PIRANA: Faster Multi-query PIR via Constant-weight Codes

Jian Liu* Zhejiang University jian.liu@zju.edu.cn
Jingyu Li* Zhejiang University jingyuli@zju.edu.cn
Di Wu Zhejiang University wu.di@zju.edu.cn
Kui Ren Zhejiang University kuiren@zju.edu.cn

Abstract—**Private information retrieval (PIR) is a cryptographic protocol that enables a wide range of privacy-preserving applications. Despite being extensively studied for decades, it is still not efficient enough to be used in practice. In this paper, we propose a novel PIR protocol named PIRANA, based on the recent advances in constant-weight codes. It is up to 188.6× faster than the original constant-weight PIR (presented in Usenix SEC '22). Most importantly, PIRANA naturally supports multi-query. It allows a client to retrieve a batch of elements from the server with a very small extra-cost compared to retrieving a single element, which results in up to an 14.4× speedup over the state-of-the-art multi-query PIR (presented in Oakland '23). We also discuss a way to extend PIRANA to labeled private set intersection (LPSI). Compared with existing LPSI protocols, PIRANA is more friendly to the scenarios where the database updates frequently.**

1. Introduction

Private information retrieval (PIR) allows a client $\mathcal{C}$ to retrieve an element from a server $\mathcal{S}$ by index, without revealing which element was retrieved. It enables a wide range of privacy-preserving applications such as private contact discovery [24], private contact tracing [48], private navigation [49], anonymous messaging [40], [35], safe browsing [31] and so on. Despite being extensively studied for decades, PIR is still a hot research topic in cryptography.

The PIR protocols can be roughly categorized into multi-server PIR [17] and single-server PIR [34]. The multi-server protocols are much more efficient in both computation and communication, and can achieve information-theoretic security. However, their reliance of multiple non-colluding servers is an unrealistic assumption in practice. In contrast, the single-server protocols do not need this strong assumption, at the cost of incurring a huge performance overhead.

Constant-weight PIR. Most of the single-server PIR protocols structure a database of n payloads as a D-dimensional hypercube [33], [38], [4], so that the communication overhead could be $O(D \cdot \sqrt[D]{n})$. Recently, Mahdavi-Kerschbaum [37] introduced a new direction for PIR. It uses *constant-weight code* to encode all indices and queries into codewords of the same Hamming weight, simplifying the homomorphic equality test. However, this research direction is still in its early stage of development. For example, the protocol proposed by Mahdavi-Kerschbaum [37], known as CwPIR, requires $(k-1)n$ ciphertext-ciphertext multiplications, making it unable to support databases with a large number of elements. Their experimental results also confirm this (cf. Table 13 in [37]).

Multi-query PIR. In many scenarios, $\mathcal{C}$ may want to retrieve a batch of elements from $\mathcal{S}$ at once [24], [35], [32], [43], which motivates *multi-query* PIR [4], [41]. Most of the multi-query PIR protocols use *batch codes* [30] to encode a collection of n elements into M codewords distributed among B buckets; any L of the n elements can be recovered by fetching at most one codeword from each bucket. Therefore, $\mathcal{C}$ could use a separate single-query PIR to query each bucket; to answer these B queries, $\mathcal{S}$ computes over $M < L \cdot n$ codewords exactly once, which significantly reduces the computational overhead. However, given that $B > L$, this may introduce more communication overhead than running L instances of single-query PIR (cf. Figure 12 in [4]). Therefore, existing multi-query PIR protocols offer an unattractive trade-off: they reduce computation but add communication overhead.

Labeled PSI. *Labeled private set intersection* (LPSI) can be considered as multi-query PIR with a stronger security guarantee: it protects both $\mathcal{C}$'s queries and $\mathcal{S}$'s database; and the queries are keywords instead of indices. To the best of our knowledge, all existing LPSI protocols [13], [18] have $\mathcal{S}$ homomorphically evaluate a large degree polynomial that interpolates the payloads. Such protocols require several hours to setup due to the expensive polynomial interpolations, which makes them undesirable for scenarios where the database updates frequently.

Our contribution. In this paper, we propose a *novel* constant-weight PIR named PIRANA. Our starting point is to replace the homomorphic equality operator in CwPIR with an SIMD-based one for batch processing: it can perform N (the number of slots in an SIMD ciphertext, typically $N = 4\ 096$ or $8\ 192$) equality checks with a single ciphertext-ciphertext multiplication. As a result, $\mathcal{S}$ in PIRANA only needs to run $(k-1)\lceil n/N \rceil$ ciphertext-ciphertext multiplications, instead of $(k-1)n$. We remark that plugging SIMD into CwPIR is non-trivial, since CwPIR uses oblivious expansion, which is incompatible with SIMD. To this end, we completely changed the way constant-weight codes are used.

*Jian Liu and Jingyu Li are co-first authors.

Most importantly, PIRANA naturally supports multi-query. It allows $\mathcal{C}$ to retrieve a batch of elements from $\mathcal{S}$ with a very small extra-cost in both communication and computation, compared to retrieving a single element. In particular, if we use 3-way cuckoo hashing as the batch code, we are able to retrieve up to $\lfloor N/1.5 \rfloor$ elements with only $3\times$ higher computational cost and almost the same communication cost compared to running the single-query PIRANA once. This is not the case for other PIR protocols.

Furthermore, PIRANA can be easily extended to support LPSI. To protect $\mathcal{S}$'s elements, we borrow the idea of using oblivious PRF (OPRF) from [13]: $\mathcal{S}$ and $\mathcal{C}$ apply OPRF to their queries and keywords; $\mathcal{S}$ uses the OPRF values to mask the corresponding payloads; $\mathcal{C}$ retrieves the masked payloads using multi-query PIRANA; $\mathcal{C}$ can get a correct payload only if the corresponding query is in the intersection. Compared with the state-of-the-art LPSI protocols, we successfully get rid of the expensive polynomial interpolations, resulting in a much faster setup phase.

We summarize our contribution as follows:

- We design a *novel* constant-weight PIR named PIRANA. It is up to 188.6× faster than CwPIR [37]. (Section 3)
- We extend PIRANA to support multi-query, achieving up to an 14.4× speedup over the state-of-the-art multi-query PIR [41]. (Section 4)
- We further extend PIRANA to support LPSI, achieving a 331-fold reduction in setup time compared with the state-of-the-art LPSI [18]. (Section 5)
- We provide a full-fledged implementation and extensive evaluation. (Section 6)

2. Preliminaries

In this section, we describe some building blocks that will be used in our protocols. A summary of notations is shown in Table 1.

2.1. Homomorphic encryption

Fully Homomorphic Encryption (FHE) is an encryption scheme that allows arbitrary operations to be performed over encrypted data [27]. In practice, it is usually used in a leveled fashion: the operations can only be performed for a limited times, o.w., the ciphertexts cannot be decrypted. In most FHE cryptosystems [9], [7], [25], [15], [8], plaintexts are encoded as polynomials from the quotient ring $\mathbb{Z}_p[x]/(x^N+1)$, where N is a power of 2, and p is the plaintext modulus. The plaintext polynomials are then encrypted into ciphertext polynomials $\mathbb{Z}_q[x]/(x^N+1)$, where q is the ciphertext modulus that determines the security level, as well as how many times the operations can be performed.

Most FHE cryptosystems support the *single instruction multiple data* (SIMD) technique, which allows one to encrypt a vector of elements into a single ciphertext and process these encrypted elements in a batch without introducing any extra cost. An SIMD ciphertext has N slots, hence it can be viewed as an encryption of a length-N vector over $\mathbb{Z}_p^N$. Furthermore, an SIMD ciphertext also supports cyclic rotations of its slots.

Notation	Description
$\mathcal{C}$	client
$\mathcal{S}$	server
n	DB size
pl	a payload
$CW(m,k)$	binary constant-weight code with length m and Hamming weight k
N	polynomial modulus degree in FHE number of slots in a SIMD ciphertext
p	bit-size of plaintext modulus, SIMD slot size
$\mathbf{x}$	a vector
$\widetilde{\mathbf{x}}$	an SIMD ciphertext
t	$\lceil n/N \rceil$
%	modulo
L	number of queries
$BC(n,M,L,B)$	a batch code encoding n elements into M codewords in B buckets, supporting L queries
C	a batch code codeword
s	$\lfloor N/B \rfloor$

TABLE 1: Summary of frequent notations.

The homomorphic operations used in this paper are summarized as follows:

- $\widetilde{\mathbf{x}} \leftarrow \mathsf{SIMDEnc}(\mathsf{pk}, \mathbf{x})$. The encryption algorithm takes a plaintext vector $\mathbf{x} = [x_1, ..., x_N]$ and outputs an SIMD ciphertext denoted by $\widetilde{\mathbf{x}}$.
- $\mathbf{x} \leftarrow \mathsf{SIMDDec}(\mathsf{sk}, \widetilde{\mathbf{x}})$. The decryption algorithm takes an SIMD ciphertext $\widetilde{\mathbf{x}}$ and outputs a plaintext vector $\mathbf{x}$.
- $\widetilde{\mathbf{z}} \leftarrow \mathsf{SIMDAdd}(\widetilde{\mathbf{x}}, \widetilde{\mathbf{y}})$. The addition algorithm takes two SIMD ciphertexts $\widetilde{\mathbf{x}}$ and $\widetilde{\mathbf{y}}$; outputs an encryption of $[x_1+y_1, ..., x_N+y_N]$.
- $\widetilde{\mathbf{z}} \leftarrow \mathsf{SIMDPmul}(\widetilde{\mathbf{x}}, \mathbf{y})$. The ciphertext-plaintext multiplication takes a SIMD ciphertext $\widetilde{\mathbf{x}}$ and a plaintext vector $\mathbf{y}$; outputs an encryption of $[x_1y_1, ..., x_Ny_N]$.
- $\widetilde{\mathbf{z}} \leftarrow \mathsf{SIMDMul}(\widetilde{\mathbf{x}}, \widetilde{\mathbf{y}})$. The ciphertext-ciphertext multiplication takes two SIMD ciphertexts $\widetilde{\mathbf{x}}$ and $\widetilde{\mathbf{y}}$; outputs an encryption of $[x_1y_1, ..., x_Ny_N]$.
- $\widetilde{\mathbf{x}}' \leftarrow \mathsf{SIMDRotate}(\widetilde{\mathbf{x}}, c)$. The rotation algorithm takes an SIMD ciphertext $\widetilde{\mathbf{x}}$ and an integer $c \in [N]$; outputs an encryption of $\left[x_{(1+c)\%N}, ..., x_{(N+c)\%N}\right]$.

2.2. Constant-weight code and CwPIR

Constant-weight code is a form of error detecting code where its codewords are binary strings that share the same Hamming weight k. Its code length m is the bit-length of its codewords and code size n is the number of distinct codewords. Clearly, for a fixed Hamming weight k, to construct a constant-weight code of size n, one must choose its code length m, s.t., $\binom{m}{k} \geq n$. Then, we have

$$m \in O(\sqrt[k]{k!n} + k).$$

We denote the constant-weight code with length m and Hamming weight k by $CW(m, k)$. Algorithm 1 shows how Mahdavi-Kerschbaum [37] map indices (i.e., $i \in [n]$) to $CW(m, k)$. Intuitively, it maps each i to the i-th valid codeword from a sorted list of codewords, with a complexity $O(m + k)$.

Algorithm 1 Mapping indices to constant-weight codewords

```
Input $i \in [n], m, k \in \mathbb{N}$ with $\binom{m}{k} \geq n$
Output $\mathbf{x} \in CW(m, k)$
1: $j := i$; $l := k$; $\mathbf{x} := 0^m$
2: for $m' := m - 1, ..., 0$ do
3:     if $j \geq \binom{m'}{l}$ then
4:         $\mathbf{x}[m'] := 1$
5:         $j := j - \binom{m'}{l}$
6:         $l := l - 1$
7:     end if
8:     if $l = 0$ then
9:         break
10:    end if
11: end for
12: return $\mathbf{x}$
```

Suppose an equality operator for checking the equality of two values is defined as follows:

Definition 1 (Equality Operator)**.** *A function f is an equality operator over a domain D if $\forall x, y \in D$,*

$$f(x, y) = \begin{cases} 1 & \text{if } x = y \\ 0 & o.w. \end{cases}$$

An equality operator for constant-weight codes could be:

$$f(\mathbf{x}, \mathbf{y}) = \prod_{\mathbf{y}[i]=1} \mathbf{x}[i] \tag{1}$$

Mahdavi-Kerschbaum [37] propose a *homomorphic equality operator* for constant-weight code based on SealPIR's oblivious expansion [4]. In more detail, to encrypt a codeword $\mathbf{x}$ of length m, they separately encrypt the k indices that correspond to 1s in $\mathbf{x}$ using FHE. To check equality between an encrypted $\mathbf{x}$ and a plain codeword $\mathbf{y}$, they first expand the encrypted indices such that each bit of $\mathbf{x}$ is in a separate ciphertext; then run the equality operator (cf. equation 1) on the encrypted bits. The oblivious expansion requires $\frac{2m(N-1)}{N}$ substitutions and m ciphertext-plaintext multiplications [4]; and the equality operator requires $(k - 1)$ ciphertext-ciphertext multiplications.

