# Endemic Oblivious Transfer via Random Oracles, Revisited*

Zhelei Zhou† Bingsheng Zhang‡ Hong-Sheng Zhou§ Kui Ren¶

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**Abstract**

The notion of Endemic Oblivious Transfer (EOT) was introduced by Masny and Rindal (CCS'19). EOT offers a weaker security guarantee than the conventional random OT; namely, the malicious parties can fix their outputs arbitrarily. The authors presented a 1-round UC-secure EOT protocol under a tailor-made and non-standard assumption, Choose-and-Open DDH, in the RO model.

In this work, we systematically study EOT in the UC/GUC framework. We present the first UC-secure 1-round EOT protocol in the RO model under the DDH assumption in both the static and the adaptive security setting. Under the GUC framework, we propose the first 1-round EOT construction under the CDH assumption in the Global Restricted Observable RO (GroRO) model proposed by Canetti *et al.* (CCS'14). We also provide an impossibility result, showing there exists *no* 1-round GUC-secure EOT protocols in the Global Restricted Programmable RO (GrpRO) model proposed by Camenisch *et al.* (Eurocrypt'18). Subsequently, we provide the first round-optimal (2-round) EOT protocol with adaptive security under the DDH assumption in the GrpRO model. Finally, we investigate the relations between EOT and other cryptographic primitives.

As side products, we present the first 2-round GUC-secure commitment in the GroRO model as well as a separation between the GroRO and the GrpRO model, which may be of independent interest.

*Corresponding authors: Bingsheng Zhang *bingsheng@zju.edu.cn*, and Hong-Sheng Zhou *hszhou@vcu.edu*.

†Zhejiang University, and ZJU-Hangzhou Global Scientific and Technological Innovation Center.

‡Zhejiang University, and ZJU-Hangzhou Global Scientific and Technological Innovation Center.

§Virginia Commonwealth University.

¶Zhejiang University, and ZJU-Hangzhou Global Scientific and Technological Innovation Center.

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# 1 Introduction

The security of a cryptographic protocol is typically analyzed under the simulation paradigm [GMW87], where the "formal specification" of the security requirements is modeled as an ideal process, and a real-world protocol is said to securely realize the specification if it "emulates" the ideal process. In the past decades, many variants were proposed: Initially, protocol security was considered in the *standalone* setting, in the sense that the challenged protocol is executed in isolation. Later, *Universal Composibility* (UC) [Can01] was introduced to analyze protocol security in arbitrary execution environments; in particular, multiple protocol sessions may be executed concurrently in an adversarially coordinated way. Note that protocols in the UC framework must be *subroutine respecting*, in the sense that all the underlying subroutines are only created for the challenged protocol instance and cannot be directly accessed by any other protocols or even the other instances of the same protocol. To address this drawback, Canetti *et al.* [CDPW07] proposed the Generalized Universal Composibility (GUC) framework.

**Endemic Oblivious Transfer.** The notion of *Endemic Oblivious Transfer* (EOT) was introduced by Masny and Rindal [MR19a] as a weaker version of Random OT (ROT). In an EOT protocol, the sender has no input, and the receiver inputs a choice bit $b \in \{0, 1\}$; at the end of EOT, the sender outputs two random elements $(m_0, m_1)$, and the receiver outputs $m_b$. Although EOT looks similar to the conventional ROT, EOT offers a weaker security guarantee — the malicious sender can fix its output $(m_0, m_1)$ arbitrarily, and the malicious receiver can fix its output $m_b$ arbitrarily. The first 1-round[1] (a.k.a. non-interactive) EOT/ROT protocol was proposed by Bellare and Micali [BM90]. It achieves standalone security against semi-honest adversaries under the DDH assumption in the Common Reference String (CRS) model. The scheme can also be transformed to achieve malicious security using the Groth-Sahai proof [GS17]. Later, Garg *et al.* proposed several 1-round UC-secure EOT protocols under the well-understood assumptions, (e.g., Decisional Diffie-Hellman (DDH), Quadratic Residuosity (QR) and Learning With Errors (LWE)), in the CRS model [GIS18]. Recently, Masny and Rindal [MR19a] demonstrated a generic construction for 1-round EOT by using any non-interactive key exchange scheme in the Random Oracle (RO) model; however, their generic construction only achieves standalone security. Masny and Rindal [MR19a, MR19b] then provided a 1-round *UC-secure* EOT protocol but under a tailor-made computational assumption called "*Choose-and-Open DDH* (CODDH)", in the RO model. We remark that, different from the DDH, the CODDH is a new assumption, and its hardness is yet to be further studied.

**(Global) Random Oracle Models.** Random oracle (RO) model [BR93] is a popular idealized setup model that has been widely used to justify the security of efficient cryptographic protocols. In spite of its known inability to provide provable guarantees when RO is instantiated with a real-world hash function [CGH98], RO is still a promising setup since it is generally accepted that security analysis in the RO model does provide strong evidence to the resilience of the protocol in question to practical attacks [CJS14]. In fact, RO model draws increasing attention along with recent advancement of the blockchain technology. It is generally viewed as a *transparent* setup that can be easily deployed with no reliance on any trusted party in the distributed system setting.

*Local RO Model vs. Global RO Model.* The "local" RO model is often used in the UC framework where the simulator is allowed to simulate it in the ideal world, and it grants the simulator two advantages: (i) observability: the simulator can see what values the parties query the RO on; (ii) programability: the simulator can program RO query responses as long as they "look" indistinguishable from the real ones. In the GUC framework, a "global" RO is external to the simulator; to facilitate simulation, some "extra

[1] In this work, we consider the simultaneous communication model with a rushing adversary, where both parties can send messages to each other within the same round. The rushing adversary can delay sending messages on behalf of corrupted parties in a given round until the messages sent by all the uncorrupted parties in that round have been received. Note that this is different from the simultaneous messaging requirement in [Kat07], which deals with a non-rushing adversary.

power" needs to be granted to the simulator. In the literature, two main strengthened variants of global RO model were proposed: global RO with restricted observability (GroRO) model proposed by Canetti *et al.* [CJS14] and global RO with restricted programmability (GrpRO) model proposed by Camenisch *et al.* [CDG+18]. Here, the restricted observability and programmability stand for the "extra power" that the simulator has but the adversary does not have.

### 1.1 Problem Statement

**Constructing EOT in (Global) RO Models.** As mentioned above, it is known that one can build a 1-round UC-secure EOT protocol under the well-known assumptions in the CRS model [GIS18]; however, it is still an open problem on whether it is possible to achieve a 1-round UC-secure EOT protocol under well-understood assumptions, e.g. DDH, in the RO model. Note that recent work by Masny and Rindal [MR19a, MR19b] was based on the tailor-made CODDH assumption without further investigation.

We emphasize that building a 1-round UC-secure EOT protocol in the RO model is essentially more difficult than that in the CRS model. In the CRS model, both parties can utilize the common cryptographic components, i.e., the CRS, to generate the correlated messages. In other words, *the CRS can be viewed as an extra communication move* during the protocol execution. However, the situation is different in the RO model. The (global) RO is a transparent model which is typically instantiated with a predefined hash function, and it contains "no secret"; there is no such common structural elements to be used as a coordination in the (global) RO model, and thus the messages in the only simultaneous round from both parties are somewhat independent. This leads us to a natural research question:

> *In the UC framework, does there exist a 1-round (a.k.a. non-interactive) EOT protocol under well-understood assumptions in the RO model?*

New techniques are needed to answer the above question. In addition, if there exists such a UC-secure EOT protocol in the (local) RO model, we are also wondering what is the situation when less idealized setups (i.e., the GroRO and the GrpRO model) are used under the GUC framework. Note that global RO setup often provides less advantages to the simulator, which increases construction difficulty:

> *In the GUC framework, does there exist a 1-round (a.k.a. non-interactive) EOT protocol under well-understood assumptions in the GroRO/GrpRO model?*

**Understanding the Complexity of EOT.** In addition to the concrete protocol constructions, we are also interested in understanding the complexity, including the power and the limits, of the cryptographic task of EOT. More precisely, what are the relations between EOT and the other well-known secure computation tasks? For example, is EOT fundamentally different from ROT or (1-out-of-2) OT? In [MR19a], Masny and Rindal have already initialized the investigation of this interesting problem: They proposed a new OT notion called Uniform OT (UOT) which also looks similar to the conventional ROT, except that it offers a strong security guarantee that no adversary can bias the distribution of the ROT outputs. They showed that it is possible to build UOT based on EOT and a coin-tossing protocol; however, it is unclear if a coin-tossing protocol can be built from an EOT protocol. We thus ask the following question:

> *What is the relation between the EOT and other cryptographic primitives (such as coin-tossing and UOT etc.)?*

**Understanding the Complexity of Global RO Models.** Finally, let us go back to the global setups we used in this work. Recall that, the GroRO and the GrpRO models provide different aspects of "extra power" to the simulator. Are these two different global RO models, essentially equivalent? Or one is strictly stronger than the other? It raises our last question:

*What is the relation between the GroRO model and the GrpRO model?*

Our goal is to provide a comprehensive and thorough investigation of constructing EOT via ROs. From a practical point of view, if the above questions could be answered, we would see highly efficient constructions for EOT. From a theoretical point of view, if (some of) the above questions could be answered, we would have a better understanding of the relation between EOT and many secure computation tasks; we could also have a better understanding of the power and limits of different global RO models.

## 1.2 Our Results

In this work, we investigate the above problems. Our results can be summarized as follows.

### 1.2.1 Constructing EOT in the (Global) Random Oracles

Table 1 depicts a selection of our new constructions, and our main results can be summarized as follows.

| Protocol | #Round | Security | Computational Assumption | Setup Assumption |
|---|---|---|---|---|
| [GIS18][a,b] | 1 | UC+Static | DDH | CRS |
| [MR19a, MR19b] | 1 | UC+Static | CODDH[c] | RO |
| [CSW20a] | 1 | UC+Adaptive | DDH | GrpRO+CRS |
| $\Pi_{\mathsf{sEOT\text{-}RO}}$(Sec. 3.1) | 1 | UC+Static | DDH | RO |
| $\Pi_{\mathsf{aEOT\text{-}RO}}$(Sec. 3.2) | 1 | UC+Adaptive | DDH | RO |
| $\Pi_{\mathsf{EOT\text{-}GroRO}}$(Sec. 5.1.1) | 1 | GUC+Static | CDH | GroRO |
| $\Pi_{\mathsf{EOT\text{-}GrpRO}}$(Sec. 5.2.2)[d] | 2 | GUC+Adaptive | DDH | GrpRO |

[a] It can be instantiated from other assumptions (e.g., LWE and QR assumptions), but here we mainly focus on the assumptions that related to cyclic groups.
[b] The work only proves static security formally, but the authors remarked that some of their protocols can achieve adaptive security.
[c] CODDH refers to Choose-and-Open DDH assumption which is not known to be reducible to DDH assumption.
[d] Round-optimal due to Theorem 10, below.

Table 1: Comparison with state-of-the-art round-optimal EOT protocols under computational assumptions that related to the cyclic groups.

Next, we provide the technical overview for our EOT protocol constructions in the (global) RO models. We first show how to construct a 1-round UC-secure EOT protocol under DDH assumption in the RO model against static adversaries, then we show how to improve it to achieve adaptive security. After that, we turn to the global RO model and show how to construct a 1-round GUC-secure EOT protocol under CDH assumption in the GroRO model. Note that, the situation in the GrpRO model is complicated: We find that there exists no 1-round GUC-secure EOT protocols in the GrpRO model even with static security, and we give a round-optimal (2-round) EOT protocol under DDH assumption against adaptive adversaries. We leave the discussion about the EOT protocols in the GrpRO model in Section 1.2.2.

**New technique: 1-round UC-secure EOT protocol in the RO model.** We present a new technique that enables the first UC-secure 1-round EOT protocol in the RO model under the DDH assumption (cf. Section 3.1). The basic scheme achieves static security. Intuitively, our technique is as follows. We start with the two-round standalone ROT/EOT protocol in the RO model proposed in [CO15]. In the 1st round, the sender sends $h := g^s$ to the receiver; in the 2nd round, the receiver uses sender's message

to compute $B := g^r h^b$ and sends $B$ back, where $b \in \{0,1\}$ is the choice bit; finally, the sender outputs $m_0 := \mathtt{Hash}(B^s)$ and $m_1 := \mathtt{Hash}((\frac{B}{h})^s)$, where $\mathtt{Hash}$ is a predefined hash function and it is modeled as a RO; the receiver outputs $m_b := \mathtt{Hash}(h^r)$. Although this protocol is simple and efficient, it cannot achieve UC security [LM18].

Our technique is presented as follows. The dependence of the sender's message in [CO15] can be eliminated such that the receiver's message can be produced simultaneously in the same round. The idea is to let the receiver produce the commitment key $h$ instead of waiting it from the sender. How to generate a random group element and be oblivious to its discrete logarithm? This can be achieved by setting $h := \mathtt{Hash}(\mathsf{seed})$, where $\mathsf{seed}$ is some randomly sampled string. Similar technique can be found in [CSW20a]. Now the 1-round (non-interactive) version of [CO15] roughly works as follows. The sender sends $z := g^s$ to the receiver; meanwhile, the receiver picks $h := \mathtt{Hash}(\mathsf{seed})$ and computes $B := g^r h^b$, and then it sends $(\mathsf{seed}, B)$ to the sender; finally, the sender computes $h := \mathtt{Hash}(\mathsf{seed})$ and outputs $m_0 := \mathtt{Hash}(B^s)$ and $m_1 := \mathtt{Hash}((\frac{B}{h})^s)$; the receiver outputs $m_b := \mathtt{Hash}(z^r)$.

Further, to make the protocol UC-secure, certain *extractability* is needed: (i) when the sender is malicious, the simulator should be able to extract the sender's private randomness $s$, so the simulator can compute both $m_0$ and $m_1$; (ii) when the receiver is malicious, the simulator should be able to extract the receiver's choice bit. In order to extract the sender's $s$, we let the sender additionally generate a RO-based straight-line extractable NIZK argument [Pas03, Fis05, Ks22]. In order to extract the receiver's choice bit $b$, we let the receiver computes the ElGamal encryption of $b$ instead of the Pedersen commitment. We then let the receiver additionally generate a NIZK argument to ensure the correctness of the ElGamal encryption. Note that, we do not need the straight-line extractability here, since the simulator can program the RO to obtain $\log_g h$ and thus be able to decrypt the ElGamal ciphertext to extract $b$.

**1-round UC-secure EOT protocol in the RO model with adaptive security.** The above EOT protocol can be transformed to achieve adaptive UC security under the DDH assumption (cf. Section 3.2). Unlike the static security where the adversary can only corrupt the parties at the beginning of the protocol execution, the adaptive adversaries can corrupt the parties at any time. Therefore, additional mechanisms need to be introduced to allow the simulator to handle the case where the sender/receiver gets corrupted after its message has been sent. In that case, the simulator needs to output the "fake" internal state that makes the simulated message as if it is the output of a real execution.

Now we describe our strategy of constructing UC-secure EOT with adaptive security. We observe that the sender's algorithm can remain the same as the one in the static-secure protocol, since the sender has no input, and the simulator can simply run the sender's algorithm to simulate the honest sender. Thus when the sender gets corrupted, the simulator simply reveals the internal states that used to generate the sender's message. Different from the sender, the receiver has a private input, i.e., the choice bit $b$. Therefore, we have to design a non-committing mechanism that the simulated receiver's message can be revealed to 0 or 1 as desired. Inspired by the techniques in [GOS06, CWS22], we take the following approaches. we let the receiver commit to the choice bit $b$ using a Pedersen commitment $B := g^r h^b$. The receiver then computes a lifted ElGamal encryption of the randomness $r$, i.e., $(u_b, v_b) := (h^t, g^t h^r)$ using a random $t \leftarrow \mathbb{Z}_q$. In addition, we let the receiver randomly select $(u_{1-b}, v_{1-b}) \leftarrow \mathbb{G} \times \mathbb{G}$, and generates an NIZK argument with Honest Prover State Reconstruction (HPSR) showing one of $(u_b, v_b)$ and $(u_{1-b}, v_{1-b})$ is a corrected encryption. In terms of the simulation, since the simulator knows $\log_g h$ (by simulating RO), it can open the Pedersen commitment $B$ to both 0 and 1, denoting the openings as $(0, r_0)$ and $(1, r_1)$, respectively. Then the simulator computes two ElGamal encryptions – $(u_0, v_0)$ as the encryption of $r_0$ and $(u_1, v_1)$ as the encryption of $r_1$. Since the ElGamal ciphertext looks pseudorandom from the random group elements, when the receiver gets corrupted, depending on the choice bit $b$, the simulator can "explain" the view and internal state of the receiver accordingly.

**1-round GUC-secure EOT protocol in the GroRO model.** Turning to the GUC setting, we propose the first 1-round EOT construction under the CDH assumption in the GroRO model (cf. Section 5.1.1). Compared to our UC-secure construction, this one requires *weaker* a computational assumption.

Recall that, in our UC-secure EOT protocol, we let the sender sends $z := g^s$ together with a straight-line extractable NIZK argument. The straight-line extractable NIZK argument gives the simulator the ability to extract $s$. However, Pass showed that it is impossible to construct NIZK arguments in observable RO model [Pas03], let alone NIZK arguments with straight-line extractability. The good news is that straight-line extractable NIWH argument is sufficient for our purpose, and it exists in the GroRO model [Pas03]. Therefore, we let the sender generate a straight-line extractable NIWH argument of $s$ such that $z = g^s$ instead. Next, to extract the receiver's choice bit, our UC-secure construction utilizes the programmability of RO; however, $\mathcal{G}_{\mathsf{roRO}}$ does not offer programability, so a different approach shall be taken. In particular, we let the receiver compute a Pedersen commitment to the choice bit $B := g^r h^b$, and generate a straight-line extractable NIWH argument of $(r, b)$ such that $B = g^r h^b$. Analogously to the sender side, the straight-line extractable NIWH argument gives the simulator extractability.

### 1.2.2 Understanding the Power/Limits of Different Global Random Oracles

**A separation between the GroRO model and the GrpRO model.** To show this separation, we first give a new impossibility result, showing that there exists *no* 1-round GUC-secure EOT protocol in the GrpRO model even with static security (cf. Section 5.2.1). By combining this negative result in the GrpRO model and the aforementioned positive result in the GroRO model, we demonstrate a separation between the GroRO model and the GrpRO model. More precisely, let $\mathcal{G}_{\mathsf{roRO}}, \mathcal{G}_{\mathsf{rpRO}}$ be the functionalities of the GroRO and the GrpRO model, we present the relation of these global RO models in Figure 1.

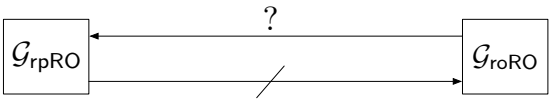



Figure 1: The relation between the $\mathcal{G}_{\mathsf{roRO}}$ model and the $\mathcal{G}_{\mathsf{rpRO}}$ model. Here, "A $\nrightarrow$ B" denotes that A does not imply B. In addition, "A $\stackrel{?}{\rightarrow}$ B" denotes that whether A implies B remains unknown.

**New impossibility and feasibility results in the GrpRO model.** Here we will present more details about the aforementioned impossibility result in the GrpRO model. Furthermore, to complete the picture, we also provide a round-optimal GUC-secure EOT protocol with adaptive security in the GrpRO model.

*New impossibility results in the GrpRO model.* The impossibility is proven by contradiction (cf. Section 5.2.1). Suppose that there exists such a 1-round GUC-secure EOT protocol in the GrpRO model. Let us first consider the case where the receiver is corrupted, and the simulator needs to extract the choice bit of the receiver from its message. Recall that, the GrpRO only grants the simulator the restricted programmability: the simulator can program the unqueried points without being detected. More importantly, unlike local RO, the simulator cannot program a global RO on the fly, as it cannot see which point is queried at this moment. Thus, the simulator needs to find a way to enforce the corrupt receiver to query the simulator's programmed points. However, in a one simultaneous round protocol, the messages between parties have no dependency. Hence the simulator cannot enforce the corrupt receiver to produce its message on the programmed points, and has no advantages. If the simulator still succeeds to extract the corrupted receiver's choice bit, then we have the following attack. The adversary corrupts the sender, and instructs the sender to run the simulator algorithm above to extract the choice bit from the message sent by the receiver/simulator. However, the simulator has no idea about the real choice bit, thus with 1/2 probability the simulation would fail.

Our impossibility result for EOT can be extended to the 1-out-of-2 OT. More precisely, we show that there exists *no* 2-round GUC-secure 1-out-of-2 OT protocols in the GrpRO model even with static security; note that, here we do not consider simultaneous rounds. Notice that Canetti *et al.* [CSW20a, CSW20b] and Byali *et al.* [BPRS17] all claimed that they proposed a 2-round 1-out-of-2 OT protocol in the GrpRO model. Recall that, in the security analysis under the GUC framework, both the ideal world simulator and the real world adversary are only allowed to query the global setup. However, in the security analysis against a malicious receiver in [CSW20b, Page 16, Figure 6] and [BPRS17, Page 19], the ideal world simulator must emulate the interface of RO functionality for the real world adversary to see which point is queried and answer to this query (UC style). Thus we remark that their protocols only achieves UC security rather than GUC security.

*New feasibility: Round-optimal GUC-secure EOT protocol in the GrpRO model.* To complete the picture, we also give a round-optimal (2-round) EOT protocol with adaptive security under the DDH assumption in the GrpRO model (cf. Section 5.2.2). Here, we don't consider simultaneous messaging in the same round. Our intuition comes from the UC-secure EOT protocol in the CRS+GrpRO model proposed by Canetti *et al.* [CSW20a]. In their protocol, the CRS consists of two group elements $g, h \in \mathbb{G}$, and the simulator knows $\log_g h$. The sender computes $z := g^r h^s$, while the receiver generates $(G, H) := \mathtt{Hash}(\mathsf{seed})$ and computes two Pedersen commitments to the choice bit using two sets of the parameter, i.e., $(g, G)$ and $(h, H)$, and the same randomness.

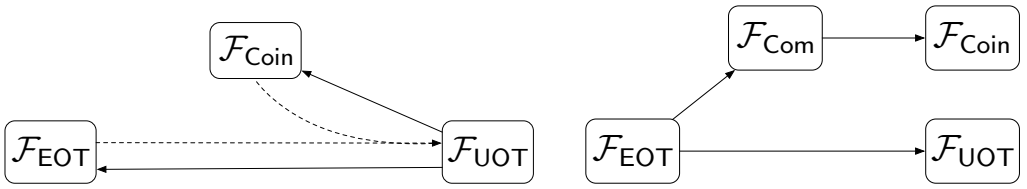
To eliminate the CRS, we let the sender generate the first set of the parameter $(g, h) := \mathtt{Hash}(\mathsf{seed}_1)$ where $\mathsf{seed}_1$ is an uniformly sampled string. At the same time, the sender computes $z := g^r h^s$ using random $r, s \leftarrow \mathbb{Z}_q$ and sends $\mathsf{seed}, z$ to the receiver in the first round. In the second round, the receiver first checks if $\mathsf{seed}_1$ is a programmed point. If not, the receiver generates the second set of the parameter $(G, H) := \mathtt{Hash}(\mathsf{seed}_2)$ where $\mathsf{seed}_2$ is an uniformly sampled string. Then the receiver can compute two Pedersen commitments to the choice bit, i.e., $(B_1, B_2) := (g^x G^b, h^x H^b)$ using random $x \leftarrow \mathbb{Z}_q$. Finally, we let the receiver send $(\mathsf{seed}_2, B_1, B_2)$ to the sender. How to make the protocol simulatable in the GrpRO model? We show the simulation strategy as follows: when the receiver is malicious (and the sender is honest), the simulator can extract the receiver's choice bit $b$ by programming the GrpRO (the simulator always succeeds to program the GrpRO since $\mathsf{seed}_1$ is sampled by the honest sender itself) and knowing $\alpha$ such that $h = g^\alpha$; when the sender is malicious (and the receiver is honest), the simulator can compute both $m_0$ and $m_1$ by programming the GrpRO (the simulator always succeeds to program the GrpRO since $\mathsf{seed}_2$ is sampled by the honest receiver itself) such that $(g, h, G, H)$ is a DDH tuple.

### 1.2.3 Understanding the Relation between EOT and Other Cryptographic Primitives

**EOT implies UOT and commitment.** In [MR19a], the authors showed that UOT implies EOT. But the work on the opposite direction is incomplete. Let $\mathcal{F}_{\mathsf{EOT}}$, $\mathcal{F}_{\mathsf{UOT}}$ and $\mathcal{F}_{\mathsf{Coin}}$ be the ideal functionalities of EOT, UOT and coin-tossing protocol, respectively. They showed that a UOT protocol can be constructed in the $\{\mathcal{F}_{\mathsf{EOT}}, \mathcal{F}_{\mathsf{Coin}}\}$-hybrid world with unconditional security, and they constructed $\mathcal{F}_{\mathsf{Coin}}$ via only $\mathcal{F}_{\mathsf{UOT}}$. However, it remains unclear whether $\mathcal{F}_{\mathsf{Coin}}$ can be constructed via only $\mathcal{F}_{\mathsf{EOT}}$; therefore, it is still an open question on whether EOT implies UOT? We present the relations that they claimed in Figure 2(a).

Recall that, Brzuska *et al.* proved that bit commitment can be constructed via 1-out-of-2 OT with unconditional security [BFSK11]. What about EOT? We first show there is a separation between the EOT and 1-out-of-2 OT (cf. Section 4.1), namely, there exists 1-round UC-secure EOT but no 1-round UC-secure 1-out-of-2 OT protocol, assuming synchronize communication. Nevertheless, surprisingly, we show that bit commitment can be constructed via a weaker primitive, i.e., EOT with unconditional security (cf. Section 4.2).

Our key observation is that the receiver's message can be viewed as the commitment to the choice bit $b$, and the locally computed message $m_b$ together with $b$ can be viewed as the opening. Typically, a commitment protocol requires both hiding and binding properties. The hiding property holds since




(a) The relations claimed in [MR19a] (b) The relations in this work.

Figure 2: The relations between EOT and other primitives. "A $\rightarrow$ B" denotes that A implies B. "A $\dashrightarrow$ B" denotes that A can be transformed into B.

the malicious receiver in the EOT cannot learn $m_{1-b}$, even if it can influence the distribution of $m_b$. The binding property holds since the malicious sender in the EOT cannot know which message is received by the receiver, even if it can influence the distributions of both $m_0$ and $m_1$.

Since it is well-known how to construct $\mathcal{F}_{\mathsf{Coin}}$ via only $\mathcal{F}_{\mathsf{Com}}$, where $\mathcal{F}_{\mathsf{Com}}$ is the commitment functionality, we show that EOT implies UOT and completes the relation between EOT and UOT (cf. Section 4.3). We present the relations that explored in this work in Figure 2(b).

Furthermore, as a side product, we present the first 2-round GUC-secure commitment in the GroRO model (cf. Section 5.1.2), which may be of independent interest. The previous state-of-the-art protocols needs 3 rounds [MRS17, ZZZR22]. Note that this result does not contradict Zhou *et al.*'s impossibility result [ZZZR22], as their work did not consider simultaneous communication.

### 1.3 Related Work

We first review the EOT (and 1-out-of-2 OT) protocols in the CRS model. For simplicity, we often use OT to refer to the 1-out-of-2 OT in this work. Bellare and Micali gave an efficient and 1-round (standalone) EOT protocol under DDH assumption in the CRS model against semi-honest adversaries [BM90]. Later, Garg and Srinivasan compiled Bellare and Micali's protocol to against malicious adversaries [GS17] by additionally utilizing Groth-Sahai proofs [GS08]. As for the UC security, Canetti *et al.* proposed the first UC-secure OT protocol [CLOS02], but their protocol is quite inefficient. Next, Peikert *et al.* provided a dual-mode framework for constructing UC-secure OT protocols along with efficient instantiations under DDH, QR and LWE assumptions in the CRS model [PVW08]. Basing on Peikert *et al*'s dual mode framework, Garg *et al.* proposed the 1-round UC-secure EOT protocols under DDH, QR and LWE assumptions in the CRS model [GIS18].

We mainly focus on the EOT (and OT) protocols in the different variants of RO models, i.e. the local RO model, the GroRO model and the GrpRO model.

In terms of the local RO model, Chou and Orlandi proposed a 3-round OT protocol called "the simplest OT protocol" [CO15]. This protocol and the protocol proposed in [HL17] have been found to suffer from a number of issues [BPRS17, LM18, GIR20] and are not UC-secure. In the following, Masny and Rindal showed how to construct EOT protocols from the key exchange schemes in the local RO model [MR19a]. In particular, they provided a 1-round UC-secure construction under a non-standard assumption, i.e., Choose-and-Open DDH (CODDH) assumption [MR19a, MR19b].

Regarding the GroRO model, Canetti *et al.* proposed a 2-round OT protocol under DDH assumption [CJS14], but their protocol is only one-sided GUC-simulatable. Later, fully GUC-secure OT protocols in the GroRO model are proposed [DKLs18, DD20]. Their protocols only need CDH assumption but require no less than 5 rounds of communication. To achieve round-optimal, Canetti *et al.* proposed a 2-round GUC-secure OT protocol in the GroRO model [CSW20a], but their protocol requires a stronger assumption, i.e., DDH assumption.

As for the GrpRO model, Canetti *et al.* proposed an adaptive-secure 1-round EOT protocol in the GrpRO+CRS hybrid model [CSW20a], but their protocol is only UC-secure since their simulator must

know the trapdoor of the CRS. The lower bounds on round complexity in the GrpRO model have been explored by Zhou *et al.* [ZZZR22], however, they mainly focused on commitments and ZK protocols and they did not consider the simultaneous communication.

### 1.4 Organization

In Section 2, we present the preliminaries that will be used in this work. We propose the first 1-round UC-secure EOT protocol in the RO model against static adversaries in Section 3.1, and then extend it to achieve adaptive security without increasing the round complexity in Section 3.2. We investigate the relations between the EOT and other primitives in Section 4, demonstrating a separation between EOT and 1-out-of-2 OT in Section 4.1 and showing EOT implies commitment and UOT in Section 4.2 and Section 4.3 respectively. We also explore the feasibility and the impossibility for EOT in the global random oracle models. As for the GroRO model, we propose the first 1-round GUC-secure EOT protocolin Section 5.1.1. Basing on that, we propose the first 2-round GUC-secure commitment protocol in the GroRO model in Section 5.1.2. Regarding the GrpRO model, we show that it is impossible to construct a 1-round GUC-secure EOT protocol even with static security in Section 5.2.1 and subsequently propose the 2-round (round-optimal) GUC-secure EOT protocol in Section 5.2.2.

## 2 Preliminaries

### 2.1 Notations

We denote by $\lambda \in \mathbb{N}$ the security parameter. We say that a function $\mathsf{negl} : \mathbb{N} \to \mathbb{N}$ is negligible if for every positive polynomial $\mathrm{poly}(\cdot)$ and all sufficiently large $\lambda$, it holds that $\mathsf{negl}(\lambda) < \frac{1}{\mathrm{poly}(\lambda)}$. We use the abbreviation PPT to denote probabilistic polynomial-time. For an NP relation $\mathcal{R}$, we denote by $\mathcal{L}$ its associate language, i.e. $\mathcal{L} = \{x \mid \exists w \text{ s.t. } (x, w) \in \mathcal{R}\}$. We say that two distribution ensembles $\mathcal{X} = \{\mathcal{X}_\lambda\}_{\lambda\in\mathbb{N}}$ and $\mathcal{Y} = \{\mathcal{Y}_\lambda\}_{\lambda\in\mathbb{N}}$ are identical (resp. computationally indistinguishable), which we denote by $\mathcal{X} \equiv \mathcal{Y}$ (resp., $\mathcal{X} \overset{c}{\approx} \mathcal{Y}$), if for any unbounded (resp., PPT) distinguisher $\mathcal{D}$ there exists a negligible function $\mathsf{negl}$ such that $|\Pr[\mathcal{D}(\mathcal{X}_\lambda) = 1] - \Pr[\mathcal{D}(\mathcal{Y}_\lambda) = 1]| = 0$ (resp., $\mathsf{negl}(\lambda)$).

We denote by $y := \mathsf{Alg}(x; r)$ the event where the algorithm $\mathsf{Alg}$ on input $x$ and randomness $r$, outputs $y$. We denote by $y \leftarrow \mathsf{Alg}(x)$ the event where $\mathsf{Alg}$ selects a randomness $r$ and sets $y := \mathsf{Alg}(x; r)$. We denote by $y \leftarrow S$ the process for sampling $y$ uniformly at random from the set $S$. Let $q$ be a $\lambda$-bit prime, and $p = 2q + 1$ also be a prime. Let $\mathbb{G}$ be a subgroup of order $q$ of $\mathbb{Z}_p^*$ with the generator $g$.

### 2.2 Universal Composability

We formalize and analyze the security of our protocols in the Canetti's Universal Composability (UC) framework [Can01] and Canetti *et al*'s Generalized UC (GUC) framework [CDPW07].

**UC Framework.** The UC framework was proposed by Canetti [Can01], and it lays down a solid foundation for designing the protocols and analyzing the security against attacks in an arbitrary and complex network execution environment.

In the UC framework, a protocol $\Pi$ is defined to be a computer program (or several programs) which is intended to be executed by multiple interconnected parties. Furthermore, we call a protocol, the one for which we want to prove security, the *challenged* protocol. We only consider the *terminating* protocols in this work which terminates in polynomial time. Each party is identified by a unique identity pair $(\mathsf{pid}, \mathsf{sid})$, where $\mathsf{pid}$ is the Party ID (PID) and $\mathsf{sid}$ is the Session ID (SID). Parties running with the same code and the same SID are said to be a part of the same protocol session, and the PIDs are used to distinguish among the various parties in a particular protocol session. Typically, all SIDs are unique,

i.e. no two protocol sessions share the same SID. The adversarial behaviors in the protocol are captured by a adversary $\mathcal{A}$ who can control the network and corrupt the parties. When a party is corrupted, the adversary $\mathcal{A}$ receives its secret input and its internal state.

The UC framework is based on the "simulation paradigm" (a.k.a., the ideal/real world paradigm). In the ideal world, there is an ideal functionality $\mathcal{F}$ communicating a set of parties $\mathcal{P} = \{P_1, \ldots, P_n\}$ and an adversary $\mathcal{S}$ (a.k.a., the simulator) and computing a task in a trusted manner. The corrupted parties are controlled by the simulator $\mathcal{S}$. In the real world, a set of parties $\mathcal{P} = \{P_1, \ldots, P_n\}$ communicate with each other and the adversary $\mathcal{A}$ to execute the protocol $\Pi$. The corrupted parties are controlled by the adversary $\mathcal{A}$.

The presence of arbitrary protocols running in the network is modeled via the concept of the environment $\mathcal{Z}$, which determines the inputs to parties and see the outputs generated by those parties. The environment $\mathcal{Z}$ can also communicate with the adversary $\mathcal{A}$/simulator $\mathcal{S}$ and corrupts parties through it. The crucial part of the ideal/real world paradigm is that for every PPT adversary $\mathcal{A}$ attacking an execution of $\Pi$, there is a PPT simulator $\mathcal{S}$ attacking the ideal process that interacts with $\mathcal{F}$ (by corrupting the same set of parties), such that the executions of $\Pi$ with $\mathcal{A}$ and that of $\mathcal{F}$ with $\mathcal{S}$ makes no difference to the environment $\mathcal{Z}$. We denote by $\mathsf{EXEC}_{\mathcal{F},\mathcal{S},\mathcal{Z}}$ (resp. $\mathsf{EXEC}_{\Pi,\mathcal{A},\mathcal{Z}}$) the output of the environment $\mathcal{Z}$ in the ideal world (resp. real world) execution. Formally, we define a protocol to be UC-secure through the following definition.

**Definition 1.** *We say a protocol $\Pi$ UC-realizes the functionality $\mathcal{F}$, if for any PPT environment $\mathcal{Z}$ and any PPT adversary $\mathcal{A}$, there exists a PPT simulator $\mathcal{S}$ s.t.* $\mathsf{EXEC}_{\Pi,\mathcal{A},\mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}_{\mathcal{F},\mathcal{S},\mathcal{Z}}$.

Furthermore, we describe the *modularity* which is a crucial aspect of the UC framework: when a protocol calls subroutines, these subroutines can be treated as separate entities and can be analyzed separately for their security by way of realizing an ideal functionality. We here introduce the notion of "hybrid world", and a protocol $\Pi$ is said to be realized "in the $\mathcal{G}$ hybrid world" if $\Pi$ invokes the ideal functionality $\mathcal{G}$ as a subroutine.

**Definition 2.** *We say protocol $\Pi$ UC-realizes the functionality $\mathcal{F}$ in the $\mathcal{G}$ hybrid world, if for any PPT environment $\mathcal{Z}$ and any PPT adversary $\mathcal{A}$, there exists a PPT simulator $\mathcal{S}$ s.t.* $\mathsf{EXEC}^{\mathcal{G}}_{\Pi,\mathcal{A},\mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}_{\mathcal{F},\mathcal{S},\mathcal{Z}}$.

We note that, in the UC framework, the environment $\mathcal{Z}$ cannot have the direct access to $\mathcal{G}$, but it can do so through the adversary. More precisely, in the real world (resp. ideal world), the adversary $\mathcal{A}$ (resp. the simulator $\mathcal{S}$) can access the ideal functionality $\mathcal{G}$ directly. It queries $\mathcal{G}$ for $\mathcal{Z}$ and forwards the answers. This implicitly means that the ideal functionality $\mathcal{G}$ is local to the challenge protocol instance. Therefore, the simulator $\mathcal{S}$ is allowed to simulate $\mathcal{G}$ in the ideal world as long as it "looks" indistinguishable from $\mathcal{G}$ hybrid world.

**GUC Framework.** In the UC framework, the environment $\mathcal{Z}$ is constrained: it cannot have the direct access to the setup. In other words, the setups in the UC framework are not global. This is an impractical assumption in real life applications, where it is more plausible that there is a global setup published and used by many protocol instances.

