# MUSE: Secure Inference Resilient to Malicious Clients


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## Abstract

The increasing adoption of machine learning inference in applications has led to a corresponding increase in concerns surrounding the privacy guarantees offered by existing mechanisms for inference. Such concerns have motivated the construction of efficient *secure inference* protocols that allow parties to perform inference without revealing their sensitive information. Recently, there has been a proliferation of such proposals, rapidly improving efficiency. However, most of these protocols assume that the client is semi-honest, that is, the client does not deviate from the protocol; yet in practice, clients are many, have varying incentives, and can behave arbitrarily.

To demonstrate that a malicious client can completely break the security of semi-honest protocols, we first develop a new *model-extraction attack* against many state-of-the-art secure inference protocols. Our attack enables a malicious client to learn model weights with 22×–312× fewer queries than the best black-box model-extraction attack [CJM20] and scales to much deeper networks.

Motivated by the severity of our attack, we design and implement MUSE, an efficient two-party secure inference protocol resilient to *malicious clients*. MUSE introduces a novel cryptographic protocol for *conditional disclosure of secrets* to switch between authenticated additive secret shares and garbled circuit labels, and an improved *Beaver's triple generation* procedure which is 8×–12.5× faster than existing techniques.

These protocols allow MUSE to push a majority of its cryptographic overhead into a preprocessing phase: compared to the equivalent *semi-honest* protocol (which is close to state-of-the-art), MUSE's online phase is only 1.7×–2.2× slower and uses 1.4× more communication. Overall, MUSE is 13.4×–21× faster and uses 2×–3.6× less communication than existing secure inference protocols which defend against malicious clients.


## 1 Introduction

The past few years have seen increasing deployment of neural network inference in popular applications such as image classification [Liu+17b] and voice assistants [Bar18]. However, the use of inference in such applications raises privacy concerns: existing implementations either require clients to send potentially sensitive data to remote servers for classification, or require the model owner to store their proprietary neural network model on the client's device. Both of these solutions are unsatisfactory: the former harms the privacy of the client, while the latter can harm a business model or reveal information about the training data and model weights.

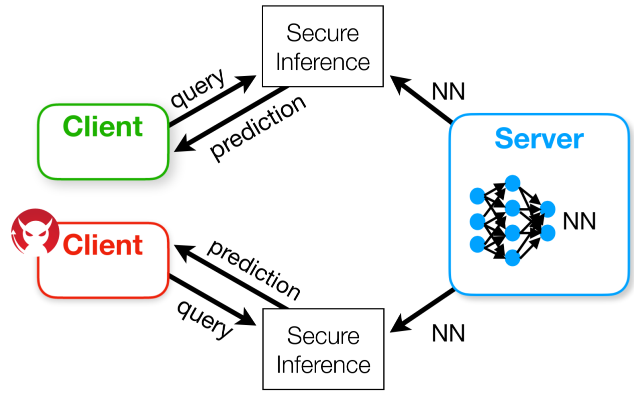


**Figure 1:** MUSE's system setup. Some of MUSE's clients may be malicious.

To resolve this tension, the community has focused on constructing specialized protocols for *secure inference*, as we depict in Table 1. A secure inference protocol enables users and model owners to interact so that the user obtains the prediction result while ensuring that neither party learns any other information about the user input or the model weights. Many of these works implement these guarantees using secure two-party computation [Gil+16; MZ17; HTG17; Liu+17a; BEG18; Cho+18; San+18; JVC18; LJ19; Bou+18; RRK18; Bal+19; Ria+19; Dat+19; Mis+20; Rat+20]. However, as we can see from Table 1, all of these two-party works assume that both the client and the server follow the protocol rules, that is, they are *semi-honest*.

While it is common in the literature to assume a semi-honest server, it is fundamentally less likely that *all* clients will behave correctly. The server is hosted at a single service provider, and existing cloud providers deploy competent intrusion detection systems, rigid access control, physical measures, and logging/tracking of the software installed [Goo17]. It is highly non-trivial to bypass these protections. Additionally, if a service provider is caught acting maliciously, the consequences may be dire due to public accountability.

In contrast, clients are many, run on a variety of setups under the control of users, users have various motives, and it suffices for only a single one of them to misbehave. The

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| | | | vulnerable to malicious clients | requires network modification[4] |
|---|---|---|---|---|
| 2PC | HE | CHET [Dat+19], CryptoDL [HTG17], LoLa [BEG18], TAPAS [San+18], Faster CryptoNets [Cho+18], FHE-DiNN [Bou+18] | ●[2] | ● |
| | GC | DeepSecure [RRK18], [Bal+19] | ●[2] | ◐ |
| | | XONN [Ria+19] | ●[2] | ● |
| | Mixed | SecureML [MZ17] | ●[1] | ● |
| | | Gazelle [JVC18], MiniONN [Liu+17a], CrypTFlow2 [Rat+20] | ●[1] | ○ |
| | | DELPHI [Mis+20] | ●[1] | ◐ |
| | | **MUSE** | ○ | ○ |
| 3PC | Mixed | Chameleon [Ria+18] | ●[1,3] | ○ |
| | | ABY$^3$ [MR18] | ○[3] | ○ |
| | SS | SecureNN [WGC19], Falcon [Wag+21], CrypTFlow [Kum+20] | ○[3] | ○ |

**Table 1:** Related work on secure convolutional neural network (CNN) inference. See Section 7 for more details. This table compares specialized secure inference protocols, not generic frameworks for MPC. We compare against generic frameworks in Section 6. HE = Homomorphic Encryption, GC = Garbled Circuits, SS = Secret Sharing.
◐ Network modifications are optional [1]See Section 2.1 [2] See Remark 2.1 [3]Requires that two of the three parties act honestly
[4]Polynomial activations or binarized/discretized weights—may reduce network accuracy

incentives for a client to cheat are high: service providers expend vast amounts of effort and money to accumulate data, clean it, design model architectures, and to train the final model. If a client wishes to obtain a similar model, it would be much easier to steal the server's than to try and train an equivalent one; this makes *model-extraction attacks* attractive.

To illustrate the threat of a malicious client, in Section 2 we demonstrate a new model-extraction attack against semi-honest secure inference protocols whereby a malicious client can learn the server's entire model in a number of inference queries *linear* in the number of parameters in a network *regardless of its depth*. This attack outperforms the best model-extraction attacks for plaintext inference by 22×–312× [Jag+20; CJM20], and demonstrates that using semi-honest secure inference protocols can *significantly amplify* a malicious client's ability to steal a model.

A natural approach to defend against such an amplification is to leverage state-of-the-art generic secure computation tools providing *malicious security*. This approach guarantees that if *either* party acts maliciously, they will be caught and the protocol aborted, preserving privacy. However, such methods for achieving malicious security add a large overhead due to the use of heavy cryptographic primitives (e.g. zero-knowledge proofs [GMR89] or cut-and-choose [LB15; Zhu+16]). In Section 6, we compare against such techniques.

To reduce this overhead, we propose MUSE, a secure inference protocol that works in the *client-malicious* threat model. In this model, the server is presumed to behave semi-honestly, but the client is allowed to deviate arbitrarily from the protocol description. As we will show in Section 6, working in this model enables MUSE to achieve much better performance than a fully malicious baseline.

**Our contributions.** To summarize, in this paper we make the following contributions:

- We devise a novel model-extraction attack against secure inference protocols that rely on additive secret sharing. This attack allows a malicious client to *perfectly extract all* the weights of a model with 22×–312× fewer queries than the state-of-the-art [CJM20]. The complexity of our attack depends *only* on the number of parameters, and not on other factors like the depth of the network.
- We present MUSE[1], an efficient two-party cryptographic inference protocol that is resilient to malicious clients. In designing MUSE, we develop a novel protocol for *conditional disclosure of secrets* to switch between authenticated additive secret shares and garbled circuit labels. Additionally, we formulate new client-malicious techniques for *triple generation* and *input authentication* in SPDZ-style MPC frameworks which improve performance by up to 12.5× and 37.8× respectively.
- Our implementation of MUSE is able to achieve an online phase that is only 1.7×–2.2× slower and uses 1.4× more communication than DELPHI [Mis+20], a recent protocol for *semi-honest* inference. When compared to fully-malicious secure inference protocols, MUSE is 13.4×–21× faster and uses 2×–3.5× less communication.

**Remark 1.1.** While MUSE's online phase is competitive with some of the best semi-honest protocols [Mis+20; Rat+20], the communication cost of preprocessing is up to 10× higher than in these semi-honest protocols. Hence, we view MUSE as a first step in constructing secure inference protocols that achieve client-malicious security, and anticipate that future works will rapidly lower this cost (the same has occurred for semi-honest secure inference protocols). MUSE already improves performance over current techniques for client-malicious inference by 13.4×–21× (see Section 6.4).

[1]MUSE is an acronym for Malicious-User Secure Inference

We now give a high-level overview of our techniques.

## 1.1 Our attack

**What can a malicious client do?** We start off by examining the power of malicious clients in secure inference protocols that rely on secret sharing. We noticed that many protocols of interest, such as [MZ17; JVC18; Liu+17a; Mis+20; Rat+20], have a similar structure, and we exploit this structure in our attack. The structure is as follows. These protocols "evaluate" the neural network in a layer-by-layer fashion, so that at the end of each layer, the client and the server both hold 2-out-of-2 secret shares of the output of that layer. At the end of the protocol, the server sends its share of the final output to the client, who uses it to reconstruct the final output.

Our attack relies on the following crucial observation: because the shares at the end of a layer are not authenticated, a malicious client can *additively malleate* them without detection. In more detail, let $\langle m \rangle_C$ be the client's share, and $\langle m \rangle_S$ be the server's share of a message $m \in \mathbb{F}$, so that $\langle m \rangle_C + \langle m \rangle_S = m$. Then, a malicious client can add an arbitrary shift $r$ to a secret share to change the shared value from $m$ to $m + r$. In Section 2, we show how one can leverage this malleability to learn the model weights.

## 1.2 Our protocol

We now explain how MUSE protects against malicious client attacks. We begin by describing our starting point: the semi-honest secure inference protocol DELPHI [Mis+20].

**Starting point: DELPHI.** We design MUSE by following the paradigm laid out in DELPHI [Mis+20]: since a convolutional neural network consists of alternating linear and non-linear layers, one should use subprotocols that are efficient for computing each type of layer, and then translate the output of one subprotocol to the input of the next. DELPHI instantiates these subprotocols by using additive secret sharing to evaluate linear layers, and garbled circuits to evaluate non-linear layers.

**Attempt 1: Preventing malleability via MACs.** The key insight in our attack in Sections 1.1 and 2 is that the client can malleate shares without detection. To prevent this, one can try to use standard techniques for authenticating the client's share via *information-theoretic homomorphic message authentication codes (MACs)*. This technique is employed by the state-of-the-art protocols for malicious security [KPR18; Che+20; Esc+20]. However, applying this technique directly to DELPHI runs into problems. For example, when switching between secret shares and garbled circuits, the server must ensure that the labels obtained by the client correspond to the authenticated secret share, and not to a different share. Doing this in a straightforward manner entails checking the share's MAC inside the garbled circuit, which is expensive. Furthermore, this check would need to be done in the online phase, which is undesirable.

**Attempt 2: Separating authentication from computation.** To remedy this, we make the following observation: garbled circuits already achieve malicious security against garbled circuit evaluators (clients in our setting). This means that, if we had a specialized protocol that could output labels for the client's secret shares *only if* the corresponding MACs were valid, then we could compose this protocol with the garbled circuits to achieve an end-to-end client-malicious secure inference protocol.

In Section 5.1, we design exactly such a protocol for "conditional disclosure of secrets" (CDS). Unfortunately, executing our CDS protocol using existing frameworks for malicious MPC [KPR18] proves to be extremely expensive. To address this, in Section 5.3 we devise a number of techniques to improve [KPR18] in the client-malicious setting, and use the optimized framework to execute our CDS procedure. While the resulting protocol is much more efficient than checking MACs inside garbled circuits, it still imposes a significant cost on the online phase.

**Our final protocol.** To remedy this, our final insight in MUSE is that the secret shares and MACs that the client feeds into the CDS protocol *do not depend* on the client's input in the online phase. This allows us to move the execution of the CDS protocol *entirely to the preprocessing phase*, resulting in an online phase that is almost identical to that of DELPHI.

To summarize, in order to defend against malicious clients we first enforce authentication for the linear layers by using homomorphic MACs, then ensure that the client only receives garbled circuit labels corresponding to these authenticated shares via a novel CDS protocol, then develop new techniques for efficiently executing the CDS protocol, and finally move all these protocols to the preprocessing phase. For details, see Section 5.

# 2 Attacks on semi-honest inference protocols

We now describe how a malicious client can leverage the additive malleability of additive secret shares to learn the weights of a server's convolutional neural network in semi-honest secure inference protocols that rely on additive secret sharing. We begin in Section 2.1 by describing the kinds of protocols that are vulnerable to our attack. Then, in Section 2.2, we provide a detailed overview of our attack. Finally, in Section 2.3, we discuss the theoretical and empirical query complexity achieved by our attack.

## 2.1 Attack threat model

Our attack recovers the weights of neural networks consisting of alternating linear (that is, fully-connected or convolutional) and non-linear ReLU layers.[2] Our attack works against semi-honest secure inference protocols that have the following properties:

[2] Our attack also supports networks with average pooling layers, as these are linear layers, but don't contain any weights that need to be recovered

- The protocol should evaluate the network iteratively by applying subprotocols for evaluating linear and non-linear layers.
- For each subprotocol, the input and output of the client and the server should be secret shares of the actual layer input and output, respectively.
- The client's final output should be the plaintext output of the final linear layer.

A number of two-party and multi-party secure inference protocols have these properties [MZ17; Liu+17a; Ria+18; JVC18; Cha+19; Mis+20; Rat+20].

**Remark 2.1** (Other semi-honest protocols)**.** Our attack does not affect semi-honest secure inference protocols based on fully-homomorphic encryption (FHE) [Gil+16; Cho+18; BEG18; San+18; Dat+19] or garbled circuits (GC) [RRK18; Bal+19; Ria+19]. However, this does not immunize these protocols against other kinds of malicious client attacks:

- *FHE-based protocols* use *noise flooding* [Gen09a] to hide the server's model. This technique is inherently semi-honest as it requires the pre-existing noise to be honestly bounded; if this does not hold, the noise term can reveal information about the server's model *despite noise flooding*.
- *GC-based protocols* use oblivious transfer (OT) [Rab81] to transfer labels for the client's input. However, if this OT is only semi-honest secure, a malicious client can attack it to learn both labels for the same input wire, which breaks the privacy guarantees of the garbled circuit, and leads to a leak of the server's model.

## 2.2 Attack strategy

**Notation.** Let NN be an $\ell$-layer network convolutional neural network that classifies an image into one of $m$ classes. That is, NN consists of $\ell$ matrices $M_1,\dots,M_\ell$ so that $\mathsf{NN}(x) = M_\ell(\mathsf{ReLU}(\dots M_2(\mathsf{ReLU}(M_1(x)))))$ where $M_\ell \in \mathbb{R}^{m\times t}$ and the image of $M_\ell$ is $\mathbb{R}^m$. We denote by $\mathsf{NN}_i(x)$ the partial evaluation of NN up to the $i$-th linear layer. That is, $\mathsf{NN}_i(x) := M_i(\mathsf{ReLU}(\dots M_2(\mathsf{ReLU}(M_1(x)))))$. Below we denote by $\mathbf{e}_j$ the $j$-th unit vector (the vector whose $j$-th entry is 1, and other entries are 0). Finally, for simplicity of exposition, we assume that biases are zero,[3] and that the network contains only fully-connected layers; for details on how to recover convolutional layers, see Remark 2.2 and Appendix C.

**Prelude.** Our attack proceeds in a bottom-up fashion: the client first recovers the parameters of the last linear layer $M_\ell \in \mathbb{R}^{m\times t}$, and then iteratively recovers previous layers. We describe the subroutine for recovering the last layer in Section 2.2.1, and then describe our subroutine for recovering intermediate layers in Section 2.2.2. In both subroutines, the client sets its initial input to the network be the all-zero vector.

[3]One can handle a bias $b$ in a linear layer $L(x) = Mx + b$ by treating it as a simply another column in the modified matrix $M' = M||b$, so that the linear layer becomes $L(x) = M' \cdot (x||1)$.