Mahdavi-Kerschbaum also propose a constant-weight PIR named CwPIR [37] based on this homomorphic equality operator. In a nutshell, it works as follows:

1) $\mathcal{C}$ maps its query to a constant-weight codeword $\mathbf{x}$;
2) $\mathcal{S}$ maps each index i to a constant-weight codeword $\mathbf{y}_i$;
3) $\mathcal{C}$ homomorphically encrypts the indices that correspond to 1s in $\mathbf{x}$, and sends the ciphertexts to $\mathcal{S}$;
4) $\mathcal{S}$ obliviously expands them into m ciphertexts, which correspond to the m bits of $\mathbf{x}$;
5) For each $\mathbf{y}_i$, $\mathcal{S}$ runs the homomorphic equality operator between $\mathbf{x}$ and $\mathbf{y}_i$; it results in a selection vector of length n;
6) $\mathcal{S}$ returns the inner product between the selection vector and the payloads.

Notice that the payloads get involved only when computing the inner product in the last step, and that step only requires ciphertext-plaintext multiplications. In contrast, the mainstream PIR protocols [4], [42], [39], [3] represent the database as a D-dimensional hypercube; they need to compute inner products (involving the payloads) for each dimension, and require ciphertext-ciphertext multiplications from the second dimension and on (cf. Section 7). Therefore, CwPIR is more friendly to databases with large payloads. However, the selection vector generation (Step 5) in CwPIR requires $(k - 1)n$ ciphertext-ciphertext multiplications, which is disastrous when n is large.

2.3. Batch code

A *batch code* [30] (n, M, L, B)-BC encodes a collection of n elements into M codewords distributed among B buckets; any L of the n elements can be recovered by fetching at most one codeword from each bucket. It is an essential building block for *multi-query* PIR: $\mathcal{C}$ issues one PIR query to each of the B buckets and receives B responses; to answer these B queries, $\mathcal{S}$ computes over all M codewords exactly once. Notice that smaller M leads to lower computation, and smaller B leads to lower communication. Given $M < L \cdot n$, the total computation done by $\mathcal{S}$ is lower than running L instances of single-query PIR separately.

Angel et al. [4] introduce the notion of *probabilistic batch code* (PBC) that differs from the traditional batch codes in that it fails to be complete with probability p, in exchange for a smaller M and B. They provide a PBC construction based on 3-way cuckoo hashing, which encodes n elements into $M = 3n$ codewords distributed among $B = 1.5L$ buckets, with a failure probability of $p = 2^{-40}$.

We summarize the operations of a (probabilistic) batch code as follows:

- $[\mathbf{C}_1, ..., \mathbf{C}_B] \leftarrow \mathsf{Encode}([pl_1, ..., pl_n])$, where $\mathbf{C}_i$ denotes vectors of codewords in the i-th bucket.
- $[i'_1, ..., i'_B] \leftarrow \mathsf{GenSchedule}([i^*_1, ..., i^*_L])$, which takes a set of L queries and outputs a query for each of the B buckets.

- $[pl_{i_1^*},...,pl_{i_L^*}] \leftarrow$ Decode$([C_1[i'_1],...,C_B[i'_B]])$, which takes B codewords and outputs L payloads.

2.4. Labeled PSI

Private set intersection (PSI) allows two parties, a sender and a receiver, to compute the intersection of their private sets X and Y. The receiver learns only the intersection $X \cap Y$, while the sender learns nothing. Most PSI protocols are *balanced*, assuming sets of similar size and communication costs that depend on the larger set. *Unbalanced* PSI protocols [14], [13], [18], in contrast, focus on cases where the receiver's set is much smaller and aim for communication complexity that depends on the size of receiver's set. In certain scenarios, the sender has labels for each item in its set, and the receiver wants to learn the corresponding labels for the intersection. Labeled and unbalanced PSI (LPSI) can be considered as a multi-query PIR protocol that supports keyword queries and protects $\mathcal{S}$'s database. To this end, we denote the sender by $\mathcal{S}$ and denote the receiver by $\mathcal{C}$; we use the term "payload" instead of "label".

Existing LPSI protocols [13], [18] work by having $\mathcal{C}$ encrypt the elements in Y using FHE and sending the ciphertexts to $\mathcal{S}$. For each encrypted y_i, $\mathcal{S}$ homomorphically evaluates a polynomial that interpolates the payloads and returns the results to $\mathcal{C}$. In this way, $\mathcal{C}$ obtains pl_i if $y_i = x_i$. However, naively running this protocol would reveal information about $\mathcal{S}$'s elements that are not in the intersection. To avoid this, an *oblivious pseudorandom function* (OPRF) [26] is used to mask the elements in Y. Specifically, $\mathcal{S}$ and $\mathcal{C}$ apply an OPRF to the elements in X and Y, respectively. $\mathcal{S}$ then uses the OPRF values to mask the corresponding payloads, and $\mathcal{C}$ retrieves the masked payloads through polynomial evaluation. $\mathcal{C}$ uses its OPRF values to unmask the payloads and obtain the intersection. SIMD is used to improve the online performance of homomorphic polynomial evaluations, but the polynomial interpolations may still take several hours, making such LPSI protocols unsuitable for scenarios with frequent database updates.

3. Single-query PIRANA

In this section, we show how we design a faster constant-weight PIR.

3.1. Single-query PIRANA for small payloads

We first consider the case that the payload size is smaller than the slot size, $|pl| \leq p$.

Intuition. We arrange the n elements of the database into a matrix of N rows and $t := \lceil n/N \rceil$ columns (for simplicity, we assume N divides n), s.t., the i-th element is in row $r := i\%N$ and column $c := \lceil i/N \rceil$.

Naively, we can have $\mathcal{C}$ send t SIMD ciphertexts, where only the r-th slot of the c-th ciphertext is 1 and all other slots are 0s; $\mathcal{S}$ simply multiplies each column to each ciphertext and adds the products together. Then, the payload pl_{i^*} that $\mathcal{C}$

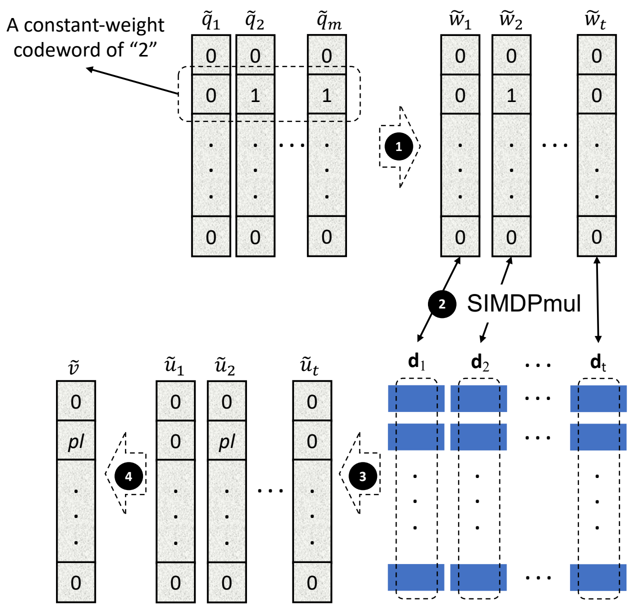


Figure 1: Workflow of single-query PIRANA for small payloads (suppose the element $\mathcal{C}$ wants to retrieve is in the second row and second column).

wants to retrieve is in the r-th slot of the resulting ciphertext. However, in this way, $\mathcal{C}$ needs to send t ciphertexts to $\mathcal{S}$. We want to reduce the number of ciphertexts to be sent.

To this end, we encode the column $c \in [1,...,t]$ as a constant-weight codeword $\mathbf{x} \in CW(m,k)$. Similarly, we encode the column indices $[1,...,t]$ as t constant-weight codewords $[\mathbf{y}_1,...,\mathbf{y}_t]$. Then, $\mathcal{C}$ only needs to send m SIMD ciphertexts: the r-th slot of each ciphertext corresponds to each bit of $\mathbf{x}$, and other slots are 0s. After running the equality operators for $[\mathbf{y}_1,...,\mathbf{y}_t]$, $\mathcal{S}$ will get the same t SIMD ciphertexts as in the aforementioned naive solution. That is to say, we can achieve the same goal with the naive solution by only sending $m \in O(\sqrt[k]{k!t} + k)$ ciphertexts. Figure 1 visualizes this process.

The protocol. The single-query PIRANA (for small payloads) works as follows:

- **Setup.** Parameters for the homomorphic encryption are chosen and keys are generated. $\mathcal{S}$ arranges its database into a matrix of N rows and $t := n/N$ columns (for simplicity, we assume N divides n).
- **Query.** Algorithm 2 shows how $\mathcal{C}$ generates a query for retrieving the i^* payload. It first maps i^* to the row index r and column index c of the matrix (Line 2-3). Then, it maps $c \in [1,...,t]$ to a constant-weight codeword $\mathbf{x} \in CW(m,k)$ (Line 4) and generates the m SIMD ciphertexts based on $\mathbf{x}$ and r (Line 6-15).
- **Answer.** $\mathcal{S}$ runs Algorithm 3 to answer a query. For each column, $\mathcal{S}$ maps the column index $j \in [1,...,t]$ to a constant-weight codeword $\mathbf{y}_j \in CW(m,k)$ (Line 3);

Algorithm 2 Single-query PIRANA: Query

```
Input $i^*$
Output $[\widetilde{\mathbf{q}}_1, ..., \widetilde{\mathbf{q}}_m]$
 1: $t := n/N$    ▷ For simplicity, we assume $N$ divides $n$
 2: $r := i^* \% N$
 3: $c := \lceil i^*/N \rceil$
 4: $\mathbf{x} \in CW(m, k) \leftarrow$ run Algorithm 1 with $c \in [1, ..., t]$
 5: Find the $k$ positions $[i_1, ..., i_k]$ in $\mathbf{x}$, where $\mathbf{x}[i] = 1$
 6: for $j := 1, ..., m$ do
 7:     for $h := 1, ..., N$ do
 8:         if $j \in [i_1, ..., i_k]$ and $h = r$ then
 9:             $\mathbf{q}_j[h] := 1$
10:         else
11:             $\mathbf{q}_j[h] := 0$
12:         end if
13:     end for
14:     $\widetilde{\mathbf{q}}_j \leftarrow \mathsf{SIMDEnc}(\mathbf{q}_j)$
15: end for
```

Algorithm 3 Single-query PIRANA: Answer (small payloads)

```
Input $[\widetilde{\mathbf{q}}_1, ..., \widetilde{\mathbf{q}}_m]$, $[pl_1, ..., pl_n]$    ▷ $|pl| \leq p$
Output $\widetilde{\mathbf{v}}$ : $\mathbf{v}[i^* \% N] = pl_{i^*}$ and other slots of $\mathbf{v}$ are 0s
 1: Init $[\mathbf{d}_1, ..., \mathbf{d}_t]$ : each $\mathbf{d}_j$ is $[pl_{(j-1)N+1}, ..., pl_{jN}]$
 2: for $j := 1, ..., t$ do
 3:     $\mathbf{y}_j \in CW(m, k) \leftarrow$ run Algorithm 1 with $j \in [t]$
 4:     Find $k$ positions $[i_1, ..., i_k]$ in $\mathbf{y}_j$, where $\mathbf{y}_j[i] = 1$
 5:     $\widetilde{\mathbf{w}}_j \leftarrow \mathsf{SIMDMul}(\widetilde{\mathbf{q}}_{i_1}, ..., \widetilde{\mathbf{q}}_{i_k})$
 6:     $\widetilde{\mathbf{u}}_j \leftarrow \mathsf{SIMDPmul}(\widetilde{\mathbf{w}}_j, \mathbf{d}_j)$
 7: end for
 8: $\widetilde{\mathbf{v}} \leftarrow \mathsf{SIMDAdd}(\widetilde{\mathbf{u}}_1, ..., \widetilde{\mathbf{u}}_t)$
 9: return $\widetilde{\mathbf{v}}$
```

runs the equality operator between $\mathbf{y}_j$ and $\mathbf{x}$, resulting in a *selection vector* $\widetilde{\mathbf{w}}_j$ (Line 4-5); and multiplies the j-th column of the database to $\widetilde{\mathbf{w}}_j$ (Line 6). Then, it adds the products of all columns together (Line 8) and returns the *inner product* $\widetilde{\mathbf{v}}$ to $\mathcal{C}$. Notice that $\mathbf{v}[r] = pl_{i^*}$ and other elements of $\mathbf{v}$ are 0s.