Motivated by solving problems caused by modeling setup as a local subroutine, the Generalized UC (GUC) framework was introduced by Canetti *et al.* [CDPW07], which can be used for properly analyzing concurrent execution of protocols in the presence of global setups. Compared to the UC framework, the environment $\mathcal{Z}$ in the GUC framework is unconstrained: it is allowed to access the setup directly without going through the simulator/adversary. Furthermore, the environment $\mathcal{Z}$ can invoke arbitrary protocols alongside the challenge protocol. Formally, Canetti *et al.* introduced the notion of *shared functionality*: it is completely analogous to an ideal functionality, except that it may interact with more than one protocol sessions. The global setup can be modeled as a shared functionality. This indeed captures the fact that any protocol in the network can use the same global setup.

To distinguish from the basic UC execution, we denote the output of the unconstrained PPT environment $\mathcal{Z}$ in the real world (resp. ideal world) execution as $\mathsf{GEXEC}_{\Pi,\mathcal{A},\mathcal{Z}}$ (resp. $\mathsf{GEXEC}_{\mathcal{F},\mathcal{S},\mathcal{Z}}$).

**Definition 3.** *We say a protocol $\Pi$ GUC-realizes functionality $\mathcal{F}$, if for any unconstrained PPT environment $\mathcal{Z}$ and any PPT adversary $\mathcal{A}$ there exists a PPT simulator $\mathcal{S}$ s.t.* $\mathsf{GEXEC}_{\Pi,\mathcal{A},\mathcal{Z}} \overset{c}{\approx} \mathsf{GEXEC}_{\mathcal{F},\mathcal{S},\mathcal{Z}}$.

Since the unconstrained environment $\mathcal{Z}$ we described above is granted a high-level of flexibility (i.e., $\mathcal{Z}$ is allowed to invoke arbitrary protocols in parallel with the challenge protocol and cause arbitrary interactions with shared functionalities), it becomes extremely difficult to prove the GUC security. Therefore, a simplified framework called Externalized UC (EUC) framework was introduced in [CDPW07]. In the EUC framework, the environment $\mathcal{Z}$ has direct access to the shared functionality $\mathcal{G}$ but does not initiate any new protocol sessions except the challenge protocol session. We call such an environment is $\mathcal{G}$-externalized constrained. We say a protocol $\Pi$ is $\mathcal{G}$-subroutine respecting if it only shares state information via a single shared functionality $\mathcal{G}$.

**Definition 4.** *Let the protocol $\Pi$ be $\mathcal{G}$-subroutine respecting. We say a protocol $\Pi$ EUC-realizes the functionality $\mathcal{F}$ with respect to shared functionality $\mathcal{G}$, if for any PPT $\mathcal{G}$-externalized constrained environment $\mathcal{Z}$ and any PPT adversary $\mathcal{A}$ there exists a PPT simulator $\mathcal{S}$ s.t.* $\mathsf{EXEC}^{\mathcal{G}}_{\Pi,\mathcal{A},\mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}^{\mathcal{G}}_{\mathcal{F},\mathcal{S},\mathcal{Z}}$.

Furthermore, Canetti *et al.* showed that for any $\mathcal{G}$-subroutine respecting protocol $\Pi$, proving $\Pi$ EUC-realizes $\mathcal{F}$ with respect to $\mathcal{G}$ is equivalent to proving $\Pi$ GUC-realizes $\mathcal{F}$ [CDPW07]. For that reason, when we want to prove the GUC security of a protocol, we always turn to EUC security for the sake of simplicity.

**Adversarial Model.** In this work, we consider both static corruption (where the adversary corrupts the parties at the beginning of the protocol) and adaptive corruption (where the adversary corrupts the parties at any time). We also consider *rushing* adversaries, who may delay sending messages on behalf of corrupted parties in a given round until the messages sent by all the uncorrupted parties in that round have been received [Kat07].

**Secure Communication Model.** Many UC-secure protocols assume the parties are interconnected with secure channels or authenticated channels [CLOS02, CDPW07, CDG+18]. The secure channel and authenticated channel can be modeled as ideal functionalities $\mathcal{F}_{\mathsf{SC}}$ and $\mathcal{F}_{\mathsf{Auth}}$ respectively [CK02, Can01]. In this work, most of our protocols are designed in the simultaneous communication model with rushing adversaries, which is different from that [Kat07] deals with non-rushing adversaries. For this reason, we often assume the synchronous channel which can be modeled as $\mathcal{F}_{\mathsf{SYN}}$ [Can01]. For readability, we will mention which secure communication model is used in the context and omit it in the protocol description.

### 2.3 Sigma Protocols

A Sigma-protocol is a 3-move public coin protocol [Dam02], where both the prover $P$ and the verifier $V$ hold a statement $x$, and the prover $P$ additionally holds a private witness $w$ such that $(x, w) \in \mathcal{R}$. In the first move, the prover $P$ generates the message $a$ by invoking $(a, \mathsf{st}) \leftarrow \mathsf{P1}(x, w)$ on a statement-witness pair $(x, w)$. In the second move, the verifier $V$ samples an uniformly random string $e \leftarrow \mathsf{V1}(1^\lambda)$ as the challenge. In the last move, the prover $P$ computes the response $z \leftarrow \mathsf{P2}(x, w, e, \mathsf{st})$ using the statement-witness pair $(x, w)$, the received challenge $e$ and the previously computed state $\mathsf{st}$. Finally, the verifier outputs a bit $b := \mathsf{Verify}(x, a, e, z)$ indicating acceptance $(b = 1)$ or rejection $(b = 0)$.

The Sigma-protocol should satisfy the completeness, special soundness and Special Honest Verifier Zero-Knowledge (SHVZK). The completeness requires that the honest prover will always make the verifier accept. The special soundness requires that there exists a PPT extractor algorithm Ext that given

two distinct accepting transcripts (i.e., $(a, e, z)$ and $(a, e', z')$ where $e \neq e'$), it can output a valid witness $w$. The SHVZK property requires that there exists a PPT simulator algorithm Sim that given the challenge $e$ ahead, it can output $(a, z)$ that are computational indistinguishable from the real ones. Formally, we have the following definition.

**Definition 5.** *Fix an NP relation $\mathcal{R}$ and its associate language $\mathcal{L}$. We say a protocol $\Pi = (\Pi.\mathsf{P1}, \Pi.\mathsf{V1}, \Pi.\mathsf{P2}, \Pi.\mathsf{Verify}, \Pi.\mathsf{Ext}, \Pi.\mathsf{Sim})$ is a Sigma-protocol for $\mathcal{R}$ if the following conditions hold:*

1. ***(Completeness)** For any $(x, w) \in \mathcal{R}$, we say it is complete if*

$$\Pr\left[\begin{array}{l}(a, \mathsf{st}) \leftarrow \mathsf{P1}(x, w); e \leftarrow \mathsf{V1}(1^\lambda); \\ z \leftarrow \mathsf{P2}(x, w, e, \mathsf{st})\end{array} : \mathsf{Verify}(x, a, e, z) = 1\right] = 1$$

2. ***(Special Soundness)** For any $x \in \mathcal{L}$, we say it is special sound if for any PPT $\mathcal{A}$,*

$$\Pr\left[\begin{array}{l}(a, e, z, e', z') \leftarrow \mathcal{A}(x); \\ e \neq e'\end{array} : \begin{array}{l}\mathsf{Verify}(x, a, e, z) = \mathsf{Verify}(x, a, e', z') = 1 \\ \wedge\ w \leftarrow \mathsf{Ext}(x, a, e, z, e', z') \ \wedge\ (x, w) \in \mathcal{R}\end{array}\right] = 1$$

3. ***(Special Honest Verifier Zero-Knowledge)** Given the challenge $e$ ahead, for any $(x, w) \in \mathcal{R}$, we say it is SHVZK if*

$$\{(a, z) \mid (a, \mathsf{st}) \leftarrow \mathsf{P1}(x, w); z \leftarrow \mathsf{P2}(x, w, e, \mathsf{st})\} \overset{c}{\approx} \{(a, z) \mid (a, z) \leftarrow \mathsf{Sim}(x, e)\}$$

**Augmented Sigma-Protocols.** The augmented Sigma-protocols [CDPW07] are Sigma-protocols that additionally satisfy the "reverse state construction" property: given the statement-witness pair $(x, w)$, the simulated transcript $(a, z)$ produced by the simulator algorithm Sim and the internal states of Sim (i.e., the challenge $e$ and the randomness $r$), there exists an efficient algorithm StateRec that can construct the "fake" state information st that makes $(a, z)$ appear as if they were the output of the honest runs of the prover algorithm, instead of the output of the simulator algorithm. Formally, we have the following definition.

**Definition 6.** *Fix an NP relation $\mathcal{R}$ and its associate language $\mathcal{L}$. We say a protocol $\Pi = (\Pi.\mathsf{P1}, \Pi.\mathsf{V1}, \Pi.\mathsf{P2}, \Pi.\mathsf{Verify}, \Pi.\mathsf{Ext}, \Pi.\mathsf{Sim}, \Pi.\mathsf{StateRec})$ is an augmented Sigma-protocol for $\mathcal{R}$ if it satisfies the completeness, special soundness, SHVZK and the following property:*

1. ***(Reverse State Construction)** For any $(x, w) \in \mathcal{R}$ and any challenge $e$, we say it has reverse state construction if the following condition holds:*

$$\begin{aligned}&\left\{(a, z, \mathsf{st}) \;\middle|\; (a, \mathsf{st}) \leftarrow \mathsf{P1}(x, w); z \leftarrow \mathsf{P2}(x, w, e, \mathsf{st})\right\} \\ \overset{c}{\approx} &\left\{(a, z, \mathsf{st}) \;\middle|\; (a, z) := \mathsf{Sim}(x, e; r); \mathsf{st} \leftarrow \mathsf{StateRec}(x, w, a, e, z, r)\right\}\end{aligned}$$

## 2.4 Non-Interactive Arguments in the Random Oracle Model

In this section, we introduce different variants of non-interactive argument systems in the RO model. In a non-interactive argument $\Pi = (\Pi.\mathsf{Prove}^{\mathcal{O}}, \Pi.\mathsf{Verify}^{\mathcal{O}})$ in the RO model, both prover and verifier are allowed to query the RO $\mathcal{O}$ at any time, during the protocol execution. A non-interactive argument system allow the prover to generate the proof $\pi$ using the statement-witness pair $(x, w)$. Upon receiving the proof $\pi$, the verifier can decide whether to accept or not. Formally, a non-interactive argument systems has the following algorithms:

- $\pi \leftarrow \mathsf{Prove}^{\mathcal{O}}(x, w)$ takes input as a statement-witness pair $(x, w)$, and it outputs a proof $\pi$.

- $b := \mathsf{Verify}^{\mathcal{O}}(x, \pi)$ takes input as a statement $x$ and a proof $\pi$, and it outputs a bit $b$ indicating acceptance ($b = 1$) or not ($b = 0$).

The non-interactive argument systems should satisfies completeness and computational soundness. The former property requires that honest provers can always make the verifiers accept. The latter property requires that any computationally bounded provers cannot convince the verifiers that a false statement is true.

**Definition 7.** *Fix an NP relation $\mathcal{R}$ and its associate language $\mathcal{L}$. Consider a random oracle $\mathcal{O}$. We say $\Pi = (\Pi.\mathsf{Prove}^{\mathcal{O}}, \Pi.\mathsf{Verify}^{\mathcal{O}})$ is a non-interactive argument system for $\mathcal{R}$ in the RO model if the following conditions hold:*

1. ***(Completeness)** For any $(x, w) \in \mathcal{R}$, we say it is complete if*

$$\Pr\left[\pi \leftarrow \mathsf{Prove}^{\mathcal{O}}(x, w) : \mathsf{Verify}^{\mathcal{O}}(x, \pi) = 1\right] = 1$$

2. ***(Computational Soundness)** For any $x \notin \mathcal{L}$, we say it is computational sound if for any PPT adversary $\mathcal{A}$, there exists a negligible function $\mathsf{negl}$ such that*

$$\Pr\left[\pi^* \leftarrow \mathcal{A}^{\mathcal{O}}(x) : \mathsf{Verify}^{\mathcal{O}}(x, \pi^*) = 1\right] \leq \mathsf{negl}(\lambda)$$

### 2.4.1 NIWH Arguments in the Random Oracle Model

An Non-Interactive Witness Hiding (NIWH) argument $\Pi = (\Pi.\mathsf{Prove}^{\mathcal{O}}, \Pi.\mathsf{Verify}^{\mathcal{O}})$ in the RO model is a non-interactive argument in the RO model that additionally satisfies the witness hiding property. The witness hiding property means that any computationally bounded adversary cannot extract the witness $w$ from any accepting proof $\pi$. Before giving the formal definition of witness hiding, we have to define the hard instance ensembles, as in [Pas03].

**Definition 8** (Hard Instance Ensembles)**.** *Let $\mathcal{R}$ be an NP relation and $\mathcal{L}$ be its associate language, and $\mathcal{X} = \{\mathcal{X}_\lambda\}_{\lambda \in \mathbb{N}}$ be a probability ensemble such that $\mathcal{X}_\lambda$ ranges over $\mathcal{L} \cap \{0,1\}^\lambda$. We say $\mathcal{X}$* a hard instance ensemble *for $\mathcal{R}$ if for any PPT adversary $\mathcal{A}$ and any $x \in \mathcal{X}$, there exists a negligible function $\mathsf{negl}$ such that $\Pr[(x, \mathcal{A}(x)) \in \mathcal{R}] \leq \mathsf{negl}(\lambda)$.*

Now we can formally define the NIWH argument in the RO model.

**Definition 9.** *Fix an NP relation $\mathcal{R}$ and its associate language $\mathcal{L}$. Consider a random oracle $\mathcal{O}$. We say $\Pi = (\Pi.\mathsf{Prove}^{\mathcal{O}}, \Pi.\mathsf{Verify}^{\mathcal{O}})$ is an NIWH argument system for $\mathcal{R}$ in the RO model if it satisfies completeness, computational soundness and the following property:*

1. ***(Witness Hiding)** For any $(x, w) \in \mathcal{R}$ and $x \in \mathcal{X}$ where $\mathcal{X}$ a hard instance ensemble for $\mathcal{R}$, we say it is* witness hiding *if for any PPT adversary $\mathcal{A}$, there exists a negligible function $\mathsf{negl}$ such that*

$$\Pr\left[\pi \leftarrow \mathsf{Prove}(x, w) : w' \leftarrow \mathcal{A}(x, \pi) \ \wedge \ (x, w') \in \mathcal{R}\right] \leq \mathsf{negl}(\lambda)$$

### 2.4.2 NIZK Arguments in the Random Oracle Model

Now let us consider another strengthened version of non-interactive argument in the RO model: Non-Interactive Zero-Knowledge (NIZK) arguments $\Pi = (\Pi.\mathsf{Prove}^{\mathcal{O}}, \Pi.\mathsf{Verify}^{\mathcal{O}}, \Pi.\mathsf{Sim})$ in the RO model. It is known that the NIZK argument systems imply NIWH argument systems in the RO model [Pas03]. An NIZK argument is a non-interactive argument that additionally satisfies the zero-knowledge property. The zero-knowledge property requires that there exists a PPT simulator algorithm $\Pi.\mathsf{Sim}$ that simulates the RO itself and outputs the simulated proof $\pi$. Formally, we have the following definition.

**Definition 10.** *Fix an NP relation $\mathcal{R}$ and its associate language $\mathcal{L}$. Consider a RO $\mathcal{O}$. We say $\Pi = (\Pi.\mathsf{Prove}^{\mathcal{O}}, \Pi.\mathsf{Verify}^{\mathcal{O}}, \Pi.\mathsf{Sim})$ is an NIZK argument system for $\mathcal{R}$ in the RO model if it satisfies completeness, computational soundness and the following property:*

1. ***(Computational Zero-Knowledge)** For any $(x, w) \in \mathcal{R}$, we say it is* computational zero-knowledge *if the following condition holds:*

$$\{(\pi, \mathcal{O}) \mid \pi \leftarrow \mathsf{Prove}^{\mathcal{O}}(x, w)\} \overset{c}{\approx} \{(\pi, \mathcal{O}) \mid (\pi, \mathcal{O}) \leftarrow \mathsf{Sim}(x)\}$$

**NIZK Arguments with Honest Prover State Reconstruction (HPSR) in the RO Model.** Similar to the reverse state construction of the augmented Sigma-protocol, Groth *et al.* defined a security property for the NIZK proof/argument in the CRS model [GOS06]: Honest Prover State Reconstruction (HPSR). In this work, we extend this security definition to the RO model. More precisely, we require that there exists an efficient algorithm $\mathsf{StateRec}^{\mathcal{O}}$ that given the query access to the random oracle $\mathcal{O}$ simulated by $\mathsf{Sim}$, on input a simulated proof $\pi$ and its corresponding internal states that used to generate the $\pi$, it can output the convincing randomness $r$ so that it looks like the simulated proof $\pi$ was constructed by an honest prover using this randomness $r$.

**Definition 11.** *Fix an NP relation $\mathcal{R}$ and its associate language $\mathcal{L}$. Consider a RO $\mathcal{O}$. We say a protocol $\Pi = (\Pi.\mathsf{Prove}^{\mathcal{O}}, \Pi.\mathsf{Verify}^{\mathcal{O}}, \Pi.\mathsf{Sim}, \Pi.\mathsf{StateRec}^{\mathcal{O}})$ is an NIZK argument system with honest prover state reconstruction for $\mathcal{R}$ in the RO model if it satisfies the completeness, computational soundness, computational zero-knowledge and the following property:*

1. ***(Honest Prover State Reconstruction)** For any $(x, w) \in \mathcal{R}$, we say it has honest prover state reconstruction if the following condition holds:*

$$\{(\pi, r, \mathcal{O}) \mid \pi := \mathsf{Prove}^{\mathcal{O}}(x, w; r)\} \overset{c}{\approx} \left\{(\pi, r, \mathcal{O}) \;\middle|\; \begin{array}{l} (\pi, \mathcal{O}) := \mathsf{Sim}(x; \rho) \\ r \leftarrow \mathsf{StateRec}^{\mathcal{O}}(x, w, \pi, \rho) \end{array}\right\}$$

### 2.4.3 Straight-Line Extractability in the Random Oracle Model

Now let us consider a stronger security property, i.e. *(straight-line)* extractability, and our extractability definition is taken from that by Pass [Pas03]. The extractability means that there exists a PPT extractor algorithm Ext which can extract the witness from a maliciously generated and accepting proof. To enable straight-line extractability, typically, the extractor Ext can be developed by simulating the RO for the prover and the verifier, and thus Ext has full control of the RO. Note that, by the simulating the RO, the extractor Ext is granted two advantages: (i) *observability:* the simulator can see the query-answer list of the RO; (ii) *programmability:* the simulator can answer the queries to the RO.

In this work, we consider the straight-line extractability in a much more restricted RO, that is, the extractor Ext is granted only the observability of RO. For that reason, we write $\mathsf{Ext}^{\mathcal{O}}$ to indicate that, the extractor Ext does not have the full control of the RO and is only allowed to see the query-answer list of the RO. Now we can formally define the straight-line extractability.

**Definition 12** (Straight-line extractability)**.** *Fix an NP relation $\mathcal{R}$ and its associate language $\mathcal{L}$. Consider a RO $\mathcal{O}$, and a non-interactive argument system for $\mathcal{R}$ in the RO model $\Pi = (\Pi.\mathsf{Prove}^{\mathcal{O}}, \Pi.\mathsf{Verify}^{\mathcal{O}}, \Pi.\mathsf{Ext}^{\mathcal{O}})$. For any $x \in \mathcal{L}$, we say the non-interactive argument $\Pi$ is* straight-line extractable *if for any PPT adversary $\mathcal{A}$, there exists a negligible function $\mathsf{negl}$ such that*

$$\Pr\left[\begin{array}{l} \pi^* \leftarrow \mathcal{A}^{\mathcal{O}}(x); b := \mathsf{Verify}^{\mathcal{O}}(x, \pi^*); \\ w^* \leftarrow \mathsf{Ext}^{\mathcal{O}}(x, \pi^*) \end{array} : b = 1 \;\wedge\; (x, w^*) \in \mathcal{R}\right] \geq 1 - \mathsf{negl}(\lambda)$$

Note that, the NIWH argument and NIZK argument are developed on top of the non-interactive argument. Hence the Definition 12 can be easily extended to the straight-line extractable NIZK (NIWH) argument in the RO model.

**Definition 13.** *Fix an NP relation $\mathcal{R}$ and its associate language $\mathcal{L}$. Consider a RO $\mathcal{O}$. We say* $\Pi = (\Pi.\mathsf{Prove}^{\mathcal{O}}, \Pi.\mathsf{Verify}^{\mathcal{O}}, \Pi.\mathsf{Ext}^{\mathcal{O}})$ *is a* straight-line extractable NIWH argument system *for $\mathcal{R}$ in the RO model if it satisfies completeness, computational soundness, witness hiding and straight-line extractability.*

**Definition 14.** *Fix an NP relation $\mathcal{R}$ and its associate language $\mathcal{L}$. Consider a RO $\mathcal{O}$. We say* $\Pi = (\Pi.\mathsf{Prove}^{\mathcal{O}}, \Pi.\mathsf{Verify}^{\mathcal{O}}, \Pi.\mathsf{Sim}, \Pi.\mathsf{Ext}^{\mathcal{O}})$ *is a* straight-line extractable NIZK argument system *for $\mathcal{R}$ in the RO model if it satisfies completeness, computational soundness, computational zero-knowledge and straight-line extractability.*

## 2.5 Ideal Functionalities

In this section, we provide ideal functionalities that will be used in UC/GUC security analysis.

### 2.5.1 Coin-Tossing

Coin-tossing is one of the most minimal building blocks for cryptographic protocols. Coin-tossing is a two-party protocol which allows each party to receive an uniformly random string at the end of the protocol execution. Formally, we put the coin-tossing functionality $\mathcal{F}_{\mathsf{Coin}}$ in Figure 3.

**Functionality $\mathcal{F}_{\mathsf{Coin}}$**

The functionality interacts with two parties $P_1, P_2$ and an adversary $\mathcal{S}$.

**Toss.** Upon receiving $(\textsc{Toss}, \mathsf{sid}, P_1, P_2)$ from $P_1$ and $P_2$, do:

- Sample $r \leftarrow \{0,1\}^\lambda$, send $(\textsc{Tossed}, \mathsf{sid}, P_1, P_2, r)$ to $P_1$ and $(\textsc{Proceed?}, \mathsf{sid}, P_1, P_2)$ to $\mathcal{S}$.
- Upon receiving $(\textsc{Proceed}, \mathsf{sid})$ from $\mathcal{S}$, send $(\textsc{Tossed}, \mathsf{sid}, P_1, P_2, r)$ to $P_2$.
- Ignore any subsequent Toss command.

Figure 3: The Ideal Functionality $\mathcal{F}_{\mathsf{Coin}}$ for Coin-Tossing

### 2.5.2 OT, UOT and EOT

We start with the Oblivious Transfer (OT). we often use OT to refer to the 1-out-of-2 OT. In a OT protocol, there is a sender $S$ holding two private input $m_0, m_1 \in \{0,1\}^\lambda$ and a receiver $R$ holding a choice bit $b \in \{0,1\}$. At the end of the honest execution of the OT protocol, the receiver $R$ will compute $m_b$. At the same time, the sender should learn nothing about $b$ while the receiver should learn nothing about $m_{1-b}$. We present the OT functionality $\mathcal{F}_{\mathsf{OT}}$ in Figure 4.

In [MR19a], Masny and Rindal proposed two notions that called Uniform OT (UOT) and Endemic OT (EOT). Both of them are similar to OT, except that the senders have no inputs. The main difference between the UOT and the EOT is that they provide different levels of security guarantees. We describe the UOT first. The UOT functionality samples two uniformly random strings $m_0, m_1$, and outputs $m_0, m_1$ to the (potentially malicious) sender and $m_b$ to the (potentially malicious) receiver. The UOT gives a strong security guarantee that any malicious party cannot influence the distribution of the OT messages. Formally, we put the UOT functionality $\mathcal{F}_{\mathsf{UOT}}$ in Figure 5.

Now let us turn to EOT. Compared to UOT, the EOT functionality gives a weak security guarantee: no matter whether the sender or the receiver is malicious, the malicious party can always determine the distribution of the OT messages. Roughly speaking, if both sender and receiver are honest, the EOT functionality acts as the UOT functionality. If the sender is malicious and the receiver is honest, the EOT functionality lets the adversary determine the message strings $m_0, m_1$, and it returns the adversarial chosen $m_b$ to the honest receiver after receiving $b$. If the receiver is malicious and the sender is honest, the EOT functionality lets the adversary determine the message string $m_b$, and it returns the

**Functionality $\mathcal{F}_{\mathsf{OT}}$**

The functionality interacts with two parties $S$, $R$ and an adversary $\mathcal{S}$.

**Transfer.** Upon receiving $(\textsc{Send}, \mathsf{sid}, S, R, m_0, m_1)$ from $S$, do:

- Record the tuple $(\mathsf{sid}, S, R, m_0, m_1)$, and send $(\textsc{Send}, \mathsf{sid}, S, R)$ to $R$ and $\mathcal{S}$.
- Ignore any subsequent $\textsc{Send}$ commands.

**Choose.** Upon receiving $(\textsc{Receive}, \mathsf{sid}, S, R, b)$ from $R$ where $b \in \{0, 1\}$, do:

- Record the tuple $(\mathsf{sid}, S, R, b)$, and send $(\textsc{Receive}, \mathsf{sid}, S, R)$ to $S$ and $\mathcal{S}$.
- Ignore any subsequent $\textsc{Receive}$ commands.

**Process.** When both $(\mathsf{sid}, S, R, m_0, m_1)$ and $(\mathsf{sid}, S, R, b)$ are recorded, do:

- Send $(\textsc{Proceed?}, \mathsf{sid}, R)$ to the adversary $\mathcal{S}$.
- Upon receiving $(\textsc{Proceed}, \mathsf{sid}, R)$ from $\mathcal{S}$, output $(\textsc{Received}, \mathsf{sid}, S, R, m_b)$ to $R$ and $(\textsc{Received}, \mathsf{sid}, S, R)$ to $S$.

Figure 4: The Ideal Functionality $\mathcal{F}_{\mathsf{OT}}$ for Oblivious Transfer

**Functionality $\mathcal{F}_{\mathsf{UOT}}$**

The functionality interacts with two parties $S$, $R$ and an adversary $\mathcal{S}$.

**Transfer.** Upon receiving $(\textsc{Send}, \mathsf{sid}, S, R)$ from $S$, do:

- Sample $m_0, m_1 \leftarrow \{0,1\}^\lambda$, record the tuple $(\mathsf{sid}, S, R, m_0, m_1)$, and send $(\textsc{Send}, \mathsf{sid}, S, R)$ to $R$ and $\mathcal{S}$.
- Ignore any subsequent $\textsc{Send}$ commands.

**Choose.** Upon receiving $(\textsc{Receive}, \mathsf{sid}, S, R, b)$ from $R$ where $b \in \{0, 1\}$, do:

- Record the tuple $(\mathsf{sid}, S, R, b)$, and send $(\textsc{Receive}, \mathsf{sid}, S, R)$ to $S$ and $\mathcal{S}$.
- Ignore any subsequent $\textsc{Receive}$ commands.

**Process.** When both $(\mathsf{sid}, S, R, m_0, m_1)$ and $(\mathsf{sid}, S, R, b)$ are recorded, do:

- Send $(\textsc{Proceed?}, \mathsf{sid}, S, R)$ to the adversary $\mathcal{S}$.
- Upon receiving $(\textsc{Proceed}, \mathsf{sid}, S)$ from $\mathcal{S}$, output $(\textsc{Received}, \mathsf{sid}, S, R, m_0, m_1)$ to $S$. Upon receiving $(\textsc{Proceed}, \mathsf{sid}, R)$ from $\mathcal{S}$, output $(\textsc{Received}, \mathsf{sid}, S, R, m_b)$ to $R$.

Figure 5: The Ideal Functionality $\mathcal{F}_{\mathsf{UOT}}$ for Uniform Oblivious Transfer

**Functionality $\mathcal{F}_{\mathsf{EOT}}$**

The functionality interacts with two parties $S$, $R$ and an adversary $\mathcal{S}$.

**Transfer/Choose.** Upon receiving $(\textsc{Send}, \mathsf{sid}, S, R)$ from $S$ or $(\textsc{Receive}, \mathsf{sid}, S, R, b)$ from $R$, do the same as $\mathcal{F}_{\mathsf{UOT}}$ that depicted in Figure 5.

**Process.** When both $(\mathsf{sid}, S, R, m_0, m_1)$ and $(\mathsf{sid}, S, R, b)$ are recorded, do:

- If both $S$ and $R$ are honest, output $(\textsc{Received}, \mathsf{sid}, S, R, m_0, m_1)$ to $S$, $(\textsc{Received}, \mathsf{sid}, S, R, m_b)$ to $R$ and $(\textsc{Received}, \mathsf{sid}, S, R)$ to $\mathcal{S}$.
- Else if $S$ is corrupted and $R$ is honest, send $(\textsc{Send}, \mathsf{sid}, S, R)$ to $\mathcal{S}$. Upon receiving $(\textsc{Send}, \mathsf{sid}, S, R, m_0^*, m_1^*)$ from $\mathcal{S}$, set $m_0 := m_0^*, m_1 := m_1^*$. Output $(\textsc{Received}, \mathsf{sid}, S, R, m_0, m_1)$ to $S$, $(\textsc{Received}, \mathsf{sid}, S, R, m_b)$ to $R$.
- Else if $S$ is honest and $R$ is corrupted, send $(\textsc{Receive}, \mathsf{sid}, S, R)$ to $\mathcal{S}$. Upon receiving $(\textsc{Receive}, \mathsf{sid}, S, R, m_b^*)$ from $\mathcal{S}$, set $m_b := m_b^*$. Output $(\textsc{Received}, \mathsf{sid}, S, R, m_0, m_1)$ to $S$, $(\textsc{Received}, \mathsf{sid}, S, R, m_b)$ to $R$.
- Else if both $S$ and $R$ are corrupted, halt.

Figure 6: The Ideal Functionality $\mathcal{F}_{\mathsf{EOT}}$ for Endemic Oblivious Transfer

adversarial chosen $m_b$ and an uniformly sampled $m_{1-b}$ to the honest sender. If both sender and receiver are malicious, the EOT functionality simply aborts. Formally, we put the EOT functionality $\mathcal{F}_{\mathsf{EOT}}$ in Figure 6.

### 2.5.3 Commitment

A commitment protocol allows a committer to compute a commitment message $c$ to a message $m$ towards a receiver in the committing phase. Later in the opening phase, the committer can open $c$ to $m$ by sending the opening message to the receiver. The commitment protocol should satisfies two properties: (i) hiding property, i.e., the receiver should learn nothing about the committed message from $c$; (ii) binding property, i.e., the committer cannot open $c$ to another message $m' \neq m$. We put the commitment functionality $\mathcal{F}_{\mathsf{Com}}$ in Figure 7, and both hiding and binding properties are captured by $\mathcal{F}_{\mathsf{Com}}$.

**Functionality $\mathcal{F}_{\mathsf{Com}}$**

The functionality interacts with two parties $C, R$ and an adversary $\mathcal{S}$.

**Commit.** Upon receiving $(\textsc{Commit}, \mathsf{sid}, C, R, m)$ from $C$, do:

- Record the tuple $(\mathsf{sid}, C, R, m)$, and send $(\textsc{Receipt}, \mathsf{sid}, C, R)$ to $R$ and $\mathcal{S}$.
- Ignore any subsequent $\textsc{Commit}$ command.

**Open.** Upon receiving $(\textsc{Decommit}, \mathsf{sid}, C, R)$ from $C$, do:

- If there is a tuple $(\mathsf{sid}, C, R, m)$ recorded, send $(\textsc{Decommit}, \mathsf{sid}, C, R, m)$ to $R$ and $\mathcal{S}$, and halt.
- Otherwise, ignore the message.

Figure 7: The Ideal Functionality $\mathcal{F}_{\mathsf{Com}}$ for Commitment

### 2.5.4 Random Oracles

We introduce the local RO model, and two well-known global RO models: Global Restricted Observable Random Oracle (GroRO) model proposed by Canetti *et al.* [CJS14] and Global Restricted Programmable Random Oracle (GrpRO) model proposed by Camenisch *et al.* [CDG+18].

**The Local RO Model.** Typically, the local RO is modeled as an ideal functionality $\mathcal{F}_{\mathsf{RO}}$. As depicted in Figure 8, upon receiving $(\textsc{Query}, \mathsf{sid}, x)$ from any party, $\mathcal{F}_{\mathsf{RO}}$ first checks whether the query $(\mathsf{sid}, x)$ has been queried before. If not, $\mathcal{F}_{\mathsf{RO}}$ selects a random value of pre-specified length $v \leftarrow \{0,1\}^{\ell_{\mathsf{out}}(\lambda)}$, answers with the value $v$ and records the tuple $(\mathsf{sid}, x, v)$; otherwise, the previously chosen value $v$ is returned again, even if the earlier query was made by another party.

**Ideal Functionality $\mathcal{F}_{\mathsf{RO}}$**

The functionality interacts with a set of parties $\mathcal{P} = \{P_1, \ldots, P_n\}$ and an adversary $\mathcal{S}$. It is parameterized by the output length $\ell_{\mathsf{out}}(\lambda)$. It maintains an initially empty list $\mathsf{List}$.

**Query.** Upon receiving $(\textsc{Query}, \mathsf{sid}', x)$ from a party $P_i \in \mathcal{P}$ where $P_i = (\mathsf{pid}, \mathsf{sid})$, or the adversary $\mathcal{S}$:

- Check if $\exists\, v \in \{0,1\}^{\ell_{\mathsf{out}}(\lambda)}$ such that $(\mathsf{sid}, x, v) \in \mathsf{List}$. If not, select $v \leftarrow \{0,1\}^{\ell_{\mathsf{out}}(\lambda)}$ and record the tuple $(\mathsf{sid}', x, v)$ in $\mathsf{List}$.
- Return $(\textsc{QueryConfirm}, \mathsf{sid}', v)$ to the requestor.

Figure 8: The Local Random Oracle Model $\mathcal{F}_{\mathsf{RO}}$

**The GroRO Model.** Compared to $\mathcal{F}_{\mathsf{RO}}$, the GroRO is modeled as a shared functionality $\mathcal{G}_{\mathsf{roRO}}$ which may interact with more than one protocol sessions. The $\mathcal{G}_{\mathsf{roRO}}$ answers to the queries in the same way as $\mathcal{F}_{\mathsf{RO}}$. The simulator is only granted the restricted observability: some of the queries can be marked as "illegitimate" and potentially disclosed to the simulator. As depicted in Figure 9, the $\mathcal{G}_{\mathsf{roRO}}$ interacts with a list of ideal functionalities $\bar{\mathcal{F}} = \{\mathcal{F}_1, \ldots, \mathcal{F}_n\}$, where $\mathcal{F}_1, \ldots, \mathcal{F}_n$ are the ideal functionalities for protocols. For any query $(\mathsf{sid}', x)$ from any party $P = (\mathsf{pid}, \mathsf{sid})$ where $\mathsf{sid}'$ is the content of the SID field, if $\mathsf{sid}' \neq \mathsf{sid}$, then this query is considered "illegitimate". After that, $\mathcal{G}_{\mathsf{roRO}}$ adds the tuple $(\mathsf{sid}', x, v)$ to the list of illegitimate queries for SID $\mathsf{sid}'$, which we denote as $\mathcal{Q}_{\mathsf{sid}'}$. The illegitimate queries $\mathcal{Q}_{\mathsf{sid}'}$ may be disclosed to an instance of ideal functionality $\mathcal{F} \in \bar{\mathcal{F}}$ whose SID is the one of the illegitimate queries, and the ideal functionality instance $\mathcal{F}$ may leak the illegitimate queries $\mathcal{Q}_{\mathsf{sid}'}$ to the simulator.

**Share Functionality $\mathcal{G}_{\mathsf{roRO}}$**

The functionality interacts with a set of parties $\mathcal{P} = \{P_1, \ldots, P_n\}$ and an adversary $\mathcal{S}$. It is parameterized by the output length $\ell_{\mathsf{out}}(\lambda)$ and a list of ideal functionalities $\bar{\mathcal{F}} := \{\mathcal{F}_1, \ldots, \mathcal{F}_n\}$. It maintains an initially empty list List.

**Query.** Upon receiving $(\textsc{Query}, \mathsf{sid}', x)$ from a party $P_i \in \mathcal{P}$ where $P_i = (\mathsf{pid}, \mathsf{sid})$, or the adversary $\mathcal{S}$, do the same as $\mathcal{F}_{\mathsf{RO}}$ depicted in Figure 8, except when $\mathsf{sid} \neq \mathsf{sid}'$, add the tuple $(\mathsf{sid}', x, v)$ to the (initially empty) list of illegitimate queries for SID $\mathsf{sid}'$, which we denote by $\mathcal{Q}_{\mathsf{sid}'}$.

**Observe.** Upon receiving a request from an instance of an ideal functionality $\mathcal{F}_i \in \bar{\mathcal{F}}$ with SID $\mathsf{sid}'$, return the list of illegitimate queries $\mathcal{Q}_{\mathsf{sid}'}$ for SID $\mathsf{sid}'$ to this instance $\mathcal{F}_i$.

Figure 9: The Global Restricted Observable Random Oracle Model $\mathcal{G}_{\mathsf{roRO}}$

**Share Functionality $\mathcal{G}_{\mathsf{rpRO}}$**

The functionality interacts with a set of parties $\mathcal{P} = \{P_1, \ldots, P_n\}$ and an adversary $\mathcal{S}$. It is parameterized by the output length $\ell_{\mathsf{out}}(\lambda)$. It maintains two initially empty lists $\mathsf{List}, \mathsf{Prog}$.

**Query.** Upon receiving $(\textsc{Query}, \mathsf{sid}', x)$ from a party $P_i \in \mathcal{P}$ where $P_i = (\mathsf{pid}, \mathsf{sid})$, or the adversary $\mathcal{S}$, do the same as $\mathcal{F}_{\mathsf{RO}}$ depicted in Figure 8.

**Program.** Upon receiving $(\textsc{Program}, \mathsf{sid}, x, v)$ with $v \in \{0,1\}^{\ell_{\mathsf{out}}(\lambda)}$ from the adversary $\mathcal{S}$:

- Check if $\exists\, v' \in \{0,1\}^{\ell_{\mathsf{out}}(\lambda)}$ s.t. $(\mathsf{sid}, x, v') \in \mathsf{List}$ and $v \neq v'$. If so, ignore this input.
- Set $\mathsf{List} := \mathsf{List} \cup \{(\mathsf{sid}, x, v)\}$ and $\mathsf{Prog} := \mathsf{Prog} \cup \{(\mathsf{sid}, x)\}$.
- Return $(\textsc{ProgramConfirm}, \mathsf{sid})$ to $\mathcal{S}$.

