SNARKBlock: Federated Anonymous Blocklisting from Hidden Common Input Aggregate Proofs

Michael Rosenberg*
Department of Computer Science,
University of Maryland
micro@umd.edu

Mary Maller
Ethereum Foundation
mary.maller@ethereum.org

Ian Miers
Department of Computer Science,
University of Maryland
imiers@umd.edu

***Abstract*—Moderation is an essential tool to fight harassment and prevent spam. The use of strong user identities makes moderation easier, but trends towards strong identity pose serious privacy issues, especially when identities are linked across social media platforms. Zero-knowledge blocklists allow cross-platform blocking of users but, counter-intuitively, do not link users identities inter- or intra-platform, or to the fact they were blocked. Unfortunately, existing approaches (Tsang et al. '10), require that servers do work linear in the size of the blocklist for each verification of a non-membership proof.**

We design and implement SNARKBLOCK, a new protocol for zero-knowledge blocklisting with server-side verification that is logarithmic in the size of the blocklist. SNARKBLOCK is also the first approach to support ad-hoc, federated blocklisting: websites can mix and match their own blocklists from other blocklists and dynamically choose which identity providers they trust.

Our core technical advance, of separate interest, is HICIAP**, a zero-knowledge proof that aggregates n Groth16 proofs into one $O(\log n)$-sized proof which also shows that the input proofs share a common hidden input.**

I. INTRODUCTION

Moderation is a powerful tool for combating online harassment, trolling and spam messages. But banning an account on one platform has an obvious problem: it leaves the user free to post under other accounts and on other platforms. As a result, moderation tends towards stronger centralized *identity providers* (e.g., Facebook's real-name policy [Fac]) and the linking of disparate pseudonymous identities within and across platforms. Tying users' online speech to a centralized identity provider poses major problems for the decentralized web and user privacy, and can have a chilling effect on free speech.

Providing both privacy and moderation is a challenge: a *user* posting anonymously on a forum presents two problems to the forum operator, termed *service provider*: access control and revocation. First, because the user's identity is unknown at post submission, the service provider cannot verify that the user is authorized to post (i.e, isn't blocked). Second, because the user's identity is not linked to the post, and posts are not linked together, the service provider cannot revoke the user's posting permissions (i.e., block the user) if their current post violates forum policies.Linking posts together raises privacy concerns that may be undesirable on a single forum and are intolerable if applied across the web.

A. Zero-knowledge proofs of blocklist non-membership

BLAC [TAKS10] introduces the first solution to anonymous blocklisting without a trusted third party. It provides users with long-term identities and allows them to prove, in zero-knowledge, that they are not on a blocklist.

The approach introduced by BLAC, which we formalize as a *zero-knowledge blocklist* (ZKBL), is conceptually simple. A user's identity is a random PRF key k signed by an identity provider to ensure Sybil resistance. Anonymous comments and posts are associated with a tag $\mathsf{tag} := \mathsf{Prf}_k(\mathsf{nonce})$. A blocklist $\mathscr{L}$ consists of tuples of $(\mathsf{tag}, \mathsf{nonce})$ from offending posts. A user *attests* that they are not blocked by presenting a fresh $(\mathsf{tag}, \mathsf{nonce})$ pair and a zero-knowledge proof that 1) tag is computed correctly; 2) k is signed by a valid party; and 3) none of the blocklisted tags were generated by their k, i.e., $\forall (\mathsf{tag}', \mathsf{nonce}') \in \mathscr{L} : \mathsf{Prf}_k(\mathsf{nonce}') \neq \mathsf{tag}'$. A user is blocked by placing an offending $(\mathsf{tag}, \mathsf{nonce})$ pair on $\mathscr{L}$.

At its core, a ZKBL is a specialized zero-knowledge proof on the PRF evaluation, tag inequalities, and identity signature. Both security and privacy depend, mainly, on the zero-knowledge proof. This gives ZKBLs their main advantage: because the proofs are over arbitrary ban lists, the system is ad-hoc. We do not need a central party to coordinate bans as in [BCD$^+$17], [CL02],

*Work supported by a National Defense Science and Engineering Graduate (NDSEG) Fellowship.

[LLX07], [VB20], or worse, a trusted third party who can deanonymize people [Cha85], [Cv91], [BMW03]. If ZKBLs also support private federated identity, this is a major advantage for deployment.

B. Existing ZKBLs are impractical for both clients and servers

Unfortunately, all existing approaches for ZKBLs require the server to do linear work in the size of the blocklist when verifying a non-membership attestation. If the size of the blocklist and the number of authentications per second is proportional to the number of total users, then the service-provider's workload grows *quadratically* as their site scales. This is costly under normal circumstances and can be a major denial of service vector if an attacker can make concurrent posts or obtain Snbil accounts that are later banned.

Almost as problematically, proof sizes are also linear in the size of the blocklist. At 144B per list entry, a single non-membership proof for a 4M-entry blocklist would require a client to upload 549MiB of data over a residential or mobile connection.

C. Our contribution

We design, implement, and benchmark SNARKBLOCK, a protocol for zero-knowledge blocklists which improves on the state of the art by offering log-sized proofs and log-time verification.

Beyond improved performance for ZKBLs, SNARKBLOCK makes ZKBLs fully ad-hoc and resolves a privacy and organization problem with deployment. While ban lists can be operated by anyone, ZKBLs—like any even non-anonymous ban system—require Sybil-resistant identities. Existing ZKBLs assume a single trusted issuer for credentials. In reality, the existence of different issuers will lead to fragmentation of user's identities and also ban lists, reducing anonymity sets and hindering adoption.

SNARKBLOCK removes the need for a single centralized identity provider by allowing service providers to dynamically pick the identity providers that they support. This avoids coordination concerns and allows different providers to adopt different levels of Sybil resistance ranging from CAPTCHAs, to cryptocurrency payment, to real-world identity verification. Crucially, during attestation, the service provider learns only that the user's identity was issued by *some* party in their *accepted identity provider* set.

The core of SNARKBLOCK is a new type of zero-knowledge proof, called HIdden Common Input Aggregate Proofs, or HICIAP (pronounced "high-chop"). HICIAP is a zero-knowledge proof that aggregates n Groth16 [Gro16] proofs (of the same underlying circuit) into a single $O(\log n)$-sized proof, and shows that the aggregated circuits share a common input that is not revealed to the verifier. It is also possible to link multiple HICIAP proofs to show in zero-knowledge that they all share a common input. SNARKBLOCK uses HICIAP to aggregate *chunk proofs*—Groth16 proofs of non-membership in equally sized non-overlapping portions of the blocklist.

II. INTUITION FOR A ZKBL CONSTRUCTION

Zero-knowledge Succinct Non-interactive Arguments of Knowledge (zkSNARKs), appear to offer a path to ZKBLs with fast verification, but limitations on prover performance—a common problem for nearly all zkSNARKs—make this challenging. This is clear when one examines the costs of using Groth16, a zkSNARK scheme with notably fast verification times.

Existing zkSNARKs can only handle pieces of a blocklist. Producing a zero-knowledge proof of knowledge is, fundamentally, at least linear in the size of the input, i.e., the blocklist. But for Groth16 and other zkSNARKs, the concrete constants are high. Looking ahead, for a blocklist of 256 entries, a single proof of non-membership is 63k constraints and takes 2.84s. A blocklist of 2^{21} entries would yield a proof with 2^{29} constraints. With modern techniques, generating a 2^{29} constraint Groth16 proof takes over 3 hours on a 256-core cluster with 4TB of memory [WZC+18]. To use zkSNARKs for a ZKBL, we cannot have the prover do work linear in the size of the blocklist for each attestation.

Decomposing blocklists by chunk. We observe that a blocklist, mostly, does not change. While total prover workload is inevitably linear in the size of the blocklist, this work does not need to be recomputed from scratch every time. By breaking the list up into non-overlapping *chunks* we can both reuse work and limit the amount of recomputation required when the list changes.

A zero-knowledge proof for consistency between chunks. A sequence of chunk non-membership proofs for a blocklist $\mathscr{L}$ poses three problems:

1) The server would need to verify $O(|\mathscr{L}|)$ chunks.
2) Reusing a chunk proof across blocklist non-membership attestations would identify the client.
3) A malicious client could use a different identity when proving non-membership in a specific chunk, avoiding a block in that segment of the blocklist.

To address the above problems, we need a *compact* proof that a sequence of chunk non-membership proofs verifies with respect to the same identity. Further, that proof must be zero-knowledge to ensure that the chunk proofs can be safely reused across blocklist non-membership proofs.

The (im)practicality of IVC. Incrementally verifiable computation (IVC) is a natural solution here. In IVC, a SNARK verifies some state transition and checks proof(s) about previous state transitions. As we will see, making changes to the blocklist is impractical when using standard recursive IVC for Groth16 and similar zkSNARKs: an expired or reversed ban invalidates a portion of the previous state transitions, forcing costly proof recomputation.[1]

For Groth16 and similar zkSNARKs, IVC is realized as a tree of proofs where each node checks its children recursively, and the leaves are proofs of the underlying state transitions (in our case, a non-membership chunk proof). The shape of the tree determines the cost of a removal or addition from the blocklist.

In simple IVC(e.g., [BCG+13]), we have a maximally unbalanced tree (i.e., a chain) where each state transition proof checks the previous proof, plus one chunk non-membership proof. Additions to the end of the list are $O(1)$, but removals require worst case $O(|\mathscr{L}|)$ proof recomputations.

In *tree-based* IVC, the tree is balanced, and additions or removals from anywhere in the list require $O(\log|\mathscr{L}|)$ proof recomputations.

Finally, a special case, sometimes called *depth-1* IVC limits cryptographic costs by having a single parent that verifies all leaves(e.g., [CFH+15]).[2]Additions and removals both cost $O(|\mathscr{L}|)$ proof recomputations.

For Groth16-based IVC, these costs prove prohibitive. For a blocklist of 2^{21} elements with chunks of size 8,192, removing an element with simple IVC takes $\frac{2^{21}}{2^{13}} \times 16.58\text{s} = 71\text{min}$ in the worst case.[3] For depth-1 IVC on the same blocklist, a removal or addition costs 17min of recomputation.[4] For tree-based IVC on the same blocklist, a removal or addition costs 11min.[5]

Beyond generic IVC and aggregate proofs. We observe that IVC is not necessary to verify a sequence of chunk non-membership proofs. There no intermediate state in our computation, rather we only require that all proofs must share the same input private input. Recent advances in inner product proofs [BMM+20] give a succinct proof that n Groth16 zkSNARK proofs verify in aggregate. However, this aggregate approach has two critical shortcomings: it is neither zero-knowledge nor does it ensure consistency.

A natural approach for consistency would be to commit to the hidden value and use it as a public input to each Groth16 proof. But if the same commitment is used across multiple anonymous attestations, it forms a persistent identifier. On the other hand, when a fresh commitment is used for each attestation, we must regenerate every chunk proof.

We use [BMM+20] as a starting point and have a single *public* input to each chunk proof, then blind it in the aggregate proof so it is not revealed to the verifier. The resulting scheme reuses the same blinders in multiple parts of the zero-knowledge protocol. This unusual property made proving honest-verifier zero-knowledge challenging.

III. Preliminaries

We write $x := z$ to denote variable assignment, and $y \leftarrow S$ to denote sampling uniformly from a set S. For an arbitrary, efficiently computable predicate P, we say that *a proof of knowledge of a relation* $R = \{(\mathbf{x};\mathbf{w}) : P(\mathbf{x},\mathbf{w})\}$ with respect to an *instance* $\mathbf{x}$ is a proof of knowledge of the *witness* $\mathbf{w}$ such that $P(\mathbf{x},\mathbf{w})$ is satisfied. We will often refer to $\mathbf{x}$ as a *public input* and $\mathbf{w}$ as a *private input*, and we will use zero-knowledge proofs of knowledge for various relations in order to hide $\mathbf{w}$ from the verifier. For 2-party protocols, $\langle \mathsf{A},\mathsf{B}\rangle$ denotes the transcript of a protocol execution between algorithms A and B. The security parameter of our system is denoted by λ.

A. Notation for Groups and Pairings

We will work exclusively with prime-order groups and their associated scalar fields. Group elements are denoted with capital letters $G \in \mathbb{G}$, while field elements are lowercase $r \in \mathbb{F}$. *Vectors* are bolded: $\mathbf{A} \in \mathbb{G}^n$, and $\mathbf{r} \in \mathbb{F}^n$. We write $\mathbf{A}_{[:k]}$ to denote the first k elements of $\mathbf{A}$, and $\mathbf{A}_{[k:]}$ to denote the last k elements. Following convention, we use additive notation for $\mathbb{G}_1$ and $\mathbb{G}_2$, and multiplicative notation for $\mathbb{G}_T$. We say that a bilinear function $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ is a *type-3 bilinear pairing* if there is no efficiently computable group homomorphism from $\mathbb{G}_2$ to $\mathbb{G}_1$. We say e is *degenerate* if there is a non-identity $G \in \mathbb{G}_1$ such that $e(G,H) = 1$ for all $H \in \mathbb{G}_2$.