### 2.2.1 Recovering the last layer

At a high level, to recover $M_\ell$, the client proceeds column-by-column as follows: for each $j \in [t]$, the client provides as initial input the all-zero vector, and then honestly follows the secure inference protocol until the $\ell$-th layer. At the $\ell$-th layer, the client malleates its share of the input to $M_\ell$ so that it becomes $\mathbf{e}_j$. This means that result $M_\ell \cdot \mathbf{e}_j$ is the $j$-th column of $M_\ell$. We illustrate this graphically for the first column below.

$$\overbrace{\begin{bmatrix}0\\0\end{bmatrix}}^{\mathbf{x}_{\ell-1}} \xrightarrow[+\mathbf{e}_1]{\text{malleate}} \overbrace{\begin{bmatrix}1\\0\end{bmatrix}}^{\mathbf{x}'_{\ell-1}} \xrightarrow{\text{query } M_\ell} \overbrace{\begin{bmatrix}-0.1 & 0.2\\-1.1 & 1.2\end{bmatrix}}^{M_\ell}\begin{bmatrix}1\\0\end{bmatrix} = \overbrace{\begin{bmatrix}-0.1\\-1.1\end{bmatrix}}^{\text{first column of } M_\ell}$$

### 2.2.2 Recovering intermediate layers

The foregoing algorithm works for recovering the last layer because the client can directly read off $M_\ell$ column-by-column by "solving" a linear system. However, this approach does not work as is for recovering the weights of intermediate linear layers, as we now demonstrate by considering the case of recovering the $\ell-1$-th linear layer $M_{\ell-1}$. We then describe how to resolve the issues that arise. (The case of the remaining layers follows similarly).

**Problem 1: Intervening ReLUs are lossy and non-linear.** ReLUs between $M_{\ell-1}$ and $M_\ell$ disrupt the linearity of the system, preventing the use of linear system solvers.

**Solution 1: Force ReLUs to behave linearly.** To resolve this issue, we recall the fact that ReLU behaves like the identity function on inputs that are positive. We use malleability to exploit this property and *force* the remaining $M_\ell(\mathsf{ReLU}(\cdot))$ computation to behave linearly, which means that we can once again solve a linear system to learn information about $M_{\ell-1}$.

In more detail, let $\langle \mathbf{y}_{\ell-2}\rangle_C$ be the client's share after applying $M_{\ell-1}$. The client malleates $\langle \mathbf{y}_{\ell-2}\rangle_C$ by setting it to $\langle \mathbf{y}'_{\ell-2}\rangle_C := \langle \mathbf{y}_{\ell-2} + \delta\rangle_C$, where $\delta$ is a constant vector whose elements are all greater than the magnitude of the largest element in $\mathbf{y}_{\ell-2}$.[4] This forces all entries of $\mathbf{y}'_{\ell-2}$ to be positive, which means ReLU acts like the identity function. Then, after evaluating the ReLU and obtaining $\langle \mathbf{x}'_{\ell-1}\rangle_C$, the client "undoes" the malleation by subtracting $\delta$. The following equation provides a graphical illustration of this process.

$$\overbrace{\begin{bmatrix}-0.1 & 0.2\\-1.1 & 1.2\end{bmatrix}}^{M_{\ell-1}}\begin{bmatrix}1\\0\end{bmatrix} = \overbrace{\begin{bmatrix}-0.1\\-1.1\end{bmatrix}}^{y_{\ell-2}} \xrightarrow[\delta := 10]{\text{malleate}} \overbrace{\begin{bmatrix}9.9\\8.9\end{bmatrix}}^{y'_{\ell-2}} \xrightarrow{\mathsf{ReLU}} \overbrace{\begin{bmatrix}9.9\\8.9\end{bmatrix}}^{x'_{\ell-1}} \xrightarrow{\text{unmalleate}} \overbrace{\begin{bmatrix}-0.1\\-1.1\end{bmatrix}}^{x_{\ell-1}}$$

**Problem 2: Underconstrained linear system.** While the foregoing technique enables us to force the network to behave like a linear function, we have no guarantees that the resulting linear system is solvable. Indeed, neural networks necessarily

[4]Note that since model weights are usually small (in the range $[-1,1]$), we can set $\delta$ to be a large value (say, $\sim 10$) to ensure that all entries of $\mathbf{y}'_{\ell-2}$ are positive.

map a high-dimensional feature to a low-dimensional classification, and so the resulting "linearized" neural network *must* be lossy. The following figure illustrates this graphically:

$$\underbrace{\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}}_{M_2}\cdot\underbrace{\begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{bmatrix}}_{M_1}=\underbrace{\begin{bmatrix}a_1+3b_1 & a_2+3b_2 & a_3+3b_3\\ 2a_1+4b_1 & 2a_2+4b_2 & 2a_3+4b_3\end{bmatrix}}_{M_3}$$

Here, $M_1$ and $M_2$ are the first and last layers of the network, respectively. We have used the technique in Section 2.2.1 to recover $M_2$, and now must recover $M_1$. If we try to do this by querying $M_3$, we get three (independent) equations for six variables, which is insufficient:

$$M_3\mathbf{e}_1=\begin{bmatrix}3a_1+6b_1\\ 0\\ 0\end{bmatrix}\text{ and } M_3\mathbf{e}_2=\begin{bmatrix}0\\ 3a_2+6b_2\\ 0\end{bmatrix}\text{ and } M_3\mathbf{e}_3=\begin{bmatrix}0\\ 0\\ 3a_3+6b_3\end{bmatrix}$$

**Solution 2: Masking variables.** The issue is that $M_3$ *does* contain sufficient information to recover $M_1$, but querying it naively loses that information. To resolve this, we use malleability again: the client uses the intervening ReLUs to "zero" out all but $m$ entries of intermediate state, as follows:

$$M_1\cdot\begin{bmatrix}1\\ 1\\ 0\end{bmatrix}=\begin{bmatrix}a_1+a_2\\ b_1+b_2\end{bmatrix}\xrightarrow[\text{+ mask}]{\text{malleate}}\begin{bmatrix}a_1+a_2+\delta\\ b_1+b_2-\delta\end{bmatrix}\xrightarrow[\text{unmalleate}]{\text{ReLU +}}\begin{bmatrix}a_1+a_2\\ 0\end{bmatrix}$$

Now, the client can obtain $M_2\cdot[a_1+a_2,0]$, and can solve the resulting equations to learn $a_1$ and $a_2$. It can then repeat this process with different queries and "masks" to learn all of $M_1$.

For a detailed description of our algorithm, see Appendix C.

**Remark 2.2** (recovering convolutional layers)**.** To recover the kernel of a convolutional linear layer, we can reuse the foregoing ideas, but must change how we malleate the input to the target layer: we instead sample a random input and query the kernel via linearly independent columns of (the `im2col` transform of) this input. See Appendix C for details. Note that for simplicity of exposition, our description in Appendix C assumes that the number of channels and number of filters in each convolutional layer are both 1, and that the number of parameters in the kernel is less than the number of classes; these restrictions are easy to lift by adapting the masking techniques from above.

### 2.3 Efficiency and evaluation

**Efficiency.** Given a neural network where the $i$-th linear layer has dimension $m_i \times t_i$, and the number of classes is $m_\ell = m$, the foregoing algorithm learns the model parameters in just $\sum_{i=1}^{\ell}\lceil\frac{m_i}{m}\rceil\cdot t_i$ queries. Furthermore, the complexity of our attack depends *only on the number of parameters*, and not on other factors such as the depth.[5] (This is not the case for other model-extraction attacks, which fail when extracting deep models that have few parameters.)

**Evaluation.** In Table 2, we compare our work to the state-of-the-art prior work on model extraction [CJM20], which does not rely on the existence of a secure inference protocol (and hence does not exploit properties of such protocols). Our experiments match the query complexity derived above.

| network dimensions | # params | # queries: us | # queries: [CJM20] | speedup |
|---|---|---|---|---|
| FC-only networks | | | | |
| 784-128-1 | 100,480 | 100,480 | $2^{21.5}$ | 29.5× |
| 784-32-1 | 25,120 | 25,120 | $2^{19.2}$ | 24× |
| 10-10-10-1 | 210 | 210 | $2^{16}$ | 312× |
| 10-20-20-1 | 620 | 620 | $2^{17.1}$ | 226.5× |
| 40-20-10-10-1 | 1,110 | 1,110 | $2^{21.5}$ | 205× |
| 80-40-20-1 | 4,020 | 4,020 | $2^{17.1}$ | 92× |
| 80×5-40-20-1 | 29,620 | 29,620 | — | n/a |
| 1000-500-1 | 500,500 | 500,500 | — | n/a |
| 1000-500-10 | 505,000 | 50,500 | — | n/a |
| 2000×2-1000-100 | 6,100,000 | 61,000 | — | n/a |
| 1000×2-40×8-20-10 | 1,052,200 | 105,220 | — | n/a |
| networks with convolutions | | | | |
| $32\times(3,3)$-P-100-10 | 158,088 | 15,809 | — | n/a |

**Table 2:** Query complexity of our attack vs. that of [CJM20]. The notation *l*-*m*-*n*-… indicates a series of fully-connected layers of dimension $l\times m$, $m\times n$, and so on, while $32\times(3,3)$ indicates a convolutional layer consisting of 32 $3\times3$ filters, and $P$ indicates a $2\times2$ average pooling layer.

## 3 Threat model and privacy goals

In our system, there are two parties: the client and the service provider (or server). The server holds a neural network model, and the client holds some data that it wants classified by the server's model. To achieve this goal, the two parties interact via a protocol for *secure inference*. This protocol takes as input the server's model and the client's data, and computes the classification so that neither party learns any information except this final classification. Below we clarify the security guarantees we aim for when designing our secure inference protocol MUSE.

### 3.1 Threat model

There are two standard notions of security for multiparty computation: security against semi-honest adversaries, and security against malicious adversaries. A semi-honest adversary follows the protocol perfectly but inspects messages it receives to learn information about other parties' inputs. A malicious adversary, on the other hand, may arbitrarily deviate from the protocol.

We design MUSE for a new threat model called "security against *malicious clients*" or *client-malicious security*. In this

[5] Note that any implementation of our attack will have to contend with errors due to limited floating point precision, but our experiments did not encounter such failures.

setting, either a malicious adversary corrupts the client, or a semi-honest adversary corrupts the server. [6]

## 3.2 Privacy goals

MUSE's goal is to enable the client to learn at most the following information: the architecture of the neural network, and the result of the inference; all other information about the client's private inputs and the parameters of the server's neural network model should be hidden. Concretely, we aim to achieve a strong simulation-based definition of security as follows:

**Definition 3.1.** *A protocol $\Pi$ between a server and a client is said to securely compute a function $f$ against a malicious client and semi-honest server if it satisfies the following properties:*

- ***Correctness.*** *For any server's input* $\mathbf{y}$ *and client's input* $\mathbf{x}$*, the probability that at the end of the protocol, the client outputs* $f(\mathbf{y},\mathbf{x})$ *is* 1.
- ***Semi-Honest Server Security.*** *For any server $S$ that follows the protocol, there exists a simulator* $\mathsf{Sim}_S$ *such that for any input* $\mathbf{y}$ *of the server and* $\mathbf{x}$ *of the client, we have:*

$$\mathsf{view}_S(\mathbf{y},\mathbf{x}) \approx_c \mathsf{Sim}_S(\mathbf{y})$$

*In other words,* $\mathsf{Sim}_S$ *is able to generate a view of the semi-honest server without knowing the client's private input.*

- ***Malicious Client Security.*** *For any malicious client $C$ (that might deviate arbitrarily from the protocol specification), there exists a simulator* $\mathsf{Sim}_C$ *such that for any input* $\mathbf{y}$ *of the server, we have:*

$$\mathsf{view}_C(\mathbf{y}) \approx_c \mathsf{Sim}_C^{f(\mathbf{y},\cdot)}$$

*In other words, the* $\mathsf{Sim}_C$ *is able to generate the view of a malicious client with only access to an ideal functionality that accepts a client's input and outputs the result of the function $f$. This modeling is used in cryptographic literature to capture the cases where a malicious client may substitute its actual input with any other input of its choice.*

**Definition 3.2.** *We say that $\Pi$ is a secure inference protocol against malicious clients and semi-honest servers if it securely computes* $\mathsf{NN}(\cdot,\cdot)$ *with the server input being* $\mathbf{M}$ *and the client input being* $\mathbf{x}$*.*

Like most prior work, MUSE does not hide information that is revealed by the result of the prediction. See Section 7.1 for a discussion of attacks that leverage this information, as well as potential mitigations.

[6]One can generalize this threat model to $n$ parties by considering two fixed subsets of parties: one of which can be corrupted by a malicious adversary, and the other which can be corrupted by a semi-honest adversary

# 4 Building blocks

MUSE uses the following cryptographic building blocks.

**Garbling Scheme.** A *garbling scheme* [Yao86; BHR12] is a tuple of algorithms $\mathsf{GS} = (\mathsf{Garble}, \mathsf{Eval})$ with the following syntax:

- $\mathsf{Garble}(1^\lambda, C, \{\mathsf{lab}_{i,0}, \mathsf{lab}_{i,1}\}_{i\in[n]}) \to \tilde{C}$. On input the security parameter, a boolean circuit $C$ (with $n$ input wires) and a set of labels $\{\mathsf{lab}_{i,0}, \mathsf{lab}_{i,1}\}_{i\in[n]}$, $\mathsf{Garble}$ outputs a *garbled circuit* $\tilde{C}$. Here $\mathsf{lab}_{i,b}$ represents assigning the value $b \in \{0,1\}$ to the $i$-th input wire.
- $\mathsf{Eval}(\tilde{C}, \{\mathsf{lab}_{i,x_i}\}_{i\in[n]}) \to y$. On input a garbled circuit $\tilde{C}$ and labels $\{\mathsf{lab}_{i,x_i}\}_{i\in[n]}$ corresponding to an input $x \in \{0,1\}^n$, $\mathsf{Eval}$ outputs a string $y = C(X)$.

We provide a formal definition in Appendix A.1, and briefly describe here the key properties satisfied by garbling schemes. First, $\mathsf{GS}$ must be *correct*: the output of $\mathsf{Eval}$ must equal $C(x)$. Second, it must be *private*: given $\tilde{C}$ and $\{\mathsf{lab}_{i,x_i}\}$, the evaluator should not learn anything about $C$ or $x$ except the size of $|C|$ (denoted by $1^{|C|}$) and the output $C(x)$.

**Leveled Fully Homomorphic public-key encryption.** A *leveled fully-homomorphic* encryption scheme $\mathsf{HE} = (\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec}, \mathsf{Eval})$ [Reg09; FV12] is a public key encryption scheme that additionally supports homomorphically evaluating any depth-$D$ arithmetic circuit on encrypted messages. Formally, $\mathsf{HE}$ satisfies the following syntax and properties:

- $\mathsf{KeyGen}(1^\lambda) \to (\mathsf{pk}, \mathsf{sk})$: On input a security parameter, $\mathsf{KeyGen}$ outputs a public key $\mathsf{pk}$ and a secret key $\mathsf{sk}$.
- $\mathsf{Enc}(\mathsf{pk}, m) \to c$: On input the public key $\mathsf{pk}$ and a message $m$, the encryption algorithm $\mathsf{Enc}$ outputs a ciphertext $c$. We assume that the message space is $\mathbb{Z}_p$ for some prime $p$.
- $\mathsf{Dec}(\mathsf{sk}, c) \to m$: On input a secret key $\mathsf{sk}$ and a ciphertext $c$, the decryption algorithm $\mathsf{Dec}$ outputs a message $m$.
- $\mathsf{Eval}(\mathsf{pk}, c_1, c_2, f) \to c'$: On input a public key $\mathsf{pk}$, ciphertexts $c_1$ and $c_2$ encrypting $m_1$ and $m_2$ respectively, and a depth-$D$ arithmetic circuit $f$, $\mathsf{Eval}$ outputs a new ciphertext $c'$.

Besides the standard correctness and semantic security properties, we require $\mathsf{HE}$ to satisfy the following properties:

- *Homomorphism.* If $c_1 := \mathsf{Enc}(\mathsf{pk}, m_1)$, $c_2 := \mathsf{Enc}(\mathsf{pk}, m_2)$, and $c := \mathsf{Eval}(\mathsf{pk}, c_1, c_2, f)$, then $\mathsf{Dec}(\mathsf{sk}, c) = f(m_1, m_2)$.
- *Function privacy.* Given a ciphertext $c$, no attacker can tell what homomorphic operations led to $c$.

See Appendix A.2 for more formal definitions.

**Additive secret sharing.** Let $p$ be a prime. A 2-of-2 *additive secret sharing* of $x \in \mathbb{Z}_p$ is a pair $(\langle x\rangle_1, \langle x\rangle_2) = (x - r, r) \in \mathbb{Z}_p^2$ for a random $r \in \mathbb{Z}_p$ such that $x = \langle x\rangle_1 + \langle x\rangle_2$. Additive secret sharing is perfectly hiding, i.e., given a share $\langle x\rangle_1$ or $\langle x\rangle_2$, the value $x$ is perfectly hidden.

**Message authentication codes.** A *message authentication code (MAC)* is a tuple of algorithms $\mathsf{MAC} = (\mathsf{KeyGen}, \mathsf{Tag}, \mathsf{Verify})$ with the following syntax:

- $\mathsf{KeyGen}(1^\lambda) \to \alpha$: On input the security parameter, KeyGen outputs a MAC key $\alpha$.
- $\mathsf{Tag}(\alpha, m) \to \sigma$: On input a key $\alpha$ and message $m$, Tag outputs a tag $\sigma$ and a secret state st.
- $\mathsf{Verify}(\alpha, \mathsf{st}, m, \sigma) \to \{0,1\}$: On input a key $\alpha$, secret state st, message $m$ and tag $\sigma$, Verify outputs 0 or 1.

