satisfies FO-SAC individually.
12. Here in column SFO-SAC $3569 A B, 1$ is under 3 means the $2^{\text {nd }}$ OPBF and $1^{\text {st }}$ OPBF both satisfies FO-SAC individually. Similarly 1 is under $5,6,9$, A and B means the $3^{\text {rd }} \mathrm{OPBF}$ and $1^{\text {st }} \mathrm{OPBF}, 3^{\text {rd }} \mathrm{OPBF}$ and $2^{\text {nd }} \mathrm{OPBF}, 4^{\text {th }}$ OPBF and $1^{\text {st }} \mathrm{OPBF}, 4^{\text {th }} \mathrm{OPBF}$ and $2^{\text {nd }} \mathrm{OPBF}$ and $4^{\text {th }} \mathrm{OPBF}$ and $2^{\text {nd }} \mathrm{OPBF}$, both satisfies FO-SAC individually respectively.
1 is under $7, \mathrm{~B}, \mathrm{D}$ and E means the $3^{\text {rd }}$ OPBF, $2^{\text {nd }}$ OPBF and $1^{\text {st }}$ OPBF, $4^{\text {th }}$ OPBF $2^{\text {nd }}$ OPBF and $1^{\text {st }} O P B F, 4^{\text {th }}$ OPBF $3^{\text {rd }} \mathrm{OPBF}$ and $1^{\text {st }} \mathrm{OPBF}$ and $4^{\text {th }} \mathrm{OPBF} 3^{\text {rd }} \mathrm{OPBF}$ and $2^{\text {nd }} \mathrm{OPBF}$, all satisfies FO-SAC individually respectively.
Here 1 is under F means $4^{\text {th }} \mathrm{OPBF}, 3^{\text {rd }} \mathrm{OPBF}, 2^{\text {nd }} \mathrm{OPBF}$ and $1^{\text {st }} \mathrm{OPBF}$ FO-SAC individually.
13. Here in SFO-SUM total numbers of 1 s in column SFO-SAC is counted.
14. Here in column MHO-SAC $3569 \mathrm{AB}, 1$ is under 3 means the $2^{\text {nd }}$ OPBF and $1^{\text {st }}$ OPBF both satisfies FO-SAC together. Similarly 1 is under $5,6,9$, A and B means the $3^{\text {rd }} \mathrm{OPBF}$ and $1^{\text {st }} \mathrm{OPBF}, 3^{\text {rd }} \mathrm{OPBF}$ and $2^{\text {nd }} \mathrm{OPBF}, 4^{\text {th }}$ OPBF and $1^{\text {st }} \mathrm{OPBF}, 4^{\text {th }}$ OPBF and $2^{\text {nd }}$ OPBF and $4^{\text {th }}$ OPBF and $2^{\text {nd }}$ OPBF, both satisfies FO-SAC together respectively.
1 is under $7, \mathrm{~B}, \mathrm{D}$ and E means the $3^{\text {rd }} \mathrm{OPBF}, 2^{\text {nd }} \mathrm{OPBF}$ and $1^{\text {st }} \mathrm{OPBF}, 4^{\text {th }}$ OPBF $2^{\text {nd }}$ OPBF and $1^{\text {st }} \mathrm{OPBF}, 4^{\text {th }} \mathrm{OPBF}$ $3^{\text {rd }} \mathrm{OPBF}$ and $1^{\text {st }} \mathrm{OPBF}$ and $4^{\text {th }} \mathrm{OPBF} 3^{\text {rd }} \mathrm{OPBF}$ and $2^{\text {nd }} \mathrm{OPBF}$, all satisfies FO-SAC together respectively.
Here 1 is under F means $4^{\text {th }} \mathrm{OPBF}, 3^{\text {rd }} \mathrm{OPBF}, 2^{\text {nd }} \mathrm{OPBF}$ and $1^{\text {st }} \mathrm{OPBF}$ FO-SAC together.
15. Here in MHO-SUM total numbers of 1 s in column MHO-SAC is counted.
16. Generation of 4-bit non-crypto S-boxes from 996 4-bit Bent BFs: The non-crypto 4-bit S-boxes generated from 4-bit Bent BFs are termed as 4-bit Bent S-boxes. Four Bent BFs are needed to generate a 4-bit Bent S-box. The MSBs of the output vector of four Bent BFs from MSB to LSB constitute a 4-bit binary number. The decimal equivalent (DE) of the 4-bit binary number is the $1^{\text {st }}$ element or MSE of a 4-bit Bent S-box. In the same manner 15 consecutive bits in rest 15 positions of the four 4-bit Bent BFs generate other 15 elements of the S-box.