For vectors $\mathbf{A} \in \mathbb{G}_1^n$ and $\mathbf{B} \in \mathbb{G}_2^n$ and a bilinear pairing we write $\mathbf{A} * \mathbf{B}$ to denote the *inner pairing product* $\prod_{i=1}^n e(A_i, B_i)$. For vectors $\mathbf{A} \in \mathbb{G}^n$ and $\mathbf{r} \in \mathbb{F}^n$ we write $\mathbf{A}^{\mathbf{r}}$ to denote the *multiscalar multiplication* (MSM) $\sum_{i=1}^n r_i A_i$, and write $\mathbf{r} \odot \mathbf{A}$ to denote the element-wise multiplication $(r_1A_1, \ldots, r_nA_n)$. For a field element $x \in \mathbb{F}$, we denote $[x]_1 := xG$ and $[x]_2 := xH$, where G and H are the canonical generators of $\mathbb{G}_1$ and $\mathbb{G}_2$, respectively.

B. Groth16

We briefly describe the trusted-setup zkSNARK scheme defined in [Gro16]. At a high level, given a

[1]For anonymity, ban removals cannot be ignored. If each user presented a proof that began at the first point they were able to attest, then each attestation would leak either the date the user created their account or the date their ban expired.

[2]Unlimited recursion has a 5-7× performance penalty for proving time [CCDW20] because of the need to switch to the MNT4/6 curve cycle. Limiting recursion to depth-1 reduces this cost.

[3]Proving that one lower proof with no inputs verifies takes 16.58s using MNT6-753 over MNT4-753.

[4]Proving that a single lower proof with no inputs verifies takes 3.97s on our on our benchmark system, using BW6-761 over BLS12-377.

[5]Proving that two lower proofs with no inputs both verify takes 50.39s on our on our benchmark system, using MNT6-753 over MNT4-753.

description of an arithmetic circuit (over the scalar field of a pairing-friendly elliptic curve), a Groth16 proof proves that a circuit is satisfied by a set of public wires (values known to the verifier) and private wires (values which are not known to the verifier, also called *witness elements*).

Let $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ be an efficiently-computable, non-degenerate, type-3 bilinear pairing, where $|\mathbb{G}_1| = |\mathbb{G}_2| = |\mathbb{G}_T|$ is a prime p and $p > 2^\lambda$. Let G denote a generator for $\mathbb{G}_1$ and H denote a generator for $\mathbb{G}_2$. We use $\mathbb{F}$ to denote the finite field $\mathbb{Z}/p\mathbb{Z}$. The Groth16 scheme defines four procedures:

$\mathsf{Setup}(\mathsf{desc}) \to \mathsf{crs}$ Generates a common reference string for the given arithmetic circuit description. crs contains the group elements necessary to compute the expressions in Groth16.Prove below.

$\mathsf{Prove}(\mathsf{crs}, \{a_i\}_{i=0}^{\ell}, \{a_i\}_{i=\ell+1}^{m}) \to \pi$ Proves the circuit described by crs is satisfied, where $a_0, \dots, a_\ell \in \mathbb{F}$ represent the circuit's public input wires and $a_{\ell+1}, \dots, a_m \in \mathbb{F}$ represent the private wires. π is of the form $([\eta]_1, [\theta]_2, [\iota]_1)$, where

$$\eta = \alpha + \sum_{i=0}^{m} a_i u_i(X) + r\delta \quad \theta = \beta + \sum_{i=0}^{m} a_i v_i(X) + s\delta$$

$$\iota = \sum_{i=\ell+1}^{m} \frac{a_i(\beta u_i(X) + \alpha v_i(X) + w_i(X)) + h(X)t(X)}{\delta} + \eta s + \theta r - rs\delta$$

and all otherwise unspecified constants and polynomials come from crs.

$\mathsf{Prepare}(\mathsf{crs}, \{a_{i_j}\}_{j=1}^{t}) \to \hat{S}$ Aggregates any subset of public inputs into a single group element called a *prepared input*: $\hat{S} = \sum_{j=1}^{t} a_{i_j} W_{i_j}$, where W_i are the CRS values whose coefficient represents the value of the i-th wire of the circuit.

$\mathsf{Vfy}(\mathsf{crs}, \pi, \{a_0\}_{i=0}^{\ell}) \to \{0,1\}$ Verifies the proof $\pi = (A, B, C)$ by checking the relation,

$$e(A,B) \stackrel{?}{=} e([\alpha]_1, [\beta]_2) \cdot e(C, [\delta]_2) \cdot \prod_{i=0}^{\ell} e(a_i W_i, [\gamma]_2),$$

where $[\alpha]_1$, $[\beta]_2$, $[\gamma]_2$, and $[\delta]_2$ come from crs. Vfy permits any subset of the public inputs to be prepared as above. The common case will be where all but the first input is prepared, i.e., calls of the form $\mathsf{Vfy}(\mathsf{crs}, \pi, (a_0, \hat{S}))$.

$\mathsf{Rerand}(\pi) \to \pi'$ Rerandomizes the proof $\pi = (A, B, C)$ by sampling $\zeta, \omega \leftarrow \mathbb{F}$ and computing

$$\pi' := (\zeta^{-1} A, \zeta B + \zeta\omega [\delta]_2, C + \omega A).$$

By Theorem 3 in [BKSV20], the output of Rerand is statistically indistinguishable from a fresh proof of the same underlying statement.

$$R_{\mathsf{TIPP}} := \left\{ \begin{array}{c} \left(\begin{array}{c} \mathsf{ck} \in \mathbb{G}_2^n \times \mathbb{G}_1^n \times \mathbb{G}_2, \\ \mathsf{com}_A, \mathsf{com}_B, \mathsf{agg}_{AB} \in \mathbb{G}_T, \\ \mathbf{r} \in \mathbb{F}^n ; \mathbf{A} \in \mathbb{G}_1^n, \mathbf{B} \in \mathbb{G}_2^n \end{array} \right) : \\ \mathsf{com}_A = \mathbf{A} * \mathsf{ck}_1 \\ \bigwedge \quad \mathsf{com}_B = \mathsf{ck}_2 * \mathbf{B} \\ \bigwedge \quad \mathsf{agg}_{AB} = \mathbf{A}^{\mathbf{r}} * \mathbf{B} \end{array} \right\}$$

$$R_{\mathsf{MIPP}-k} := \left\{ \begin{array}{c} \left(\begin{array}{c} \mathsf{ck} \in \mathbb{G}_2^n \times \mathbb{G}_1^n \times \mathbb{G}_2, \\ \mathsf{com}_C, \in \mathbb{G}_T, \mathsf{agg}_C \in \mathbb{G}_1, \\ \mathbf{r} \in \mathbb{F}^n ; \mathbf{C} \in \mathbb{G}_1^n \end{array} \right) : \\ \mathsf{com}_C = (\mathbf{C} * \mathsf{ck}_1) \bigwedge \mathsf{agg}_C = \mathbf{C}^{\mathbf{r}} \end{array} \right\}$$

$$R_{\mathsf{HMIPP}} := \left\{ \begin{array}{c} \left(\begin{array}{c} \mathsf{ck} \in \mathbb{G}_2^n \times \mathbb{G}_1^n \times \mathbb{G}_2, \\ \mathsf{com}_C, \in \mathbb{G}_T, \mathsf{agg}_C \in \mathbb{G}_1, \\ \mathbf{r} \in \mathbb{F}^n ; \mathbf{C} \in \mathbb{G}_1^n, z \in \mathbb{F} \end{array} \right) : \\ \mathsf{com}_C = e([z]_1, \mathsf{ck}_3) \cdot (\mathbf{C} * \mathsf{ck}_1) \\ \bigwedge \quad \mathsf{agg}_C = \mathbf{C}^{\mathbf{r}} \end{array} \right\}$$

Fig. 1: We directly use Bünz et al.'s definition of R_{TIPP} and $R_{\mathsf{MIPP}\text{-}k}$, and we use R_{HMIPP} to refer to the "hiding commitment" version of $R_{\mathsf{MIPP}\text{-}k}$. While R_{HMIPP} admits a zero-knowledge proof of knowledge, R_{TIPP} does not, as it fails to hide the witnesses $\mathbf{A}$ and $\mathbf{B}$. Patching this is one of the primary focuses of HICIAP.

C. *Inner product proofs*

Bünz et al. [BMM+20] introduce a proof system for various inner product relations. We will make use of the TIPP, MIPP_k, and HMIPP proof systems, whose relations are defined in Figure 1.

In short, R_{TIPP} is satisfied when $\mathbf{A}^{\mathbf{r}} * \mathbf{B} = \mathsf{agg}_{AB}$, $R_{\mathsf{MIPP}-k}$ is satisfied when $\mathbf{C}^{\mathbf{r}} = \mathsf{agg}_C$, and R_{HMIPP} is the same as $R_{\mathsf{MIPP}-k}$ except its commitment to $\mathbf{C}$ is hiding.

D. *HICIAP*

Since HICIAP is used extensively in the construction of SNARKBLOCK, we provide a brief overview of its functionality here. We defer discussion of these algorithms including their construction and security claims until Section VI.

HICIAP is a zkSNARK which aggregates multiple Groth16 proofs of the same relation. Of its aggregated proofs, it proves that 1) they verify with respect to verifier-supplied public inputs, and 2) they share a common public input element (which is hidden by the aggregate proof). In addition, HICIAP can *link* aggregate proofs: it can prove in zero-knowledge that the common input element of each HICIAP proof in a given set is equal.

Formally, HICIAP consists of six procedures:

$\mathsf{GenCk}(n) \to (\mathsf{ck}, \mathsf{srs})$ Generates a commitment key $\mathsf{ck} \in \mathbb{G}_2^n \times \mathbb{G}_1^n \times \mathbb{G}_2$ and a (short) structured verification key srs which can be used, respectively, to prove and verify HICIAP aggregates of up to $n-2$ Groth16 proofs.

$\mathsf{Com}(\mathsf{ck}, \hat{\mathbf{S}}) \to \mathsf{com}_{\mathsf{in}}$ Constructs a commitment to the prepared Groth16 public inputs $\hat{\mathbf{S}} \in \mathbb{G}_1^{n-2}$ as $\mathsf{com}_{\mathsf{in}} := \hat{\mathbf{S}} * \mathsf{ck}_{1,[:n-2]}$.

$\mathsf{Prove}((\mathsf{ck}, \mathsf{crs}), \hat{\mathbf{S}}, (a_0, \{\pi_i\}_{i=1}^{n-2})) \to (\hat{\pi}, o)$ Produces a succinct proof that each Groth16 proof π_i verifies w.r.t. the common witness element $a_0 \in \mathbb{F}$, the prepared input $\hat{S}_i \in \mathbb{G}_1$, and the given Groth16 CRS. Also produces an opening o to a commitment to a_0 contained inside $\hat{\pi}$. The opening is used in LinkProve.

$\mathsf{Vfy}(\mathsf{srs}, \hat{\pi}, \mathsf{com}_{\mathsf{in}}) \to \{0,1\}$ verifies the given aggregate proof w.r.t. the committed public input. Alternatively, a set of prepared Groth16 inputs can be passed instead of $\mathsf{com}_{\mathsf{in}}$.

$\mathsf{LinkProve}(\{\hat{\pi}_i\}_{i=1}^t, (a_0, \{o_i\}_{i=1}^t)) \to \pi_{\mathsf{link}}$ Using the openings o_i, produces a proof that the given aggregate proofs share the witness element $a_0 \in \mathbb{F}$.

$\mathsf{LinkVfy}(\pi_{\mathsf{link}}, \{\hat{\pi}_i\}_{i=1}^t) \to \{0,1\}$ Verifies the link proof w.r.t. the given aggregate proofs.

IV. Zero-knowledge blocklists

We now give our framework for zero-knowledge blocklists, taken directly from BLAC [TAKS10], but with modifications to support multiple identity providers and allow for additional precomputation.

A. Setting

A zero-knowledge blocklist allows users to attest that an *identity* issued by one of a set of *identity providers* is not in a *blocklist*. We now detail these concepts:

Identity. We use k to denote a user's private *identity*. A single user in the real world can hold arbitrarily many identities. In all cases, k will be a field element uniformly selected by the user. Other similar schemes refer to k as a user's "nym," "pseudonym," or "credential."

Identity providers. Blocking users fundamentally depends on identities being Sybil-resistant. As with previous work, SnarkBlock depends on issuers to ensure Sibyl resistance, but unlike previous work, SnarkBlock does not assume a single issuer. Instead each service provider is allowed to maintain its own list of accepted identity providers, which we call the *AIP set* and denote by $\mathscr{I}$. Identity providers are responsible for ensuring Sybil resistance. The service provider is allowed to update this set over time, and should distribute it via the same channels it uses to distribute its blocklist.