- **Extract.** $\mathcal{C}$ decrypts $\widetilde{\mathbf{v}}$ and outputs its r-th slot.

In this way, $\mathcal{S}$ needs to run $\lceil n/N \rceil$ ciphertext-plaintext multiplications and $\lceil n/N \rceil \cdot (k-1)$ ciphertext-ciphertext multiplications (recall that the main overhead of CwPIR [37] is to run $n \cdot (k-1)$ ciphertext-ciphertext multiplications).

3.2. Single-query PIRANA for large payloads

Notice that the returned ciphertext $\widetilde{\mathbf{v}}$ in Algorithm 3 has only one non-empty slot. We could make full use of other empty slots to return a large payload via *rotate-and-sum* [2]. Suppose a payload is as large as l ciphertexts, i.e., $|pl| = l \cdot N \cdot p$. We could split each payload into $l \cdot N$ small blocks and compute $l \cdot N$ ciphertexts: the combination of their non-empty slots is the payload to be retrieved. Then, for every N ciphertexts, $\mathcal{S}$ could rotate them so that their non-empty

Algorithm 4 Single-query PIRANA: Answer (large payloads)

```
Input $[\widetilde{\mathbf{q}}_1, ..., \widetilde{\mathbf{q}}_m]$, $[pl_1, ..., pl_n]$    ▷ $|pl| = l \cdot N \cdot p$
Output $[\widetilde{\mathbf{v}}_1, ..., \widetilde{\mathbf{v}}_l]$ : $\mathbf{v}_1[1]||...||\mathbf{v}_l[N] = pl_{i^*}$
 1: Split each $pl$ into $pl^1||...||pl^{l \cdot N}$
 2: Init $\begin{bmatrix} \mathbf{d}_{1,1} & ... & \mathbf{d}_{1,l\cdot N} \\ \vdots & \ddots & \vdots \\ \mathbf{d}_{t,1} & ... & \mathbf{d}_{t,l\cdot N} \end{bmatrix}$ : $\mathbf{d}_{j,i}$ is $\left[pl^i_{(j-1)N+1}, ..., pl^i_{jN}\right]$
 3: Init $[\widetilde{\mathbf{u}}_1, ..., \widetilde{\mathbf{u}}_{l\cdot N}]$: each $\widetilde{\mathbf{u}}_i$ is an encryption of $0^N$
 4: for $j := 1, .., t$ do
 5:     $\widetilde{\mathbf{w}}_j \leftarrow$ Run Line 3-5 of Algorithm 3
 6:     for $i := 1, ..., l \cdot N$ do
 7:         $\widetilde{\mathbf{f}}_i \leftarrow \mathsf{SIMDPmul}(\widetilde{\mathbf{w}}_j, \mathbf{d}_{j,i})$
 8:         $\widetilde{\mathbf{u}}_i \leftarrow \mathsf{SIMDAdd}(\widetilde{\mathbf{f}}_i, \widetilde{\mathbf{u}}_i)$
 9:     end for
10: end for
11: Init $[\widetilde{\mathbf{v}}_1, ..., \widetilde{\mathbf{v}}_l]$: each $\mathbf{v}_i$ is an encryption of $0^N$
12: for $i := 1, ..., l$ do
13:     for $j := 1, ..., N$ do
14:         $\widetilde{\mathbf{v}}_i \leftarrow \mathsf{SIMDAdd}(\widetilde{\mathbf{v}}_i, \widetilde{\mathbf{u}}_{(i-1)\cdot N+j})$
15:         $\widetilde{\mathbf{v}}_i \leftarrow \mathsf{SIMDRotate}(\widetilde{\mathbf{v}}_i, 1)$
16:     end for
17: end for
18: return $[\widetilde{\mathbf{v}}_1, ..., \widetilde{\mathbf{v}}_l]$
```

slots are interlaced, and add them together. As a result, $\mathcal{S}$ only needs to return l ciphertexts. Algorithm 4 shows the details and Figure 2 visualizes this process. Specifically, $\mathcal{S}$ runs as follows to answer a query:

❶ For each column, it runs Line 3-5 of Algorithm 3 to get a selection vector (Line 5).

❷ It multiplies the blocks of the N payloads in each column to each selection vector, and repeats this for all $l \cdot N$ blocks of each payload (Line 7).

❸ It adds together the ciphertexts from different columns (Line 8), resulting in $l \cdot N$ ciphertexts.

❹ For every N ciphertexts, it rotates them so that their non-empty slots are interlaced, and adds them together (Line 11-17). There will be l inner products left for the whole database. An important optimization here is that we rotate $\widetilde{\mathbf{v}}$ instead of $\widetilde{\mathbf{u}}$,[1] enabling us to rotate one slot per rotation. Notice that the rotation becomes much faster when the number of rotated slots is a power of two. Therefore, rotating one $(1 = 2^0)$ slot here is more efficient than rotating an arbitrary number of slots.

1. We rotate $\widetilde{\mathbf{u}}$ in Figure 2 because it is easy to visualize.

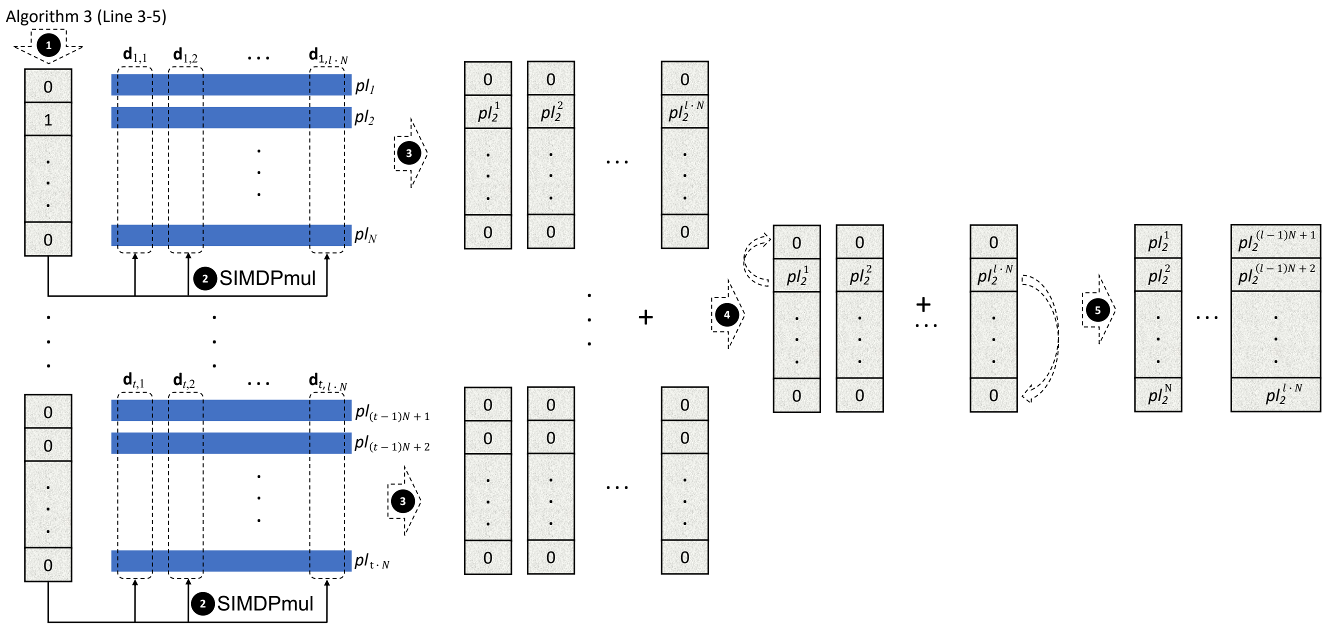


Figure 2: Workflow of single-query PIRANA for large payloads (visualization of Algorithm 4).

3.3. Single-query PIRANA for large payloads and small n

Notice that Algorithm 4 requires $l \cdot (N-1)$ rotations. In this subsection, we first present Algorithm 5, which achieves the same goal with Algorithm 4 but requires $t \cdot (N-1)$ rotations, hence it is more friendly to databases with large payloads but a small n. Then, we show how can we find a balance between Algorithm 4 and Algorithm 5.

The intuition is to pre-compute all N possible rotations for each selection vector, then it no longer needs any rotation when computing the inner product. In more detail, $\mathcal{S}$ first rotates the payloads so that the values to be selected are interlaced (Line 2). Notice that this step only needs to be done once and should be in the setup phase (we simply leave it in Algorithm 5). After running Line 3-5 of Algorithm 3, it rotates each of the t selection vectors $(N-1)$ times (Line 7-9). Then, it multiplies the rotated selection vectors with the corresponding rotated payloads (Line 11-13). In the end, it adds every $t \cdot N$ products together, resulting in l inner products (Line 14).

In practice, we could combine Algorithm 4 and Algorithm 5 to minimize the number of rotations for a specific l and t. Specifically, we could first run Algorithm 5, with two modifications:

- In Line 7-9, it rotates the selection vector for $(\alpha-1)$ times (instead of $(N-1)$ times) and rotates $\frac{N}{\alpha}$ slots per rotation.
- In Line 14, it does not add the N $\widetilde{\mathbf{f}}$s together, instead it only adds the $\widetilde{\mathbf{f}}$s from t columns together (in a way like in Line 8 of Algorithm 4), resulting in $l \cdot N$ ciphertexts.

Then, it runs Algorithm 4 from Line 11, but only rotates for $(\frac{N}{\alpha}-1)$ times in Line 13-16. Now, the total number of rotations becomes:

$$f(\alpha) = (\alpha-1)t + (\frac{N}{\alpha}-1)l.$$

Algorithm 5 Single-query PIRANA: Answer (large payloads and small n)

```
Input [$\widetilde{\mathbf{q}}_1, ..., \widetilde{\mathbf{q}}_m$], [$pl_1, ..., pl_n$]        ▷ $|pl| = l \cdot N \cdot p$
Output [$\widetilde{\mathbf{v}}_1, ..., \widetilde{\mathbf{v}}_l$] : $\mathbf{v}_1[1]||...||\mathbf{v}_l[N] = pl_{i^*}$
 1: Split each $pl$ into $pl^1||...||pl^{l \cdot N}$
 2: Init $\begin{bmatrix} \mathbf{d}_{1,1} & \cdots & \mathbf{d}_{1,l\cdot N} \\ \vdots & \ddots & \vdots \\ \mathbf{d}_{t,1} & \cdots & \mathbf{d}_{t,l\cdot N} \end{bmatrix}$ : $\mathbf{d}_{j,i}$ is $\left[pl^i_{(j-1)N+1}, ..., pl^i_{jN}\right]$
    with an rotation of $i$
 3: Init $\begin{bmatrix} \widetilde{\mathbf{u}}_{1,1} & \cdots & \widetilde{\mathbf{u}}_{1,N} \\ \vdots & \ddots & \vdots \\ \widetilde{\mathbf{u}}_{t,1} & \cdots & \widetilde{\mathbf{u}}_{t,N} \end{bmatrix}$ : $\mathbf{u}_{j,i}$ is an encryption of $0^N$
 4: Init [$\widetilde{\mathbf{v}}_1, ..., \widetilde{\mathbf{v}}_l$]: each $\mathbf{v}_i$ is an encryption of $0^N$
 5: for $j := 1, .., t$ do
 6:     $\widetilde{\mathbf{w}}_j \leftarrow$ Run Line 3-5 of Algorithm 3
 7:     for $i := 1, ..., N-1$ do
 8:         $\widetilde{\mathbf{u}}_{j,i} \leftarrow$ SIMDRotate($\widetilde{\mathbf{w}}_j, i$)
 9:     end for
10:     for $h := 1, ..., l$ do
11:         for $i := 1, ..., N$ do
12:             $\widetilde{\mathbf{f}}_i \leftarrow$ SIMDPmul($\widetilde{\mathbf{u}}_{j,i}, \mathbf{d}_{j,(h-1)N+i}$)
13:         end for
14:         $\widetilde{\mathbf{v}}_h \leftarrow$ SIMDAdd($\widetilde{\mathbf{v}}_h, \widetilde{\mathbf{f}}_1, ..., \widetilde{\mathbf{f}}_N$)
15:     end for
16: end for
17: return [$\widetilde{\mathbf{v}}_1, ..., \widetilde{\mathbf{v}}_l$]
```