The 4-bit Bent BF that has complement bits in all positions w.r.t bits in all corresponding positions of a certain 4-bit Bent BF is called as the complement 4-bit Bent BF of the said 4-bit Bent BF. In table.3.1 the 4-bit Bent BF with DE 64681, 64678, 64666 and 64661 are the complement 4-bit Bent BFs of the 4-bit Bent BFs with DE 854, 857, 869 and 874 respectively. So the 2, 4-bit Bent S-boxes generated from 4, 4-bit Bent BFs and their complement 4-bit Bent BFs are complement $S$-boxes i.e. elements in a certain position of the complement $S$-box is equal to $F$ - elements in that particular position of the $S$-box in hex.

| Non-crypto S-box |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | Complement Non-crypto S-box |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DEs | Output 4-bit Bent BFs |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | DEs | Complement Output 4-bit Bent BFs |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 00854 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 64681 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 00857 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 64678 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 00869 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 64666 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 00874 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 64661 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| S-box | 0 | 0 | 0 | 0 | 0 | 0 | F | F | 0 | F | 3 | C | 5 | A | 9 | 6 | S-box | F | F | F | F | F | F | 0 | 0 | F | 0 | C | 3 | A | 5 | 6 | 9 |

Table.3.1: 4-bit Bent S-box generation from 4-bit Bent BFs.
5. Cryptanalysis of 4-bit Bent BFs: Here 3 well known cryptanalysis techniques are used to analyze the 4-bit Bent S-boxes. They are '(Output) Bit Independence Criterion' (BIC) [1] 'Linear Cryptanalysis of 4-bit S-boxes' (LC) [9][10] and 'Differential Cryptanalysis of 4-bit S-boxes' (DC) [9][10]. The numbers of 0s in Linear Approximation Table (LAT) and Difference Distribution Table (DDT) are used to conclude the security. With increase in numbers of 0 s in LAT and DDT the security of the non-crypto 4-bit S-boxes increases [9][10]. The results are shown in the table 4.1 in appendix which is a part of the c language program generated text file entitled "Analysis of the noncrypto 4-bit Bent S-boxes" enclosed with this paper as supplementary material. Description of the table and the file is given below.