Blocklists and session tuples. A ZKBL *blocklist* consists of pairs containing a *session nonce* nonce and *session tag* tag, where tag is bound to the user's identity by $\mathsf{tag} := \mathsf{Prf}_k(\mathsf{nonce})$ for some fixed pseudorandom function Prf. Blocklist entries can support *context binding* via structured auxiliary data. By computing nonce as $\mathsf{nonce} := \mathsf{H}(\mathsf{aux}, r)$ for some hash H, *aux* is bound to the attestation. This data can be used to, for example, bind attestation to an action (e.g., to prove that the blocked user is the action's author) or to a particular blocklist or policy (e.g., to enforce which lists a banned tuple can be transfered to).

Finally, in a departure from BLAC, we allow blocklists to be split into *chunks*—equally sized non-overlapping segments—whose sizes are decided by the service provider. Blocklists are chunked so that users can precompute non-membership proofs over individual chunks rather than the entire blocklist at once.

Formalizing non-membership proofs. A non-membership proof π_{zkbl} is a zero-knowledge proof of three distinct properties.

1) *Issuance.* That the user's identity k is signed by an identity provider.
2) *Tag well-formedness.* That tag and nonce are honestly computed, i.e., $\mathsf{tag} = \mathsf{Prf}_k(\mathsf{nonce})$.
3) *Blocklist non-membership.* That the user's identity k did not generate any tuples already on a blocklist, i.e., $\forall(\mathsf{tag}', \mathsf{nonce}') \in \mathscr{L} : \mathsf{tag}' \neq \mathsf{Prf}_k(\mathsf{nonce}')$.

B. ZKBL functionality

A zero-knowledge blocklist consists of five algorithms.

CRS-Setup Generates system-wide parameters.

IdP-Keygen The identity provider generates a keypair $(\mathsf{sk}, \mathsf{pk})$ for a digital signature scheme.

IssueReq/Issue A protocol for users to register and receive a signature on their identity from an identity provider.

Sync A user periodically syncs with the service providers it uses, and performs some of its proof computations offline. To sync with a service provider, a user fetches the latest additions to the blocklist and then precomputes cryptographic material for them.

Attest Attestation is a non-interactive protocol executed by a user. First, the user constructs a session-specific tuple $(\mathsf{tag}, \mathsf{nonce})$ as $\mathsf{tag} := \mathsf{Prf}_k(\mathsf{nonce})$, where nonce is pseudorandom and optionally binds a context aux. This tuple can be used by the service provider to block the user at any point in the future by simply including it in the blocklist. The user then produces a zero-knowledge proof π_{zkbl} that proves well-formedness of the tuple and that their (signed) identity did not generate any tuples already on a blocklist. The session tuple and zero-knowledge proof are then sent to the service provider as $(\pi_{\mathsf{zkbl}}, \mathsf{tag}, \mathsf{nonce})$.

Verify The service provider accepts an attestation if and only if π_{zkbl} verifies with respect to the supplied session tuple $(\mathsf{tag}, \mathsf{nonce})$, the blocklist $\mathscr{L}$, the chunk size schedule, and the AIP set $\mathscr{I}$, and the optional context-binding string aux.

Separately, we assume two non-cryptographic operations for blocklist management.

Blocklist-Add Adds a token to a blocklist

Blocklist-Remove Removes a token from a blocklist

We stress that the Add and Remove routines are distinct from the cryptographic scheme, and can be run by anyone. How parties decide to manage their blocklists is wholly orthogonal to SNARKBLOCK.

BLAC as a ZKBL. The authors of BLAC construct their scheme using BBS+ signatures [BBS04] and a Camenisch-Shoup Σ-protocol [CS03]. This is the same PRF approach we take but with a different PRF and ZKP. Their tag function is $\mathsf{nonce} \mapsto \mathsf{H}(\mathsf{nonce})^k$, and it is done in two steps with the hash evaluation outside the ZKP and only the exponentiation witnessed inside. Conceptually, the entire question for designing a practical ZKBL is how to co-design a PRF and zero-knowledge proof protocol to make an efficient non-membership proof.

C. Security requirements

Our desired security properties are taken from BLAC. For the complete definitions see [TAKS10]. Note the following aesthetic changes in our description: blocklistability encompasses misauthentication resistance; and anonymity is described as a distinguishability notion as opposed to a simulatability notion which we believe better captures the actual security properties achieved by BLAC's game-based definition.

Blocklistability A coalition of dishonest service providers and users can only successfully authenticate to an honest service provider if that user holds a valid credential issued by an identity provider that is not included in the blocklist.

Non-Frameability A coalition of dishonest identity providers, service providers and users cannot prevent an honest, non-blocklisted user from successfully authenticating with an honest service provider.

Anonymity A coalition of dishonest identity providers, service providers, and users cannot distinguish attestation transcripts associated with any two honest users. Further, no such coalition can link any given authentication transcript with the registration in which an identity provider issued the associated credential.

V. SNARKBLOCK DESIGN AND OVERVIEW

The full design of SNARKBLOCK is detailed in Figure 3. The core relations we use are used are defined in Figure 2. In words, the R_{isu} relation is satisfied when a user's identity is signed by an issuer in the AIP set, R_{tag} is satisfied when is computed correctly, and R_{chunk} is satisfied when a user did not produce any of the tags in a chunk.

$$R_{\mathsf{isu}} := \left\{ \begin{array}{c} (k, (\mathsf{pk}_i)_{i=1}^{\ell}\,;\, i^*, \sigma, r) : \\ \mathsf{Schnorr.Ver}_{\mathsf{pk}_{i^*}}(\mathsf{Com}(k, r), \sigma) \end{array} \right\}$$

$$R_{\mathsf{tag}} := \{(k, \mathsf{tag}, \mathsf{nonce}) : \mathsf{Prf}_k(\mathsf{nonce}) = \mathsf{tag}\}$$

$$R_{\mathsf{chunk}} := \left\{ (k, \mathsf{chunk}) : \bigwedge_{(\mathsf{tag},\mathsf{nonce}) \in \mathsf{chunk}} \mathsf{Prf}_k(\mathsf{nonce}) \neq \mathsf{tag} \right\}$$

$$R_{\mathsf{zkbl}} := \left\{ \begin{array}{ll} & (\mathscr{L}, \mathscr{I}, \mathsf{tag}, \mathsf{nonce}\,;\, k, i^*, \sigma, r) : \\ & R_{\mathsf{isu}}(k, \mathscr{I}\,;\, i^*, \sigma, r) \\ \bigwedge & R_{\mathsf{tag}}(k, \mathsf{tag}, \mathsf{nonce}) \\ \bigwedge_{i=1}^{c} & R_{\mathsf{chunk}}(k, \mathsf{chunk}_i) \end{array} \right\}$$

Fig. 2: R_{zkbl} is the relation which the attestation procedure in SNARKBLOCK attests to. $\mathscr{I}$ is the AIP set $\{\mathsf{pk}_1, \ldots, \mathsf{pk}_\ell\}$, and $\mathscr{L}$ is the set of chunks $\{\mathsf{chunk}_1, \ldots, \mathsf{chunk}_c\}$. Note that k is a public (rather than private) input to the three sub-relations R_{isu}, R_{tag}, and R_{chunk}. This is because the implementation of HICIAP requires that the hidden common input be a public input in the underlying Groth16 proof.

We omit textual descriptions of the full set of algorithms and detail the two key ones: Sync and Attest.

Sync. Sync is the offline phase of attestation. During Sync a client fetches the most recent versions of the service provider's blocklist, chunk schedule, and AIP set. The client then precomputes Groth16 chunk proofs π_{chunk_i} of the relation $R_{\mathsf{chunk}}(k, \mathsf{chunk}_i)$ for every new chunk_i received from the service provider. The client also precomputes π_{isu}, by computing a Groth16 proof π_{isu} of $R_{\mathsf{isu}}((k, \mathscr{I}), (i^*, \sigma, r))$ where i^* is the chosen identity provider in the AIP set $\mathscr{I} = \{\mathsf{pk}_1, \ldots, \mathsf{pk}_\ell\}$, σ is the identity provider's signature of the identity commitment, and r is the randomness used to commit to k.

Attest. To attest to blocklist non-membership, the client must combine a series of proofs about the user's identity k. First the client computes fresh session tuple $(\mathsf{tag}, \mathsf{nonce})$ and proves it is well-formed with respect to k using a Groth16 proof π_{tag} for the relation $R_{\mathsf{tag}}(k, \mathsf{tag}, \mathsf{nonce})$.

Ideally, the client would combine this proof about k with the precomputed π_{isu} proof and chunk proofs from Sync. But a single HICIAP instance only works for proofs over the same relation. Thus, each proof is wrapped in a HICIAP instance and then a linking proof, π_{link}, is used to show each aggregate is made with respect to the same identity k.

$\underline{\mathsf{IdPKeyGen}()}$
$(\mathsf{sk}, \mathsf{pk}) := \mathsf{Schnorr.KeyGen}()$
$\textbf{return } (\mathsf{sk}, \mathsf{pk})$

$\underline{\mathsf{IssueReq}(k)}$
$r \leftarrow \mathbb{F}$
$\mathsf{com} := \mathsf{Com}(k, r)$
$\textbf{return } \mathsf{com}$

$\underline{\mathsf{Issue}(\mathsf{sk}, \mathsf{com})}$
$\sigma := \mathsf{Schnorr.Sign}_{\mathsf{sk}}(\mathsf{com})$
$\textbf{return } \sigma$

$\underline{\mathsf{CrsSetup}(n)}$
$\mathsf{crs}_{\mathsf{isu}} := \mathsf{Groth16.Setup}(R_{\mathsf{isu}})$
$\mathsf{crs}_{\mathsf{tag}} := \mathsf{Groth16.Setup}(R_{\mathsf{tag}})$
$\mathsf{crs}_{\mathsf{chunk}} := \mathsf{Groth16.Setup}(R_{\mathsf{chunk}})$
$\mathsf{ck}, \mathsf{srs} := \mathsf{HICIAP.GenCk}(n)$
$\textbf{return } (\mathsf{ck}, \mathsf{srs})$

$\underline{\mathsf{Attest}(k, \pi_{\mathsf{isu}}, \{\pi_{\mathsf{chunk}_i}\}_{i=1}^{c})}$
$\mathsf{nonce} \leftarrow \{0,1\}^{\lambda}$
$\mathsf{tag} := \mathsf{Prf}_k(\mathsf{nonce})$
$\pi_{\mathsf{tag}} := \mathsf{Groth16.Prove}(\mathsf{crs}_{\mathsf{tag}}, (k, \mathsf{tag}, \mathsf{nonce}), \cdot)$
$\hat{\pi}_{\mathsf{isu}} := \mathsf{HICIAP.Prove}((\mathsf{ck}, \mathsf{crs}_{\mathsf{isu}}), \mathscr{I}, (k, \{\pi_{\mathsf{isu}}\}))$
$\hat{\pi}_{\mathsf{tag}} := \mathsf{HICIAP.Prove}((\mathsf{ck}, \mathsf{crs}_{\mathsf{tag}}), (\mathsf{tag}, \mathsf{nonce}), (k, \{\pi_{\mathsf{tag}}\})))$
$\hat{\pi}_{\mathsf{chunk}} := \mathsf{HICIAP.Prove}\left(\begin{array}{c} (\mathsf{ck}, \mathsf{crs}_{\mathsf{chunk}}), \mathscr{L}, \\ (k, \{\pi_{\mathsf{chunk}_i}\}_{i=1}^{c}) \end{array} \right)$
$\pi_{\mathsf{link}} := \mathsf{HICIAP.LinkProve}(k, (\pi_{\mathsf{isu}}, \pi_{\mathsf{chunk}}, \pi_{\mathsf{tag}}), k)$
$\pi_{\mathsf{zkbl}} := (\pi\mathsf{link}, \hat{\pi}_{\mathsf{isu}}, \hat{\pi}_{\mathsf{tag}}, \hat{\pi}_{\mathsf{chunk}})$
$\textbf{return } (\pi_{\mathsf{zkbl}}, \mathsf{tag}, \mathsf{nonce})$

$\underline{\mathsf{Sync}(\{\mathsf{chunk}_i\}_{i=s'}^{c}, \mathscr{I}, i^*, k, \sigma, r)}$
$\textbf{for } s' \leq j \leq s:$
$\quad \pi_{\mathsf{chunk}_j} := \mathsf{Groth16.Prove}(\mathsf{crs}_{\mathsf{chunk}}, (k, \mathsf{chunk}_j), \cdot)$
$\pi_{\mathsf{isu}} := \mathsf{Groth16.Prove}(\mathsf{crs}_{\mathsf{isu}}, (k, \mathscr{I}), (i^*, \sigma, r))$
$\textbf{return } \{\pi_{\mathsf{chunk}_1}, \ldots, \pi_{\mathsf{chunk}_c}\}$

$\underline{\mathsf{PrepBlocklist}(\{\mathsf{chunk}_i\}_{i=1}^{c})}$
$\textbf{for } 1 \leq i \leq s$
$\quad \hat{S}_{\mathsf{chunk}_i} := \mathsf{Groth16.Prepare}(\mathsf{crs}_{\mathsf{chunk}}, \mathsf{chunk}_i)$
$\mathsf{com}_{\mathscr{L}} := \mathsf{HICIAP.Com}(\mathsf{ck}, \{\hat{S}_{\mathsf{chunk}_i}\}_{i=1}^{c})$
$\textbf{return } \mathsf{com}_{\mathscr{L}}$