We require MAC to satisfy the following properties:

- *Correctness.* For any message $m$, $\alpha \leftarrow \mathsf{KeyGen}(1^\lambda)$, and $(\sigma, \mathsf{st}) \leftarrow \mathsf{Tag}(\alpha, m)$, $\mathsf{Verify}(\alpha, \mathsf{st}, m, \sigma) = 1$.
- *One-time Security.* Given a valid message-tag pair, no adversary can forge a different, valid message-tag pair.

See Appendix A.3 for formal definitions. In this work, we will use the following construction of MACs:

1. The message space is $\mathbb{Z}_p^n$ for some $p^n \geq 2^\lambda$.
2. KeyGen samples a uniform element $\alpha \leftarrow \mathbb{Z}_p$.
3. $\mathsf{Tag}(\alpha, m)$ outputs $\sigma = \langle \alpha \cdot m \rangle_1$ and $\mathsf{st} = \langle \alpha \cdot m \rangle_2$.
4. $\mathsf{Verify}(\alpha, \mathsf{st}, m, \sigma)$ checks if $\sigma + \mathsf{st} = \alpha \cdot m$.

We prove the one-time security of this scheme in Lemma A.1.

**Beaver's multiplicative triples.** A multiplication triple is a triple $(a, b, c) \in \mathbb{Z}_p^3$ such that $ab = c$. A triple generation procedure is a two-party protocol that outputs secret shares of a triple $(a, b, c)$ to two parties.

**Authenticated secret shares.** For any prime $p$, an element $x \in \mathbb{Z}_p$, and a MAC key $\delta \in \mathbb{Z}_p$ an *authenticated share* of $x$ is a tuple $(\varepsilon, [\![x]\!]_1, [\![x]\!]_2) := (\varepsilon, (\langle x \rangle_1, \langle \delta \cdot x \rangle_1), (\langle x \rangle_2, \langle \delta \cdot x \rangle_2))$. An authenticated share naturally supports local evaluation of addition and multiplication by public constants, as well as addition with another authenticated share. To multiply two authenticated shares, one needs to use multiplication triples. For simplicity of exposition, in the rest of the paper we omit $\varepsilon$, as it is merely used for bookkeeping when adding public constants.

**Zero-knowledge proofs.** Let $\mathcal{R}$ be any NP relation. A *zero-knowledge proof* for $\mathcal{R}$ is a protocol between a prover $P$ and a verifier $V$ that both have a common input $x$, where $P$ tries to convince $V$ that it "knows" a secret witness $w$ such that $(x, w) \in \mathcal{R}$. At the end of the protocol, $V$ should have learnt no additional information about $w$. We want our zero-knowledge proof system to satisfy the standard definitions of *completeness*, *soundness*, *proof of knowledge*, and *zero-knowledge*. See Appendix A.4 for formal definitions of these properties.

# 5 The MUSE protocol

In this section, we describe MUSE, our secure inference protocol that is secure against a malicious client and a semi-honest server. Like the DELPHI protocol (see Fig. 2) [Mis+20], MUSE's protocol consists of two phases: an offline preprocessing phase, and an online inference phase. The offline preprocessing phase is independent of the client's input (which regularly changes), but assumes that the server's model is static; if this model changes, then both parties have to re-run the preprocessing phase. After preprocessing, during the online inference phase, the client provides its input to our specialized secure two-party computation protocol, and eventually learns the inference result. Below, we expand on the high level overview in Section 1.2 and provide a detailed description of both phases of our protocol.

**Preprocessing phase.** During preprocessing, the client and the server pre-compute data for the online execution. This phase can be executed independently of the input values, i.e., DELPHI can run this phase before either party's input is known. Preprocessed data can only be used for a single inference.

1. *Linear correlations generator:* The client and server interact with a functionality that, for each $i \in [\ell]$, outputs to them secret shares of $M_i \mathbf{r}_i$, where $\mathbf{r}_i$ is a random masking vector.
2. *Preprocessing for ReLUs:* The server constructs a garbled circuit $\widetilde{C}$ for a circuit $C$ computing ReLU. It sends $\widetilde{C}$ to the client and then uses OT to send to the client the input wires corresponding to $\mathbf{r}_{i+1}$ and $\mathbf{M}_i \cdot \mathbf{r}_i - \mathbf{s}_i$.

**Online phase.** The online phase is divided into two stages:

1. *Preamble:* On input $\mathbf{x}$, the client sends $\mathbf{x} - \mathbf{r}_1$ to the server. The server and the client now hold an additive secret sharing of $\mathbf{x}$.
2. *Layer evaluation:* Let $\mathbf{x}_i$ be the result of evaluating the first $(i-1)$ layers of the neural network on $\mathbf{x}$. At the beginning of the $i$-th layer, the client holds $\mathbf{r}_i$, and the server holds $\mathbf{x}_i - \mathbf{r}_i$, which means that they possess secret shares of $\mathbf{x}_i$.
   - *Linear layer:* The server computes $M_i \cdot (\mathbf{x}_i - \mathbf{r}_i)$, which means that the client and the server hold an additive secret sharing of $M_i \mathbf{x}_i$.
   - *ReLU layer:* After the linear layer, the client and server hold secret shares of $M_i \mathbf{x}_i$. The server sends to the client the labels corresponding to its secrete share, and the client then evaluates the GC to obtain a secret share of the ReLU output.

**Figure 2:** High-level overview of the DELPHI protocol [Mis+20].

**Notation.** The server holds a model $\mathbf{M}$ consisting of $\ell$ linear layers $\mathbf{M}_1, \ldots, \mathbf{M}_\ell$ and the client holds an input vector $\mathbf{x} \in \mathbb{Z}_p^n$. We use $\mathsf{NN}(\mathbf{M}, \mathbf{x})$ to denote the output of the neural network when the server's input is $\mathbf{M}$ and the client's input is $\mathbf{x}$. We assume that the algorithm computing NN is public and is known to both the client and the server.

## 5.1 Preprocessing phase

In the preprocessing phase, the client and the server precompute data that can be used during the online execution. This phase is independent of the client's input values, and can be run before the client's input is known. However, this phase cannot be reused and has to be run once for each client input.

### 5.1.1 Intuition

As explained in Section 1.2, MUSE follows the approach of DELPHI, and uses different cryptographic primitives to produce preprocessed material for linear and non-linear layers. Below we describe these primitives at a high level.

**Linear layers.** Like in DELPHI, our goal is to produce shares

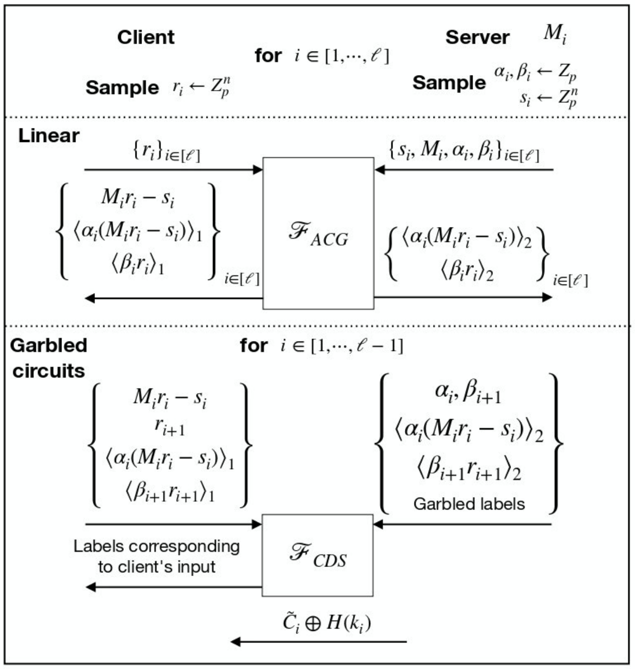



**Figure 3:** MUSE preprocessing phase.

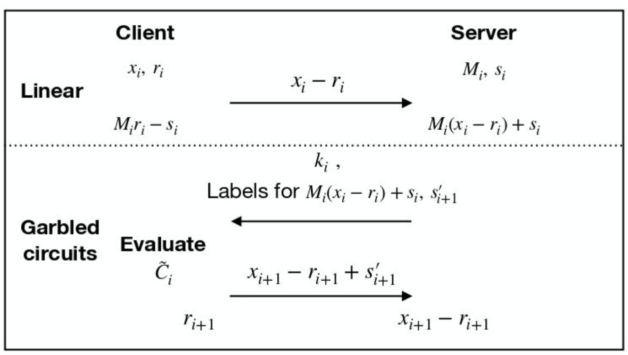



**Figure 4:** MUSE online phase.

of $M\mathbf{r}$ for a linear layer $M$. This enables us to efficiently compute linear layer operations in the online phase. Unlike DELPHI, we additionally need to prevent tampering by malicious clients. To this end, we extend the linear correlations generator (CG) used in DELPHI (see Fig. 2) to additionally support authentication. We formalize this via functionality $\mathcal{F}_{\text{ACG}}$ for generating *authenticated correlations*. See Fig. 11 for a formal description.

To construct a protocol $\Pi_{\text{ACG}}$ that realizes $\mathcal{F}_{\text{ACG}}$, we extend the techniques based on the leveled fully-homomorphic encryption used in DELPHI to additionally authenticate and secret share the relevant ciphertexts (see Fig. 5). We also require the client to provide zero-knowledge proofs that assert that their input ciphertexts are well-formed.

**Non-linear layers.** Like in DELPHI, we use garbled circuits to efficiently evaluate ReLUs. However, unlike DELPHI, we can no longer use oblivious transfer to send garbled labels to the client, because we have no way to check that the input to the oblivious transfer corresponds to the output from $\mathcal{F}_{\text{ACG}}$. Instead we introduce a functionality which conditionally outputs these labels if the inputs match the output of $\mathcal{F}_{\text{ACG}}$. We call this functionality *Conditional Disclosure of Secrets*, and denote it by $\mathcal{F}_{\text{CDS}}$.

To construct a protocol $\Pi_{\text{CDS}}$ that realizes $\mathcal{F}_{\text{CDS}}$, we have two options: use 2PC protocols specialized for boolean computation, or 2PC protocols specialized for arithmetic computation. Indeed, because this operation fundamentally reasons about boolean values, it would seem reasonable to use a protocol like garbled circuits. However, checking validity of the client's input requires modular multiplications, which are extremely expensive when expressed as boolean circuits. Since even the simplest neural networks oftentimes have thousands of activations, the resulting communication and computation cost is unacceptable.

Instead, we implement this functionality via MPC for arithmetic circuits, as modular multiplication is cheap here. However, now the boolean operations are expensive. To overcome this, we take further advantage of the client-malicious setting to improve the MPC protocol we use to securely execute the arithmetic circuit, as we describe in Section 5.3.

By designing efficient protocols for $\mathcal{F}_{\text{ACG}}$ and $\mathcal{F}_{\text{CDS}}$, MUSE achieves client-malicious security with an online phase design *identical to that of the semi-honest* DELPHI *protocol*. In our implementation, there are a few differences we detail in Remarks 5.2 and 5.3.

### 5.1.2 Protocol

We now present the full protocol for the preprocessing phase of MUSE (see Fig. 3 for a graphical overview).

1. For every $i \in [\ell]$, denote $n_i, m_i$ as the input and output sizes of the $i$-th linear layer respectively. The client samples a random layer input mask $\mathbf{r}_i \leftarrow \mathbb{Z}_p^{n_i}$ and the server samples a random layer output mask $\mathbf{s}_i \leftarrow \mathbb{Z}_p^{m_i}$. Additionally, the server samples random MAC keys $\alpha_i, \beta_i \leftarrow \mathbb{Z}_p$.
2. **Authenticated correlations generator:** The client and server invoke functionality $\mathcal{F}_{\text{ACG}}$ with the client input $\{\mathbf{r}_i\}_{i\in[\ell]}$, and with server input $\{\mathbf{s}_i, \mathbf{M}_i, \alpha_i, \beta_i\}_{i\in[\ell]}$. For each $i \in [\ell]$, the client obtains $\{\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i\}_{i\in[\ell]}$ along with $\{\langle\beta_i \cdot \mathbf{r}_i\rangle_1, \langle\alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_1\}$ whereas the server receives $\{\langle\beta_i \cdot \mathbf{r}_i\rangle_2, \langle\alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_2\}$. In Fig. 11 we describe the ideal functionality in more detail, and give a protocol for achieving it in Fig. 5.
3. For each $i \in [\ell]$, let $\mathsf{inp}_i := (\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i, \mathbf{r}_{i+1})$ denote the client's input to the $i$-th non-linear layer and $|\mathsf{inp}_i|$ denote its size in bits. The server chooses a set of random garbled circuit input labels $\{\mathsf{lab}^C_{i,k,0}, \mathsf{lab}^C_{i,k,1}\}_{k\in[|\mathsf{inp}_i|]}$.
4. **Conditional disclosure of secrets:** For each $i \in [\ell]$, the client and the server invoke functionality $\mathcal{F}_{\text{CDS}}$ on the client's input $\mathsf{inp}_i$ and the MAC shares received from $\mathcal{F}_{\text{ACG}}$. If the client honestly inputs the correct shares, the functionality outputs the garbled input labels $\{\mathsf{lab}^C_{i,k,\mathsf{inp}_i}\}_{k\in[|\mathsf{inp}_i|]}$ corresponding to $\mathsf{inp}_i$ to the client. In Fig. 12, we describe

the ideal functionality in more detail, and give a protocol for securely computing this functionality in Fig. 6.

5. For each $i \in [\ell]$, the server chooses random labels $\{\mathsf{lab}^S_{i,k,0}, \mathsf{lab}^S_{i,k,1}\}_{k\in[|\mathsf{inp}_i|]}$ for its input to the $i^{\text{th}}$ non-linear layer.[7]
6. **Offline garbling:** For each $i \in [\ell]$, the server garbles the circuit $C_i$ (described in Fig. 7) using $\{\mathsf{lab}^C_{i,k,0}, \mathsf{lab}^C_{i,k,1}, \mathsf{lab}^S_{i,k,0}, \mathsf{lab}^S_{i,k,1}\}_{k\in[|\mathsf{inp}_i|]}$ as the input labels to obtain the garbled circuit $\widetilde{C}_i$. It chooses a key $k_i \leftarrow \{0,1\}^\lambda$ and sends $H(k_i) \oplus \widetilde{C}_i$ to the client where $H$ is a random oracle.

## 5.2 Online phase

The online phase is divided into two stages: the *preamble* and the *layer evaluation*. (See Fig. 4 for a graphical overview.)

### 5.2.1 Preamble

The client sends $(\mathbf{x} - \mathbf{r}_1)$ to the server.

### 5.2.2 Layer evaluation

At the beginning of evaluating the $i$-th layer, the client holds $\mathbf{r}_i$ and the server holds $\mathbf{x}_i - \mathbf{r}_i$ where $\mathbf{x}_i$ is the vector obtained by evaluating the first $(i-1)$ layers of the neural network on input $\mathbf{x}$ (with $\mathbf{x}_1 = \mathbf{x}$). This invariant will be maintained for each layer. We now describe the protocol for evaluating the $i$-th layer, which consists of linear functions and activation functions:

**Linear layer.** The server computes $\mathbf{M}_i(\mathbf{x}_i - \mathbf{r}_i) + \mathbf{s}_i$ which ensures that the client and server hold an additive secret share of $\mathbf{M}_i\mathbf{x}_i$.

**Non-linear layer.** After the linear layer, the server holds $\mathbf{M}_i(\mathbf{x}_i - \mathbf{r}_i) + \mathbf{s}_i$ and the client holds $\mathbf{M}_i\mathbf{r}_i - \mathbf{s}_i$. The parties evaluate the non-linear garbled circuit layer as follows:

1. The server chooses a random masking vector $\mathbf{s}'_{i+1}$ and sends the labels from the set $\{\mathsf{lab}^S_{i,k,0}, \mathsf{lab}^S_{i,k,1}\}_{k\in[|\mathsf{inp}_i|]}$ corresponding to its input $\mathbf{M}_i(\mathbf{x}_i - \mathbf{r}_i) + \mathbf{s}_i$ and $\mathbf{s}'_{i+1}$ to the client along with the key $k_i$.
2. The client uses $k_i$ to unmask $H(k_i) \oplus \widetilde{C}_i$ to obtain $\widetilde{C}_i$. The client evaluates the garbled circuit $\widetilde{C}_i$ using its input labels obtained in the preprocessing phase and labels obtained from the server in the online phase. The client decodes the output labels and sends $\mathbf{x}_{i+1} - \mathbf{r}_{i+1} + \mathbf{s}'_{i+1}$ along with the hash of the output labels to the server.
3. The server checks if the hash computation is correct and recovers $\mathbf{x}_{i+1} - \mathbf{r}_{i+1}$.