The first-order derivative of $f(\alpha)$ is:

$$\frac{df(\alpha)}{d\alpha} = t - \frac{N \cdot l}{\alpha^2}.$$

The second-order derivative of $f(\alpha)$ is:

$$\frac{d^2 f(\alpha)}{d\alpha^2} = \frac{2lN}{\alpha^3} > 0.$$

As the second-order derivative is always positive, we have:

1) If the first-order derivative is always negative (i.e. $t - \frac{N \cdot l}{\alpha^2} < 0$), $f(\alpha)$ is monotonically decreasing, hence it is minimal when α is maximal (i.e, $\alpha = N$). In this case, $t < \frac{l}{N}$.
2) If the first-order derivative is always positive (i.e. $t - \frac{N \cdot l}{\alpha^2} > 0$), $f(\alpha)$ is monotonically increasing, hence it is minimal when α is minimal (i.e, $\alpha = 1$). In this case, $t > l \cdot N$.
3) If the first-order derivative is zero (i.e. $t - \frac{N \cdot l}{\alpha^2} = 0$), $f(\alpha)$ is minimal when $\alpha = \sqrt{Nl/t}$.

Formally, we have:

$$\underset{1 \le \alpha \le N}{\operatorname{argmin}} f(\alpha) = \begin{cases} N & t < \frac{l}{N} \\ \sqrt{Nl/t} & \frac{l}{N} \le t \le l \cdot N \\ 1 & t > l \cdot N \end{cases}$$

which implies that:

- when the payload size is large enough s.t., $l \ge tN$, we should run Algorithm 5;
- when the number of elements is large enough s.t., $t \ge lN$, we should run Algorithm 4;
- in the middle ground case, we should combine Algorithm 4 and Algorithm 5 as aforementioned, with $\alpha = \sqrt{Nl/t}$.

Remark. To rotate a ciphertext for a specific number of slots, $\mathcal{S}$ needs to know the corresponding rotation key. Therefore, $\mathcal{C}$ needs to transfer the rotation keys to $\mathcal{S}$. This is not a problem for either Algorithm 4 or Algorithm 5, because both Algorithms only require $\mathcal{S}$ to rotate one slot per rotation, hence $\mathcal{C}$ only needs to transfer one rotation key. If we combine them, $\mathcal{C}$ has to transfer one more rotation key for rotating $\frac{N}{\alpha}$ slots.

4. Multi-query PIRANA

In this section, we show how we extend PIRANA to support multi-query with a very small extra-cost in both communication and computation, compared to retrieving a single element.

4.1. Multi-query PIRANA for small payloads

Intuition. Recall that in our single-query PIRANA, $\mathcal{C}$ sends m SIMD ciphertexts to $\mathcal{S}$; only one slot in each ciphertext is useful and other slots are empty. Then, a natural question to ask is that can we use other slots to batch more queries? The answer is no because there may be multiple desired elements reside on the same row, rendering the query invalid. Thanks to the batch code, we could encode $\mathcal{S}$'s database into a $(n, M, L, B) - BC$ and treat each bucket as a "row"; then, the desired elements are for sure in different rows. If $N > B$, we could split a bucket into multiple rows to reduce the bucket size and make full use of the slots.

If we use 3-way cuckoo hashing as the batch code (cf. Section 2.3), we are able to retrieve up to $\lfloor N/1.5 \rfloor$ elements with only $3\times$ higher computational cost and almost the same communication cost compared to running the single-query PIRANA once (for small payloads with $|pl| \le p$). In more detail, recall that 3-way cuckoo hashing encodes n elements into $M = 3n$ codewords distributed among $B = 1.5L$ buckets; if we set $B = N$, we could maximize the number queries i.e. $L = N/1.5$ and minimize the bucket size i.e. $3n/N$. The number of columns determines both computational and communication costs, and it increases from n/N (in single-query PIRANA) to $3n/N$. Therefore, the computational cost increases exactly 3 times; the communication cost increases from $(\sqrt[k]{k!n/N} + k)$ to $(\sqrt[k]{k!3n/N} + k)$, which is insignificant.

The protocol. The multi-query PIRANA (for small payloads) works as follows:

Algorithm 6 Multi-query PIRANA: Query

```
Input $[i_1^*, ..., i_L^*]$
Output $[\widetilde{\mathbf{q}}_1, ..., \widetilde{\mathbf{q}}_m]$
1: $[i'_1, ..., i'_N] \leftarrow \mathsf{GenSchedule}([i_1^*, ..., i_L^*])$
2: for $h := 1, ..., N$ do
3:     $\mathbf{x}_h \in CW(m,k) \leftarrow$ run Algorithm 1 with $i'_h \in [M/N]$    ▷ For simplicity, we assume $N$ divides $M$
4:     Find $k$ positions $[i_1, ..., i_k]$ in $\mathbf{x}_h$, where $\mathbf{x}_h[i] = 1$
5:     for $j := 1, ..., m$ do
6:         if $j \in [i_1, ..., i_k]$ then
7:             $\mathbf{q}_j[h] := 1$
8:         else
9:             $\mathbf{q}_j[h] := 0$
10:        end if
11:    end for
12: end for
13: for $j := 1, ..., m$ do
14:     $\widetilde{\mathbf{q}}_j \leftarrow \mathsf{SIMDEnc}(\mathbf{q}_j)$
15: end for
```

- **Setup.** Parameters for the homomorphic encryption are chosen and keys are generated. $\mathcal{S}$ encodes its database using a batch code $BC(n, M, L, B)$:
$$[\mathbf{C}_1, ..., \mathbf{C}_B] \leftarrow \mathsf{Encode}([pl_1, ..., pl_n]),$$
where $\mathbf{C}_i$ denotes vectors of codewords in the i-th bucket. If $B < N$, it splits each bucket into $s := N/B$ small buckets (for simplicity, we assume B divides N). That means the database is in fact encoded as:
$$[\mathbf{C}_1, ..., \mathbf{C}_N] \leftarrow \mathsf{Encode}([pl_1, ..., pl_n]),$$
- **Query.** $\mathcal{C}$ runs Algorithm 6 to generate the query ciphertexts. More specifically, given L original indices $[i_1^*, ..., i_L^*]$, $\mathcal{C}$ first runs GenSchedule of the batch code to generate indices $[i''_1, ..., i''_B]$ for each of the B buckets. If the buckets has been split into N small buckets,

Algorithm 7 Multi-query PIRANA: Answer (small payloads)

```
Input $[\widetilde{\mathbf{q}}_1, ..., \widetilde{\mathbf{q}}_m]$, $[\mathbf{C}_1, ..., \mathbf{C}_N]$
Output $\widetilde{\mathbf{v}} : \mathbf{v}[j] = \mathbf{C}_j[i'_j]$
 1: $t := M/N$   ▷ For simplicity, we assume $N$ divides $M$
 2: Init $[\mathbf{d}_1, ..., \mathbf{d}_t]$ : each $\mathbf{d}_j$ is $[\mathbf{C}_1[j], ..., \mathbf{C}_N[j]]$
 3: for $j := 1, ..., t$ do
 4:     $\mathbf{y}_j \in CW(m, k) \leftarrow$ run Algorithm 1 with $j \in [t]$
 5:     Find $k$ positions $[i_1, ..., i_k]$ in $\mathbf{y}_j$, where $\mathbf{y}_j[i] = 1$
 6:     $\widetilde{\mathbf{w}}_j \leftarrow \mathsf{SIMDMul}(\widetilde{\mathbf{q}}_{i_1}, ..., \widetilde{\mathbf{q}}_{i_k})$
 7:     $\widetilde{\mathbf{u}}_j \leftarrow \mathsf{SIMDPmul}(\widetilde{\mathbf{w}}_j, \mathbf{d}_j)$
 8: end for
 9: $\widetilde{\mathbf{v}} \leftarrow \mathsf{SIMDAdd}(\widetilde{\mathbf{u}}_1, ..., \widetilde{\mathbf{u}}_t)$
10: return $\widetilde{\mathbf{v}}$
```

the corresponding indices $[i'_1, ..., i'_N]$ should be calculated (Line 1). Given that a bucket has been split into $s := N/B$ rows, we could either group these s indices together (e.g., $[i'_1, ..., i'_s]$ are the indices for the first bucket), or group the indices for different buckets together (e.g., $\left[i'_1, i'_{B+1}, ..., i'_{(s-1)B+1}\right]$ are the indices for the first bucket). We choose the later as it is more friendly to large payloads (cf. Section 4.2)
Then, we run in a similar way as in Algorithm 2, except that it needs to compute a constant-weight codeword $\mathbf{x}_h$ for each of the N slots (Line 4) and uses $\mathbf{x}_h$ to determine the value for that slot.

- **Answer.** $\mathcal{S}$ runs Algorithm 7 to answer these L queries in a batch. It is mostly the same as Algorithm 3, except that $\mathbf{d}_j$ is a combination of the j-th elements in each bucket (Line 2).
- **Extract.** $\mathcal{C}$ decrypts $\widetilde{\mathbf{v}}$ and gets $[C_1[i'_1], ..., C_B[i'_N]]$. Then, it runs

$$\left[pl_{i^*_1}, ..., pl_{i^*_L}\right] \leftarrow \mathsf{Decode}([C_1[i'_1], ..., C_B[i'_N]]).$$

4.2. Multi-query PIRANA for large payloads

Notice that only B out of N slots in $\widetilde{\mathbf{v}}$ (the returned ciphertext of Algorithm 7) are non-zero and other slots are empty. When the payloads are large (i.e., $|pl| = l \cdot N \cdot p$) and B is small, it is desirable to combine multiple ciphertexts into a single one in a similar way as in Algorithm 4. However, this time, there are B (instead of one) non-zero values in $\widetilde{\mathbf{v}}$, hence we cannot directly rotate-and-sum. Fortunately, the positions for the non-zero values are somehow regular, e.g., there is only one non-zero value at positions $\left[i'_1, i'_{B+1}, ..., i'_{(s-1)B+1}\right]$ of $\mathbf{v}$. Therefore, we could still use rotate-and-sum to combine multiple ciphertexts as we did in Algorithm 4, except that we need to rotate B slots (instead of one) each time and we could only combine s ciphertexts (instead of N). Algorithm 8 shows the details.

For each column, $\mathcal{S}$ runs in a similar way as in Algorithm 7 to generate a selection vector (Line 9-11). Then, it multiplies the blocks of N payloads in each column to each selection vector, and repeats this for all $l \cdot N$ blocks of each payload (Line 12-14). Next, it combines every s ciphertexts (Line 19-22) and this process will be repeated for $l \cdot B$ times. In this way, it requires $l \cdot N$ rotations in total. However, we could rotate $\widetilde{\mathbf{w}}$s first to enumerate all rotations, and then rotate the payload in the same way (as we did in Algorithm 5). This optimization reduces the number of rotations to $3n/B$, which is independent to the payload size $l \cdot N$. Algorithm 9 shows this optimization.

