Zero-Knowledge for Homomorphic Key-Value Commitments with Applications to Privacy-Preserving Ledgers

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Abstract. Commitments to key-value maps (or, authenticated dictionaries) are an important building block in cryptographic applications, including cryptocurrencies and distributed file systems.
In this work we study short commitments to key-value maps with two additional properties: full-hiding (both keys and values should be hidden) and homomorphism (we should be able to combine two commitments to obtain one that is the "sum" of their key-value openings). Furthermore, we require these commitments to be short and to support efficient transparent zero-knowledge arguments (i.e., without a trusted setup).
As our main contribution, we show how to construct commitments with the properties above as well as efficient zero-knowledge arguments over them. We additionally discuss a range of practical optimizations that can be carried out depending on the application domain.
Finally, we show a specific application of commitments to key-value maps to scalable anonymous ledgers. Our contribution there is to formalize multi-type anonimity ledgers and show how to extend QuisQuis (Fauzi et al. ASIACRYPT 2019). This results in an efficient, confidential multi-type system with a state whose size is independent of the number of transactions.

1 Introduction

In this work we propose constructions for efficient committments to key-value maps (with specific features) and for efficient zero-knowledge arguments that can prove properties on committed key-value maps.

Key-Value Maps. We can loosely consider a key-value map as the equivalent of a dictionary in some programming languages (e.g., Python): a way to map arbitrary keys—e.g., strings—to values—e.g., scalars. For example, the balance of a user in a wallet application could be represented by a key-value map as $\mathsf{kv} = \{(\mathsf{USD}, 100), (\mathsf{BTC}, 10)\}$, where each of the different asset types (the keys) are associated to an amount (the values). In this paper we will generally assume that values are in an algebraic group endowed with an addition operation $+$.

Our Focus: Short, Homomorphic, Fully-Hiding Commitments. A commitment to a key-value map is roughly similar to an ordinary commitment: it cannot be opened to two different key-value maps (binding) and it should not leak anything about neither the keys nor the values in it. In the case of key-value maps, however, we are interested in some additional functional and efficiency-related requirements:

- *Large key universe:* our commitments should support a large universe of keys, potentially super-polynomial in the security parameter. This implies that the algorithms of the commitment scheme should have a running time independent of (or logarithmic in) the size of the key universe.
- *Short commitments:* our commitments should have size independent not only of the size of the key universe, but also of the *density* of the key-value map. The density is the number of elements whose value is not zero (e.g., the density of kv in the example above was 2).
- *Homomorphic commitments:* we require our commitments to support an homomorphic operation $\circ$. For example if a commitment encodes a wallet and

  $$c = \mathsf{Com}(\{(\mathsf{USD}, 100), (\mathsf{BTC}, 10)\}) \text{ and } c' = \mathsf{Com}(\{(\mathsf{USD}, 20), (\mathsf{ETH}, 1)\})$$

  then we can compute

  $$c^* = c \circ c' = \mathsf{Com}(\{(\mathsf{USD}, 120), (\mathsf{BTC}, 10), (\mathsf{ETH}, 1)\})$$

  without knowing the opening of any of the commitments. Requiring homomorphism rules out Merkle Trees as a solution. Homomorphic properties of commitments to "structured objects" have wide applications in cryptography (see, e.g., [KZG10] for homomorphic polynomial commitments). We discuss our main application to privacy-preserving cryptocurrencies in Section 7, but we believe homomorphic commitments to key-value maps will find more applications in the future.

– *Efficient and transparent*[1] *zero-knowledge proofs*: we should be able to prove (and verify) efficiently arbitrary properties over commitments of key-value maps. We are interested in zero-knowledge proofs—which allow to prove properties over a secret value without leaking it—and where both keys and values are part of the secret. For example, one can prove that two committed key-value maps hold the same value for some key $\tilde{k}$. More formally, given as public input commitments c, c' and a public function f, one can prove knowledge of a key $\tilde{k}$ such that c, c' are commitments to key-value maps $\mathsf{kv}, \mathsf{kv}'$ respectively and $\mathsf{kv}[\tilde{k}] = f(\mathsf{kv}'[\tilde{k}])$.

While different subsets of these properties have been studied in literature, our contribution is to investigate constructions that require them *all*. Our goal is to provide concretely efficient tools useful in different application domains.

Key-Hiding Properties. Here we clarify what we mean by key-hiding properties and discuss how existing solution fail to solve our problem. We have three *key sets* of interest: the set of all the keys in the universe (which we will assume to be $\{0,1\}^*$ or a field $\mathbb{F}$ from now on), the set K_{active} of *active keys*, defined as all the keys that are being used in the system, and the set K_{com} of *committed keys*, defined as the non-zero keys in any *given* commitment. For example, in a wallet setting, K_{active} consists of all the keys (asset types) encoded in *some* wallet, while K_{com} would consist of those encoded in a *specific given* wallet. Depending on whether we want to hide the active or the committed keys or both we get four different settings, which we discuss below (see also Fig. 1).

Public Active keys. In the case where both the active keys and the committed keys are public, Pedersen commitments are already a solution to our problem. The system parameters will contain group elements $h, g_1, \ldots, g_n$ where there is a known association between k_i and g_i for all active keys k_i. We commit by computing $c = h^r \prod_i g_i^{v_i}$, and proving properties of values is trivial to do using existing sigma protocols since the verifier is allowed to learn the keys. In the case in which the active keys are public but the committed keys are private, Pedersen commitment can still be used but the (proving) complexity of the ZK proof would be linear in the number of active keys[2]. One of our contributions is to show how to bring this down to the size of the committed set.

Private Active keys. It does not make sense to consider the case where the set of active keys are private but the committed keys are public. The most interesting case is the one in which both of these sets are private. In this setting, it would be possible to commit using a non-homomorphic version of Pedersen commitment. We thus have $2n+1$ generators $(h, g_1, f_1, \ldots, g_n, f_n)$ and we commit computing $c = h^r \Pi g_i^{v_i} f_i^{k_i}$. Now it is possible to efficiently prove statements but the commitment is not homomorphic (and therefore not applicable in our settings of interest). Our main contribution is to provide a better solution for this case.

Application: Multi-Type QuisQuis. One application which uses the homomorphic property of commitments is the privacy-preserving transaction system QuisQuis [FMMO19]. It builds upon accounts to which tokens are deposited in a transaction without interaction of the receiver. Compared to existing privacy-preserving transaction systems like Zcash [BCG+14] or Monero [NM+16], QuisQuis's design achieves a state size linear in the number of participants instead of monotonically growing over time. Our goal is to integrate a notion of types into the system such that different currencies share a common anonymity set. Additionally, it allows for dynamic creation of confidential tokens by any participant without setting up a full separate system. For this application, we also present a secret key based key-value map commitment. In combination with effizient NIZKs, to show that transactions conserve all value, we achieve small transaction sizes.

1.1 Technical Overview

Our Construction of Key-Value Map Commitments. In order to commit to a key-value map $\{v_k\}_{k \in K}$ we assume a group $\mathbb{G}$ where the discrete logarithm is hard and a hash function $\mathcal{H}$ modeled as a random oracle mapping keys to group elements. We then compute a commitment as $\mathsf{c} = \prod_{k \in K} \mathcal{H}(k)^{v_k} h^r$ where h is a random generator of the group and r is a random scalar. This can be seen as a (vector)

[1] In a *transparent* argument system the setup does not need to be produced by a trusted party. This property is interesting in the case of *non-interactive* argument systems, which are the focus of this work.

[2] This is true for the aforementioned approach with sigma-protocols as well as for other straightforward applications of NIZKs.

Pedersen commitment with random key-dependent generators and it has short homomorphic commitments. In the next paragraphs, we show how we can construct efficient zero-knowledge proofs for circuits over such commitments.

	Public K_{active}	Private K_{active}
Public K_{com}	$c = h^r \prod_i g_i^{v_i}$	*(Uninteresting case)*
Private K_{com}	Section 5	Section 4

Fig. 1: Constructions for committments to key-value maps satisfying our requirements and their different settings. K_{active} is the set of "*active*" keys (all the keys committed somewhere); K_{com} is set of *committed* keys (those that open the commitments we are using in a proof *right now*).

Modular Transparent Zero-Knowledge Arguments for Committed Key-Value Maps. Fix a (large) field $\mathbb{F}$ and consider a circuit C over key-value maps (we assume that $\mathbb{F}$ is also both the key and the value space of the key-value map). We assume the syntax of C to be of the type $C(\mathsf{kv}_1, \ldots, \mathsf{kv}_\ell, \omega)$, the kv_i-s as private key value maps and ω as an additional private witness (ω is a vector of field elements). Given such a circuit we are interested in proving an augmented circuit that take as public input commitments to the ℓ key-value maps and proves their opening in addition to the relation from circuit C. More specifically, we propose an argument system for:

$$\begin{aligned} & C^*(\mathsf{c}_1, \ldots, \mathsf{c}_\ell; (\mathsf{kv}_1, \rho_1), \ldots, (\mathsf{kv}_\ell, \rho_\ell), \omega) \\ := & C(\mathsf{kv}_1, \ldots, \mathsf{kv}_\ell, \omega) \wedge \bigwedge_{i \in [\ell]} \mathsf{c}_i = \mathsf{Com}(\mathsf{kv}_i, \rho_i) \end{aligned} \tag{1}$$

where the part after the semicolon is considered the private witness. A more concrete intuition on the circuit above is: given committed key–value pairs we can prove properties of their values, their keys and any relation between these and other private values (contained in ω).

Our Template for Zero-Knowledge Arguments on Committed Key-Value Maps. We now describe how to prove properties on committed key-value maps. The following refers to the setting with private-active-keys/private-committed-keys (see lower-right quadrant in Figure 1) (in Section 5 we extend this approach to the public-active-key/private-committed-keys scenario; see lower-left quadrant in Figure 1).

Our construction follows a basic blueprint, which we now exemplify through a concrete case. Consider a committed key-value map $\{v_k\}_{k \in K}$ and the problem of proving in zero-knowledge that all its values v_k are in some range $\{0, \ldots 2^\mu - 1\}$. We proceed in two steps: we first let the prover send the verifier what we call *key tags*, these are masked version of the non-zero keys in the committed key-value map. The prover will also show that they are valid maskings of some set of keys. By providing key tags, we can then break the rest of the relation in two parts: *a)* showing knowledge of values (and randomness) that combined with the key tags produce the commitment c (part of the public input); *b)* showing that these values are in range. We now elaborate on each of these steps.

Given a key-value map $\{v_k\}_{k \in K}$ with density n (the number of non-zero keys[3]) we provide n key tags by sending $b_k = \mathcal{H}(k) h^{r_k}$ for a random r_k. The prover should also prove that each of them is of the prescribed form. We stress that, in order to do this, use the heuristic technique of proving a random oracle in zero-knowledge. Next, the prover would show knowledge of values $v_1, \ldots, v_n$ and an appropriate r' such that $\mathsf{c} = b_{k_1}^{v_1} \cdots b_{k_n}^{v_n} h^{r'}$ and $v_i \in \{0, \ldots 2^\mu - 1\}$ for all $i \in [n]$. The latter relation—comprising reconstructing the commitment from the key tags and the range proof—can for example be performed through a system like generalized Bulletproofs [LMR19] or compressed Σ-protocols [ACR20]. These provide interfaces to prove bilinear circuits, of which we only use the non-bilinear gates, with

[3] Here we consider the case where leaking the density of the key-value map is not a problem. We will also adapt our construction where this leakage does not occur if an upper bound on this density is known.

logarithmic proof sizes. We stress that our focus is on *transparent* solutions, i.e., without a trusted setup; all our constructions can be instantiated in a fully transparent manner.

We remark on two properties of the template of our construction above. First, we can easily reduce its amortized cost by splitting it into an offline stage (independent of the commitments on which we are carrying out proofs) and an online stage. We further discuss these improvements in Section 6. Second, we can adapt and optimize our construction to the scenario with *public* active keys and private committed keys (lower-left quadrant in Figure 1). We describe this next.

A Construction with Registration of Active Keys. In some settings, although the whole universe of keys can be extremely large, the set of active keys K_{active} at any given time can have a manageable size and be publicly known. Consider for example applications (e.g. multi-asset transaction system) where there is an exponentially large set of potential asset types (keys), but only a manageable subset of them are present in the system (active) at any given time. Moreover, before becoming active in the system they plausibly need to be registered (for example, through a first "genesis" transaction for that specific asset type). In such settings we can leverage the partial knowledge on existing keys to improve efficiency. We do this by introducing an operation that preprocesses the parameters of the system (or CRS) specializing them for a specific set of active keys. One example construction, which we discuss later in more detail, is one in which the specialized CRS for the set K_{active} is an accumulator[4] to the set of (unmasked) key tags corresponding to K_{active}, i.e., to the set $B_{\text{active}} = \{\mathcal{H}(k) : k \in K_{\text{active}}\}$. Thus, in the online stage, we can produce a proof on a commitment $\mathsf{c} = \mathsf{Com}(\{v_k\}_{k \in K})$ by: 1) producing masked key tags $B' = \{\mathcal{H}(k)h^{r_k} : k \in K\}$ produced with some fresh randomness r_k; 2) showing that each $b'_k \in B'$ is of the form $b \cdot h^{r_k}$ for some b in the accumulator; 3) showing knowledge of v_k-s such that $\mathsf{c} = h^r \prod_k b_k'^{v_k}$.