$\underline{\mathsf{Vfy}(\pi_{\mathsf{zkbl}}, (\mathsf{tag}, \mathsf{nonce}), \mathscr{I}, \mathsf{com}_{\mathscr{L}})}$
$\hat{S}_{\mathsf{tag}} = \mathsf{Groth16.Prepare}(\mathsf{crs}_{\mathsf{tag}}, (\mathsf{tag}, \mathsf{nonce}))$
$\hat{S}_{\mathsf{isu}} = \mathsf{Groth16.Prepare}(\mathsf{crs}_{\mathsf{isu}}, \mathscr{I})$
$\textbf{return } \mathsf{HICIAP.LinkVfy}(\pi_{\mathsf{link}}, (\hat{\pi}_{\mathsf{isu}}, \hat{\pi}_{\mathsf{tag}}, \hat{\pi}_{\mathsf{chunk}}))$
$\bigwedge \quad \mathsf{HICIAP.Vfy}(\mathsf{srs}, \hat{\pi}_{\mathsf{isu}}, \{\hat{S}_{\mathsf{isu}}\})$
$\bigwedge \quad \mathsf{HICIAP.Vfy}(\mathsf{srs}, \hat{\pi}_{\mathsf{tag}}, \{\hat{S}_{\mathsf{tag}}\})$
$\bigwedge \quad \mathsf{HICIAP.Vfy}(\mathsf{srs}, \hat{\pi}_{\mathsf{chunk}}, \mathsf{com}_{\mathscr{L}})$

Fig. 3: A pseudocode definition of the SNARKBLOCK system

The client's output is thus $(\pi_{\mathsf{zkbl}}, \mathsf{tag}, \mathsf{nonce})$, where $\pi_{\mathsf{zkbl}} := (\hat{\pi}_{\mathsf{isu}}, \hat{\pi}_{\mathsf{tag}}, \hat{\pi}_{\mathsf{chunk}}, \pi_{\mathsf{link}})$.

Buffering recent blocklist additions and deletions. When a ban is added or removed from the blocklist, the user must redo the corresponding chunk proof. It is inevitable between Sync operations that some number of additions and deletions will occur, thus requiring recomputation during attest and adding the corresponding amount of latency. The larger the chunk size, the longer the latency. While we can avoid this for deletions by having bans expire in batches, this is undesirable for additions—we want bans to take affect as soon as possible.

To avoid a tradeoff between chunk size and attestation latency, we have the tail of the list be a buffer of smaller chunks and have a separate instance of HICIAP prove they are correct. Because the circuit is different from the circuit used for larger chunks, this optimization increases the number of distinct HICIAP proofs passed to the verifier, while decreasing the overall attestation time.

A. *Security argument*

Security of SNARKBLOCK depends on it correctly instantiating the PRF+Sig+ZKP paradigm using HICIAP. We state the theorem of security for SNARKBLOCK here and give a proof sketch in Appendix B. This proof depends on the security of HICIAP as a building block, and so HICIAP is the main focus of our security analysis over subsequent sections and appendices.

Theorem 1 (SNARKBLOCK Security)**.** SNARKBLOCK *described in Figure 2 is blocklistable, anonymous and non-frameable provided that Groth16 and* HICIAP *proofs are knowledge sound and subversion zero-knowledge; Schnorr signatures are unforgeable;* Prf *is pseaudorandom; and* Com *is binding and hiding.*

Looking ahead, in the concrete instantiation, this in turn assumes the key-prefixed Poseidon hash function is a PRF and, for Groth16 that the Algebraic Group Model [FKL18], q-SDH [BB04] and q-DDH [BB04] assumptions hold. For HICIAP we also depend on the Auxiliary Structured Double Pairing assumption [BMM+20].

B. Trusted setup

Our protocol and security proof assumes a trusted party generates a CRS both for each Groth16 circuit and HICIAP. The CRSs are similar, being of the form $g^s, g^{s^2}, \ldots$ for several bases. It seems plausible that parameters generated by a reputable party would be trusted for this application. If necessary, protocols [BGM17], [BCG+15] for multiparty setup have been used for commercial cryptocurrencies such as Zcash [Rad21] and Filecoin, where failure would allow the forgery of billions of dollars.

VI. HICIAP

We now introduce the core of SNARKBLOCK: HIdden Common-Input Aggregate Proofs (HICIAP), a novel zkSNARK scheme which we use to generate the zero-knowledge proof of blocklist non-membership π_{zkbl}.

Recall the purpose of HICIAP is to aggregate multiple Groth16 proofs of the same relation. Of its aggregated proofs, it proves that 1) they verify with respect to verifier-supplied public inputs, and 2) they share a common public input element (which is hidden by the aggregate proof). In the case of SNARKBLOCK, the relation is chunk non-membership, the verifier-supplied public inputs are the (prepared) chunk contents, and the common input element is the user's identity.

In addition, HICIAP can *link* aggregate proofs: it can prove in zero-knowledge that the common input element of each HICIAP proof in a given set is equal. In the case of SNARKBLOCK, there are three aggregate proofs that are linked: chunk non-membership, issuance, and tag well-formedness.[6]

In this section, we provide intuition for the design of HICIAP and then describe each of its components in detail.

A. Intution

To explain HICIAP, we start with a naïve verifier who takes a full (non-succinct and non-hiding) set of Groth16 proofs $\pi_i = (A_i, B_i, C_i)$ and checks they verify with respect to a common public input. We transform this into a succinct zero-knowledge proof that vector commitments to A, B and C contain proofs that verify with respect to a hidden input. For simplicity, we omit the blinding factors from discussion, and leave their detailed accounting to the proof of honest verifier zero-knowledge in Appendix B.

The HICIAP verifier must be convinced that there is some hidden wire value $a_0 \in \mathbb{F}$ for which a set Groth16 proofs (A_i, B_i, C_i) verify given a set of prepared public inputs $\hat{\mathbf{S}}$, i.e.,

$$e(A_i, B_i) = e([\alpha]_1, [\beta]_2) \cdot e(C_i, [\delta]_2) \cdot e\left(a_0 W_0 + \hat{S}_i, [\gamma]_2\right)$$

for all $i = 1 \ldots, n-2$.

Our first step is to combine the above $n-2$ checks into a single polynomial equation. Checking this would require the verifier to do linearly many equality checks, so the verifier picks a random $r \leftarrow \mathbb{F}$ and evaluates the polynomial "in the exponent" at the random point. Combining these two steps, the new verifier equation is

$$\begin{aligned} &\prod e(A_i, B_i)^{r^i} \\ &\stackrel{?}{=} \prod e([\alpha]_1, [\beta]_2)^{r^i} \cdot \prod e(C_i, [\delta]_2)^{r^i} \\ &\quad \cdot \prod e\left(a_0 W_0 + \hat{S}_i, [\gamma]_2\right)^{r^i}. \end{aligned}$$

By the Schwartz-Zippel lemma, equality here implies the equality of the initial $n-2$ equations with overwhelming probability. We now have one equality check.

The next step is to make the verifier oblivious to a_0. To do that, we split the $e(a_0 W_0 + \hat{S}_i, [\gamma]_2)^{r^i}$ term in two. The prover sends W, a blinded version of $\sum r^i a_0 W_0$, to the verifier. It proves that W is computed correctly using an instance of the Σ-protocol HWW. The verifier equation is now

$$\begin{aligned} &\prod e(A_i, B_i)^{r^i} \\ &\stackrel{?}{=} \prod e([\alpha]_1, [\beta]_2)^{r^i} \cdot \prod e(C_i, [\delta]_2)^{r^i} \cdot e(W, [\gamma]_2) \\ &\quad \cdot \prod e\left(\hat{S}_i, [\gamma]_2\right)^{r^i}. \end{aligned}$$

For both succinctness and privacy, the prover cannot give the verifier every Groth16 proof. Instead, the prover gives only succinct commitments, $\mathsf{com}_A, \mathsf{com}_B, \mathsf{com}_C$ to the proof vectors $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, respectively. This requires the prover to calculate $\mathsf{agg}_{AB} := \prod e(A_i, B_i)^{r^i}$ and $\mathsf{agg}_C := \prod e(C_i, [\delta]_2)^{r^i}$ itself and send them to the verifier. Since these calculations can be expressed as inner product operations, the prover shows they are correct using instances of TIPP and HMIPP, respectively. The verifier equation is now

$$\begin{aligned} &\mathsf{agg}_{AB} \\ &\stackrel{?}{=} \prod e([\alpha]_1, [\beta]_2)^{r^i} \cdot \mathsf{agg}_C \cdot e(W, [\gamma]_2) \cdot \prod e\left(\hat{S}_i, [\gamma]_2\right)^{r^i}. \end{aligned}$$

This equation is now verifiable by the HICIAP verifier, but, it is not fully succinct—the verifier must still do linear work in order to compute the products containing the r^i exponents. The verifier can avoid this for the term $\prod e([\alpha]_1, [\beta]_2)^{r^i}$ by simply using the geometric sum formula: $\sum_{i=0}^{n} r^i = (r^{n+1} - 1)/(r-1)$. The second optimization, due to Bünz et al. [BMM+20], moves the aggregation of the prepared inputs $\mathsf{agg}_{\mathsf{in}} := \sum r^i \hat{S}_i$ to the prover. The prove sends $\mathsf{agg}_{\mathsf{in}}$ and proves it was

[6] Since issuance and tag well-formedness are only a single Groth16 proof each, we "aggregate" them by simply duplicating them and computing the HICIAP proof over the resulting vector. The inefficiencies incurred here are overshadowed by the time and space used by the blocklist non-membership proof.

constructed correctly using an instance of MIPP. The verifier checks this with respect to $\mathsf{com}_{\mathsf{in}}$, which it can can compute from public inputs independently of the proof-specific r values. With these two optimizations, the final verifier equation is

$$\mathsf{agg}_{AB} \stackrel{?}{=} e([\alpha]_1, [\beta]_2)^{\frac{r^{n+1}-1}{r-1}} \cdot \mathsf{agg}_C \cdot e(W, [\gamma]_2) \cdot e(\mathsf{agg}_{\mathsf{in}}, [\gamma]_2).$$

It is important to reiterate that, while this resembles HICIAP's verification equation,[7] the protocol outlined above is not zero-knowledge. W leaks a_0; agg_{AB}, com_A, and com_B leak parts of $\mathbf{A}$ and $\mathbf{B}$; and agg_C and com_C leak parts of $\mathbf{C}$. In order to achieve zero-knowledge, we blind all of these values using the explicit blinders $z_1, \ldots, z_4 \in \mathbb{F}$ and the implicit blinders in the Groth16.Rerand subprocedure.

B. HICIAP details

We now give the formal definitions of the HICIAP relations and procedures. The HICIAP relation is defined to be

$$R_{\mathsf{HICIAP}} = \left\{ \begin{array}{c} \left(\begin{array}{c} \mathsf{ck}, \mathsf{crs}, \mathsf{com}_{\mathsf{in}}, \{\hat{S}\}_{i=1}^{n-2}; \\ a_0, \{\pi_i\}_{i=1}^{n-2} \end{array} \right) : \\ \mathsf{com}_{\mathsf{in}} = \hat{\mathbf{S}} * \mathsf{ck}_1 \\ \bigwedge_{i=1}^{n-2} \mathsf{Groth16.Vfy}(\mathsf{crs}, \pi_i, (a_0, \hat{S}_i)) \end{array} \right\}.$$

The associated protocol is given in Figure 4, and is made noninteractive by applying the Fiat-Shamir transform [FS87].

Note that Prove outputs an opening $o = (z_1, z_3)$ of com_{a_0}. This opening is used for linkage proofs, which show that multiple HICIAP proofs share the same a_0. Formally, this relation is

$$R_{\mathsf{link}} = \left\{ \begin{array}{c} \left(\{\hat{\pi}_i\}_{i=1}^t; a_0, \{o_i\}_{i=1}^t \right) : \\ \bigwedge_{i=1}^t \mathsf{com}_{a_0}^{(i)} = a_0 P_1 + z_1^{(i)} P_2 + z_3^{(i)} P_3 \end{array} \right\}$$

where $\mathsf{com}_{a_0}^{(i)}$ comes from $\hat{\pi}_i$, $(z_1^{(i)}, z_3^{(i)})$ come from o_i, and $P_1, P_2, P_3 \in \mathbb{G}_1$ is a Pedersen basis. The LinkProve and LinkVfy algorithms are constructed using the generic Σ-protocol framework defined by Camenisch and Stadler [CS97]. We defer their full description and security analysis to the extended version of this paper.

For the last relation, recall from the intuition that the value W in HICIAP proofs must be proven to represent the value a_0 on the wire W_0 and no more (i.e., it must not allow the prover to cancel other wire values). We call this the *hidden wire well-formedness* (HWW) relation:

$$R_{\mathsf{HWW}} := PK \left\{ \begin{array}{c} \left(\begin{array}{c} U, V, \{G_i\}_{i=1}^5 \in \mathbb{G}_1; \\ w, x, y \in \mathbb{F} \end{array} \right) : \\ U = wG_1 + xG_2 + yG_3 \\ \bigwedge \quad V = wG_4 + xG_5 \end{array} \right\}$$

Like Link, the HWW proof system is a Σ-protocol constructed using the Camenisch-Stadler framework. The protocol is described and proven secure in the extended version of this paper.