Algorithm 8 Multi-query PIRANA: Answer (large payloads)

```
Input $[\widetilde{\mathbf{q}}_1, ..., \widetilde{\mathbf{q}}_m]$, $[\mathbf{C}_1, ..., \mathbf{C}_N]$   ▷ $|pl| = l \cdot N \cdot p$
Output $[\widetilde{\mathbf{v}}_1, ..., \widetilde{\mathbf{v}}_d]$
 1: $t := M/N$   ▷ we assume $N$ divides $M$
 2: $s := N/B$   ▷ we assume $B$ divides $N$
 3: Split each $\mathbf{C}[j]$ into $\mathbf{C}[j][1]||...||\mathbf{C}[j][l \cdot N]$
 4: Init $\begin{bmatrix} \mathbf{d}_{1,1} & ... & \mathbf{d}_{1,l\cdot N} \\ \vdots & \ddots & \vdots \\ \mathbf{d}_{t,1} & ... & \mathbf{d}_{t,l\cdot N} \end{bmatrix}$: $\mathbf{d}_{j,h}$ is $[\mathbf{C}_1[j][h], ..., \mathbf{C}_N[j][h]]$
 5: for $h := 1, ..., l \cdot N$ do
 6:     Init $\widetilde{\mathbf{u}}_h$: $\mathbf{u}_h = 0^N$
 7: end for
 8: for $j := 1, ..., t$ do
 9:     $\mathbf{y}_j \in CW(m, k) \leftarrow$ run Algorithm 1 with $j \in [t]$
10:     Find $k$ positions $[i_1, ..., i_k]$ in $\mathbf{y}_j$, where $\mathbf{y}_j[i] = 1$
11:     $\widetilde{\mathbf{w}}_j \leftarrow \mathsf{SIMDMul}(\widetilde{\mathbf{q}}_{i_1}, ..., \widetilde{\mathbf{q}}_{i_k})$
12:     for $h := 1, ..., l \cdot N$ do
13:         $\widetilde{\mathbf{u}}_h := \mathsf{SIMDAdd}(\widetilde{\mathbf{u}}_h, \mathsf{SIMDPmul}(\widetilde{\mathbf{w}}_j, \mathbf{d}_{j,h}))$
14:     end for
15: end for
16: Init $[\widetilde{\mathbf{v}}_1, ..., \widetilde{\mathbf{v}}_{l\cdot B}]$: each $\mathbf{v}_i$ is an encryption of $0^N$
17: for $i := 1, ..., l$ do
18:     for $j := 1, ..., B$ do
19:         for $h := 1, ..., s$ do
20:             $\widetilde{\mathbf{v}}_{(i-1)B+j} \leftarrow \mathsf{SIMDAdd}(\widetilde{\mathbf{v}}_{(i-1)B+j}, \widetilde{\mathbf{u}}_{(i-1)\cdot N+(j-1)\cdot s+h})$
21:             $\widetilde{\mathbf{v}}_{(i-1)B+j} \leftarrow \mathsf{SIMDRotate}(\widetilde{\mathbf{v}}_{(i-1)B+j}, B)$
22:         end for
23:     end for
24: end for
25: return $[\widetilde{\mathbf{v}}_1, ..., \widetilde{\mathbf{v}}_{l\cdot B}]$
```

We further apply some common optimizations, including secret-key for encryption/decryption [3] and modulus switching [10] to our protocol. The detail of these optimizations can be found in Appendix A.

4.3. Comparison with Mughees-Ren [41]

A concurrent and independent work was proposed by Mughees-Ren [41] to improve the performance of multi-query PIR. They encode the elements in each bucket (of a batch code) as a 3-dimensional hypercube and batch the queries for different buckets into a single SIMD ciphertext. They further use a rotate-and-sum approach to batch the

Algorithm 9 Multi-query PIRANA: Answer with less rotations (large payloads)

```
Input $[\widetilde{\mathbf{q}}_1, ..., \widetilde{\mathbf{q}}_m]$, $[\mathbf{C}_1, ..., \mathbf{C}_N]$          $\triangleright$ $|pl| = l \cdot N \cdot p$
Output $[\widetilde{\mathbf{v}}_1, ..., \widetilde{\mathbf{v}}_d]$
 1: Run Line 1-4 of Algorithm 8 to init $t$, $s$ and $\mathbf{d}$
 2: for $j := 1, ..., t$ do
 3:     $\mathbf{y}_j \in CW(m,k) \leftarrow$ run Algorithm 1 with $j \in [t]$
 4:     Find $k$ positions $[i_1, ..., i_k]$ in $\mathbf{y}_j$, where $\mathbf{y}_j[i] = 1$
 5:     $\widetilde{\mathbf{w}}_{j,1} \leftarrow \mathsf{SIMDMul}(\widetilde{\mathbf{q}}_{i_1}, ..., \widetilde{\mathbf{q}}_{i_k})$
 6:     for $g := 2, ..., s$ do
 7:         $\widetilde{\mathbf{w}}_{j,g} \leftarrow \mathsf{SIMDRotate}(\widetilde{\mathbf{w}}_{j,g-1}, B)$
 8:     end for
 9: end for
10: Init $[\widetilde{\mathbf{v}}_1, ..., \widetilde{\mathbf{v}}_{l \cdot B}]$: each $\mathbf{v}_i$ is an encryption of $0^N$
11: for $i := 1, ..., l$ do
12:     for $j := 1, ..., B$ do
13:         for $g := 1, ..., s$ do
14:             for $h := 1, ..., t$ do
15:                 $\mathbf{a} \leftarrow \mathsf{Rotate}(\mathbf{d}_{h,(i-1)N+(j-1)s+g}, (h-1)B)$
16:                 $\widetilde{\mathbf{u}} \leftarrow \mathsf{SIMDPmul}(\widetilde{\mathbf{w}}_{h,g}, \mathbf{a})$
17:                 $\widetilde{\mathbf{v}}_{(i-1)B+j} \leftarrow \mathsf{SIMDAdd}(\widetilde{\mathbf{v}}_{(i-1)B+j}, \widetilde{\mathbf{u}})$
18:             end for
19:         end for
20:     end for
21: end for
22: return $[\widetilde{\mathbf{v}}_1, ..., \widetilde{\mathbf{v}}_{l \cdot B}]$
```

responses. However, in this protocol, most of $\mathcal{S}$'s operations need to involve the payloads, which incurs a large computational overhead when the payload size is large. In contrast, the expensive operations (ciphertext-ciphertext multiplications) in PIRANA are independent of the payloads.

Table 2 provides a theoretical comparison between Mughees-Ren [41] and multi-query PIRANA. For a large payload size (i.e., $|pl| = l \cdot N \cdot p$ with $l > 1$), PIRANA is basically better than Mughees-Ren's protocol in all aspects. In particular, PIRANA saves $(\frac{3n(l \cdot N - d)}{d \cdot N} + l \cdot B \cdot d)$ ciphertext-ciphertext multiplications and $\frac{Bd^2}{N} + \frac{3n(Bl-d)}{Bd} + lBd(\log d - 1)$ rotations. Notice that the query size of PIRANA remains the same as long as $N > 1.5L$, whereas the query size of Mughees-Ren increases linearly with L.

The right-most column of Table 2 provides some concrete numbers for a specific database configuration. The advantage will be more prominent when l is larger. We will provide a more detailed experimental comparison in Section 6.3.

5. LPSI-PIRANA

To extend multi-query PIRANA to LPSI (dubbed LPSI-PIRANA), we need to improve it to (1) support keyword queries and (2) protect $\mathcal{S}$'s database. Ali et al. [3] describe a general reduction from LPSI to PIR via OPRF and cuckoo hashing. This reduction is applicable to PIRANA as well:

- **Setup.**

	# elements	n	2^{20}
	payload size	$l \cdot N \cdot p$ $(l \geq 1)$	20KB
	# queries	$L \leq N$	256
MR [41]	# SIMDPmul	$3l \cdot n$	3 145 728
	# SIMDMul	$\frac{3l \cdot n}{d} + l \cdot B \cdot d$	110 592
	# SIMDRotate	$\frac{Bd^2}{N} + \frac{3ln}{d} + lBd(\log d - 1)$	147 504
	# ciphers (query)	$\frac{3Bd}{N}$	6
	# ciphers (answer)	$l \cdot B$	384
PIRANA	# SIMDPmul	$3l \cdot n$	3 145 728
	# SIMDMul	$3n/N$	384
	# SIMDRotate	$3n/B$	8 192
	# ciphers (query)	$\sqrt{6n/N} + 2$	30
	# ciphers (answer)	$l \cdot B$	384

TABLE 2: Comparison of answer generation time between Mughees-Ren [41] and multi-query PIRANA. ($N = 8\ 192$, $p = 20$; the batch code is based on cuckoo hashing, hence $B = 1.5L$; d is a power of two and larger than $(3n/B)^{1/3}$: $d = 32$ for $n = 2^{20}$ and $B = 384$)

1) $\mathcal{S}$ computes $(x'_i, x''_i) = PRF_k(x_i)$ for all x_i in its set X. The payload pl_i is masked by x''_i: $pl'_i := pl_i \oplus x''_i$.
2) $\mathcal{S}$ uses cuckoo hashing to assign $\{x'_1, ..., x'_n\}$ to B buckets.
3) Inside each bucket, $\mathcal{S}$ uses cuckoo hashing again to map each x to indices and sorts the masked payloads accordingly.

- **Query.**

1) For each $y_i \in [y_1, ..., y_L]$, $\mathcal{S}$ and $\mathcal{C}$ run OPRF so that $\mathcal{C}$ gets (y'_i, y''_i) without learning the key.
2) $\mathcal{C}$ runs Algorithm 6 with input $[y'_1, ..., y'_L]$,[2] and sends the output $[\widetilde{\mathbf{q}}_1, ..., \widetilde{\mathbf{q}}_m]$ to $\mathcal{S}$.

- **Answer.** $\mathcal{S}$ runs Algorithm 7 (for small payloads) or Algorithm 8 (for large payloads) and returns the results.
- **Extract.** $\mathcal{C}$ runs the "Extract" of multi-query PIRANA, and unmask the results using y''s.

Compared with the state-of-the-art LPSI protocol [18], we have the following advantages:

1) Whenever the database updates, $\mathcal{S}$ in [18] has to interpolate the polynomials, which takes several hours. We successfully get rid of the expensive polynomial interpolations, making it more friendly to scenarios with frequent database updates.
2) Recall that when $N > B$, we could use the empty slots to batch more responses (cf. Algorithm 8), which is not the case for [18]: for $x \notin Y$, our protocol results in 0s in the corresponding slots, whereas [18] results in dummy random values (hence [18] cannot use such slots to carry responses).
3) In [18], polynomial evaluation happens in an SIMD-modulus p with 20-30 bits, but the hashed keyword is much larger (80-128 bits). This requires the hashed keyword to be split into multiple sequential slots for separate polynomial evaluation. Our protocol avoids this overhead by mapping keywords to indices.

2. Algorithm 6 takes indices as input, but it naturally supports keywords.

	# elements n	2^8	2^9	2^{10}	2^{11}	2^{12}	2^{13}	2^{14}	2^{15}	2^{16}
	DB Size (MB)	5.2	10	21	42	84	170	340	670	1 300
CwPIR [37]	Selection Vec. (s)	3.9	7.8	15.5	31.0	61.7	123.1	246.2	492.7	983.3
	Inner Product (s)	0.2	0.4	0.8	1.6	3.3	6.5	13.1	26.2	52.3
	Total server (s)	4.1	8.2	16.3	32.6	65.0	129.7	259.4	518.9	1 035.6
PIRANA (single-query)	Selection Vec. (s)	0.001	0.001	0.001	0.001	0.001	0.001	0.027	0.05	0.1
	Inner Product (s)	0.22	0.24	0.28	0.36	0.52	0.86	1.57	2.86	5.39
	Total server (s)	0.22	0.24	0.28	0.36	0.52	0.86	1.6	2.9	5.49
	Speedup	18.6×	34.2×	58.2×	90.6×	125×	151×	162.1×	178.9×	188.6×

TABLE 3: Microbenchmark of PIRANA and CwPIR ($k = 2$, $N = 2^{13}$, payload size is 20KB).

6. Evaluation

In this section, we provide a full-fledged implementation for PIRANA and systematically evaluate its performance.

6.1. Implementation

We fully implement PIRANA in C++ based on the Microsoft SEAL homomorphic encryption library (version 4.0)[3]. We use the Brakerski-Fan-Vercauteren (BFV) [7], [25] scheme, with $N \in \{4096, 8192\}$ and the default parameters in SEAL for 128-bit security. All ciphertext-plaintext multiplications and ciphertext-ciphertext additions are implemented using number theoretic transform (NTT). To this end, we encode all payloads into NTT forms so that they can be multiplied directly to the NTTed selection vectors. Such payload encoding only needs to be done once in the setup phase and can be used for all queries.

We use index queries to evaluate all PIR protocols and we use keyword queries to evaluate all LPSI protocols. We run all experiments on an Intel Xeon Cooper Lake (with a base frequency of 3.4 GHz and turbo frequency of 3.8 GHz) server running Ubuntu 20.04. This setup is similar to the setting of CwPIR [37]. All experiments were repeated 5 times and average values (the variances are very small) were reported.