The main advantage of the construction above is that it does not require the hashing $\mathcal{H}(k)$ for the key-tags to be proven in zero-knowledge. This feature can improve efficiency in several settings (see also Section 5), such as the multi-type transaction systems Multi-type QuisQuis, described in Section 7. Yet, we remark that our techniques in Section 5 are compatible with other frameworks for transaction systems besides QuisQuis: they could for example be straightforwardly applied to obtain a multi-type version of 𝕍eksel[CHA21].

1.2 Related Work

Authenticated Data Structures Besides the aforementioned straw-man schemes based on Pedersen, a common approach to succinct key-value commitments uses Merkle trees. They are not homomorphic and opening them in zero-knowledge requires proving a number of hashes logarithmic in the number of committed elements, which can be expensive.

There is a large body of work on succinct commitments to key-value maps, e.g.[5], [AR20, CFG+20, BBF19, TXN20]. [6] Differently from our work, constructions in literature are not homomorphic and do not directly support hiding of keys/values. We observe, however, that if one could do without homomorphism the latter problem could be mitigated for some of these constructions by applying masking of keys/values and zero-knowledge. This is true for example for some of the works based on groups of unknown-order [CFG+20, BBF19] where we can use techniques to compose algebraic accumulators proofs and succinct zero-knowledge proof systems described in [BCFK19].

The work in [GPR+21] studies "oblivious key-value stores". They are setting is, however, different from ours: these data-structures hide the keys only if the values are random (not applicable in our setting) and are not homomorphic.

Other works on efficient Zero-Knowledge and key-value maps include Spice [SAGL18]. The authors use data-structures that hide the key but that are not homomorphic. The efficient solutions they describe requires a trusted setup.

[4] An accumulator is a cryptographic data structure that allows to commit to a set in a binding manner and to prove membership of an element efficiently.

[5] We refer the reader to [TXN20] for a survey of this rapidly growing field.

[6] The constructions in [CFG+20] and [BBF19] focus on vector commitments but they can be both be compiled into commitments to key-value maps: both the construction in [BBF19] and the first construction in [CFG+20] can be using compiled the techniques described in Section 5.4 of [BBF19]; the second construction of [CFG+20] is very similar to that in [AR20] and can be turned into a key-value commitments with similar tricks.

Confidential Transaction Systems and Multiple Token Types Here we describe works related to our main application, a multi-type version of QuisQuis.

Works on confidential transaction systems include Zcash [BCG+14], Monero [NM+16], Omniring [LRR+19], and 𝕍eksel [CHA21]. We now compare these works against the QuisQuis framework, which we extend in this work. The most critical aspect is sender anonymity. Zcash obtains the largest anonymity set among these works (as large as the UTXO set), but it does not have plausible deniability[7] and requires a trusted setup. Monero does not have these limitations; it is unclear how the anonymity in Monero fares against that in QuisQuis (see Discussion in [FMMO19]). Omniring improves the transaction size from being linear in the size of the anonymity set (Monero) to a logarithmic size. Both Zcash and Monero style systems, however, have transaction outputs that can (essentially) never be removed from the UTXO set. The payment system in 𝕍eksel, like QuisQuis, does not have this drawback. Differently from QuisQuis, 𝕍eksel achieves $O(1)$ transaction sizes, but at the price of weaker anonymity guarantees. All of these systems provide amount confidentiality but lack a notion of type or currency.

The work in [PBF+19] introduces confidential types by using homomorphic commitments whose construction is the "single key" version of ours. Their design has also been used in SwapCT [EMP+21] and integrated in MimbleWimble [YYD+19]. Another construction of confidential types is that of Cloaked Assets developed by Stellar, which separates types and values in two different data structures, similar to our non-homomorphic example. Therefore a transactor requires the openings of all inputs to create a conservation proof, providing no sender anonymity.

2 Notation and Preliminaries

Preliminaries on (Sparse) Key-Value Maps We assume a universe of keys $\mathcal{K}$ and a universe of values $\mathcal{V}$ such that in a key-value map, keys are a subset of $\mathcal{K}$ and values are any element in $\mathcal{V}$; they may be of size superpolynomial in a security parameter λ. We assume $\mathcal{V}$ to be an additive group endowed w.r.t. some operation $+$. A key-value map is defined as a function $\mathsf{kv} : \mathcal{K} \to \mathcal{V}$. We call its *density* the number of elements that are mapped to a non-zero value in $\mathcal{V}$. Our focus will be on *sparse* key-value maps, i.e., those whose density grows asymptotically as a polynomial in λ (and in practice may be concretely small). We can represent a sparse key-value map as a set of pairs $\{(k, v_k)\}_{k\in K}$ where $K \subseteq \mathcal{K}$: this maps each element $k \in K$ to v_k and any other element to $0 \in \mathcal{V}$. We often use the more succinct notation $\{v_k\}_{k\in K}$ for a key-value map $\{(k, v_k)\}_{k\in K}$ over the set K (we assume that the set K is implicitly part of the description of $\{v_k\}_{k\in K}$). Consequently the empty set $\emptyset$ represents the key-value map with all elements in the universe initialized to zero; we denote the latter empty key-value map $\emptyset_{kv}$ to be explicit. We denote by $-\mathsf{kv}$ the key-value map associating to each key k the value $-\mathsf{kv}(k)$ and we define a sum of key-value maps as follows: $\{v_k\}_{k\in K} + \{v'_{k'}\}_{k'\in K'}$ is defined as $\left(k, v_k + v'_{k'}\right)_{k\in K\cup K'}$. A partition of a key-value map $\{v_k\}_{k\in K}$ is a pair of key-value maps $(\{v'_{k'}\}_{k'\in K'}, \{v''_{k''}\}_{k''\in K''})$ such that (K', K'') is a partition of K.

Cryptographic Assumptions

Assumption 1 (Interactive Generalized DLOG) *For all PPT $\mathcal{A}$ and all $n \geq 1$*

$$\Pr\left[\begin{array}{rl} \mathbb{G} \leftarrow \mathcal{G}(1^\lambda) & \\ (g_1, \ldots, g_n, \mathsf{aux}) \leftarrow \mathcal{A}(\mathbb{G}) & : \exists j^* \in \{n+1, \ldots, 2n\}\ a_{j^*} \neq 0 \ \wedge \\ g_{n+1} \leftarrow\!\!\$\ \mathbb{G} & \prod_{j\in[2n]} g_j^{a_j} = 1_\mathbb{G} \\ (a_1, \ldots, a_{2n}, g_{n+2}, \ldots, g_{2n}) \leftarrow \mathcal{A}(g_{n+1}, \mathsf{aux}) & \end{array}\right] \leq \mathsf{negl}(\lambda)$$

Assumption 2 (Generalized DLOG [BBB+18]) *For all PPT $\mathcal{A}$ and all $m \geq 2$*

$$\Pr\left[\begin{array}{rl} \mathbb{G} \leftarrow \mathcal{G}(1^\lambda) & \exists j^* \in [m]\ a_{j^*} \neq 0 \ \wedge \\ (g_1, \ldots, g_m) \leftarrow\!\!\$\ \mathbb{G} & : \prod_{j\in[m]} g_j^{a_j} = 1_\mathbb{G} \\ (a_1, \ldots, a_m) \leftarrow \mathcal{A}(\mathbb{G}, g_1, \ldots, g_m) & \end{array}\right] \leq \mathsf{negl}(\lambda)$$

[7] Plausible deniability: no one can tell if a user meant to be involved in a transaction.

Lemma 1. *Assumption 2 implies Assumption 1.*

Proof. TODO. To be written down. It uses some rewinding of the adversary. NB: there may exist a proof of this in Russel Lai's notes.

NIZKs Here we describe the basic notion of non-interactive zero-knowledge. In Section 4.1 we provide explicit syntax for the specific setting of NIZKs over committed key-value maps.

Definition 1. *A NIZK for a relation family* $\mathcal{R} = \{\mathcal{R}_\lambda\}_{\lambda\in\mathbb{N}}$ *is a tuple of algorithms* $\mathsf{NIZK} = (\mathsf{Setup}, \mathsf{Prove}, \mathsf{VerProof})$ *with the following syntax:*

- $\mathsf{NIZK.Setup}(1^\lambda) \to \mathsf{crs}$ *outputs a common-reference string* crs *for the relation family* $\mathcal{R}$*. If the argument system is transparent , this can consist of uniform random elements.*
- $\mathsf{NIZK.Prove}(\mathsf{crs}, R, \mathbb{w}) \to \pi$ *takes as input a string* crs*, a relation description* R *(in which we embed the whole public input), a witness* w *such that* $R(\mathbb{w})$*; it returns a proof* π*.*
- $\mathsf{NIZK.VerProof}(\mathsf{crs}, R, \pi) \to b \in \{0,1\}$ *takes as input a string* crs*, a relation description* R *a proof* π*; it accepts or rejects the proof.*

Whenever the relation family is obviously defined, we talk about a "NIZK for a relation R". We require a NIZK to be *complete*, that is, for any $\lambda \in \mathbb{N}$, $R \in \mathcal{R}$ and $\mathbb{w} \in R$ it holds with overwhelming probability that $\mathsf{VerProof}(\mathsf{crs}, R, \pi)$ where $\mathsf{crs} \leftarrow\!\$\ \{0,1\}^{\mathsf{poly}(\lambda)}$ and $\pi \leftarrow \mathsf{Prove}(\mathsf{crs}, R, \mathbb{w})$. Other properties we require are: *knowledge-soundness* and *zero-knowledge*. Informally, the former states we can efficiently "extract" a valid witness from a proof that passes verification; the latter states that the proof leaks nothing about the witness (this is modeled through a simulator that can output a valid proof for an input in the language without knowing the witness). Notationally, we separate public and private inputs in relations and proving algorithm through a semicolon. These definitions are standard and we defer formal details to Appendix A.

On efficiency of NIZKs. The efficiency (proving/verification runtimes and proof size) of a NIZK often depends on the size of the description of a relation in *constraints* (these roughly correspond to the multiplication gates of its circuit representation). We will refer to this notion later in the text. See also [BBB+18].

3 Key-Value Commitments

Here we define homomorphic commitments to key-value maps where both keys and values are hidden. Although we will use the extensions in Section C.2 for our construction of Multi-Type Quis-Quis, we will present a simplified version first since it is arguably easier to grasp first and it has independent interest.

Definition 2 (Commitment to Key-Value Maps (kvC)). *The following is a syntax for our key-value maps*

$\mathsf{Setup}(1^\lambda) \to \mathsf{pp}$ *generates public parameters.*
$\mathsf{Com}(\mathsf{pp}, \{v_k\}_{k\in K}; r) \to \mathsf{c}$ *commits to the key value map with randomness* r*. We keep the randomness implicit whenever it does not affect clarity and we assume it to be sampled from an additive group.*[8]

Definition 3 (Hiding). *A key-value map commitment is hiding if for all key-value maps* $\{v'_{k'}\}_{k'\in K'}, \{v''_{k''}\}_{k''\in K''}$ *(even of different size), for* $\mathsf{pp} \leftarrow \mathsf{Setup}(1^\lambda)$ *the following two distributions are computationally indistinguishable*

$$\{\mathsf{Com}(\mathsf{pp}, \{v'_{k'}\}_{k'\in K'})\} \approx \{\mathsf{Com}(\mathsf{pp}, \{v''_{k''}\}_{k''\in K''})\}$$

Definition 4 (Binding). *A key-value map commitment is (computationally) binding if for any PPT adversary* $\mathcal{A}$*, it holds that*

$$\Pr\left[\begin{array}{rl} \mathsf{pp} \leftarrow \mathsf{Setup}(1^\lambda) & \mathsf{c} = \mathsf{Com}(\mathsf{pp}, \{v'_{k'}\}_{k'\in K'}, r') \wedge \\ (\mathsf{c}, \{v'_{k'}\}_{k'\in K'}, r', & : \ \mathsf{c} = \mathsf{Com}(\mathsf{pp}, \{v''_{k''}\}_{k''\in K''}, r'') \wedge \\ \{v''_{k''}\}_{k''\in K''}, r'') \leftarrow \mathcal{A}(\mathsf{pp}) & \{v'_{k'}\}_{k'\in K'} \neq \{v''_{k''}\}_{k''\in K''} \end{array}\right] \leq \mathsf{negl}(\lambda)$$

[8] We will use this when defining the homomorphic property of commitments.