**Output phase.** The server sends $\mathbf{s}'_{\ell+1}$ to the client and the client unmasks the output of the garbled circuit using this to learn the output of the inference $\mathbf{y}$.

**Theorem 5.1.** *Assuming the security of garbled circuits and the protocols for securely computing $\mathcal{F}_{ACG}$ (see Lemma B.1)*

[7] We slightly abuse the notation and use $|\mathsf{inp}_i|$ to also denote the size of the server input to the garbled circuit.

Protocol $\Pi_{\text{ACG}}$

1. Both parties engage in a two-party computation protocol with security against malicious clients and semi-honest servers to generate $(\mathsf{pk}, \mathsf{sk})$ for HE. The client learns $\mathsf{pk}$ and $\mathsf{sk}$ whereas the server only learns $\mathsf{pk}$.
2. The client sends $\{\mathsf{Enc}(\mathsf{pk}, \mathbf{r}_i)\}_{i\in[\ell]}$ to the server along with a zero-knowledge proof of well-formedness of the ciphertext. The server verifies this proof before continuing.
3. For every $i \in [\ell]$,
   (a) The server homomorphically computes $\mathsf{Enc}(\mathsf{pk}, \mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)$, $\mathsf{Enc}(\mathsf{pk}, \alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i))$, and $\mathsf{Enc}(\mathsf{pk}, \beta_i \cdot \mathbf{r}_i)$.
   (b) The server randomly samples $\langle \alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_2$ and $\langle \beta_i \cdot \mathbf{r}_i\rangle_2$, homomorphically creates additive shares of the MAC values, and sends $(\mathsf{Enc}(\mathsf{pk}, \mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i), \mathsf{Enc}(\mathsf{pk}, \langle \alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_1), \mathsf{Enc}(\mathsf{pk}, \langle \beta_i \cdot \mathbf{r}_i\rangle_1))$ to the client.
   (c) The client decrypts the above ciphertexts and obtains $(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)$, $\langle \alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_1$, and $\langle \beta_i \cdot \mathbf{r}_i\rangle_1$. The server holds $\langle \alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_2$ and $\langle \beta_i \cdot \mathbf{r}_i\rangle_2$.

**Figure 5:** Our construction of an authenticated correlations generator (ACG).

Protocol $\Pi_{\text{CDS}}$

Denote the bit decomposition of $\mathsf{inp}_i = (\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i, \mathbf{r}_{i+1})$ as $\{b^i_k\}_{k\in[|\mathsf{inp}_i|]}$.

1. The client and server input securely compute the following function $f_{\text{CDS}}$ using a secure two-party computation protocol against malicious clients and semi-honest servers:
   (a) The client's input consists of $(\{b^i_k\}_{k\in[|\mathsf{inp}_i|]}, \langle \alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_1, \langle \beta_{i+1} \cdot \mathbf{r}_{i+1}\rangle_1)$ and the server's input consists of $(\alpha_i, \beta_{i+1}, \langle \alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_2, \langle \beta_{i+1} \cdot \mathbf{r}_{i+1}\rangle_2, \{\mathsf{lab}^C_{i,k,0}, \mathsf{lab}^C_{i,k,1}\}_{k\in[|\mathsf{inp}_i|]})$ for some $i \in [\ell - 1]$.
   (b) Denote $g_k = \mathsf{lab}^C_{i,k,0} - (\mathsf{lab}^C_{i,k,0} - \mathsf{lab}^C_{i,k,1}) \cdot b^i_k$. Additively secret share $g_k$ into $(\langle g_k\rangle_1, \langle g_k\rangle_2)$.
   (c) Reconstruct $\overline{\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i}$ and $\overline{\mathbf{r}_{i+1}}$ from $\{b^i_k\}_{k\in[|\mathsf{inp}_i|]}$.
   (d) Sample random vectors $\mathbf{c}_1 \leftarrow \mathbb{Z}_p^{|\mathbf{r}_{i+1}|}, \mathbf{c}_2 \leftarrow \mathbb{Z}_p^{|\mathbf{r}_{i+1}|}$
   (e) Denote $\rho = \alpha_i(\mathbf{c}_1^T \cdot \overline{\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i}) - \mathbf{c}_1^T \cdot (\langle \alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_1 + \langle \alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_2)$ and $\sigma = \beta_{i+1}(\mathbf{c}_2^T \cdot \overline{\mathbf{r}_{i+1}}) - \mathbf{c}_2^T \cdot (\langle \beta_{i+1} \cdot \mathbf{r}_{i+1}\rangle_1 + \langle \beta_{i+1} \cdot \mathbf{r}_{i+1}\rangle_2)$.
   (f) Output $\rho$, $\sigma$, $\{\langle g_k\rangle_2\}_{k\in[|\mathsf{inp}_i|]}$ to the server and $\{\langle g_k\rangle_1\}_{k\in[|\mathsf{inp}_i|]}$ to the client.
2. If either of them are non-zero, the server aborts the protocol. Else, it sends $\{\langle g_k\rangle_2\}_{k\in[|\mathsf{inp}_i|]}$ to the client. The client reconstructs $\{g_k\}_{k\in[|\mathsf{inp}_i|]}$ and outputs it.

**Figure 6:** Our protocol for conditional disclosure of secrets.

**Server's input:** $\mathbf{M}_i(\mathbf{x}_i - \mathbf{r}_i) + \mathbf{s}_i, \mathbf{s}'_{i+1}$.
**Client's input:** $\mathbf{r}_{i+1}$, $\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i$

1. Compute $\mathbf{M}_i(\mathbf{x}_i) = \mathbf{M}_i(\mathbf{x}_i - \mathbf{r}_i) + \mathbf{s}_i + \mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)$.
2. Compute $ReLU(\mathbf{M}_i(\mathbf{x}_i))$ to obtain $\mathbf{x}_{i+1}$.
3. Output $\mathbf{x}_{i+1} - \mathbf{r}_{i+1} + \mathbf{s}'_{i+1}$.

**Figure 7:** Description of circuit $C_i$.

*and $\mathcal{F}_{CDS}$ (see Lemma B.3), the protocol described above is a private inference protocol against malicious clients and semi-honest servers (see Definition 3.2) in the random oracle model.*

Correctness follows from inspection; we prove security in Appendix B.4.

**Remark 5.2** (ACG for subsequent linear layers)**.** Many network architectures contain consecutive linear layers between two ReLU activations. For simplicity of exposition, we have composed these linear layers in our protocol description. However, doing so in the actual implementation would be inefficient since our homomorphic algorithms are highly specialized for specific layer types. As a result, in practice $\Pi_{\mathrm{ACG}}$ must be modified so that on consecutive linear layers the client only receives MAC shares of the layer output on the *final* linear layer in the sequence. In the online phase, the client additionally sends MAC shares of their input on intermediate linear layers since they are not checked inside of the CDS.

**Remark 5.3** (Checking client CDS inputs)**.** $f_{\mathrm{CDS}}$ must check that the client's bit decomposition is correct. That is, it must check that the claimed bit decomposition (a) consists of boolean values, (b) corresponds to an integer with value less than $p$. For efficiency reasons, we perform only the first check in the preprocessing phase, and move the second check to our garbled circuits in the online phase.

**Remark 5.4** (Fixed-point arithmetic in finite fields)**.** Neural networks work over the real numbers, but our cryptographic protocols work over finite prime fields. To emulate real arithmetic, we rely on fixed-point arithmetic. However, to maintain precision, one needs to occasionally *truncate* intermediate values to ensure that the result does not wrap around the field.

In DELPHI, both parties perform truncation directly on their local secret shares following the technique of [MZ17] which correctly truncates the shared value with a small, additional error. While this error does not greatly impact accuracy, it is unacceptable in the client-malicious setting as it would invalidate the MAC of the share. As a result, MUSE must perform truncation directly on the shared value using a secure MPC. We perform this truncation for free within our garbled circuits by always returning zero labels for the upper bits of the ReLU output.

## 5.3 An efficient protocol for computing $f_{\mathrm{CDS}}$

To securely compute the function $f_{\mathrm{CDS}}$, MUSE adapts the state-of-the-art arithmetic MPC framework Overdrive [KPR18] (which achieves malicious security) to the simpler client-malicious 2PC setting. Doing so results in great efficiency improvements, as we now explain.[8]

[8] While the protocol of [Che+20] offers better performance than Overdrive, at the time of writing the source code for it was unavailable, and so we could not build upon it. Our optimizations in this section apply also to the [Che+20] protocol, so it is plausible that in the future MUSE could instead rely on it.

The heaviest cryptographic costs when using Overdrive for $f_{\mathrm{CDS}}$ are due to (a) MAC key generation, (b) triples generation, and (c) authentication of secret client and server inputs. We now describe how we optimize all of these procedures in the client-malicious setting.

**MAC key generation.** In Overdrive, a MAC key must be secret-shared among the parties, since any party may be malicious and could use knowledge of the key to cheat. In the client-malicious setting, the server will never cheat so they can simply generate and hold the MAC key themselves.

**Triples generation.** In order to generate multiplication triples in Overdrive, all parties must generate ciphertexts of their shares, prove knowledge of these ciphertexts in zero-knowledge, homomorphically compute a triple from the ciphertexts, and run a distributed decryption algorithm so all parties receive a share of the result. Note that the distributed decryption allows a malicious adversary to inject an authenticated additive shift, so parties must "sacrifice" a triple in order to ensure correctness [Dam+12], which harms performance.

In the client-malicious setting, we can avoid distributed decryption, triple sacrifice, and a number of zero-knowledge proofs by taking advantage of the fact that the server knows the MAC key. In particular, we devise the following efficient protocol: the client sends the encryption of their shares directly to the server (along with a zero-knowledge proof of plaintext knowledge). The server homomorphically computes the shares of the triple, and returns it to the client. Since the server performs the computation, correctness is guaranteed and no distributed decryption or triple sacrifice is necessary. We provide benchmarks of our optimized generation in Section 6.5. See Fig. 17 for a full description of $\Pi_{\mathrm{Triple}}$.

**Input authentication.** Overdrive [KPR18] optimizes the input sharing method of [Dam+12], by assuming that the encryption scheme they employ achieves *linear-targeted malleability* (LTM) [Bit+13]. The LTM assumption for an encryption scheme informally states that only affine transformations can be computed on ciphertexts. This assumption is non-falsifiable, and, when applied to the encryption schemes used in Overdrive, has received insufficient scrutiny.

In our protocol, we avoid relying on this strong assumption by observing that the majority of secret inputs originate with the server, and because the server holds the MAC keys, it can easily authenticate its inputs *without cryptography*. In more detail, the protocol proceeds as follows.

- The server shares their inputs by producing a random authenticated share of their input using the MAC key and sends it to the client.
- The client shares their input by following the same methodology as [Dam+12]. Note that generating random authenticated shares can be implemented using our triple generation procedure from above, thus inheriting the same speedups.

We benchmark these techniques in Section 6.5. See Fig. 14 for a full description of $\Pi_{\mathrm{InputAuth}}$.

| | | | system | threads | MNIST time (s) | MNIST comm. (GB) | CIFAR-10 time (s) | CIFAR-10 comm. (GB) |
|---|---|---|---|---|---|---|---|---|
| **Preprocessing** | Linear | CG | DELPHI | 1 | 3.93 | 0.03 | 36.21 | 0.05 |
| | | ACG | MUSE | 1 | 4.74 | 0.04 | 40.78 | 0.07 |
| | Non-linear | Garbling | DELPHI | 2 | 1.81 | 0.18 | 19.31 | 2.95 |
| | | Garbling | MUSE | 2 | 4.34 | 0.51 | 62.19 | 7.45 |
| | | OT | DELPHI | 8 | 1.67 | 0.02 | 5.2 | 0.35 |
| | | CDS Triple Gen. | MUSE | 6 | 7.32 | 2.66 | 112.51 | 44.36 |
| | | CDS Input Auth. | MUSE | 2 | 4.10 | 0.43 | 59.579 | 7.00 |
| | | CDS Evaluation | MUSE | 2 | 2.17 | 0.53 | 31.34 | 8.79 |
| **Online** | | Online | DELPHI | 8 | 0.48 | 0.01 | 3.74 | 0.16 |
| | | Online | MUSE | 8 | 0.80 | 0.01 | 8.37 | 0.23 |

**Table 3:** Latency and communication cost of the individual components of MUSE and DELPHI. See Section 6.2 for more information on the network architectures and number of threads used.

# 6 Evaluation

We divide the evaluation into three sections which answer the following questions:

- Section 6.3: What are the latency and communication costs of MUSE's individual components when performing inference and how do they compare to the semi-honest DELPHI?
- Section 6.4: How does MUSE compare to other inference protocols secure against malicious clients?
- Section 6.5: How do our client-malicious Overdrive subprotocols compare to standard Overdrive?

## 6.1 System implementation

We implemented MUSE in Rust and C++. We use the SEAL homomorphic encryption library [Sea] to implement HE, the `fancy-garbling` library[9] to implement garbled circuits, and MP-SPDZ[10] [Kel20] to implement zero-knowledge proofs. MUSE achieves 128 bits of computational security, and 40 bits of statistical security. Our implementation is available online at https://github.com/mc2-project/muse.

## 6.2 Evaluation setup

All experiments were carried out on AWS `c5.9xlarge` instances possessing an Intel Xeon 8000 series CPU at 3.6 GHz. The client and server instances were located in the `us-west-1` (Northern California) and `us-west-2` (Oregon) regions respectively with 21 ms round-trip latency. The client and server executions used 8 threads each. We evaluate MUSE on the following datasets and network architectures:

1. *MNIST* is a standardized dataset consisting of $(28 \times 28)$ greyscale images of the digits 0–9. The training set contains $60,000$ images, while the test set has $10,000$ images. Our experiments use the 2-layer CNN architecture specified in MiniONN [Liu+17a] with average pooling in place of max pooling.
2. *CIFAR-10* is a standardized dataset consisting of $(32 \times 32)$ RGB images separated into 10 classes. The training set contains $50,000$ images, while the test set has $10,000$ images. Our experiments use the 7-layer CNN architecture specified in MiniONN [Liu+17a].

In our experiments, MUSE runs all of its various preprocessing components in parallel using a work-stealing threadpool with 8 threads. For simplicity, in Table 3 we provide microbenchmarks for each component using a static thread allocation that closely reflects the allocation used during actual execution.

**Baselines.** Since there are no specialized protocols for client-malicious secure inference, we chose to use generic MPC frameworks as our baselines to compare MUSE against: maliciously-secure Overdrive [KPR18] *and* Overdrive with our client-malicious optimizations. We used MP-SDPZ's implementation of the maliciously-secure Overdrive protocol, and estimated the total runtime and communication costs of client-malicious Overdrive using microbenchmarks from MUSE's triple generation, MUSE's input authentication, and MP-SPDZ.

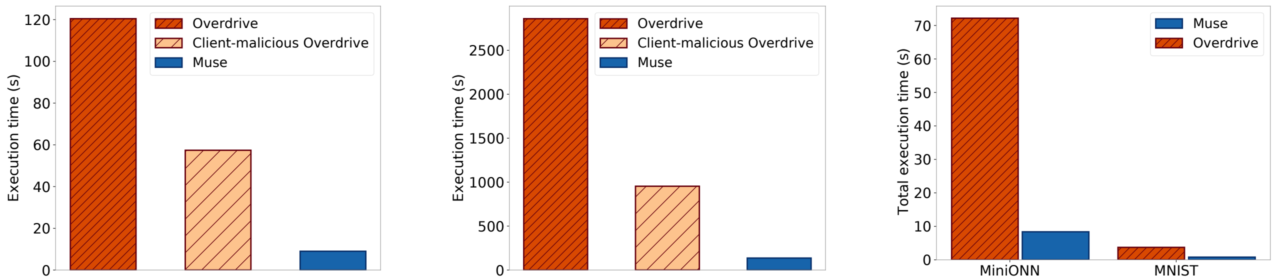
Additionally, we use microbenchmarks in Table 3 to demonstrate the concrete costs of strengthening each individual component of MUSE from semi-honest to client-malicious security. As a semi-honest baseline, we chose to compare against DELPHI and not against the more recent work [Rat+20] which offers better performance because (1) this work uses different techniques for both linear and non-linear layers, which would make isolating the cost of upgrading security difficult, and (2) it is unclear how to upgrade their semi-honest protocols to achieve client-malicious security in an efficient way.

## 6.3 Microbenchmarks

In Table 3 we compare microbenchmarks for MUSE and DELPHI on the MNIST and CIFAR-10 networks using a similar number of threads to demonstrate the concrete costs of strengthening each component of DELPHI to client-malicious security.