**IsProgramed.** Upon receiving $(\textsc{IsProgramed}, \mathsf{sid}', x)$ from a party $P_i \in \mathcal{P}$ where $P_i = (\mathsf{pid}, \mathsf{sid})$, or the adversary $\mathcal{S}$:

- If the input was given by $P_i = (\mathsf{pid}, \mathsf{sid})$ and $\mathsf{sid} \neq \mathsf{sid}'$, ignore this input.
- If $(\mathsf{sid}', x) \in \mathsf{Prog}$, set $b := 1$; otherwise, set $b := 0$.
- Return $(\textsc{IsProgramed}, \mathsf{sid}', b)$ to the requester.

Figure 10: The Global Restricted Programmable Random Oracle Model $\mathcal{G}_{\mathsf{roRO}}$

**The GrpRO Model.** The GrpRO is also modeled as a share functionality $\mathcal{G}_{\mathsf{rpRO}}$, and it answers to the queries in the same way as $\mathcal{F}_{\mathsf{RO}}$. The simulator is only granted the restricted programmability: both the adversary and the simulator are allowed to program the unqueried points of the random oracle, but only the simulator can program it without being detected. More precisely, as depicted in Figure 10, upon receiving $(\textsc{Program}, \mathsf{sid}, x, v)$ from the simulator/adversary, $\mathcal{G}_{\mathsf{rpRO}}$ first checks whether $(\mathsf{sid}, x)$ has been queried before. If not, $\mathcal{G}_{\mathsf{rpRO}}$ stores $(\mathsf{sid}, x, v)$ in the query-answer lists. Any honest party can check whether a point has been programmed or not by sending the $(\textsc{IsProgramed}, \mathsf{sid}, x)$ to $\mathcal{G}_{\mathsf{rpRO}}$. Thus, in the real world, the programmed points can always be detected. However, in the ideal world,

the simulator $\mathcal{S}$ can escape the detection since it can return $(\text{IsProgramed}, \mathsf{sid}, 0)$ when the adversary invokes $(\text{IsProgramed}, \mathsf{sid}, x)$ to verify whether a point $x$ has been programmed or not.

### 2.6 Computational Assumptions

Recall that, throughout the work, we let $q$ be a $\lambda$-bit prime, and $p = 2q + 1$ also be a prime. We also let $\mathbb{G}$ be a subgroup of order $q$ of $\mathbb{Z}_p^*$ with the generator $g$. We then present the formal descriptions about the computation assumptions that will be used in this work, i.e., Computational Diffie-Hellman (CDH) assumption and Decisional Diffie-Hellman (DDH) assumption [DH76], as follows.

**Definition 15** (Computational Diffie-Hellman Assumption)**.** *We say that the Computational Diffie-Hellman (CDH) assumption holds in a group $\mathbb{G}$ if there is a negligible function $\mathsf{negl}$ such that for any PPT adversary $\mathcal{A}$, we have*

$$\Pr[\mathcal{A}(\mathbb{G}, p, q, g, g^r, g^s) = g^{rs}] \leq \mathsf{negl}(\lambda)$$

*where $r, s \leftarrow \mathbb{Z}_q$.*

**Definition 16** (Decisional Diffie-Hellman Assumption)**.** *We say that the Decisional Diffie-Hellman (DDH) assumption holds in a group $\mathbb{G}$ if there is a negligible function $\mathsf{negl}$ such that for any PPT adversary $\mathcal{A}$, we have*

$$|\Pr[\mathcal{A}(\mathbb{G}, p, q, g^r, g^s, g^{rs}) = 1] - \Pr[\mathcal{A}(\mathbb{G}, p, q, g, g^r, g^s, Z) = 1]| \leq \mathsf{negl}(\lambda)$$

*where $Z \leftarrow \mathbb{G}$ and $r, s \leftarrow \mathbb{Z}_q$.*

## 3 UC-Secure Endemic OT via Random Oracles

In this section, we construct two 1-round UC-secure EOT protocols under standard assumptions in the RO model. The first construction achieves static security while the second one achieves adaptive security. Note that, in our protocol construction, we make use of the well-known Pedersen commitment [Ped92] and the ElGamal encryption [ElG84], and we refer interesting readers to see formal description about them in Appendix A.

### 3.1 Our Statically Secure EOT

We start with the two-round standalone EOT protocol in the RO model proposed in [CO15]: in the first round, the sender sends $h := g^s$ using $s \leftarrow \mathbb{Z}_q$; in the second round, the receiver uses sender's message to compute $B := g^r h^b$ based on its choice bit $b$ and its secret randomness $r \leftarrow \mathbb{Z}_q$; finally, the sender computes and outputs $m_0 := \mathcal{F}_{\mathsf{RO}}(B^s)$ and $m_1 := \mathcal{F}_{\mathsf{RO}}((\frac{B}{h})^s)$ while the receiver outputs $m_b := \mathcal{F}_{\mathsf{RO}}(h^r)$. Here we use to notation $y := \mathcal{F}_{\mathsf{RO}}(x)$ to describe the process for querying $x$ to the random oracle $\mathcal{F}_{\mathsf{RO}}$ and obtaining the output $y$, which aligns with the notation in [CSW20a]. Since $B$ can be viewed as the Pedersen commitment of the choice bit $b$, $b$ is perfectly hidden in $B$ and thus a malicious sender cannot learn $b$ from the receiver's message. On the other hand, since the sender's randomness $s$ is kept secret from the receiver, a malicious receiver cannot compute both messages as it requires computing $h^s$ (as it involves querying $B^s, (\frac{B}{h})^s$ to the RO). Such a computation is hard by CDH assumption. This completes the standalone security of this protocol. Although this protocol is very simple and efficient, it cannot achieve UC security [LM18, GIR20].

Our goals are: (i) reduce the round complexity of this protocol to one simultaneous round; (ii) add new mechanisms to make this protocol UC-secure. In order to reduce the round complexity, we let the receiver generate $h$ by invoking the RO on a randomly sampled string $\mathsf{seed}$. In this way, the receiver can compute its message without the sender's message, thus only one simultaneous round is needed. This technique can be found in [CSW20a].

We then discuss how to provide UC security. The UC-secure EOT protocol requires *extractability*: (i) when the sender is malicious, the simulator should be able to extract the sender's secret randomness, so the simulator can compute both $m_0$ and $m_1$; (ii) when the receiver is malicious, the simulator should be able to extract the receiver's choice bit $b$. In order to extract the sender's secret randomness $s$, we let the sender additionally generate a straight-line extractable NIZK argument [Pas03, Fis05, Ks22] of $s$ such that $z = g^s$. The straight-line extractability relies on the observability of the RO model. In this way, the simulator can extract the malicious sender's secret randomness. In order to extract the receiver's choice bit $b$, we let the receiver generate an ElGamal encryption of $b$ instead of a Pedersen commitment of $b$, i.e., the receiver computes $(u, v) := (h^r, h^b g^r)$ using $r \leftarrow \mathbb{Z}_q$. We also let the receiver generate a NIZK argument of $(b, r)$ such that $(u, v) = (h^r, h^b g^r)$ to ensure that $(u, v)$ is an ElGamal encryption of a bit $b$. In this way, the simulator knows $\log_g h$ by making use of the programmability of the RO model, and thus be able to extract $b$ from $(u, v)$.

Let $g$ be the generator of $\mathbb{G}$. Let $\mathcal{F}_{\mathsf{RO1}} : \{0,1\}^* \to \mathbb{G}$ and $\mathcal{F}_{\mathsf{RO2}} : \{0,1\}^* \to \{0,1\}^\lambda$. Let $\mathcal{R}_{\mathsf{ENC}} := \{((g, h, u, v), (r, b)) \mid (b = 0 \wedge (u, v) = (h^r, g^r)) \vee (b = 1 \wedge (u, v) = (h^r, g^r h))\}$ and $\mathcal{R}_{\mathsf{DL}} := \{((g, z), s) \mid z = g^s\}$. We denote by $\Pi_{\mathsf{sleNIZK}}$ the straight-line extractable NIZK argument in the $\mathcal{F}_{\mathsf{RO3}}$-hybrid world. We denote by $\Pi_{\mathsf{NIZK}}$ the NIZK argument in the $\mathcal{F}_{\mathsf{RO4}}$-hybrid world. We note that, the domain and range of $\mathcal{F}_{\mathsf{RO3}}$ and $\mathcal{F}_{\mathsf{RO4}}$ depend on the concrete instantiations of the protocols, for that reason, we do not write them explicitly. We assume synchronous channel $\mathcal{F}_{\mathsf{SYN}}$. Formally, we present our protocol $\Pi_{\mathsf{sEOT\text{-}RO}}$ in Figure 11 and prove the security through Theorem 1.

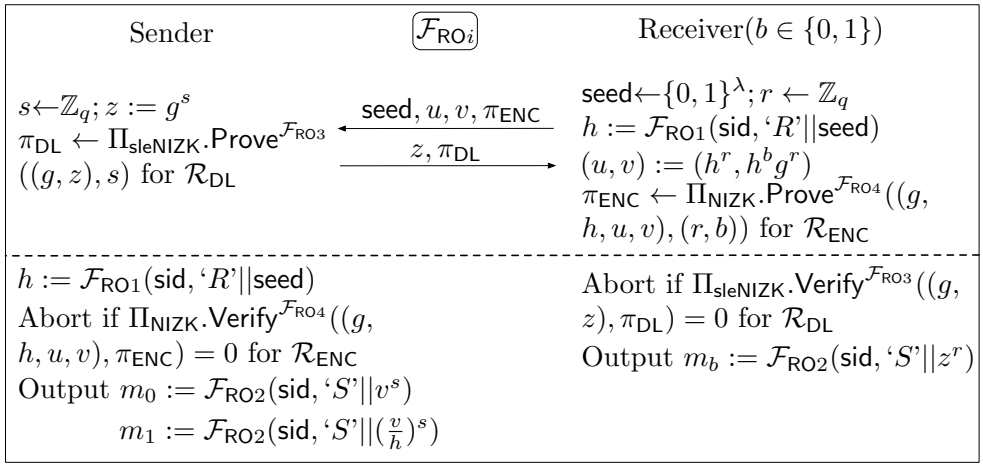



Figure 11: 1-round EOT protocol $\Pi_{\mathsf{sEOT\text{-}RO}}$ in the $\{\mathcal{F}_{\mathsf{RO}}, \mathcal{F}_{\mathsf{SYN}}\}$-hybrid world, where $\mathcal{F}_{\mathsf{RO}} = \{\mathcal{F}_{\mathsf{RO}i}\}_{i\in[4]}$. Let $g$ be the generator of $\mathbb{G}$. Let $\mathcal{F}_{\mathsf{RO1}} : \{0,1\}^* \to \mathbb{G}$ and $\mathcal{F}_{\mathsf{RO2}} : \{0,1\}^* \to \{0,1\}^\lambda$. Let $\mathcal{R}_{\mathsf{ENC}} := \{((g, h, u, v), (r, b)) \mid (b = 0 \wedge (u, v) = (h^r, g^r)) \vee (b = 1 \wedge (u, v) = (h^r, g^r h))\}$ and $\mathcal{R}_{\mathsf{DL}} := \{((g, z), s) \mid z = g^s\}$.

**Theorem 1.** *Assume the DDH assumption holds in group $\mathbb{G}$. Let $\mathcal{F}_{\mathsf{RO1}} : \{0,1\}^* \to \mathbb{G}$ and $\mathcal{F}_{\mathsf{RO2}} : \{0,1\}^* \to \{0,1\}^\lambda$ be the random oracles. Let $\Pi_{\mathsf{NIZK}}$ be an NIZK argument in the $\mathcal{F}_{\mathsf{RO3}}$-hybrid world. Let $\Pi_{\mathsf{sleNIZK}}$ be a straight-line extractable NIZK argument in the $\mathcal{F}_{\mathsf{RO4}}$-hybrid world. The protocol $\Pi_{\mathsf{sEOT\text{-}RO}}$ depicted in Figure 11 UC-realizes the functionality $\mathcal{F}_{\mathsf{EOT}}$ depicted in Figure 6 in the $\{\mathcal{F}_{\mathsf{RO}}, \mathcal{F}_{\mathsf{SYN}}\}$-hybrid world against static malicious corruption, where $\mathcal{F}_{\mathsf{RO}} = \{\mathcal{F}_{\mathsf{RO}i}\}_{i\in[4]}$.*

*Proof.* We leave the formal proof in Appendix C.1. □

**Instantiation.** We instantiate $\Pi_{\mathsf{sleNIZK}}$ for relation $\mathcal{R}_{\mathsf{DL}}$ with the Schnorr's protocol [Sch90] and the recently proposed Kondi-shelat transform [Ks22] which improves the Fischlin transform [Fis05]. We present the formal description of the Schnorr's protocol and the Kondi-shelat transform in Appendix B.1 and Appendix B.2 respectively.

We instantiate $\Pi_{\mathsf{NIZK}}$ for relation $\mathcal{R}_{\mathsf{ENC}}$ with the following techniques: we first employ the OR-composition [CDS94] to the Chaum-Pedersen protocols [CP93] to prove either $(g, h, v, u)$ is a DDH tuple (which means $b = 0$) or $(g, h, \frac{v}{h}, u)$ is a DDH tuple (which means $b = 1$), we then apply the the Fiat-Shamir transform [FS87] to remove the interaction. We present the formal description of the Chaum-Pedersen protocols and the OR-composition of the Sigma-protocols in Appendix B.1.

### 3.2 Our Adaptively Secure EOT

In this section, we try to modify our previous construction into an adaptive-secure one. Unlike the static adversaries, the adaptive adversaries can corrupt the parties at any time. Therefore, we have to add mechanisms such that the simulator can handle the case where the sender/receiver gets corrupted after its message is sent. In that case, the simulator needs to output the "fake" internal states that makes the simulated message as if it is generated by the honest sender/receiver algorithm.

Note that, the sender algorithm can remain the same as our static-secure construction. The intuition behind is very simple: the sender has no inputs, and the simulator runs the honest sender's algorithm to simulate the sender. Therefore, when the sender gets corrupted, the simulator simply reveals the internal states that used to generate the sender's message.

Now we turn our attention to the receiver. Different from the sender, the receiver has a private input, namely the choice bit $b \in \{0, 1\}$ that the simulator has no idea about. Therefore, we have to design a mechanism that the simulated receiver's message can be revealed to 0 or 1 as desired. Inspired by the techniques in [GOS06, CWS22], we take the following approaches: we let the receiver generate $h$ by invoking the RO on a randomly sampled string seed, and commit to the choice bit $b$ using a Pedersen commitment $B := g^r h^b$ where $r \leftarrow \mathbb{Z}_q$. The receiver then computes an ElGamal encryption of the randomness $r$, i.e., $(u_b, v_b) := (h^t, g^t h^r)$ using $t \leftarrow \mathbb{Z}_q$. Finally the receiver randomly selects $(u_{1-b}, v_{1-b}) \leftarrow \mathbb{G} \times \mathbb{G}$, and computes an NIZK argument with Honest Prover State Reconstruction (HPSR) of $(b, r, t)$ such that $B = g^r h^b$ and $(u_b, v_b) = (h^t, g^t h^r)$. Now we describe the strategy of the simulator: Since the simulator knows $\log_g h$ and the Pedersen commitment is a trapdoor commitment, the simulator can open $B$ to both 0 and 1, and we denote the two openings as $(0, r_0)$ and $(1, r_1)$. Then the simulator computes two ElGamal encryptions, i.e., $(u_0, v_0)$ (which is the encryption of $r_0$) and $(u_1, v_1)$ (which is the encryption of $r_1$). Finally the simulator simulates the NIZK proof $\pi$. Since the ElGamal ciphertext looks pseudorandom from the random group elements, when the receiver gets corrupted, the simulator simply opens the ciphertext containing the opening we want and claim that the other ciphertext was chosen randomly. Guaranteed by the honest prover state reconstruction property, the simulator can output the "fake" randomness as if the NIZK proof $\pi$ is generated by the honest prover algorithm.

Let $g$ be the generator of $\mathbb{G}$. Let $\mathcal{F}_{\mathsf{RO1}} : \{0,1\}^* \to \mathbb{G}$ and $\mathcal{F}_{\mathsf{RO2}} : \{0,1\}^* \to \{0,1\}^\lambda$. Let $\mathcal{R}_{\mathsf{HYB}} := \{((g, h, B, u_0, v_0, u_1, v_1), (b, r, t)) \mid (b = 0 \wedge B = g^r \wedge (u_0, v_0) = (h^t, g^t h^r)) \vee (b = 1 \wedge B = g^r h \wedge (u_0, v_0) = (h^t, g^t h^r))\}$ and $\mathcal{R}_{\mathsf{DL}} := \{((g, z), s) \mid z = g^s\}$. We denote by $\Pi_{\mathsf{sleNIZK}}$ the straight-line extractable NIZK argument in the $\mathcal{F}_{\mathsf{RO3}}$-hybrid world. We denote by $\Pi_{\mathsf{aNIZK}}$ the NIZK argument with HPSR in the $\mathcal{F}_{\mathsf{RO4}}$-hybrid world. We note that, the domain and range of $\mathcal{F}_{\mathsf{RO3}}$ and $\mathcal{F}_{\mathsf{RO4}}$ depend on the concrete instantiations of the protocols, for that reason, we do not write them explicitly. We assume the synchronous channel $\mathcal{F}_{\mathsf{SYN}}$. Formally, we present our protocol $\Pi_{\mathsf{aEOT\text{-}RO}}$ in Figure 12 and prove the security through Theorem 2.

**Theorem 2.** *Assume the DDH assumption holds in group $\mathbb{G}$. Let $\mathcal{F}_{\mathsf{RO1}} : \{0,1\}^* \to \mathbb{G}$ and $\mathcal{F}_{\mathsf{RO2}} : \{0,1\}^* \to \{0,1\}^\lambda$ be the random oracles. Let $\Pi_{\mathsf{aNIZK}}$ be an NIZK argument with honest prover state reconstruction in the $\mathcal{F}_{\mathsf{RO3}}$-hybrid world. Let $\Pi_{\mathsf{sleNIZK}}$ be a straight-line extractable NIZK argument in the $\mathcal{F}_{\mathsf{RO4}}$-hybrid world. The protocol $\Pi_{\mathsf{aEOT\text{-}RO}}$ depicted in Figure 12 UC-realizes the functionality $\mathcal{F}_{\mathsf{EOT}}$ depicted in Figure 6 in the $\{\mathcal{F}_{\mathsf{RO}}, \mathcal{F}_{\mathsf{SYN}}\}$-hybrid world against adaptive malicious corruption, where $\mathcal{F}_{\mathsf{RO}} = \{\mathcal{F}_{\mathsf{RO}i}\}_{i \in [4]}$.*

*Proof.* We leave the formal proof in Appendix C.2. □

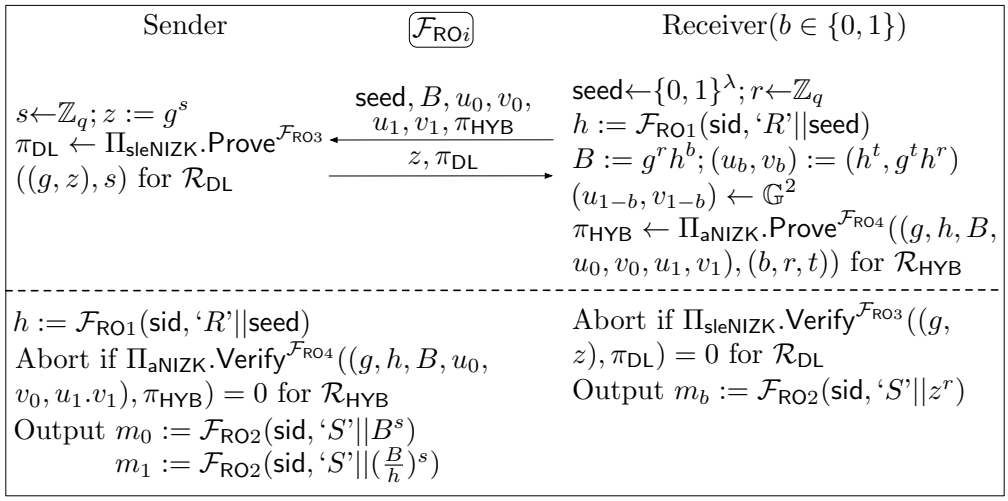



Figure 12: 1-round EOT protocol $\Pi_{\mathsf{aEOT\text{-}RO}}$ in the $\{\mathcal{F}_{\mathsf{RO}}, \mathcal{F}_{\mathsf{SYN}}\}$-hybrid world, where $\mathcal{F}_{\mathsf{RO}} = \{\mathcal{F}_{\mathsf{RO}i}\}_{i\in[4]}$. Let $g$ be the generator of $\mathbb{G}$. Let $\mathcal{F}_{\mathsf{RO1}} : \{0,1\}^* \to \mathbb{G}$ and $\mathcal{F}_{\mathsf{RO2}} : \{0,1\}^* \to \{0,1\}^\lambda$. Let $\mathcal{R}_{\mathsf{HYB}} := \{((g,h,B,u_0,v_0,u_1,v_1),(b,r,t)) \mid (b = 0 \wedge B = g^r \wedge (u_0,v_0) = (h^t, g^t h^r)) \vee (b = 1 \wedge B = g^r h \wedge (u_0,v_0) = (h^t, g^t h^r))\}$ and $\mathcal{R}_{\mathsf{DL}} := \{((g,z),s) \mid z = g^s\}$.

**From Augmented Sigma-Protocols to NIZK Arguments with HPSR.** One main building block of our protocol $\Pi_{\mathsf{aEOT\text{-}RO}}$ is NIZK arguments with HPSR in the RO model. Here we discuss how to instantiate it with augmented Sigma-protocols.

It is well-known that one can obtain an NIZK argument in the RO model by applying Fiat-Shamir transform to the (standard) Sigma-protocol. It is natural for us to ask: what if we apply the Fiat-Shamir transform to the augmented Sigma-protocol? Can we get an NIZK argument with HPSR in the RO model? In fact, we get the positive answers to these questions. Formally, we present the protocol in Figure 13 and prove security through Theorem 3. Note that, here the session ID sid is not important, thus we omit it for simplicity.

**Protocol $\Pi_{\mathsf{FS\text{-}NIZK}}$**

**Primitive**: Augmented Sigma-protocol $\Pi_\Sigma$ for relation $\mathcal{R}$.
**Random Oracle**: $\mathcal{F}_{\mathsf{RO}} : \{0,1\}^* \to \{0,1\}^*$.
**Common Input**: Statement $x$.
**Private Input of the Prover**: Witness $w$ such that $(x,w) \in \mathcal{R}$.

- $\mathsf{Prove}^{\mathcal{F}_{\mathsf{RO}}}(x,w)$: It invokes $(a,\mathsf{st}) \leftarrow \Pi_\Sigma.\mathsf{P1}(x,w)$, then computes $e := \mathcal{F}_{\mathsf{RO}}(x,a)$, finally computes $z \leftarrow \Pi_\Sigma.\mathsf{P2}(x,w,e,\mathsf{st})$ and outputs $\pi := (a,z)$.
- $\mathsf{Verify}^{\mathcal{F}_{\mathsf{RO}}}(x,\pi)$: It computes $e := \mathcal{F}_{\mathsf{RO}}(x,a)$, then invokes $b := \Pi_\Sigma.\mathsf{Verify}(x,a,e,z)$, finally outputs $b$.
- $\mathsf{Sim}(x)$: It simulates $\mathcal{F}'_{\mathsf{RO}}$ itself, then invokes $e' \leftarrow \Pi_\Sigma.\mathsf{V1}(1^\lambda)$ and $(a',z') := \Pi_\Sigma.\mathsf{Sim}(x,e';r)$ using the randomness $r$, finally programs $\mathcal{F}'_{\mathsf{RO}}$ such that $e' = \mathcal{F}'_{\mathsf{RO}}(x,a')$. It outputs $\pi' := (a',z')$ and $\mathcal{F}'_{\mathsf{RO}}$.
- $\mathsf{StateRec}^{\mathcal{F}'_{\mathsf{RO}}}(x,w,\pi',r)$: It computes $e' := \mathcal{F}'_{\mathsf{RO}}(x,a')$, then invokes $\mathsf{st}' \leftarrow \Pi_\Sigma.\mathsf{StateRec}(x,w,a',e',z',r)$, finally outputs $\mathsf{st}'$.

Figure 13: NIZK Argument with HPSR $\Pi_{\mathsf{FS\text{-}NIZK}}$ from Augmented Sigma-Protocol

**Theorem 3.** *Assume $\Pi_\Sigma$ is an augmented Sigma-protocol for relation $\mathcal{R}$ with negligible soundness error, then the protocol $\Pi_{\mathsf{FS\text{-}NIZK}}$ depicted in Figure 13 is an NIZK argument with HPSR for $\mathcal{R}$ in the RO model.*

*Proof.* The perfect completeness, computational soundness and computational zero-knowledge are triv-

ial, the proof comes directly from the well-known fact that one can obtain an NIZK argument in the RO model by applying Fiat-Shamir transform to a (standard) Sigma-protocol.

The only thing left is to show the Honest Prover State Reconstruction (HPSR). Let $\mathcal{F}_{\mathsf{RO}}$ be the random oracle, $\pi := (a, z)$ be the real proof and st be the prover's randomness. Let $\mathcal{F}'_{\mathsf{RO}}$ be the random oracle that simulated by $\Pi_{\mathsf{FS\text{-}NIZK}}.\mathsf{Sim}$, $\pi' := (a', z')$ be the simulated proof and st′ be the simulator's randomness. The honest prover state reconstruction requires that no PPT adversary $\mathcal{A}$ can distinguish the $\{\pi, \mathsf{st}, \mathcal{F}_{\mathsf{RO}}\}$ from the $\{\pi', \mathsf{st}', \mathcal{F}'_{\mathsf{RO}}\}$ with non-negligible probability. Note that, the computational zero-knowledge property guarantees that $\{\pi, \mathcal{F}_{\mathsf{RO}}\}$ and $\{\pi', \mathcal{F}'_{\mathsf{RO}}\}$ are computationally indistinguishable. Therefore, we only have to show no PPT adversary $\mathcal{A}$ can distinguish $\{\mathsf{st}\}$ from $\{\mathsf{st}'\}$ with non-negligible probability.

We observe that if such a PPT adversary $\mathcal{A}$ exists, then we can build a PPT reduction algorithm $\mathcal{B}$ that breaks the reverse state construction of $\Pi_\Sigma$. Our $\mathcal{B}$ simulates $\mathcal{F}_{\mathsf{RO}}$ and interacts with the reverse state construction game challenger $\mathcal{C}$. Our $\mathcal{B}$ starts the protocol $\Pi_{\mathsf{FS\text{-}NIZK}}$ with the adversary $\mathcal{A}$ by internally running $\mathcal{A}$ as a black-box. Upon receiving $(x, w)$ from $\mathcal{A}$, the reduction algorithm $\mathcal{B}$ checks if $(x, w) \in \mathcal{R}$. If so, our $\mathcal{B}$ forwards $(x, w)$ with a randomly sampled $e$ to the challenger $\mathcal{C}$. Upon receiving $(x, w, a, e, z, \mathsf{st})$ from $\mathcal{C}$, $\mathcal{B}$ programs $\mathcal{F}_{\mathsf{RO}}$ such that $e = \mathcal{F}_{\mathsf{RO}}(x, a)$ and forwards the message to $\mathcal{A}$. When $\mathcal{A}$ outputs a bit $b$, $\mathcal{B}$ simply forwards it to the challenger $\mathcal{C}$. Clearly, $\mathcal{B}$ wins whenever $\mathcal{A}$ wins. Hence, we prove that $\Pi_{\mathsf{FS\text{-}NIZK}}$ is an NIZK argument with HPSR for $\mathcal{R}$ in the RO model. □

Another interesting question to ask is: what if we apply the OR-composition to the augmented Sigma-protocols? Can we get an augmented Sigma-protocol? The answers are still positive. Formally, we present the resulting protocol in Figure 14 and prove the security through Theorem 4.

**Protocol $\Pi^{\vee}_{\Sigma}$**

**Primitive**: Augmented Sigma-protocol $\Pi^0_\Sigma$ for relation $\mathcal{R}_0$, augmented Sigma-protocol $\Pi^1_\Sigma$ for relation $\mathcal{R}_1$.
**Common Input**: Statement $x := (x_0, x_1)$.
**Private Input of the Prover**: Witness $w$ such that $(x_b, w) \in \mathcal{R}_b$.

---

- $\mathsf{P1}(x, w)$: It first computes $(a_b, \mathsf{st}_b) \leftarrow \Pi^b_\Sigma.\mathsf{P1}(x_b, w)$, then selects $s_{1-b} \leftarrow \Pi^{1-b}_\Sigma.\mathsf{V1}(1^\lambda)$ and computes $(a_{1-b}, z_{1-b}) := \Pi^{1-b}_\Sigma.\mathsf{Sim}(x_{1-b}, s_{1-b}; \mathsf{st}_{1-b})$ using randomness $\mathsf{st}_{1-b}$. It outputs $a := (a_0, a_1), \mathsf{st} := (\mathsf{st}_b, \mathsf{st}_{1-b})$.
- $\mathsf{V1}(1^\lambda)$: It selects an uniformly random $e$ and outputs $e$.
- $\mathsf{P2}(x, w, e, \mathsf{st})$: It first sets $s_b := e \oplus s_{1-b}$, then computes $z_b \leftarrow \Pi^b_\Sigma.\mathsf{P2}(x_b, w, s_b, \mathsf{st}_b)$, finally outputs $z := (s_0, z_0, s_1, z_1)$.
- $\mathsf{Verify}(x, a, e, z)$: It checks if $e = s_0 \oplus s_1$ and $\Pi^0_\Sigma.\mathsf{Verify}(x_0, a_0, s_0, z_0) = \Pi^1_\Sigma.\mathsf{Verify}(x_1, a_1, s_1, z_1) = 1$ hold. If so, it outputs 1; otherwise, it outputs 0.
- $\mathsf{Ext}(x, a, e, \tilde{e}, z, \tilde{z})$: It checks if there exists a $b \in \{0, 1\}$ such that $s_b = \tilde{s}_b$. If so, it sets $\beta := 1 - b$; otherwise, it selects $\beta \leftarrow \{0, 1\}$. It invokes $w \leftarrow \Pi^\beta_\Sigma.\mathsf{Ext}(x_\beta, a_\beta, s_\beta, \tilde{s}_\beta, z_\beta, \tilde{z}_\beta)$ and outputs $w$.
- $\mathsf{Sim}(x, e')$: It first selects $s'_0 \leftarrow \Pi^0_\Sigma.\mathsf{V1}(1^\lambda)$, then computes $s'_1 := e' \oplus s'_0$, and invokes $(a'_0, z'_0) := \Pi^0_\Sigma.\mathsf{Sim}(x_0, s'_0; r_0)$ using randomness $r_0$ and invokes $(a'_1, z'_1) := \Pi^1_\Sigma.\mathsf{Sim}(x_1, s'_1; r_1)$ using randomness $r_1$. It outputs $(a' := (a'_0, a'_1), z' := (z'_0, z'_1))$.
- $\mathsf{StateRec}(x, w, a', e', z', (r_0, r_1))$: It determines $b$ such that $(x_b, w) \in \mathcal{R}_b$, then invokes $\mathsf{st}'_b \leftarrow \Pi^b_\Sigma(x_b, w_b, a'_b, s'_b, z'_b, r_b)$, finally outputs $\mathsf{st}' := (\mathsf{st}'_b, r_{1-b})$.

Figure 14: Augmented Sigma-Protocol $\Pi^{\vee}_{\Sigma}$ for Relation $\mathcal{R}_0 \vee \mathcal{R}_1$

**Theorem 4.** *Assume $\Pi^0_\Sigma$ is an augmented Sigma-protocol for relation $\mathcal{R}_0$ and $\Pi^1_\Sigma$ is an augmented Sigma-protocol for relation $\mathcal{R}_1$, then the protocol $\Pi^{\vee}_{\Sigma}$ depicted in Figure 14 is an augmented Sigma-protocol for relation $\mathcal{R}_0 \vee \mathcal{R}_1$.*

*Proof.* Analogously to Theorem 3, we only have to show the reverse state construction. Let $\pi := (a, z)$ be the real transcript and st be the prover's randomness. Let $\pi' := (a', z')$ be the simulated transcript and st′ be the simulator's randomness. The reverse state construction requires that no PPT adversary $\mathcal{A}$

can distinguish the $\{a, z, \mathsf{st}\}$ from the $\{a', z', \mathsf{st}'\}$ with non-negligible probability. Note that, the SHVZK property guarantees that $\{a, z\}$ and $\{a', z'\}$ are computationally indistinguishable. Therefore, we only have to show no PPT adversary $\mathcal{A}$ can distinguish $\{\mathsf{st}\}$ from $\{\mathsf{st}'\}$ with non-negligible probability.

We observe that if such a PPT adversary $\mathcal{A}$ exists, then we can build a PPT reduction algorithm $\mathcal{B}$ that breaks the reverse state construction of $\Pi_\Sigma^b$. Our $\mathcal{B}$ interacts with the reverse state construction game challenger $\mathcal{C}$. Then $\mathcal{B}$ starts the protocol $\Pi_\Sigma^\vee$ with the adversary $\mathcal{A}$ by internally running $\mathcal{A}$ as a black-box. Upon receiving $(x, w, e)$ from $\mathcal{A}$, $\mathcal{B}$ checks if there exists a $b \in \{0, 1\}$ such that $(x_b, w) \in \mathcal{R}_b$. If so, $\mathcal{B}$ selects a random $s_{1-b}$, computes $s_b := e \oplus s_{1-b}$ and forwards $(x_b, w, e)$ to the challenger $\mathcal{C}$. Upon receiving $(a_b, z_b, \mathsf{st}_b)$ from $\mathcal{C}$, $\mathcal{B}$ invokes $(a_{1-b}, z_{1-b}) := \Pi_\Sigma^{1-b}.\mathsf{Sim}(x_{1-b}, s_{1-b}; \mathsf{st}_{1-b})$ using randomness $\mathsf{st}_{1-b}$ and forwards $(a := (a_0, a_1), z := (s_0, z_0, s_1, z_1), \mathsf{st} := (\mathsf{st}_0, \mathsf{st}_1))$ to $\mathcal{A}$. When $\mathcal{A}$ outputs a bit $\beta$, $\mathcal{B}$ forwards it to $\mathcal{C}$. Clearly, $\mathcal{B}$ wins whenever $\mathcal{A}$ wins. Hence, we prove that $\Pi_\Sigma^\vee$ has reverse state construction, and is an augmented Sigma-protocol for relation $\mathcal{R}_0 \vee \mathcal{R}_1$. □

**Instantiation.** We instantiate $\Pi_{\mathsf{sleNIZK}}$ for relation $\mathcal{R}_{\mathsf{DL}}$ with the Schnorr's protocol and the Kondi-shelat transform as in Section 3.1.

With Theorem 3 and Theorem 4, we can instantiate $\Pi_{\mathsf{aNIZK}}$ for $\mathcal{R}_{\mathsf{HYB}}$ easily. Note that, the relation $\mathcal{R}_{\mathsf{HYB}}$ can be represented as $\mathcal{R}_0 \vee \mathcal{R}_1$, where $\mathcal{R}_b := \{((g, h, B, u_0, v_0, u_1, v_1), (b, r, t)) \mid B = g^r h^b \wedge (u_0, v_0) = (h^t, g^t h^r))\}$ for $b \in \{0, 1\}$. We then perform the following approaches: we first use the OR-composition of the augmented Sigma-protocol for $\mathcal{R}_0$ and the one for $\mathcal{R}_1$, then apply Fiat-Shamir transform to remove the interaction. We put the augmented Sigma-protocol $\Pi_\Sigma^0$ for relation $\mathcal{R}_0$ in Figure 15 and prove the security through Theorem 5; note that, the augmented Sigma-protocol $\Pi_\Sigma^1$ for relation $\mathcal{R}_1$ can be designed in a similar way, and we omit it here.

Protocol $\Pi_\Sigma^0$

**Common Input:** Statement $x := (\mathbb{G}, p, q, g, h, B, u_0, v_0, u_1, v_1)$, where $p, q$ describe the group $\mathbb{G}$ and $g, h, B, u_0, v_0, u_1, v_1$ are the group elements of $\mathbb{G}$.
**Private Input of the Prover:** Witness $w := (0, r, t)$ s.t. $B = g^r$ and $(u_0, v_0) = (h^t, g^t h^r)$.

- $\mathsf{P1}(x, w)$: It first selects $s_0, s_1 \leftarrow \mathbb{Z}_q$, then computes $a_0 := g^{s_0} \mod p$, $a_1 := h^{s_1} \mod p$, $a_2 := g^{s_1} h^{s_0} \mod p$, finally outputs $a := (a_0, a_1, a_2), \mathsf{st} := (s_0, s_1)$.
- $\mathsf{V1}(1^\lambda)$: It selects $e \leftarrow \mathbb{Z}_q$ and outputs $e$.
- $\mathsf{P2}(x, w, e, \mathsf{st})$: It first sets $z_0 := s_0 + e \cdot r \mod q$, $z_1 := s_1 + e \cdot t \mod q$, then outputs $z := (z_0, z_1)$.
- $\mathsf{Verify}(x, a, e, z)$: It checks if $g^{z_0} = a_0 \cdot B^e \mod p$, $h^{z_1} = a_1 \cdot u_0^e \mod p$ and $g^{z_1} h^{z_0} = a_2 \cdot v_0^e \mod p$ hold. If so, it outputs 1; otherwise, it outputs 0.
- $\mathsf{Ext}(x, a, e, \tilde{e}, z, \tilde{z})$: It computes $r := (z_0 - \tilde{z}_0)(e - \tilde{e})^{-1} \mod q$ and $t := (z_1 - \tilde{z}_1)(e - \tilde{e})^{-1} \mod q$, then outputs $w := (r, t)$.
- $\mathsf{Sim}(x, e')$: It first selects $r'_0, r'_1 \leftarrow \mathbb{Z}_q$, then sets $z'_0 := r'_0, z'_1 := r'_1$, finally computes $a'_0 := g^{z'_0} / B^{e'} \mod p$, $a'_1 := h^{z'_1} / u_0^{e'} \mod p$, $a'_2 := g^{z'_1} h^{z'_0} / v_0^{e'} \mod p$. It outputs $a' := (a'_0, a'_1, a'_2), z' := (z'_0, z'_1)$.
- $\mathsf{StateRec}(x, w, a', e', z', (r'_0, r'_1))$: It first computes $s'_0 := r'_0 - e' \cdot r \mod q$, $s'_1 := r'_1 - e' \cdot t \mod q$, then outputs $\mathsf{st}' := (s'_0, s'_1)$.