6.2. Comparison with constant-weight PIR

We first compare PIRANA (a combination of Algorithm 4 and Algorithm 5 with α depending on the number of elements n and payload sizes) with CwPIR [37] in terms of constant-weight PIRs. To this end, we reproduce the results of CwPIR (i.e., Table 7 in [37]) reported in their original paper, by running their open-sourced implementation[4]. We set ($k = 2$, $N = 8\ 192$) for both CwPIR and PIRANA, and choose the minimal m that satisfies $\binom{m}{k} > n$ and $\binom{m}{k} > n/N$ respectively. Our reproduced results of CwPIR are better than their original results, for two reasons: (1) our CPU is more advanced, and (2) we use a newer version of the SEAL library (they use version 3.6).

We measure PIRANA by the same metric and list the comparison results in Table 3. As expected, PIRANA's advantage in selection vector generation is significant compared to CwPIR. Recall that CwPIR needs to run $(k-1)\cdot n$ ciphertext-ciphertext multiplications to generate a selection vector, whereas PIRANA only needs to run $\lceil n/N \rceil \cdot (k-1)$ times. Surprisingly, inner product calculation in PIRANA is also much faster than that in CwPIR. The reason is that the selection vector generated by CwPIR has n ciphertexts, which need to be transformed to NTT to be multiplied to the payloads, hence they need to run NTT for n times. In contrast, our selection vector only has $\lceil n/N \rceil$ ciphertexts, so we save upto N times of NTT. Given our advantages in both selection vector generation and inner product calculation, we achieve up to 188.6× speedup over CwPIR.

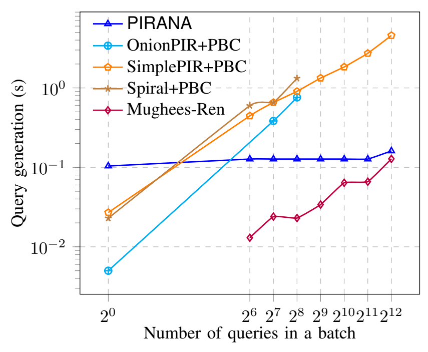
On the other hand, the query size of PIRANA is up to 2.5× larger than CwPIR (the response sizes are roughly equal). For example, when $n = 2^{16}$, the query size is 432KB in CwPIR and 1 080KB in PIRANA. Furthermore, $\mathcal{C}$ in PIRANA needs to transfer additional 6.2MB rotation keys during setup. However, as we will show next, this communication overhead in PIRANA can be utilized to batch up to $\lfloor N/1.5 \rfloor$ queries for free.

6.3. Comparison with the state-of-the-art PIR

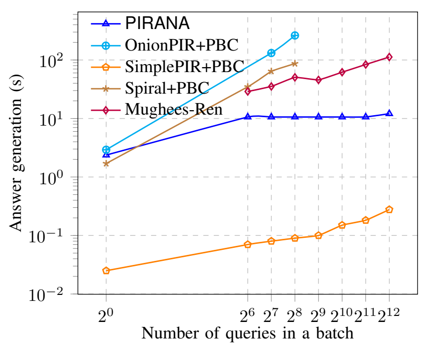
We compare PIRANA with OnionPIR[5] [42], SimplePIR[6] [1], Spiral[7] [39], and Mughees-Ren[8] [41], in terms of both single-query and multi-query PIR. It is worth mentioning that SimplePIR is inherently more efficient than others due to its stateful nature, which requires clients to maintain a large state. However, this approach may not be applicable to scenarios where the database updates frequently, as the server would have to contact all clients to update their states whenever the database changes. We compare with SimplePIR only to show that, in some aspects (e.g., query generation and answer extraction), our scheme even outperforms it.

We consider a database of 2^{20} elements ($n = 2^{20}$) and we consider two types of payloads: 256-byte and 20KB. For multi-query, we scale the query batch size from 64 to 4 096. Notice that OnionPIR, SimplePIR and Spiral are designed

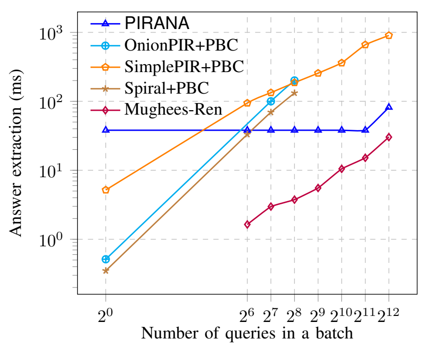
3. https://github.com/Microsoft/SEAL
4. https://github.com/RasoulAM/constant-weight-pir
5. https://github.com/mhmughees/Onion-PIR
6. https://github.com/ahenzinger/simplepir
7. https://github.com/menonsamir/spiral
8. By the time we submitted this paper, the protocol of Mughees-Ren [41] was not open-sourced. Thus, we implemented it by ourselves and the evaluation results are consistent with those reported in their paper.



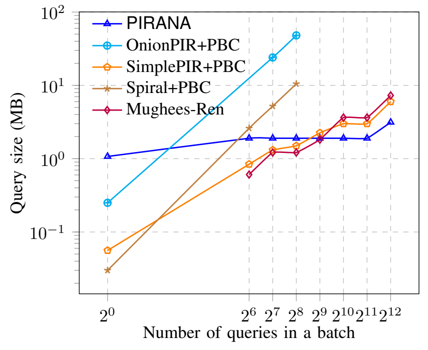
(a) Query generation time.



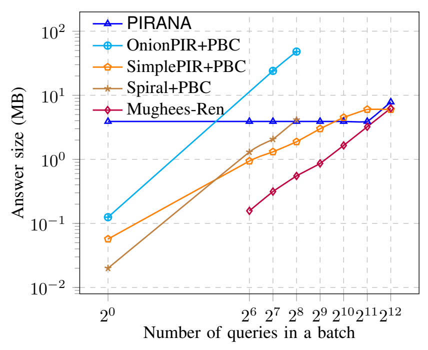
(b) Answer generation time.



(c) Answer extraction time.



(d) Query size.



(e) Answer size.

Figure 3: Evaluation of PIRs (2^{20} elements, 256-byte payloads).

for single-query, but we could plug them into PBC [4] to support multi-query. Recall that PBC is constructed based on 3-way cuckoo hashing, which encodes n elements into $M = 3n$ codewords distributed among $B = 1.5L$ buckets, with a failure probability of $p = 2^{-40}$. We could use these single-query PIR schemes to query each bucket, but unlike PIRANA and Mughees-Ren, they cannot batch the queries for different buckets into a single ciphertext.

Figure 3 and Figure 4 show the benchmark results for 256-byte and 20KB payloads respectively.

To answer a single query, PIRANA is mostly slower than others (except Mughees-Ren[9]):

- It takes 2.4s to answer a single query for a 256-byte payload, 1.2× faster than OnionPIR, but 1.4× slower than Spiral and 94.8× slower than SimplePIR.
- It takes 243s to answer a single query for a 20KB payload, 1.1× slower than OnionPIR, 3.1× slower than Spiral, and 116× slower than SimplePIR.

However, the costs of PIRANA begin to amortize when the number of queries L increases:

- It takes 12.1s to answer 4 096 queries for 256-byte payloads (812-fold amortization).
- It takes 583s to answer 4 096 queries for 20KB payloads (1 707-fold amortization).

When $L \leq 2\,048$, the PBC bucket size for 256-byte payloads is always $\frac{M}{N}$, hence the overhead is stable. For 20KB payloads, we set $B = 1.5L$ and use the rest slots to carry payloads, hence the PBC bucket size $\frac{M}{B}$ decreases when L increases, which explains the decreased overhead. When L increases from 2 048 to 4 096, $\mathcal{C}$ in PIRANA needs to send two ciphertexts instead of one, hence $L = 4\,096$ would be the scaling up point. However, the answer generation time will not double, because the PBC bucket size also decreases.

The open-sourced code of OnionPIR and Spiral cannot support 4 096 queries. Therefore, for these two schemes, we measure up to 256 queries for 256-byte payloads[10] and 2 048 queries for 20KB payloads.

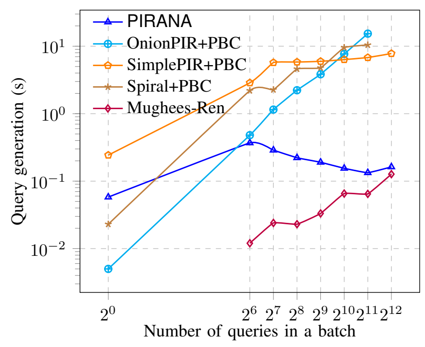
- To answer 256 queries for 256-byte payloads, PIRANA is 24.8× faster than OnionPIR and 8.2× faster than Spiral.
- To answer 2 048 queries for 20KB payloads, PIRANA is 5.4× faster than OnionPIR and 3.6× faster than Spiral.

When considering the client time (i.e., query generation and answer extraction), the advantages of PIRANA and Mughees-Ren become more prominent:

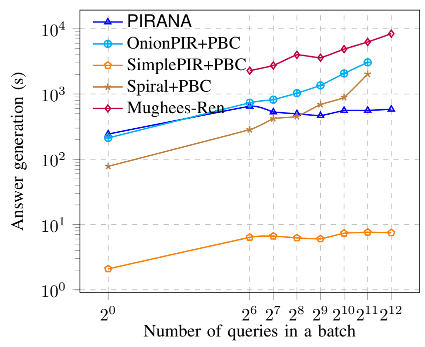
- To generate queries for retrieving 4 096 256-byte or 20KB payloads, PIRANA takes 0.16s, 28.4× faster

9. We left out Mughees-Ren for single-query because it is designed for multi-query and finding appropriate parameters for single-query is difficult.

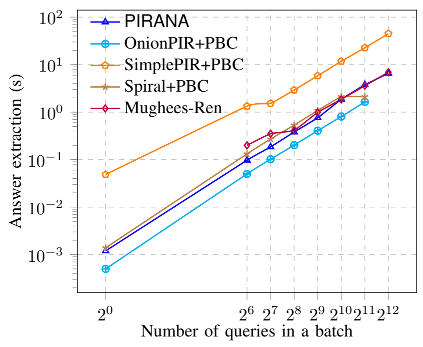
10. When the payload size is small, OnionPIR and Spiral will concatenate multiple payloads into one to improve performance. Therefore, when $B = 1.5L$ is large, there will not be enough elements in each bucket.



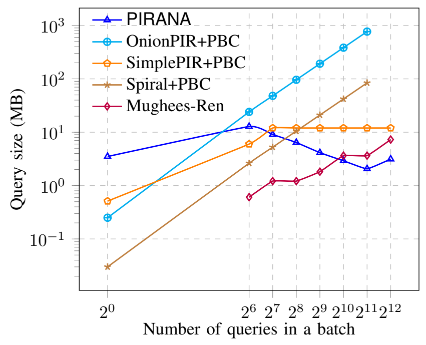
(a) Query generation time.



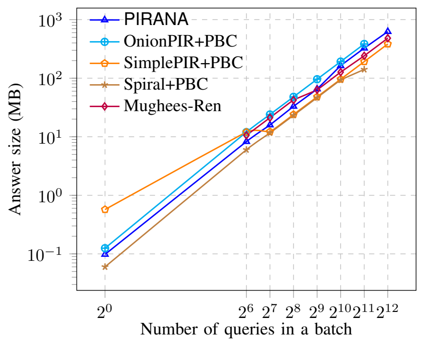
(b) Answer generation time.



(c) Answer extraction time.



(d) Query size.



(e) Answer size.

Figure 4: Evaluation of PIRs (2^{20} elements, 20KB payloads).

than SimplePIR.

- To extract 4 096 256-byte payloads, PIRANA takes 81.7ms, 11× faster than SimplePIR.
- To extract 4 096 20KB payloads, PIRANA takes 6.6s, 6.8× faster than SimplePIR.

Table 4 provides a detailed comparison between PIRANA and Mughees-Ren. It is evident that PIRANA significantly outperforms Mughees-Ren in answer generation, achieving 14.4× speedup for retrieving 4 096 20KB payloads (notice that the answer generation time dominates the overall latency). This is somehow consistent with our theoretical analysis in Section 4.3. In other aspects, they are evenly matched.