## Description of the table 4.1 or the file "Analysis of the non-crypto 4-bit Bent S-boxes":

1. Here in column Sl. No. the numbers shows the permuted [2] 4-bit Bent S-boxes in ascending order of their hexadecimal equivalents and their corresponding Complement 4-bit Bent S-boxes in descending order of their hexadecimal equivalents.
2. In column DEs of BFs-crypto S-box SB(DBF1,DBF2,DBF3,DBF4) DBF1 shows the decimal equivalent of $4^{\text {th }}$ OPBF of the 4-bit Bent S-boxes. Similarly DBF2, DBF3 and DBF4 show decimal equivalents of $3^{\text {rd }}$ OPBF, $2^{\text {nd }}$ OPBF and $1^{\text {st }}$ OPBF of the same.
3. In column S-box in Hex the permuted [2] 4-bit Bent S-boxes are shown with the hexadecimal equivalents of their elements.
4. In column DEs of BFs-Complement crypto S-box CSB(DBF1,DBF2,DBF3,DBF4) DBF1 shows the decimal equivalent of $4^{\text {th }}$ OPBF of the 4-bit Complement Bent S-boxes. Similarly DBF2, DBF3 and DBF4 show decimal equivalents of $3^{\text {rd }} \mathrm{OPBF}, 2^{\text {nd }} \mathrm{OPBF}$ and $1^{\text {st }} \mathrm{OPBF}$ of the same.
5. In column Complement S-box in Hex the permuted [2] 4-bit Complement Bent S-boxes are shown with the hexadecimal equivalents of their elements.
6. In columns $(\mathbf{1}, 2),(1,3),(1,4),(2,3),(2,4)$ and $(3,4)$ the decimal equivalents of the xor 4 -bit BFs of the (DBF1, DBF2), (DBF1, DBF3), (DBF1, DBF4), (DBF2, DBF3), (DBF2, DBF4) and (DBF3, DBF4) respectively are shown. And immediate next 10s are used to store the number of 1 s under 1 and 0 s under 0 of the xored 4 -bit BFs respectively.
7. Here the column $\mathbf{8 8}$ is used to store the numbers of xored BFs that are balanced or have 81 s and 8 0s in its binary output vector. Similarly column $\mathbf{9 7}$ and A6 are used to store the numbers of xored BFs that are unbalanced and have 91 s and 70 s and 101 s and 60 s in its binary output vector respectively.
8. Here L $\mathbf{0}$ and D $\mathbf{0}$ shows numbers of 0 s in LAT and DDT of the said 4-bit Bent S-boxes and their corresponding complement 4-bit Bent S-boxes respectively.
9. Results and discussion: In this analysis it is found that the numbers of 0s in LAT and DDT of the 4-bit Bent Sboxes and their corresponding 4-bit Complement Bent S-boxes are adequate to use the generated 4-bit Bent Sboxes in encryption and decryption algorithms of block ciphers. Since S-boxes are used in nonlinear substitution and many S-boxes can be used in a particular block cipher so the hurdle to use non-crypto 4-bit S-boxes in block ciphers can easily be overcome.

Conclusion: It can be concluded from the results that the non-crypto 4-bit S-boxes can be used in block ciphers. The Bent 4-bit S-boxes are the best option to replace crypto 4-bit S-boxes with the non-crypto 4-bit S-boxes. The said Bent 4-bit S-boxes have a very high security and very fine design methodology. So Bent non-crypto 4-bit S-boxes are the better choice than low secure crypto 4-bit S-boxes.

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## Appendix:

|  | BF | BF | CBF | CBF |  |  | FO-SAC | SFO-SAC | SFO | MHO-SAC | MHO |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sl.No | (Dec) | (BInary) | 10 L (Dec) | (BInary) | 10 L | ANF Coefficients | Mm 84 21------3569AC--7BDE-F---SUM------3569AC-7BDE--F--- SUM |  |  |  |  |
| 00001 | 00854 | 00000011010 | 6 a 264681 | 111100101 | 64 | -001100-0000 | A6 111 | 1111-1 | - | 111111 | 1-1---011 |
| 00002 | 00857 | 000000110101 | 6a 26467 | 1110010 | 6 | -001101-000 | A6 111 | 1111- | -1- | 111111 | 1-1---010 |
| 00003 | 00869 | 000000110110 | 6a 26466 | 11100100 | 6 | -001110-0000 | A6 111 | 1111-1 | - | 111111 | 1-1---010 |
| 00004 | 00874 | 000000110110 | 6a 264661 | 1110010 | a6 | -001111-0000 | A6 111 | 1111-1 | -1 | 111111 | 1-1---011 |
| 00005 | 00917 | 00000011100 | 6a 264618 | 1110001 | 6 | -001111-000 | A6 111 | 1111-1 | -1- | 111111 | 1-1---011 |
| 00006 | 00922 | 000000111001 | 6a 264613 | 1110001 | a6 | -001110-0000 | A6 111 | 1111-1 | -1- | 111111-1 | 1-1---010 |
| 00007 | 00934 | 000000111010 | 6 2 264601 | 11100010 | a | -001101-0000 | A6 111 | 1111-1 | -1-- | 111111- | 1-1---010 |
| 00008 | 00937 | 000000111010 | a 264598 | 11100010 | 6 | -001100-000 | A6 111 | 1111-1 | -1- | 111111-1 | $1-1--011$ |
| 00009 | 01334 | 00000101001 | a 264201 | 11010110 | a6 4 | -010010-000 | A6 111 | 1111-1 | -1 | -111111 | 1-1---011 |
| 00010 | 01337 | 000001010011 | 6a 264198 | 1110101100 | a6 4 | 0-010011-0000 | A6 111 | 11111-11 | 1-1-- | -111111- | 11-1---010 |