Figure 15: Augmented Sigma-protocol $\Pi_\Sigma^0$ for relation $\mathcal{R}_0$, where $\mathcal{R}_0 := \{((g, h, B, u_0, v_0, u_1, v_1), (0, r, t)) \mid B = g^r \wedge (u_0, v_0) = (h^t, g^t h^r))\}$.

**Theorem 5.** *Assume the Decisional Diffie-Hellman assumption holds in group $\mathbb{G}$, then the protocol $\Pi_\Sigma^0$ depicted in Figure 15 is an augmented Sigma-protocol for relation $\mathcal{R}_0$.*

*Proof. Perfect Completeness.* It is straightforward.

*Special Soundness.* We have described the extractor $\Pi_\Sigma^0.\mathsf{Ext}$ in Figure 15. More precisely, we first extract $z_0, \tilde{z}_0$ from $z, \tilde{z}$. Since $z_0, \tilde{z}_0$ are verified, we have the following equations: $g^{z_0} = a_0 \cdot B^e \mod p$, $g^{\tilde{z}_0} =$

$a_0 \cdot B^{\tilde{e}} \mod p$. Dividing one equation by the other, we get $g^{z_0-\tilde{z}_0} = B^{e-\tilde{e}} \mod p$. Now by condition $e - \tilde{e} \neq 0 \mod q$, so it has a multiplicative inverse modulo $q$. Therefore, we can get $B = g^{(z_0-\tilde{z}_0)(e-\tilde{e})^{-1}} \mod p$, in other words $r = (z_0 - \tilde{z}_0)(e - \tilde{e})^{-1} \mod q$. In the similar approach, we can extract $t = (z_1 - \tilde{z}_1)(e - \tilde{e})^{-1} \mod q$. In conclusion, given two distinct accepting transcripts, we can extract the witness $w := (r, t)$ efficiently.

*SHVZK.* We have described the simulator $\Pi^0_\Sigma$.Sim in Figure 15. More precisely, given the challenge $e'$ ahead, first selects $r'_0, r'_1 \leftarrow \mathbb{Z}_q$, then sets $z'_0 := r'_0, z'_1 := r'_1$, finally computes $a'_0 := g^{z'_0}/B^{e'} \mod p$, $a'_1 := h^{z'_1}/u_0^{e'} \mod p$, $a'_2 := g^{z'_1}h^{z'_0}/v_0^{e'} \mod p$. It outputs $a' := (a'_0, a'_1, a'_2), z' := (z'_0, z'_1)$. It is easy to see that the simulated $(a', e', z')$ is an accepting transcript. Now we argue the indistinguishability. Note that, in the real transcript, $z_0 = s_0 + e \cdot r \mod q, z_1 = s_1 + e \cdot t \mod q$ where $s_0, s_1$ are uniformly random in $\mathbb{Z}_q$, thus the real $z_0, z_1$ are also uniformly random in $\mathbb{Z}_q$. In the simulated transcript, $z'_0, z'_1$ are uniformly sampled in $\mathbb{Z}_q$. Thus $z$ and $z'$ are perfectly indistinguishable. Conditioned on $e, z$ (resp. $e', z'$), $a$ (resp. $a'$) is determined. In conclusion, given $e = e'$, $\{a, z\}$ are perfectly indistinguishable from $\{a', z'\}$.

*Reverse State Construction.* We have described the state reconstruction algorithm $\Pi^0_\Sigma$.StateRec in Figure 15. More precisely, given the statement-witness pair $(x, w)$, the simulated proof $(a', e', z')$ and the simulator's randomness $r'_0, r'_1$, it first computes $s'_0 := r'_0 - e' \cdot r \mod q$, $s'_1 := r'_1 - e' \cdot t \mod q$, then outputs $\mathsf{st}' := (s'_0, s'_1)$. Since $a'_0 = g^{s'_0} \mod p$, $a'_1 = h^{s'_1} \mod p$ and $a'_2 = g^{s'_1}h^{s'_0} \mod p$ hold, the state $\mathsf{st}' = (s'_0, s'_1)$ can be viewed as the honest prover's state. Now we argue the indistinguishability. Since we have already argued that given $e = e'$, $\{a, z\}$ are perfectly indistinguishable from $\{a', z'\}$, we only have to show $\{\mathsf{st} = (s_0, s_1)\}$ and $\{\mathsf{st}' = (s'_0, s'_1)\}$ are indistinguishable. In the real life scenario, $s_0, s_1$ are uniformly random in $\mathbb{Z}_q$. In the simulation scenario, $s'_0 = r'_0 - e' \cdot r \mod q$, $s'_1 = r'_1 - e' \cdot t \mod q$ where $r'_0, r'_1$ are uniformly random in $\mathbb{Z}_q$, thus $s'_0, s'_1$ are also uniformly random in $\mathbb{Z}_q$. In other words, $\{\mathsf{st} = (s_0, s_1)\}$ and $\{\mathsf{st}' = (s'_0, s'_1)\}$ are perfectly indistinguishable. □

# 4 The Relations between Endemic OT and Other Primitives

In this section, we first reveal a separation between 1-out-of-2 OT and EOT. Then we show how to construct a bit commitment protocol via EOT with unconditional security. Subsequently, we complete the picture of OT relations in [MR19a], showing that UOT can be constructed via EOT with unconditional security.

## 4.1 The Separation between Endemic OT and OT

First of all, we show it is impossible to construct a 1-round UC-secure 1-out-of-2 OT protocol in the synchronous channel model $\mathcal{F}_{\mathsf{SYN}}$ via Theorem 6.

**Theorem 6.** *There exists* no *1-round protocols* $\Pi$ *that UC-realizes functionality* $\mathcal{F}_{\mathsf{OT}}$ *depicted in Figure 4 with static security in the* $\mathcal{F}_{\mathsf{SYN}}$*-hybrid world.*

*Proof.* We proceed by contraction. Suppose that there exists such a protocol $\Pi$ that UC-realizes $\mathcal{F}_{\mathsf{OT}}$ in the $\mathcal{F}_{\mathsf{SYN}}$-hybrid world. Then there must exist a PPT simulator $\mathcal{S}$ such that $\mathsf{EXEC}_{\mathcal{F}_{\mathsf{EOT}},\mathcal{S},\mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}^{\mathcal{F}_{\mathsf{SYN}}}_{\Pi,\mathcal{A},\mathcal{Z}}$ for any PPT adversary $\mathcal{A}$ and any PPT environment $\mathcal{Z}$.

Let $\mathcal{A}$ be a dummy adversary that simply forwards protocol flows between the corrupt party and the environment $\mathcal{Z}$. We consider the following case: the environment $\mathcal{Z}$ corrupts the receiver $R^*$ at first and feeds the honest sender $S$ with two messages $m_0, m_1 \leftarrow \{0,1\}^\lambda$. Then $\mathcal{Z}$ waits for $S$ to send its message $\pi_S$. Note that, once the message $\pi_S$ is given to $\mathcal{F}_{\mathsf{SYN}}$, $\mathcal{Z}$ is allowed to see it. After $\mathcal{Z}$ obtaining $\pi_S$, $\mathcal{Z}$ chooses a random bit $b \leftarrow \{0,1\}$ and instructs $R^*$ to execute the honest receiver's algorithm, i.e., generates and sends receiver's message $\pi_R$. Finally, $\mathcal{Z}$ performs the local computation to obtain $m_{b'}$. If $m_b = m_{b'}$, $\mathcal{Z}$ outputs 1; otherwise, $\mathcal{Z}$ outputs 0.

In order to make the UC experiment above remain indistinguishable, the simulator $\mathcal{S}$ needs to determine $m_b$ before sending $\pi_S$. However, the adversary instructs $R^*$ to execute its algorithm until $\pi_S$ is obtained. There are only two ways for $\mathcal{S}$ to obtain $m_b$: (i) obtain the $m_b$ from $\mathcal{F}_{\mathsf{OT}}$ by guessing the choice bit $b$ of the receiver correctly, which would happen at probability $\frac{1}{2}$; (ii) guess the message $m_b$ chosen by $\mathcal{Z}$ correctly, which would happen at probability $2^{-\lambda}$. Therefore, in the ideal world, our $\mathcal{Z}$ outputs 1 with probability $2^{-1-\lambda}$; in the real world, our $\mathcal{Z}$ always outputs 1. In other words, our $\mathcal{Z}$ can distinguish the real world and the ideal world with probability $\frac{1}{2} - \frac{1}{2^\lambda}$ which is non-negligible, contradicting our assumption that $\Pi$ is UC-secure. □

**Corollary 1.** *There exists* no *1-round protocols* $\Pi$ *that UC-realizes functionality* $\mathcal{F}_{\mathsf{OT}}$ *depicted in Figure 4 in the* $\{\mathcal{F}_{\mathsf{SYN}}, \mathcal{F}_{\mathsf{RO}}\}$*-hybrid world.*

By combining the negative result in Corollary 1 and the positive results in Section 3, we demonstrate a separation between the EOT and OT.

### 4.2 From Endemic OT to Commitment

Recall that, Brzuska *et al.* proved that bit commitment can be constructed via 1-out-of-2 OT with unconditional security [BFSK11]. What about EOT? Although there is a separation between the EOT and 1-out-of-2 OT (cf. Section 4.1), we show a surprising fact: bit commitment can also be constructed via EOT with unconditional security.

Our key observation is that the receiver's message can be viewed as the commitment to the receiver's choice bit $b$, and the locally computed message $m_b$ together with $b$ can be viewed as the opening. Typically, a commitment protocol requires hiding and binding property. The hiding property comes from the fact: even if the malicious receiver in the EOT can influence the distribution of $m_b$, it cannot learn the other message $m_{1-b}$. The binding property comes from the fact: even if the malicious sender in the EOT can influence the distributions of both $m_0$ and $m_1$, it cannot tell which one is received by the receiver. Furthermore, if we use a UC-secure EOT protocol as the building block, the resulting commitment protocol is also UC-secure. Formally, we put our protocol $\Pi_{\mathsf{Com}}$ in Figure 16 and prove the security through Theorem 7. Here we only assume the authenticated channel $\mathcal{F}_{\mathsf{Auth}}$.

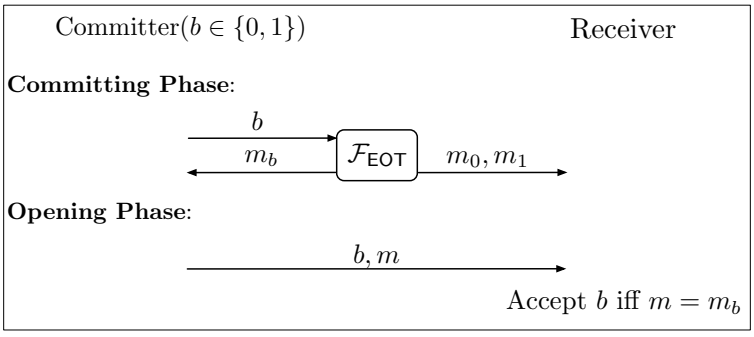



Figure 16: Bit Commitment Protocol $\Pi_{\mathsf{Com}}$ in the $\{\mathcal{F}_{\mathsf{EOT}}, \mathcal{F}_{\mathsf{Auth}}\}$-Hybrid World

**Theorem 7.** *The protocol* $\Pi_{\mathsf{Com}}$ *depicted in Figure 16 UC-realizes the functionality* $\mathcal{F}_{\mathsf{Com}}$ *depicted in Figure 7 with unconditional security in the* $\{\mathcal{F}_{\mathsf{EOT}}, \mathcal{F}_{\mathsf{Auth}}\}$*-hybrid world against static malicious corruption.*

*Proof.* We leave the formal proof in Appendix C.3. □

### 4.3 From Endemic OT to Uniform OT

In [MR19a], the Masny and Rindal showed how to construct UOT with unconditional security in the $\{\mathcal{F}_{\mathsf{EOT}}, \mathcal{F}_{\mathsf{Coin}}, \mathcal{F}_{\mathsf{Auth}}\}$-hybrid world, and we recall their protocol construction in Figure 17. However,

they only showed how to construct the coin-tossing protocol via UOT. Therefore, whether EOT implies UOT remains an open question.

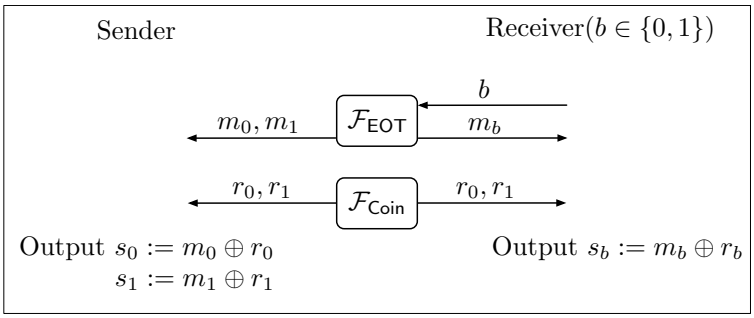



Figure 17: UOT Protocol $\Pi_{\mathsf{UOT}}$ in the $\{\mathcal{F}_{\mathsf{EOT}}, \mathcal{F}_{\mathsf{Coin}}, \mathcal{F}_{\mathsf{Auth}}\}$-Hybrid World from [MR19a]

**Lemma 1** ([MR19a]). *The protocol $\Pi_{\mathsf{UOT}}$ depicted in Figure 17 UC-realizes $\mathcal{F}_{\mathsf{UOT}}$ depicted in Figure 5 with unconditional security in the $\{\mathcal{F}_{\mathsf{EOT}}, \mathcal{F}_{\mathsf{Coin}}, \mathcal{F}_{\mathsf{Auth}}\}$-hybrid world against static malicious corruption.*

In this section, we provide a positive answer to this unsolved question. Our solution is as follows: we have showed that EOT implies commitment in Section 4.2, and the coin-tossing protocol can be easily constructed via only commitment; putting things together, we show that EOT implies UOT. Formally, we put our coin-tossing protocol $\Pi_{\mathsf{Coin}}$ in Figure 18 and prove the security through Theorem 8. We only assume authenticated channel $\mathcal{F}_{\mathsf{Auth}}$.

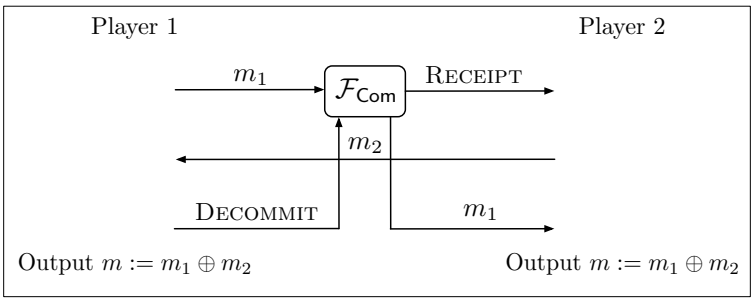



Figure 18: Coin-Tossing Protocol $\Pi_{\mathsf{Coin}}$ in the $\{\mathcal{F}_{\mathsf{Com}}, \mathcal{F}_{\mathsf{Auth}}\}$-Hybrid World

**Theorem 8.** *The protocol $\Pi_{\mathsf{Coin}}$ depicted in Figure 18 UC-realizes the functionality $\mathcal{F}_{\mathsf{Coin}}$ depicted in Figure 3 with unconditional security in the $\{\mathcal{F}_{\mathsf{Com}}, \mathcal{F}_{\mathsf{Auth}}\}$-hybrid world against static malicious corruption.*

*Proof.* We leave the formal proof in Appendix C.4. □

Formally, we prove that EOT implies UOT through Corollary 2. The security proof of Corollary 2 directly comes from Lemma 1, Theorem 7 and Theorem 8, and thus we omit the trivial proof here.

**Corollary 2.** *The protocol $\Pi_{\mathsf{UOT}}$ depicted in Figure 17 UC-realizes $\mathcal{F}_{\mathsf{UOT}}$ depicted in Figure 5 with unconditional security in the $\{\mathcal{F}_{\mathsf{EOT}}, \mathcal{F}_{\mathsf{Auth}}\}$-hybrid world against static malicious corruption.*

## 5 GUC-Secure Endemic OT via Global Random Oracles

In this section, we turn to global RO models to seek a stronger variant of UC security, i.e., GUC security. As for the GroRO model, we construct the *first* 1-round GUC-secure EOT protocol under CDH assumption against static adversaries. Using our 1-round GUC-secure EOT protocol as the main building block, we propose the *first* 2-round GUC-secure commitment protocol in the GroRO model.

Regarding the GrpRO model, we prove that there exists *no* 1-round GUC-secure EOT protocol in the GrpRO model even with static security. By combining this negative result in the GrpRO model and the positive result in the GroRO model, we reveal a separation between these two models. Furthermore, we construct the *first* 2-round (round-optimal) GUC-secure EOT protocol under DDH assumption in the GrpRO model against adaptive adversaries.

## 5.1 Feasibility Results in the GroRO Model

### 5.1.1 Our EOT Protocol

We start with our UC-secure EOT protocol $\Pi_{\mathsf{sEOT\text{-}RO}}$ depicted in Figure 11. Recall that, we let the sender sends $z := g^s$ using $s \leftarrow \mathbb{Z}_q$, together with a straight-line extractable NIZK argument of $s$ such that $z = g^s$ in $\Pi_{\mathsf{sEOT\text{-}RO}}$. The straight-line extractable NIZK argument gives the simulator chance of extracting the sender's secret randomness. However, Pass showed that it is impossible to construct NIZK arguments in observable RO model [Pas03], let alone NIZK arguments with straight-line extractability. The good news is that we find that straight-line extractable NIWH argument is sufficient for our purpose, and it is possible in the GroRO model [Pas03]. Therefore, we let the sender generate a straight-line extractable NIWH argument of $s$ such that $z = g^s$. Now let us consider the receiver. In order to extract the receiver's choice bit, we make full use of the programmability of random oracles in $\Pi_{\mathsf{sEOT\text{-}RO}}$. Since $\mathcal{G}_{\mathsf{roRO}}$ does not permit anyone to program the random oracle, we need to take a different strategy: we let the receiver generate $h$ by invoking the $\mathcal{G}_{\mathsf{roRO}}$ on a randomly sampled string $\mathsf{seed}$, compute a Pedersen commitment to the choice bit $B := g^r h^b$ using $r \leftarrow \mathbb{Z}_q$, and generate a straight-line extractable NIWH argument of $(r, b)$ such that $B = g^r h^b$. Analogously to the sender side, the simulator can extract the malicious receiver's choice bit $b$.

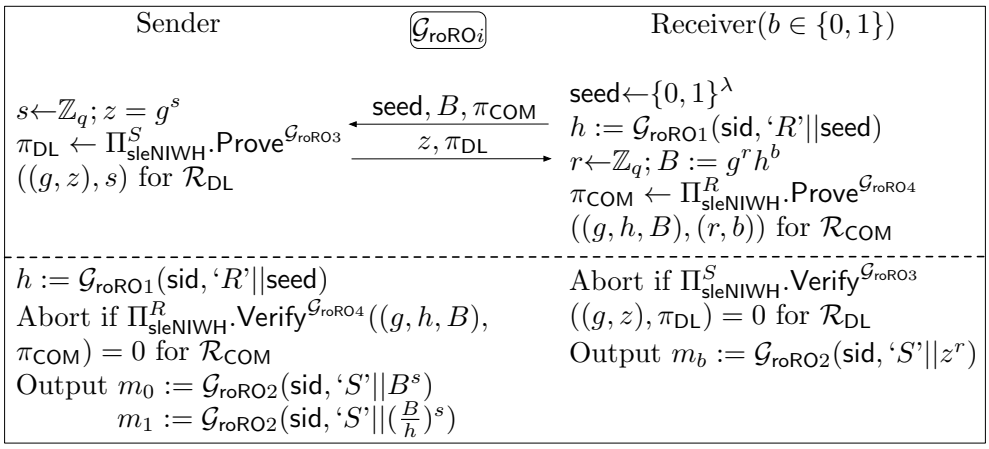



Figure 19: 1-round EOT protocol $\Pi_{\mathsf{EOT\text{-}GroRO}}$ in the $\{\mathcal{G}_{\mathsf{roRO}}, \mathcal{F}_{\mathsf{SYN}}\}$-hybrid world, where $\mathcal{G}_{\mathsf{roRO}} = \{\mathcal{G}_{\mathsf{roRO}i}\}_{i\in[4]}$. Let $g$ be the generator of $\mathbb{G}$. Let $\mathcal{G}_{\mathsf{roRO1}} : \{0,1\}^* \to \mathbb{G}$ and $\mathcal{G}_{\mathsf{roRO2}} : \{0,1\}^* \to \{0,1\}^\lambda$. Let $\mathcal{R}_{\mathsf{Com}} := \{((g,h,B),(r,b)) \mid B = g^r h^b\}$ and $\mathcal{R}_{\mathsf{DL}} := \{((g,z),s) \mid z = g^s\}$.

Let $g$ be the generator of $\mathbb{G}$. Let $\mathcal{G}_{\mathsf{roRO1}} : \{0,1\}^* \to \mathbb{G}$ and $\mathcal{G}_{\mathsf{roRO2}} : \{0,1\}^* \to \{0,1\}^\lambda$. Let $\mathcal{R}_{\mathsf{Com}} := \{((g,h,B),(r,b)) \mid B = g^r h^b\}$ and $\mathcal{R}_{\mathsf{DL}} := \{((g,z),s) \mid z = g^s\}$. We denote by $\Pi^S_{\mathsf{sleNIWH}}$ the straight-line extractable NIWH argument in the $\mathcal{G}_{\mathsf{roRO3}}$-hybrid world which is used for generating the proof by sender. We denote by $\Pi^R_{\mathsf{sleNIWH}}$ the straight-line extractable NIWH argument in the $\mathcal{G}_{\mathsf{roRO4}}$-hybrid world which is used for generating the proof by receiver. We assume synchronous channel $\mathcal{F}_{\mathsf{SYN}}$. Formally, we put our protocol $\Pi_{\mathsf{EOT\text{-}GroRO}}$ in Figure 19 and prove the security through Theorem 9.

Before giving the theorem, we have to give the transferable EOT functionality $\mathcal{F}_{\mathsf{tEOT}}$ in Figure 20. The main difference with the traditional EOT functionality is that in $\mathcal{F}_{\mathsf{tEOT}}$, the simulator can request the list of illegitimate queries, which fits the $\mathcal{G}_{\mathsf{roRO}}$ model.

**Functionality $\mathcal{F}_{\mathsf{tEOT}}$**

The functionality interacts with two parties $S$, $R$ and an adversary $\mathcal{S}$.

**Transfer/Choose/Process.** Same as $\mathcal{F}_{\mathsf{EOT}}$ depicted in Figure 6.

**Observe.** When asked by the adversary $\mathcal{S}$, obtain from $\mathcal{G}_{\mathsf{roRO}}$ the list of illegitimate queries $\mathcal{Q}_{\mathsf{sid}}$ that pertain to SID sid, and send $\mathcal{Q}_{\mathsf{sid}}$ to the adversary $\mathcal{S}$.

Figure 20: The Transferable Ideal Functionality $\mathcal{F}_{\mathsf{tEOT}}$ for Endemic Oblivious Transfer

**Theorem 9.** *Assume the CDH assumption holds in group $\mathbb{G}$. Let $\mathcal{G}_{\mathsf{roRO1}} : \{0,1\}^\lambda \to \mathbb{G}$ and $\mathcal{G}_{\mathsf{roRO2}} : \mathbb{G} \to \{0,1\}^\lambda$ be the random oracles. Let $\Pi^S_{\mathsf{sleNIWH}}$ be a straight-line extractable NIWH argument in the $\mathcal{G}_{\mathsf{roRO3}}$-hybrid world. Let $\Pi^R_{\mathsf{sleNIWH}}$ be a straight-line extractable NIWH argument in the $\mathcal{G}_{\mathsf{roRO4}}$-hybrid world. The protocol $\Pi_{\mathsf{EOT\text{-}GroRO}}$ depicted in Figure 19 GUC-realizes the functionality $\mathcal{F}_{\mathsf{tEOT}}$ depicted in Figure 20 in the $\{\mathcal{G}_{\mathsf{roRO}}, \mathcal{F}_{\mathsf{SYN}}\}$-hybrid world against static malicious corruption, where $\mathcal{G}_{\mathsf{roRO}} = \{\mathcal{G}_{\mathsf{roRO}i}\}_{i\in[4]}$.*

*Proof.* We leave the formal proof in Appendix C.5. □

**Instantiation.** We instantiate $\Pi^S_{\mathsf{sleNIZK}}$ for relation $\mathcal{R}_{\mathsf{DL}}$ with the Schnorr's protocol and the Kondi-shelat transform as in Section 3.1. Note that, although we use the same instantiation as in Section 3.1, we only obtain a straight-line extractable NIWH argument, since here we use a observable RO model [Pas03].

We instantiate $\Pi^R_{\mathsf{sleNIWH}}$ for relation $\mathcal{R}_{\mathsf{Com}}$ with the Okamoto's protocol [Oka93] and the Kondi-shelat transform. We present the formal description of the Okamoto's protocol and the Kondi-shelat transform in Appendix B.1 and Appendix B.2 respectively.

### 5.1.2 Our Commitment Protocol

At the very beginning, we introduce the transferable commitment functionality $\mathcal{F}_{\mathsf{tCom}}$ from [CJS14] in Figure 21. The main difference with the traditional commitment functionality is that in $\mathcal{F}_{\mathsf{tCom}}$, the simulator can request the list of illegitimate queries, which fits the $\mathcal{G}_{\mathsf{roRO}}$ model.

**Functionality $\mathcal{F}_{\mathsf{tCom}}$**

The functionality interacts with two parties $S$, $R$ and an adversary $\mathcal{S}$.

**Commit/Open.** Same as $\mathcal{F}_{\mathsf{Com}}$ depicted in Figure 7.

**Observe.** When asked by the adversary $\mathcal{S}$, obtain from $\mathcal{G}_{\mathsf{roRO}}$ the list of illegitimate queries $\mathcal{Q}_{\mathsf{sid}}$ that pertain to SID sid, and send $\mathcal{Q}_{\mathsf{sid}}$ to the adversary $\mathcal{S}$.

Figure 21: The Transferable Ideal Functionality $\mathcal{F}_{\mathsf{tCom}}$ for Commitment

Recall that, we construct a commitment protocol $\Pi_{\mathsf{Com}}$ depicted in Figure 16 in the $\{\mathcal{F}_{\mathsf{EOT}}, \mathcal{F}_{\mathsf{Auth}}\}$-hybrid world with unconditional security (cf. Section 4.2). It is easy to see that if we replace the ideal functionality $\mathcal{F}_{\mathsf{EOT}}$ with transferable ideal functionality $\mathcal{F}_{\mathsf{tEOT}}$ and call the resulting protocol $\Pi_{\mathsf{tCom}}$, then the protocol $\Pi_{\mathsf{tCom}}$ will GUC-realize $\mathcal{F}_{\mathsf{tCom}}$ in the $\{\mathcal{F}_{\mathsf{tEOT}}, \mathcal{F}_{\mathsf{Auth}}\}$-hybrid world with unconditional security. Formally, we have the following corollary, and its security proof is analogously to the proof of Theorem 7.

**Corollary 3.** *The protocol $\Pi_{\mathsf{tCom}}$ GUC-realizes the functionality $\mathcal{F}_{\mathsf{tCom}}$ depicted in Figure 21 with unconditional security in the $\{\mathcal{F}_{\mathsf{tEOT}}, \mathcal{F}_{\mathsf{Auth}}\}$-hybrid world against static malicious corruption.*

**Instantiation.** We instantiate $\mathcal{F}_{\mathsf{tEOT}}$ with our 1-round GUC-secure EOT protocol depicted in Figure 19. Then we immediate obtain a 2-round GUC-secure commitment protocol $\Pi_{\mathsf{tCom}}$ in the GroRO model; note that, the first round messages are communicated over the synchronous channel $\mathcal{F}_{\mathsf{SYN}}$ and the second round message is communicated over the authenticated channel $\mathcal{F}_{\mathsf{Auth}}$. The security is guaranteed by Theorem 9 and Corollary 3.

**Comparison.** Our commitment protocol is the *first* 2-round GUC-secure commitment in the GroRO model, while the previous state-of-the-art protocols achieves 3 rounds [MRS17, ZZZR22]. Note that, Zhou *et al.* proved that it is impossible to construct 2-round GUC-secure commitment protocol in the GroRO model even with static security [ZZZR22]; but they did not consider the simultaneous rounds. Our 2-round commitment protocol contains a simultaneous round, so we do not contradict their impossibility result. We also note that, our protocol and protocols in [MRS17, ZZZR22] are all 3-move static-secure protocols, but ours is the only one whose first two moves can be executed in one simultaneous round; hence, ours is the only one that can achieve 2-round. The details of the comparison are presented in Table 2.

| Protocol | #Round | Computational Assumption |
|---|---|---|
| [MRS17] | 3 | DL |
| [ZZZR22] | 3 | OWF |
| $\Pi_{\mathsf{tCom}}$ | 2 | CDH |

Table 2: Comparison with state-of-the-art GUC-secure commitment against static adversaries in the GroRO model.

## 5.2 Impossibility and Feasibility Results in the GrpRO Model

### 5.2.1 Our Impossibility Result

Here we show that there exists *no* 1-round GUC-secure EOT protocols against static adversaries in the GrpRO model.

We prove this impossibility by contradiction. Suppose that there exists such a 1-round GUC-secure EOT protocol. Let us first consider the case where the receiver is corrupted, and the simulator needs to extract the choice bit of the receiver from its message. Recall that, the $\mathcal{G}_{\mathsf{rpRO}}$ only grants the simulator the restricted programmability: although the simulator can program the unqueried points without being detected, the simulator is external to the $\mathcal{G}_{\mathsf{rpRO}}$ and it can not know in real time what queries other parties are sending to $\mathcal{G}_{\mathsf{rpRO}}$. Thus, the simulator needs to program the points in advance and find a way to enforce the corrupt receiver to generate its message on the simulator's programmed points. In that way, the simulator can have the chance of extracting the choice bit of the receiver. However, in a one simultaneous round protocol, the messages between parties have no dependency. Hence the simulator cannot enforce the corrupt receiver to produce its message on the programmed points, and has no advantages over the real world adversary. If the simulator still succeeds to extract the corrupted receiver's choice bit, then distinctions will be revealed when the adversary performs the following attacks. The adversary corrupts the sender, and instructs the sender to run the simulator algorithm above to extract the choice bit from the message sent by the receiver/simulator. However, the receiver/simulator has no idea about the real choice bit, thus with high probability the simulation would fail. Formally, we prove this impossibility through Theorem 10.

**Theorem 10.** *There exists no terminating 1-round protocol* $\Pi$ *that GUC-realizes* $\mathcal{F}_{\mathsf{EOT}}$ *depicted in Figure 6 with static security in the* $\{\mathcal{G}_{\mathsf{rpRO}}, \mathcal{F}_{\mathsf{SYN}}\}$*-hybrid world.*

*Proof.* We proceed by contradiction. Suppose there exists such a protocol $\Pi$ that GUC-realizes $\mathcal{F}_{\mathsf{EOT}}$ in the $\{\mathcal{G}_{\mathsf{rpRO}}, \mathcal{F}_{\mathsf{SYN}}\}$-hybrid world. Then there must exist a PPT simulator $\mathcal{S}$ such that $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}}}_{\mathcal{F}_{\mathsf{EOT}},\mathcal{S},\mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}},\mathcal{F}_{\mathsf{SYN}}}_{\Pi,\mathcal{A},\mathcal{Z}}$ for any PPT adversary $\mathcal{A}$ and any PPT $\mathcal{G}_{\mathsf{rpRO}}$-externally constrained environment $\mathcal{Z}$.

In particular, let us first consider the session with SID $\mathsf{sid}_1$, and let $\mathcal{A}$ be a dummy adversary that simply forwards protocol flows between the corrupt party and the environment $\mathcal{Z}$. Our $\mathcal{Z}$ proceeds by corrupting the receiver $R^*$ at first. Then $\mathcal{Z}$ waits for $S$ to send its message $\pi_S$. Note that, once the message $\pi_S$ is given to $\mathcal{F}_{\mathsf{SYN}}$, $\mathcal{Z}$ is allowed to see it. After obtaining $\pi_S$, $\mathcal{Z}$ chooses a random bit $b \leftarrow \{0,1\}$ and instructs $R^*$ to perform the receiver algorithm on input $b$, and send its message $\pi_R$ to $S$. Finally, $\mathcal{Z}$ performs the local computation of $R^*$ to obtain $m_b$, and waits for $S$ to output $m'_0, m'_1$ at the end of the protocol. If $m_b = m'_b$, $\mathcal{Z}$ outputs 1; otherwise, $\mathcal{Z}$ outputs 0.

In order to make the GUC experiment above remain indistinguishable, the simulator $\mathcal{S}$ needs to perform the following strategy. First, $\mathcal{S}$ extracts the choice bit $b$ of the receiver from message $\pi_R$. Then $\mathcal{S}$ performs the local computation of $S$ to obtain $m_0, m_1$. Finally, $\mathcal{S}$ sends $(\textsc{Receive}, \mathsf{sid}, b, m_b)$ to $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted receiver. It is easy to see that the crucial part lies in the extraction of the choice bit $b$ of the receiver. Recall that, the main advantage of $\mathcal{S}$ is that it can program the $\mathcal{G}_{\mathsf{rpRO}}$ on unqueried points without being detected, but the simulator is external to the $\mathcal{G}_{\mathsf{rpRO}}$ and it can not know in real time what queries other parties are sending to $\mathcal{G}_{\mathsf{rpRO}}$. For notation convenience, we denote by $\mathsf{Prog}_{\mathsf{sid}_1}$ the queries programmed by $\mathcal{S}$. If the adversary happens to use the points that belongs to $\mathsf{Prog}_{\mathsf{sid}_1}$, then $\mathcal{S}$ has the chance of extracting the private information of the malicious party. We also note that, the simulator $\mathcal{S}$ also can query $\mathcal{G}_{\mathsf{rpRO}}$ just like other parties. In order to describe the process of querying to $\mathcal{G}_{\mathsf{rpRO}}$, we denote by $\mathcal{G}^*_{\mathsf{rpRO}}$ the simplified version of the $\mathcal{G}_{\mathsf{rpRO}}$, i.e., the $\mathcal{G}_{\mathsf{rpRO}}$ without the $\textsc{Program}$ interface. We write $\mathcal{S}^{\mathcal{G}^*_{\mathsf{rpRO}}}$ to denote the event that $\mathcal{S}$ is given query access to $\mathcal{G}_{\mathsf{rpRO}}$ and can continuously query to $\mathcal{G}_{\mathsf{rpRO}}$. With notations above, we denote by $b \leftarrow \mathcal{S}^{\mathcal{G}^*_{\mathsf{rpRO}}}(\mathsf{sid}_1, \pi_R, \mathsf{Prog}_{\mathsf{sid}_1})$ the event where $\mathcal{S}$ extracts the choice bit $b$ from $\pi_R$ using $\mathsf{Prog}_{\mathsf{sid}_1}$. Recall that, (i) the simulator $\mathcal{S}$ should be able to handle any PPT adversary $\mathcal{A}$ and any PPT environment $\mathcal{Z}$; (ii) there should be no dependency between $\pi_S$ and $\pi_R$ in one simultaneous round protocol, i.e., the computation of $\pi_R$ is totally independent of $\pi_S$. Hence we can consider the following case: the environment $\mathcal{Z}$ queries $\mathcal{G}_{\mathsf{rpRO}}$ everything that will be needed to compute the receiver's message $\pi_R$ in advance (we denote these queries as $\mathcal{Q}_{\mathsf{sid}_1,\mathcal{Z}}$), then $\mathcal{Z}$ starts the protocol $\Pi$ and instructs $R^*$ to run the receiver algorithm on those previously queried points. In this case, we have $\Pr[\mathsf{Prog}_{\mathsf{sid}_1} \cap \mathcal{Q}_{\mathsf{sid}_1,\mathcal{Z}} = \emptyset] = 0$ where $\mathcal{Q}_{\mathsf{sid}_1,\mathcal{Z}}$ is the queries used for generating the receiver's message $\pi_R$. However, the simulator $\mathcal{S}$ should still be able to extract the choice bit; otherwise, $\mathcal{Z}$ will distinguish the ideal world from the real world. In other words, the algorithm $b \leftarrow \mathcal{S}^{\mathcal{G}^*_{\mathsf{rpRO}}}(\mathsf{sid}_1, \pi_1, \emptyset)$ should work even if we replace the programmed queries $\mathsf{Prog}_{\mathsf{sid}_1}$ with an empty set $\emptyset$. We note that $\mathcal{S}^{\mathcal{G}^*_{\mathsf{rpRO}}}(\mathsf{sid}_1, \pi_R, \emptyset)$ works as long as the appropriate inputs are provided, even if we switch to the session with a different SID.

Next, we show that the existence of the simulator $\mathcal{S}$ above contradicts the security of $\Pi$ against static corruptions, by creating a particular $\mathcal{Z}'$ which distinguishes $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}}}_{\mathcal{F}_{\mathsf{EOT}},\mathcal{S}',\mathcal{Z}'}$ from $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}}}_{\Pi,\mathcal{A}',\mathcal{Z}'}$ after a static corruption operation for any PPT simulator $\mathcal{S}'$. Let us consider the session with SID $\mathsf{sid}_2$. We let $\mathcal{Z}'$ corrupt the sender $S^*$ at first. Then $\mathcal{Z}'$ feeds the honest receiver with a random bit $b$, and waits for the arrival of message $\pi_R$. Note that, $b$ is the hidden input of the honest receiver, and the receiver sends $b$ only to $\mathcal{F}_{\mathsf{EOT}}$ which hides it from $\mathcal{S}$ information theoretically. Therefore, the entire computation of $\pi_R$ is totally independent of $b$. Now $\mathcal{Z}'$ instructs $S^*$ to invoke $b' \leftarrow \mathcal{S}^{\mathcal{G}^*_{\mathsf{rpRO}}}(\mathsf{sid}_2, \pi_R, \emptyset)$. In the real world, we always have $b' = b$. In the ideal world, we have $b' = b$ with probability at most $\frac{1}{2}$ since $\pi_R$ is totally independent of $b$. Therefore, $\mathcal{Z}'$ can distinguish between the real world and the ideal world with a non-negligible probability, contradicting our assumption that $\Pi$ is GUC-secure. □

By combining this negative result in the GrpRO model and the positive result in the GroRO model depicted in Section 5.1.1, we demonstrate a separation between the GroRO and the GrpRO model.