We claim that HICIAP is a zkSNARK for the R_{HICIAP} relation. The proofs of the below theorems can be found in Appendix B. Lastly, we note that the $n \geq 16$ requirement is not a barrier to usage. One may simply pad a HICIAP input with placeholder proofs in order to fulfill the minimum.

Theorem 2 (HICIAP Soundness)**.** HICIAP *on $n-2$ proofs has witness-extended emulation against algebraic adversaries under the* DL*, n-*ASDBP*, and $2n$-*SDH *assumptions.*

Theorem 3 (HICIAP Perfect Honest Verifier Zero Knowldege)**.** *The* HICIAP *protocol is perfect HVZK, provided that $n \geq 16$.*

VII. Implementation and evaluation

We now detail the design and evaluation of SnarkBlock.

A. Implementation and setup

Hardware. All benchmarks were peformed on a desktop computer with a 2021 Intel i9-11900KB CPU with 8 physical cores and 64GiB RAM running Ubuntu 20.04 with kernel 5.11.0-40-generic.

Code. SnarkBlock consists of 4.3k lines of Rust code using the Arkworks[Ar21] zkSNARK crates and Rayon for parallelization where possible. The Criterion-rs crate was used for all benchmarks and statistics.

Statistics. Performance measurments are for medians and include error bars for 95% confidence interval. These are usually not vissible: over all benchmarks, the maximum relative standard error of the median is 1.6%.

Instantiating cryptographic primitives. We set $\lambda = 128$. We use BLS12-381 [Bow17] for our Groth16 and HICIAP proofs, and Jubjub [ZCa19] for Schnorr signatures. We use hash functions $\mathsf{H}_1, \mathsf{H}_2, \mathsf{H}_3$ for identity registration, tag computation, and Schnorr signatures, respectively. Specifically, the commitment scheme used for R_{isu} is $\mathsf{Com}(m, r) := \mathsf{H}_1(r, m)$ and the PRF scheme used for R_{chunk} is $\mathsf{Prf}_k(m) := \mathsf{H}_2(k, m)$. Each H_i is a domain-separated instantiation of the Poseidon family

[7] For clarity's sake, however, the Vfy algorithm in Figure 4 does not include the geometric sum formula, or any other arithmetic optimizations.

$$\underline{\mathsf{HICIAP.Prove}\begin{pmatrix}(\mathsf{ck},\mathsf{crs}),\hat{\mathbf{S}},\\ (a_0,\mathbf{A}',\mathbf{B}',\mathbf{C}')\end{pmatrix}} \qquad\qquad \underline{\mathsf{HICIAP.Vfy}((\mathsf{srs},\mathsf{crs}),\mathsf{com}_{\mathsf{in}})}$$

$(\mathbf{A}',\mathbf{B}',\mathbf{C}') \leftarrow \mathsf{Groth16.Rerand}^{\mathbb{M}}(\mathbf{A}',\mathbf{B}',\mathbf{C}')$

$z_1,z_2,z_3,z_4 \leftarrow \mathbb{F}$

$\mathbf{A} := \mathbf{A}' \,\|\, [z_1]_1 \,\|\, [z_2]_1 \in \mathbb{G}_1^n$

$\mathbf{B} := \mathbf{B}' \,\|\, [\gamma]_2 \,\|\, [\delta]_2 \in \mathbb{G}_2^n$

$\mathbf{C} := \mathbf{C}' \,\|\, [1]_1 \,\|\, [z_2]_1 \in \mathbb{G}_1^n$

$\mathsf{com}_{a_0} := a_0P_1 + z_1P_2 + z_3P_3$

$\mathsf{com}_A := \mathbf{A} * \mathsf{ck}_1$

$\mathsf{com}_B := \mathsf{ck}_2 * \mathbf{B}$

$\mathsf{com}_C := e\left([z_4]_1,\mathsf{ck}_3\right)\cdot(\mathbf{C} * \mathsf{ck}_1)$ $\quad \xrightarrow{\mathsf{com}_{a_0},\mathsf{com}_A,\mathsf{com}_B,\mathsf{com}_C}$

$\xleftarrow{r}$ $\quad r \leftarrow \mathbb{F}$

$\mathbf{r} := (r,r^2,\ldots,r^n)$

$\mathbf{r}' := \mathbf{r}_{[:n-2]}$

$\mathsf{agg}_{\mathsf{in}} := \hat{\mathbf{S}}^{\mathbf{r}'}$

$\mathsf{agg}_C := \mathbf{C}^{\mathbf{r}}$

$W := \left[z_1r^{n-1}\right]_1 + \sum_{i=1}^{n-2} r^i a_0 W_0$ $\quad \xrightarrow{\mathsf{agg}_{\mathsf{in}},\mathsf{agg}_C,W}$

$\xleftrightarrow{\mathsf{MIPP}_k((\mathsf{com}_{\mathsf{in}},\mathsf{agg}_{\mathsf{in}},\mathbf{r}'),\hat{\mathbf{S}})}$ $\quad J := e(\mathsf{agg}_{\mathsf{in}},[\gamma]_2)$

$\xleftrightarrow{\mathsf{HMIPP}((\mathsf{com}_C,\mathsf{agg}_C,\mathbf{r}),(\mathbf{C},z_4))}$

$G_1 := \sum_{i=1}^{n-2} r^iW_0, \quad G_2 := \left[r^{n-1}\right]_1$ $\quad \xleftrightarrow{\mathsf{HWW}\begin{pmatrix}(\mathsf{com}_{a_0},W,P_1,P_2,P_3,G_1,G_2),\\(a_0,z_1,z_3)\end{pmatrix}}$ $\quad G_1 := \sum_{i=1}^{n-2} r^iW_0, \quad G_2 := \left[r^{n-1}\right]_1$

$\mathsf{agg}_{AB} := \mathbf{A}^{\mathbf{r}} * \mathbf{B}$ $\quad \xleftrightarrow{\mathsf{TIPP}((\mathsf{com}_A,\mathsf{com}_B,\mathsf{agg}_{AB},\mathbf{r}),(\mathbf{A},\mathbf{B}))}$ $\quad \mathsf{agg}_{AB} := \prod_{i=1}^{n} e([\alpha]_1,[\beta]_2)^{r^i}\cdot J$

return $o := (z_1,z_3)$ $\quad \cdot\, e(W,[\gamma]_2)\cdot e(\mathsf{agg}_C,[\delta]_2)$

Fig. 4: The (interactive) HICIAP protocol. Vfy accepts iff all subprotocols MIPP, HMIPP, HWW, and TIPP accept. The group elements $W_j, [\alpha]_1, [\beta]_2, [\delta]_2, [\gamma]_2$ are supplied by crs. The values P_1,P_2,P_3 used to compute com_{a_0} are unrelated Pedersen commitment bases. The Rerand procedure is only executed on the indices in the (log-sized) masking set $\mathbb{M}$, defined in the proof of Lemma 4. It is the identity function everywhere else.

[GKK+19], configured to be compatible with BLS12-381 and a 128-bit security level (i.e., $\alpha = 5$ and capacity $= 1$).

B. Evaluation

Benchmarks are given in Figures 5 and 6. Lines marked *buf* were benchmarked using a size-14 buffer of 16-element chunks. Lines marked *nobuf* used no buffer. The *cs* parameter refers to chunk size.

Figure 5a gives the time clients take to attest to non-membership on a blocklist that has recently changed. Specifically, this is the time it takes for a user to re-compute the last Groth16 chunk proof; compute HICIAP proofs over the blocklist, buffer (if the buffer exists), issuance, and well-formedness proofs;[8] and compute the link proof over those. Separately, Figure 5b gives the

[8]For speed, we combine R_{isu} and R_{tag} into a single circuit in our implementation. Thus there are only 2 proofs to link in an unbuffered SNARKBLOCK attestation.

offline computation a client must do as a function of the number of additions/removals to the blocklist (e.g. per day). This includes syncing chunks and precomputing its Groth16 issuance and well-formedness proofs for the next attestation.

Figure 6 gives proof sizes and throughput for server verification. these graphs, which are semi-log scale, shows that, unlike previous work, SNARKBLOCK proofs scale logarithmically with the number of elements in the blocklist.

VIII. DISCUSSION

We now discuss real-world performance and possible extensions.

A. *Is* SNARKBLOCK *practical?*

Attestation latency. How long can attestation take in practice? A client that computes an attestation in the background while a user drafts their post or comment adds no latency to the user's workflow. When the expected time to write a comment is lower than attestation time (e.g., a tweet), then the comment must be queued and posted by the client software when attestation is complete. While this is acceptable in many cases, it renders SNARKBLOCK impractical for real time chat when blocklists are large.

Operating Costs. SNARKBLOCK can be used when 1) logging in to a pseudonymous session, or 2) posting or commenting anonymously. The latter puts more load on a server. English language Wikipedia had 2 edits per second in the past year [Wik] and Reddit had 64 comments per second in 2020 [Red20].

An Amazon EC2 c5.4xLarge costs about $10 USD per day if reserved for a year.[9] For a blocklist of 2^{24} entries, SNARKBLOCK handles at least 35 attestations a second. At Reddit's scale, deployed in the more resource-intensive attestation-per-comment mode, SNARKBLOCK costs on the order of $20 per day. Allowing for CPU differences and virtualization overhead and, pessimistically, assuming full EC2 retail pricing at scale, this is $200 per day in the worst case. For reference, Facebook pays US moderators $120 a day. [Sal19].

B. *Client side perfomrance vs BLAC*

SNARKBLOCK's main advantage over BLAC is logarithmic server side scaling. Nonetheless, we briefly discuss client-side performance. The biggest problem for BLAC, surprisingly, is proof size. A blocklist with 4M bans uses 549MiB. In contrast, a SNARKBLOCK attestation is less than 200KiB 134M-entry list. On a 50 Mbps connection, which is 5× the upstream bandwidth of the median US household [FCC20],[10] uploading a BLAC attestation would take 90s. Even with 100Mbps fiber, SNARKBLOCK can compute and upload the attestation before a BLAC proof would upload.

What if we ignore proof size? Although Tsang et al. give benchmarks for BLAC, they are on 10+ year old hardware using the very dated PBC library [Lyn] for pairings. Luckily, Tsang et al also characterize their systems performance in terms of group operations. In lieu of a reimplementation, we report these measurements and give the equivalent values for SNARKBLOCK in Table I.

SNARKBLOCK pays an initial overhead in terms of upfront costs (e.g., the 244 pairings). The major advantage for SNARKBLOCK is that its operations are per chunk as opposed to per element. Ignoring constants, SNARKBLOCK is faster for proving whenever $2nM_{\mathbb{G}_1} > \frac{10nM_{\mathbb{G}_1}}{s} + \frac{10nM_{\mathbb{G}_2}}{s} + \frac{15nP}{s}$, where $s = n/c$ is the chunk size. Thus, as the blocklist size grows, SNARKBLOCK will outperform as long as $s > 5 + 12.5o$ where $o = \frac{P}{M_{G_1}}$ is the overhead for pairings relative to G_1 multiplications.[11] On our benchmark system, G_2 multiplications are about 3 times G_1, and pairings twice that. i.e., $o \approx 6$.

Unfortunately, the exact transition point is impossible to estimate for group operations: we need to compare runtimes to a full reimplementation of BLAC. Real-world performance will differ significantly from group operation counts due to parallelization and other optimizations. Indeed, SNARKBLOCK outperforms estimates based on group operations and benchmarked operation times.

SNARKBLOCK has one substantial cost that BLAC does not: SNARKBLOCK requires periodic sync computations for blocklist additions and removals. Per Fig. 5, this is less than 200 seconds for every 12,000 additions, with appropriate batching or buffering. BLAC would have an initial advantage on faster growing lists. But at > 12k additions per day, we will exceed 2^{22} bans within a year. But, given its proof size limitation, BLAC cannot deal with blocklists of this magnitude.