Table.2.1 Properties of 10 4-bit Bent BFs with their Complement 4-bit Bent BFs.

| Es of BFs-crypto S-box | DEs of BFs-complement crypto S-box Compleme | Balancedness of the six xored 4-bit BFs Total |
| :---: | :---: | :---: |
| Sl No. SB(DBF1,DBF2,DBF3,DBF4 | $S$-box in He | 3) $10(1,4) 10(2,3) 10(2,4) 10(3,4) 108897$ A6LOD 0 |
| $001.01 \mathrm{SB}(00854,00857,00869,00874) 000000 f f 0 f 3 \mathrm{c} 5 \mathrm{a} 96 \mathrm{CSB}(64681,64678,64666,64661)$ ffffff00f0c3a569 00015 4c $000514 \mathrm{4c} 00060$ 4c $000604 \mathrm{4c} 000514 \mathrm{c} 000154 \mathrm{c} 000000102165$ |  |  |
| 001.02 SB(00857,00854,00869,00874) 000000ff0f3c965a CSB(64678,64681,64666,64661) ffffff00f0c369a5 00015 4c 00060 4c 00051 4c 00051 4c $000604 \mathrm{4c} 00015$ 4c 000000102165 |  |  |
| 001.03 SB(00854,00869,00857,00874) 000000ff0f5a3c96 CSB(64681,64666,64678,64661) ffffff00f0a5c369 00051 4c 00015 4c 00060 4c 00060 4c 00015 4c 00051 4c 000000102165 |  |  |
| 001.04 SB(00857,00869,00854,00874) 000000ff0f5a963c CSB(64678,64666,64681,64661) ffffff00f0a569c3 00060 4c 00015 4c 00051 4c 00051 4c 00015 4c 00060 4c 000000102165 |  |  |
| 001.05 SB(00869,00854,00857,00874) 000000ff0f963c5a CSB(64666,64681,64678,64661) ffffff00f069c3a5 00051 4c 00060 4c 00015 4c 00015 4c 00060 4c 00051 4c 000000102165 |  |  |
| 001.06 SB(00869,00857,00854,00874) 000000ff0f965a3c CSB(64666,64678,64681,64661) ffffff00f069a5c3 00060 4c 00051 4c 00015 4c 00015 4c 00051 4c 00060 4c 000000102165 |  |  |
| 001.07 SB(00854,00857,00874,00869) 000000ff0f3c69a5 CSB(64681,64678,64661,64666) ffffff00f0c3965a 00015 4c 00060 4c 00051 4c 00051 4c $000604 \mathrm{4c} 00015$ 4c 000000102165 |  |  |
| 001.08 SB(00857,00854,00874,00869) 000000ff0f3ca569 CSB(64678,64681,64661,64666) ffffff00f0c35a96 00015 4c 00051 4c 00060 4c 00060 4c 00051 4c 00015 4c 000000102165 |  |  |
| 001.09 SB(00854,00869,00874,00857) 000000ff0f693ca5 CSB(64681,64666,64661,64678) ffffff00f096c35a 00051 4c 00060 4c 00015 4c 00015 4c 00060 4c 00051 4c 000000102165 |  |  |
| 001.10 SB(00857,00869,00874,00854) 000000ff0f69a53c CSB(64678,64666,64661,64681) ffffff00f0965ac3 00060 4c 00051 4c 00015 4c 00015 4c 00051 4c 00060 4c 000000102165 |  |  |
| 001.11 SB(00869,00854,00874,00857) 000000ff0fa53c69 CSB(64666,64681,64661,64678) ffffffoof05ac396 00051 4c 00015 4c 00060 4c 00060 4c 00015 4c 00051 4c 000000102165 |  |  |
| 001.12 SB(00869,00857,00874,00854) 000000ff0fa5693c CSB(64666,64678,64661,64681) ffffff00f05a96c3 00060 4c 00015 4c 00051 4c 00051 4c 00015 4c 00060 4c 000000102165 |  |  |
| 001.13 SB(00854,00874,00857,00869) 000000ff0f5a69c3 CSB(64681,64661,64678,64666) ffffff00f0a5963c 00060 4c 00015 4c 00051 4c 00051 4c 00015 4c 00060 4c 000000102165 |  |  |
| 001.14 SB(00857,00874,00854,00869) 000000ff0f5ac369 CSB(64678,64661,64681,64666) ffffff00f0a53c96 00051 4c 00015 4c 00060 4c 00060 4c 00015 4c 00051 4c 000000102165 |  |  |
| 001.15 SB(00854,00874,00869,00857) 000000ff0f695ac3 CSB(64681,64661,64666,64678) ffffff00f096a53c 00060 4c 00051 4c 00015 4c 00015 4c 00051 4c 00060 4c 000000102165 |  |  |
| 001.16 SB(00857,00874,00869,00854) 000000ff0f69c35a CSB(64678,64661,64666,64681) ffffff00f0963ca5 00051 4c 00060 4c 00015 4c 00015 4c 000604 c 000514 c 000000102165 |  |  |