**Remark 1.** *Our separation result is fundamentally different from the separation between the RO and the Non-Programmable RO (NPRO) model revealed by Nielsen [Nie02], where Nielsen showed that there exists a Non-Interactive Non-Committing Encryption (NINCE) in the RO model and there exists no NINCEs in the NPRO model. First, the setups are different, our separation is between the GroRO and the GrpRO model under the GUC framework; second, the proof technique is different: Nielsen's proof explores the message length issue w.r.t. the secret key length, while our proof is a non-trivial extension of the Canetii-Fischlin approach [CF01].*

Our impossibility result for EOT can be extended to OT. More precisely, we show that there exists *no* 2-round GUC-secure OT protocols in the $\{\mathcal{G}_{\mathsf{rpRO}}, \mathcal{F}_{\mathsf{Auth}}\}$-hybrid world even with static security; note that, here we do not consider simultaneous rounds. We also note that, Canetti *et al.* [CSW20a] and Byali *et al.* [BPRS17] proposed 2-round 1-out-of-2 OT protocols in the GrpRO model. Recall that, in the security analysis under the GUC framework, both the ideal world simulator and the real world adversary are only allowed to query the global setup. However, in the security analysis against a malicious receiver in [CSW20a, Page 16, Figure 6] and [BPRS17, Page 19], the ideal world simulator must emulate the interface of RO functionality for the real world adversary to see which point is queried and answer to this query (UC style). Thus we remark that their protocols only achieves UC security rather than GUC security. Formally, we present our impossibility result in Theorem 11.

**Theorem 11.** *There exists no terminating 2-round protocol* $\Pi$ *that GUC-realizes* $\mathcal{F}_{\mathsf{OT}}$ *depicted in Figure 4 with static security in the* $\{\mathcal{G}_{\mathsf{rpRO}}, \mathcal{F}_{\mathsf{Auth}}\}$*-hybrid world.*

*Proof.* We leave the formal proof in Appendix D. □

### 5.2.2 Our EOT Protocol

Theorem 10 rules out the possibility of 1-round GUC-secure EOT protocols in the $\{\mathcal{G}_{\mathsf{rpRO}}, \mathcal{F}_{\mathsf{SYN}}\}$-hybrid world. It makes us wonder if we do not let the sender and the receiver send their messages simultaneously but in a specific order, can we get a 2-move (also 2-round) GUC-secure protocol?

We start with the UC-secure EOT protocol in the CRS+GrpRO hybrid model proposed by Canetti *et al.* [CSW20a]. Their CRS consists of two group elements $g, h \in \mathbb{G}$, and the simulator knows $\log_g h$. They let the receiver generate parameter $G, H$ by invoking the RO on a randomly sampled string seed, and compute two instances of Pedersen commitment to the choice bit $(B_1, B_2) := (g^x G^b, h^x H^b)$ using two sets of different parameters $(g, G), (h, H)$ and the same randomness $x \leftarrow \mathbb{Z}_q$. As for the sender, they let the sender compute $z := g^r h^s$ using randomness $r, s \leftarrow \mathbb{Z}_q$. Finally, the sender outputs $m_0 := \mathcal{G}_{\mathsf{rpRO}}(B_1^r B_2^s)$ and $m_1 := \mathcal{G}_{\mathsf{rpRO}}((\frac{B_1}{G})^r (\frac{B_2}{H})^s)$ while the receiver outputs $m_b := \mathcal{G}_{\mathsf{rpRO}}(z^x)$.

Our goals are: (i) remove the CRS setup of this protocol; (ii) make this protocol GUC-secure in the GrpRO model. To achieve the former goal, we let the sender generate $g, h$ by invoking random oracle on a randomly sampled string $\mathsf{seed}_1$. Then the sender computes $z := g^r h^s$ using $r, s \leftarrow \mathbb{Z}_q$, and sends $\mathsf{seed}_1, z$ to the receiver. On the other hand, we let the receiver generate $G, H$ by invoking $\mathcal{G}_{\mathsf{rpRO}}$ on another randomly sampled string $\mathsf{seed}_2$, computes two instances of Pedersen commitment to the choice bit $(B_1, B_2) := (g^x G^b, h^x H^b)$ using the same randomness $x \leftarrow \mathbb{Z}_q$. The local computation is the same as Canetti *et al*'s protocol. In order to show that our modified protocol achieves the latter goal, we show the simulation strategy as follows: when the receiver is malicious, the simulator can extract the receiver's choice bit $b$ by programming the $\mathcal{G}_{\mathsf{rpRO}}$ and knowing $\alpha$ such that $h = g^\alpha$. Then the simulator can extract $b$ by the following strategy: if $B_2 = B_1^\alpha$, it sets $b := 0$; else if $\frac{B_2}{H} = (\frac{B_1}{G})^\alpha$, it sets $b := 1$; else, it sets $b := \perp$. Note that, when $B_1, B_2$ are not correctly constructed (i.e., the simulator sets $b := \perp$), the malicious receiver cannot compute either $m_0$ or $m_1$. When the sender is malicious, the simulator can compute both $m_0$ and $m_1$ by programming the $\mathcal{G}_{\mathsf{rpRO}}$ such that $(g, h, G, H)$ is a DDH tuple, i.e., $G = g^t, H = h^t$. In this way, the simulator can compute $m_0 := \mathcal{G}_{\mathsf{rpRO}}(z^x)$ and $m_1 := \mathcal{G}_{\mathsf{rpRO}}(z^{x-t})$.

Let $\mathcal{G}_{\mathsf{rpRO1}} : \{0,1\}^* \to \mathbb{G}^2$ and $\mathcal{G}_{\mathsf{rpRO2}} : \{0,1\}^* \to \{0,1\}^\lambda$. Here we assume authenticated channels $\mathcal{F}_{\mathsf{Auth}}$. Formally, we put our protocol $\Pi_{\mathsf{EOT\text{-}GrpRO}}$ in Figure 22 and prove the security through Theorem 12.

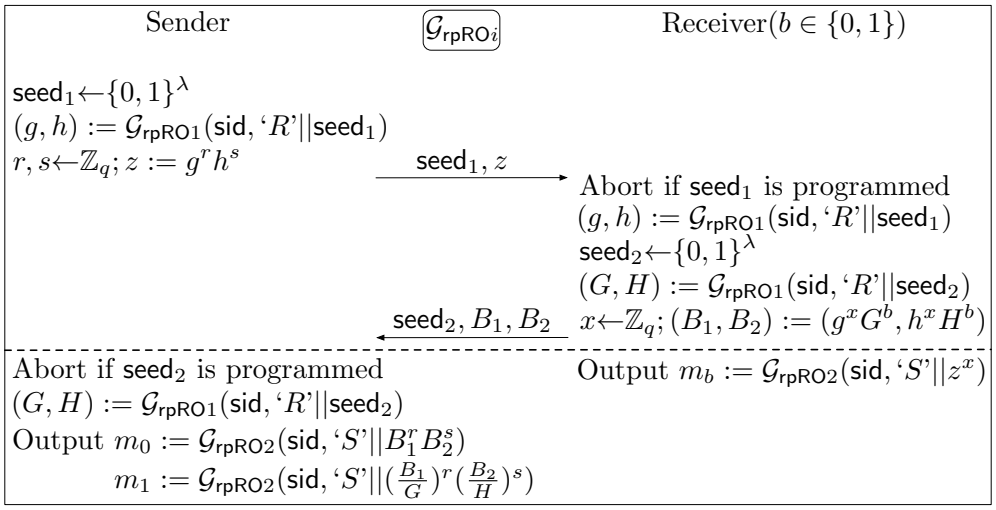



Figure 22: 2-round EOT protocol $\Pi_{\mathsf{EOT\text{-}GrpRO}}$ in the $\{\mathcal{G}_{\mathsf{rpRO}}, \mathcal{F}_{\mathsf{Auth}}\}$-hybrid world, where $\mathcal{G}_{\mathsf{rpRO}} = \{\mathcal{G}_{\mathsf{rpRO1}}, \mathcal{G}_{\mathsf{rpRO2}}\}$. Let $\mathcal{G}_{\mathsf{rpRO1}} : \{0,1\}^* \to \mathbb{G}^2$ and $\mathcal{G}_{\mathsf{rpRO2}} : \{0,1\}^* \to \{0,1\}^\lambda$.

**Theorem 12.** *Assume the DDH assumption holds in group $\mathbb{G}$. Let $\mathcal{G}_{\mathsf{rpRO1}} : \{0,1\}^\lambda \to \mathbb{G}^2$ and $\mathcal{G}_{\mathsf{rpRO2}} : \mathbb{G} \to \{0,1\}^\lambda$ be the random oracles. The protocol $\Pi_{\mathsf{EOT\text{-}GrpRO}}$ depicted in Figure 22 GUC-realizes the functionality $\mathcal{F}_{\mathsf{EOT}}$ depicted in Figure 6 in the $\{\mathcal{G}_{\mathsf{rpRO}}, \mathcal{F}_{\mathsf{Auth}}\}$-hybrid world against static malicious corruption, where $\mathcal{G}_{\mathsf{rpRO}} = \{\mathcal{G}_{\mathsf{rpRO1}}, \mathcal{G}_{\mathsf{rpRO2}}\}$.*

*Proof.* We leave the formal proof in Appendix C.6. □

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# A Additional Preliminaries

## A.1 Pedersen Commitment

The Pedersen commitment is one of the most common trapdoor commitment scheme [Ped92]. In order to formally define the Pedersen commitment, we first introduce the trapdoor commitment and then describe how to instantiate it with Pedersen commitment.

A trapdoor commitment scheme $\Pi = (\Pi.\mathsf{KeyGen}, \Pi.\mathsf{Commit}, \Pi.\mathsf{ComVer}, \Pi.\mathsf{Equiv})$ allows the committer to compute the commitment $c$ to any value $m$ using the commitment key $\mathsf{ck}$ and the randomness $r$. Later, the committer can open $c$ to $m$ by sending the the opening $d$ to the receiver who verifies it. Furthermore, if the committer somehow obtains the trapdoor key $\mathsf{td}$ with respect to $\mathsf{ck}$, it can open the previous commitment $c$ to any other message $\tilde{m} \neq m$. Formally, the trapdoor commitment has the following algorithms:

- $(\mathsf{ck}, \mathsf{td}) \leftarrow \mathsf{KeyGen}(1^\lambda)$ takes input as the security parameter $\lambda$, and outputs a commitment key $\mathsf{ck}$ and a trapdoor key $\mathsf{td}$.
- $(c, d) := \mathsf{Commit}(\mathsf{ck}, m; r)$ takes input as a commitment key $\mathsf{ck}$, a message $m$ and a randomness $r$. It outputs the commitment $c$ and the opening $d$. We assume that there exists a deterministic algorithm that can extract $m$ from $d$. When $r$ is not important, we use $\mathsf{Commit}(\mathsf{ck}, m)$ for simplicity.
- $b := \mathsf{ComVer}(\mathsf{ck}, c, d)$ takes input as a commitment key $\mathsf{ck}$, and a commitment-opening pair $(c, d)$. It outputs a bit $b$ indicating acceptance ($b = 1$) or rejection ($b = 0$).
- $(\tilde{d}, \tilde{r}) := \mathsf{Equiv}(\mathsf{ck}, \mathsf{td}, c, d, \tilde{m})$ takes input as a commitment key $\mathsf{ck}$, a trapdoor key $\mathsf{td}$, a commitment $c$ and its opening $d$, and an arbitrary message $\tilde{m}$ for which equivocation is required. It outputs the new opening $\tilde{d}$ and a new randomness $\tilde{r}$ such that $(c, \tilde{d}) = \mathsf{Commit}(\mathsf{ck}, \tilde{m}; \tilde{r})$.

The trapdoor commitment requires the following properties: perfect correctness, perfect hiding, computational binding and trapdoor property. Perfect correctness means that any commitment produced by the honest committer can always be verified. Perfect hiding means that the commitment $c$ reveals nothing about the message $m$. Computational binding means that it is infeasible for the PPT committer to output the commitment $c$ that can be opened in two different ways. Trapdoor property means that given the trapdoor key $\mathsf{td}$, one can open a previously constructed commitment $c$ that corresponds to the message $m$ to any other message $\tilde{m} \neq m$. Formally, we have the following definition:

**Definition 17.** *We say a scheme* $\Pi = (\Pi.\mathsf{KeyGen}, \Pi.\mathsf{Commit}, \Pi.\mathsf{ComVer}, \Pi.\mathsf{Equiv})$ *is a trapdoor commitment scheme if the following conditions hold:*

- ***(Perfect Correctness)*** *For any message $m$, we say it is perfect correct if*

$$\Pr[(\mathsf{ck}, \mathsf{td}) \leftarrow \mathsf{KeyGen}(1^\lambda); (c, d) \leftarrow \mathsf{Commit}(\mathsf{ck}, m) : \mathsf{ComVer}(\mathsf{ck}, c, d) = 1] = 1$$

- ***(Perfect Hiding)*** *We say it is perfect hiding if for any adversary $\mathcal{A}$*

$$\Pr\left[\begin{array}{l|l} (\mathsf{ck}, \mathsf{td}) \leftarrow \mathsf{KeyGen}(1^\lambda); (m_0, m_1, \mathsf{st}) \leftarrow \mathcal{A}(\mathsf{ck}); & b' \leftarrow \mathcal{A}(c, \mathsf{st}) \\ b \leftarrow \{0,1\}; (c, d) \leftarrow \mathsf{Commit}(\mathsf{ck}, m_b) & \wedge\ b = b' \end{array}\right] = \frac{1}{2}$$

- ***(Computational Binding)*** *We say it is computational binding if for any PPT adversary $\mathcal{A}$, there exists a negligible function* $\mathsf{negl}$ *such that*

$$\Pr\left[\begin{array}{l|l} (\mathsf{ck}, \mathsf{td}) \leftarrow \mathsf{KeyGen}(1^\lambda); & \mathsf{ComVer}(\mathsf{ck}, c, d_0) = \mathsf{ComVer}(\mathsf{ck}, c, d_1) = 1 \\ (c, d_0, d_1) \leftarrow \mathcal{A}(\mathsf{ck}) & \wedge\ d_0 \neq d_1 \end{array}\right] \leq \mathsf{negl}(\lambda)$$

- ***(Trapdoor Property)*** *For any message pair $(m, \tilde{m})$, we say it has trapdoor property if*

$$\left\{(\mathsf{ck}, c, d, r) \,\middle|\, (\mathsf{ck}, \mathsf{td}) \leftarrow \mathsf{KeyGen}(1^\lambda); (c, d) := \mathsf{Commit}(\mathsf{ck}, m; r)\right\}$$
$$\overset{c}{\approx} \left\{(\mathsf{ck}, c, d, r) \,\middle|\, \begin{array}{l} (\mathsf{ck}, \mathsf{td}) \leftarrow \mathsf{KeyGen}(1^\lambda); (c, \tilde{d}) := \mathsf{Commit}(\mathsf{ck}, \tilde{m}; \tilde{r}); \\ (d, r) := \mathsf{Equiv}(\mathsf{ck}, \mathsf{td}, c, \tilde{d}, \tilde{r}) \end{array}\right\}$$

Now we show how to instantiate the trapdoor commitment scheme with the famous Pedersen commitment. We present the formal description of the Pedersen commitment in Figure 23. It is well-known that the Pedersen commitment is a trapdoor commitment scheme under the Discrete Logarithm (DL) assumption.

Pedersen Commitment

- $\mathsf{KeyGen}(1^\lambda)$: It outputs $\mathsf{ck} := (\mathbb{G}, p, q, g, h)$ and $\mathsf{td} := \alpha$, where $q = \frac{p-1}{2}$ and $p$ are primes, $\mathbb{G}$ is a subgroup of order $q$ of $\mathbb{Z}_p^*$, $g$ and $h = g^\alpha$ are the generators of $\mathbb{G}$.
- $\mathsf{Commit}(\mathsf{ck}, m; r)$: It outputs $c := g^m h^r \mod p$ and $d := (m, r)$.
- $\mathsf{ComVer}(\mathsf{ck}, c, d)$: It checks if $c = g^m h^r$ holds. If so, it outputs 1; otherwise, outputs 0.
- $\mathsf{Equiv}(\mathsf{ck}, \mathsf{td}, c, d, \tilde{m})$: It computes $\tilde{r} := \frac{m-\tilde{m}}{\alpha} + r \mod q$, and outputs $\tilde{d} := (\tilde{m}, \tilde{r})$ and $\tilde{r}$.

Figure 23: Pedersen Commitment

## A.2 ElGamal Encryption

The ElGamal encryption is a pseudorandom Public Key Encryption (PKE) system [GOS06]. Analogously to the Section A.1, we first introduce the pseudorandom PKE system, then describe how to instantiate it with the ElGamal encryption.

A pseudorandom PKE system $\Pi = (\Pi.\mathsf{KeyGen}, \Pi.\mathsf{Enc}, \Pi.\mathsf{Dec}, \Pi.\mathsf{oEnc}, \Pi.\mathsf{Invert})$ allows anyone to compute an encryption $E$ of the message $m$ using the public key $\mathsf{pk}$ and the randomness $r$. The one who has the decryption key $\mathsf{dk}$ can compute $m$ from $E$. Anyone who has the public key $\mathsf{pk}$ can sample a random ciphertext $\tilde{E}$ that looks indistinguishable from the real ciphertext $E$. Given a ciphertext $E$ and the internal states that used to generate it (i.e., $\mathsf{ck}, m, r$ such that $E = \mathsf{Enc}(\mathsf{ck}, m; r)$), anyone who has the decryption key $\mathsf{dk}$ can compute a randomness $\tilde{r}$ such that $E = \mathsf{oEnc}(1^\lambda; \tilde{r})$. Formally, the pseudorandom PKE system has the following algorithms:

- $(\mathsf{pk}, \mathsf{dk}) \leftarrow \mathsf{KeyGen}(1^\lambda)$ takes input as the security parameter $\lambda$, and outputs a public key $\mathsf{pk}$ and a decryption key $\mathsf{dk}$.
- $E := \mathsf{Enc}(\mathsf{pk}, m; r)$ takes input as a public key $\mathsf{pk}$, a message $m$ and a randomness $r$. It outputs the ciphertext $E$. When $r$ is not important, we use $\mathsf{Enc}(\mathsf{pk}, m)$ for simplicity.
- $m \leftarrow \mathsf{Dec}(\mathsf{pk}, \mathsf{dk}, E)$ takes input as a public key $\mathsf{pk}$, a decryption key $\mathsf{dk}$ and a ciphertext $E$. It outputs the plaintext message $m$.
- $\tilde{E} := \mathsf{oEnc}(\mathsf{pk}; r)$ takes input as the public key $\mathsf{pk}$ and a randomness $r$. It outputs the ciphertext $\tilde{E}$ that looks indistinguishable from the real ciphertext $E$. When $r$ is not important, we use $\mathsf{oEnc}(\mathsf{pk}, m)$ for simplicity.
- $\tilde{r} := \mathsf{Invert}(\mathsf{pk}, \mathsf{dk}, m, r, E)$ takes input as the public-decryption key pair $(\mathsf{pk}, \mathsf{dk})$, a message $m$, a randomness $r$ and its corresponding ciphertext $E$. It outputs a randomness $\tilde{r}$ such that $E = \mathsf{oEnc}(1^\lambda; \tilde{r})$.

The pseudorandom PKE system requires the following properties: perfect correctness, semantic security and oblivious ciphertext sampling. Perfect correctness is trivial. Semantic security means that the ciphertext $E$ reveals nothing about the message $m$. Oblivious ciphertext sampling means that given the decryption key $\mathsf{dk}$, one can output a randomness that makes the previously constructed ciphertext look as if it is randomly sampled. Formally, we have the following definition:

**Definition 18.** *We say a scheme* $\Pi = (\Pi.\mathsf{KeyGen}, \Pi.\mathsf{Enc}, \Pi.\mathsf{Dec}, \Pi.\mathsf{oEnc}, \Pi.\mathsf{Invert})$ *is a pseudorandom PKE system if the following conditions hold:*

- ***(Perfect Correctness)*** *For any message* $m$*, we say it is perfect correct if*

$$\Pr[(\mathsf{pk}, \mathsf{dk}) \leftarrow \mathsf{KeyGen}(1^\lambda); E \leftarrow \mathsf{Enc}(\mathsf{pk}, m) : m' \leftarrow (\mathsf{pk}, \mathsf{dk}, E) \wedge m = m'] = 1$$

- ***(Semantic Security)*** *We say it is semantic secure if for any PPT adversary $\mathcal{A}$, there exists a negligible function* $\mathsf{negl}$ *such that*

$$\left|\Pr\left[\begin{array}{l|l}(\mathsf{pk},\mathsf{dk})\leftarrow\mathsf{KeyGen}(1^\lambda);(m_0,m_1,\mathsf{st})\leftarrow\mathcal{A}(\mathsf{ck}); & b'\leftarrow\mathcal{A}(E,\mathsf{st})\\ b\leftarrow\{0,1\};E\leftarrow\mathsf{Enc}(\mathsf{ck},m_b) & \wedge\ b=b'\end{array}\right]-\frac{1}{2}\right|\leq\mathsf{negl}(\lambda)$$

- ***(Oblivious Ciphertext Sampling)*** *For any message $m$, we say it has oblivious ciphertext sampling if*

$$\begin{aligned}&\left\{(\mathsf{pk},E,m,r)\;\middle|\;(\mathsf{pk},\mathsf{dk})\leftarrow\mathsf{KeyGen}(1^\lambda);E:=\mathsf{oEnc}(\mathsf{pk};r)\right\}\\ \overset{c}{\approx}&\left\{(\mathsf{pk},E,m,r)\;\middle|\;\begin{array}{l}(\mathsf{pk},\mathsf{dk})\leftarrow\mathsf{KeyGen}(1^\lambda);E:=\mathsf{Enc}(\mathsf{pk},m;\tilde{r});\\ r:=\mathsf{Invert}(\mathsf{pk},\mathsf{dk},m,\tilde{r},E)\end{array}\right\}\end{aligned}$$

Here we show how to instantiate the pseudorandom PKE system with the well-known ElGamal encryption. We present the formal description of the ElGamal encryption in Figure 24. It is well-known that the ElGamal encryption is a pseudorandom PKE system under the DDH assumption.

**ElGamal Encryption**

- $\mathsf{KeyGen}(1^\lambda)$: It outputs $\mathsf{pk}:=(\mathbb{G},p,q,g,h)$ and $\mathsf{dk}:=\alpha$, where $q=\frac{p-1}{2}$ and $p$ are primes, $\mathbb{G}$ is a subgroup of order $q$ of $\mathbb{Z}_p^*$, $g$ and $h=g^\alpha$ are the generators of $\mathbb{G}$.
- $\mathsf{Enc}(\mathsf{pk},m;r)$: It computes $(u,v):=(g^r \mod p, g^m h^r \mod p)$ and outputs $E:=(u,v)$.
- $\mathsf{Dec}(\mathsf{ck},\mathsf{dk},E)$: It computes $\frac{v}{u^\alpha}$ and searches exhaustively for $m$.
- $\mathsf{oEnc}(\mathsf{pk};r)$: It derives $r_0,r_1$ from $r$ and computes $(\tilde{u},\tilde{v}):=(g^{r_0} \mod p, g^{r_1} \mod p)$.
- $\mathsf{Invert}(\mathsf{pk},\mathsf{dk},m,r,E)$: It sets $\tilde{r}_0:=r$ and $\tilde{r}_1:=m+\alpha\cdot r \mod q$ and outputs $\tilde{r}:=\tilde{r}_0||\tilde{r}_1$.

Figure 24: ElGamal Encryption

# B Building Blocks of (Straight-Line Extractable) NIZK/NIWH Arguments

In this section, we introduce the building blocks of our (straight-line extractable) NIZK/NIWH arguments. More precisely, we first introduce several Sigma-protocols that are previously mentioned in our instantiations. We then introduce the Kondi-shelat transform [Ks22] which compiles a Sigma-protocol into a straight-line extractable NIZK (resp. NIWH) arguments in the RO (resp. observable RO) model.

## B.1 Concrete Examples of Sigma-Protocols

**Schnorr's protocol [Sch90].** The Schnorr's protocol aims to prove the knowledge of a discrete logarithm. Let $\mathcal{R}_{\mathsf{DL}}:=\{((g,h),w)\mid h=g^w\}$. We present the Sigma-protocol for relation $\mathcal{R}_{\mathsf{DL}}$ in Figure 25.

**Chaum-Pedersen protocol [CP93].** The Chaum-Pedersen protocol aims to prove the knowledge of a DDH tuple. Let $\mathcal{R}_{\mathsf{DDH}}:=\{((g,h,u,v),w)\mid u=g^w \wedge v=h^w\}$. We present the Sigma-protocol for relation $\mathcal{R}_{\mathsf{DDH}}$ in Figure 26.

**Okamoto's protocol [Oka93].** The Okamoto's protocol aims to prove the knowledge of a Pedersen commitment's opening. Let $\mathcal{R}_{\mathsf{Com}}:=\{((g,h,C),(m,r))\mid C=g^m h^r\}$. We present the Sigma-protocol for relation $\mathcal{R}_{\mathsf{Com}}$ in Figure 27.

**Schnorr's Protocol**

**Common Input:** Statement $x := (\mathbb{G}, p, q, g, h)$, where $p, q$ describe the group $\mathbb{G}$ and $g, h \in \mathbb{G}$.
**Private Input of the Prover:** Witness $w$ such that $h = g^w$.

---

- $\mathsf{P1}(x, w)$: Select $r \leftarrow \mathbb{Z}_q$ and output $a := g^r \mod p, \mathsf{st} := r$.
- $\mathsf{V1}(1^\lambda)$: Select $e \leftarrow \mathbb{Z}_q$ and output $e$.
- $\mathsf{P2}(x, w, e, \mathsf{st})$: Compute $z := r + e \cdot w \mod q$ and output $z$.
- $\mathsf{Verify}(x, a, e, z)$: Check if $g^z = a \cdot h^e$ holds. If so, output 1; otherwise, output 0.

Figure 25: Sigma-Protocol for Relation $\mathcal{R}_{\mathsf{DL}}$

**Chaum-Pedersen Protocol**

**Common Input:** Statement $x := (\mathbb{G}, p, q, g, h, u, v)$, where $p, q$ describe the group $\mathbb{G}$ and $g, h, u, v \in \mathbb{G}$.
**Private Input of the Prover:** Witness $w$ such that $u = g^w$ and $v = h^w$.

---

- $\mathsf{P1}(x, w)$: Select $r \leftarrow \mathbb{Z}_q$, compute $a_0 := g^r \mod p, a_1 := h^r \mod p$ and output $a := (a_0, a_1), \mathsf{st} := r$.
- $\mathsf{V1}(1^\lambda)$: Select $e \leftarrow \mathbb{Z}_q$ and output $e$.
- $\mathsf{P2}(x, w, e, \mathsf{st})$: Compute $z := r + e \cdot w \mod q$ and output $z$.
- $\mathsf{Verify}(x, a, e, z)$: Check if $g^z = a_0 \cdot u^e$ and $h^z = a_1 \cdot v^e$ hold. If so, output 1; otherwise, output 0.

Figure 26: Sigma-Protocol for Relation $\mathcal{R}_{\mathsf{DDH}}$

**Okamoto's Protocol**

**Common Input:** Statement $x := (\mathbb{G}, p, q, g, h, C)$, where $p, q$ describe the group $\mathbb{G}$ and $g, h, C \in \mathbb{G}$.
**Private Input of the Prover:** Witness $w := (m, r)$ such that $C = g^m h^r$.

---

- $\mathsf{P1}(x, w)$: Select $s, t \leftarrow \mathbb{Z}_q$, compute $a := g^s h^t \mod p$ and output $a, \mathsf{st} := (s, t)$.
- $\mathsf{V1}(1^\lambda)$: Select $e \leftarrow \mathbb{Z}_q$ and output $e$.
- $\mathsf{P2}(x, w, e, \mathsf{st})$: Compute $z_0 := s + e \cdot m \mod q, z_1 := t + e \cdot r \mod q$ and output $z := (z_0, z_1)$.
- $\mathsf{Verify}(x, a, e, z)$: Check if $g^{z_0} h^{z_1} = a \cdot C^e$ holds. If so, output 1; otherwise, output 0.

Figure 27: Sigma-Protocol for Relation $\mathcal{R}_{\mathsf{Com}}$

**OR-Composition**

**Common Input**: Statement $x := (x_0, x_1)$.
**Private Input of the Prover**: Witness $w$ such that $(x_b, w) \in \mathcal{R}_b$.

---

- $\mathsf{P1}(x, w)$: Compute $(a_b, \mathsf{st}) \leftarrow \Pi_b.\mathsf{P1}(x_b, w)$. Select $s_{1-b} \leftarrow \Pi_b.\mathsf{V1}(1^\lambda)$ and compute $(a_{1-b}, z_{1-b}) \leftarrow \Pi_{1-b}.\mathsf{Sim}(x_{1-b}, s_{1-b})$. Output $a := (a_0, a_1), \mathsf{st}$.
- $\mathsf{V1}(1^\lambda)$: Select $e \leftarrow \mathbb{Z}_q$ and output $e$.
- $\mathsf{P2}(x, w, e, \mathsf{st})$: Set $s_b := e \oplus s_{1-b}$, compute $z_b \leftarrow \Pi_b.\mathsf{P2}(x_b, w, s_b, \mathsf{st})$ and output $z := (s_0, z_0, s_1, z_1)$.
- $\mathsf{Verify}(x, a, e, z)$: Check if $e = s_0 \oplus s_1$ and $\Pi_0.\mathsf{Verify}(x_0, a_0, s_0, z_0) = \Pi_1.\mathsf{Verify}(x_1, a_1, s_1, z_1) = 1$ hold. If so, output 1; otherwise, output 0.

Figure 28: Sigma-Protocol for Relation $\mathcal{R}_0 \vee \mathcal{R}_1$

**OR-composition [CDS94].** Let $(x_0, x_1)$ be a pair of the statements.The OR-composition of the $\Sigma$-protocol allows the prover to prove that it knows a witness $w$ such that either $(x_0, w) \in \mathcal{R}_0$ or $(x_1, w) \in \mathcal{R}_1$ without revealing which is the case. Let $\Pi_0$ be the Sigma-protocol for relation $\mathcal{R}_0$ and $\Pi_1$ be the one for relation $\mathcal{R}_1$. We present the Sigma-protocol for relation $\mathcal{R}_0 \vee \mathcal{R}_1$ in Figure 28.

## B.2 Kondi-shelat Transform

Recently, Kondi and shelat revisited the famous Fischlin transform [Fis05], which complies a Sigma-protocol into a straight-line extractable NIZK (resp. NIWH) arguments in the RO (resp. observable RO) model, and improved it in both efficiency and applicability. We call their construction *Kondi-shelat transform* [Ks22], and review the Kondi-shelat transform $\Pi_{\mathsf{Ks\text{-}NIZK}}$ in Figure 29.

**Kondi-shelat Transform**

**Common Input:** Statement $x$.
**Private Input of the Prover:** Witness $w$ such that $(x, w) \in \mathcal{R}$.
**Parameter:** $r, \ell, t$ such that $r$ is an even number, $r \cdot \ell = 2^\lambda$ and $t = \lceil \log \lambda \rceil \cdot \ell$.
**Primitive:** Sigma-protocol $\Pi_\Sigma$ for relation $\mathcal{R}$.
**Random Oracle:** $\mathcal{F}_{\mathsf{RO}} : \{0,1\}^* \to \{0,1\}^{2\ell}$.

---

- $\mathsf{Prove}^{\mathcal{F}_{\mathsf{RO}}}(x, w)$:
  1. For $i \in [r]$: compute $(a_i, \mathsf{st}_i) \leftarrow \Pi_\Sigma.\mathsf{P1}(x, w)$.
  2. Set $a := (a_i)_{i \in [r]}$.
  3. For $i \in [r/2]$: do the following:
     (a) Set $\xi_i := \emptyset$ and $\mathsf{flag} := 0$.
     (b) Sample $e_i \leftarrow \Pi_\Sigma.\mathsf{V1}(1^\lambda) \setminus \xi_i$ and compute $z_i \leftarrow \Pi_\Sigma.\mathsf{P2}(x, w, e_i, \mathsf{st}_i)$.
     (c) For $j \in [2^{2\ell}]$:
        i. Set $\xi_{i+r/2} := \emptyset$.
        ii. Sample $e_{i+r/2} \leftarrow \Pi_\Sigma.\mathsf{V1}(1^\lambda) \setminus \xi_{i+r/2}$ and compute $z_{i+r/2} \leftarrow \Pi_\Sigma.\mathsf{P2}(x, w, e_{i+r/2}, \mathsf{st}_{i+r/2})$.
        iii. If $\mathcal{F}_{\mathsf{RO}}(a, i, e_i, z_i) \neq \mathcal{F}_{\mathsf{RO}}(a, i + r/2, e_{i+r/2}, z_{i+r/2})$, repeat Step 3(c)ii; else, set $\mathsf{flag} := 1$ and break the loop.
     (d) If $\mathsf{flag} \neq 1$, repeat Step 3b.
  4. Output $\pi := (a_i, e_i, z_i)_{i \in [r]}$.
- $\mathsf{Verify}^{\mathcal{F}_{\mathsf{RO}}}(x, \pi)$:
  1. Output 1 if the following conditions hold:
     (a) For $i \in [r/2]$: Check if $\mathcal{F}_{\mathsf{RO}}(a, i, e_i, z_i) = \mathcal{F}_{\mathsf{RO}}(a, i + r/2, e_{i+r/2}, z_{i+r/2})$ holds.
     (b) For $i \in [r]$: Check if $\Pi_\Sigma.\mathsf{Verify}(x, a_i, e_i, z_i) = 1$ holds.

Figure 29: Kondi-shelat Transform $\Pi_{\mathsf{Ks\text{-}NIZK}}$ from [Ks22]

Recall that, the Fischlin transform requires the Sigma-protocol to additionally satisfy quasi-unique responses [Fis05]. Roughly speaking, this means that no PPT prover can compute a statement $x$ and $a, e, z, z'$ such that $(a, e, z)$ and $(a, e, z')$ are both accepting transcripts for $x$. Kondi and shelat showed that a simpler property, i.e., *strong special soundness*, would be sufficient for their purpose. More precisely, they defined a variant of Sigma-protocols, i.e., *strong special sound Sigma-protocols* and proved that their transform can be applied to these Sigma-protocols securely. We present the definition of strong special sound Sigma-protocols as follows.

**Definition 19.** *Fix an NP relation $\mathcal{R}$ and its associate language $\mathcal{L}$. We say a protocol $\Pi = (\Pi.\mathsf{P1}, \Pi.\mathsf{V1}, \Pi.\mathsf{P2}, \Pi.\mathsf{Verify}, \Pi.\mathsf{Ext}, \Pi.\mathsf{Sim})$ is a strong special sound Sigma-protocol for relation $\mathcal{R}$ if it satisfies completeness and SHVZK that have been defined in Definition 5 and the following property:*

1. ***(Strong Special Soundness)*** *For any $x \in \mathcal{L}$, we say it is a strong special sound if for any PPT adversary $\mathcal{A}$,*

$$\Pr\left[\begin{array}{l|l} (T := (a,e,z), T' := (a,e',z')) \leftarrow \mathcal{A}(x); & \mathsf{Verify}(x,T) = \mathsf{Verify}(x,T') = 1 \\ T \neq T' & \wedge\ w \leftarrow \mathsf{Ext}(x,T,T') \ \wedge\ (x,w) \in \mathcal{R} \end{array}\right] = 1$$

In the main body of our paper, we apply Kondi-shelat transform to Schnorr's protocol [Sch90] and Okamoto's protocol [Oka93] to instantiate straight-line extractable NIZK (resp. NIWH) arguments in the RO (resp. observable RO) model. It is easy to see that both Schnorr's protocol and Okamoto's protocol are strong special sound Sigma-protocols. Thus the security of our instantiations is guaranteed.

# C Additional Security Proofs

## C.1 Proof of Theorem 1

**Theorem 1.** *Assume the DDH assumption holds in group $\mathbb{G}$. Let $\mathcal{F}_{\mathsf{RO1}} : \{0,1\}^* \to \mathbb{G}$ and $\mathcal{F}_{\mathsf{RO2}} : \{0,1\}^* \to \{0,1\}^\lambda$ be the random oracles. Let $\Pi_{\mathsf{NIZK}}$ be an NIZK argument in the $\mathcal{F}_{\mathsf{RO3}}$-hybrid world. Let $\Pi_{\mathsf{sleNIZK}}$ be a straight-line extractable NIZK argument in the $\mathcal{F}_{\mathsf{RO4}}$-hybrid world. The protocol $\Pi_{\mathsf{sEOT\text{-}RO}}$ depicted in Figure 11 UC-realizes the functionality $\mathcal{F}_{\mathsf{EOT}}$ depicted in Figure 6 in the $\{\mathcal{F}_{\mathsf{RO}}, \mathcal{F}_{\mathsf{SYN}}\}$-hybrid world against static malicious corruption, where $\mathcal{F}_{\mathsf{RO}} = \{\mathcal{F}_{\mathsf{RO}i}\}_{i\in[4]}$.*

*Proof.* We now prove the security of our protocol $\Pi_{\mathsf{sEOT\text{-}RO}}$ by showing it is a UC-secure realization of $\mathcal{F}_{\mathsf{EOT}}$. We describe the workflow of $\mathcal{S}$ in the ideal-world with $\mathcal{F}_{\mathsf{EOT}}$, and give a proof that the simulation in the ideal world execution $\mathsf{EXEC}_{\mathcal{F}_{\mathsf{EOT}},\mathcal{S},\mathcal{Z}}$ is computationally indistinguishable from a real world execution $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{RO}},\mathcal{F}_{\mathsf{SYN}}}_{\Pi_{\mathsf{sEOT\text{-}RO}},\mathcal{A},\mathcal{Z}}$ for any PPT adversary $\mathcal{A}$ and any PPT environment $\mathcal{Z}$. Note that, the simulator $\mathcal{S}$ simply forwards the communication between $\mathcal{A}$ and $\mathcal{Z}$.