$\mathsf{Setup}(1^\lambda) \to \mathsf{pp}$: samples a group $\mathbb{G}$ and $g, h \leftarrow\!\!\$\, \mathbb{G}$ and return $\mathsf{pp} = (\mathbb{G}, g, h)$.
$\mathsf{Com}(\mathsf{pp}, \{v_k\}_{k\in K}; r) \to \mathsf{c}$: return $\prod_{k\in K} \mathcal{H}(k)^{v_k} h^r$.

Fig. 2: Our construction for kvC.

Definition 5 (Homomorphism). *We say a commitment to a key-value map is homomorphic if there exists an operation $\circ$ such that* Setup *always produces* pp *such that for all maps $\{v_k\}_{k\in K}, \{v'_{k'}\}_{k'\in K'}$ and randomness r, r' it holds that*

$$\mathsf{Com}(\{v_k\}_{k\in K}; r) \circ \mathsf{Com}(\{v'_{k'}\}_{k'\in K'}; r') = \mathsf{Com}(\{v^*_{k^*}\}_{k^*\in K^*}; r + r')$$

where $K^* = K \cup K'$ and $(k^*, v^*_{k^*}) = \begin{cases} (k^*, v_{k^*}) \text{ if } k^* \in K \setminus K' \\ (k^*, v'_{k^*}) \text{ if } k^* \in K' \setminus K \\ (k^*, v_{k^*} + v'_{k^*}) \text{ if } k^* \in K \cap K' \end{cases}$

3.1 Construction

We recap some of the properties we are interested in obtaining in our construction: (i) support large key universe; (ii) small commitments and small parameters; (iii) homomorphic (Definition 5); (iv) support efficient non-interactive zero-knowledge proof of knowledge of opening (in particular they should run in time linear in the density of the key-value map).

In Figure 2 we propose a construction based on random-oracle with the properties above. Given a prime p we consider the universe of values $\mathcal{V} = \mathbb{Z}_p$, a group $\mathbb{G}$ isomorphic to it and for which the GDLOG assumption holds, a hash function $\mathcal{H}$ modeled as a random oracle and an arbitrary key universe $\mathcal{K}$ such that $\mathcal{H} : \mathcal{K} \to \mathbb{G}$.

Theorem 1. *If $\mathcal{H}$ is a random oracle and under the GDLOG assumption the construction in Figure 2 is a kvC with value universe $\mathbb{Z}_p$.*

Proof. As for standard Pedersen, the construction is perfectly hiding because of the masking factor h^r. To show why the construction is binding we argue as follows. Assume an adversary is able to provide two distinct openings for the same commitment. Each of these openings consists of both keys and values. The case where the keys are the same for both openings can be argued as in standard (vector) Pedersen. Consider the case where the two openings differ in the set of keys. We can write $\mathsf{c} = \prod_{k\in K} \mathcal{H}(k)^{v_k} h^r = \prod_{k'\in K'} \mathcal{H}(k')^{v'_{k'}} h^{r'}$ which reduces to

$$\prod_{k\in K} \mathcal{H}(k)^{v_k} \prod_{k\in K'} \mathcal{H}(k')^{-v'_{k'}} h^{r-r'} = 1_{\mathbb{G}} \ .$$

Since the random oracle evaluations are distributed as random group generators we can use this adversary to break Assumption 2.

4 Arguments on Key-Value Commitments (Fully Private Setting)

Here we formalize and construct zero-knowledge arguments over key-value map commitments for the setting in which there is no information on the keys available in the system and those we are using in our proof.

4.1 Circuits over Key-Value Maps

To support arbitrary computation on committed key-value maps, we provide an interface which supports any arithmetic circuit of the following form. The keys and values kv as well as an additional witness ω are field elements in $\mathbb{F}$. The circuit consists of multiplication gates of the form $\mathbb{F} \times \mathbb{F} \to \mathbb{F}$. They have

an unbounded outbound degree and any linear relations are directly expressed between outputs and inputs.

We write any circuit using multiplication gates in the domain $\mathcal{KV}^\ell \times \mathbb{F}^{n_\omega}$ as $C^{\mathbb{F}}(\mathsf{kv}_1, \ldots, \mathsf{kv}_\ell, \omega)$. This circuit depends on the desired property the openings should have. Here ω is an additional private witness that may not depend on the opening to the key-value maps.

4.2 Arguments for Circuits over Committed Key-Value Maps

Here we present the overview of our argument system which works over committed key value maps and takes an arbitrary inner circuit operating on the openings of the commitments. To be clear, we spell out its formalization explicitly but it is a special case of Definition 1.

Given an inner circuit $C^{\mathbb{F}}$ as described above, our high level interface for proofs has the following form:

$\mathsf{kvNIZK.Setup}(1^\lambda) \to \mathsf{crs}$ takes the security parameter λ and outputs a crs.
$\mathsf{kvNIZK.Prove}(\mathsf{crs}, C^{\mathbb{F}}, c_1, \ldots, c_\ell, (\mathsf{kv}_1, \rho_1), \ldots, (\mathsf{kv}_\ell, \rho_\ell), \omega) \to \pi$ takes the crs and a circuit $C^{\mathbb{F}}$ as well as ℓ commitments c_i and their openings (kv_i, ρ_i) and an auxiliary witness ω. It outputs a proof π
$\mathsf{kvNIZK.VerProof}(\mathsf{crs}, C^{\mathbb{F}}, c_1, \ldots, c_\ell, \pi) \to b \in \{0, 1\}$ takes the crs, a circuit $C^{\mathbb{F}}$, and ℓ commitments c_i. It outputs a bit b depending on the validity of the proof π.

The relation we want to prove is defined by the circuit in Equation (1). To clarify our notation we re-define correctness for arguments for committed key-value maps.

Definition 6 (Correctness). *An argument for committed key-value maps is* correct*, if for any $\lambda \in \mathbb{N}$ with $\mathsf{crs} \in \mathsf{kvNIZK.Setup}(1^\lambda)$, any circuit $C^{\mathbb{F}}$, any key-value maps $\vec{\mathsf{kv}} \in \mathcal{KV}^\ell$ and randomness $\vec{\rho} \in \mathbb{F}^\ell$ with $\forall i \in [\ell] : \mathsf{c}_i = \mathsf{Com}(\mathsf{kv}_i, \rho_i)$ and any $\omega \in \mathbb{F}^{n_\omega}$ for which*

$$C^*(\mathsf{c}_1, \ldots, \mathsf{c}_\ell; (\mathsf{kv}_1, \rho_1), \ldots, (\mathsf{kv}_\ell, \rho_\ell), \omega) = 1$$

it holds that

$$\pi \leftarrow \mathsf{kvNIZK.Prove}(\mathsf{crs}, C^{\mathbb{F}}, c_1, \ldots, c_\ell, (\mathsf{kv}_1, \rho_1), \ldots, (\mathsf{kv}_\ell, \rho_\ell), \omega)$$
$$\mathsf{kvNIZK.VerProof}(\mathsf{crs}, C^{\mathbb{F}}, c_1, \ldots, c_\ell, \pi) = 1$$

As for NIZKs, we require knowledge-soundness and zero-knowledge.

4.3 Construction with intermediate Key Tags

Our construction has two stages. First the prover creates some key tags b_k and proves that they are well formed (i.e., they are obtained by hashing a key and masking with a random group element, both known to the prover). These key tags are then used in a subsequent proof for the opening of the commitment and the actual relation. Intuitively, since the prover knows how the key tags have been produced, the prover is able to compute openings of the input commitments under the new "base" $(h, b_1, \ldots, b_n)$ – as opposed to the the original base $(g, \mathcal{H}(k_1), \ldots, \mathcal{H}(k_n)$ – by appropriately computing the randomizers as a function of the values in the commitment and the randomness used for producing the key tags. This allows to avoid proving properties of the hash function in the second part of the proof. The full construction is presented in Figure 3. For sake of presentation and w.l.o.g., in the construction we assume that all our key-value maps include the same keys $k_1, \ldots, k_n$.

Theorem 2. *Under the GDLOG assumption, if NIZK_{tags} and NIZK_R are secure (correct, zero-knowledge, knowledge-sound) NIZKs for their required relation families, then the construction in Fig. 3 is a secure kvNIZK for arbitrary circuits over the key-value map commitment scheme in Fig. 2.*

Proof. Correctness: Correctness follows by inspection. In particular, note that when proving R_{R} the prover "opens" the commitments c_i under the base defined by the key tags b_k's. Since the b_k's are generated with the same h as the original commitment and the prover knows the openings of the b_k's, the prover can find the right value to be used as exponent for h.

The protocol is described in the random oracle model where all parties have access to $\mathcal{H} : \{0,1\}^* \to \mathbb{G}$.

$\mathsf{Setup}(1^\lambda)$:

1. compute $\mathsf{crs}_{\text{tags}} \leftarrow \mathsf{NIZK.Setup}_{\text{tags}}(1^\lambda)$ and $\mathsf{crs}_R \leftarrow \mathsf{NIZK.Setup}(1^\lambda)$.
2. generate the commitment key $h \xleftarrow{\$} \mathbb{G}$ (setup for kv commitments).
3. sample random generators $\vec{g} \xleftarrow{\$} \mathbb{G}^n$ where n is the maximum number of keys in a commitment.
4. Output $\mathsf{crs} = (\mathsf{crs}_{\text{tags}}, \mathsf{crs}_R, h, \vec{g})$

$\mathsf{Prove}(\mathsf{crs}, C^{\mathbb{F}}, c_1, \ldots, c_\ell, (\mathsf{kv}_1, \rho_1), \ldots, (\mathsf{kv}_\ell, \rho_\ell), \omega)$

1. From all key value maps kv_i, extract the set of non-zero keys K_i and define $(k_1, \ldots, k_n) := \bigcup_{i\in[\ell]}\{k \in K_i\}$. Parse $\mathsf{kv}_i = \{(k, v_{i,k}) : k = k_1, \ldots, k_n\}$ for each $i = 1, \ldots, \ell$
2. Sample random values $\vec{r} \xleftarrow{\$} \mathbb{F}^n$ and create the key tags $b_{k_i} = \mathcal{H}(k_i)h^{r_i}$. Additionally, create a vector Pedersen commitment to all keys $c^* = h^s \prod_{i=1}^n g_i^{k_i}$ with randomness $s \xleftarrow{\$} \mathbb{F}$.
3. Generate a proof that the key tags are well formed by proving knowledge of the pre images k_i i.e., generate a proof
$$\pi_{\text{tags}} \leftarrow \mathsf{NIZK.Prove}_{\text{tags}}(\mathsf{crs}, b_{k_1}, \ldots, b_{k_n}, c^*; k_1, \ldots, k_n, \vec{r}, s)$$
for the relation
$$R_{\text{tags}}(b_{k_1}, \ldots, b_{k_n}, c^*; k_1, \ldots, k_n, \vec{r}, s) :=$$
$$(c^* = h^s \prod_{i=1}^n g_i^{k_i} \wedge \forall i \in [n] : b_{k_i} = \mathcal{H}(k_i)h^{r_i})$$
4. Generate a proof
$$\pi_R \leftarrow \mathsf{NIZK.Prove}_{\text{R}}(\mathsf{crs}, C^{\mathbb{F}}, c_1, \ldots, c_\ell, b_{k_1}, \ldots, b_{k_n}, c^*; (\mathsf{kv}_1, \rho_1), \ldots, (\mathsf{kv}_\ell, \rho_\ell), \vec{r}, s, \omega)$$
for the relation
$$R_{\text{R}}(C^{\mathbb{F}}, c_1, \ldots, c_\ell, b_{k_1}, \ldots, b_{k_n}, c^*; (\mathsf{kv}_1, \rho_1), \ldots, (\mathsf{kv}_\ell, \rho_\ell), \vec{r}, s, \omega) :=$$
$$c^* = h^s \prod_{i=1}^n g_i^{k_i} \wedge \forall i \in [\ell] : c_i = h^{\rho_i - \sum_{j=1}^n r_j v_{i,k_j}} \prod_{j=1}^n b_{k_j}^{v_{i,k_j}} \tag{2}$$
$$\wedge C^{\mathbb{F}}((\mathsf{kv}_i)_{i\in[\ell]}, \omega) = 1$$
5. Return both proofs including the key-tags and the pre-image commitment $\pi := (b_{k_1}, \ldots, b_{k_n}, c^*, \pi_{\text{tags}}, \pi_R)$