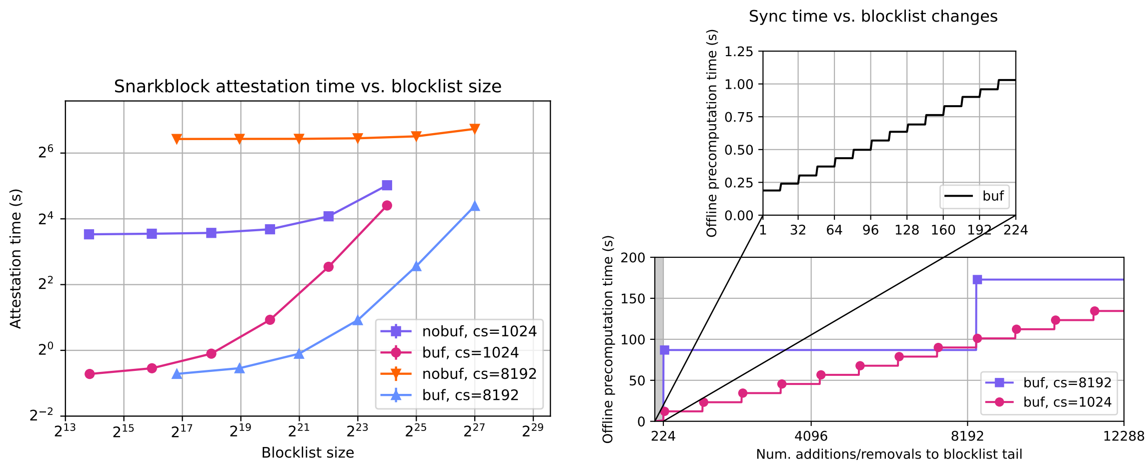
C. *Choice of hash functions*

For efficiency, our system uses the Poseidon [GKK+19] for signatures and the PRF. The PRF can be replaced with the BLAC's construction: $\mathsf{tag} := kH$ where, outside the circuit, we compute $H := \mathsf{HashToCurve}(\mathsf{nonce})$This change would result in an estimated 3.3× increase in π_{chunk} proving time. Similarly, replacing Poseidon with Blake2s [Aum15] in Schnorr signature verification would result in an estimated 3.5×

[9] With 16 virtual Xeon CPUs and 30GB of memory, this is a decent analog to our test system since in testing, SNARKBLOCK never exceeded 20 GB of memory for verification.

[10] FCC measuments are a trailing indicator. The latest report, released in Sep. 2020 [FCC20], is for data as of Dec. 2018. For Oct. 2021, Sp eedtest.net reports it's US users have upstream *averages* of 19.18Mbps for wired connections and 8.81 Mbps for mobile.

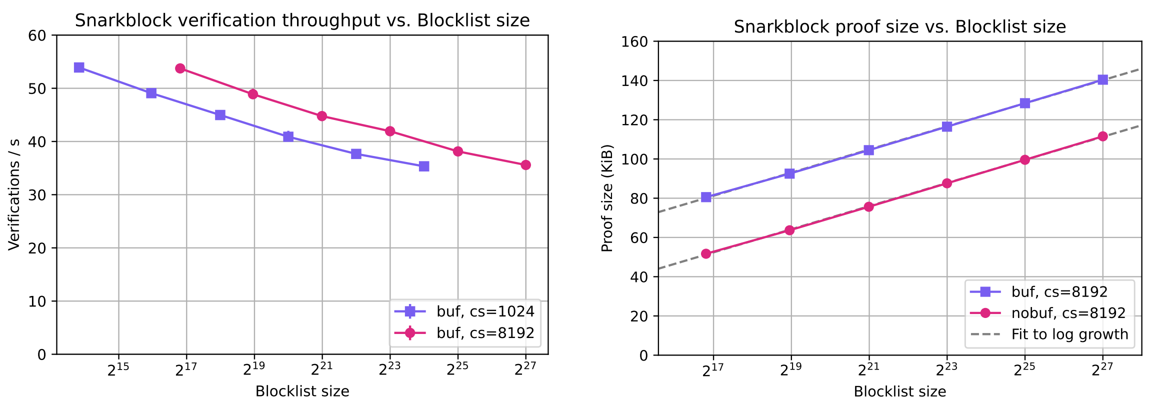
[11] Since $M_{G_2} < P$, we can approximate them as the same.



(a) Online costs for proving non-membership vs. blocklist size

(b) Client precomputation cost vs. additions/removals to the blocklist

Fig. 5: Client-side performance for SNARKBLOCK



(a) SNARKBLOCK attestation verifications per second.

(b) SNARKBLOCK attestation proof size. Varying chunk size only shifts the x-axis.

Fig. 6: Server-side performance for SNARKBLOCK

increase in π_{isu} proving time, when the AIP set size is 10 and would not affect attestation or verification time.

D. Cold Start

One significant caveat for SNARKBLOCK is that a new user of a system with a pre-existing ban list must compute significant work to sync the entire blocklist and compute the chunk proofs.

One option is to leverage the issuance date for a user's identity and allow them to skip proving membership in blocklist chunks whose last entry is before they joined. This can be done directly now, albeit at the cost of leaking the user's approximate join time for, e.g., a particular forum. Specifically, a given service provider can use a custom CRS for their Groth16 chunk proofs. They can then, using the CRS trapdoor, give each new user non-membership chunk proofs for earlier portions of the blocklist. Crucially, proving with a trapdoor is constant time, so this process efficient.

We leave to future work the question of how to build a general trapdoor for cold start. In particular, it should be possible skip chunk proofs who's last entry was inserted before the issuance data of the user's identity.

IX. RELATED WORK

For a full formalization of privacy preserving blocklists, we refer the reader to excellent SoK of Henry and Goldberg [HG11]. This also describes a number of interesting hybrid systems that can be constructed, in a black-box way, from either SNARKBLOCK or BLAC and allow for pruning of blocklists.

A. Blocklists

The work closest to ours is the ZKBL approach introduced in BLAC [TAKS10]. As discussed in prior

	Client Attestation	Server Verification	Proof Size	
BLAC [TAKS10]	$2nM_{\mathbb{G}_1}$	$(2n+4)M_{\mathbb{G}_1}+2P$	Abs.	$(3n+12)\lvert\mathbb{F}\rvert+(n+3)\lvert\mathbb{G}_1\rvert$
			Real	$528\text{B}+n\cdot 144\text{B}$
SNARKBLOCK	$(197+10c)M_{\mathbb{G}_1}$ $+(160+10c)M_{\mathbb{G}_2}+2M_{\mathbb{G}_T}$ $+(244+15c)P$	$25M_{\mathbb{G}_1}+38P$ $(46+10\log_2(c))M_{\mathbb{G}_T}$	Abs.	$8\lvert\mathbb{F}\rvert+29\lvert\mathbb{G}_1\rvert+14\lvert\mathbb{G}_2\rvert$ $+(48+10\log_2 c)\lvert\mathbb{G}_T\rvert$
			Real	$29.3\text{KiB}+\log_2(c)\cdot 5.6\text{KiB}$

Legend: $n=$ Blocklist length, $c=$ Num. chunks, $M_{\mathbb{G}}=$ Var. base MSM in $\mathbb{G}$, $P=$ Pairing op., $\lvert\mathbb{G}\rvert=$ Size of group elem., $\lvert\mathbb{F}\rvert=$ Size of scalar field elem.

TABLE I: BLAC and SNARKBLOCK operation counts and proof sizes. SNARKBLOCK operation counts assume a fully synchronized client and an unbuffered blocklist. The top subcell in the Proof Size column represents abstract element counts. The bottom subcell represents the byte count when instantiated with BLS12-381.

sections, by replacing the zero-knowledge proofs in BLAC with our novel proving system HICIAP, we get a system that offers logarithmic verification time and proof size instead of linear. Further, we extend the system to support federated identities.

Also close to our work is the windowed approach from PEREA [TAKS08], also by the authors of BLAC. In PEREA, users are issued a finite number of one-time-use identity tickets every for use during a revocation window, e.g., one month, and must prove none of those tickets are in the blocklist to take an action. They compute the same proof to get the next set of tickets. Verification time is proportional to size of the revocation window, not the total size of the blocklist. It has a number of drawbacks for broad deployment on the web:

1) Issuing users a small number of tickets is feasible for individual low volume sites, but the limit would apply to all sites in a federated system.
2) The approach is inherently centralized. All blocklists must be registered with the single identity provider to ensure non-membership before reissuing identities.
3) Service providers must react quickly to ban users, since bans expire once the user gets new identities. The exact time depends on configuration; PEREA gives the example of a 1-hour window for a site like Wikipedia.

Finally, a number of systems provide weaker anonymity. One line of work relies on a trusted third party to revoke anonymity, e.g, [Cha85], [Cv91], [BMW03]. Another approach is to leverage blind signatures to remove the linkage between, e.g., an IP address, and the pseudonym, e.g., [JKTS07], [TKCS11], [LH10]. These schemes only provide pseudonymity, allowing the linking of pseudonymous posts across different platforms. In contrast, SNARKBLOCK provides anonymity and does not trust a third party to safeguard user identities.

B. Zero-knowledge proofs

Our HICIAP protocol consists of a non-membership proof and a proof that a revocation tag has been computed correctly. Bayer and Groth design a non-membership proof [BG13] with logarithmic proof size and no trusted setup, but they have (quasi-)linear prover and verifier costs. Non-membership proofs can also be constructed in groups of unknown order [CL02], [BCFK19], and have constant verifier time and prover time. However, it is not obvious how to apply these techniques to a blocklist without requiring a finite number of tickets per user as in PEREA.

An alternative and thus far unexplored direction for proving blocklist non-membership is recent advances in recursive zero-knowledge proofs using techniques first introduced by Bowe et al. for Halo[BGH19]. Halo-like schemes, formalized in [BCMS20] as "accumulator schemes", have been extended to a wider variety of polynomial commitment schemes in [BDFG21]. These use Bulletproofs [BBB$^+$18] as a building block, which introduces a linear verification time component to the constructions. To get good verification time, this cost most be amortised over all the aggregated proofs. Bünz et al. [BCL$^+$21] and Kothapalli et al. [KST21] improve upon these results.

One key challenge to using Halo-like techniques is keeping the recursion threshold low to prevent a blow up in prover costs. With SNARKBLOCK, aggregation costs are less than $8\times$ the cost of native verification, keeping online costs low. There are a few different ways to apply Halo-like techniques to our problem domain and its an interesting question if any of them offer comparatively competitive proving cost.

A second key challenge to Halo-like approaches is handling changes to the blocklist. Recall from Section II that adding or removing entries from the blocklist requires costly recomputation in order to preserve the privacy of recently unbanned users. For Groth16, depending on approach, this required redoing all proof computation from scratch or, for the best approach, redoing a logarithmic portion of the computation with impractical concrete costs. A Halo-like scheme would face similar challenges, though neither the tradeoffs nor their concrete efficiencies are clear at this time.

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APPENDIX

A. Cryptographic Definitions

We leave the full definitions of the cryptographic assumptions we rely on for the extended version of this paper. We will use the definition of the Discrete Log (DL) assumption from [FKL18] and the definitions of the q-ASDBP and q-SDH assumptions from [BMM+20].

We will use the definitions of *computation knowledge soundness* and *perfect honest-verifier zero-knowledge* (HVZK) from [BMM+20] in the following proofs of soundness and zero-knowledge.

B. Deferred proofs

Recall R_{HMIPP} is a zero-knowledge ("hiding") version of the $R_{\mathsf{MIPP}-k}$ relation. Both relations are detailed in Figure 1, and their proof systems are given in [BMM+20]. We defer the proof of knowledge soundness to the extended version of this paper.

Lemma 4. *The* HMIPP *protocol is perfect HVZK. (Lemma 5 in [BMM+20])*

Lemma 5 (HMIPP Computational Knowledge Soundness)**.** HMIPP *on n elements is computationally knowledge sound against algebraic adversaries under the n-*ASDBP *and $2n$-*SDH *assumptions.*

Recall R_{HWW} is the proof of hidden wire well-formedness described in Section VI. We state its theorems of soundness and zero-knowledge below, and defer the proofs and full description of the algorithms to the extended version of this paper.

Lemma 6. HWW *is Perfect HVZK.*

Lemma 7. HWW *is statistically knowledge-sound.*

Theorem 1 (SNARKBLOCK Security)**.** SNARKBLOCK *described in Figure 2 is blocklistable, anonymous and non-frameable provided that Groth16 and* HICIAP *proofs are knowledge sound and subversion zero-knowledge; Schnorr signatures are unforgeable;* Prf *is pseaudorandom; and* Com *is binding and hiding.*

Proof Sketch. **Blocklistability.** Let A be an adversary that breaks blocklistability. Then A generates a verifying attestation $(\pi_{\mathsf{zkbl}}, \mathsf{tag}, \mathsf{nonce})$. Either an extractor can output k such that $\mathsf{tag} = \mathsf{Prf}_k(n)$; or $\hat{\pi}_{\mathsf{tag}}$ is a forgery for HICIAP and we cannot extract a verifying Groth16 statement and proof k, π_{tag} breaking knowledge soundness; or π_{tag} is a forgery for Groth16 and $\mathsf{tag} \neq \mathsf{Prf}_k(\mathsf{nonce})$ breaking knowledge soundness.

Either an extractor can output a verifying signature σ under some identity providers public key pk_{i^*} on the message $\mathsf{com} = \mathsf{Com}(k,r)$ the same k and some r; or $\hat{\pi}_{\mathsf{link}}$ is a forgery for Link breaking knowledge soundness; or the adversary can find $\mathsf{com} = \mathsf{Com}(k',r')$ for different k',r' breaking binding; or $\hat{\pi}_{\mathsf{isu}}$ is a forgery for HICIAP and we cannot extract a verifying Groth16 proof π_{isu} for k breaking knowledge soundness; or $((k,\mathscr{I}),\pi_{\mathsf{isu}})$ is a forgery for Groth16 breaking knowledge soundness. If

σ is a verifying signature then either pk_{i^*} authenticated com at some point, or we break unforgeability of the signature scheme.