**The sender is honest while the receiver is statically corrupted.** Here the simulator $\mathcal{S}$ needs to extract the choice bit of the receiver from the message sent from $R^*$. Note that, the simulator $\mathcal{S}$ needs to send its message on behalf of the honest sender before seeing the adversary $\mathcal{A}$'s message. We describe the strategy of $\mathcal{S}$ as follows:

- Generate $(z, \pi_{\mathsf{DL}})$ honestly, and send $(z, \pi_{\mathsf{DL}})$ to $R^*$.
- Whenever $R^*$ invokes $\mathcal{F}_{\mathsf{RO1}}$ on candidate seed values, select different $\alpha \in \mathbb{Z}_q$ and return $h = g^\alpha$ as the output of $\mathcal{F}_{\mathsf{RO1}}$.
- Wait for $R^*$ to send $(\mathsf{seed}, u, v, \pi_{\mathsf{ENC}})$, then do the following:
  - Abort if $\Pi_{\mathsf{NIZK}}.\mathsf{Verify}^{\mathcal{F}_{\mathsf{RO4}}}((g,h,u,v), \pi_{\mathsf{ENC}}) = 0$ for $\mathcal{R}_{\mathsf{ENC}}$.
  - If $u = v^\alpha$, set $b := 0$; else if $u = (\frac{v}{h})^\alpha$, set $b := 1$; else, abort.
  - Compute $m_0, m_1$ honestly, and send $(\textsc{Receive}, \mathsf{sid}, S, R^*, b)$ to $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted receiver. Return $(\textsc{Receive}, \mathsf{sid}, S, R^*, b, m_b)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\textsc{Receive}, \mathsf{sid}, S, R^*)$.

We prove the indistinguishability through the following hybrid experiments:

- $\mathcal{H}_0$: This is the real world execution $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{RO}},\mathcal{F}_{\mathsf{SYN}}}_{\Pi_{\mathsf{sEOT\text{-}RO}},\mathcal{A},\mathcal{Z}}$.
- $\mathcal{H}_1$: Same as $\mathcal{H}_0$, except that $\mathcal{S}$ selects different $\alpha \in \mathbb{Z}_q$ and returns $h = g^\alpha$ as the output of $\mathcal{F}_{\mathsf{RO1}}$ whenever $R^*$ invokes $\mathcal{F}_{\mathsf{RO1}}$ on candidate seed values. Indistinguishability follows from the fact that the tuple $(g,h)$ is randomly selected in both $\mathcal{H}_0$ and $\mathcal{H}_1$.

- $\mathcal{H}_2$: Same as $\mathcal{H}_1$, except that $\mathcal{S}$ aborts when $\pi_{\mathsf{ENC}}$ is not valid. Indistinguishability follows from the fact that honest sender would always abort when $\pi_{\mathsf{ENC}}$ is not valid.

- $\mathcal{H}_3$: Same as $\mathcal{H}_2$, except that $\mathcal{S}$ aborts when it fails to extract $b \in \{0,1\}$.

  **Lemma 2.** *Assume the DDH assumption holds in $\mathbb{G}$. Let $\Pi_{\mathsf{NIZK}}$ be an NIZK argument in the RO model. Hybrid $\mathcal{H}_3$ is computationally indistinguishable from $\mathcal{H}_1$.*

  *Proof.* Since the proof $\pi_{\mathsf{ENC}}$ has already been verified, the tuple $(u, v)$ should be an ElGamal encryption of a bit $b \in \{0,1\}$. In this case, due to the computational soundness of $\Pi_{\mathsf{NIZK}}$, the simulator $\mathcal{S}$ fails to extract $b \in \{0,1\}$ only at a negligible probability. In conclusion, $\mathcal{H}_3$ is computationally indistinguishable from $\mathcal{H}_2$. □

- $\mathcal{H}_4$: Same as $\mathcal{H}_3$, except that $\mathcal{S}$ extracts $b \in \{0,1\}$, computes $m_0, m_1$ honestly, sends $(\text{RECEIVE}, \mathsf{sid}, S, R^*, b)$ to the $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted receiver, and return $(\text{RECEIVE}, \mathsf{sid}, S, R^*, m_b)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\text{RECEIVE}, \mathsf{sid}, S, R^*)$. The simulator $\mathcal{S}$ aborts when $R^*$ computes both $m_0$ and $m_1$.

  **Lemma 3.** *Assume the CDH assumption holds in $\mathbb{G}$, then $\mathcal{H}_4$ is computationally indistinguishable from $\mathcal{H}_3$.*

  *Proof.* The simulator $\mathcal{S}$ aborts when $R^*$ computes $m_0$ and $m_1$ by setting $b = 0$ ($b = 1$) and querying $z^r$, $\frac{z^r}{h^s}$ ($z^r h^s$) to $\mathcal{F}_{\mathsf{RO2}}$. We discuss the case where $R^*$ sets $b = 0$ and queries $z^r$, $\frac{z^r}{h^s}$ to $\mathcal{F}_{\mathsf{RO2}}$ here, and the other case is similar.

  We observe that if there exists such a $R^*$, then there is a reduction $\mathcal{B}$ which breaks the CDH assumption. The reduction $\mathcal{B}$ interacts with the CDH game challenger $\mathcal{C}$ and receives $(A_1, A_2, A_3)$ from $\mathcal{C}$, and we assume that $A_1 := g, A_2 := g^t, A_3 := g^s$. Then $\mathcal{B}$ simulates $\mathcal{F}_{\mathsf{RO1}}, \mathcal{F}_{\mathsf{RO2}}, \mathcal{F}_{\mathsf{RO3}}, \mathcal{F}_{\mathsf{RO4}}$ and starts the protocol $\Pi_{\mathsf{sEOT\text{-}RO}}$ with $R^*$ by running $R^*$ internally as a black-box. Note that $\mathcal{B}$ has full control over $\{\mathcal{F}_{\mathsf{RO}i}\}_{i\in[4]}$. First $\mathcal{B}$ sets $z := A_3$ and simulates the straight-line extractable NIZK proof $\pi_{\mathsf{DL}}$ by programming the random oracles $\mathcal{F}_{\mathsf{RO3}}$, and sends $z, \pi_{\mathsf{DL}}$ to $R^*$. When $R^*$ queries a $\lambda$-bit string $\mathsf{seed}_i$ to $\mathcal{F}_{\mathsf{RO1}}$, $\mathcal{B}$ returns $A_2 \cdot g^{a_i}$ where $a_i \leftarrow \mathbb{Z}_q$. Upon receiving $(\mathsf{seed}, u, v, \pi_{\mathsf{ENC}})$ from $R^*$, $\mathcal{B}$ looks up the query-answer table of $\mathcal{F}_{\mathsf{RO1}}$ and finds the index $j$ such that $\mathsf{seed} = \mathsf{seed}_j$ and $h = A_2 \cdot g^{a_j}$. Finally, $\mathcal{B}$ randomly selects two queries $q_1, q_2$ made to $\mathcal{F}_{\mathsf{RO2}}$ by $R^*$ and sends $A_4 := \frac{q_1}{q_2 A_3^{a_j}}$ to $\mathcal{C}$. If there are $Q$ queries made to $\mathcal{F}_{\mathsf{RO2}}$ by $R^*$ in total, then $A_4 = \frac{q_1}{q_2 A_3^{a_j}} = \frac{z^r}{\frac{z^r}{h^s} A_3^{a_j}} = \frac{A_2^s \cdot g^{s a_j}}{A_3^{a_j}} = g^{st}$ happens at probability $\frac{1}{2\cdot\binom{Q}{2}}$. Therefore, $\mathcal{B}$ wins the CDH game at probability $\frac{1}{2\cdot\binom{Q}{2}}$ which is non-negligible. In conclusion, $\mathcal{H}_3$ is computationally indistinguishable from $\mathcal{H}_2$. □

Hybrid $\mathcal{H}_4$ is identical to the ideal world execution $\mathsf{EXEC}_{\mathcal{F}_{\mathsf{EOT}}, \mathcal{S}, \mathcal{Z}}$. In conclusion, when the sender $S$ is honest and the receiver $R^*$ is statically corrupted, $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{RO}}, \mathcal{F}_{\mathsf{SYN}}}_{\Pi_{\mathsf{sEOT\text{-}RO}}, \mathcal{A}, \mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}_{\mathcal{F}_{\mathsf{EOT}}, \mathcal{S}, \mathcal{Z}}$ holds.

**The receiver is honest while the sender is statically corrupted.** Here the simulator $\mathcal{S}$ needs to extract the secret randomness of the sender from the message sent from $S^*$ and computes both messages. Note that, the simulator $\mathcal{S}$ needs to send its message on behalf of the honest receiver before seeing the adversary $\mathcal{A}$'s message. We describe the strategy of $\mathcal{S}$ as follows:

- Select a random bit $b \in \{0,1\}$ and generate $(\mathsf{seed}, u, v, \pi_{\mathsf{ENC}})$ honestly. Send $(\mathsf{seed}, u, v, \pi_{\mathsf{ENC}})$ to $S^*$.

- Wait for $S^*$ to send $(\mathsf{seed}, z, \pi_{\mathsf{DL}})$, then do the following:

  – Abort if $\Pi_{\mathsf{sleNIZK}}.\mathsf{Verify}^{\mathcal{F}_{\mathsf{RO3}}}((g, z), \pi_{\mathsf{DL}}) = 0$ for $\mathcal{R}_{\mathsf{DL}}$.

– Invoke the straight-line extractor $\Pi_{\mathsf{sleNIZK}}.\mathsf{Ext}^{\mathcal{F}_{\mathsf{RO3}}}$ to obtain $s$ such that $z = g^s$ (note that, the proof $\pi_{\mathsf{DL}}$ is verified, and the simulator $\mathcal{S}$ can see the query-answer list of $\mathcal{F}_{\mathsf{RO3}}$ since $\mathcal{F}_{\mathsf{RO3}}$ is simulated by $\mathcal{S}$, and thus $\mathcal{S}$ is able to invoke the extractor algorithm).
– Compute $m_0 := \mathcal{F}_{\mathsf{RO2}}(\mathsf{sid}, `S^{*\prime}||v^r)$ and $m_1 := \mathcal{F}_{\mathsf{RO2}}(\mathsf{sid}, `S^{*\prime}||(\frac{v}{h})^r)$.
– Send $(\textsc{Send}, \mathsf{sid}, S^*, R)$ to $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted sender. Return $(\textsc{Send}, \mathsf{sid}, S^*, R, m_0, m_1)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\textsc{Receive}, \mathsf{sid}, S^*, R)$.

We prove the indistinguishability through the following hybrid experiments:

- $\mathcal{H}_0$: This is the real world execution $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{RO}}, \mathcal{F}_{\mathsf{SYN}}}_{\Pi_{\mathsf{sEOT\text{-}RO}}, \mathcal{A}, \mathcal{Z}}$.
- $\mathcal{H}_1$: Same as $\mathcal{H}_0$, except that $\mathcal{S}$ aborts when $\pi_{\mathsf{DL}}$ is not valid. Indistinguishability follows from the fact that honest receiver would always abort when $\pi_{\mathsf{DL}}$ is not valid.
- $\mathcal{H}_2$: Same as $\mathcal{H}_1$, except that $\mathcal{S}$ extracts $s$ by invoking the straight-line extractor $\Pi_{\mathsf{sleNIZK}}.\mathsf{Ext}^{\mathcal{F}_{\mathsf{RO3}}}$ and computes $m_0, m_1$. Indistinguishability follows from the straight-line extractability of $\Pi_{\mathsf{sleNIZK}}$.
- $\mathcal{H}_3$: Same as $\mathcal{H}_2$, except that $\mathcal{S}$ sends $(\textsc{Send}, \mathsf{sid}, S^*, R)$ to $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted sender, and returns $(\textsc{Send}, \mathsf{sid}, S^*, R, m_0, m_1)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\textsc{Send}, \mathsf{sid}, S^*, R)$. The simulator $\mathcal{S}$ aborts when $S^*$ extracts $b$ from the messages sent by $\mathcal{S}$.

  **Lemma 4.** *Assume the DDH assumption holds in $\mathbb{G}$. Let $\Pi_{\mathsf{NIZK}}$ be an NIZK argument in the RO model. Hybrid $\mathcal{H}_3$ is computationally indistinguishable from $\mathcal{H}_1$. Hybrid $\mathcal{H}_3$ is computationally indistinguishable from $\mathcal{H}_2$.*

  *Proof.* We show that any PPT $S^*$ cannot extract $b$ from $(u, v)$ or $\pi_{\mathsf{ENC}}$ except with a negligible probability. It is easy to see that $(u, v)$ is the ElGammal encryption of $b$, so $b$ is computationally hidden in $(u, v)$. The zero-knowledge property of $\Pi_{\mathsf{NIZK}}$ guarantees that no PPT $S^*$ can learn any information about $b$ from $\pi_{\mathsf{ENC}}$. Therefore, $\mathcal{S}$ aborts at a negligible probability. In conclusion, $\mathcal{H}_3$ is computationally indistinguishable from $\mathcal{H}_2$. □

Hybrid $\mathcal{H}_3$ is identical to the ideal world execution $\mathsf{EXEC}_{\mathcal{F}_{\mathsf{EOT}}, \mathcal{S}, \mathcal{Z}}$. In conclusion, when the receiver $R$ is honest and the sender $S^*$ is statically corrupted, $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{RO}}, \mathcal{F}_{\mathsf{SYN}}}_{\Pi_{\mathsf{sEOT\text{-}RO}}, \mathcal{A}, \mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}_{\mathcal{F}_{\mathsf{EOT}}, \mathcal{S}, \mathcal{Z}}$ holds. □

## C.2 Proof of Theorem 2

**Theorem 2.** *Assume the DDH assumption holds in group $\mathbb{G}$. Let $\mathcal{F}_{\mathsf{RO1}} : \{0,1\}^* \to \mathbb{G}$ and $\mathcal{F}_{\mathsf{RO2}} : \{0,1\}^* \to \{0,1\}^\lambda$ be the random oracles. Let $\Pi_{\mathsf{aNIZK}}$ be an NIZK argument with honest prover state reconstruction in the $\mathcal{F}_{\mathsf{RO3}}$-hybrid world. Let $\Pi_{\mathsf{sleNIZK}}$ be a straight-line extractable NIZK argument in the $\mathcal{F}_{\mathsf{RO4}}$-hybrid world. The protocol $\Pi_{\mathsf{aEOT\text{-}RO}}$ depicted in Figure 12 UC-realizes the functionality $\mathcal{F}_{\mathsf{EOT}}$ depicted in Figure 6 in the $\{\mathcal{F}_{\mathsf{RO}}, \mathcal{F}_{\mathsf{SYN}}\}$-hybrid world against adaptive malicious corruption, where $\mathcal{F}_{\mathsf{RO}} = \{\mathcal{F}_{\mathsf{RO}i}\}_{i \in [4]}$.*

*Proof.* We now prove the security of our protocol $\Pi_{\mathsf{aEOT\text{-}RO}}$ by showing it is a UC-secure realization of $\mathcal{F}_{\mathsf{EOT}}$. We describe the workflow of $\mathcal{S}$ in the ideal-world with $\mathcal{F}_{\mathsf{EOT}}$, and give a proof that the simulation in the ideal world execution $\mathsf{EXEC}_{\mathcal{F}_{\mathsf{EOT}}, \mathcal{S}, \mathcal{Z}}$ is computationally indistinguishable from a real world execution $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{RO}}, \mathcal{F}_{\mathsf{SYN}}}_{\Pi_{\mathsf{aEOT\text{-}RO}}, \mathcal{A}, \mathcal{Z}}$ for any PPT adversary $\mathcal{A}$ and any PPT environment $\mathcal{Z}$. Note that, the simulator $\mathcal{S}$ simply forwards the communication between $\mathcal{A}$ and $\mathcal{Z}$. We first argue the static security, then discuss the adaptive corruptions of the parties.

**The sender is honest while the receiver is statically corrupted.** Here the simulator $\mathcal{S}$ needs to extract the choice bit of the receiver from the message sent by the malicious receiver $R^*$. Note that, the simulator $\mathcal{S}$ needs to send its message on behalf of the honest sender before seeing the adversary $\mathcal{A}$'s message. We describe the strategy of $\mathcal{S}$ as follows:

- Generate $(z, \pi_{\mathsf{DL}})$ honestly, and send $(z, \pi_{\mathsf{DL}})$ to $R^*$.
- Whenever $R^*$ invokes $\mathcal{F}_{\mathsf{RO1}}$ on candidate seed values, select different $\alpha \in \mathbb{Z}_q$ and return $h = g^\alpha$ as the output of $\mathcal{F}_{\mathsf{RO1}}$.
- Wait for $R^*$ to send $(\mathsf{seed}, B, u_0, v_0, u_1, v_1, \pi_{\mathsf{HYB}})$, then do the following:
  - Abort if $\Pi_{\mathsf{aNIZK}}.\mathsf{Verify}^{\mathcal{F}_{\mathsf{RO4}}}((g, h, B, u_0, v_0, u_1, v_1), \pi_{\mathsf{HYB}}) = 0$ for $\mathcal{R}_{\mathsf{HYB}}$.
  - For $b = 0, 1$:
    - Compute $\frac{v_b^\alpha}{u_b} = h^{\alpha r}$ and search exhaustively for $r$. If cannot find $r$, start over with a new $b$.
    - Check if $B = g^r h^b$. If not, abort.
  - Abort if it computes both $r_0$ (such that $B = g^{r_0}$ and $(u_0, v_0) = (h^t, g^t h^{r_0})$) and $r_1$ (such that $B = g^{r_1} h$ and $(u_1, v_1) = (h^t, g^t h^{r_1})$) from the message sent by $R^*$.
  - Compute $m_0, m_1$ honestly, and send $(\text{RECEIVE}, \mathsf{sid}, b, m_b)$ to $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted receiver. Return $(\text{RECEIVE}, \mathsf{sid}, S, R^*, b, m_b)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\text{RECEIVE}, \mathsf{sid}, S, R^*, b)$.

We prove the indistinguishability through the following hybrid experiments:

- $\mathcal{H}_0$: This is the real world execution $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{RO}}, \mathcal{F}_{\mathsf{SYN}}}_{\Pi_{\mathsf{aEOT\text{-}RO}}, \mathcal{A}, \mathcal{Z}}$.
- $\mathcal{H}_1$: Same as $\mathcal{H}_0$, except that $\mathcal{S}$ selects different $\alpha \in \mathbb{Z}_q$ and returns $h = g^\alpha$ as the output of $\mathcal{F}_{\mathsf{RO1}}$ whenever $R^*$ invokes $\mathcal{F}_{\mathsf{RO1}}$ on candidate seed values. Indistinguishability follows from the fact that the tuple $(g, h)$ is randomly selected in both $\mathcal{H}_0$ and $\mathcal{H}_1$.
- $\mathcal{H}_2$: Same as $\mathcal{H}_1$, except that $\mathcal{S}$ aborts when $\pi_{\mathsf{HYB}}$ is not valid. Indistinguishability follows from the fact that honest sender would always abort when $\pi_{\mathsf{ENC}}$ is not valid.
- $\mathcal{H}_3$: Same as $\mathcal{H}_2$, except that $\mathcal{S}$ aborts when it fails to extract $b \in \{0, 1\}$.

  **Lemma 5.** *If $\Pi_{\mathsf{aNIZK}}$ is an NIZK argument with HPSR in the RO model,, then $\mathcal{H}_3$ is computationally indistinguishable from $\mathcal{H}_2$.*

  *Proof.* Since $\pi_{\mathsf{HYB}}$ has already been verified, by the computational soundness of $\Pi_{\mathsf{aNIZK}}$, either $(u_0, v_0)$ or $(u_1, v_1)$ should be an ElGammal encryption of randomness $r$ such that $B = g^r$ or $B = g^r h$ holds. By the perfect correctness of the ElGamal encryption, the simulator $\mathcal{S}$ should be able to decrypt the ciphertext to obtain $b$. Therefore, the simulator $\mathcal{S}$ fails to extract $b \in \{0, 1\}$ only at a negligible probability. In conclusion, $\mathcal{H}_3$ is computationally indistinguishable from $\mathcal{H}_2$. □
- $\mathcal{H}_4$: Same as $\mathcal{H}_3$, except that $\mathcal{S}$ aborts when it computes both $r_0$ (such that $B = g^{r_0}$ and $(u_0, v_0) = (h^t, g^t h^{r_0})$) and $r_1$ (such that $B = g^{r_1} h$ and $(u_1, v_1) = (h^t, g^t h^{r_1})$) the message sent by $R^*$.

  **Lemma 6.** *Assume the DL assumption holds in $\mathbb{G}$, then $\mathcal{H}_4$ is computationally indistinguishable from $\mathcal{H}_3$.*

  *Proof.* The simulator $\mathcal{S}$ aborts when the malicious receiver $R^*$ computes two different openings of the same commitment $B$. Since the DL assumption holds in $\mathbb{G}$, by the computational binding property of the Pedersen commitment, this would happen only at a negligible probability. In conclusion, $\mathcal{H}_4$ is computationally indistinguishable from $\mathcal{H}_3$. □

- $\mathcal{H}_5$: Same as $\mathcal{H}_4$, except that $\mathcal{S}$ extracts $b \in \{0,1\}$, computes $m_0, m_1$ honestly, sends $(\text{RECEIVE}, \mathsf{sid}, S, R, b)$ to the $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted receiver, and returns $(\text{RECEIVE}, \mathsf{sid}, S, R^*, b, m_b)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\text{RECEIVE}, \mathsf{sid}, S, R^*, b)$. The simulator $\mathcal{S}$ aborts when $R^*$ computes both $m_0$ and $m_1$.

  **Lemma 7.** *Assume the CDH assumption holds in $\mathbb{G}$, then $\mathcal{H}_5$ is computationally indistinguishable from $\mathcal{H}_4$.*

  *Proof.* The simulator $\mathcal{S}$ aborts when $R^*$ computes $m_0$ and $m_1$ by setting $b = 0$ ($b = 1$) and querying $z^r, \frac{z^r}{h^s}$ ($z^r h^s$) to $\mathcal{F}_{\mathsf{RO2}}$. We discuss the case where $R^*$ sets $b = 0$ and queries $z^r, \frac{z^r}{h^s}$ to $\mathcal{F}_{\mathsf{RO2}}$ here, and the other case is similar.

  We observe that if there exists such a $R^*$, then there is a reduction $\mathcal{B}$ which breaks the CDH assumption. The reduction $\mathcal{B}$ interacts with the CDH game challenger $\mathcal{C}$ and receives $(A_1, A_2, A_3)$ from $\mathcal{C}$, and we assume that $A_1 := g, A_2 := g^t, A_3 := g^s$. Then $\mathcal{B}$ simulates $\mathcal{F}_{\mathsf{RO1}}, \mathcal{F}_{\mathsf{RO2}}, \mathcal{F}_{\mathsf{RO3}}, \mathcal{F}_{\mathsf{RO4}}$ and starts the protocol $\Pi_{\mathsf{aEOT\text{-}RO}}$ with $R^*$ by running $R^*$ internally as a black-box. Note that $\mathcal{B}$ has full control over $\{\mathcal{F}_{\mathsf{RO}i}\}_{i\in[4]}$. First $\mathcal{B}$ sets $z := A_3$ and simulates the straight-line extractable NIZK proof $\pi_{\mathsf{DL}}$ by programming the random oracles $\mathcal{F}_{\mathsf{RO3}}$, and sends $z, \pi_{\mathsf{DL}}$ to $R^*$. When $R^*$ queries a $\lambda$-bit string $\mathsf{seed}_i$ to $\mathcal{F}_{\mathsf{RO1}}$, $\mathcal{B}$ returns $A_2 \cdot g^{a_i}$ where $a_i \leftarrow \mathbb{Z}_q$. Upon receiving $(\mathsf{seed}, B, u_0, v_0, u_1, v_1, \pi_{\mathsf{HYB}})$ from $R^*$, $\mathcal{B}$ looks up the query-answer table of $\mathcal{F}_{\mathsf{RO1}}$ and finds the index $j$ such that $\mathsf{seed} = \mathsf{seed}_j$ and $h = A_2 \cdot g^{a_j}$. Finally, $\mathcal{B}$ randomly selects two queries $q_1, q_2$ made to $\mathcal{F}_{\mathsf{RO2}}$ by $R^*$ and sends $A_4 := \frac{q_1}{q_2 A_3^{a_j}}$ to $\mathcal{C}$. If there are $Q$ queries made to $\mathcal{F}_{\mathsf{RO2}}$ by $R^*$ in total, then $A_4 = \frac{q_1}{q_2 A_3^{a_j}} = \frac{z^r}{\frac{z^r}{h^s} A_3^{a_j}} = \frac{A_2^s \cdot g^{s a_j}}{A_3^{a_j}} = g^{st}$ happens at probability $\frac{1}{2\cdot\binom{Q}{2}}$. Therefore, $\mathcal{B}$ wins the CDH game at probability $\frac{1}{2\cdot\binom{Q}{2}}$ which is non-negligible. In conclusion, $\mathcal{H}_5$ is computationally indistinguishable from $\mathcal{H}_4$. □

Hybrid $\mathcal{H}_5$ is identical to the ideal world execution $\mathsf{EXEC}_{\mathcal{F}_{\mathsf{EOT}},\mathcal{S},\mathcal{Z}}$. In conclusion, when the sender $S$ is honest and the receiver $R^*$ is statically corrupted, $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{RO}},\mathcal{F}_{\mathsf{SYN}}}_{\Pi_{\mathsf{aEOT\text{-}RO}},\mathcal{A},\mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}_{\mathcal{F}_{\mathsf{EOT}},\mathcal{S},\mathcal{Z}}$ holds.

**The receiver is honest while the sender is statically corrupted.** Here the simulator $\mathcal{S}$ needs to extract the secret randomness of the sender from the message sent by the malicious sender $S^*$ and computes both messages. Note that, the simulator $\mathcal{S}$ needs to send its message on behalf of the honest receiver before seeing the adversary $\mathcal{A}$'s message. We describe the strategy of $\mathcal{S}$ as follows:

- Select an uniformly random string $\mathsf{seed} \leftarrow \{0,1\}^\lambda$, compute $h := g^\alpha$ using $\alpha \leftarrow \mathbb{Z}_q$ and program the RO such that $h = \mathcal{F}_{\mathsf{RO1}}(\mathsf{sid}, `R'||\mathsf{seed})$.
- Select an uniformly random $r_0 \leftarrow \mathbb{Z}_q$ and compute $B := g^{r_0}$.
- Set $r_1 := r_0 - \alpha$, so that $B = g^{r_1} h$ holds.
- Select uniformly random $t_0, t_1 \leftarrow \mathbb{Z}_q$ and compute $(u_0, v_0) := (h^{t_0}, g^{t_0} h^{r_0})$, $(u_1, v_1) := (h^{t_1}, g^{t_1} h^{r_1})$.
- Invoke the simulator algorithm of $\Pi_{\mathsf{aNIZK}}$ to output $\pi_{\mathsf{HYB}} \leftarrow \Pi_{\mathsf{aNIZK}}.\mathsf{Sim}(g, h, B, u_0, v_0, u_1, v_1)$.
- Send $(\mathsf{seed}, B, u_0, v_0, u_1, v_1, \pi_{\mathsf{HYB}})$ to $S^*$.
- Wait for $S^*$ to send $(\mathsf{seed}, z, \pi_{\mathsf{DL}})$, then do the following:
  - Abort if $\Pi_{\mathsf{sleNIZK}}.\mathsf{Verify}^{\mathcal{F}_{\mathsf{RO3}}}((g, z), \pi_{\mathsf{DL}}) = 0$ for $\mathcal{R}_{\mathsf{DL}}$.
  - Invoke the straight-line extractor $\Pi_{\mathsf{sleNIZK}}.\mathsf{Ext}^{\mathcal{F}_{\mathsf{RO3}}}$ to obtain $s$ such that $z = g^s$ (note that, the proof $\pi_{\mathsf{DL}}$ is verified, and the simulator $\mathcal{S}$ can see the query-answer list of $\mathcal{F}_{\mathsf{RO3}}$ since $\mathcal{F}_{\mathsf{RO3}}$ is simulated by $\mathcal{S}$, and thus $\mathcal{S}$ is able to invoke the extractor algorithm).

– Compute $m_0 := \mathcal{F}_{\mathsf{RO2}}(\mathsf{sid}, `S^{*\prime}||B^r)$ and $m_1 := \mathcal{F}_{\mathsf{RO2}}(\mathsf{sid}, `S^{*\prime}||(\frac{B}{h})^r)$.

– Send $(\textsc{Send}, \mathsf{sid}, S, R)$ to $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted sender. Return $(\textsc{Send}, \mathsf{sid}, S^*, R, m_0, m_1)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\textsc{Receive}, \mathsf{sid}, S^*, R)$.

We prove the indistinguishability through the following hybrid experiments:

- $\mathcal{H}_0$: This is the real world execution $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{RO}}, \mathcal{F}_{\mathsf{SYN}}}_{\Pi_{\mathsf{aEOT\text{-}RO}}, \mathcal{A}, \mathcal{Z}}$.
- $\mathcal{H}_1$: Same as $\mathcal{H}_0$, except that $\mathcal{S}$ generates $B := g^{r_0}$ using $r_0 \leftarrow \mathbb{Z}_q$. Indistinguishability follows from the perfect hiding property of the Pedersen commitment.
- $\mathcal{H}_2$: Same as $\mathcal{H}_1$, except that $\mathcal{S}$ sets $r_1 := r_0 - \alpha$, selects $t_0, t_1 \leftarrow \mathbb{Z}_q$ and computes two ElGamal ciphertext, i.e., $(u_0, v_0) := (h^{t_0}, g^{t_0} h^{r_0}), (u_1, v_1) := (h^{t_1}, g^{t_1} h^{r_1})$. Indistinguishability follows from the oblivious ciphertext sampling of the ElGamal encryption.
- $\mathcal{H}_3$: Same as $\mathcal{H}_2$, except that $\mathcal{S}$ simulates the proof $\pi_{\mathsf{HYB}} \leftarrow \Pi_{\mathsf{aNIZK}}.\mathsf{Sim}(g, h, B, u_0, v_0, u_1, v_1)$. Indistinguishability follows from the zero-knowledge property of $\Pi_{\mathsf{aNIZK}}$.
- $\mathcal{H}_4$: Same as $\mathcal{H}_3$, except that $\mathcal{S}$ aborts when $\pi_{\mathsf{DL}}$ is not valid. Indistinguishability follows from the fact that honest receiver would always abort when $\pi_{\mathsf{DL}}$ is not valid.
- $\mathcal{H}_5$: Same as $\mathcal{H}_4$, except that $\mathcal{S}$ extracts $s$ by invoking the straight-line extractor $\Pi_{\mathsf{sleNIZK}}.\mathsf{Ext}^{\mathcal{F}_{\mathsf{RO3}}}$ and computes $m_0, m_1$. Indistinguishability follows from the straight-line extractability of $\Pi_{\mathsf{sleNIZK}}$.
- $\mathcal{H}_6$: Same as $\mathcal{H}_5$, except that $\mathcal{S}$ sends $(\textsc{Send}, \mathsf{sid}, S^*, R)$ to $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted sender, and returns $(\textsc{Send}, \mathsf{sid}, S^*, R, m_0, m_1)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\textsc{Send}, \mathsf{sid}, S^*, R)$. The simulator $\mathcal{S}$ aborts when $S^*$ extracts $b$ from the messages sent by $\mathcal{S}$.

  **Lemma 8.** *Assume the DDH assumption holds in $\mathbb{G}$. Let $\Pi_{\mathsf{NIZK}}$ be an NIZK argument in the RO model. Hybrid $\mathcal{H}_6$ is computationally indistinguishable from $\mathcal{H}_5$.*

  *Proof.* We show that any PPT malicious sender $S^*$ cannot extract $b$ from $(B, u_0, v_0, u_1, v_1, \pi_{\mathsf{HYB}})$ except with a negligible probability. It is easy to see that $B$ is the Pedersen commitment of $b$, so $b$ is perfectly hidden in $(u, v)$. Since the DDH assumption holds in $\mathbb{G}$, by the semantic security of the ElGamal encryption, the malicious $S^*$ cannot learn anything from $(u_0, v_0, u_1, v_1)$. The zero-knowledge property of $\Pi_{\mathsf{aNIZK}}$ guarantees that any PPT $S^*$ cannot learn any information about $b$ from $\pi_{\mathsf{ENC}}$. Therefore, $\mathcal{S}$ aborts at a negligible probability. In conclusion, $\mathcal{H}_6$ is computationally indistinguishable from $\mathcal{H}_5$. □

Hybrid $\mathcal{H}_6$ is identical to the ideal world execution $\mathsf{EXEC}_{\mathcal{F}_{\mathsf{EOT}}, \mathcal{S}, \mathcal{Z}}$. In conclusion, when the receiver $R$ is honest and the sender $S^*$ is statically corrupted, $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{RO}}, \mathcal{F}_{\mathsf{SYN}}}_{\Pi_{\mathsf{aEOT\text{-}RO}}, \mathcal{A}, \mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}_{\mathcal{F}_{\mathsf{EOT}}, \mathcal{S}, \mathcal{Z}}$ holds.

The proof of static security is completed. Now let us turn to the adaptive security.

**The sender/receiver gets corrupted adaptively.** We present the different simulation cases in Figure 30. We present the according strategy of the simulator $\mathcal{S}$ as follows:

- Online Phase:
  - *Case 1-3*: If $S$ is honest, $\mathcal{S}$ performs the same strategy when $S$ is honest in the static security proof. More precisely, the simulator $\mathcal{S}$ works as follows:
    * Generate $(z, \pi_{\mathsf{DL}})$ honestly, and send $(z, \pi_{\mathsf{DL}})$ to the receiver.
    * Whenever the receiver invokes $\mathcal{F}_{\mathsf{RO1}}$ on candidate seed values, select different $\alpha \in \mathbb{Z}_q$ and return $h = g^\alpha$ as the output of $\mathcal{F}_{\mathsf{RO1}}$.

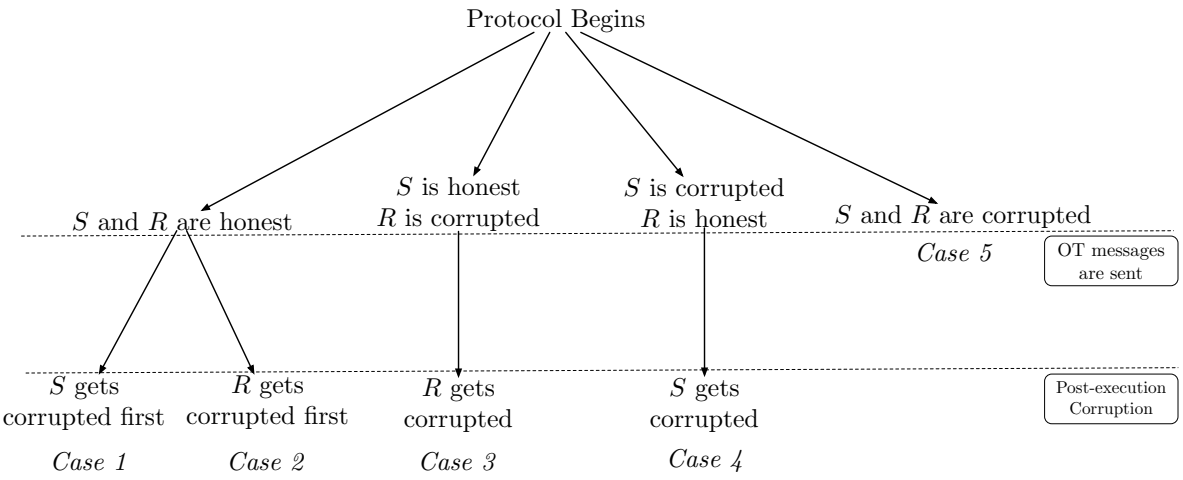


Figure 30: Simulation Cases for Adaptive Corruptions in Protocol $\Pi_{\mathsf{aEOT\text{-}RO}}$

- *Case 1,2,4*: If $R$ is honest, $\mathcal{S}$ performs the same strategy when $R$ is honest in the static security proof. More precisely, the simulator $\mathcal{S}$ works as follows:
  - Select an uniformly random string $\mathsf{seed} \leftarrow \{0,1\}^\lambda$, compute $h := g^\alpha$ using $\alpha \leftarrow \mathbb{Z}_q$ and program the RO such that $h = \mathcal{F}_{\mathsf{RO1}}(\mathsf{sid}, `R'||\mathsf{seed})$.
  - Select an uniformly random $r_0 \leftarrow \mathbb{Z}_q$ and compute $B := g^{r_0}$.
  - Set $r_1 := r_0 - \alpha$, so that $B = g^{r_1} h$ holds.
  - Select uniformly random $t_0, t_1 \leftarrow \mathbb{Z}_q$ and compute $(u_0, v_0) := (h^{t_0},\ g^{t_0} h^{r_0}), (u_1, v_1) := (h^{t_1}, g^{t_1} h^{r_1})$.
  - Invoke the simulator algorithm to output $\pi_{\mathsf{HYB}} \leftarrow \Pi_{\mathsf{aNIZK}}.\mathsf{Sim}(g, h, B, u_0, v_0, u_1, v_1)$.
  - Send $(\mathsf{seed}, B, u_0, v_0, u_1, v_1, \pi_{\mathsf{HYB}})$ to the sender.
- *Case 5*: If $S^*$ and $R^*$ are corrupted, $\mathcal{S}$ ends the simulation.