$\mathsf{VerProof}(\mathsf{crs}, C^{\mathbb{F}}, c_1, \ldots, c_\ell, \pi)$

1. Parse π as $(b_{k_1}, \ldots, b_{k_n}, c^*, \pi_{\text{tags}}, \pi_R)$.
2. Verify $b_0 \leftarrow \mathsf{NIZK.VerProof}_{\text{tags}}(\mathsf{crs}, b_{k_1}, \ldots, b_{k_n}, c^*, \pi_{\text{tags}})$
3. Verify $b_1 \leftarrow \mathsf{NIZK.VerProof}_R(\mathsf{crs}, C^{\mathbb{F}}, c_1, \ldots, c_\ell, b_{k_1}, \ldots, b_{k_n}, c^*, \pi_R)$
4. return $b_0 \wedge b_1$

Fig. 3: Our construction for kvNIZK

Zero-Knowledge: For zero-knowledge, we build our simulator $\mathcal{S}^*$ using the simulators for the two subprotocols, namely $\mathcal{S}_{\text{tags}}, \mathcal{S}_{\text{R}}$. On input a valid statement $(\mathsf{crs}, C^{\mathbb{F}}, c_1, \ldots, c_\ell)$, $\mathcal{S}^*$ outputs $\pi = (b_1, \ldots, b_n, c^*, \pi_{\text{tags}}, \pi_R)$ where: $b_1, \ldots, b_n, c^*$ are random group elements, and we simulate $\pi_{\text{tags}} \leftarrow \mathcal{S}_{\text{tags}}(\mathsf{crs}_{\text{tags}}, b_1, \ldots, b_n, c^*)$ and $\pi_{\text{R}} \leftarrow \mathcal{S}_{\text{R}}(\mathsf{crs}_{\text{R}}, C^{\mathbb{F}}, c_1, \ldots, c_\ell, b_1, \ldots, b_n, c^*)$

We can prove indistinguishability of the simulated and the real transcript through the following hybrids:

- *Hybrid 0.* This is the view in the real protocol.
- *Hybrid 1.* This is the same as the previous hybrid, but we replace π_{tags} with a simulated proof. Indistinguishability follows from the ZK property of the subprotocol for R_{tags}.
- *Hybrid 2.* This is the same as the previous hybrid, but we replace π_{R} with a simulated proof. Indistinguishability follows from the ZK property of the subprotocol for R_{R}.
- *Hybrid 3.* This is the same as the previous hybrid, but we replace $b_1, \ldots, b_n, c^*$ with random group elements. Indistinguishability follows since these group elements are distributed identically in both hybrid 2 and 3 due to the choice of the randomizers $s, \vec{r}$ in the real protocol/Hybrid 2. Note moreover that when replacing these commitments with random group elements we do not change the valdity of the statements to be proven by the two simulators $\mathcal{S}_{\text{tags}}$ and $\mathcal{S}_{\text{R}}$. That is, $(b_1, \ldots, b_n, c^*)$ and $(C^{\mathbb{F}}, c_1, \ldots, c_\ell, b_1, \ldots, b_n, c^*)$ are true statements for the two sub-relations also when the group elements are randomly sampled. This is due to the fact that both relations are trivial: since Pedersen commitments are perfectly hiding, for any $\mathsf{kv}_1, \ldots, \mathsf{kv}_\ell$ there exist $s, \vec{r}$ such that the relation holds.

Since the distribution in Hybrid 3 is identical to a simulated execution, this concludes the proof of zero-knowledge.

Knowledge-Soundness: To prove knowledge-soundness, assume the existence of extractors $\mathcal{E}_{\text{tags}}, \mathcal{E}_{\text{R}}$ for the two sub-relations. We build an extractor $\mathcal{E}^*$ that on input a statement $(C^{\mathbb{F}}, c_1, \ldots, c_\ell)$ and accepting proof $(b_1, \ldots, b_n, c^*, \pi_{\text{tags}}, \pi_{\text{R}})$, outputs $((\mathsf{kv}_1, \rho_1), \ldots, (\mathsf{kv}_\ell, \rho_\ell), \omega)$. Our $\mathcal{E}^*$ works as follows. It first runs $(k'_1, \ldots, k'_n, \vec{r}', s') \leftarrow \mathcal{E}_{\text{tags}}(b_1, \ldots, b_n, c^*, \pi_{\text{tags}})$ and $((\mathsf{kv}_1, \rho'_1), \ldots, (\mathsf{kv}_\ell, \rho'_\ell), \vec{r}, s, \omega) \leftarrow \mathcal{E}_{\text{R}}(C^{\mathbb{F}}, c_1, \ldots, c_\ell, b_1, \ldots, b_n, c^*, \pi_{\text{R}})$ such that the two relations hold for the extracted witnesses. We argue first that $(k'_1, \ldots, k'_\ell, s') = (k_1, \ldots, k_\ell, s)$, since otherwise we can construct an adversary that breaks the binding property of the Pedersen commitment c^*. Then, we show how to extract valid openings for the input commitments c_i. Remember that thanks to the knowledge-soundness of the second proof system (for relation R_{R}) we know that for all commitments c_i it holds that (we remove the index i here to improve readability)

$$c = h^{\rho' - \sum_{j=1}^{n} r_j v_{k_j}} \prod_{j=1}^{n} b_{k_j}^{v_{k_j}}$$

Thanks to the knowledge soundness of the first proof system (for relation R_{tags}) we know that $b_{k_j} = \mathcal{H}(k_j) h^{r'_j}$, thus we can rewrite c as

$$c = h^{\rho' - \sum_{j=1}^{n} r_j v_{k_j}} \prod_{j=1}^{n} \mathcal{H}(k_j)^{v_{k_j}} h^{r'_j \cdot v_{k_j}} \qquad = h^{\rho' - \sum_{j=1}^{n} (r_j - r'_j) v_{k_j}} \prod_{j=1}^{n} \mathcal{H}(k_j)^{v_{k_j}}$$

Thus our extractor can output $((\mathsf{kv}_1, \rho_1), \ldots, (\mathsf{kv}_\ell, \rho_\ell), \omega)$ with $\rho_i = \rho'_i - \sum_{j=1}^{n} (r_j - r'_j) v_{k_{i,j}}$ as the witness for the overall relation. Note that the proof does not guarantee that $\vec{r} = \vec{r'}$. However this is not a problem since we are still guaranteed that the extracted witness for the overall relation is a valid one.

4.4 How to Instantiate the Subprotocols

To instantiate the well formedness of the tags, i.e. the relation R_{tags}, we propose to use a cryptographic hash function such as MiMC [AGR+16], Poseidon [GKR+21], GMiMC [AGP+19], or Marvellous [AAB+19] which are optimized for zero-knowledge proofs. They provide hashing to a field element $(\mathcal{H}^{\mathbb{F}} : \{0,1\}^* \to \mathbb{F})$. A subsequent circuit then proves that the field element is equivalent to the group element in the key tags. For the example of elliptic curves, this is relation $[b_{k_i}] = [(\mathcal{H}^{\mathbb{F}}(k_i), 0)] + r_i \cdot [h]$

where brackets enclose group elements and $(\mathcal{H}^{\mathbb{F}}(k_i), 0)$ is an explicit description of the curve point from a field element $\mathcal{H}^{\mathbb{F}}(k_i)$. The combined constraints are then proven by e.g. Bulletproofs [BBB+18].

The circuit for the relation R_{R} can be implemented through a generalization of Bulletproofs [LMR19] or compressed Σ-protocols [ACR20]. They provide an interface supporting bilinear circuits with five gates to enable arbitrary computation of which we use a subset of gates for non-bilinear circuits only. Given a circuit C constructed from the available gates, they then provide an efficient protocol with communication complexity $6\lceil \log_2(|C|) \rceil + 28$.

5 Arguments on Key-Value Commitments with Public Active Keys

In this section we consider the case where the set of active keys is known by both the prover and the verifier. We can then add a feature by which in a setup phase it is possible to register a set of keys which are going to be used for later proofs. This can improve the amortized efficiency when several proofs share the same set of active keys.

5.1 NIZKs with Key-Registration

We define NIZK for key-value maps which allow a key-registration phase. To this goal an algorithm $\mathsf{RegisterActiveSet}$ to the NIZK interface *deterministically* specializes a crs produces one specific to a set of active keys. Similar ideas have been used in previous work e.g., [BCFK19, CFG+20, CHA21] where a set-dependent specialized CRS is used.

Definition 7. *A NIZK for key-value maps with registration stage* kvNIZK^{reg} *consists of the following algorithms:*

$\mathsf{kvNIZK}^{\mathbf{reg}}.\mathsf{Setup}(1^\lambda) \to \mathsf{crs}$ *on input* λ *it outputs a crs* crs.

$\mathsf{kvNIZK}^{\mathbf{reg}}.\mathsf{RegisterActiveSet}(\mathsf{crs}, K_{\mathbf{active}}) \to \mathsf{crs}^{\mathbf{reg}}$ *is a deterministic algorith that takes as input a* crs*, an active set* K_{active} *and outputs a specialized CRS* crs^{reg} [9].

$\mathsf{kvNIZK}^{\mathbf{reg}}.\mathsf{Prove}(\mathsf{crs}^{\mathbf{reg}}, K_{\mathbf{active}}, C^{\mathbb{F}}, c_1, \ldots, c_\ell, (\mathsf{kv}_1, \rho_1), \ldots, (\mathsf{kv}_\ell, \rho_\ell), \omega) \to \pi$ *takes* crs^{reg}*, a circuit* $C^{\mathbb{F}}$*,* ℓ *commitments* c_i*, their openings* (kv_i, ρ_i) *and auxiliary witness* ω*. It outputs a proof* π*. Each of the* kv_i *should use keys that are subset of* K_{active}*. .*

$\mathsf{kvNIZK}^{\mathbf{reg}}.\mathsf{VerProof}(\mathsf{crs}^{\mathbf{reg}}, C^{\mathbb{F}}, c_1, \ldots, c_\ell, \pi) \to b \in \{0,1\}$ *takes the* crs^{reg}*, a circuit* $C^{\mathbb{F}}$*, and* ℓ *commitments* c_i*.*

We require properties as in Definition 1: correctness (completeness), knowledge soundness, zero-knowledge. Definition 7 is *almost* a special case of the notion for kvNIZKs, but not quite. We could express it as a NIZK including K_{active} as input to verification and require whatever $\mathsf{RegisterActiveSet}$ does as part of the relation. However, as it's standard in NIZKs, that would require the verifier to run linearly in $|K_{\mathrm{active}}|$. By taking it as input only a (potentially smaller) processing of it, $\mathsf{crs}^{\mathrm{reg}}$, the verifier can even run sublinearly in the set of active keys (which is the case in our construction). This motivates the special interface we introduce in Definition 7. Otherwise, the required properties are straightforward variants of those for NIZKs.

The relation we want to prove is again that defined by the circuit in Equation (1). For sake of clarity, we spell out correctness.

Definition 8 (Correctness). *We say that a NIZK for key-value maps with registration stage* kvNIZK^{reg} *over key-value map commitment scheme* $\mathsf{C_{kv}}$ *is correct if for any* $\lambda \in \mathbb{N}$ *with* $\mathsf{crs} \in \mathsf{kvNIZK.Setup}(1^\lambda)$*, any* $\mathsf{crs} \in \mathsf{C_{kv}.Setup}(1^\lambda)$*, any set of keys* K_{active}*, any circuit* $C^{\mathbb{F}}$*, any key-value maps* $\vec{\mathsf{kv}} \in \mathcal{KV}^\ell$ *with keys all in* K_{active} *and randomness* $\vec{\rho} \in \mathbb{F}^\ell$ *with* $\forall i \in [\ell] : \mathsf{c}_i = \mathsf{C_{kv}.Com}(\mathsf{crs}, \mathsf{kv}_i, \rho_i)$ *and any* $\omega \in \mathbb{F}^{n_\omega}$ *for which*

$$C(\mathsf{c}_1, \ldots, \mathsf{c}_\ell; (\mathsf{kv}_1, \rho_1), \ldots, (\mathsf{kv}_\ell, \rho_\ell), \omega) = 1$$

[9] It is w.l.o.g. that we require the registration algorithm to register in batch the whole set of active keys. It would be possible to define a version where we incrementally register a new key to a previously preprocessed CRS. Several constructions, e.g. algebraic accumulators, would efficiently support this syntax.

it holds that

$$\pi \leftarrow \mathsf{kvNIZK.Prove}(\mathsf{crs}^{reg}, C^{\mathbb{F}}, c_1, \ldots, c_\ell, (\mathsf{kv}_1, \rho_1), \ldots, (\mathsf{kv}_\ell, \rho_\ell), \omega)$$
$$\mathsf{kvNIZK.VerProof}(\mathsf{crs}^{reg}, C^{\mathbb{F}}, c_1, \ldots, c_\ell, \pi) = 1$$

where $\mathsf{crs}^{reg} \leftarrow \mathsf{RegisterActiveSet}(\mathsf{crs}, K_{active})$

5.2 Accumulator-based Construction

We require the two following building blocks (described below): *accumulators* and *NIZKs for membership over accumulators and rereandomization.*

The intuition behind our construction is to register the *already hashed* keys in an accumulator (without masking). This process can be performed publicly since producing the accumulator is deterministic. In order to output a proof, we produce a set of key tags (masked version of the keys committed in the accumulator) and that these are appropriate rerandomized versions of the group elements in the accumulator.