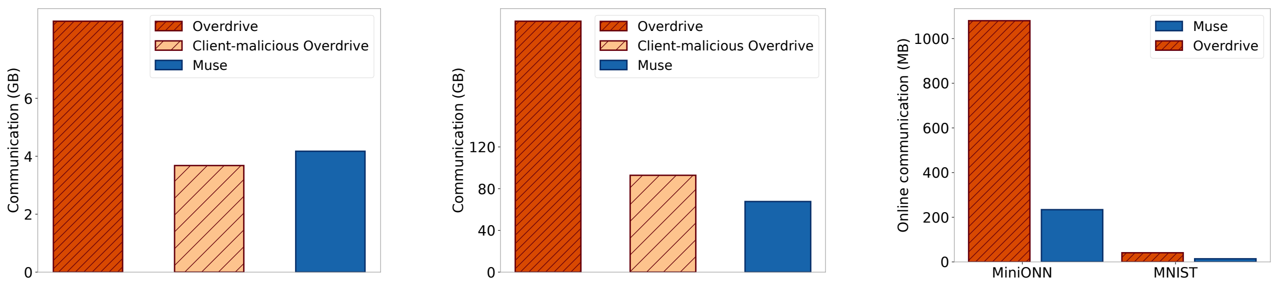
[9] https://github.com/GaloisInc/fancy-garbling/

[10] https://github.com/data61/MP-SPDZ




**(a)** MNIST Preprocessing Time **(b)** CIFAR-10 Preprocessing Time **(c)** Online Time

**Figure 8:** Comparison of execution times between MUSE, Overdrive, and client-malicious Overdrive




**(a)** MNIST Preprocessing Communication **(b)** CIFAR-10 Preprocessing Communication **(c)** Online Communication

**Figure 9:** Comparison of communication cost between MUSE, Overdrive, and client-malicious Overdrive

#### 6.3.1 Preprocessing phase

The primary difference between MUSE and DELPHI occurs in the preprocessing phase.

**Linear layers.** As discussed in Section 5.1, the primary difference in how DELPHI and MUSE preprocess linear layers lies in the fact that the former uses a plain correlations generator (CG), while MUSE use an *authenticated correlations generator* (ACG). Because the ACG requires additional homomorphic operations and zero-knowledge proofs, we should expect MUSE to be slightly slower than DELPHI and require slightly more communication. In Table 3 we observe that this is precisely the case.

**Non-linear layers.** To preprocess the non-linear layers in DELPHI, the server garbles a circuit corresponding to ReLU and sends to the client. The two parties than engage in an oblivious transfer whereby the client learns the garbled labels corresponding to their input.

In MUSE, a number of modifications to this procedure must be made. First, MUSE cannot use simple oblivious transfer and must opt for the much more expensive CDS protocol to ensure the client receives the correct garbled labels. Second, as detailed in Remark 5.3, MUSE pushes some checks from the CDS to the online garbled circuits which roughly doubles the number of AND gates in the circuit.

As a result, we should expect a $2\times$–$3\times$ increase in latency and communication for the garbling in MUSE when compared to DELPHI, and a much higher cost for the CDS compared to oblivious transfer. Table 3 validates these hypotheses.

#### 6.3.2 Online phase

MUSE retains the same structure for the online phase, but has a few small additions. For subsequent linear layers, MUSE requires the client to send additional MAC shares (see Remark 5.2). For non-linear layers, MUSE requires an extra hash key to be sent, and the circuit being evaluated is roughly twice the size as the one in DELPHI (see Remark 5.3).

As a result, we should expect the garbled circuit evaluation time to be the only significant difference in online runtime between MUSE and DELPHI, and for MUSE to have slightly higher communication. In Table 3, we see that the difference in online runtime is $1.7\times$–$2.2\times$ and the communication difference is approximately $1.4\times$.

In conclusion, MUSE's overhead when compared to DELPHI is minimal in every component *except* the CDS. MUSE's online phase outperforms *all prior two-party semi-honest* works listed in Table 1 besides DELPHI, CrypTFlow2 [Rat+20], and XONN [Ria+19].

### 6.4 Full system comparisons

Fig. 8 and Fig. 9 demonstrate how MUSE performs against malicious Overdrive and client-malicious Overdrive. Note that our client-malicious optimizations for Overdrive don't affect the online phase which is why we exclude client-malicious Overdrive in the online figures.

In summary, MUSE's preprocessing is $13.4\times$–$21\times$ faster and reduces communication by $2\times$–$3.6\times$ compared to standard Overdrive. For client-malicious Overdrive, MUSE's preprocessing phase is 6.4×–7× faster. For the smaller MNIST network, the communication cost of MUSE is slightly higher than that of client-malicious Overdrive (due to a constant overhead from the garbled circuits), but our techniques scale better and achieve a 1.4× reduction for the larger CIFAR-10 network.

For the online phase, we observe a 7.8×–8.6× latency improvement and 3.4×–4.6× communication improvement when comparing MUSE to Overdrive.

## 6.5 Improvements to Overdrive

In this section we demonstrate the effectiveness of our optimizations to Overdrive in the client-malicious setting. In particular, we show that in client-malicious Overdrive *without the LTM assumption*:

- Triple generation is significantly more efficient.
- Client input authentication is slightly more efficient.
- Server input authentication is significantly more efficient.

These improvements are of independent interest and can easily be extended to support more parties.

**Triple generation.** In Fig. 10 we benchmark the generation of triples on a variable number of threads. In summary, client-malicious Overdrive achieves a 8×–12.5× latency improvement and 1.7× communication reduction (the latter is not shown in the graph) over standard Overdrive.

**Input authentication.** In Table 4 we show benchmarks for input authentication for the client and server. Our protocol for client inputs achieves a 1.6× speed improvement *without the LTM assumption*, but increases communication by 3.6×. We observe a 37.8× improvement in latency and 4.5× improvement in communication for server inputs.

# 7 Related work

## 7.1 Model extraction attacks

A number of recent works extract convolutional neural networks (CNNs) [Tra+16; Mil+19; Jag+20; RK20; CJM20] given oracle access to a neural network. Unlike our attack, these works do not exploit properties of any secure inference protocol (and indeed do not rely on the existence of these), but require a much larger number of queries. We compare against the state-of-the-art attack [CJM20] in Section 2.3.

**Mitigations.** While MUSE protects against our attack, it does *not* defend against attacks that leverage only the prediction result. Mitigations fall into two camps: those that inspect prediction queries [Kes+18; Juu+19], and those that try to instead modify the network to make it resilient to extraction [Tra+16; Lee+19]. While the latter kind of defense can be applied independently of secure inference, adapting the first kind of defense to work with secure inference protocols is tricky, because it requires inspecting the client's queries, which can violate privacy guarantees.

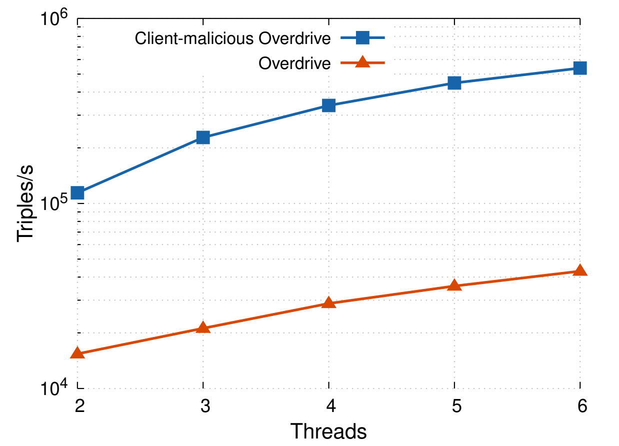



**Figure 10:** Triple Generation amortized on a batch of 10,000,000 triples over a 44 bit prime field with 40 bit statistical security.

| Threat Model | Client Inputs time (s) | Client Inputs comm. (MB) | Server Inputs time (s) | Server Inputs comm. (MB) |
|---|---|---|---|---|
| Malicious | 12.11 | 90 | 12.11 | 90 |
| Client-mal. | 7.406 | 320 | 0.32 | 20 |

**Table 4:** Input Authentication on 1,000,000 inputs over a 44 bit prime field with 40 bit statistical security using a single thread. ■ Relies on the LTM Assumption.

## 7.2 Secure inference protocols

A number of recent works have attempted to design specialized protocols for performing secure inference. These protocols achieve efficiency by combining secure computation techniques such as homomorphic encryption [Gen09b], Yao's garbled circuits [Yao86], and homomorphic secret sharing [Boy+17] with various modifications such as approximating ReLU activations with low-degree polynomials or binarizing (quantizing to one bit of accuracy) network weights. See Table 1 for a high-level overview of these protocols.

While these works have improved on latency and communication costs by orders of magnitude, *all* of the two-party protocols in Table 1 assume a semi-honest adversary. Currently-existing maliciously-secure inference protocols generally fall into the following categories: 3PC-based protocols, generic MPC frameworks, TEE-based protocols, and GC-based protocols. In the remainder of the section we discuss each of these categories:

**3PC-based protocols.** Recent works have explored how the addition of a third party can greatly improve efficiency for secure machine learning applications [MZ17; Ria+18; MR18; WGC19; Wag+21; Kum+20]. Many of these protocols also allow for easy extensions to handle malicious adversaries [MR18; WGC19; Wag+21]. These extensions are made possible by the fact that these works assume only *one* of the parties is corrupted. In other words, these works consider *honest majority* malicious security. On the other hand, MUSE addresses the fundamentally more difficult problem of a dishonest majority. While having three non-colluding parties is convenient from a protocol design perspective, in practice, it is difficult to setup such a third party running in a separate trust domain out of the control of the server or client.

Chameleon [Ria+18] proposed a slightly weaker threat model where a semi-honest third server assists in the preprocessing phase but is not needed for the online phase. If such a setup is feasible, MUSE could naturally take advantage of this threat model by having the semi-honest third server assist in triple generation for the CDS protocol. This augmentation improves latency and bandwidth of MUSE's preprocessing phase by roughly 3×.

**TEE-based protocols.** Generally speaking, TEE-based protocols [TB19; Top+18; Han+18; App19] provide better efficiency than protocols relying on purely cryptographic techniques. However, this improved efficiency comes at the cost of a weaker threat model that requires trust in hardware vendors and the implementation of the enclave. Indeed, the past few years have seen a number of powerful side-channel attacks [Bra+17; HCP17; Göt+17; MIE17; Sch+17; Wan+17; Van+18] against popular enclaves like Intel SGX and ARM TrustZone.

**Generic frameworks.** Maliciously-secure MPC frameworks exist for computing arithmetic circuits [Dam+12; KPR18; Che+20], binary circuits [Kat+18], and mixed circuits [RW19; Esc+20; MR18]. Before MUSE, these were the only existing cryptographic mechanisms for two-party client-malicious secure inference. While [Che+20] is the most efficient of these for inference, an implementation was not available at the time of writing so we compared against [KPR18] in Section 6.4. From the results of Section 6.4 and the experiments provided in [Che+20], we can roughly estimate that the preprocessing communication of [Che+20] is similar to MUSE, but MUSE is superior on all other accounts.

**GC-based protocols.** DeepSecure [RRK18], the protocol of Ball et al. [Bal+19], and XONN [Ria+19], all use circuit garbling schemes to implement constant-round secure inference protocols. While DeepSecure supports general neural networks, the protocol of [Bal+19] operates on discretized neural networks, which have integer weights, while XONN is optimized for binarized neural networks [CBD15], which have boolean weights. These quantized networks allow for improved performance by avoiding computing expensive fixed-point multiplication in favor of integer multiplication or binary XNOR gates.

While neural network inference is commonly performed on quantized networks [Kri18], in practice quantization is never done below 8-bits since inference accuracy begins to suffer [Ban+18]. To combat this accuracy drop, XONN increases the number of neurons in its linear layers, gaining increased accuracy at the cost of a slower evaluation time. While this technique appears to work well for the datasets XONN evaluates, additional techniques are needed to scale to more difficult datasets like Imagenet as the current best-known quantization techniques for Imagenet requires 2 bit weights and 4 bit activations [Don+19]. Consequently, it is our opinion that it is still important to focus on supporting secure inference for general neural networks even though BNNs appear promising.

Any GC-based protocol can be upgraded to malicious security through a combination of cut-and-choose techniques [Zhu+16] and malicious OT-extension [KOS15], and client-malicious security for the evaluator by using malicious OT-extension. Thus, it would follow that all of these GC-based inference protocols can be transformed into malicious and fixed-subset malicious protocols. Note that DeepSecure would provide *server-malicious* security since the client garbles the circuit. XONN uses a specialized protocol to evaluate the first layer of the network since the client's input is an un-quantized integer. In order for these malicious/client-malicious transformations to work, XONN would need to evaluate this layer within the more-expensive garbled circuit, instead of their optimized protocol. Furthermore, an implementation of XONN was not available at the time of writing, so we could not benchmark a client-malicious version of their protocol on our experimental setup. Finally, MUSE's online speed is already superior to the *semi-honest* versions of DeepSecure and the protocol of [Bal+19].

## 8 Conclusion

In this paper, we introduce a novel model-extraction attack against many semi-honest secure inference protocols which outperforms existing attacks by orders of magnitude. In response, we design and implement MUSE, an efficient two-party secure inference protocol resilient to *malicious clients*. MUSE achieves online performance close to existing semi-honest protocols, and greatly outperforms alternate solutions for client-malicious secure inference. As part of MUSE's design, we introduce a novel cryptographic protocol for *conditional disclosure of secrets* and improved procedures for generic MPC in the client-malicious setting. We hope that MUSE is a first step towards achieving practical two-party secure inference in a strong threat model.

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## A Security properties of our building blocks

Let $\lambda$ denote the security parameter. For a finite set $S$, we denote $x \leftarrow S$ as the process of sampling $x$ uniformly from the set $S$. We say that a function $\mu : \mathbb{N} \rightarrow [0,1]$ is negligible if for every polynomial $p(\cdot)$, there exists $\lambda_0$ such that for all $\lambda > \lambda_0$, $\mu(\lambda) < 1/p(\lambda)$. We denote an unspecified negligible function with negl and an unspecified polynomial with poly. We say that two distribution ensembles $X = \{X_\lambda\}_{\lambda\in\mathbb{N}}$ and $Y = \{Y_\lambda\}_{\lambda\in\mathbb{N}}$ are computationally indistinguishable (denoted by $X \approx_c Y$) if for every polynomial time distinguisher $D$, there exists a negligible function $\mu(\cdot)$ such that for every $\lambda$, we have $|\Pr[D(X_\lambda) = 1] - \Pr[D(Y_\lambda) = 1]| \leq \mu(\lambda)$.

### A.1 Garbling scheme

We would like a garbling scheme GS to satisfy two properties: *correctness* and *security*.

- *Correctness:* The correctness requirement says that the string $y$ output by the Eval algorithm is same as $C(x)$ with probability 1.
- *Security:* The security requirement says that given $\widetilde{C}$ and $\{\mathsf{lab}_{i,x_i}\}$, the evaluator does not learn anything about $C$ or $x$ except the size of the circuit $C$ (denoted by $1^{|C|}$) and the output $C(x)$. Specifically, we require the existence of a simulator $\mathsf{Sim}_{\mathsf{GS}}$ such that for any adversary $\mathcal{A}$,

$$\Pr[\mathsf{Expt}_{\mathsf{GS}}(\mathcal{A}, \mathsf{GS}) = 1] \leq 1/2 + \mathsf{negl}(\lambda)$$

where $\mathsf{Expt}_{\mathsf{GS}}$ is defined below.

$$(C, x, \{\mathsf{lab}_{i,x_i}\}_{i\in[n]}) \leftarrow \mathcal{A}(1^\lambda)$$
$$\mathsf{lab}_{i,1-x_i} \leftarrow \{0,1\}^\lambda \text{ for } i \in [n]$$
$$b \leftarrow \{0,1\}$$
$$\text{If } b = 0 : \widetilde{C} \leftarrow \mathsf{GS.Garble}(1^\lambda, C, \{\mathsf{lab}_{i,0}, \mathsf{lab}_{i,1}\}_{i\in[n]})$$
$$\text{Else, } \widetilde{C} \leftarrow \mathsf{Sim}_{\mathsf{GS}}(1^\lambda, 1^{|C|}, C(x), \{\mathsf{lab}_{i,x_i}\}_{i\in[n]})$$
$$b' \leftarrow \mathcal{A}(\widetilde{C})$$
$$\text{Output 1 iff } b = b'$$

The above requirement where $\mathcal{A}$ chooses the labels corresponding to its input is stronger than the standard definition where in both the labels for each input wire are chosen uniformly. However, we note that the existing constructions of garbled circuits (such as the one described in [LP09]) satisfies this stronger definition.