If σ has been authenticated by pk_{i^*} that gets blocked then $(\mathsf{tag}, \mathsf{nonce})$ gets added to $\mathscr{L}$. If A later generates a verifying attestation with respect to the same σ then either $\hat{\pi}_{\mathsf{link}}$ is a forgery for Link breaking knowledge soundness; or the adversary can find $\mathsf{com} = \mathsf{Com}(k', r')$ for different k', r' breaking binding; or $\hat{\pi}_{\mathsf{chunk}}$ is a forgery for HICIAP and we cannot extract verifying Groth16 proofs π_{chunk_j} for k breaking knowledge soundness; or π_{chunk_j} is a forgery for Groth16 for some j and $\mathsf{Prf}_k(\mathsf{nonce}^*) = \mathsf{tag}^*$ for some $(\mathsf{nonce}^*, \mathsf{tag}^*) \in \mathscr{L}$ breaking knowledge soundness; or σ is never associated with a blocked session and A does not break blocklistability.

Non-Frameability. If an adversarial identity provider prevents an honest user from authenticating then they must get some $(\mathsf{nonce}, \mathsf{tag})$ added to $\mathscr{L}$ such that $\mathsf{tag} = \mathsf{Prf}_k(\mathsf{nonce})$ for an honest user's k. By the pseudorandomness of Prf and the anonymity of SNARKBLOCK, the probability that they guess any such tag is negligible.

Anonymity. We claim that the transcript between an honest user and any number of identity providers and service providers is uncorrelated. By the hiding of Com we have that com reveals no information about k (and uses distinct r each registration). By the zero-knowledge of HICIAP we have that π_{zkbl} reveals no information (even to the identity providers). nonce is chosen uniformly at random for each session associated with k. By the pseudorandomness of Prf, tag is indistinguishable from random for users that don't know k and thus reveals no information about k. Thus the scheme is anonymous.

Issuer Anonymity. By the same argument above, this scheme is issuer-anonymous. □

Theorem 2 (HICIAP Soundness)**.** *HICIAP on $n-2$ proofs has witness-extended emulation against algebraic adversaries under the* DL*,* n-ASDBP*, and* $2n$-SDH *assumptions.*

Proof. We wish to show that, there exists an expected polynomial time HICIAP extractor $\mathsf{E}_{\mathsf{HICIAP}}(\mathsf{tr}, \mathsf{ck}, \mathsf{crs}, \mathsf{com}_{\mathsf{in}})$ for some $\mathsf{com}_{\mathsf{in}} := \hat{\mathbf{S}} * \mathsf{ck}_1$, which outputs a witness $(a_0, \mathbf{A}, \mathbf{B}, \mathbf{C}, \hat{\mathbf{S}}')$ such that $\hat{\mathbf{S}}' = \hat{\mathbf{S}}$ and for all $i = 1, \ldots, n$,

$$\mathsf{Groth16.Vfy}\left(\mathsf{crs}, (A_i, B_i, C_i), (a_0, \hat{S}_i)\right).$$

By Theorem 3 of [BMM+20], there is an expected polynomial time extractor $\mathsf{E}_{\mathsf{TIPP}}$ for $\mathsf{TIPP}(\mathsf{com}_A, \mathsf{com}_B, \mathsf{agg}_{AB}, \mathbf{r})$ which extracts $(\mathbf{A}, \mathbf{B})$ such that $\mathsf{com}_A = \mathbf{A} * \mathsf{ck}_1$, $\mathsf{com}_B = \mathsf{ck}_2 * \mathbf{B}$, and $\mathsf{agg}_{AB} = \mathbf{A}^{\mathbf{r}} * \mathbf{B}$. By Theorem 6 of [BMM+20], there is an expected polynomial time extractor $\mathsf{E}_{\mathsf{MIPP}-k}$ for $\mathsf{MIPP}_k(\mathsf{com}_{\mathsf{in}}, \mathsf{agg}_{\mathsf{in}}, \mathbf{r})$ which extracts $\hat{\mathbf{S}}'$ such that $\hat{\mathbf{S}}' * \mathsf{ck}_1 = \mathsf{com}_{\mathsf{in}}$ and $(\hat{\mathbf{S}}')^{\mathbf{r}} = \mathsf{agg}_{\mathsf{in}}$. By Lemma 5, there is an expected polynomial time extractor $\mathsf{E}_{\mathsf{HMIPP}}$ for $\mathsf{HMIPP}(\mathsf{com}_C, \mathsf{agg}_C, \mathbf{r})$ which extracts $(\mathbf{C}, z_4)$ such that $\mathbf{C}^{\mathbf{r}} = \mathsf{agg}_C$ and $\mathsf{com}_C = e(z_4 G, \mathsf{ck}_3) + (\mathbf{C} * \mathsf{ck}_1)$. By Lemma 7, there is an expected polynomial time extractor $\mathsf{E}_{\mathsf{HWW}}(\mathsf{com}_{a_0}, W, P_1, P_2, P_3, G_1, G_2)$ for HWW which extracts (a_0, z_1, z_3) such that $\mathsf{com}_{a_0} = a_0 P_1 + z_1 P_2 + z_3 P_3$ and $W = a_0 G_1 + z_1 G_2$.

Let P^* be a probabilistic prover with fixed randomness and unknown probability ε of producing an argument that accepts. We define an extractor $\mathsf{E}_{\mathsf{HICIAP}}(\mathsf{tr}, \mathsf{ck}, \mathsf{crs}, \hat{\mathbf{S}})$, which extracts the witness $(a_0, \mathbf{A}, \mathbf{B}, \mathbf{C}, \hat{\mathbf{S}}')$, as follows.

First, run HICIAP with a random $r \leftarrow \mathbb{F}$, and run all the subprotocols honestly. Note that, by the definition of witness-extended emulation, if P^* does not produce an accepting transcript tr on the first run, the extractor is allowed to exit early with $(\mathsf{tr}, \perp)$. If tr is accepting, rewind to the point after r is chosen and run $\mathsf{E}_{\mathsf{MIPP}-k}$, $\mathsf{E}_{\mathsf{HMIPP}}$, $\mathsf{E}_{\mathsf{HWW}}$, and $\mathsf{E}_{\mathsf{TIPP}}$ to extract $(a_0, z_1, z_3, z_4, \mathbf{A}, \mathbf{B}, \mathbf{C}, \hat{\mathbf{S}}')$. Finally, output $(a_0, \mathbf{A}_{[:n]}, \mathbf{B}_{[:n]}, \mathbf{C}_{[:n]}, \hat{\mathbf{S}}')$.

Note that $\mathsf{E}_{\mathsf{HICIAP}}$ algorithm is expected polynomial time, since its runtime is at most the sum of the runtimes of $\mathsf{E}_{\mathsf{HMIPP}}$, $\mathsf{E}_{\mathsf{HWW}}$, and $\mathsf{E}_{\mathsf{TIPP}}$, which are assumed to be expected polynomial time.

To prove the claimed relations hold, first note that the commitment $\mathsf{com}_{\mathsf{in}}$ is computationally binding under the $(n-2)$-ASDBP assumption, and so, with overwhelming probability, $\hat{\mathbf{S}}' = \hat{\mathbf{S}}$.

It remains to show that, with overwhelming probability, the extracted witness satisfies the Groth16 verification condition. That is, for all $i = 1, \ldots, n$,

$$e(A_i, B_i) = e([\alpha]_1, [\beta]_2) \cdot e(C_i, [\delta]_2) \cdot e\left(\hat{S}_i, [\gamma]_2\right).$$

The commitments $\mathsf{com}_{\mathsf{in}}, \mathsf{com}_A, \mathsf{com}_B, \mathsf{com}_C$ are computationally binding under the $(n-2)$- and n-ASDBP assumptions. Further, since P_1, P_2, P_3 are unrelated by assumption, the Pedersen commitment com_{a_0} is computationally binding by the DL hardness assumption. Thus, with overwhelming probability, the formal product being evaluated in TIPP is the one committed to by $\mathsf{com}_A, \mathsf{com}_B, \mathsf{com}_C$, i.e.,

$$\begin{aligned}\prod_{i=1}^{n} & e(A_i, B_i)^{x_i} \\ &= \prod_{i=1}^{n-2} e(a_0 W_0, [\gamma]_2)^{x_i} + \prod_{i=1}^{n} \cdot e([\alpha]_1, [\beta]_2)^{x_i} \\ &\quad + \prod_{i=1}^{n-2} \cdot e\left(\hat{S}_i, [\gamma]_2\right)^{x_i} + \prod_{i=1}^{n} e(C_i, [\delta]_2)^{x_i}. \\ &\quad + e([z_1]_1, [\gamma]_2)^{x^{n-1}}\end{aligned}$$

Then by the Schwartz-Zippel lemma, the above relation holds with probability at least $1 - (n+2)/p$. Since the above equality directly implies the Groth16 verification condition, the theorem is proved. □

Theorem 3 (HICIAP Perfect Honest Verifier Zero Knowldege)**.** *The* HICIAP *protocol is perfect HVZK, provided that* $n \geq 16$*.*

Proof. A HICIAP proof consists of the values

$$\mathsf{com}_{\mathsf{in}}, \mathsf{com}_{a_0}, \mathsf{com}_A, \mathsf{com}_B, \mathsf{com}_C, \mathsf{agg}_{\mathsf{in}}, \mathsf{agg}_C, W, \mathsf{tr}_{\mathsf{MIPP}\text{-}k}, \mathsf{tr}_{\mathsf{HMIPP}}, \mathsf{tr}_{\mathsf{HWW}}, \mathsf{tr}_{\mathsf{TIPP}}.$$

We construct a simulator that knows a Groth16 simulation trapdoor τ to crs and which can choose the verifier's randomness in advance, such that the simulated transcript is indistinguishable from an honest prover's transcript. The simulator will also use the simulators described in Lemmas 4 and 6 which generate transcripts for subprotocols HMIPP and HWW,

$$\mathsf{Sim}_{\mathsf{HMIPP}}(\mathsf{crs}, \mathsf{com}_C, \mathsf{agg}_c, \mathbf{r}) \rightarrow \mathsf{tr}_{\mathsf{HMIPP}}$$
$$\mathsf{Sim}_{\mathsf{HWW}}(\mathsf{com}_{a_0}, W, P_1, P_2, P_3, G_1, G_2) \rightarrow \mathsf{tr}_{\mathsf{HWW}}.$$

The simulator is given the Groth16 prepared public inputs $\hat{\mathbf{S}}$ and behaves as follows.

1) The simulator computes the first prover message $\mathsf{com}_{a_0}, \mathsf{com}_A, \mathsf{com}_B, \mathsf{com}_C$. It chooses randomness $a_0, z_1, z_2 \leftarrow \mathbb{F}$ and $\mathsf{com}_{a_0} \leftarrow \mathbb{G}_1$ and $\mathsf{com}_C \leftarrow \mathbb{G}_T$. It runs
$$(A'_i, B'_i, C'_i) = \mathsf{Sim}_{\mathsf{Groth16}}(\mathsf{crs}, \tau, (a_0, a_{i,1}, \ldots, a_{\ell,i}))$$
for $1 \leq i \leq n-2$. It sets
$$\mathbf{A} := \mathbf{A}' \,||\, [z_1]_1 \,||\, [z_2]_1$$
$$\mathbf{B} := \mathbf{B}' \,||\, [\gamma]_2 \,||\, [\delta]_2$$
$$\mathbf{C} := \mathbf{C}' \,||\, [1]_1 \,||\, [z_2]_1$$
$$\mathsf{com}_A := \mathbf{A} * \mathsf{ck}_1$$
$$\mathsf{com}_B := \mathsf{ck}_2 * \mathbf{B}$$
2) The simulator computes the first verifier message honestly and chooses $r \leftarrow \mathbb{F}$ randomly.
3) The simulator uses $\hat{\mathbf{S}}$ to construct $\mathsf{agg}_{\mathsf{in}}$ and $\mathsf{tr}_{\mathsf{MIPP}\text{-}k}$ honestly.
4) The simulator computes the second prover message (agg_C, W) honestly as
$$\mathsf{agg}_C := \mathbf{C}^{\mathbf{r}}$$
$$W := [z_1 r^{n-1}]_1 + \sum_{i=1}^{n-2} r^i a_0 W_0$$
for $\mathbf{r} = (r, r^2, \ldots, r^n)$.
5) The simulator generates a transcript $\mathsf{tr}_{\mathsf{HMIPP}}$ for the HMIPP protocol by running
$$\mathsf{tr}_{\mathsf{HMIPP}} := \mathsf{Sim}_{\mathsf{HMIPP}}(\mathsf{crs}, \mathsf{com}_C, \mathsf{agg}_c, \mathbf{r})$$
6) The simulator generates a transcript $\mathsf{tr}_{\mathsf{HWW}}$ for the HWW protocol by running
$$\mathsf{tr}_{\mathsf{HWW}} := \mathsf{Sim}_{\mathsf{HWW}}(\mathsf{com}_{a_0}, W, P_1, P_2, P_3, \sum_{i=1}^{n-2} r^i W_0, [r^{n-1}]_1)$$
7) The simulator generates a transcript $\mathsf{tr}_{\mathsf{TIPP}}$ by running the prover
$$\mathsf{tr}_{\mathsf{TIPP}} := \mathsf{TIPP}(\mathsf{com}_A, \mathsf{com}_B, \mathbf{A}^{\mathbf{r}} * \mathbf{B}, \mathbf{r}; \mathbf{A}, \mathbf{B})$$
honestly.