5.2.1 Building Block: Accumulators

Definition 9 (Accumulator scheme). *An accumulator scheme* AccScm *over universe* $\mathcal{U}_\lambda(\mathsf{AccScm})$ *(where* λ *is a security parameter) consists of a quadruple of PPT algorithms* $\mathsf{AccScm} = (\mathsf{Setup}, \mathsf{Accum}, \mathsf{PrvMem}, \mathsf{VfyMem})$ *with the following syntax:*

$\mathsf{Setup}(1^\lambda) \to \mathsf{pp}_{\mathbf{acc}}$ *generates public parameters* pp_{acc}.
$\mathsf{Accum}(\mathsf{pp}_{\mathbf{acc}}, S) \to \mathsf{acc}$ *deterministically computes accumulator* acc *for set* $S \subseteq \mathcal{U}_\lambda(\mathsf{AccScm})$.
$\mathsf{PrvMem}(\mathsf{pp}_{\mathbf{acc}}, S, S') \to \pi_{acc}$ *computes witness* π_{acc} *that shows* $S' \subseteq S$.
$\mathsf{VfyMem}(\mathsf{pp}_{\mathbf{acc}}, \mathsf{acc}, S', \pi_{acc}) \to b \in \{0,1\}$ *verifies through witness whether subset* S' *is in the set accumulated in* acc. *We do not require parameter* S *to be in* $\mathcal{U}_\lambda(\mathsf{AccScm})$ *from the syntax.*

An accumulator scheme should satisfy correctness—the accumulator works as expected—and soundness—no efficient adversary can pick set S *and find a witness that checks on* $\mathsf{AccScm.Accum}(\mathsf{pp}_{acc}, S)$ *and* $S' \not\subseteq S$[10].

We assume the universe of the accumulator to be a group of prime order.

5.2.2 Building Block: NIZKs for membership and rerandomization We will use a NIZK for the following relation, which intuitively states that a set of group elements (fresh key tags) are a rerandomization of elements accumulated in the set.

$$R_{\text{mem-rand}}((\mathsf{pp}_{\text{acc}}, h \in \mathbb{G}, \mathsf{acc}); (B = (b_1, \ldots, b_n) \in \mathbb{G}^n, S = \{s_1, \ldots, s_n\} \in \mathbb{G}^n, \pi_{acc}, (r_1, \ldots, r_n) \in \mathbb{Z}_p^n))$$
$$\text{s.t. } b_i = s_i \cdot h^{r_i} \text{ for } i \in [n] \text{ and } \mathsf{VfyMem}(\mathsf{pp}_{\text{acc}}, \mathsf{acc}, S) = 1$$

We refer to the NIZK for the relation above as $\mathsf{NIZK}_{\text{mem-rand}}$.

5.2.3 Construction We describe our construction in Fig. 4. In our description we assume the buildig blocks above and a general-purpose zero-knowledge scheme kvNIZK like the one assumed in Section 4.2.

The security of our construction can be argued analogously to the security proof of our construction in Fig. 3 given the security of $\mathsf{NIZK}_{\text{mem-rand}}$. The latter NIZK proves membership of the keys in the accumulator without revealing anything about them and simulation is straighforward from this observation. By the binding property of the accumulator and the knowledge-soundness of $\mathsf{NIZK}_{\text{mem-rand}}$ we are guaranteed that keys at proving time belong to the accumulated set.

[10] These definitions are standard; see [BBF19] for a formal treatment.

$\mathsf{Setup}(1^\lambda)$:

1. compute accumulator parameters $\mathsf{pp}_{\text{acc}} \leftarrow \mathsf{AccScm.Setup}(1^\lambda)$
2. compute CRS for membership&randomize relation $\mathsf{crs}_{\text{mem-rand}} \leftarrow \mathsf{NIZK}_{\text{mem-rand}}.\mathsf{Setup}(1^\lambda)$
3. compute CRS for general-purpose NIZK $\mathsf{crs}_R \leftarrow \mathsf{NIZK.Setup}(1^\lambda)$
4. generate the commitment key $h \xleftarrow{\$} \mathbb{G}$.
5. sample random generators $\vec{g} \xleftarrow{\$} \mathbb{G}^n$ where n is the maximum number of keys in a commitment.
6. Output $\mathsf{crs} = (\mathsf{pp}_{\text{acc}}, \mathsf{crs}_{\text{mem-rand}}, \mathsf{crs}_R, h, \vec{g})$

$\mathsf{RegisterActiveSet}(\mathsf{crs}, K_{\mathbf{active}})$:

1. add hashes to an accumulator inside the new CRS by computing $\mathsf{acc} = \mathsf{AccScm.Accum}(\mathsf{pp}_{\text{acc}}, H)$ where $H := \{\mathcal{H}(k) : k \in K_{\text{active}}\}$
2. output $\mathsf{crs}^{\text{reg}} = (\mathsf{crs}, \mathsf{acc})$

$\mathsf{Prove}(\mathsf{crs}^{\mathbf{reg}}, K_{\mathbf{active}}, C^{\mathbb{F}}, c_1, \ldots, c_\ell, (\mathsf{kv}_1, \rho_1), \ldots, (\mathsf{kv}_\ell, \rho_\ell), \omega)$:

1. Run steps 1,2 and 4 in $\mathsf{NIZK.Prove}$ as described in Figure 3, that is: 1) parse the input; 2) create the key tags $b_{k_i} = \mathcal{H}_{\mathbb{G}}(k_i)h^{r_i}$ and commit to all keys $c^* = h^s \prod_{i=1}^n g_i^{k_i}$; and 4) Generate the proof π_R for the relation

$$\begin{aligned} R_{\text{R}}(C^{\mathbb{F}}, c_1, \ldots, c_\ell, b_{k_1}, \ldots, b_{k_n}, c^*; (\mathsf{kv}_1, \rho_1), \ldots, (\mathsf{kv}_\ell, \rho_\ell), \vec{r}, s, \omega) := \\ c^* = h^s \prod_{i=1}^{n} g_i^{k_i} \wedge \forall i \in [\ell] : c_i = h^{\rho_i - \sum_{j=1}^n r_j v_{i,k_j}} \prod_{j=1}^{n} b_{k_j}^{v_{i,k_j}} \\ \wedge C^{\mathbb{F}}((\mathsf{kv}_i)_{i\in[\ell]}, \omega) = 1 \end{aligned} \tag{3}$$

2. Prove membership&randomize relation:

$$\pi_{\text{mem-rand}} \leftarrow \mathsf{NIZK}_{\text{mem-rand}}.\mathsf{Prove}(\mathsf{crs}_{\text{mem-rand}}, R; \mathbb{w})$$

where

$$R = (\mathsf{pp}_{\text{acc}}, h, \mathsf{acc}) \text{ and } \mathbb{w} = (B = (b_1, \ldots, b_n), K_{\text{active}}, \pi_{acc}, (r_1, \ldots, r_n)))$$

3. Return $\pi := (B, c^*, \pi_{\text{mem-rand}}, \pi_R)$

$\mathsf{VerProof}(\mathsf{crs}^{\mathbf{reg}}, C^{\mathbb{F}}, c_1, \ldots, c_\ell, \pi)$

1. Parse π as $(B, c^*, \pi_{\text{mem-rand}}, \pi_R)$
2. Verify $b_0 \leftarrow \mathsf{NIZK}_{\text{mem-rand}}.\mathsf{VerProof}(\mathsf{crs}_{\text{mem-rand}}, (\mathsf{pp}_{\text{acc}}, h, \mathsf{acc}))$
3. Verify $b_1 \leftarrow \mathsf{NIZK.VerProof}_R(\mathsf{crs}, C^{\mathbb{F}}, c_1, \ldots, c_\ell, B, c^*, \pi_R)$
4. return $b_0 \wedge b_1$

Fig. 4: Accumulator-based construction for $\mathsf{kvNIZK}^{\text{reg}}$

5.3 Instantiation from Algebraic Accumulators in Groups of Unknown Order

It is possible to instantiate our construction above from accumulators in groups of unknown order [BBF19]. While we describe our instantiation in detail in the appendix (see Appendix B), here we provide a high-level view of the proof system $\mathsf{NIZK}_{\text{mem-rand}}$. Our techniques are adaptations of those in [BCFK19] and their concrete improvements in [CHA21]. The proof system consists of two parts. One part proves in zero-knowledge the bulk of accumulator membership. This consists of a low communication-complexity protocol where the proof partly consists of elements in $\mathbb{G}_?$ (by which we denote the unknown-order group). The other component is a Bulletproof where we both perform some sanity check on the values in the first proof (for example, some hidden values claimed by the prover should be in a certain range) and we perform a rerandomization of the group elements. This construction can be instantiated in a completely transparent manner using class groups[BH11][11].

A breakdown of the communication complexity of the proof is as follows. Let n be the maximum number of keys in any of the key-value commitments input to the proof algorithm. The first component provides roughly δn elements in $\mathbb{G}_?$ and in the prime order group, for a small constant δ. The second component consists of roughly $\log(n|\mathbb{G}_{\text{add}}||C|)$ where $|C|$ is the size of the circuit representing the general relation on key-value maps and $|\mathbb{G}_{\text{add}}|$ is the cost of representing an addition in the prime-order group. The latter can be in the order of 1K constraints using some of the instantiations in [CHA21]. Concretely, for $n \approx 30$ we estimate the size of the proof to be lower than 180 KB plus the cost of a Bulletproof on the circuit C (of logarithmic size).

5.4 Discussion on Practical Benefits

Intuitively, using an accumulator with a public set of active keys is beneficial whenever the "cost" of a proof of membership & randomization (step 1 in Fig. 4) is lower than that of a proof for hashing & randomization (steps 1–3 in Fig. 3). Here we discuss when this can be the case; see also Table 1. *The bottomline is: for proving time and proof size, Fig. 4 may not improve efficiency, but it does for verification time.*

For proving time Fig. 4 is a worse choice because proving membership introduces a linear dependency in $|K_{\text{active}}|$, not present in our construction from Fig. 3. This is not just a matter of asymptotics: by our estimates, this would also be the case for common concrete parameters for accumulators in unknown-order groups. Similar observations hold for proof size.

Where we may see substantial gains from applying Fig. 4 is in verification time. Ignoring the cost of the circuit C and only considering the additive overhead we achieve the following verification times. Without registration (Fig. 3): we would have n times: one hashing subcircuit—each of approximately 5k constraints—plus rerandomization (additional 1k contraints). This gives a total of $6n$k constraints. With registration (Fig. 4): n group exponentiations in a group of unknown-order plus (proved with a special-purpose "accumulator-membership" NIZK; see also Appendix B), n additional rerandomizations and additional sanity check constraints (for a total around 1.5k constraints). To assess which solution is best one needs to compare the concrete costs of the n exponentiations in groups of unknown order vs the $n \times 4.5$k constraints of the hash function. We know the first to be cheaper and in particular to be an order of magnitude so (each exponentiation requiring tens of milliseconds, vs hundreds of milliseconds for the case of verifying a single hash function). This time difference in verification can be substantial. Our time estimates refer to RSA-groups and are derived from [GKR+21, CHA21, CFG+20].

6 Improvements in Practice: Offline/Online Stages

The proving algorithms of our constructions (both in Fig. 3 and Fig. 4) follow a two-step template. In step *(a)* the prover provides key-tags and proves that they are valid, and in step *(b)* it computes a proof that actually depends on the commitments, using the key-tags produced before as as anchors for the keys. We observe that we can exploit the fact that *(a)* does not depend on the commitments to the key-value maps (but only on the keys that they will contain) to preprocess this step.

[11] The more efficient instantiation based on RSA groups would require the existence of a trusted RSA modulus. Nonetheless, all the other components of the construction remain transparent.