001.17 SB(00869,00874,00854,00857) 000000ff0fc35a69 CSB(64666,64661,64681,64678) ffffff00f03ca596 00015 4c 00051 4c 00060 4c 00060 4c 00051 4c 00015 4c 000000102165
001.18 SB( $00869,00874,00857,00854$ ) 000000ff0fc 3695 a CSB(64666,64661,64678,64681) ffffff00f03c96a5 00015 4c 00060 4c 00051 4c 00051 4c 000604 c 00015 4c 000000102165
001.19 SB(00874,00854,00857,00869) 000000ff0f96a5c3 CSB(64661,64681,64678,64666) ffffff00f0695a3c 00060 4c 00051 4c 00015 4c 00015 4c 00051 4c 00060 4c 000000102165
001.20 SB( $00874,00857,00854,00869$ ) 000000ff0f96c3a5 CSB( $64661,64678,64681,64666$ ) ffffff00f0693c5a 00051 4c 00060 4c 00015 4c 00015 4c 00060 4c 00051 4c 000000102165
001.21 SB( $00874,00854,00869,00857$ ) 000000ff0fa596c3 CSB(64661,64681,64666,64678) ffffff00f05a693c 000604 c 00015 4c 00051 4c 00051 4c 00015 4c 000604 c 000000102165
001.22 SB(00874,00857,00869,00854) 000000ff0fa5c396 CSB(64661,64678,64666,64681) ffffff00f05a3c69 00051 4c 00015 4c 00060 4c 00060 4c 00015 4c 00051 4c 000000102165
001.23 SB( $00874,00869,00854,00857$ ) 000000ff0fc $396 a 5 \operatorname{CSB}(64661,64666,64681,64678)$ ffffff00f03c695a 000154 c 000604 c 000514 c 000514 cc 000604 c 000154 c 000000102165
001.24 SB(00874,00869,00857,00854) 000000ff0fc 3a596 CSB(64661,64666,64678,64681) ffffff00f03c5a69 00015 4c 00051 4c 00060 4c 00060 4c 00051 4c 00015 4c 000000102165

Table.4.1 BIC, LC and DC analysis of 4! or 24 permuted non-crypto 4-bit S-boxes.