We will show that the simulator's transcript is indistinguishable from an honest provers. We look at the distribution of each of the proof components.

The client-side MSM. We first note that this optimization is simulated perfectly, since it involves no witness values. Specifically, $\mathsf{com}_{\mathsf{in}}$, $\mathsf{agg}_{\mathsf{in}}$, and $\mathsf{tr}_{\mathsf{HMIPP}\text{-}k}$ are simulated perfectly, since both the prover and simulator have access to the Groth16 public prepared inputs $\hat{\mathbf{S}}$.

The first prover message. We look at com_{a_0}, com_A, com_B, and com_C. In the real prover execution: com_{a_0} is distributed uniformly at random because it is randomized by z_3; com_A is distributed uniformly at random because it is randomized by z_2; com_B is distributed uniformly at random because it is randomized by B'_{n-2};com_C is distributed uniformly at random because it is randomized by z_4. In the simulated execution: com_{a_0} is chosen uniformly at random; com_A is distributed uniformly at random because it is randomized by z_2; com_B is distributed uniformly at random because it is randomized by B'_{n-2}; com_C is chosen uniformly at random. Thus both the provers and the simulators first messages are distributed randomly and are indistinguishable.

The second prover message. We second look at agg_C, W. In the real prover execution: agg_C is distributed uniformly at random because it is randomized by C'_{n-2}; W is distributed uniformly at random because it is randomized by z_1. In the simulated execution: com_{a_0} is chosen uniformly at random; agg_C is distributed uniformly at random because it is randomized by C'_{n-2}; com_W is distributed uniformly at random because it is randomized by z_1. Thus both the provers and the simulators second messages are distributed randomly and are indistinguishable.

The hidden MSM argument. We see that $\mathsf{tr}_{\mathsf{HMIPP}}$ generated by the prover and simulator are indistinguishable by the zero-knowledge of HMIPP (Lemma 4).

The HWW argument. We see that $\mathsf{tr}_{\mathsf{HWW}}$ generated by the prover and simulator are indistinguishable by the perfect zero-knowledge of HWW (Lemma 6).

The TIPP argument. In order to argue that $\mathsf{tr}_{\mathsf{TIPP}}$ generated by the prover and simulator are indistinguishable we must look at the rerandomisations of each (A_i, B_i, C_i). The bulk of the following argument consists of demonstrating that enough values in the HICIAP protocol are independent and uniformly distributed. To do this, we associate each iid uniform blinding factor to at most one transcript variable. One thing to be careful about here is enforcing the "at most one" requirement. To do this cleanly, we make heavy use of a decomposition trick.

Following [HKR19] we define a masking set $\mathbb{M}$ of size $3\log_2(n-2)$ that defines a position of randomized values that will ensure the transcripts appear random in the recursion. We track the parts of the TIPP transcript which are functions of $\mathbf{A}, \mathbf{B}, \mathsf{ck}_1, \mathsf{ck}_2$ (where we let $\mathbf{A}$ represent $\mathbf{r} \odot \mathbf{A}$ and ck_1 represent $\mathbf{r}^{-1} \odot \mathsf{ck}_1$ for simplicity). In each round of the TIPP protocol (of which there are $\log n$), the prover sends six values:

$$\begin{aligned} \mathsf{com}_{LA} &:= \mathbf{A}_{[:h]} * \mathsf{ck}_A & \mathsf{com}_{RA} &:= \mathbf{A}_{[h:]} * \mathsf{ck}_A \\ \mathsf{com}_{LB} &:= \mathsf{ck}_B * \mathbf{B}_{[:h]} & \mathsf{com}_{RB} &:= \mathsf{ck}_B * \mathbf{B}_{[h:]} \\ \mathsf{agg}_{LR} &:= \mathbf{A}_{[:h]} * \mathbf{B}_{[h:]} & \mathsf{agg}_{RL} &:= \mathbf{A}_{[h:]} * \mathbf{B}_{[:h]} \end{aligned}$$

The verifier sends a challenge x, which defines the prover's values for the next round:

$$\begin{aligned} \mathbf{A}' &:= \mathbf{A}_{[:h]} + x \cdot \mathbf{A}_{[h:]} & \mathsf{ck}_1' &:= \mathsf{ck}_{1,[:h]} + x^{-1} \cdot \mathsf{ck}_{1,[h:]} \\ \mathbf{B}' &:= \mathbf{B}_{[:h]} + x^{-1} \cdot \mathbf{B}_{[h:]} & \mathsf{ck}_2' &:= \mathsf{ck}_{2,[:h]} + x \cdot \mathsf{ck}_{2,[h:]} \end{aligned}$$

Note that a randomized A_i value in round k will yield a uniform value of A_j in round $k+1$, where $j \equiv i \pmod{2^{k-1}}$, and similarly for B_i.

With 6 proof elements in each round, we need to ensure there are at least 6 randomizers per round, and that one unique randomizer appears in each proof element. We divide them as 3 randomizers in $\mathbf{A}$ (to randomize $\mathsf{com}_{LA}, \mathsf{com}_{RA}, \mathsf{agg}_{LR}$) and 3 in $\mathbf{B}$ (to randomize $\mathsf{com}_{LB}, \mathsf{com}_{RB}, \mathsf{agg}_{RL}$). We define the masking set

$$\mathbb{M} = \{2^k, 2^k+1\}_{k=0}^{\ell-1} \cup \{2^k-1\}_{k=2}^{\ell-1}$$

The two sets making up $\mathbb{M}$ are non-overlapping. Note that because $\log_2(n) \geq 4$ we have that $\mathbb{M}$ also does not overlap with the blinders B'_{n-2} or C'_{n-2}.

For the components agg_{LR} and agg_{RL} in the TIPP argument, we must use the fact that with overwhelming probability, none of the components of a Groth16 proof (A_i, B_i, C_i) equals 0. This implies that the rerandomisation (A'_i, B'_i, C'_i) is a uniformly proof of the same statement, and also contains no zeros.

With this in mind we argue that $\mathbb{M}$ is sufficient to randomise the distribution of the com_{LA}, com_{RA}, agg_{LR} components of TIPP. To see this, observe that in round k with verifier challenges $x_0, \ldots, x_{k-1}$

$$\mathsf{com}_{LA} = \prod_{\mathbf{b} \in \{0,1\}^{\ell-k-1}} e\left(\sum_{\mathbf{s} \in \{0,1\}^k} A_{(\mathbf{s},0,\mathbf{b})} f_{k,\mathbf{x}}(\mathbf{s}), \sum_{\mathbf{s} \in \{0,1\}^k} \mathsf{ck}_{A,(\mathbf{s},1,\mathbf{b})} f_{k,\mathbf{x}^{-1}}(\mathbf{s}) \right),$$

$$\mathsf{com}_{RA} = \sum_{\mathbf{b} \in \{0,1\}^{\ell-k-1}} e\left(\sum_{\mathbf{s} \in \{0,1\}^k} A_{(\mathbf{s},1,\mathbf{b})} f_{k,\mathbf{x}}(\mathbf{s}), \sum_{\mathbf{s} \in \{0,1\}^k} \mathsf{ck}_{A,(\mathbf{s},0,\mathbf{b})} f_{k,\mathbf{x}^{-1}}(\mathbf{s}) \right),$$

$$\mathsf{agg}_{LR} = \prod_{\mathbf{b} \in \{0,1\}^{\ell-k-1}} e\left(\sum_{\mathbf{s} \in \{0,1\}^k} A_{(\mathbf{s},0,\mathbf{b})} f_{k,\mathbf{x}}(\mathbf{s}), \sum_{\mathbf{s} \in \{0,1\}^k} B_{(\mathbf{s},1,\mathbf{b})} f_{k,\mathbf{x}^{-1}}(\mathbf{s}) \right),$$

where $f_{k,\mathbf{x}}(\mathbf{s}) := \prod_{j=0}^{k-1} \left(s_j x_{k-j-1} + 1 - s_j \right)$ for $\mathbf{s} \in \{0,1\}^k$ and $A_{\mathbf{b}}$ represents A_i when $\mathbf{b}$ is the binary representation of the integer i. Thus

- $A_{\mathbf{0},0,1,\mathbf{0},1}$ is included in com_{LA} in the kth round and corresponds to the blinder $2^{\ell-k-2}+1$
- $A_{\mathbf{0},1,\mathbf{0}}$ is included in com_{RA} in the kth round and corresponds to the blinder $2^{\ell-k-1}$.
- $A_{\mathbf{0},0,\mathbf{1}}$ is included in agg_{LR} in the kth round and corresponds to the blinder $2^{\ell-k-1}-1$.

Denote

$$R_{LA,k,\boldsymbol{b}} = \sum_{\mathbf{s} \in \{0,1\}^k} \mathsf{ck}_{A,(\mathbf{s},1,\mathbf{b})} f_{k,\mathbf{x}^{-1}}(\mathbf{s})$$

and

$$R_{RA,k,\boldsymbol{b}} = \sum_{\mathbf{s} \in \{0,1\}^k} \mathsf{ck}_{A,(\mathbf{s},0,\mathbf{b})} f_{k,\mathbf{x}^{-1}}(\mathbf{s})$$

and

$$R_{LR,k,\boldsymbol{b}} = \sum_{\mathbf{s} \in \{0,1\}^k} B_{\mathbf{s},1,\mathbf{b}} f_{k,\mathbf{x}^{-1}}(\mathbf{s})$$

Observe that with overwhelming probability $\{R_{LR,k-1,(\mathbf{0},0,\mathbf{1})}\}_{k=1}^{\ell-1}$ are non-zero, depend non-trivially on x_{k-1}, and have no dependence on x_k Thus the $R_{LR,0,(0,\mathbf{0},1)}, \ldots, R_{LR,\ell-2,(0,\mathbf{0},1)}$ are pairwise independent and ensure that: the $A_{0,0,\mathbf{1}}$ term in $\mathsf{agg}_{LR,2}$ (denoting the second round's agg_{LR}) is randomised by x_1 and thus is independent from the $A_{0,0,\mathbf{1}}$ terms in $\mathsf{agg}_{LR,1}$; the $A_{0,0,0,\mathbf{1}}$ term in $\mathsf{agg}_{LR,3}$ is randomised by x_2 and thus is independent from the $A_{0,0,0,\mathbf{1}}$ terms in $\mathsf{agg}_{LR,1}$ and $\mathsf{agg}_{LR,2}$; etc. Thus $A_{\mathbf{0},0,\mathbf{1}}$ perfectly blinds agg_{LR} except with negligible probability.

By the same argument, the $R_{LA,k,(\mathbf{0},0,1,\mathbf{0},1)}$ terms are pairwise independent and the $R_{LB,k,(\mathbf{0},1,\mathbf{0})}$ terms are pairwise independent ensuring independence between the $A_{(\mathbf{0},0,1,\mathbf{0},1)}$ terms in $\mathsf{com}_{LA,k}$ and $A_{(\mathbf{0},1,\mathbf{0})}$ terms in $\mathsf{com}_{LB,k}$ respectively. Thus $A_{\mathbf{0},0,1,\mathbf{0},1}$, $A_{\mathbf{0},1,\mathbf{0}}$ terms perfectly blinds com_{LA}, com_{RA} except with negligible probability.

By a symmetric argument we see that $B_{\mathbf{0},0,1,\mathbf{0},1}$, $B_{\mathbf{0},1,\mathbf{0}}$, $B_{\mathbf{0},0,\mathbf{1}}$ perfectly blinds com_{LB}, com_{RB}, agg_{RL} except with negligible probability.

We now consider the penultimate round (round $\ell-1$). Here the proof elements com_{LA}, com_{LB}, com_{RA} and com_{RB} of both the honest prover and simulator take the form

$$\begin{aligned} \mathsf{com}_{LA} = e(A_{\ell-1,1}, H_2), \mathsf{com}_{RA} = e(A_{\ell-1,2}, H_1), \\ \mathsf{com}_{LB} = e(G_1, B_{\ell-1,2}), \mathsf{com}_{RB} = e(G_2, A_{\ell-1,1}) \end{aligned}$$

for $\mathsf{ck}_{A,\ell-1} = (H_1, H_2)$ and $\mathsf{ck}_{B,\ell-1} = (G_1, G_2)$. Thus the proof elements

$$\mathsf{agg}_{LR} = e(A_{\ell-1,1}, B_{\ell-1,2}), \mathsf{agg}_{RL} = e(A_{\ell-1,2}, B_{\ell-1,1})$$

are uniquely detemined given com_{LA}, com_{LB}, com_{RA} and com_{RB}. Hence they are sampled from the same distribution.

□