Consider, for instance, the running example from the introduction where the commitments contain multi-currency wallets. Assume that the prover knows that *in the future* they will want to prove some properties about some of their wallets (which are expected to keep changing between now and the proving time). Moreover, while a large set K_{active} of asset types might be circulating in the system, the prover knows that they will only hold a very specific and relatively small subset K_{pre} of these keys (e.g., maybe only ETH, USD and EUR). If that is the case they can preemptively perform an "offline proving stage" that would be valid for all of the *online* proofs they will have to carry out later. Specifically, for the construction in Fig. 3, the prover performs step *(a)* above offline as follows. On input set K_{pre}, the prover provides a set of $|K_{\text{pre}}|$ key tags $B = \{b_k : k \in K_{\text{pre}}\}$, defined as usual as $b_k = \mathcal{H}(k)h^{r_k}$ together with a proof π_{tags} that they were constructed honestly. The output of this step is therefore $\pi_{\text{offl}} = (B, \pi_{\text{tags}})$. At a later time, when input commitments $\mathsf{c}_1, \ldots, \mathsf{c}_\ell$ are available, the prover uses the pre-computed set of key-tags B to produce a step (a) (the production of key tags) for each of the commitments c_i. In order to preserve zero-knowledge, step (b) is modified to rerandomize the related b_k-s first. The reranomization hides the mapping between online proofs, however the verifier learns that all commitments of the online phase contain the same set of keys. We can similarly adapt the construction in Fig. 4 by performing membership and masking (step 1 in Fig. 4) before hand.

The advantage of this approach is to use a single offline stage for many proofs. The efficiency savings of this stage (both for proving and verification time) can be significant since it involves proving/verifying hashing in zero-knowledge. For example, we conservatively estimate approximately 5k constraints using Poseidon hashing on a Ristretto curve [GKR+21]. Each of these hashes can be proved in the order of hundreds of milliseconds (see, e.g., Table 5 in [GKR+21]). For $n \approx 100$ this involves for example saving half a million constraints[12] amounting to around half a minute of proving time. Savings for verification time are comparable.

Naturally the offline stage preprocesses an *upper bound* U on the number of committed keys, since each commitment may have openings to key-value maps of different density. This may incur a high overhead cost if U is far from the actual densities (because we still need to process U key tags as input to the circuit). The gains from an offline stage can differ accordingly and should be weighed depending on the application.

Efficiency Summary of Our Constructions We summarize the efficiency of our constructions in Table 1. We present it for the case with offline processing, but summing the offline and online columns corresponds to the setting without an offline stage.

7 Application: Multi-Type QuisQuis

QuisQuis [FMMO19] is a privacy-preserving transaction system which allows for pruning old transactions, keeping the state of each participant linear in the number of users. This is a major advantage over other privacy-preserving transaction systems, which require a state size linear in the number of transactions. A QuisQuis transaction is a redistribution of tokens among a set of accounts. An account belongs to an owner and stores their tokens. Instead of consuming accounts and creating new ones, QuisQuis updates the accounts. These update operations need to change the balance of a peer without knowing their total balance. This is achieved with homomorphic commitments.

In constrast to the original QuisQuis where an account stored a scalar value representing an amount, we generalize accounts to store tokens of different types in a key value map. Each key corresponds to a type, also known as currency, and the value specifies how many tokens of the specific type are held by the account. An account acct then belongs to a secret key sk and holds a balance kv, represented as a key value map. To transfer tokens from one account to another, a transaction includes both accounts as active inputs, denoted by the set $\mathcal{P}$. The transaction subtracts tokens from one account with a secret index s and adds them to the other one. The output of the transaction are two updated accounts belonging to the same users as the inputs; the input accounts are discarded. To achieve anonymity, the input consists of $\mathcal{P}$ together with a potentially large anonymity set of accounts $\mathcal{A}$ which keep their balance unchanged but provide set anonymity for the active accounts.

[12] For some applications this can be huge—for comparison, the ZCash circuit [HBHW21] has approximately 100k constraints.

Table 1: Efficiency of our constructions and comparison with non-homomorphic solutions (when they are applicable). Above we describe proof sizes, proving time and verification times during offline and online stage. We also describe the additional costs for proving the homomorphism in zero-knowledge with a non-homomorphic solution ($\mathsf{kv}_{\mathrm{add}}(n)$). All values are implicitly in big O notation and denote operations in a prime-order group unless underlined. Rows marked with $\star$ refer to this work. "Acc" refers to instantiations of our registration-based construction with accumulators in unknown-order groups. "Pedersen (Non-Hom.)" refers to the non-homomorphic solution based on Pedersen described in the introduction. Typical values for our parameters could be $M \approx 1000$ and $n \approx 100$.

$K_{\mathbf{active}}$	$K_{\mathbf{com}}$	**Construction**	$\lvert\pi_{\mathbf{offl}}\rvert$	$\lvert\pi_{\mathbf{onl}}\rvert$	$\lvert\pi(\mathsf{kv}_{\mathbf{add}}(n))\rvert$
priv	priv	Section 4$\star$	$\log(n(\lvert\mathcal{H}\rvert + \lvert\mathbb{G}_{\mathrm{add}}\rvert))$	$\log(\lvert C\rvert)$	—
priv	priv	Pedersen (Non-Hom.)	—	$\log(\lvert C\rvert)$	$\log(n)$
publ	priv	Section 5(Acc) $\star$	$n + \log(n\lvert\mathbb{G}_{\mathrm{add}}\rvert)$	$\log(\lvert C\rvert)$	—

$K_{\mathbf{active}}$	$K_{\mathbf{com}}$	**Construction**	$\mathcal{V}_{\mathbf{offl}}$	$\mathcal{V}_{\mathbf{onl}}$	$\mathcal{V}(\mathsf{kv}_{\mathbf{add}}(n))$
priv	priv	Section 4$\star$	$n(\lvert\mathcal{H}\rvert + \lvert\mathbb{G}_{\mathrm{add}}\rvert)$	$\lvert C\rvert$	—
priv	priv	Pedersen (Non-Hom.)	—	$\lvert C\rvert$	n
publ	priv	Section 5(Acc) $\star$	$\underline{n\lvert\mathbb{G}_?\rvert} + n\lvert\mathbb{G}_{\mathrm{add}}\rvert$	$\lvert C\rvert$	—

$K_{\mathbf{active}}$	$K_{\mathbf{com}}$	**Construction**	$\mathcal{P}_{\mathbf{offl}}$	$\mathcal{P}_{\mathbf{onl}}$	$\mathcal{P}(\mathsf{kv}_{\mathbf{add}}(n))$
priv	priv	Section 4$\star$	$n(\lvert\mathcal{H}\rvert + \lvert\mathbb{G}_{\mathrm{add}}\rvert)$	$\lvert C\rvert$	—
priv	priv	Pedersen (Non-Hom.)	—	$\lvert C\rvert$	n
publ	priv	Section 5(Acc) $\star$	$\underline{(M - n + n\log n)\lvert\mathbb{G}_?\rvert} + n\lvert\mathbb{G}_{\mathrm{add}}\rvert$	$\lvert C\rvert$	—

N :	Size of key universe $\mathcal{K}$
M :	Number of active / registered keys (K_{active})
n :	Number of keys in the opening of a key-value map commitment
	$N >> M \geq n$
$\lvert\mathcal{H}\rvert$:	Number of constraints for hashing to a group element
$\lvert\mathbb{G}_{\mathrm{add}}\rvert$:	Number of constraints for summing two group elements
$\lvert\mathbb{G}_?\rvert$:	Cost of exponentiation in unknown-order group
$\mathsf{kv}_{\mathrm{add}}(n)$:	Sum operation among key-value maps of size n
C :	Circuit computed on key-value map.

As the central building block of our multi-type QuisQuis system, we present an updatable account based on our key-value commitments.

7.1 Multi-Type QuisQuis: Syntax

The original QuisQuis transaction protocol consists of the three algorithms $(\mathsf{Setup}, \mathsf{Trans}, \mathsf{Verify})$. Their multi-type equivalent is as follows:

$\mathsf{Setup}(1^\lambda, \vec{\mathsf{kv}}) \to \mathsf{state}$**:** takes the security parameter λ and a vector of key value balances $\vec{\mathsf{kv}}$ and outputs an initial state state. One part of the state is a set of unspent accounts where each key value balance has an account.

$\mathsf{Trans}(\mathsf{sk}, \mathcal{P}, \mathcal{A}, \delta\vec{\mathsf{kv}}) \to \mathsf{tx}$**:** takes a secret key sk which corresponds to one account in the set of active accounts $\mathcal{P}$ and an anonymity set $\mathcal{A}$ with a vector of update key value maps $\delta\vec{\mathsf{kv}}$ to redistribute wealth. Trans outputs a transaction tx.

$\mathsf{Verify}(\mathsf{state}, \mathsf{tx}) \to \bot/\mathsf{state}'$**:** takes a state and a transaction tx and outputs a new state' or fails with $\bot$.

To support dynamic registration of new types, we require an additional algorithm $\mathsf{Register}$, which is defined as:

$\mathsf{Register}(\mathsf{acct}, k, v_k) \to \mathsf{tx}$ takes an account acct and a new type k with amount v_k and outputs a transaction tx.

A registration transaction is accepted by Verify if the type k has not been registered before. We define the correctness of a transaction system more formally. Let for all $\lambda \in \mathbb{N}$ and $\vec{\mathsf{kv}}$ with $R^{\mathsf{kv}}_{\mathsf{rng}}(\mathsf{kv}_i) = 1$ be $\mathsf{state} \leftarrow \mathsf{Setup}(1^\lambda, \vec{\mathsf{kv}})$. For all accounts in $\mathcal{P}, \mathcal{A}$ with index sets $\mathcal{P}^* := \{i \in [|\mathsf{Sort}(\mathcal{P} \cup \mathcal{A})|] : \mathsf{acct}_i \in \mathcal{P}\}$ and $\mathcal{A}^*$ accordingly in a canonically ordered form with Sort. All accounts in $\mathcal{P} \cup \mathcal{A}$ are part of the UTXO set in state, all $\delta\vec{\mathsf{kv}}$ with $R^{\mathsf{kv}}_{\mathsf{rng}}(-\mathsf{kv}_s) = 1$ and $R^{\mathsf{kv}}_{\mathsf{rng}}(\mathsf{kv}_i) = 1$ for $i \in \mathcal{P}^*\{s\}$ and $\mathsf{kv}_i = \emptyset_{kv}$ for $i \in \mathcal{A}^*$ and sk corresponding to an account $\mathsf{acct}_s \in \mathcal{P}$ with enough tokens such that after the transaction there is no negative type $R^{\mathsf{kv}}_{\mathsf{rng}}(\mathsf{kv}_s + \delta\mathsf{kv}_s) = 1$, it holds that $\mathsf{Verify}(\mathsf{state}, \mathsf{Trans}(\mathsf{sk}, \mathcal{P}, \mathcal{A}, \delta\vec{\mathsf{kv}})) = \mathsf{state}'$ where $\mathsf{state}' \neq \bot$ and contains an updated UTXO set with all inputs $\mathcal{P} \cup \mathcal{A}$ removed and the transaction outputs added.

Multi-Type QuisQuis: Security The security of a QuisQuis-like transaction system consists of two main properties. The first property we need to achieve is *anonymity*. A transaction system is anonymous if an adversary cannot successfully distinguish two transactions. The transactions are created according to malicious instructions after the adversary has interacted with an oracle signing transactions on behalf of uncompromised participants.

The second property is *theft prevention*. This entails that *(i)* the adversary cannot steal tokens from uncompromised accounts; *(ii)* the adversary cannot create tokens out of thin air. Slightly more formally, we model these properties as follows. While interacting with the aforementioned signing oracle the balance of honest accounts (not controlled by the adversary) must not decrease. Additionally, the total amount of tokens must not increase from transaction to transaction. Notice, however, the number of tokes may increase as the result of mining or a token registration—the latter counts as a "genesis" transaction.

Our variant of QuisQuis with multiple token types shares many of the same properties as the non-type aware system. For lack of space we refer the reader to the original QuisQuis paper [FMMO19] for details.

7.2 Construction

We construct the multi token QuisQuis scheme following the original QuisQuis but with two main adaptations: each account holds tokens in multiple types and making sure a transaction guarantees that the amounts of tokens are balanced for each of the token types.

The details of updatable accounts for key value maps are presented in Appendix C. In a nutshell they have the same algorithms as the original QuisQuis construction but allow for multiple kind of tokens.

Setup The setup algorithm generates a list of updatable accounts, one for each initial balance key value map.