6.4. Comparison with the state-of-the-art LPSI

We evaluate PIRANA in terms of labeled PSI and compare it with the state-of-the-art LPSI[11] [18]. We took the OPRF implementation directly from the source code of [18]: it uses OMGDH-based OPRF, with the Elligator 2 map [6] for the FourQ elliptic curve [21].

The setup phase of [18] is prohibitively slow due to the expensive polynomial interpolation: 1 256.3s with 32 threads and 14.4 hours with a single thread. On the other hand, the most time-consuming part in the setup phase of LPSI-PIRANA is encoding the payloads into NTT forms, which takes only 157s with a single thread, 331× faster than [18]. Therefore, LPSI-PIRANA is more friendly to the scenarios where the database updates frequently.

Cong et al. [18] introduce several algorithmic optimizations for the polynomial evaluation. For example, they split each bucket into subsets and evaluate a polynomial for each subset separately, which reduces the number of ciphertext-ciphertext multiplications at the cost of increasing the number of ciphertexts to be returned. They also use the windowing technique to reduce the multiplicative depth, but it requires $\mathcal{C}$ to send more monomials to $\mathcal{S}$. All these optimizations require fine-tuning the parameters to find the best trade-off between computation and communication, which has been done in the source code of [18]. We simply follow their parameters, which leads to lower performance for us: 3.72× slower in runtime and 2.6× larger in bandwidth, for 1 024 queries. We leave it as future work to optimize our performance for LPSI-PIRANA.

7. Related Work

Early single-server PIR. Most of the early single-server PIR protocols follow the blueprint of Kushilevitz and Ostrovsky [33]: representing the database as a D-dimensional hypercube. The original protocol proposed by Kushilevitz and Ostrovsky is based on additively homomorphic encryption, with a query size of $O(\sqrt{N}\log N)$ and a re-

11. https://github.com/microsoft/APSI/

	# queries L	Mughees-Ren [41] 2^6	2^8	2^{10}	2^{12}	PIRANA 2^6	2^8	2^{10}	2^{12}
2^{20} elements 256-byte payloads	Query gen. (s)	0.01	0.02	0.06	0.13	0.12	0.12	0.12	0.16
	Answer gen. (s)	28.9	50.4	61.8	112	10.6	10.6	10.6	12.1
	Answer extr. (ms)	1.6	3.8	10.5	30.3	38.1	38.1	38.1	81.7
	Query size (MB)	0.6	1.2	3.6	7.3	1.9	1.9	1.9	3.1
	Answer size (MB)	0.16	0.55	1.6	6.3	3.9	3.9	3.9	7.8
2^{20} elements 20KB payloads	Query gen. (s)	0.01	0.02	0.07	0.13	0.37	0.22	0.15	0.16
	Answer gen. (s)	2 276	3 979	4 871	8 401	648	498	558	583
	Answer extr. (s)	0.20	0.41	1.9	6.9	0.10	0.38	1.9	6.6
	Query size (MB)	0.60	1.2	3.6	7.3	12.8	6.4	2.9	3.1
	Answer size (MB)	10.6	42.3	127	482	8.3	33.1	162	625

TABLE 4: Comparison between PIRANA and Mughees-Ren [41].

sponse size of $O(\sqrt{N})$. Cachin et al. [11] instead use the ϕ-Hiding assumption to achieve $O(\log^4 N)$ query size and $O(\log^D N)$ response size. Gentry and Ramazan [28] further reduce the query size of Cachin et al.'s approach to $O(\log^{3-o(1)} N)$. Chang [12] instantiates Kushilevitz-Ostrovsky's approach with Paillier homomorphic encryption to achieve $O(\sqrt{N}\log N)$ query size and $O(\log N)$ response size. This protocol was later generalized by Lipmaa [36] with Damgard-Jurik encryption [22] to achieve $O(\log^2 N)$ query size and $O(\log N)$ response size. As it has been pointed by Sion and Carbunar [47], such protocols are even slower than downloading the entire database.

Expansion-based PIR. SealPIR [4] improves the hypercube-based PIR by introducing an oblivious expansion technique. In more detail, (1) $\mathcal{C}$ sends $\mathcal{S}$ a ciphertext that homomorphically encrypts its desired index i; (2) $\mathcal{S}$ obliviously expands it into a selection vector of n ciphertexts where the i-th ciphertext encrypts 1 and others encrypt 0; (3) $\mathcal{S}$ returns the inner product between the selection vector and the payloads. However, in this way, $\mathcal{S}$ needs to compute the inner products between selection vectors and *encrypted* payloads from the second dimension and on. To avoid the expensive ciphertext-ciphertext multiplications, SealPIR [4] treat the encrypted payloads as "plaintexts", and run plaintext-ciphertext multiplications instead. This technique trades one ciphertext-ciphertext multiplication to multiple plaintext-ciphertext multiplications, leading to a large expansion factor for the responses. OnionPIR [42] realizes ciphertext-ciphertext multiplication via *external product* [16], which reduces the response sizes but incurs a large computational overhead. Spiral [39] further improves this idea by composing Regev encryption [45] with GSW encryption [29] to achieve a faster external product. MulPIR [3] uses alternative ways to reduce the communication of SealPIR. Namely, it uses symmetric key FHE to reduce the upload size and uses modulus switching to reduce the expansion factor. It also introduces a new query expansion scheme to halve the upload size for some specific parameter sets. However, as shown by the benchmarks in [37], MulPIR requires a large server runtime for answer generation.

SIMD-based PIR. FastPIR [2] uses the SIMD technique the batch the one-hot encoding of the index, which is somehow similar to single-query PIRANA. However, compared to FastPIR, our solution is superior in three aspects: (1) we use constant-weight code to encode the column index, which significantly reduces the query size; (2) we propose a customized rotation technique for large payloads; (3) we use the empty slots to batch more queries. Another recent SIMD-based PIR is from Mughees-Ren [41]. We refer to Section 4.3 for a detailed comparison.

PIR with preprocessing. Beimel et al. [5] propose the notion of PIR with preprocessing, in which the database is processed in an encoded form. However, the scheme proposed by Beimel et al. is only applicable to multi-server PIR. Patel et al. [44] propose a single-server solution, where $\mathcal{C}$ retrieves some helper data during preprocessing and uses them to run online queries in sublinear time. The computation cost of preprocessing is still linear but they are mostly symmetric-key operations. However, the preprocessing stage requires linear communication, which is less desirable. In a recent breakthrough [20], Corrigan-Gibbs and Kogan propose a PIR that has sublinear time in both online and preprocessing, but it requires running black-box PIR for multiple times during preprocessing. This idea was further explored in [19], [46], but none of these protocols has an overall efficiency that is better than PIR without preprocessing. Another line of work in PIR with preprocessing is by Hengzinger et al. [1] and Davidson et al. [23]. The key observation is that in a LWE-based PIR, the bulk of $\mathcal{S}$'s computation is independent of the $\mathcal{C}$s' queries and can be preprocessed. Compared with [20], [19], [46], this preprocessing only needs to be done once and can be used for all queries. However, each $\mathcal{C}$ still needs to download and maintain a large state, clearly unfriendly to the scenarios where the database updates frequently.

8. Conclusion

In this paper, we propose a novel PIR protocol named PIRANA, based on the recent advances in constant-weight codes. It is significantly faster than the original constant-weight PIR. It allows a client to retrieve a batch of elements from the server with a very small extra-cost in both communication and computation, compared with retrieving a single element. We also discuss a way to extend PIRANA to labeled private set intersection (LPSI). Compared to existing

LPSI protocols, PIRANA is more friendly to the scenarios where the database updates frequently.

References

[1] One server for the price of two: Simple and fast Single-Server private information retrieval. In *32nd USENIX Security Symposium (USENIX Security 23)*, Anaheim, CA, August 2023. USENIX Association.

[2] Ishtiyaque Ahmad, Yuntian Yang, Divyakant Agrawal, Amr El Abbadi, and Trinabh Gupta. Addra: Metadata-private voice communication over fully untrusted infrastructure. In Angela Demke Brown and Jay R. Lorch, editors, *15th USENIX Symposium on Operating Systems Design and Implementation, OSDI 2021, July 14-16, 2021*. USENIX Association, 2021.

[3] Asra Ali, Tancrède Lepoint, Sarvar Patel, Mariana Raykova, Phillipp Schoppmann, Karn Seth, and Kevin Yeo. Communication-computation trade-offs in PIR. In Michael Bailey and Rachel Greenstadt, editors, *30th USENIX Security Symposium, USENIX Security 2021, August 11-13, 2021*, pages 1811–1828. USENIX Association, 2021.

[4] Sebastian Angel, Hao Chen, Kim Laine, and Srinath T. V. Setty. PIR with compressed queries and amortized query processing. In *2018 IEEE Symposium on Security and Privacy, SP 2018, Proceedings, 21-23 May 2018, San Francisco, California, USA*, pages 962–979. IEEE Computer Society, 2018.

[5] Amos Beimel, Yuval Ishai, and Tal Malkin. Reducing the servers computation in private information retrieval: PIR with preprocessing. In Mihir Bellare, editor, *Advances in Cryptology - CRYPTO 2000, 20th Annual International Cryptology Conference, Santa Barbara, California, USA, August 20-24, 2000, Proceedings*, volume 1880 of *Lecture Notes in Computer Science*, pages 55–73. Springer, 2000.

[6] Daniel J. Bernstein, Mike Hamburg, Anna Krasnova, and Tanja Lange. Elligator: elliptic-curve points indistinguishable from uniform random strings. In Ahmad-Reza Sadeghi, Virgil D. Gligor, and Moti Yung, editors, *2013 ACM SIGSAC Conference on Computer and Communications Security, CCS'13, Berlin, Germany, November 4-8, 2013*, pages 967–980. ACM, 2013.

[7] Zvika Brakerski. Fully homomorphic encryption without modulus switching from classical gapsvp. *IACR Cryptol. ePrint Arch.*, page 78, 2012.

[8] Zvika Brakerski, Craig Gentry, and Vinod Vaikuntanathan. (leveled) fully homomorphic encryption without bootstrapping. *ACM Trans. Comput. Theory*, 6(3):13:1–13:36, 2014.

[9] Zvika Brakerski and Vinod Vaikuntanathan. Fully homomorphic encryption from ring-lwe and security for key dependent messages. In Phillip Rogaway, editor, *Advances in Cryptology - CRYPTO 2011 - 31st Annual Cryptology Conference, Santa Barbara, CA, USA, August 14-18, 2011. Proceedings*, volume 6841 of *Lecture Notes in Computer Science*, pages 505–524. Springer, 2011.

[10] Zvika Brakerski and Vinod Vaikuntanathan. Efficient fully homomorphic encryption from (standard) lwe. *SIAM Journal on computing*, 43(2):831–871, 2014.

[11] Christian Cachin, Silvio Micali, and Markus Stadler. Computationally private information retrieval with polylogarithmic communication. In Jacques Stern, editor, *Advances in Cryptology - EUROCRYPT '99, International Conference on the Theory and Application of Cryptographic Techniques, Prague, Czech Republic, May 2-6, 1999, Proceeding*, volume 1592 of *Lecture Notes in Computer Science*, pages 402–414. Springer, 1999.

[12] Yan-Cheng Chang. Single database private information retrieval with logarithmic communication. In Huaxiong Wang, Josef Pieprzyk, and Vijay Varadharajan, editors, *Information Security and Privacy: 9th Australasian Conference, ACISP 2004, Sydney, Australia, July 13-15, 2004. Proceedings*, volume 3108 of *Lecture Notes in Computer Science*, pages 50–61. Springer, 2004.

[13] Hao Chen, Zhicong Huang, Kim Laine, and Peter Rindal. Labeled PSI from fully homomorphic encryption with malicious security. In David Lie, Mohammad Mannan, Michael Backes, and XiaoFeng Wang, editors, *Proceedings of the 2018 ACM SIGSAC Conference on Computer and Communications Security, CCS 2018, Toronto, ON, Canada, October 15-19, 2018*, pages 1223–1237. ACM, 2018.

[14] Hao Chen, Kim Laine, and Peter Rindal. Fast private set intersection from homomorphic encryption. In Bhavani Thuraisingham, David Evans, Tal Malkin, and Dongyan Xu, editors, *Proceedings of the 2017 ACM SIGSAC Conference on Computer and Communications Security, CCS 2017, Dallas, TX, USA, October 30 - November 03, 2017*, pages 1243–1255. ACM, 2017.