### A.2 Linearly homomorphic public key encryption

We require a linear homomorphic public key encryption to satisfy the following properties:

- *Correctness.* With probability 1, the output of the decryption algorithm Dec on input *sk* and a ciphertext $c = \mathsf{Enc}(\mathsf{pk}, m)$ is equal to $m$. Here, the probability is over the random coins of KeyGen and Enc algorithms.
- *Homomorphic Evaluation Correctness.* With probability 1, the output of the decryption algorithm Dec on input *sk* and a ciphertext $c' = \mathsf{Eval}(\mathsf{pk}, \mathsf{Enc}(\mathsf{pk}, m_1), \mathsf{Enc}(\mathsf{pk}, m_2), L)$ is equal to $L(m_1, m_2)$. Here, the probability is over the random coins of KeyGen and Enc algorithms.
- *Semantic Security.* For any two messages $m, m' \in \mathbb{Z}_p$, we require

$$\{\mathsf{pk}, \mathsf{Enc}(\mathsf{pk}, m)\} \approx_c \{\mathsf{pk}, \mathsf{Enc}(\mathsf{pk}, m')\}$$

where the two distributions are over the random choice of pk and the random coins of the encryption algorithm.
- *Function Privacy.* There exists a simulator $\mathsf{Sim}_{\mathsf{FP}}$ such that for every polynomial time adversary $\mathcal{A}$ and every linear function $L$, we have:

$$\Pr[\mathsf{Expt}_{\mathsf{FP}}(\mathcal{A}, \mathsf{HE}, L) = 1] \leq 1/2 + \mathsf{negl}(\lambda)$$

where $\mathsf{Expt}_{\mathsf{FP}}$ is defined below.

1. $(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(1^\lambda; r)$.
2. $(m_1, r_1), (m_2, r_2) \leftarrow \mathcal{A}(\mathsf{pk}, \mathsf{sk})$.
3. $c_i \leftarrow \mathsf{Enc}(\mathsf{pk}, m_i; r_i)$ for $i \in \{1,2\}$.
4. $b \leftarrow \{0,1\}$.
5. If $b = 0 : c' \leftarrow \mathsf{Eval}(\mathsf{pk}, c_1, c_2, L)$.
6. Else, $c' \leftarrow \mathsf{Sim}_{\mathsf{FP}}(\mathsf{pk}, (m_1, r_1), (m_2, r_2))$.
7. $b' \leftarrow \mathcal{A}(c')$.
8. Output 1 iff $b = b'$.

### A.3 Message authentication codes

We want the MAC scheme to satisfy the following two properties:

- *Correctness.* For any message $m$, we want the probability that $\mathsf{Verify}(\alpha, \mathsf{st}, m, \sigma) = 1$ is 1 for $\alpha \leftarrow \mathsf{KeyGen}(1^\lambda)$, $(\sigma, \mathsf{st}) \leftarrow \mathsf{Tag}(\alpha, m)$.
- *One-time Security.* For any message $m$ and for any adversary $\mathcal{A}$, we have:

$$\Pr[\mathsf{Expt}_{\mathsf{MAC}}(\mathcal{A}, m, \mathsf{MAC}) = 1] \leq \mathsf{negl}(\lambda)$$

where $\mathsf{Expt}_{\mathsf{MAC}}$ is defined below.

$$\alpha \leftarrow \mathsf{KeyGen}(1^\lambda)$$
$$(\sigma, \mathsf{st}) \leftarrow \mathsf{Tag}(\alpha, m)$$
$$(m', \sigma') \leftarrow \mathcal{A}(m, \sigma)$$
$$\text{Output 1 iff } (m', \sigma') \neq (m, \sigma) \wedge \mathsf{Verify}(\alpha, \mathsf{st}, m', \sigma') = 1.$$

**Lemma A.1.** *The scheme described in Section 4 is a one-time secure MAC.*

*Proof.* The correctness is easy to observe. Assume for the sake of contradiction that adversary $\mathcal{A}$ produces a $(m', \sigma')$ such that $(m', \sigma') \neq (m, \sigma) \wedge \mathsf{Verify}(\alpha, \mathsf{st}, m', \sigma') = 1$. This means that $\sigma' + \mathsf{st} = \alpha \cdot m'$. By definition of Tag, it follows that $\sigma + \mathsf{st} = \alpha \cdot m$. Subtracting these two equations, we get

$\alpha(m - m') = \sigma - \sigma'$. If $m = m'$, then this equation is satisfied only if $\sigma = \sigma'$ which contradicts $(m', \sigma') \neq (m, \sigma)$. Else, we get $\alpha = (\sigma_i - \sigma'_i)/(m_i - m'_i)$ for some $i \in [n]$ such that $m_i \neq m'_i$ and the probability that $\alpha$ satisfies this equation is $1/p \leq 2^{-\lambda}$. □

## A.4 Zero-knowledge proofs

We want our zero-knowledge proof system to satisfy the following properties:

- *Completeness.* For any $x \in L$, the prover always convinces the verifier.
- *Soundness.* For any $x \notin L$, the verifier rejects with overwhelming probability.
- *Proof of Knowledge.* For every polynomial time malicious prover $P^*$, there exists an extractor Ext such that for any statement $x$, if the verifier accepts the proof on interaction with the prover $P^*$ with probability at least $1/\mathsf{poly}(\lambda)$ then there exists a negligible function $\mu$ such that $\Pr[\mathsf{Ext}(x) = w \wedge (x, w) \in L] \geq 1/\mathsf{poly}(\lambda) - \mu(\lambda)$.
- *Zero-Knowledge:* For any $x \in L$ and any polynomial time malicious verifier $V^*$, there exists a simulator $\mathsf{Sim}_{\mathsf{ZK}}$ such that the transcript of the interaction between the honest $P$ and $V^*$ is computationally indistinguishable to the transcript generated by $\mathsf{Sim}_{\mathsf{ZK}}$.

# B Security proofs

Recall the formal definition of a protocol that is secure against malicious clients and semi-honest server provided in Definition 3.1. We first prove security of the various components of MUSE before proving security of the entire protocol.

## B.1 ACG

**Lemma B.1.** *Assuming a secure two-party computation protocol against malicious clients and semi-honest servers for* KeyGen*, a leveled fully-homomorphic encryption scheme (see Section 4) and a zero-knowledge proof system for well-formedness of ciphertexts (see Section 4), the protocol described in Fig. 5 securely computes $\mathcal{F}_{ACG}$ against malicious clients and semi-honest servers.*

*Proof.* The correctness follows from observation and we now prove semi-honest server security and then show security against malicious clients.

**Semi-Honest server security.** The simulator $\mathsf{Sim}_S$ samples $(pk, sk) \leftarrow \mathsf{HE.KeyGen}(1^\lambda)$. It then runs the simulator for the semi-honest server in the two-party computation protocol that generates the public-key, secret-key pair for the homomorphic encryption scheme. When this simulator access the ideal functionality, $\mathsf{Sim}_S$ provides $pk$ as the output. For each $i \in [\ell]$, $\mathsf{Sim}_S$ sends $\{\mathsf{HE.Enc}(pk, \mathbf{0})\}_{i \in [\ell]}$ and with a simulated proof of well-formedness of ciphertexts. We now show the indistinguishability of the real view from the simulated view using a hybrid argument.

- $\mathsf{Hyb}_1$ : This corresponds to the real execution of the protocol.
- $\mathsf{Hyb}_2$ : In this hybrid, we use the simulator for the semi-honest server in the two-party computation protocol for generating the public key, secret key pair. When this simulator queries its ideal functionality, we sample a $(pk, sk) \leftarrow \mathsf{HE.KeyGen}(1^\lambda)$ and give $pk$ to it. This hybrid is computationally indistinguishable to $\mathsf{Hyb}_1$ from the security of this protocol.
- $\mathsf{Hyb}_3$ : In this hybrid, we start using the simulator for the zero-knowledge proofs of plaintext knowledge. This hybrid is indistinguishable to the previous one from the zero-knowledge property of these proof systems.
- $\mathsf{Hyb}_4$ : In this hybrid, we switch from sending $\mathsf{HE.Enc}(pk, \mathbf{r}_i)$ for each $i \in [\ell]$ to sending $\mathsf{HE.Enc}(pk, \mathbf{0})$. This hybrid is indistinguishable from the previous one from the semantic security of the homomorphic encryption. Note that view of server in $\mathsf{Hyb}_4$ is identical to the output of $\mathsf{Sim}_S$.

**Malicious Client Security.** $\mathsf{Sim}_C$ samples a $(pk, sk) \leftarrow \mathsf{HE.KeyGen}(1^\lambda)$. $\mathsf{Sim}_C$ runs the simulator for the malicious client in the two-party computation protocol that generates the public-key, secret-key pair for the homomorphic encryption scheme. When this simulator access the ideal functionality, $\mathsf{Sim}_C$ provides $(pk, sk)$ as the output. On receiving the ciphertext $\{\mathsf{HE.Enc}(pk, \mathbf{r}_i)\}_{i \in [\ell]}$ and the proof of plaintext knowledge from the client, $\mathsf{Sim}_C$ first checks if the proof is valid. If yes, $\mathsf{Sim}_C$ then runs the knowledge extractor Ext on the ciphertext and the proof to extract $\mathbf{r}_i$ and the randomness used for generating the ciphertext encrypting $\mathbf{r}_i$ for each $i \in [\ell]$. $\mathsf{Sim}_C$ queries the ideal functionality $\mathcal{F}_{\mathsf{ACG}}$ on input $\mathbf{r}_i$ and obtains $(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i, \langle \alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i) \rangle_1, \langle \beta_i \cdot \mathbf{r}_i \rangle_1)$. It then runs $\mathsf{Sim}_{\mathsf{FP}}$ on the output $(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i, \langle \alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i) \rangle_1, \langle \beta_i \cdot \mathbf{r}_i \rangle_1)$ along with $\mathbf{r}_i$ and the randomness used to generate the ciphertext and obtains the simulated ciphertexts. It sends the simulated ciphertexts to the client.

We now show the view of the malicious client in the real protocol is computationally indistinguishable to the view generated by $\mathsf{Sim}_C$ via a hybrid argument.

- $\mathsf{Hyb}_1$ : This corresponds to the view of the malicious client in the real execution of the protocol.
- $\mathsf{Hyb}_2$ : In this hybrid, we use the simulator for the malicious client in the two-party computation protocol for KeyGen. When this simulator queries its ideal functionality, we sample a $(pk, sk) \leftarrow \mathsf{HE.KeyGen}(1^\lambda)$ and give $(pk, sk)$ to it. This hybrid is computationally indistinguishable to $\mathsf{Hyb}_1$ from the security of this protocol.
- $\mathsf{Hyb}_3$ : In this hybrid, the server checks if the zero-knowledge proofs are valid and in that case, it uses the

extractor Ext to extract $\mathbf{r}_i$ and the randomness used for generating the ciphertext encrypting $\mathbf{r}_i$ for each $i \in [\ell]$. Since the verifier in a non-interactive zk proof is deterministic, it follows from the proof of knowledge property that the extractor Ext succeeds in extracting a witness except with negligible probability.

- $\mathsf{Hyb}_4$ : In this hybrid, we use the simulator for the function privacy of the homomorphic encryption in generating $\mathsf{HE.Enc}(\mathsf{pk}, \mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)$, $\mathsf{HE.Enc}(\mathsf{pk}, \langle \alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_1)$ and $\mathsf{HE.Enc}(\mathsf{pk}, \langle \beta_i \cdot \mathbf{r}_i\rangle_1)$ for each $i \in [\ell]$. This hybrid is computationally indistinguishable to the previous hybrid from the function privacy of the HE scheme. Note that view of the client in $\mathsf{Hyb}_4$ is identical to the view generated by $\mathsf{Sim}_C$.

□

## B.2 MPC Engine

**Lemma B.2.** *Assuming a secure two-party computation protocol against malicious clients and semi-honest servers for* KeyGen*, a leveled fully-homomorphic encryption scheme which supports depth-1 arithmetic circuits (see Section 4) and a zero-knowledge proof system for* NP *(see Section 4), the protocol described in Fig. 17 securely computes $\mathcal{F}_{Triple}$ against malicious clients and semi-honest servers.*

*Proof.* We now prove semi-honest server security and then show security against malicious clients.

**Semi-Honest server security.** The simulator $\mathsf{Sim}_S$ samples $(pk, sk) \leftarrow \mathsf{HE.KeyGen}(1^\lambda)$. It then runs the simulator for the semi-honest server in the two-party computation protocol that generates the public-key, secret-key pair for the homomorphic encryption scheme. When this simulator access the ideal functionality, $\mathsf{Sim}_S$ provides $pk$ as the output. $\mathsf{Sim}_S$ sends $\{\mathsf{HE.Enc}(pk, \mathbf{0})\}_{i\in[3]}$ and with a simulated proof of well-formedness of ciphertexts. We now show the indistinguishability of the real view from the simulated view using a hybrid argument.

- $\mathsf{Hyb}_1$ : This corresponds to the real execution of the protocol.
- $\mathsf{Hyb}_2$ : In this hybrid, we use the simulator for the semi-honest server in the two-party computation protocol for generating the public key, secret key pair. When this simulator queries its ideal functionality, we sample a $(pk, sk) \leftarrow \mathsf{HE.KeyGen}(1^\lambda)$ and give $pk$ to it. This hybrid is computationally indistinguishable to $\mathsf{Hyb}_1$ from the security of this protocol.
- $\mathsf{Hyb}_3$ : In this hybrid, we start using the simulator for the zero-knowledge proofs of well-formedness of ciphertexts. This hybrid is indistinguishable to the previous one from the zero-knowledge property of these proof systems.
- $\mathsf{Hyb}_4$ : In this hybrid, we switch from sending $\mathsf{HE.Enc}(pk, a_1), \mathsf{Enc}(\mathsf{pk}, b_1)$ to sending $\{\mathsf{HE.Enc}(pk, \mathbf{0})\}_{i\in[2]}$. This hybrid is indistinguishable from the previous one from the semantic security of the homomorphic encryption. Note that view of server in $\mathsf{Hyb}_4$ is identical to the output of $\mathsf{Sim}_S$.

**Malicious Client Security.** $\mathsf{Sim}_C$ samples a $(pk, sk) \leftarrow \mathsf{HE.KeyGen}(1^\lambda)$. $\mathsf{Sim}_C$ runs the simulator for the malicious client in the two-party computation protocol that generates the public-key, secret-key pair for the homomorphic encryption scheme. When this simulator access the ideal functionality, $\mathsf{Sim}_C$ provides $(pk, sk)$ as the output. On receiving the ciphertext $\mathsf{Enc}(\mathsf{pk}, a_1), \mathsf{Enc}(\mathsf{pk}, b_1)$ and the corresponding zero-knowledge proofs from the client, $\mathsf{Sim}_C$ first checks if the proofs are valid. If yes, $\mathsf{Sim}_C$ then runs the knowledge extractor Ext on the ciphertext and the proof to extract $a_1, b_1$ and the randomness used for generating these ciphertexts. $\mathsf{Sim}_C$ samples random $a_2, b_2$, queries the ideal functionality $\mathcal{F}_{\text{Triple}}$ on input $a_1, b_1, a_2, b_2$, and obtains $[\![a_1 + a_2]\!]_1 = (a_1 + a_2 - u_1, \delta(a_1 + a_2) - u_2), [\![b_1 + b_2]\!]_1 = (b_1 + b_2 - v_1, \delta(b_1 + b_2) - v_2), [\![(a_1 + a_2)\cdot(b_1 + b_2)]\!]_1 = (a_1 + a_2)\cdot(b_1 + b_2) - t_1, \delta\cdot(a_1 + a_2)\cdot(b_1 + b_2) - t_2)$. It then runs $\mathsf{Sim}_{\mathsf{FP}}$ on the six outputs and the randomness used to generate the initial ciphertexts and obtains the six simulated ciphertexts. It sends the six simulated ciphertexts to the client.

We now show the view of the malicious client in the real protocol is computationally indistinguishable to the view generated by $\mathsf{Sim}_C$ via a hybrid argument.

- $\mathsf{Hyb}_1$ : This corresponds to the view of the malicious client in the real execution of the protocol.
- $\mathsf{Hyb}_2$ : In this hybrid, we use the simulator for the malicious client in the two-party computation protocol for KeyGen. When this simulator queries its ideal functionality, we sample a $(pk, sk) \leftarrow \mathsf{HE.KeyGen}(1^\lambda)$ and give $(pk, sk)$ to it. This hybrid is computationally indistinguishable to $\mathsf{Hyb}_1$ from the security of this protocol.
- $\mathsf{Hyb}_3$ : In this hybrid, the server checks if the zero-knowledge proofs are valid and in that case, it uses the extractor Ext to extract $a_1, b_1$ and the randomness used for generating these ciphertexts. Since the verifier in a non-interactive zk proof is deterministic, it follows from the proof of knowledge property that the extractor Ext succeeds in extracting a witness except with negligible probability.
- $\mathsf{Hyb}_4$ : In this hybrid, we use the simulator for the function privacy of the homomorphic encryption in generating $\mathsf{Enc}(\mathsf{pk}, a_1 + a_2 - u_1)$, $\mathsf{Enc}(\mathsf{pk}, b_1 + b_2 - v_1)$ and $\mathsf{Enc}(\mathsf{pk}, (a_1 + a_2)\cdot(b_1 + b_2) - t_1)$ along with $\mathsf{Enc}(\mathsf{pk}, \delta(a_1 + a_2) - u_2)$, $\mathsf{Enc}(\mathsf{pk}, \delta(b_1 + b_2) - v_2)$ and $\mathsf{Enc}(\mathsf{pk}, \delta\cdot(a_1 + a_2)\cdot(b_1 + b_2) - t_2)$. This hybrid is computationally indistinguishable to the previous hybrid from the function privacy

of the HE scheme. Note that view of the client in $\mathsf{Hyb}_4$ is identical to the view generated by $\mathsf{Sim}_C$.