- Local Computation:
  - *Case 1,2*: If $S$ and $R$ are honest, $\mathcal{S}$ performs nothing.
  - *Case 3*: If $R^*$ is corrupted and $S$ is honest, $\mathcal{S}$ performs the same strategy when $S$ is honest in the static security proof. More precisely, the simulator $\mathcal{S}$ works as follows:
    - Abort if $\Pi_{\mathsf{aNIZK}}.\mathsf{Verify}^{\mathcal{F}_{\mathsf{RO4}}}((g, h, B, u_0, v_0, u_1, v_1), \pi_{\mathsf{HYB}}) = 0$ for $\mathcal{R}_{\mathsf{HYB}}$.
    - For $b = 0, 1$:
      - Compute $\frac{v_b^\alpha}{u_b} = h^{\alpha r}$ and search exhaustively for $r$. If cannot find $r$, start over with a new $b$.
      - Check if $B = g^r h^b$. If not, abort.
    - Abort if it computes both $r_0$ (such that $B = g^{r_0}$ and $(u_0, v_0) = (h^t, g^t h^{r_0})$) and $r_1$ (such that $B = g^{r_1} h$ and $(u_1, v_1) = (h^t, g^t h^{r_1})$) from the message sent by $R^*$.
    - Compute $m_0, m_1$ honestly, and send $(\textsc{Receive}, \mathsf{sid}, b, m_b)$ to $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted receiver. Return $(\textsc{Receive}, \mathsf{sid}, S, R^*, m_b)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\textsc{Receive}, \mathsf{sid}, S, R^*, b)$.
  - *Case 4*: If $S^*$ is corrupted and $R$ is honest, $\mathcal{S}$ performs the same strategy when $R$ is honest in the static security proof. More precisely, the simulator $\mathcal{S}$ works as follows:
    - Abort if $\Pi_{\mathsf{sleNIZK}}.\mathsf{Verify}^{\mathcal{F}_{\mathsf{RO3}}}((g, z), \pi_{\mathsf{DL}}) = 0$ for $\mathcal{R}_{\mathsf{DL}}$.

* Invoke the straight-line extractor $\Pi_{\mathsf{sleNIZK}}.\mathsf{Ext}^{\mathcal{F}_{\mathsf{RO3}}}$ to obtain $s$ such that $z = g^s$ (note that, the proof $\pi_{\mathsf{DL}}$ is verified, and the simulator $\mathcal{S}$ can see the query-answer list of $\mathcal{F}_{\mathsf{RO3}}$ since $\mathcal{F}_{\mathsf{RO3}}$ is simulated by $\mathcal{S}$, and thus $\mathcal{S}$ is able to invoke the extractor algorithm).
* Compute $m_0 := \mathcal{F}_{\mathsf{RO2}}(\mathsf{sid}, `S'||B^r)$ and $m_1 := \mathcal{F}_{\mathsf{RO2}}(\mathsf{sid}, `S'||(\frac{B}{h})^r)$.
* Send $(\textsc{Send}, \mathsf{sid}, S, R)$ to $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted sender. Return $(\textsc{Send}, \mathsf{sid}, S^*, R, m_0, m_1)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\textsc{Receive}, \mathsf{sid}, S, R^*)$.

- Post-Execution Corruption:
  - When $S$ gets corrupted, $\mathcal{S}$ reveals $s$ as the honest sender's internal state.
  - When $R$ gets corrupted, $\mathcal{S}$ performs the following:
    * Corrupt the dummy receiver in the ideal world, and obtain the real choice bit $b$.
    * Invoke $\mathsf{st} \leftarrow \Pi_{\mathsf{aNIZK}}.\mathsf{StateRec}^{\mathcal{F}_{\mathsf{RO4}}}((g, h, B, u_0, v_0, u_1, v_1), (b, r_b, t_b), \pi_{\mathsf{HYB}}, \rho)$ where $\rho$ is the randomness used to generate $\pi_{\mathsf{HYB}}$.
    * Output the following elements as the internal states:
      · $(b, r_b)$: used to generate the Pedersen commitment of $b$, i.e., $B = g^{r_b}h^b$.
      · $(r_b, t_b)$: used to generate the ElGamal encryption of $r_b$, i.e., $(u_b, v_b) = (h^{t_b}, g^{t_b}h^{r_b})$.
      · $(s_0 := \alpha \cdot t_{1-b}, s_1 := t_{1-b} + \alpha \cdot r_{1-b})$: used to sample the random group elements, i.e., $(u_{1-b}, v_{1-b}) = (g^{s_0}, g^{s_1}) = (h^{t_{1-b}}, g^{t_{1-b}}h^{r_{1-b}})$.
      · st: used to generate the augmented NIZK proof $\pi$.
  - In addition to reveal the internal states, $\mathcal{S}$ needs to perform the following:
    * *Case 1,2*: If $S$ and $R$ are honest, $\mathcal{S}$ performs the following based on the sequence of corruption:
      · *Case 1*: If $R$ is honest and $S$ gets corrupted first, $\mathcal{S}$ obtains $(m_0, m_1)$ from $\mathcal{F}_{\mathsf{EOT}}$ and programs $\mathcal{F}_{\mathsf{RO2}}$ s.t. $m_0 = \mathcal{F}_{\mathsf{RO2}}(\mathsf{sid}, `S'||B^s)$, $m_1 = \mathcal{F}_{\mathsf{RO2}}(\mathsf{sid}, `S'||(\frac{B}{h})^s)$.
      · *Case 2*: If $S$ is honest and $R$ gets corrupted first, $\mathcal{S}$ obtains $(b, m_b)$ from $\mathcal{F}_{\mathsf{EOT}}$ and programs $\mathcal{F}_{\mathsf{RO2}}$ s.t. $m_b = \mathcal{F}_{\mathsf{RO2}}(\mathsf{sid}, `S'||B^s(h^s)^{-b})$. When the sender $S$ gets corrupted, $\mathcal{S}$ obtains $(m_0, m_1)$ from $\mathcal{F}_{\mathsf{EOT}}$, programs $\mathcal{F}_{\mathsf{RO2}}$ such that $m_{1-b} = \mathcal{F}_{\mathsf{RO2}}(\mathsf{sid}, `S'||B^s(h^s)^{b-1})$.
    * *Case 3*: If $R^*$ is corrupted and $S$ gets corrupted, $\mathcal{S}$ obtains $(m_0, m_1)$ from $\mathcal{F}_{\mathsf{EOT}}$, programs $\mathcal{F}_{\mathsf{RO2}}$ such that $m_{1-b} = \mathcal{F}_{\mathsf{RO2}}(\mathsf{sid}, `S'||B^s(h^s)^{b-1})$.
    * *Case 4*: If $S^*$ is corrupted and $R$ gets corrupted, $\mathcal{S}$ performs nothing additionally.

We prove the indistinguishability through the following hybrid experiments:

- $\mathcal{H}_0$: This is the real world execution $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{RO}}, \mathcal{F}_{\mathsf{SYN}}}_{\Pi_{\mathsf{aEOT-RO}}, \mathcal{A}, \mathcal{Z}}$.
- $\mathcal{H}_1$: Same as $\mathcal{H}_0$, except if $S$ is honest before the sender's message is sent, the simulator $\mathcal{S}$ performs the same strategy when $S$ is honest in the static security proof. Indistinguishability has been proven in the static security proof. This happens in *Case 1-3* of Figure 30.
- $\mathcal{H}_2$: Same as $\mathcal{H}_1$, except if $S$ gets corrupted after its message is sent, the simulator $\mathcal{S}$ simply reveals $s$ as the internal states. Indistinguishability follows from the fact that the sender's message is generated by invoking honest sender's algorithm. This happens in *Case 1-3* of Figure 30.
- $\mathcal{H}_3$: Same as $\mathcal{H}_2$, except if $R$ is honest before the receiver's message is sent, the simulator $\mathcal{S}$ performs the same strategy when $S$ is honest in the static security proof. Indistinguishability has been proven in the static security proof. This happens in *Case 1,2,4* of Figure.
- $\mathcal{H}_4$: Same as $\mathcal{H}_3$, except if $R$ gets corrupted after its message is sent, the simulator $\mathcal{S}$ reveals $(b, r_b, t_b, s_0, s_1, \mathsf{st})$ as the internal states. This happens in *Case 1,2,4* of Figure 30.

**Lemma 9.** *Assume the DDH assumption holds in $\mathbb{G}$. Let $\Pi_{\mathsf{aNIZK}}$ be an NIZK argument with HPSR in the RO model. Hybrid $\mathcal{H}_4$ is computationally indistinguishable from $\mathcal{H}_3$.*

*Proof.* Since the DDH assumption holds in $\mathbb{G}$, the Pedersen commitment is a secure trapdoor commitment and the ElGamal encryption is a secure pseudorandom PKE system. Due to the trapdoor property of the Pedersen commitment, no PPT adversary can find the distinction when one reveals $(b, r_b)$ as the internal states that used to generate the Pedersen commitment $B$. Since $(u_b, v_b)$ is generated by using $(r_b, t_b)$, revealing it will not cause distinction. Due to the oblivious ciphertext sampling of the ElGamal encryption, no PPT adversary can find the distinction when one reveals $(s_0, s_1)$ as internal states that used to sample the random group elements $(u_{1-b}, v_{1-b})$. Due to the the honest prover state reconstruction of the NIZK argument, no PPT adversary can find the distinction when one reveals st as internal states that used to generate the NIZK proof $\pi$. In conclusion, $\mathcal{H}_4$ is computationally indistinguishable from $\mathcal{H}_3$. □

- $\mathcal{H}_5$: Same as $\mathcal{H}_4$, except if $S$ and $R$ are honest during the protocol and $S$ gets corrupted first in post-execution and then $R$ gets corrupted, in this case the simulator $\mathcal{S}$ obtains $(m_0, m_1)$ from $\mathcal{F}_{\mathsf{EOT}}$ and programs $\mathcal{F}_{\mathsf{RO2}}$ such that $m_0 = \mathcal{F}_{\mathsf{RO2}}(\mathsf{sid}, `S'||B^s)$, $m_1 = \mathcal{F}_{\mathsf{RO2}}(\mathsf{sid}, `S'||(\frac{B}{h})^s)$. This completes *Case 1* of Figure 30.

  **Lemma 10.** *Assume the CDH assumption holds in $\mathbb{G}$, then $\mathcal{H}_5$ is computationally indistinguishable from $\mathcal{H}_4$.*

  *Proof.* Here we argue that the simulator $\mathcal{S}$ can program $\mathcal{F}_{\mathsf{RO2}}$ successfully, i.e., the external adversary $\mathcal{A}$ cannot query $B^s$ or $(\frac{B}{h})^s$ to $\mathcal{F}_{\mathsf{RO2}}$ without corrupting any party and without knowing $s$ or $r$. Here we only discuss the case where the PPT adversary $\mathcal{A}$ cannot query $B^s$ to $\mathcal{F}_{\mathsf{RO2}}$, the case concerning $(\frac{B}{h})^s$ is similar.

  Formally, we observe that if such adversary $\mathcal{A}$ exists, then we can build a reduction $\mathcal{B}$ which breaks the CDH assumption. The reduction $\mathcal{B}$ interacts with the CDH game challenger $\mathcal{C}$ and receives $(A_1, A_2, A_3)$ from $\mathcal{C}$, and we assume that $A_1 := g, A_2 := g^s, A_3 := g^r$. Then $\mathcal{B}$ simulates $\mathcal{F}_{\mathsf{RO1}}, \mathcal{F}_{\mathsf{RO2}}, \mathcal{F}_{\mathsf{RO3}}, \mathcal{F}_{\mathsf{RO4}}$ and starts the protocol $\Pi_{\mathsf{aEOT\text{-}RO}}$ with $\mathcal{A}$ by running $\mathcal{A}$ internally as a black-box. Our $\mathcal{B}$ sets $z := A_2$ and simulates $\pi_{\mathsf{DL}}$ by programming the random oracle, and sends $(z, \pi_{\mathsf{DL}})$ as the sender's message. Meanwhile, our $\mathcal{B}$ sets $B := A_3$, samples $(\mathsf{seed}, u_0, v_0, u_1, v_1)$ randomly, simulates $\pi_{\mathsf{HYB}}$ by programming the random oracle, and finally sends $(\mathsf{seed}, B, u_0, v_0, u_1, v_1, \pi_{\mathsf{HYB}})$ as the receiver's message. Finally, $\mathcal{B}$ randomly selects a query $q$ made to $\mathcal{F}_{\mathsf{RO2}}$ by $\mathcal{A}$ and sends $A_4 := q$ to $\mathcal{C}$. If there are $Q$ queries made to $\mathcal{F}_{\mathsf{RO2}}$ by $\mathcal{A}$ in total, then $A_4 = B^s = g^{rs}$ happens at probability $\frac{1}{Q}$. Therefore, $\mathcal{B}$ wins the CDH game at probability $\frac{1}{Q}$ which is non-negligible. In conclusion, if the CDH assumption holds, then the simulator $\mathcal{S}$ can program $\mathcal{F}_{\mathsf{RO2}}$ successfully. Therefore, we prove that $\mathcal{H}_5$ is computationally indistinguishable from $\mathcal{H}_4$. □

- $\mathcal{H}_6$: Same as $\mathcal{H}_5$, except if $S$ and $R$ are honest during the protocol and $R$ gets corrupted first in post-execution and then $S$ gets corrupted, in this case $\mathcal{S}$ obtains $(b, m_b)$ from $\mathcal{F}_{\mathsf{EOT}}$ and programs $\mathcal{F}_{\mathsf{RO2}}$ such that $m_b = \mathcal{F}_{\mathsf{RO2}}(\mathsf{sid}, `S'||B^s(h^s)^{-b})$. When the sender $S$ gets corrupted, $\mathcal{S}$ obtains $(m_0, m_1)$ from $\mathcal{F}_{\mathsf{EOT}}$, programs $\mathcal{F}_{\mathsf{RO2}}$ such that $m_{1-b} = \mathcal{F}_{\mathsf{RO2}}(\mathsf{sid}, `S'||B^s(h^s)^{b-1})$. This completes *Case 2* of Figure 30.

  **Lemma 11.** *Assume the CDH assumption holds in $\mathbb{G}$, then $\mathcal{H}_6$ is computationally indistinguishable from $\mathcal{H}_5$.*

  *Proof.* Here we argue that $\mathcal{S}$ can program $\mathcal{F}_{\mathsf{RO2}}$ s.t. $m_{1-b} = \mathcal{F}_{\mathsf{RO2}}(\mathsf{sid}, `S'||B^s(h^s)^{b-1})$, i.e., the adversary $\mathcal{A}$ cannot query $B^s(h^s)^{b-1}$ to $\mathcal{F}_{\mathsf{RO2}}$ even after corrupting the receiver $R$. Note that, the case

where the adversary cannot query $B^s$ or $(\frac{B}{h})^s$ to $\mathcal{F}_{\mathsf{RO2}}$ without corrupting any party and without knowing $s$ or $r$, is analogously to the Lemma 10, thus we omit it here.

We observe that if such adversary $\mathcal{A}$ exists, then we can build a reduction $\mathcal{B}$ which breaks the CDH assumption. The reduction $\mathcal{B}$ interacts with the CDH game challenger $\mathcal{C}$ and receives $(A_1, A_2, A_3)$ from $\mathcal{C}$, and we assume that $A_1 := g, A_2 := g^s, A_3 := g^t$. Then $\mathcal{B}$ simulates $\mathcal{F}_{\mathsf{RO1}}, \mathcal{F}_{\mathsf{RO2}}, \mathcal{F}_{\mathsf{RO3}}, \mathcal{F}_{\mathsf{RO4}}$ and $\mathcal{F}_{\mathsf{EOT}}$, then starts the protocol $\Pi_{\mathsf{aEOT\text{-}RO}}$ with $\mathcal{A}$ by running $\mathcal{A}$ internally as a black-box. Our $\mathcal{B}$ sets $z := A_2$ and simulates $\pi_{\mathsf{DL}}$ by programming the random oracle, and sends $(z, \pi_{\mathsf{DL}})$ as the sender's message. Meanwhile, our $\mathcal{B}$ samples seed randomly and programs $\mathcal{F}_{\mathsf{RO1}}$ such that it can output $h := A_3$ on input seed. It then generates $(B, u_0, u_1, v_0, v_1, \pi_{\mathsf{HYB}})$ honestly by using $b$; note that, since $\mathcal{F}_{\mathsf{EOT}}$ is simulated by $\mathcal{B}$, $\mathcal{B}$ can obtain $b$ by playing the role of $\mathcal{F}_{\mathsf{EOT}}$. Then our $\mathcal{B}$ sends $(\mathsf{seed}, B, u_0, u_1, v_0, v_1, \pi_{\mathsf{HYB}})$ as the receiver's message. Since we have already proved that the simulated receiver's message is computationally indistinguishable from the honest receiver's message, the adversary $\mathcal{A}$ will continue the protocol. When $R$ gets corrupted, our $\mathcal{B}$ simply reveals the internal states. Finally, $\mathcal{B}$ randomly selects two queries $q_1, q_2$ made to $\mathcal{F}_{\mathsf{RO2}}$ by $\mathcal{A}$ and sends $A_4 := \frac{q_1}{q_2}$ to $\mathcal{C}$. If there are $Q$ queries made to $\mathcal{F}_{\mathsf{RO2}}$ by $\mathcal{A}$ in total, then $A_4 = \frac{B^s}{B^s/h^s} = h^s = g^{st}$ happens at probability $\frac{1}{2\cdot\binom{Q}{2}}$. Therefore, $\mathcal{B}$ wins the CDH game at probability $\frac{1}{2\cdot\binom{Q}{2}}$ which is non-negligible. In conclusion, if the CDH assumption holds, then the adversary $\mathcal{A}$ cannot query $B^s(h^s)^{b-1}$ to $\mathcal{F}_{\mathsf{RO2}}$ even after corrupting the receiver $R$. Therefore, we prove that $\mathcal{H}_6$ is computationally indistinguishable from $\mathcal{H}_5$. □

- $\mathcal{H}_7$: Same as $\mathcal{H}_6$, except if $S$ is honest and $R^*$ is statically corrupted, the simulator $\mathcal{S}$ extracts the choice bit $b$ and finishes the rest of the simulation. Indistinguishability has been proven in the static security proof. This happens in *Case 3* of Figure 30.

- $\mathcal{H}_8$: Same as $\mathcal{H}_7$, except if $S$ is honest and $R^*$ is statically corrupted and finally $S$ gets corrupted post-execution, the simulator $\mathcal{S}$ obtains $(m_0, m_1)$ from $\mathcal{F}_{\mathsf{EOT}}$, programs $\mathcal{F}_{\mathsf{RO2}}$ such that $m_{1-b} = \mathcal{F}_{\mathsf{RO2}}(\mathsf{sid}, `S'||B^s(h^s)^{b-1})$. Indistinguishability follows from the CDH assumption (as explain in Lemma 7). This completes *Case 3* of Figure 30.

- $\mathcal{H}_9$: Same as $\mathcal{H}_8$, except if $R$ is honest and $S^*$ is statically corrupted, the simulator $\mathcal{S}$ extracts the randomness of $S^*$, computes $m_0, m_1$ and finishes the rest of the simulation. Indistinguishability has been proven in the static security proof. This happens in *Case 4* of Figure 30.

- $\mathcal{H}_{10}$: Same as $\mathcal{H}_9$, except if $R$ is honest and $S^*$ is statically corrupted and finally $R$ gets corrupted post-execution, the simulator performs nothing additionally; note that, we have already argued the case where $R$ gets corrupted, the simulator reveals the internal states of the receiver which is computationally indistinguishable from the real ones (as explained in Lemma 9). This completes *Case 4* of Figure 30.

- $\mathcal{H}_{11}$: Same as $\mathcal{H}_{10}$, except if $S^*$ and $R^*$ are statically corrupted, the simulator $\mathcal{S}$ simply halts. Indistinguishability follows from the fact that the distribution is identical in both hybrids since both two parties are controlled by the adversary before sending anything. This completes *Case 5* of Figure 30.

Hybrid $\mathcal{H}_{11}$ is identical to the ideal world execution $\mathsf{EXEC}_{\mathcal{F}_{\mathsf{EOT}},\mathcal{S},\mathcal{Z}}$. In conclusion, when the sender or the receiver gets corrupted adaptively, $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{RO}},\mathcal{F}_{\mathsf{SYN}}}_{\Pi_{\mathsf{aEOT\text{-}RO}},\mathcal{A},\mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}_{\mathcal{F}_{\mathsf{EOT}},\mathcal{S},\mathcal{Z}}$ holds.

The proof of adaptive security is completed. □

### C.3 Proof of Theorem 7

**Theorem 7.** *The protocol $\Pi_{\mathsf{Com}}$ depicted in Figure 16 UC-realizes the functionality $\mathcal{F}_{\mathsf{Com}}$ depicted in Figure 7 with unconditional security in the $\{\mathcal{F}_{\mathsf{EOT}}, \mathcal{F}_{\mathsf{Auth}}\}$-hybrid world against static malicious corruption.*

*Proof.* We now prove the security of our protocol $\Pi_{\mathsf{Com}}$ by showing it is a UC-secure realization of $\mathcal{F}_{\mathsf{Com}}$. We describe the workflow of $\mathcal{S}$ in the ideal-world with $\mathcal{F}_{\mathsf{Com}}$, and give a proof that the simulation in the ideal world execution $\mathsf{EXEC}_{\mathcal{F}_{\mathsf{Com}},\mathcal{S},\mathcal{Z}}$ is statistically indistinguishable from a real world execution $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{EOT}},\mathcal{F}_{\mathsf{Auth}}}_{\Pi_{\mathsf{Com}},\mathcal{A},\mathcal{Z}}$ for any adversary $\mathcal{A}$ and any environment $\mathcal{Z}$. Note that, the simulator $\mathcal{S}$ simply forwards the communication between $\mathcal{A}$ and $\mathcal{Z}$.

**The committer is honest while the receiver is statically corrupted.** Here the simulator $\mathcal{S}$ needs to generate an equivocate commitment message without knowing $b$. We describe the strategy of $\mathcal{S}$ as follows:

- Committing Phase:
  - Simulate $\mathcal{F}_{\mathsf{EOT}}$. Upon receiving $(\textsc{Send}, \mathsf{sid}, R^*, C)$ from $R^*$, sends $(\textsc{Send}, \mathsf{sid}, R^*, C)$ to the adversary on behalf of $\mathcal{F}_{\mathsf{EOT}}$. Upon receiving $(\textsc{Send}, \mathsf{sid}, R^*, C, m_0, m_1)$ from the adversary, output $(\textsc{Received}, \mathsf{sid}, R^*, C, m_0, m_1)$ to $R^*$ on behalf of $\mathcal{F}_{\mathsf{EOT}}$.
- Opening Phase:
  - Upon receiving $(\textsc{Decommit}, \mathsf{sid}, C, R^*, b)$ from $\mathcal{F}_{\mathsf{Com}}$, send $(b, m_b)$ to $R^*$ on behalf of the honest committer.

We prove the indistinguishability through the following hybrid experiments:

- $\mathcal{H}_0$: This is the real world execution $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{EOT}},\mathcal{F}_{\mathsf{Auth}}}_{\Pi_{\mathsf{Com}},\mathcal{A},\mathcal{Z}}$.
- $\mathcal{H}_1$: Same as $\mathcal{H}_0$, except that $\mathcal{S}$ obtains $(m_0, m_1)$ by playing the role of the $\mathcal{F}_{\mathsf{EOT}}$ and outputs $(m_0, m_1)$ as the commitment message to $R^*$, and $\mathcal{S}$ aborts when $R^*$ extracts $b$ from $(m_0, m_1)$. Perfect indistinguishability holds since $b$ is independent of $m_0, m_1$; in other words, any malicious malicious $R^*$ cannot learn $b$.
- $\mathcal{H}_2$: Same as $\mathcal{H}_1$, except that $\mathcal{S}$ sends $(b, m_b)$ to $R^*$ after receiving $(\textsc{Decommit}, \mathsf{sid}, C, R, b)$ from $\mathcal{F}_{\mathsf{Com}}$. Perfect indistinguishability holds since $\mathcal{S}$ does not modify anything.

Hybrid $\mathcal{H}_2$ is identical to the ideal world execution $\mathsf{EXEC}_{\mathcal{F}_{\mathsf{Com}},\mathcal{S},\mathcal{Z}}$. In conclusion, when the committer $C$ is honest and the receiver $R^*$ is statically corrupted, $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{EOT}},\mathcal{F}_{\mathsf{Auth}}}_{\Pi_{\mathsf{Com}},\mathcal{A},\mathcal{Z}} \equiv \mathsf{EXEC}_{\mathcal{F}_{\mathsf{Com}},\mathcal{S},\mathcal{Z}}$ holds.

**The receiver is honest while the committer is statically corrupted.** Here the simulator $\mathcal{S}$ needs to extract $b$ from the commitment message sent by the malicious $C^*$. We describe the strategy of $\mathcal{S}$ as follows:

- Committing Phase:
  - Simulate $\mathcal{F}_{\mathsf{EOT}}$. Upon receiving $(\textsc{Receive}, \mathsf{sid}, R, C^*, b)$ from $C^*$, sends $(\textsc{Receive}, \mathsf{sid}, R, C^*)$ to the adversary on behalf of $\mathcal{F}_{\mathsf{EOT}}$. Upon receiving $(\textsc{Send}, \mathsf{sid}, R, C^*, m_b)$ from the adversary, sample an uniformly random $m_{1-b} \leftarrow \{0,1\}^\lambda$ and output $(\textsc{Received}, \mathsf{sid}, R, C^*, m_b)$ to $C^*$ on behalf of $\mathcal{F}_{\mathsf{EOT}}$.
  - Send $(\textsc{Commit}, \mathsf{sid}, C, R, b)$ to $\mathcal{F}_{\mathsf{Com}}$ on behalf of the dummy corrupted committer.
- Opening Phase:
  - Wait for $C^*$ to send $(b', m)$, and abort if $b' = 1 - b$ and $m = m_{1-b}$.

We prove the indistinguishability through the following hybrid experiments:

- $\mathcal{H}_0$: This is the real world execution $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{EOT}},\mathcal{F}_{\mathsf{Auth}}}_{\Pi_{\mathsf{Com}},\mathcal{A},\mathcal{Z}}$.
- $\mathcal{H}_1$: Same as $\mathcal{H}_0$, except that $\mathcal{S}$ obtains $(b, m_b)$ by playing the role of the $\mathcal{F}_{\mathsf{EOT}}$ and outputs $m_b$ to $C^*$. Perfect indistinguishability holds since $\mathcal{S}$ does not modify anything.
- $\mathcal{H}_2$: Same as $\mathcal{H}_1$, except that $\mathcal{S}$ aborts when $C^*$ sends $(1-b, m_{1-b})$. Indistinguishability follows from the fact that the message $m_{1-b}$ is sampled uniformly, thus the probability of the adversary guessing the $m_{1-b}$ correctly is $2^{-\lambda}$, which is negligible. In conclusion, $\mathcal{H}_2$ is statistically indistinguishable from $\mathcal{H}_1$.

Hybrid $\mathcal{H}_2$ is identical to the ideal world execution $\mathsf{EXEC}_{\mathcal{F}_{\mathsf{Com}},\mathcal{S},\mathcal{Z}}$. In conclusion, when the receiver $R$ is honest while the committer $C^*$ is statically corrupted, $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{EOT}},\mathcal{F}_{\mathsf{Auth}}}_{\Pi_{\mathsf{Com}},\mathcal{A},\mathcal{Z}}$ is statistically indistinguishable $\mathsf{EXEC}_{\mathcal{F}_{\mathsf{Com}},\mathcal{S},\mathcal{Z}}$. □

### C.4 Proof of Theorem 8

**Theorem 8.** *The protocol $\Pi_{\mathsf{Coin}}$ depicted in Figure 18 UC-realizes the functionality $\mathcal{F}_{\mathsf{Coin}}$ depicted in Figure 3 with unconditional security in the $\{\mathcal{F}_{\mathsf{Com}}, \mathcal{F}_{\mathsf{Auth}}\}$-hybrid world against static malicious corruption.*

*Proof.* We now prove the security of our protocol $\Pi_{\mathsf{Coin}}$ by showing it is a UC-secure realization of $\mathcal{F}_{\mathsf{Coin}}$. We describe the workflow of $\mathcal{S}$ in the ideal-world with $\mathcal{F}_{\mathsf{Coin}}$, and give a proof that the simulation in the ideal world execution $\mathsf{EXEC}_{\mathcal{F}_{\mathsf{Coin}},\mathcal{S},\mathcal{Z}}$ is perfectly indistinguishable from a real world execution $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{Com}},\mathcal{F}_{\mathsf{Auth}}}_{\Pi_{\mathsf{Coin}},\mathcal{A},\mathcal{Z}}$ for any adversary $\mathcal{A}$ and any environment $\mathcal{Z}$. Note that, the simulator $\mathcal{S}$ simply forwards the communication between $\mathcal{A}$ and $\mathcal{Z}$.

**The player 1 is honest while the player 2 is statically corrupted.** Here the simulator $\mathcal{S}$ needs to generate an equivocate commitment message and later open it to any message. We describe the strategy of $\mathcal{S}$ as follows:

- Round 1: Simulate $\mathcal{F}_{\mathsf{Com}}$, and output $(\textsc{Receipt}, \mathsf{sid}, P_1, P_2)$ to $P_2^*$ on behalf of $\mathcal{F}_{\mathsf{Com}}$.
- Round 2: Wait for $P_2^*$ to send $m_2$.
- Round 3: Upon receiving $(\textsc{Tossed}, \mathsf{sid}, P_1, P_2, r)$ from $\mathcal{F}_{\mathsf{Coin}}$, compute $m_1 := r \oplus m_2$, and output $(\textsc{Decommit}, \mathsf{sid}, P_1, P_2, m_1)$ to $P_2^*$ on behalf of $\mathcal{F}_{\mathsf{Com}}$.

We prove the indistinguishability through the following hybrid experiments:

- $\mathcal{H}_0$: This is the real world execution $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{Com}},\mathcal{F}_{\mathsf{Auth}}}_{\Pi_{\mathsf{Com}},\mathcal{A},\mathcal{Z}}$.
- $\mathcal{H}_1$: Same as $\mathcal{H}_0$, except that $\mathcal{S}$ plays the role of the $\mathcal{F}_{\mathsf{Com}}$ and outputs $(\textsc{Receipt}, \mathsf{sid}, P_1, P_2)$ to $P_2^*$. Perfect indistinguishability holds since in both $\mathcal{H}_0$ and $\mathcal{H}_1$, the malicious $P_2^*$ would receive $(\textsc{Receipt}, \mathsf{sid}, P_1, P_2)$ from $\mathcal{F}_{\mathsf{Com}}$.
- $\mathcal{H}_2$: Same as $\mathcal{H}_1$, except that $\mathcal{S}$ computes $m_1 := r \oplus m_2$, where $m_2$ is sent by $P_2^*$ and $r$ is sent by $\mathcal{F}_{\mathsf{Coin}}$, and outputs $(\textsc{Decommit}, \mathsf{sid}, P_1, P_2, m_1)$ to $P_2^*$ on behalf of $\mathcal{F}_{\mathsf{Com}}$. Perfect indistinguishability holds since $\mathcal{F}_{\mathsf{Com}}$ is simulated by $\mathcal{S}$.

Hybrid $\mathcal{H}_2$ is identical to the ideal world execution $\mathsf{EXEC}_{\mathcal{F}_{\mathsf{Coin}},\mathcal{S},\mathcal{Z}}$. In conclusion, when the the player 1 $P_1$ is honest while the player 2 $P_2^*$ is statically corrupted, $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{Com}},\mathcal{F}_{\mathsf{Auth}}}_{\Pi_{\mathsf{Coin}},\mathcal{A},\mathcal{Z}} \equiv \mathsf{EXEC}_{\mathcal{F}_{\mathsf{Coin}},\mathcal{S},\mathcal{Z}}$ holds.

**The player 2 is honest while the player 1 is statically corrupted.** Here the simulator $\mathcal{S}$ needs to extract $m_1$ from the commitment message sent by $P_1$. We describe the strategy of $\mathcal{S}$ as follows:

- Round 1: Simulate $\mathcal{F}_{\mathsf{Com}}$, and receive $(\textsc{Commit}, \mathsf{sid}, P_1, P_2, m_1)$ from $P_1^*$ on behalf of $\mathcal{F}_{\mathsf{Com}}$.
- Round 2: Upon receiving $(\textsc{Tossed}, \mathsf{sid}, P_1, P_2, r)$ from $\mathcal{F}_{\mathsf{Coin}}$, compute and send $m_2 := r \oplus m_1$ to $P_1^*$.
- Round 3: Wait for $P_1^*$ to open the commitment. If $P_1^*$ does not open the commitment, do nothing when $\mathcal{F}_{\mathsf{Coin}}$ sends $(\textsc{Proceed?}, \mathsf{sid}, P_1, P_2)$; else, return $(\textsc{Proceed}, \mathsf{sid})$.

We prove the indistinguishability through the following hybrid experiments:

- $\mathcal{H}_0$: This is the real world execution $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{Com}}, \mathcal{F}_{\mathsf{Auth}}}_{\Pi_{\mathsf{Coin}}, \mathcal{A}, \mathcal{Z}}$.
- $\mathcal{H}_1$: Same as $\mathcal{H}_0$, except that $\mathcal{S}$ obtains $m_1$ by playing the role of the $\mathcal{F}_{\mathsf{Com}}$, and sends $m_2 := r \oplus m_1$ to $P_1^*$ where $r$ is sent by $\mathcal{F}_{\mathsf{Coin}}$ and $r$ is uniformly random. Perfect indistinguishability holds since $m_2$ in both $\mathcal{H}_1$ and $\mathcal{H}_0$ is uniformly random.
- $\mathcal{H}_2$: Same as $\mathcal{H}_1$, except that if $P_1^*$ does not open the commitment, $\mathcal{S}$ will do nothing when $\mathcal{F}_{\mathsf{Coin}}$ sends $(\textsc{Proceed?}, \mathsf{sid}, P_1, P_2)$. Perfect indistinguishability holds since the honest $P_2$ would always aborts if the malicious $P_1^*$ does not open the commitment.
- $\mathcal{H}_3$: Same as $\mathcal{H}_2$, except that if $P_1^*$ opens the commitment, $\mathcal{S}$ will return $(\textsc{Proceed}, \mathsf{sid})$ when $\mathcal{F}_{\mathsf{Coin}}$ sends $(\textsc{Proceed?}, \mathsf{sid}, P_1, P_2)$. Perfect indistinguishability holds since the protocol participants would receive $r$ as final output in both $\mathcal{H}_3$ and $\mathcal{H}_2$.

Hybrid $\mathcal{H}_3$ is identical to the ideal world execution $\mathsf{EXEC}_{\mathcal{F}_{\mathsf{Coin}}, \mathcal{S}, \mathcal{Z}}$. In conclusion, when the player 2 $P_2$ is honest while the player 1 $P_1^*$ is statically corrupted, $\mathsf{EXEC}^{\mathcal{F}_{\mathsf{Com}}, \mathcal{F}_{\mathsf{Auth}}}_{\Pi_{\mathsf{Coin}}, \mathcal{A}, \mathcal{Z}} \equiv \mathsf{EXEC}_{\mathcal{F}_{\mathsf{Coin}}, \mathcal{S}, \mathcal{Z}}$ holds. □

### C.5 Proof of Theorem 9

**Theorem 9.** *Assume the CDH assumption holds in group $\mathbb{G}$. Let $\mathcal{G}_{\mathsf{roRO1}} : \{0,1\}^\lambda \to \mathbb{G}$ and $\mathcal{G}_{\mathsf{roRO2}} : \mathbb{G} \to \{0,1\}^\lambda$ be the random oracles. Let $\Pi^S_{\mathsf{sleNIWH}}$ be a straight-line extractable NIWH argument in the $\mathcal{G}_{\mathsf{roRO3}}$-hybrid world. Let $\Pi^R_{\mathsf{sleNIWH}}$ be a straight-line extractable NIWH argument in the $\mathcal{G}_{\mathsf{roRO4}}$-hybrid world. The protocol $\Pi_{\mathsf{EOT\text{-}GroRO}}$ depicted in Figure 19 GUC-realizes the functionality $\mathcal{F}_{\mathsf{tEOT}}$ depicted in Figure 20 in the $\{\mathcal{G}_{\mathsf{roRO}}, \mathcal{F}_{\mathsf{SYN}}\}$-hybrid world against static malicious corruption, where $\mathcal{G}_{\mathsf{roRO}} = \{\mathcal{G}_{\mathsf{roRO}i}\}_{i\in[4]}$.*

*Proof.* We now prove the security of our protocol $\Pi_{\mathsf{EOT\text{-}GroRO}}$ by showing it is a GUC-secure realization of $\mathcal{F}_{\mathsf{tEOT}}$. We only need to prove that $\Pi_{\mathsf{EOT\text{-}GroRO}}$ EUC-realizes $\mathcal{F}_{\mathsf{tEOT}}$ with respect to the shared functionality $\mathcal{G}_{\mathsf{roRO}}$, where $\mathcal{G}_{\mathsf{roRO}} = (\{\mathcal{G}_{\mathsf{roRO}i}\}_{i\in[4]})$. Therefore, we describe the workflow of $\mathcal{S}$ in the ideal-world with $\mathcal{F}_{\mathsf{tEOT}}$, and give a proof that the simulation in the ideal world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{roRO}}}_{\mathcal{F}_{\mathsf{tEOT}}, \mathcal{S}, \mathcal{Z}}$ is computationally indistinguishable from a real world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{roRO}}, \mathcal{F}_{\mathsf{SYN}}}_{\Pi_{\mathsf{EOT\text{-}GrpRO}}, \mathcal{A}, \mathcal{Z}}$ for any PPT adversary $\mathcal{A}$ and any PPT $\mathcal{G}_{\mathsf{roRO}}$-constrained environment $\mathcal{Z}$. Note that, the simulator $\mathcal{S}$ simply forwards the communication between $\mathcal{A}$ and $\mathcal{Z}$.

**The sender is honest while the receiver is statically corrupted.** Here the simulator $\mathcal{S}$ needs to extract the choice bit of the receiver from the message sent from malicious receiver $R^*$. Note that, the simulator $\mathcal{S}$ needs to send its message on behalf of the honest sender before seeing the adversary $\mathcal{A}$'s message. We describe the strategy of $\mathcal{S}$ as follows:

- Generate $(z, \pi_{\mathsf{DL}})$ honestly, and send $(z, \pi_{\mathsf{DL}})$ to $R^*$.
- Wait for $R^*$ to send $(\mathsf{seed}, B, \pi_{\mathsf{Com}})$, then do the following:
  - Compute $h := \mathcal{G}_{\mathsf{roRO1}}(\mathsf{sid}, \text{`}R\text{'}||\mathsf{seed})$.
  - Abort if $\Pi^R_{\mathsf{sleNIWH}}.\mathsf{Verify}^{\mathcal{G}_{\mathsf{roRO4}}}((B, g, h), \pi_{\mathsf{Com}}) = 0$ for $\mathcal{R}_{\mathsf{Com}}$.