[15] Jung Hee Cheon, Andrey Kim, Miran Kim, and Yong Soo Song. Homomorphic encryption for arithmetic of approximate numbers. In Tsuyoshi Takagi and Thomas Peyrin, editors, *Advances in Cryptology - ASIACRYPT 2017 - 23rd International Conference on the Theory and Applications of Cryptology and Information Security, Hong Kong, China, December 3-7, 2017, Proceedings, Part I*, volume 10624 of *Lecture Notes in Computer Science*, pages 409–437. Springer, 2017.

[16] Ilaria Chillotti, Nicolas Gama, Mariya Georgieva, and Malika Izabachène. TFHE: fast fully homomorphic encryption over the torus. *J. Cryptol.*, 33(1):34–91, 2020.

[17] Benny Chor, Oded Goldreich, Eyal Kushilevitz, and Madhu Sudan. Private information retrieval. In *36th Annual Symposium on Foundations of Computer Science, Milwaukee, Wisconsin, USA, 23-25 October 1995*, pages 41–50. IEEE Computer Society, 1995.

[18] Kelong Cong, Radames Cruz Moreno, Mariana Botelho da Gama, Wei Dai, Ilia Iliashenko, Kim Laine, and Michael Rosenberg. Labeled PSI from homomorphic encryption with reduced computation and communication. In Yongdae Kim, Jong Kim, Giovanni Vigna, and Elaine Shi, editors, *CCS '21: 2021 ACM SIGSAC Conference on Computer and Communications Security, Virtual Event, Republic of Korea, November 15 - 19, 2021*, pages 1135–1150. ACM, 2021.

[19] Henry Corrigan-Gibbs, Alexandra Henzinger, and Dmitry Kogan. Single-server private information retrieval with sublinear amortized time. In Orr Dunkelman and Stefan Dziembowski, editors, *Advances in Cryptology - EUROCRYPT 2022 - 41st Annual International Conference on the Theory and Applications of Cryptographic Techniques, Trondheim, Norway, May 30 - June 3, 2022, Proceedings, Part II*, volume 13276 of *Lecture Notes in Computer Science*, pages 3–33. Springer, 2022.

[20] Henry Corrigan-Gibbs and Dmitry Kogan. Private information retrieval with sublinear online time. In Anne Canteaut and Yuval Ishai, editors, *Advances in Cryptology - EUROCRYPT 2020 - 39th Annual International Conference on the Theory and Applications of Cryptographic Techniques, Zagreb, Croatia, May 10-14, 2020, Proceedings, Part I*, volume 12105 of *Lecture Notes in Computer Science*, pages 44–75. Springer, 2020.

[21] Craig Costello and Patrick Longa. Fourq: four-dimensional decompositions on a q-curve over the mersenne prime. Cryptology ePrint Archive, Paper 2015/565, 2015. https://eprint.iacr.org/2015/565.

[22] Ivan Damgård, Mads Jurik, and Jesper Buus Nielsen. A generalization of paillier's public-key system with applications to electronic voting. *Int. J. Inf. Sec.*, 9(6):371–385, 2010.

[23] Alex Davidson, Gonçalo Pestana, and Sofía Celi. Frodopir: Simple, scalable, single-server private information retrieval. *IACR Cryptol. ePrint Arch.*, page 981, 2022.

[24] Daniel Demmler, Peter Rindal, Mike Rosulek, and Ni Trieu. PIR-PSI: scaling private contact discovery. *Proc. Priv. Enhancing Technol.*, 2018(4):159–178, 2018.

[25] Junfeng Fan and Frederik Vercauteren. Somewhat practical fully homomorphic encryption. *IACR Cryptol. ePrint Arch.*, page 144, 2012.

[26] Michael J. Freedman, Yuval Ishai, Benny Pinkas, and Omer Reingold. Keyword search and oblivious pseudorandom functions. In Joe Kilian, editor, *Theory of Cryptography, Second Theory of Cryptography Conference, TCC 2005, Cambridge, MA, USA, February 10-12, 2005, Proceedings*, volume 3378 of *Lecture Notes in Computer Science*, pages 303–324. Springer, 2005.

[27] Craig Gentry. *A Fully Homomorphic Encryption Scheme*. PhD thesis, Stanford, CA, USA, 2009. AAI3382729.

[28] Craig Gentry and Zulfikar Ramzan. Single-database private information retrieval with constant communication rate. In Luís Caires, Giuseppe F. Italiano, Luís Monteiro, Catuscia Palamidessi, and Moti Yung, editors, *Automata, Languages and Programming, 32nd International Colloquium, ICALP 2005, Lisbon, Portugal, July 11-15, 2005, Proceedings*, volume 3580 of *Lecture Notes in Computer Science*, pages 803–815. Springer, 2005.

[29] Craig Gentry, Amit Sahai, and Brent Waters. Homomorphic encryption from learning with errors: Conceptually-simpler, asymptotically-faster, attribute-based. In Ran Canetti and Juan A. Garay, editors, *Advances in Cryptology - CRYPTO 2013 - 33rd Annual Cryptology Conference, Santa Barbara, CA, USA, August 18-22, 2013. Proceedings, Part I*, volume 8042 of *Lecture Notes in Computer Science*, pages 75–92. Springer, 2013.

[30] Yuval Ishai, Eyal Kushilevitz, Rafail Ostrovsky, and Amit Sahai. Batch codes and their applications. In László Babai, editor, *Proceedings of the 36th Annual ACM Symposium on Theory of Computing, Chicago, IL, USA, June 13-16, 2004*, pages 262–271. ACM, 2004.

[31] Dmitry Kogan and Henry Corrigan-Gibbs. Private blocklist lookups with checklist. In *30th USENIX Security Symposium (USENIX Security 21)*, pages 875–892. USENIX Association, August 2021.

[32] Dmitry Kogan and Henry Corrigan-Gibbs. Private blocklist lookups with checklist. In *30th USENIX Security Symposium (USENIX Security 21)*, pages 875–892. USENIX Association, August 2021.

[33] E. Kushilevitz and R. Ostrovsky. Replication is not needed: Single database, computationally-private information retrieval. In *Proceedings of the 38th Annual Symposium on Foundations of Computer Science*, FOCS '97, page 364, USA, 1997. IEEE Computer Society.

[34] Eyal Kushilevitz and Rafail Ostrovsky. Replication is NOT needed: SINGLE database, computationally-private information retrieval. In *38th Annual Symposium on Foundations of Computer Science, FOCS '97, Miami Beach, Florida, USA, October 19-22, 1997*, pages 364–373. IEEE Computer Society, 1997.

[35] Albert Kwon, David Lazar, Srinivas Devadas, and Bryan Ford. Riffle: An efficient communication system with strong anonymity. *Proc. Priv. Enhancing Technol.*, 2016(2):115–134, 2016.

[36] Helger Lipmaa. An oblivious transfer protocol with log-squared communication. In Jianying Zhou, Javier López, Robert H. Deng, and Feng Bao, editors, *Information Security, 8th International Conference, ISC 2005, Singapore, September 20-23, 2005, Proceedings*, volume 3650 of *Lecture Notes in Computer Science*, pages 314–328. Springer, 2005.

[37] Rasoul Akhavan Mahdavi and Florian Kerschbaum. Constant-weight PIR: Single-round keyword PIR via constant-weight equality operators. In *31st USENIX Security Symposium (USENIX Security 22)*, pages 1723–1740, Boston, MA, August 2022. USENIX Association.

[38] Carlos Aguilar Melchor, Joris Barrier, Laurent Fousse, and Marc-Olivier Killijian. XPIR : Private information retrieval for everyone. *Proc. Priv. Enhancing Technol.*, 2016(2):155–174, 2016.

[39] Samir Jordan Menon and David J. Wu. SPIRAL: fast, high-rate single-server PIR via FHE composition. In *43rd IEEE Symposium on Security and Privacy, SP 2022, San Francisco, CA, USA, May 22-26, 2022*, pages 930–947. IEEE, 2022.

[40] Prateek Mittal, Femi Olumofin, Carmela Troncoso, Nikita Borisov, and Ian Goldberg. PIR-Tor: Scalable anonymous communication using private information retrieval. In *20th USENIX Security Symposium (USENIX Security 11)*, San Francisco, CA, August 2011. USENIX Association.

[41] M. Mughees and L. Ren. Vectorized batch private information retrieval. In *2023 2023 IEEE Symposium on Security and Privacy (SP) (SP)*, pages 1812–1827, Los Alamitos, CA, USA, may 2023. IEEE Computer Society.

[42] Muhammad Haris Mughees, Hao Chen, and Ling Ren. Onionpir: Response efficient single-server PIR. In Yongdae Kim, Jong Kim, Giovanni Vigna, and Elaine Shi, editors, *CCS '21: 2021 ACM SIGSAC Conference on Computer and Communications Security, Virtual Event, Republic of Korea, November 15 - 19, 2021*, pages 2292–2306. ACM, 2021.

[43] Muhammad Haris Mughees, Gonçalo Pestana, Alex Davidson, and Benjamin Livshits. Privatefetch: Scalable catalog delivery in privacy-preserving advertising. *arXiv preprint arXiv:2109.08189*, 2021.

[44] Sarvar Patel, Giuseppe Persiano, and Kevin Yeo. Private stateful information retrieval. In David Lie, Mohammad Mannan, Michael Backes, and XiaoFeng Wang, editors, *Proceedings of the 2018 ACM SIGSAC Conference on Computer and Communications Security, CCS 2018, Toronto, ON, Canada, October 15-19, 2018*, pages 1002–1019. ACM, 2018.

[45] Oded Regev. On lattices, learning with errors, random linear codes, and cryptography. In Harold N. Gabow and Ronald Fagin, editors, *Proceedings of the 37th Annual ACM Symposium on Theory of Computing, Baltimore, MD, USA, May 22-24, 2005*, pages 84–93. ACM, 2005.

[46] Elaine Shi, Waqar Aqeel, Balakrishnan Chandrasekaran, and Bruce M. Maggs. Puncturable pseudorandom sets and private information retrieval with near-optimal online bandwidth and time. In Tal Malkin and Chris Peikert, editors, *Advances in Cryptology - CRYPTO 2021 - 41st Annual International Cryptology Conference, CRYPTO 2021, Virtual Event, August 16-20, 2021, Proceedings, Part IV*, volume 12828 of *Lecture Notes in Computer Science*, pages 641–669. Springer, 2021.

[47] Radu Sion and Bogdan Carbunar. On the practicality of private information retrieval. In *Proceedings of the Network and Distributed System Security Symposium, NDSS 2007, San Diego, California, USA, 28th February - 2nd March 2007*. The Internet Society, 2007.

[48] Ni Trieu, Kareem Shehata, Prateek Saxena, Reza Shokri, and Dawn Song. Epione: Lightweight contact tracing with strong privacy. *IEEE Data Eng. Bull.*, 43(2):95–107, 2020.

[49] David J. Wu, Joe Zimmerman, Jérémy Planul, and John C. Mitchell. Privacy-preserving shortest path computation. In *23rd Annual Network and Distributed System Security Symposium, NDSS 2016, San Diego, California, USA, February 21-24, 2016*. The Internet Society, 2016.

Appendix

1. Further optimizations for PIRANA

To be complete, we describe some optimizations used in PIRANA. We remark that these optimizations were proposed by prior work.

Using secret-key homomorphic encryption to reduce query size. Notice that in the scenario of PIR, only $\mathcal{C}$ needs to do encryption but no one else. To this end, we could use a secret-key version of homomorphic encryption instead of the public-key version. Given that the ciphertext of homomorphic encryption is a tuple (c_0, c_1) in $\mathbb{Z}_q^2[x]$, if we use the secret-key version, c_0 is sampled uniformly at random in $\mathbb{Z}_q[x]$ (whereas in the public-key version, it depends on the public key). Therefore, instead of sending c_0, $\mathcal{C}$ can send the seed that has been used to generates c_0, and $\mathcal{S}$ can reconstruct c_0 from the seed. This reduces the query size by a factor $2\times$.

Using modulus switching to reduce answer size. Given that the returned ciphertexts will no longer be used for further computations, we could use the *modulus switching* technique [10] to reduce their sizes. Modulus switching is a public operation that can reduce the ciphertext modulus from q to q', without affecting the plaintext. Since the size of a FHE ciphertext is linear in $\log q$, modulus switching reduces it by a factor of $\frac{\log q}{\log q'}$. Therefore, this trick allows $\mathcal{S}$ to reduce the response size the the same factor.