□

Note that $\mathcal{F}_{\text{Rand}}$ can be implemented using $\mathcal{F}_{\text{Triple}}$ and that security of $\Pi_{\text{Online}}$ follows directly from the security of Beaver's protocol.

## B.3 CDS

**Lemma B.3.** *Assuming a secure two-party computation protocol for the function $f_{CDS}$ (see Fig. 6) against malicious clients and semi-honest servers, the protocol described in Fig. 6 secure computes $\mathcal{F}_{CDS}$ against malicious clients and semi-honest servers.*

*Proof.* The correctness of the protocol follows from observation and we now show semi-honest server security and then show security against malicious clients.

**Semi-honest server security.** To show this, we run the simulator for the protocol computing $f_{\text{CDS}}$ to generate the client side messages for this protocol. When this simulator queries the ideal functionality, we give $\rho = \sigma = 0$ and an uniform share $\{\langle g_k\rangle_2\}_{k\in[|\mathsf{inp}_i|]}$ as the output. It follows from the security of the protocol that implements $f_{\text{CDS}}$ that the above generated view is computationally indistinguishable to the real view.

**Malicious Client Security.** The simulator $\mathsf{Sim}_C$ against malicious client first runs the corresponding simulator for $f_{\text{CDS}}$. When this simulator access the ideal functionality $f_{\text{CDS}}$ using the client's input $(\{b_k^i\}_{k\in[|\mathsf{inp}_i|]}, \langle\alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_1, \langle\beta_{i+1}\cdot\mathbf{r}_{i+1}\rangle_1)$, the $\mathsf{Sim}_C$ queries $\mathcal{F}_{\text{CDS}}$ on input $(\{b_k^i\}_{k\in[|\mathsf{inp}_i|]}, \langle\alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_1, \langle\beta_{i+1}\cdot\mathbf{r}_{i+1}\rangle_1)$ and obtains $\{\mathsf{lab}_{i,k,b_k^i}\}_{k\in|\mathsf{inp}_i|}$ or the special symbol abort. $\mathsf{Sim}_C$ answers the simulator for $f_{\text{CDS}}$ with uniformly chosen shares and random tags as the output of the client. If the answer from $\mathcal{F}_{\text{CDS}}$ was abort, then $\mathsf{Sim}_C$ aborts at the end of step (ii). Else, in Step (ii), $\mathsf{Sim}_C$ sends $\{\mathsf{lab}_{i,k,b_k^i} - \langle g_k\rangle_1\}_{k\in|\mathsf{inp}_i|}$ as the final round message.

We now show that the view generated by $\mathsf{Sim}_C$ is computationally indistinguishable to the malicious client's view in the real protocol using a hybrid argument.

- $\mathsf{Hyb}_0$ : This corresponds to the view of the malicious client in the real protocol execution.
- $\mathsf{Hyb}_1$ : In this hybrid, we use the simulator for $f_{\text{CDS}}$ and generate the view of the client in Step (i). When this simulator queries the ideal $f_{\text{CDS}}$ functionality, we give uniformly chosen random shares $\{\langle g_k\rangle_1\}_{k\in|\mathsf{inp}_i|}$. We compute the output of the server using its actual inputs and the rest of the steps proceed exactly as in the protocol. This hybrid is computationally indistinguishable to $\mathsf{Hyb}_0$ from the security of the protocol implementing the $f_{\text{CDS}}$ functionality against malicious clients.
- $\mathsf{Hyb}_3$ : In this hybrid, we query the $\mathcal{F}_{\text{CDS}}$ functionality on client input $(\{b_k^i\}_{k\in[|\mathsf{inp}_i|]}, \langle\alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_1, \langle\beta_{i+1}\cdot\mathbf{r}_{i+1}\rangle_1)$ and obtain the output $\{\mathsf{lab}_{i,k,b_k^i}\}_{k\in|\mathsf{inp}_i|}$ or the special symbol abort. If the output was not abort, then it means that $\sigma = \rho = 0$ and thus, we set $\langle g_k\rangle_2 = \mathsf{lab}_{i,k,b_k^i} - \langle g_k\rangle_1$ and send this as the final round message from the server. Otherwise, we abort at the end of Step (ii).

  Notice that the only difference between $\mathsf{Hyb}_3$ and $\mathsf{Hyb}_2$ is when the output of the $\mathcal{F}_{\text{CDS}}$ functionality is abort. We now argue that with that protocol in $\mathsf{Hyb}_2$ also aborts at the end of Step (ii) with overwhelming probability. If the output of $\mathcal{F}_{\text{CDS}}$ is abort, then it means that either $\overline{\alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)} \neq \langle\alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_1 + \langle\alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_2$ or $\beta_{i+1}\cdot\overline{\mathbf{r}_{i+1}} \neq \langle\beta_{i+1}\cdot\mathbf{r}_{i+1}\rangle_1 + \langle\beta_{i+1}\cdot\mathbf{r}_{i+1}\rangle_2$. Let us assume without loss of generality that $\beta_{i+1}\cdot\overline{\mathbf{r}_{i+1}} \neq \langle\beta_{i+1}\cdot\mathbf{r}_{i+1}\rangle_1 + \langle\beta_{i+1}\cdot\mathbf{r}_{i+1}\rangle_2$. It now follows from Schwartz-Zippel lemma that with overwhelming probability $\sigma = \beta_{i+1}\left\langle\mathbf{c}_2, \overline{\mathbf{r}_{i+1}}\right\rangle - \left\langle\mathbf{c}_2, \langle\beta_{i+1}\cdot\mathbf{r}_{i+1}\rangle_1 + \langle\beta_{i+1}\cdot\mathbf{r}_{i+1}\rangle_2\right\rangle$ will not be equal to 0 and thus, the honest server will also abort in $\mathsf{Hyb}_2$. We finally observe that the view of malicious client in $\mathsf{Hyb}_3$ is identical to $\mathsf{Sim}_C$.

□

## B.4 MUSE Protocol

We first show security against a malicious client and then prove security against a semi-honest server.

**Malicious Client.** The simulator $\mathsf{Sim}_C$ does the following:

- **Offline Phase.** In the offline phase,
  1. **Authenticated Correlations Generator.** $\mathsf{Sim}_C$ runs the corresponding simulator for the protocol implementing $\mathcal{F}_{\text{ACG}}$. When this simulator queries the ideal functionality on input $\{\mathbf{r}_i\}_{i\in[\ell]}$, $\mathsf{Sim}_C$ stores $\{\mathbf{r}_i\}_{i\in[\ell]}$ and answers the query with randomly chosen values $\{\mathbf{s}_i, \mathbf{t}_i, \mathbf{v}_i\}_{i\in[\ell]}$.
  2. For every $i \in [\ell]$, $\mathsf{Sim}_C$ samples random input labels $\{\mathsf{lab}_{i,k}^C\}_{k\in[|\mathsf{inp}_i|]}$.
  3. **Conditional Disclosure of Secrets.** $\mathsf{Sim}_C$ runs the corresponding simulator for the protocol implementing $\mathcal{F}_{\text{CDS}}$. For every $i \in [\ell]$, when this simulator queries the ideal functionality on some input, $\mathsf{Sim}_C$ checks if this input is equal to $\mathbf{s}_i, \mathbf{t}_i, \mathbf{r}_{i+1}, \mathbf{v}_{i+1}$. If not, $\mathsf{Sim}_C$ gives $\perp$ as the answer from ideal functionality. Else, it outputs $\{\mathsf{lab}_{i,k}^C\}_{k\in[\mathsf{inp}_i]}$.
  4. **Offline Garbling.** For every $i \in [\ell]$, $\mathsf{Sim}_C$ sends a uniformly chosen random bit string $D_i$ of size $|\widetilde{C}_i|$.

- **Online Phase.** In the online phase, the simulator does the following:
  - **Preamble:** On receiving, $\mathbf{x} - \mathbf{r}_1$, $\mathsf{Sim}_C$ extracts $\mathbf{x}$ from this and queries the neural network functionality on input $\mathbf{x}$ and learns the output $\mathbf{y}$.
  - **Layer Evaluation.** For every $i \in [\ell]$, $\mathsf{Sim}_C$ does the following:
    - **Non-Linear Layer.**
      1. $\mathsf{Sim}_C$ samples random input labels $\{\mathsf{lab}^S_{i,k}\}_{k\in[\mathsf{inp}_i]}$.
      2. The simulator computes $\widetilde{C}_i \leftarrow \mathsf{Sim}_{GC}(\{\mathsf{lab}^C_{i,k}, \mathsf{lab}^S_{i,k}\}_{k\in[|\mathsf{inp}_i|]}, (z_i, \overline{\mathsf{lab}}))$ where $(z_i, \overline{\mathsf{lab}})$ is an uniformly chosen random string from the output space of $C_i$. It chooses a uniform key $k_i \leftarrow \{0,1\}^\lambda$ and programs the RO to output $D_i \oplus \widetilde{C}_i$ on input $k_i$.
      3. $\mathsf{Sim}_C$ sends $\{k_i, \mathsf{lab}^S_{i,k}\}_{k\in[\mathsf{inp}_i]}$ to the client.
      4. The client evaluates the garbled circuit $\widetilde{C}$ using the labels $\{\mathsf{lab}^C_{i,k}, \mathsf{lab}^S_{i,k}\}_{k\in[\mathsf{inp}_i]}$ and sends $(z'_i, h)$.
      5. $\mathsf{Sim}_C$ checks if $z_i = z'_i$ and if it is not the case, it aborts. Otherwise, it proceeds to the next layer.
  - **Output Phase.** The simulator sends $z_\ell - \mathbf{y}$ to the client.

**Proof of Indistinguishability.** To prove that the view of a malicious client is indistinguishable to the above generated simulated view, we use a hybrid argument.

- $\mathsf{Hyb}_1$ : This corresponds to the real execution of the protocol.

- $\mathsf{Hyb}_2$ : In this hybrid, we use the simulator for the protocol implementing $\mathcal{F}_{\text{ACG}}$. When this simulator queries the ideal functionality, we give $\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i$, $\langle \alpha_i(\mathbf{M}_i\mathbf{r}_i - \mathbf{s}_i)\rangle_1$ and $\langle \beta_i \cdot \mathbf{r}_i\rangle_1$ as the output. This hybrid is computationally indistinguishable from $\mathsf{Hyb}_1$ from the security of the sub-protocol implementing $\mathcal{F}_{\text{ACG}}$.

- $\mathsf{Hyb}_3$ : In this hybrid, we run the simulator for the protocol implementing $\mathcal{F}_{\text{CDS}}$. When this simulator queries its ideal functionality, we implement this functionality honestly and output the corresponding labels (or the special symbol $\perp$). This hybrid is computationally indistinguishable to $\mathsf{Hyb}_2$ from the security of this sub-protocol.

- $\mathsf{Hyb}_4$ : In this hybrid, when implementing the ideal functionality for the conditional disclosure of secrets, we check if the inputs provided by the simulator on behalf of the corrupted client are equal to $\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i$, $\langle \alpha_i(\mathbf{M}_i\mathbf{r}_i - \mathbf{s}_i)\rangle_1$, $\mathbf{r}_{i+1}$ and $\langle \beta_{i+1} \cdot \mathbf{r}_{i+1}\rangle_1$ instead of performing the MAC checks. This hybrid is statistically close to the previous one due to the MAC security (see Lemma A.1).

- $\mathsf{Hyb}_5$ : In this hybrid, instead of sending $\widetilde{C}_i \oplus H(k_i)$ in the preprocessing phase, we send a random string $D_i$ and in the online phase, we choose an uniform key $k_i \leftarrow \{0,1\}^\lambda$ and program the random oracle to output $D_i \oplus \widetilde{C}_i$. This hybrid is statistically close to the previous one because the probability that the adversary queries the oracle on an uniformly chosen $k_i$ is negligible.

- $\mathsf{Hyb}_6$ : In this hybrid, we make a syntactic change with respect to the previous hybrid. When the client sends $\mathbf{x} - \mathbf{r}_1$ to the server in the preamble of the online phase, we use the extracted value $\mathbf{r}_1$ and obtain $\mathbf{x}$. As a result, we find the output of the neural network in every layer, namely, $\mathbf{x}_1 = \mathbf{x}, \ldots, \mathbf{x}_{\ell+1} = \mathbf{y}$.

- $\mathsf{Hyb}_{6+i}$ : (for every $i \in [\ell]$) In this hybrid, we make the following changes when compared to $\mathsf{Hyb}_{6+i-1}$ in the specified order:
  - $\mathsf{Hyb}'_1$ : We generate $\widetilde{C}_i$ using the corresponding simulator for the garbled circuits. Specifically, we compute $\widetilde{C}_i \leftarrow \mathsf{Sim}_{GC}(\{\mathsf{lab}^C_{i,k}, \mathsf{lab}^S_{i,k}\}_{k\in[\mathsf{inp}_i]}, (z_i, \overline{\mathsf{lab}}))$ where $z_i, \overline{\mathsf{lab}}$ is the output of $C_i$ on the server's input being equal to $\mathbf{M}_i(\mathbf{x}_i - \mathbf{r}_i) + \mathbf{s}_i, \mathbf{s}'_{i+1}$ and the client's input being equal to $\mathbf{r}_{i+1}$, $\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i$. This hybrid is computationally indistinguishable to $\mathsf{Hyb}_{6+i-1}$ from the security of garbled circuits (see Appendix A.1).
  - $\mathsf{Hyb}'_2$ : In this hybrid, we sample $z_i, \overline{\mathsf{lab}}$ uniformly at random instead of fixing them to be the output of $C_i$. When $i = \ell$, we send $z_\ell - \mathbf{y}$ as the final round message from the server to the client. This hybrid is identical to the previous hybrid from the one-time pad security (with the pad $s'_{i+1}$).
  - $\mathsf{Hyb}'_3$ : In this hybrid, on receiving $z'_i, h$ from the client, the server checks if $z'_i = z_i$ and otherwise, it aborts. This hybrid is statistically close to the previous hybrid because for any position where $z'_i$ and $z_i$ differs, the label corresponding to the bit of $z'_i$ in that position is uniformly distributed. Thus, the probability that the malicious client outputs the correct hash value on an uniformly chosen input is negligible.

- $\mathsf{Hyb}_{7+\ell}$ : In this hybrid, we give randomly chosen vectors $\{\mathbf{s}_i, \mathbf{t}_i, \mathbf{v}_i\}_{i\in[\ell]}$ as output from $\mathcal{F}_{\text{ACG}}$. This hybrid is identical to the previous hybrid from the one-time pad security.

- $\mathsf{Hyb}_{8+\ell}$ : In this hybrid, instead of computing the output $\mathbf{y}$ using the server's input matrices, we query the ideal functionality and obtain $\mathbf{y}$. This change is only syntactic and this hybrid is identical to the previous hybrid. Notice that $\mathsf{Hyb}_{8+\ell}$ is identical to the simulator.

**Semi-honest server security.** The simulator $\mathsf{Sim}_S$ does the following:

- It initializes the server on an uniform random tape.

- **Offline Phase.** In the offline phase,

  1. **Authenticated Correlations Generator.** For each $i \in [\ell]$, $\mathsf{Sim}_S$ uses the simulator for the semi-honest server in the protocol implementing $\mathcal{F}_{\text{ACG}}$ by giving randomly chosen shares as the output of the server.
  2. **Conditional Disclosure of Secrets.** $\mathsf{Sim}_S$ runs the simulator for the semi-honest server in the two-party computation protocol computing the $\mathcal{F}_{\text{CDS}}$.

- **Online Phase.** In the online phase, the $\mathsf{Sim}_S$ does the following:

  – **Preamble:** Send an uniformly chosen random vector $\mathbf{r}_1$ to the server.

  – **Layer Evaluation.** For every $i \in [\ell]$, $\mathsf{Sim}_S$ does the following:

    ∗ **Non-Linear Layer.**
      1. $\mathsf{Sim}_S$ recovers the output labels of $\widetilde{C}_i$ since it had set the random tape for the semi-honest server.
      2. $\mathsf{Sim}_S$ chooses a uniform $z_i'$ and computes the hash of the output labels $h_i'$ corresponding to $z_i'$. It sends $(z_i', h_i')$ to the corrupted server.

**Proof of Indistinguishability.**

- $\mathsf{Hyb}_1$ : This corresponds to the real execution of the protocol.

- $\mathsf{Hyb}_2$ : In this hybrid, we use the simulator for the protocol implementing the $\mathcal{F}_{\mathsf{ACG}}$ functionality by giving randomly chosen shares as the output of the server. This hybrid is computationally indistinguishable to the previous one from the security of this sub-protocol.