Trans Our transaction structure follows that in QuisQuis where a "transaction" denotes a redistribution of wealth among all accounts involved ($\mathcal{P} \cup \mathcal{A}$). The transaction takes a vector of key value maps, one for each account. The account is then updated according to the key value map. Key value maps that contain only valid positive values ($R^{\mathsf{kv}}_{\mathsf{rng}}(\delta\mathsf{kv}_i) = 1$) are used to deposit tokens to receiving accounts. In order to ensure that the total number of tokens is preserved, we require that one key value map holds negative values. This is to satisfy that the sum of all key value maps is zero, or $\sum_{i=1}^{|\vec{\delta\mathsf{kv}}|} \delta\mathsf{kv}_i = \emptyset_{kv}$. For the account with the negative key value map (indexed by s in the canonically ordered set $\mathcal{P} \cup \mathcal{A}$), the transaction signature ensures that the owner of the account acct_s authorizes the spending by proving knowledge of the matching secret key sk. The algorithm $\mathsf{Trans}((s, \mathsf{sk}_s, \mathsf{kv}_s), \mathcal{P}, \mathcal{A}, \vec{\mathsf{kv}})$ performs the following steps:

1. Parse all input accounts $\mathcal{P} \cup \mathcal{A} = \{\mathsf{acct}_1, \ldots, \mathsf{acct}_n\}$ and check the spending account is valid by $\mathsf{VerifyAcct}(\mathsf{acct}_s, (\mathsf{sk}_s, \mathsf{kv}_s)) = 1$. The transaction needs to be balanced: $\sum_{i=1}^{|\vec{\delta\mathsf{kv}}|} \delta\mathsf{kv}_i = \emptyset_{kv}$ and all key value maps other than the spending account must be non-negative $\forall i \neq s : R^{\mathsf{kv}}_{\mathsf{rng}}(\delta\mathsf{kv}_i) = 1$. To support large anonymity sets $\mathcal{A}$, we choose to disclose the upper bound on active accounts by showing that $\delta\mathsf{kv}_i = \emptyset_{kv}$ for $i \in \mathcal{A}^*$ instead or a range proof. The spending account must be negative $R^{\mathsf{kv}}_{\mathsf{rng}}(-\delta\mathsf{kv}_s) = 1$ and the resulting account must be valid $R^{\mathsf{kv}}_{\mathsf{rng}}(\mathsf{kv}_s + \delta\mathsf{kv}_s) = 1$, to prevent overspending.
2. Let $\mathsf{outputs} = (\mathsf{acct}^{\mathcal{T}}_1, \ldots, \mathsf{acct}^{\mathcal{T}}_n)$ be a canonical order of the accounts generated by $\mathsf{UpdateAcct}(\mathcal{P} \cup \mathcal{A}, \vec{\delta\mathsf{kv}}; r)$.
3. Let $\psi : [n] \to [n]$ be the permutation that maps the canonically ordered inputs to the canonically ordered outputs, i.e. input i has the same secret key as output $\psi(i)$.
4. Create a zero knowledge proof π showing that the transaction is well formed, i.e. that it satisfies the following relation:

$$R_{\mathsf{tx\text{-}wf}} : \begin{cases} R = (\mathsf{inputs}, \mathsf{outputs}), \mathbb{w} = (\mathsf{sk}, \mathsf{kv}_s, \vec{\delta\mathsf{kv}}, r = (r_1, r_2), \psi) \text{ s.t.} \\ \mathsf{VerifyUpdateAcct}(\mathsf{acct}^{\mathcal{S}}_i, \mathsf{acct}^{\mathcal{T}}_{\psi(i)}, \delta\mathsf{kv}_i) = 1 \forall i \in \mathcal{P}^* \\ \wedge R^{\mathsf{kv}}_{\mathsf{rng}}(\delta\mathsf{kv}_i) = 1 \forall i \in \mathcal{P}^* / \{s\} \wedge R^{\mathsf{kv}}_{\mathsf{rng}}(-\delta\mathsf{kv}_s) = 1 \\ \wedge \delta\mathsf{kv}_i = \emptyset_{kv} \forall i \in \mathcal{A}^* \wedge \sum_{i=1}^{n} \delta\mathsf{kv}_i = \emptyset_{kv} \\ \wedge \mathsf{VerifyAcct}(\mathsf{acct}^{\mathcal{T}}_{\psi(s)}, \mathsf{sk}, \mathsf{kv}_s + \delta\mathsf{kv}_s) = 1. \end{cases}$$

The relation ensures that the permuted output account is correctly updated by the transferred balance $\delta\mathsf{kv}_i$ for all active accounts. It then ensures that the updated key value maps are valid, i.e. there is one spending account at index s and no value is taken from other accounts. The balances of the accounts in the anonymity set must not change. To ensure that the spender has enough tokens, the proof checks that the updated spender account has no negativ balance. The transaction consists of the inputs, outputs and the proof π.

Verify A transaction is valid in respect to a state if all accounts in inputs have not been used in another transaction and the proof π is valid.

Security Analysis Our key-value commitments provide the same hiding and binding properties as the commitments to single scalars used in QuisQuis. The construction is a parallel version of the single type case and thereby the theft security holds also for all keys in parallel. Regarding anonymity, we achieve the same properties as QuisQuis, if we define an upper bound of the number of types involved in a transaction. For transactions with few different types, we achieve this through padding. With a constant size transaction proof, our new transactions have the same indistinguishability as the original QuisQuis transactions.

7.3 Instantiation of our Construction

In this section we describe the instantiation of our multi token QuisQuis extension. As the spending account $\mathsf{acct}^{\mathcal{S}}_s$ requires different properties than the remaining accounts, we utilize the same method of shuffling twice. A first permutation ψ_1 places the spending account to the first position $\psi_1(s) = 1$. For

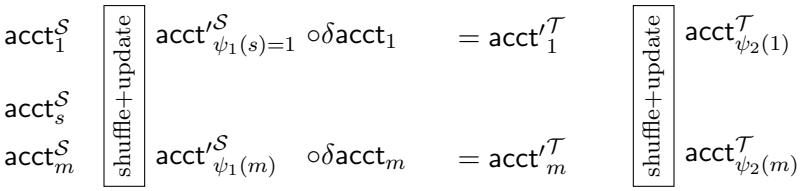

Fig. 5: transaction overview

this first account, we prove the spending authorization and the correctness of redistribution. After that is done, a second permutation $\psi_2 = \psi_1^{-1} \circ \psi$ then hides the spending account randomly in the outputs.

A full transaction overview is presented in Figure 5. The transaction zero knowledge proof is split into three parts. We have two shuffle proofs, one for the inputs and one for the outputs. The shuffle argument of QuisQuis makes no assumptions on the value committed in a QuisQuis account and our key value commitments have the same outer structure, therefore we can use the same proof system as the original QuisQuis. Formally the proof satisfies the following relation for the input/output shuffles:

$$\mathsf{VerifyUpdateAcct}(\{\mathsf{acct}_i^{\mathcal{S}}, \mathsf{acct}'^{\mathcal{S}}_{\psi_b(i)}, \emptyset_{kv}\}) = 1 \text{ for } b \in \{1, 2\}$$

To satisfy the complete transaction relation R_{tx}, the $\delta\mathsf{acct}$ have to hold all non-positive values for the first delta account $\delta\mathsf{acct}_1$ and all non-negative for the rest. In addition, all committed key value maps in $\delta\mathsf{acct}$ have to sum to zero. After homomorphically applying the update, the sender has to prove that the resulting account $\mathsf{acct}'^{\mathcal{T}}_1$ is not overdrawn.

Each $\delta\mathsf{acct}_i$ contains a commitment under the respective public key which is equal to the public key of the commitment in $\mathsf{acct}'^{\mathcal{S}}_i$ and $\mathsf{acct}'^{\mathcal{T}}_i$ is calculated with the help of the homomorphic property. This very feature is an issue for showing that the sum of all key value maps is zero because the commitments under different public keys are not homomorphic.

Therefore we use a single large circuit with the relation described in Appendix D to show that they are in range and sum to zero for each type.

8 Implementation

Our construction with two separate proving systems allows for separate optimization, however the implemented proving systems require delicate dependencies between each other. The first NIZK about the tags $Prove_{\mathsf{tags}}$ shows the well formedness of the tags and proves knowledge of the key pre-images. From a technical perspective, the tags need to reside in the same group as the key-value commitments. This is neessary to use the tags when proving the composition of the commitment. From this dependency we are limited in the proving systems when aiming for efficient implementations. Let $\mathbb{G}_\square$ be the group in which the tags are defined. We need to implement two proving systems: $Prove_{\mathsf{tags}}$ and $Prove_{\mathsf{rel}}$. The minimal requirements on the choice of proving system are as follows:

- We need a circuit description of a hash function (e.g. Poseidon) and group addition in $\mathbb{G}_\square$ compatible with the tags proof
- As the tags and the commitment are the statement to the rel proof, it should natively operate in $\mathbb{G}_\square$ with exponentiation gates explained by LMR (i.e. extended Bulletproofs with a single element-wise exponentiation gate with the product evaluating to the identity).

Given existing implementations, we estimated the running time of our two proving systems. While this does not allow end-to-end performance measurements, we show the feasibility of our construction. For tags we used the ark-works[13] implementation of groth16. The library provides a circuit for Poseidon and point addition in the BLS12_381 curves of which we chose G1. This results in a proof with 280? constraints and a proving time of 200ms. The constant verification time is 5ms. Using this circuit, we fixed $\mathbb{G}_\square$ to be BLS12_381_1.

For a performant rel proving system we need further assumptions on the relation to be proven. In most cases it is a range proof, for which Bulletproofs are well suited. The LMR18 extension directly

[13] https://ark-works.com?

accepts group elements as input for the exponentiation gate. However this requires the proving system to operate in the same group. We used a curve agnostic bulletproof implementation[14] to replace the underlaying elliptic curve from Ristretto, on which Bulletproofs were originally implemented, to BLS12_381_1. The larger size of the 380 bit elements increased the proving and verification time. For concrete numbers, we measure a single 64 bit range proof. The proving time increases 8 fold from 30ms in Ristretto to 250ms in BLS. Similarely the verifier requires 200ms instead of 10ms. Note that this Bulletproof implementation does not feature the required expoentitation gate. From the implementation used in SwapCT[15], the additional exponentiation gate increases the runtime by 30%.

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A Further Preliminaries on NIZKs

Knowledge-Soundness. For all $\lambda \in \mathbb{N}$ and for all (non-uniform) efficient adversaries $\mathcal{A}$ there exists a (non-uniform) efficient extractor $\mathcal{E}$ such that

$$\Pr\left[\begin{array}{c} \mathsf{crs} \leftarrow \mathsf{Setup}(1^\lambda) \\ (R,\pi) \leftarrow \mathcal{A}(\mathsf{crs}) \\ \mathbb{w} \leftarrow \mathcal{E}(z,\mathsf{crs}) \end{array} : \begin{array}{r} R(\mathbb{w}) \neq 1 \wedge \\ \mathsf{Vfy}(\mathsf{crs},R,\pi)=1 \end{array}\right] \leq \mathsf{negl}(\lambda)$$

Zero-Knowledge. There exists a PPT simulator $\mathcal{S}$ such that for any PPT adversary $\mathcal{A}$, auxiliary input $z \in \{0,1\}^{\mathsf{poly}(\lambda)}$, and it holds that $\Pr\left[\begin{array}{r} \mathsf{crs} \leftarrow \mathsf{Setup}(1^\lambda) \\ (R,\mathbb{w}) \leftarrow \mathcal{A}(\mathsf{crs}) \ : \ \mathcal{A}(\mathsf{crs},\pi)=1 \\ \pi \leftarrow \mathcal{S}(\mathsf{crs},R) \text{ if } R(\mathbb{w}) \text{ o.w. } \perp \end{array}\right] =$

$$\Pr\left[\begin{array}{r} \mathsf{crs} \leftarrow \mathsf{Setup}(1^\lambda) \\ (R,\mathbb{w}) \leftarrow \mathcal{A}(\mathsf{crs}) \ : \ \mathcal{A}(\mathsf{crs},\pi)=1 \\ \pi \leftarrow \mathsf{Prove}(\mathsf{crs},R) \text{ if } R(\mathbb{w}) \text{ o.w. } \perp \end{array}\right]$$

B Zero-Knowledge Arguments over Algebraic Accumulators

The numbers in our Table 1 for the registration construction refer to instantiations described in this section.

B.1 Construction of accumulators for group elements

Below we describe an instantiation of accumulator for group elements point pt on an elliptic curve. The original construction is that of accumulators in unknown-order groups from [BBF19], while the explicit description for elliptic curve points is from [CHA21]. The adaptations we describe in this section are important to obtain concretely efficient instantiations.

The permissible set $\mathcal{P}$ allows to avoid collisions and is defined as follows. The set $\mathcal{P}$ of commitments, parametrized by an integer μ, consists of points on the an elliptic curve where the x-coordinate is a μ-bit prime and the y-coordinate is the "canonically chosen" square root so that the point can be described by its x-coordinate alone.

$$\begin{aligned} \mathcal{P} = \{(\mathsf{x},\mathsf{y}) \in (\mathbb{F},\mathbb{F}) \mid \\ \mathsf{x} \in [2^{\mu-1},2^\mu) \wedge \mathsf{y} \equiv 0 \mod 2\} \end{aligned}$$

One can use $\mu = 251$ bits for concrete instantiations in the Ristretto curve (which we can use with [LMR19]).