- $\mathsf{Hyb}_3$ : In this hybrid, we use the knowledge of the input of the client, the server and the random tape to compute the correct output of every garbled circuit instead of evaluating them. This change is syntactic and is identical to the previous hybrid.

- $\mathsf{Hyb}_4$ : In this hybrid, we run the simulator for the protocol implementing $\mathcal{F}_{\text{CDS}}$. This hybrid is computationally indistinguishable to $\mathsf{Hyb}_3$ from the security of this protocol.

- $\mathsf{Hyb}_5$ : In this hybrid, instead of using the knowledge of the client's input to compute the output of each garbled circuit, we choose a uniform $z_i'$ and computes the hash of the output labels $h_i'$ corresponding to $z_i'$. It sends $(z_i', h_i')$ to the corrupted server. This hybrid is identically distributed to the previous one from the one-time pad security (with the pad $\mathbf{r}_{i+1}$). Note that $\mathsf{Hyb}_5$ is identical to $\mathsf{Sim}_S$.

Functionality $\mathcal{F}_{\text{ACG}}$

1. On client input $\{\mathbf{r}_i\}_{i\in[\ell]}$, server input $\{\mathbf{s}_i, \mathbf{M}_i, \alpha_i, \beta_i\}_{i\in[\ell]}$, compute $\{\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i, \alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i), \beta_i \cdot \mathbf{r}_i\}_{i\in[\ell]}$.
2. Secret share $\alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)$ to $\langle\alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_1, \langle\alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_2$ and $\beta_i \cdot \mathbf{r}_i$ to $\langle\beta_i \cdot \mathbf{r}_i\rangle_1, \langle\beta_i \cdot \mathbf{r}_i\rangle_2$.
3. Output $(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i, \langle\alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_1, \langle\beta_i \cdot \mathbf{r}_i\rangle_1)$ to the client, and $(\langle\alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_2, \langle\beta_i \cdot \mathbf{r}_i\rangle_2)$ to the server.

**Figure 11:** The ideal functionality for Authenticated Correlations Generator

Functionality $\mathcal{F}_{\text{CDS}}$

1. The client and server input $(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i, \langle\alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_1, \mathbf{r}_{i+1}, \langle\beta_{i+1} \cdot \mathbf{r}_{i+1}\rangle_1)$ and $(\alpha_i, \beta_{i+1}, \langle\alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_2, \langle\beta_{i+1} \cdot \mathbf{r}_{i+1}\rangle_2, \{\mathsf{lab}^C_{i,k,0}, \mathsf{lab}^C_{i,k,1}\}_{k\in[|\mathsf{inp}_i|]})$ respectively for some $i \in [\ell]$.
2. If $\alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i) = \langle\alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_1 + \langle\alpha_i(\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i)\rangle_2$ and $\beta_{i+1} \cdot \mathbf{r}_{i+1} = \langle\beta_{i+1} \cdot \mathbf{r}_{i+1}\rangle_1 + \langle\beta_{i+1} \cdot \mathbf{r}_{i+1}\rangle_2$, output the labels $\{\mathsf{lab}^C_{i,k,0}, \mathsf{lab}^C_{i,k,1}\}_{k\in[|\mathsf{inp}_i|]}$ corresponding to $\mathbf{M}_i(\mathbf{r}_i) - \mathbf{s}_i$ and $\mathbf{r}_{i+1}$ to the client. Otherwise, abort.

**Figure 12:** The ideal functionality for Conditional Disclosure of Secrets

$\mathcal{F}_{\text{InputAuth}}$

- The client's input is $\mathbf{m}_c$ and the server's input is $\mathbf{m}_s$ and a MAC key $\delta$. The client receives $[\![\mathbf{m}_c]\!]_1, [\![\mathbf{m}_s]\!]_1$ and the server receives $[\![\mathbf{m}_c]\!]_2, [\![\mathbf{m}_s]\!]_2$.

**Figure 13:** Description of $\mathcal{F}_{\text{InputAuth}}$.

Protocol $\Pi_{\mathsf{InputAuth}}$

1. Both parties invoke $\mathcal{F}_{\text{Rand}}$ to receive $|\mathbf{m}_c|$ random shares $[\![\mathbf{r}]\!]$
2. $\mathbf{r}$ is privately opened to the client.
3. The client broadcasts $\varepsilon = \mathbf{m}_c - \mathbf{r}$.
4. The server's share is $[\![\mathbf{m}_c]\!]_2 = (\varepsilon, [\![\mathbf{r}]\!]_2)$ and the client's share is $[\![\mathbf{m}_c]\!]_1 = (\varepsilon, [\![\mathbf{r}]\!]_1)$.
5. The server chooses two masking vectors $\mathbf{u}, \mathbf{v}$ and sends the client $[\![\mathbf{m}_s]\!]_1 = (\mathbf{m}_s - \mathbf{u}, \delta \cdot \mathbf{m}_s - \mathbf{v})$. The server sets its share $[\![\mathbf{m}_s]\!]_2 = (\mathbf{u}, \mathbf{v})$

**Figure 14:** The protocol for Input Authentication.

$\mathcal{F}_{\text{Triple}}$

- The client's input is $a_1, b_1$ and the server's input is $a_2, b_2$ and a MAC key $\delta$. The client receives $[\![a_1 + a_2]\!]_1, [\![b_1 + b_2]\!]_1, [\![(a_1 + a_2) \cdot (b_1 + b_2)]\!]_1$ and the server receives $[\![a_1 + a_2]\!]_2, [\![b_1 + b_2]\!]_2, [\![(a_1 + a_2) \cdot (b_1 + b_2)]\!]_2$.

**Figure 15:** Description of $\mathcal{F}_{\text{Triple}}$.

$\mathcal{F}_{\text{Rand}}$

- The client's input is $\mathbf{r}_1$ and the server's input is $\mathbf{r}_2$ and a MAC key $\delta$. The client receives $[\![\mathbf{r}_1 + \mathbf{r}_2]\!]_1$ and the server receives $[\![\mathbf{r}_1 + \mathbf{r}_2]\!]_2$.

**Figure 16:** Description of $\mathcal{F}_{\text{Rand}}$.

Protocol $\Pi_{\text{Triple}}$

i. The client and the server engage in a two-party computation protocol with security against malicious clients and semi-honest servers to generate the public key, secret key pair for $\mathsf{HE}$. At the end of the protocol, the client learns the public key $\mathsf{pk}$ and the secret key $\mathsf{sk}$ whereas the server only learns $\mathsf{pk}$.

ii. The client sends $\mathsf{Enc}(\mathsf{pk}, a_1), \mathsf{Enc}(\mathsf{pk}, b_1)$ to the server along with a zero-knowledge proof of well-formedness of the two ciphertexts. The server verifies this proof before continuing.

iii. The server homomorphically computes $\mathsf{Enc}(\mathsf{pk}, a_1 + a_2)$, $\mathsf{Enc}(\mathsf{pk}, b_1 + b_2)$ and $\mathsf{Enc}(\mathsf{pk}, (a_1 + a_2) \cdot (b_1 + b_2))$ along with $\mathsf{Enc}(\mathsf{pk}, \delta(a_1 + a_2))$, $\mathsf{Enc}(\mathsf{pk}, \delta(b_1 + b_2))$ and $\mathsf{Enc}(\mathsf{pk}, \delta \cdot (a_1 + a_2) \cdot (b_1 + b_2))$.

iv. The server chooses six random masking elements $u_1, v_1, t_1, u_2, v_2, t_2$ and computes $\mathsf{Enc}(\mathsf{pk}, a_1 + a_2 - u_1)$, $\mathsf{Enc}(\mathsf{pk}, b_1 + b_2 - v_1)$ and $\mathsf{Enc}(\mathsf{pk}, (a_1 + a_2) \cdot (b_1 + b_2) - t_1)$ along with $\mathsf{Enc}(\mathsf{pk}, \delta(a_1 + a_2) - u_2)$, $\mathsf{Enc}(\mathsf{pk}, \delta(b_1 + b_2) - v_2)$ and $\mathsf{Enc}(\mathsf{pk}, \delta \cdot (a_1 + a_2) \cdot (b_1 + b_2) - t_2)$. It sends these six ciphertexts to the client.

v. The client decrypts the above ciphertexts and obtains $[\![a_1 + a_2]\!]_1 = (a_1 + a_2 - u_1, \delta(a_1 + a_2) - u_2), [\![b_1 + b_2]\!]_1 = (b_1 + b_2 - v_1, \delta(b_1 + b_2) - v_2), [\![(a_1 + a_2) \cdot (b_1 + b_2)]\!]_1 = (a_1 + a_2) \cdot (b_1 + b_2) - t_1, \delta \cdot (a_1 + a_2) \cdot (b_1 + b_2) - t_2)$. The server outputs $[\![a_1 + a_2]\!]_2 = (u_1, u_2), [\![b_1 + b_2]\!]_2 = (v_1, v_2), [\![(a_1 + a_2) \cdot (b_1 + b_2)]\!]_2 = (t_1, t_2)$.

**Figure 17:** The protocol for Triple Generation.

## C Pseudocode for our attacks from Section 2

RecoverNetwork:

1. First, recover the last layer:
   (a) Denote by $\tilde{M}_\ell$ the recovered matrix for the last layer.
   (b) For each $j \in [t]$:
      i. Set the initial input to the network to be zero, i.e. $\mathbf{x}_1 := 0$.
      ii. Follow the inference protocol to partially evaluate the network up to the $\ell-1$-th layer: $\mathbf{x}_{\ell-1} := \mathsf{ReLU}(M_{\ell-1}(\ldots\mathsf{ReLU}(M_1\mathbf{x}_1))) = \mathbf{0}$.
      iii. Malleate the client's share of $\mathbf{x}_{\ell-1}$: $\langle\mathbf{x}'_{\ell-1}\rangle_C := \langle\mathbf{x}_{\ell-1}\rangle_C + \mathbf{e}_j$.
      iv. Complete the protocol with the server to obtain $\mathbf{x}_\ell := M_\ell\mathbf{x}'_{\ell-1} = M_\ell\mathbf{e}_j$.
      v. Set the $j$-th column of $\tilde{M}_\ell$ to be $\mathbf{x}_\ell$.
   (c) Output $\tilde{M}_\ell$.
2. Then, recover all previous layers:
   (d) For each $i \in [\ell-1,\ldots,1]$:
      i. If the $i$-th layer is a fully-connected layer, set $M_i := \mathsf{RecoverFCLayer}(M_{i+1},\ldots,M_\ell)$.
      ii. If the $i$-th layer is a convolutional layer, set $M_i := \mathsf{RecoverConvLayer}(M_{i+1},\ldots,M_\ell)$.
   (e) Output $(M_1,\ldots,M_{\ell-1})$.

MaskAndLinearizeReLU($\langle\mathbf{y}\rangle_C, S$):

1. Malleate the client's local share of $\mathbf{y}$ to obtain a share of the malleated $\mathbf{y}'$ as follows:
   (a) For all $l \in S$, set $\langle\mathbf{y}'\rangle_C[l] := \langle\mathbf{y}\rangle_C[l] + c$.
   (b) For all $l \notin S$, set $\langle\mathbf{y}'\rangle_C[l] := \langle\mathbf{y}\rangle_C[l] - c$.
2. Obtain $\langle\mathbf{x}\rangle_C := \mathsf{LinearizeReLU}(\mathbf{y}')$..
3. Invert Step 1a by malleating $\langle\mathbf{x}\rangle_C$: set $\langle\mathbf{x}'\rangle_C[S] := \langle\mathbf{x}\rangle_C[S] - c$
4. Output $\langle\mathbf{x}'\rangle_C$.

LinearizeReLU($\langle\mathbf{y}\rangle_C$):

1. Malleate the client's local share of $\mathbf{y}$ to obtain a share of the malleated $\mathbf{y}'$: set $\langle\mathbf{y}'\rangle_C := \langle\mathbf{y}\rangle_C + c$.
2. Interact with the server to obtain $\langle\mathbf{x}\rangle_C$, which is the client's share of $\mathbf{x} := \mathsf{ReLU}(\mathbf{y}')$.
3. Invert Step 1 by malleating $\langle\mathbf{x}\rangle_C$: set $\langle\mathbf{x}'\rangle_C := \langle\mathbf{x}\rangle_C - c$
4. Output $\langle\mathbf{x}'\rangle_C$.

RecoverConvLayer($M_{i+1},\ldots,M_\ell$) :

1. Let the dimensions of the convolutional kernel $K_i$ be $k_i \times k_i$.
2. Sample a random matrix $R$ having the same dimension as $\mathbf{x}_{i-1}$.
3. Apply the `im2col` transformation to $R$ to obtain $R'$.
4. Let $S$ be the indices of the pivot columns of $R'$ when it is in row-reduced echelon form. If $|S| < k_i \times k_i$, resample $R$ and retry. Then $S$ specifies the indices of the independent columns of $R'$.[a]
5. Follow the inference protocol to evaluate the network up to the $i-1$-th layer to obtain (a share of) the intermediate state $\mathbf{x}_{i-1} := \mathsf{ReLU}(M_{i-1}(\ldots\mathsf{ReLU}(M_1\mathbf{x}_1))) = \mathbf{0}$.
6. Malleate the client's share of $\mathbf{x}_{i-1}$: $\langle\mathbf{x}'_{i-1}\rangle_C := \langle\mathbf{x}_{i-1}\rangle_C + R$.
7. Interact with the server to evaluate the $i$-th linear layer to obtain a share of $\mathbf{y}_i := M_i\mathbf{x}'_{i-1}$.
8. Obtain the input for the next linear layer: $\langle\mathbf{x}_i\rangle_C := \mathsf{MaskAndLinearizeReLU}(\langle\mathbf{y}_i\rangle_C, S)$.
   The vector $\mathbf{x}_i$ is now all-zero, except at locations in $S$, where it equals the corresponding elements of $\mathbf{y}_i$.
9. Interact with the server to complete the evaluation of the rest of the network, invoking LinearizeReLU to force intervening ReLUs to behave linearly.
10. Set $K := [X_1,\ldots X_{k_i\times k_i}]$, where each $X_j$ is a formal variable.
11. Set $X$ to be the all-zero matrix of dimension equal to $R'$, except at locations in $S$, where it equals $R'$.
12. Compute $\mathbf{X}_\ell := M_\ell \cdot M_{\ell-1}\cdots M_{i+1}\cdot(KX)$.
13. Solve the linear system $\mathbf{X}_\ell = \mathbf{x}_\ell$ to learn the values of the formal variables $X_j$, and hence the kernel $K_i$.

[a]The pivot columns are linearly independent by definition, and row operations do not change linear dependence of columns.

RecoverFCLayer($M_{i+1},\ldots,M_\ell$) :

1. Let the dimension of the $i$-th linear layer be $s_i \times t_i$.
2. Set $M'_i$ to be a $s_i \times t_i$ matrix consisting of formal variables.
3. Let $s'_i := \lfloor s_i/m \rfloor$.
4. For each $j \in [t_i]$, and for each $k \in [s'_i]$:
   (a) Set the initial input to the network to be zero, i.e. $\mathbf{x}_1 := \mathbf{0}$.
   (b) Follow the inference protocol to evaluate the network up to the $i-1$-th layer to obtain (a share of) the intermediate state $\mathbf{x}_{i-1} := \mathsf{ReLU}(M_{i-1}(\ldots\mathsf{ReLU}(M_1\mathbf{x}_1))) = \mathbf{0}$.
   (c) Construct a query $\mathbf{q}_j := \mathbf{e}_j$.
   (d) Malleate the client's share of $\mathbf{x}_{i-1}$: $\langle\mathbf{x}'_{i-1}\rangle_C := \langle\mathbf{x}_{i-1}\rangle_C + \mathbf{q}_j$.
   (e) Interact with the server to evaluate the $i$-th linear layer to obtain a share of $\mathbf{y}_i := M_i\mathbf{x}'_{i-1}$.
   (f) Set $k' := k\cdot m$, and $S := \{k',\ldots,k'+m-1\}$.
   (g) Obtain the input for the next linear layer: $\langle\mathbf{x}_i\rangle_C := \mathsf{MaskAndLinearizeReLU}(\mathbf{y}_i, S)$.
   The vector $\mathbf{x}_i$ is now all-zero, except at locations in $S$, where it equals the corresponding elements of $\mathbf{y}_i$.
   (h) Interact with the server to complete the evaluation of the rest of the network, invoking LinearizeReLU to force intervening ReLUs to behave linearly.
   (i) Compute $\mathbf{X}_i$ as follows. First, compute $M'_i\cdot\mathbf{q}_j$, and then zero out all locations that are not in $S$.
   (j) Construct the $k$-th linear system $\mathbf{x}_\ell = M_\ell\cdot M_{\ell-1}\cdots M_{i+1}\cdot\mathbf{X}_i$.
5. Solve all the linear systems to recover the matrix $M_i$.