The requirement for prime numbers below is necessary for the soundness of the accumulator scheme. We observe that, when the accumulator is computed publicly (which is the case in our applications), it is not necessary to prove in zero-knowledge that its elements are primes. This can be done at registration time ensuring that, when adding a key k, its hash $\mathcal{H}(k)$ is prime. We can naturally add a padding to the key before registration time and, through rejection sampling, ensure we obtain a prime.

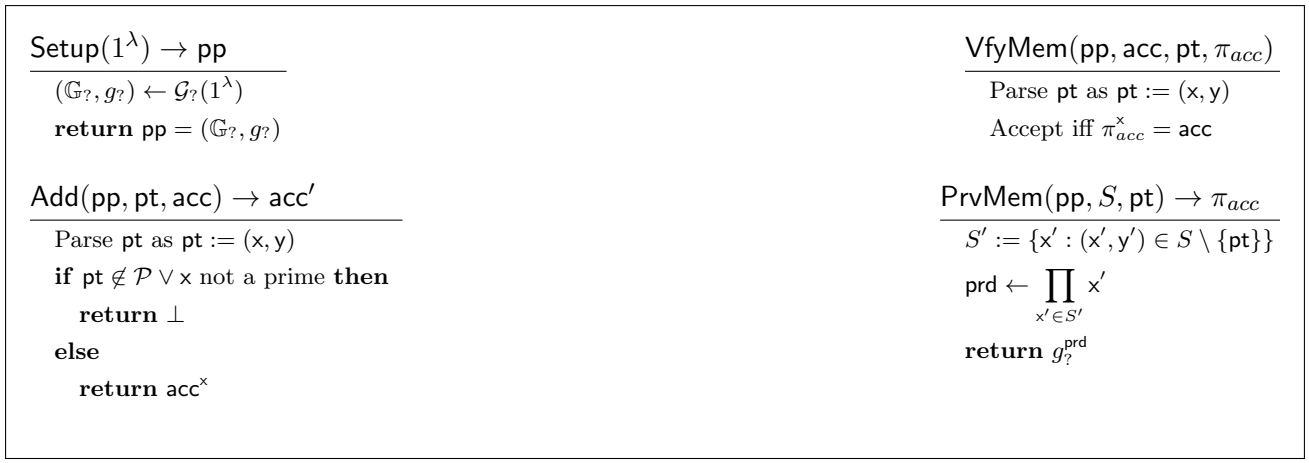

Fig. 6: Accumulator Instantiation for AccScm.

For efficiency, we observe that PrvMem can executed in time growing with $(M - n) + n \log n$ for a subset of size n and accumulated set of size M (the quasilinear factor derives from RootFactor algorithm described in [BBF19]).

B.2 Instantiating $\mathsf{NIZK}_{\text{mem-rand}}$

In Section 5, we need to efficiently prove in zero-knowledge that a key-tag (roughly, the hash of a key) is inside an accumulator ($\mathsf{NIZK}_{\text{mem-rand}}$). For our instantiations in unknown-order groups, we can adopt a variant of the solution in [CHA21] (see their Section 5.2 and instantiations in 5.4.1 and 5.4.2). Our relation $R_{\text{mem-rand}}$ has only one difference from their "one-out-of-many" relation in Section 5.2: we need to prove the same relation for multiple key-tags (which correspond to what they call an "outer commitment"). In other words, we need a "many-out-of-many" relation.

The subsequent change to their instantiation is almost straightoforward. Their one-out-of-many relation is composed of two parts: one described in 5.4.1 (roughly consisting of a Bulletproof execution on a gadget of approximately 1.5K constraints) the other in 5.4.2 (a NIZK whose proof consists of a constant number of unknown-order and prime-order group elements). To turn them into a many-out-of-many variant we do the following. The constraint system for 5.4.1 can simply be adapted to take as input n inputs, instead of just one. The batching properties of Bulletproof allow the proof size to be affected only minimally. For the second component, the one from 5.4.2, we can simply provide n runs of it. This part of the argument system grows linearly in n.

C Updatable Accounts

C.1 Preliminaries: Updatable Public Keys [FMMO19]

We need updatable public keys. Intuitively, such a key can be publicly updated and the update is indistinguishable from a newly created key. An updatable public key scheme consists of the following algorithms:

$\mathsf{Setup}(1^\lambda) \to \mathsf{pp}$ outputs the public parameters pp.
$\mathsf{Gen}(\mathsf{pp}) \to (\mathsf{sk}, \mathsf{pk})$ generates a new key pair
$\mathsf{Update}(\mathsf{pk}) \to (\mathsf{pk}')$ updates the key pk to pk'
$\mathsf{VerifyKP}(\mathsf{pk}, \mathsf{sk}) \to b \in \{0, 1\}$ verifies that a key pair is consistent.
$\mathsf{VerifyUpdate}(\mathsf{pk}, \mathsf{pk}', r) \to b \in \{0, 1\}$ verifies that the public key pk was correctly updated to pk' with some randomness r.

The correctness follows directly from QuisQuis such that given a key pair, Update generates again a valid key pair. The update process is verifiable by VerifyUpdate and the updated key must keep the

original secret key. The security requires (a) that an updated key is indistinguishable from a new, randomly generated one and (b) that the update process cannot change the secret key if the update is verified.

C.2 A secret-key version with updatable keys

We now describe an extended commitment to key-value map, which is the one we will use in our constructions for multi-type QuisQuis. In this extension we allow a party with a secret key to open the commitment even without knowing the randomness used to commit to it. This will be useful to allow to verify that an account has a certain balance. In order to do this we let the setup return a pair of public–secret keys $\mathsf{Setup}(1^\lambda) \to (\mathsf{pk}, \mathsf{sk})$ and we use the notation $\mathsf{VerifyCommSk}(\mathsf{pk}, \mathsf{c}, \mathsf{sk}, \{v_k\}_{k\in K})$. We also require the commitment to be binding also with respect to this function, that is no efficient adversary can output $(\mathsf{pk}, \mathsf{c}, \mathsf{sk}, \{v_k\}_{k\in K}, \mathsf{sk}', \{v'_{k'}\}_{k'\in K'})$ with $\{v_k\}_{k\in K} \neq \{v'_{k'}\}_{k'\in K'}$ but such that $\mathsf{VerifyCommSk}(\mathsf{pk}, \mathsf{c}, \mathsf{sk}, \{v_k\}_{k\in K}) = \mathsf{VerifyCommSk}(\mathsf{pk}, \mathsf{c}, \mathsf{sk}', \{v'_{k'}\}_{k'\in K'}) = 1$. We provide a construction reminiscent of ElGamal commitments in Figure 7.

$\mathsf{Setup}(1^\lambda) \to (\mathsf{pk}, \mathsf{sk})$**:** samples a group $\mathbb{G}$ and a generator g, $z \leftarrow\!\!\$\ [|\mathbb{G}|]$ and return $(\mathsf{pk} = (\mathbb{G}, g, h := g^z), \mathsf{sk} = z)$.
$\mathsf{Com}(\mathsf{pk}, \{v_k\}_{k\in K}; r) \to \mathsf{c}$**:** return $(g^r, \prod_{k\in K} \mathcal{H}(k)^{v_k} h^r)$.
$\mathsf{VerifyCommSk}(\mathsf{pk}, \mathsf{c} = (u, v), \mathsf{sk} = z, \{v_k\}_{k\in K})$**:** return 1 iff $\prod_{k\in K} \mathcal{H}(k)^{v_k} = v/u^z$.

Fig. 7: Secret-key variant of construction for kvC.

C.3 Formalization

With a key value commitment scheme and updatable keys, we build updatable accounts which belong to a secret key and hold a key-value map of tokens. The important feature of an updatable account is, that anyone can rerandomize the account and create a proof of correct rerandomization. An updated account is not linkable to the previous version and indistinguishable from a newly created account. In combination with a shuffling proof, a set of accounts is permuted without changing ownership and without knowledge of openings. We provide formal details in the full version.

Definition 10. *Updatable Accounts consist of the following algorithms:*

$\mathsf{GenAcct}(\mathsf{pp}, \mathsf{kv}) \to (\mathsf{acct}, \mathsf{sk})$ *takes an amount map* $\{v_k\}_{k\in K}$ *and outputs an account* acct *and a secret key* sk

$\mathsf{VerifyAcct}(\mathsf{acct}, (\mathsf{sk}, \mathsf{kv})) \to b \in \{0,1\}^*$ *checks the account* acct *for consistency with the private key* sk *and the key value map* $\{v_k\}_{k\in K}$ *such that* $R^{\mathsf{kv}}_{\mathsf{rng}}(\mathsf{kv}) = 1$.

These accounts express the same updatability as the original QuisQuis accounts, such that they remain valid for updates to the public key and homomorphic operations on the commitment. This leads to the following additional operations:

$\mathsf{UpdateAcct}(\{(\mathsf{acct}_i, \delta\mathsf{kv}_i)\}_{i=1}^n; r_1, r_2) \to \{\mathsf{acct}'_i\}_{i=1}^n$ *takes as input set of accounts* $\mathsf{acct}_i = (\mathsf{pk}_i, c_i)$ *and key value maps* $\delta\mathsf{kv}_i$*, representing the change of amount in different types, such that* $R^{\mathsf{kv}}_{\mathsf{rng}}(\delta\mathsf{kv}_i) = 1$ *or* $R^{\mathsf{kv}}_{\mathsf{rng}}(-\delta\mathsf{kv}_i) = 1$*; it outputs a new set of accounts* $\{\mathsf{acct}'_i\}_{i=1}^n$ *with* $\mathsf{acct}'_i = (\mathsf{Update}(\mathsf{pk}_i; r_1)), c \circ \mathsf{Com}(\delta\mathsf{kv}_i, \mathsf{pk}_i; r_2)$

$\mathsf{VerifyUpdateAcct}(\{(\mathsf{acct}_i, \mathsf{acct}'_i, \delta\mathsf{kv}_i)\}_{i=1}^n; r_1, r_2) \to \{0,1\}$ *outputs* 1 *if* $\{\mathsf{acct}'_i\}_{i=1}^n = \mathsf{UpdateAcct}(\{(\mathsf{acct}_i, \delta\mathsf{kv}_i)\}_{i=1}^n, \mathsf{kv}; r_1, r_2)$ *and* $R^{\mathsf{kv}}_{\mathsf{rng}}(\delta\mathsf{kv}_i) = 1$ *or* $R^{\mathsf{kv}}_{\mathsf{rng}}(-\delta\mathsf{kv}_i) = 1$*,* 0 *otherwise.*

D Multi-Type QuisQuis Transaction Relation

With $(c_1, \ldots, c_\ell) := (\delta\mathsf{acct}_2, \ldots, \delta\mathsf{acct}_m, \mathsf{acct}_1'^{\mathcal{T}}, \delta\mathsf{acct}_1)$, we construct a circuit which checks that the first update account $\delta\mathsf{acct}_1$ is all negative and the remaining ones are all positive. Last, it checks that for each type, the update values sum to zero.

$$C^{\mathbb{F}}((k_1, (v_{1,k_1}, \ldots, v_{\ell,k_1})), \ldots, (k_n, (v_{1,k_n}, \ldots, v_{\ell,k_n})), \omega) :=$$
$$\forall_{k \in K} \begin{cases} R_{\mathsf{rng}}(-v_{\ell,k}) = 1 \\ \wedge \forall_{i \in [\ell-1]} R_{\mathsf{rng}}(v_{i,k}) = 1 \\ \wedge \sum_{i \in [\ell-2] \cup \{\ell\}} v_{i,k} = 0 \end{cases}$$

Our kvNIZK construction builds upon Pedersen style key value commitments, but for the multi-type QuisQuis setting we need public key based commitments as shown in Appendix C.2. Therefore we change the relation R_{R} in Equation (3) to prove that the input to the circuit is still equivalent to the commitment openings.

$$R_{\mathrm{R}}(C^{\mathbb{F}}, ((g_1, h_1), c_1), \ldots, ((g_\ell, h_\ell), c_\ell), b_{k_1}, \ldots, b_{k_n}, c^*;$$
$$(\mathsf{kv}_1, \rho_1), \ldots, (\mathsf{kv}_\ell, \rho_\ell), r, \omega) :=$$
$$c^* = h^r \prod_{i=1}^{n} g_i^{k_i} \wedge \forall i \in [\ell] : \mathsf{c}_i = (g_i^{\rho_i}, h_i^{\rho_i - \sum_{j=1}^{n} r_j v_{i,k_j}} \prod_{j=1}^{n} b_{k_j}^{v_{i,k_j}})$$
$$\wedge C^{\mathbb{F}}((k_1, (v_{1,k_1}, \ldots, v_{\ell,k_1}), \ldots, (k_n, (v_{1,k_n}, \ldots, v_{\ell,k_n})), \omega) = 1$$