- Request illegitimate queries $\mathcal{Q}_{\mathsf{sid}}$ from $\mathcal{F}_{\mathsf{tEOT}}$.
- Invoke the straight-line extractor $\Pi^R_{\mathsf{sleNIWH}}.\mathsf{Ext}^{\mathcal{G}_{\mathsf{roRO4}}}$ to obtain $(b, r)$ such that $B = g^r h^b$.
- If $b \notin \{0, 1\}$, act as the honest sender to until the end of the protocol.
- Else, compute $m_0, m_1$ honestly, and send $(\text{RECEIVE}, \mathsf{sid}, S, R^*, b)$ to $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted receiver. Return $(\text{RECEIVE}, \mathsf{sid}, S, R^*, b, m_b)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\text{RECEIVE}, \mathsf{sid}, S, R^*)$.

We prove the indistinguishability through the following hybrid experiments:

- $\mathcal{H}_0$: This is the real world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{roRO}}, \mathcal{F}_{\mathsf{SYN}}}_{\Pi_{\mathsf{EOT\text{-}GroRO}}, \mathcal{A}, \mathcal{Z}}$.
- $\mathcal{H}_1$: Same as $\mathcal{H}_0$, except that $\mathcal{S}$ aborts when $\pi_{\mathsf{Com}}$ is not valid. Indistinguishability follows from the fact that honest sender would always abort when $\pi_{\mathsf{Com}}$ is not valid.
- $\mathcal{H}_2$: Same as $\mathcal{H}_1$, except that $\mathcal{S}$ obtains $b$ by invoking the straight-line extractor $\Pi^R_{\mathsf{sleNIWH}}.\mathsf{Ext}^{\mathcal{G}_{\mathsf{roRO4}}}$. Indistinguishability follows from the straight-line extractability of $\Pi_{\mathsf{sleNIWH}}$.
- $\mathcal{H}_3$: Same as $\mathcal{H}_2$, except that $b \notin \{0, 1\}$ and $\mathcal{S}$ acts as the honest sender to until the end of the protocol. The simulator $\mathcal{S}$ aborts when $R^*$ computes either $m_0$ or $m_1$.

  **Lemma 12.** *Assume the CDH assumption holds in $\mathbb{G}$, then $\mathcal{H}_3$ is computationally indistinguishable from $\mathcal{H}_2$.*

  *Proof.* Here we argue that when $b \notin \{0, 1\}$, the malicious receiver $R^*$ can compute $m_0$ or $m_1$ with only negligible probability. Note that, the malicious $R^*$ compute $m_0$ ($m_1$) by querying $z^r h^{s \cdot b}$ ($z^r h^{s(b-1)}$) to $\mathcal{G}_{\mathsf{roRO2}}$. We discuss the case where $R^*$ computes $m_0$ by querying $z^r h^{sb}$ to $\mathcal{G}_{\mathsf{roRO2}}$ here, and the other case is similar.

  More precisely, we observe that if there exists such a $R^*$, then there is a reduction $\mathcal{B}$ which breaks the CDH assumption. The reduction $\mathcal{B}$ interacts with the CDH game challenger $\mathcal{C}$ and receives $(A_1, A_2, A_3)$ from $\mathcal{C}$, and we assume that $A_1 := g, A_2 := g^t, A_3 := g^s$. Then $\mathcal{B}$ simulates $\{\mathcal{G}_{\mathsf{roRO}i}\}_{i \in [4]}$ and starts the protocol $\Pi_{\mathsf{EOT\text{-}GroRO}}$ with $R^*$ by running $R^*$ internally as a black-box. Note that $\mathcal{B}$ has full control over $\{\mathcal{G}_{\mathsf{roRO}i}\}_{i \in [4]}$. First $\mathcal{B}$ sets $z := A_3$ and simulates the straight-line extractable NIWH argument $\pi_{\mathsf{DL}}$ by programming the random oracles, and sends $z, \pi_{\mathsf{DL}}$ to $R^*$. When $R^*$ queries a $\lambda$-bit string $\mathsf{seed}_i$ to $\mathcal{G}_{\mathsf{roRO1}}$, $\mathcal{B}$ returns $A_2 \cdot g^{a_i}$ where $a_i \xleftarrow{\$} \mathbb{Z}_q$. Upon receiving $(B, \pi_{\mathsf{Com}}, \mathsf{seed})$ from $R^*$, $\mathcal{B}$ looks up the query-answer table of $\mathcal{G}_{\mathsf{roRO1}}$ and finds the index $j$ such that $\mathsf{seed} = \mathsf{seed}_j$ and $h = A_2 \cdot g^{a_j}$. After that, $\mathcal{B}$ invokes the straight-line extractor $\Pi^R_{\mathsf{sleNIWH}}.\mathsf{Ext}^{\mathcal{G}_{\mathsf{roRO4}}}$ to obtain $(b, r)$ such that $B = g^r h^b$. Finally, $\mathcal{B}$ randomly selects a query $q$ made to $\mathcal{G}_{\mathsf{rpRO2}}$ by $R^*$ and sends $A_4 := (\frac{q}{A_3^{r + b \cdot a_j}})^{b^{-1}}$ to $\mathcal{C}$. If there are $Q$ queries made to $\mathcal{G}_{\mathsf{roRO2}}$ by $R^*$ in total, then $A_4 = (\frac{q}{A_3^{r+b \cdot a_j}})^{b^{-1}} = (\frac{z^r h^{s \cdot b}}{z^r z^{b \cdot a_j}})^{b^{-1}} = (\frac{A_2^{s \cdot b} \cdot g^{s \cdot b \cdot a_j}}{g^{s \cdot b \cdot a_j}})^{b^{-1}} = (A_2^{s \cdot b})^{b^{-1}} = g^{st}$ happens at probability $\frac{1}{Q}$. Therefore, $\mathcal{B}$ wins the CDH game at probability $\frac{1}{Q}$ which is non-negligible. In conclusion, $\mathcal{H}_3$ is computationally indistinguishable from $\mathcal{H}_2$. □

- $\mathcal{H}_4$: Same as $\mathcal{H}_3$, except that $\mathcal{S}$ extracts $b \in \{0, 1\}$, computes $m_0, m_1$ honestly, sends $(\text{RECEIVE}, \mathsf{sid}, S, R^*, b)$ to the $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted receiver, and return $(\text{RECEIVE}, \mathsf{sid}, S, R^*, m_b)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\text{RECEIVE}, \mathsf{sid}, S, R^*)$. The simulator $\mathcal{S}$ aborts when $R^*$ computes both $m_0$ and $m_1$.

  **Lemma 13.** *Assume the CDH assumption holds in $\mathbb{G}$, $\mathcal{H}_4$ is computationally indistinguishable from $\mathcal{H}_3$.*

*Proof.* The simulator $\mathcal{S}$ aborts when $R^*$ computes $m_0$ and $m_1$ by setting $b = 0$ ($b = 1$) and querying $z^r, \frac{z^r}{h^s}$ ($z^r h^s$) to $\mathcal{G}_{\mathsf{roRO2}}$. We discuss the case where $R^*$ sets $b = 0$ and queries $z^r, \frac{z^r}{h^s}$ to $\mathcal{G}_{\mathsf{roRO2}}$ here, and the other case is similar.

We observe that if there exists such a $R^*$, then there is a reduction $\mathcal{B}$ which breaks the CDH assumption. The reduction $\mathcal{B}$ interacts with the CDH game challenger $\mathcal{C}$ and receives $(A_1, A_2, A_3)$ from $\mathcal{C}$, and we assume that $A_1 := g, A_2 := g^t, A_3 := g^s$. Then $\mathcal{B}$ simulates $\{\mathcal{G}_{\mathsf{roRO}i}\}_{i\in[4]}$ and starts the protocol $\Pi_{\mathsf{EOT\text{-}GroRO}}$ with $R^*$ by running $R^*$ internally as a black-box. Note that $\mathcal{B}$ has full control over $\{\mathcal{G}_{\mathsf{roRO}i}\}_{i\in[4]}$. First $\mathcal{B}$ sets $z := A_3$ and simulates the straight-line extractable NIWH argument $\pi_{\mathsf{DL}}$ by programming the random oracles, and sends $z, \pi_{\mathsf{DL}}$ to $R^*$. When $R^*$ queries a $\lambda$-bit string $\mathsf{seed}_i$ to $\mathcal{G}_{\mathsf{roRO1}}$, $\mathcal{B}$ returns $A_2 \cdot g^{a_i}$ where $a_i \xleftarrow{\$} \mathbb{Z}_q$. Upon receiving $(B, \pi_{\mathsf{Com}}, \mathsf{seed})$ from $R^*$, $\mathcal{B}$ looks up the query-answer table of $\mathcal{G}_{\mathsf{roRO1}}$ and finds the index $j$ such that $\mathsf{seed} = \mathsf{seed}_j$ and $h = A_2 \cdot g^{a_j}$. Finally, $\mathcal{B}$ randomly selects two queries $q_1, q_2$ made to $\mathcal{G}_{\mathsf{rpRO2}}$ by $R^*$ and sends $A_4 := \frac{q_1}{q_2 A_3^{a_j}}$ to $\mathcal{C}$. If there are $Q$ queries made to $\mathcal{G}_{\mathsf{roRO2}}$ by $R^*$ in total, then $A_4 = \frac{q_1}{q_2 A_3^{a_j}} = \frac{z^r}{\frac{z^r}{h^s} A_3^{a_j}} = \frac{A_2^s \cdot g^{s a_j}}{A_3^{a_j}} = g^{st}$ happens at probability $\frac{1}{2\cdot\binom{Q}{2}}$. Therefore, $\mathcal{B}$ wins the CDH game at probability $\frac{1}{2\cdot\binom{Q}{2}}$ which is non-negligible. In conclusion, $\mathcal{H}_4$ is computationally indistinguishable from $\mathcal{H}_3$. □

Hybrid $\mathcal{H}_4$ is identical to the ideal world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{roRO}}}_{\mathcal{F}_{\mathsf{tEOT}},\mathcal{S},\mathcal{Z}}$. In conclusion, when the sender $S$ is honest and the receiver $R^*$ is statically corrupted, $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{roRO}},\mathcal{F}_{\mathsf{SYN}}}_{\Pi_{\mathsf{EOT\text{-}GroRO}},\mathcal{A},\mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}^{\mathcal{G}_{\mathsf{roRO}}}_{\mathcal{F}_{\mathsf{tEOT}},\mathcal{S},\mathcal{Z}}$ holds.

**The receiver is honest while the sender is statically corrupted.** Here the simulator $\mathcal{S}$ needs to compute both $m_0$ and $m_1$. Note that, the simulator $\mathcal{S}$ needs to send its message on behalf of the honest receiver before seeing the adversary $\mathcal{A}$'s message. We describe the strategy of $\mathcal{S}$ as follows:

- Select $b \leftarrow \{0, 1\}$, generate $(\mathsf{seed}, B, \pi_{\mathsf{Com}})$ honestly, and send $(\mathsf{seed}, B, \pi_{\mathsf{Com}})$ to $R^*$.
- Wait for $S^*$ to send $(z, \pi_{\mathsf{DL}})$, then do the following:
  - Abort if $\Pi^S_{\mathsf{sleNIWH}}.\mathsf{Verify}^{\mathcal{G}_{\mathsf{roRO3}}}((g, z), \pi_{\mathsf{DL}}) = 0$ for $\mathcal{R}_{\mathsf{DL}}$.
  - Request illegitimate queries $\mathcal{Q}_{\mathsf{sid}}$ from $\mathcal{F}_{\mathsf{tEOT}}$.
  - Invoke the straight-line extractor $\Pi^S_{\mathsf{sleNIWH}}.\mathsf{Ext}^{\mathcal{G}_{\mathsf{roRO3}}}$ to obtain $s$ such that $z = g^s$.
  - Compute $m_0 := \mathcal{G}_{\mathsf{roRO2}}(\mathsf{sid}, `S^*'||B^s)$ and $m_1 := \mathcal{G}_{\mathsf{roRO2}}(\mathsf{sid}, `S^*'||(\frac{B}{h})^s)$.
  - Send $(\textsc{Send}, \mathsf{sid}, S^*, R)$ to $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted sender. Return $(\textsc{Send}, \mathsf{sid}, S^*, R, m_0, m_1)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\textsc{Receive}, \mathsf{sid}, S^*, R)$.

We prove the indistinguishability through the following hybrid experiments:

- $\mathcal{H}_0$: This is the real world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{roRO}},\mathcal{F}_{\mathsf{SYN}}}_{\Pi_{\mathsf{EOT\text{-}GroRO}},\mathcal{A},\mathcal{Z}}$.
- $\mathcal{H}_1$: Same as $\mathcal{H}_0$, except that $\mathcal{S}$ aborts when $\pi_{\mathsf{DL}}$ is not valid. Indistinguishability follows from the fact that honest receiver would always abort when $\pi_{\mathsf{DL}}$ is not valid.
- $\mathcal{H}_2$: Same as $\mathcal{H}_1$, except that $\mathcal{S}$ extracts $s$ by invoking the straight-line extractor $\Pi^S_{\mathsf{sleNIWH}}.\mathsf{Ext}^{\mathcal{G}_{\mathsf{roRO3}}}$ and computes $m_0, m_1$. Indistinguishability follows from the straight-line extractability of $\Pi_{\mathsf{sleNIWH}}$.
- $\mathcal{H}_3$: Same as $\mathcal{H}_2$, except that $\mathcal{S}$ sends $(\textsc{Send}, \mathsf{sid}, S^*, R)$ to $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted sender, and returns $(\textsc{Send}, \mathsf{sid}, S^*, R, m_0, m_1)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\textsc{Send}, \mathsf{sid}, S^*, R)$. The simulator $\mathcal{S}$ aborts when $S^*$ extracts $b$ from the messages sent by $\mathcal{S}$.

**Lemma 14.** *If $\Pi_{\mathsf{sleNIWH}}$ be a straight-line extractable NIWH argument in the RO model, then $\mathcal{H}_3$ is computationally indistinguishable from $\mathcal{H}_2$.*

*Proof.* We only have to show that any PPT $S^*$ cannot extract $b$ from $B$ or $\pi_1$. It is easy to see that $B$ is the Pedersen commitment of $b$, so $b$ is perfectly hidden in $B$ due to the perfect hiding property. The witness hiding property of $\Pi_{\mathsf{sleNIWH}}$ guarantees that any PPT $S^*$ cannot extract $b$ from $\pi_{\mathsf{Com}}$. Therefore, $\mathcal{S}$ aborts at a negligible probability. In conclusion, $\mathcal{H}_3$ is computationally indistinguishable from $\mathcal{H}_2$. □

Hybrid $\mathcal{H}_3$ is identical to the ideal world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{roRO}}}_{\mathcal{F}_{\mathsf{tEOT}},\mathcal{S},\mathcal{Z}}$. In conclusion, when the receiver $R$ is honest and the sender $S^*$ is statically corrupted, $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{roRO}},\mathcal{F}_{\mathsf{SYN}}}_{\Pi_{\mathsf{EOT\text{-}GroRO}},\mathcal{A},\mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}^{\mathcal{G}_{\mathsf{roRO}}}_{\mathcal{F}_{\mathsf{tEOT}},\mathcal{S},\mathcal{Z}}$ holds. □

### C.6 Proof of Theorem 12

**Theorem 12.** *Assume the DDH assumption holds in group $\mathbb{G}$. Let $\mathcal{G}_{\mathsf{rpRO1}} : \{0,1\}^\lambda \to \mathbb{G}^2$ and $\mathcal{G}_{\mathsf{rpRO2}} : \mathbb{G} \to \{0,1\}^\lambda$ be the random oracles. The protocol $\Pi_{\mathsf{EOT\text{-}GrpRO}}$ depicted in Figure 22 GUC-realizes the functionality $\mathcal{F}_{\mathsf{EOT}}$ depicted in Figure 6 in the $\{\mathcal{G}_{\mathsf{rpRO}}, \mathcal{F}_{\mathsf{Auth}}\}$-hybrid world against static malicious corruption, where $\mathcal{G}_{\mathsf{rpRO}} = \{\mathcal{G}_{\mathsf{rpRO1}}, \mathcal{G}_{\mathsf{rpRO2}}\}$.*

*Proof.* We now prove the security of our protocol $\Pi_{\mathsf{EOT\text{-}GrpRO}}$ by showing it is a GUC-secure realization of $\mathcal{F}_{\mathsf{EOT}}$. We only need to prove that $\Pi_{\mathsf{EOT\text{-}GrpRO}}$ EUC-realizes $\mathcal{F}_{\mathsf{EOT}}$ with respect to the shared functionality $\mathcal{G}_{\mathsf{rpRO}}$, where $\mathcal{G}_{\mathsf{rpRO}} = \{\mathcal{G}_{\mathsf{rpRO1}}, \mathcal{G}_{\mathsf{rpRO2}}\}$. Therefore, we describe the workflow of $\mathcal{S}$ in the ideal-world with $\mathcal{F}_{\mathsf{EOT}}$, and give a proof that the simulation in the ideal world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}}}_{\mathcal{F}_{\mathsf{EOT}},\mathcal{S},\mathcal{Z}}$ is computationally indistinguishable from a real world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}},\mathcal{F}_{\mathsf{Auth}}}_{\Pi_{\mathsf{EOT\text{-}GrpRO}},\mathcal{A},\mathcal{Z}}$ for any PPT adversary $\mathcal{A}$ and any PPT $\mathcal{G}_{\mathsf{rpRO}}$-constrained environment $\mathcal{Z}$. Note that, the simulator $\mathcal{S}$ simply forwards the communication between $\mathcal{A}$ and $\mathcal{Z}$. We first argue the static security, then discuss the adaptive corruptions of the parties.

**The sender is honest while the receiver is statically corrupted.** Here the simulator $\mathcal{S}$ needs to extract the choice bit of the receiver from the message sent in the second round. We describe the strategy of $\mathcal{S}$ as follows:

- Round 1:
  - Select $\mathsf{seed}_1 \leftarrow \{0,1\}^\lambda; \alpha \leftarrow \mathbb{Z}_q$.
  - Sample a generator $g$ of group $\mathbb{G}$ and compute $h := g^\alpha$.
  - Send $(\textsc{Program}, \mathsf{sid}, `S'||\mathsf{seed}_1, (g,h))$ to $\mathcal{G}_{\mathsf{rpRO1}}$.
  - Select $r, s \leftarrow \mathbb{Z}_q$ and compute $z := g^r h^s$. Send $(\mathsf{seed}_1, z)$ to $R^*$.
- Round 2:
  - Return $(\textsc{IsProgramed}, \mathsf{sid}, 0)$ when $R^*$ invokes $\mathcal{G}_{\mathsf{rpRO1}}$ on $(\textsc{IsProgramed}, \mathsf{sid}, `S'||\mathsf{seed}_1)$.
  - Wait for $R^*$ to send $(\mathsf{seed}_2, B_1, B_2)$.
- Local Computation:
  - Invoke $\mathcal{G}_{\mathsf{rpRO1}}$ on $(\textsc{IsProgramed}, \mathsf{sid}, `S'||\mathsf{seed}_2)$ and abort if it returns $(\textsc{IsProgramed}, \mathsf{sid}, 1)$.
  - Compute $(G,H) := \mathcal{G}_{\mathsf{rpRO1}}(\mathsf{sid}, `R'||\mathsf{seed}_2)$.
  - If $B_2 = B_1^\alpha$, set $b := 0$; else if $\frac{B_2}{H} = (\frac{B_1}{G})^\alpha$, set $b := 1$; else, set $b := \perp$.
  - Compute $m_0, m_1$ honestly. If $b = \perp$, do nothing; else, send $(\textsc{Receive}, \mathsf{sid}, S, R^*, b)$ to $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted receiver, and return $(\textsc{Receive}, \mathsf{sid}, S, R^*, b, m_b)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\textsc{Receive}, \mathsf{sid}, S, R^*)$.

We prove the indistinguishability through the following hybrid experiments:

- $\mathcal{H}_0$: This is the real world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}},\mathcal{F}_{\mathsf{Auth}}}_{\Pi_{\mathsf{EOT\text{-}GrpRO}},\mathcal{A},\mathcal{Z}}$.

- $\mathcal{H}_1$: Same as $\mathcal{H}_0$, except that $\mathcal{S}$ programs $\mathcal{G}_{\mathsf{rpRO1}}$ on $\mathsf{seed}_1$, so $\mathcal{S}$ knows $\alpha$ s.t. $h = g^\alpha$. Indistinguishability between the hybrids follows from (i) the tuple $(g, h)$ is randomly selected in both $\mathcal{H}_0$ and $\mathcal{H}_1$, and (ii) $\mathcal{S}$ would return $(\text{IsProgramed}, \mathsf{sid}, 0)$ when $R^*$ invokes $\mathcal{G}_{\mathsf{rpRO1}}$ on $(\text{IsProgramed}, \mathsf{sid}, `S'||\mathsf{seed}_1)$.

- $\mathcal{H}_2$: Same as $\mathcal{H}_1$, except that $\mathcal{S}$ extracts $b \notin \{0, 1\}$ and just does nothing.

  **Lemma 15.** *Assume the DL assumption holds in $\mathbb{G}$, then $\mathcal{H}_2$ is computationally indistinguishable from $\mathcal{H}_1$.*

  *Proof.* We first consider the case where the choice bit of $R^*$ is 0, but $B_2 \neq B_1^\alpha$. This would happen when $B_1 = g^{x_1}, B_2 = h^{x_2}$ for $x_1 \neq x_2$. We then argue that any PPT malicious receiver $R^*$ cannot obtain $m_0$, thus the indistinguishability holds. If we assume that $h = g^\alpha$, then $B_1^r B_2^s = g^{rx_1} h^{sx_2} = g^{rx_1+\alpha s x_2}, z = g^r h^s = g^{r+\alpha s}$. Since the DL assumption holds in $\mathbb{G}$, any PPT malicious receiver $R^*$ cannot know $r, s$ from $z$. Thus it is impossible for $R^*$ to compute $B_1^r B_2^s$ given only $z, x_1, x_2$. Therefore, any PPT $R^*$ cannot compute $m_0$. The case where the choice bit of $R^*$ is 1 but $\frac{B_2}{H} \neq (\frac{B_1}{G})^\alpha$ is similar. In conclusion, $\mathcal{H}_2$ is computationally indistinguishable from $\mathcal{H}_1$. □

- $\mathcal{H}_3$: Same as $\mathcal{H}_2$, except that $\mathcal{S}$ extracts $b \in \{0, 1\}$, computes $m_0, m_1$ honestly, sends $(\text{Receive}, \mathsf{sid}, S, R^*, b)$ to $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted receiver, and returns $(\text{Receive}, \mathsf{sid}, S, R^*, b, m_b)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\text{Receive}, \mathsf{sid}, S, R^*)$. The simulator $\mathcal{S}$ aborts when $R^*$ computes $m_{1-b}$.

  **Lemma 16.** *Assume DDH assumption holds in $\mathbb{G}$, then $\mathcal{H}_3$ is computationally indistinguishable from $\mathcal{H}_2$.*

  *Proof.* We first consider the case where $(g, h, G, H)$ is a DDH tuple. In this case, $R^*$ can compute $m_0$ and $m_1$ easily. Since $\mathcal{S}$ has checked that $\mathsf{seed}_2$ is not programmed by $R^*$, the probability of $(g, h, G, H)$ being a DDH tuple is negligible.

  We then consider the case where $(g, h, G, H)$ is not a DDH tuple, and $R^*$ computes both $m_0$ and $m_1$ by setting $b = 0$ ($b = 1$) and querying $z^x$, $\frac{z^x}{G^r H^s}$ ($z^x G^r H^s$) to $\mathcal{G}_{\mathsf{rpRO2}}$. We discuss the case where $R^*$ sets $b = 0$ and queries $z^x$, $\frac{z^x}{G^r H^s}$ to $\mathcal{G}_{\mathsf{rpRO2}}$ here, and the other case is similar. We observe that if there exists such a $R^*$, then there is a reduction $\mathcal{B}$ which breaks the CDH assumption. The reduction $\mathcal{B}$ interacts with the CDH game challenger $\mathcal{C}$ and receives $(A_1, A_2, A_3)$ from $\mathcal{C}$, and we assume that $A_1 := g, A_2 := g^t, A_3 := g^r$. Then $\mathcal{B}$ simulates $\mathcal{G}_{\mathsf{rpRO1}}, \mathcal{G}_{\mathsf{rpRO2}}$ and starts the protocol $\Pi_{\mathsf{EOT\text{-}GrpRO}}$ with $R^*$ by running $R^*$ internally as a black-box. Note that $\mathcal{B}$ has full control over $\mathcal{G}_{\mathsf{rpRO1}}, \mathcal{G}_{\mathsf{rpRO2}}$. In round 1, $\mathcal{B}$ samples $s, \mathsf{seed}_1, h$ randomly, programs the $\mathcal{G}_{\mathsf{rpRO1}}$ such that it can output $(g, h)$ on input $\mathsf{seed}_1$ , and sends $z := A_3 \cdot h^s$ to $R^*$. In round 2, when $R^*$ queries $\mathsf{seed}_2$ to $\mathcal{G}_{\mathsf{rpRO1}}$, $\mathcal{B}$ samples $H$ randomly and returns $(G := A_2, H)$ as the output of $\mathcal{G}_{\mathsf{rpRO1}}$. Finally, $\mathcal{B}$ randomly selects two queries $q_1, q_2$ made to $\mathcal{G}_{\mathsf{rpRO2}}$ by $R^*$ and sends $A_4 := \frac{q_1}{q_2 H^s}$ to $\mathcal{C}$. If there are $Q$ queries made to $\mathcal{G}_{\mathsf{rpRO2}}$ by $R^*$ in total, then $A_4 = \frac{q_1}{q_2 H^s} = \frac{z^x}{\frac{z^x}{G^r H^s} H^s} = G^r = g^{tr}$ happens at probability $\frac{1}{2 \cdot \binom{Q}{2}}$. Therefore, $\mathcal{B}$ wins the CDH game at probability $\frac{1}{2 \cdot \binom{Q}{2}}$ which is non-negligible. In conclusion, $\mathcal{H}_3$ is computationally indistinguishable from $\mathcal{H}_2$. □

Hybrid $\mathcal{H}_3$ is identical to the ideal world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}}}_{\mathcal{F}_{\mathsf{EOT}},\mathcal{S},\mathcal{Z}}$. In conclusion, when the sender $S$ is honest and the receiver $R^*$ is statically corrupted, $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}},\mathcal{F}_{\mathsf{Auth}}}_{\Pi_{\mathsf{EOT\text{-}GrpRO}},\mathcal{A},\mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}}}_{\mathcal{F}_{\mathsf{EOT}},\mathcal{S},\mathcal{Z}}$ holds.

**The receiver is honest while the sender is statically corrupted.** Here the simulator $\mathcal{S}$ needs to compute both $m_0$ and $m_1$. We describe the strategy of $\mathcal{S}$ as follows:

- Round 1: Wait for $S^*$ to send $(\mathsf{seed}_1, z)$.
- Round 2:
  - Invoke $\mathcal{G}_{\mathsf{rpRO1}}$ on $(\text{IsProgramed}, \mathsf{sid}, `S'||\mathsf{seed}_1)$ and abort if it returns $(\text{IsProgramed}, \mathsf{sid}, 1)$.
  - Compute $(g, h) := \mathcal{G}_{\mathsf{rpRO1}}(\mathsf{sid}, `S'||\mathsf{seed}_1)$.
  - Select $\mathsf{seed}_2 \leftarrow \{0,1\}^\lambda$ and $t \leftarrow \mathbb{Z}_q$, and compute $G := g^t, H \leftarrow h^t$.
  - Send $(\text{Program}, \mathsf{sid}, `R'||\mathsf{seed}_2, (G, H))$ to $\mathcal{G}_{\mathsf{rpRO1}}$.
  - Select $x \leftarrow \mathbb{Z}_q$, compute $B_1 := g^x$ and $H_2 := h^x$, and $(\mathsf{seed}_1, B_1, B_2)$ to $S^*$.
- Local Computation:
  - Return $(\text{IsProgramed}, \mathsf{sid}, 0)$ when $S^*$ invokes $\mathcal{G}_{\mathsf{rpRO1}}$ on $(\text{IsProgramed}, \mathsf{sid}, `R'||\mathsf{seed}_2)$.
  - Compute $m_0 := \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||z^x), m_1 := \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||z^{x-t})$, sends $(\text{Send}, \mathsf{sid}, S^*, R)$ to $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted sender, and returns $(\text{Send}, \mathsf{sid}, S^*, R, m_0, m_1)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\text{Receive}, \mathsf{sid}, S^*, R)$.

We prove the indistinguishability through the following hybrid experiments:

- $\mathcal{H}_0$: This is the real world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}}, \mathcal{F}_{\mathsf{Auth}}}_{\Pi_{\mathsf{EOT\text{-}GrpRO}}, \mathcal{A}, \mathcal{Z}}$.
- $\mathcal{H}_1$: Same as $\mathcal{H}_0$, except that $\mathcal{S}$ programs $\mathcal{G}_{\mathsf{rpRO1}}$ on $\mathsf{seed}_2$, so $\mathcal{S}$ knows $t$ s.t. $G = g^t, H = h^t$; furthermore, the simulator $\mathcal{S}$ would return $(\text{IsProgramed}, \mathsf{sid}, 0)$ when $S^*$ invokes $\mathcal{G}_{\mathsf{rpRO1}}$ on $(\text{IsProgramed}, \mathsf{sid}, `R'||\mathsf{seed}_2)$.

  **Lemma 17.** *Assume the DDH assumption holds in $\mathbb{G}$, then $\mathcal{H}_1$ is computationally indistinguishable from $\mathcal{H}_0$.*

  *Proof.* The simulator $\mathcal{S}$ would return $(\text{IsProgramed}, \mathsf{sid}, 0)$ when the malicious sender $S^*$ invokes $\mathcal{G}_{\mathsf{rpRO1}}$ on $(\text{IsProgramed}, \mathsf{sid}, `R'||\mathsf{seed}_2)$. Thus, the distinction is revealed if and only if $S^*$ can distinguish the DDH tuple $(g, h, g^t, h^t)$ from the non-DDH tuple. In other words, we observe that if there exists such a $S^*$, then there is a reduction $\mathcal{B}$ which breaks the DDH assumption. The reduction $\mathcal{B}$ interacts with the DDH game challenger $\mathcal{C}$ and receives $(A_1, A_2, A_3, A_4)$ from $\mathcal{C}$. Then $\mathcal{B}$ simulates $\mathcal{G}_{\mathsf{rpRO1}}, \mathcal{G}_{\mathsf{rpRO2}}$ and starts the protocol $\Pi_{\mathsf{EOT\text{-}GrpRO}}$ with $R^*$ by running $S^*$ internally as a black-box. Note that $\mathcal{B}$ has full control over $\mathcal{G}_{\mathsf{rpRO1}}, \mathcal{G}_{\mathsf{rpRO2}}$. In round 1, when $S^*$ queries $\mathsf{seed}_1$ to $\mathcal{G}_{\mathsf{rpRO1}}$, $\mathcal{B}$ returns $(g := A_1, h := A_2)$ as the output of $\mathcal{G}_{\mathsf{rpRO1}}$. In round 2, $\mathcal{B}$ acts as an honest receiver except that programs the output of $\mathcal{G}_{\mathsf{rpRO1}}$ as $(T_1 := A_3, T_2 := A_4)$ on input $\mathsf{seed}_2$. If $R^*$ aborts the protocol, $\mathcal{B}$ sets $b := 1$ indicating $(A_1, A_2, A_3, A_4)$ is a DDH tuple; else, $\mathcal{B}$ sets $b := 0$. Finally, $\mathcal{B}$ sends $b$ to $\mathcal{C}$ and wins the game. In conclusion, $\mathcal{H}_1$ is computationally indistinguishable from $\mathcal{H}_0$. □

- $\mathcal{H}_2$: Same as $\mathcal{H}_1$, except that $\mathcal{S}$ always sets $B_1 := g^x, B_2 := h^x$ and computes $m_0 := \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||z^x)$, and $m_1 := \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||z^{x-t})$. Indistinguishability follows from the fact that for $b \in \{0,1\}$, we can always compute $m_b := \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||z^{\alpha - bt})$.
- $\mathcal{H}_3$: Same as $\mathcal{H}_2$, except that $\mathcal{S}$ sends $(\text{Send}, \mathsf{sid}, S^*, R)$ to $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted sender, and returns $(\text{Send}, \mathsf{sid}, S^*, R, m_0, m_1)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\text{Send}, \mathsf{sid}, S^*, R)$. The simulator $\mathcal{S}$ aborts when $S^*$ extracts $b$ from the messages sent by $\mathcal{S}$. Indistinguishability follows from the perfect hiding property of the Pedersen commitment.

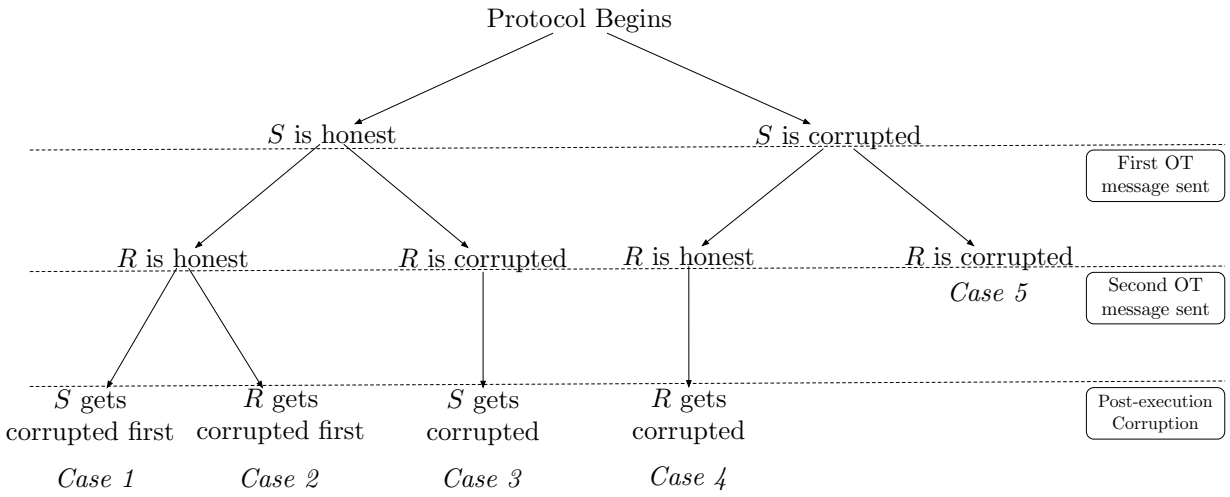


Figure 31: Simulation Cases for Adaptive Corruptions in Protocol $\Pi_{\mathsf{EOT\text{-}GrpRO}}$

Hybrid $\mathcal{H}_3$ is identical to the ideal world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}}}_{\mathcal{F}_{\mathsf{EOT}},\mathcal{S},\mathcal{Z}}$. In conclusion, when the receiver $R$ is honest and the sender $S^*$ is statically corrupted, $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}},\mathcal{F}_{\mathsf{Auth}}}_{\Pi_{\mathsf{EOT\text{-}GrpRO}},\mathcal{A},\mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}}}_{\mathcal{F}_{\mathsf{EOT}},\mathcal{S},\mathcal{Z}}$ holds.

The proof of static security is completed, and we will discuss the cases for adaptive corruptions.

**The sender/receiver gets corrupted adaptively.** We present the different simulation cases in Figure 31. We present the according strategy of the simulator $\mathcal{S}$ as follows:

- Round 1:
  - *Case 1-3*: If $S$ is honest, $\mathcal{S}$ performs the following:
    - Select $\mathsf{seed}_1 \leftarrow \{0,1\}^\lambda$; $\alpha \leftarrow \mathbb{Z}_q$.
    - Sample a generator $g$ of group $\mathbb{G}$ and compute $h := g^\alpha$.
    - Send $(\text{PROGRAM}, \mathsf{sid}, `S'||\mathsf{seed}_1, (g,h))$ to $\mathcal{G}_{\mathsf{rpRO1}}$.
    - Select $r, s \leftarrow \mathbb{Z}_q$ and compute $z := g^r h^s$. Send $(\mathsf{seed}_1, z)$ to $R$.
  - *Case 4-5*: Else, $\mathcal{S}$ simply waits for $S^*$ to send $(\mathsf{seed}_1, z)$.
- Round 2:
  - *Case 1,2*: If $S$ and $R$ are honest, $\mathcal{S}$ performs the following:
    - Select $\mathsf{seed}_2 \leftarrow \{0,1\}^\lambda$ and $t \leftarrow \mathbb{Z}_q$, and compute $G := g^t, H \leftarrow h^t$.
    - Send $(\text{PROGRAM}, \mathsf{sid}, `R'||\mathsf{seed}_2, (G,H))$ to $\mathcal{G}_{\mathsf{rpRO1}}$.
    - Select $x \leftarrow \mathbb{Z}_q$, compute $B_1 := g^x$ and $H_2 := h^x$, and $(\mathsf{seed}_1, B_1, B_2)$ to $S$.
  - *Case 3*: If $S$ is honest and $R^*$ is corrupted, $\mathcal{S}$ performs the following:
    - Return $(\text{ISPROGRAMED}, \mathsf{sid}, 0)$ when $R^*$ invokes $\mathcal{G}_{\mathsf{rpRO1}}$ on $(\text{ISPROGRAMED}, \mathsf{sid}, `S'||\mathsf{seed}_1)$.
    - Wait for $R^*$ to send $(\mathsf{seed}_2, B_1, B_2)$.
  - *Case 4*: If $R$ is honest and $S^*$ is corrupted, $\mathcal{S}$ performs the following:
    - Invoke $\mathcal{G}_{\mathsf{rpRO1}}$ on $(\text{ISPROGRAMED}, \mathsf{sid}, `S'||\mathsf{seed}_1)$ and abort if it returns $(\text{ISPROGRAMED}, \mathsf{sid}, 1)$.
    - Compute $(g,h) := \mathcal{G}_{\mathsf{rpRO1}}(\mathsf{sid}, `S'||\mathsf{seed}_1)$.
    - Select $\mathsf{seed}_2 \leftarrow \{0,1\}^\lambda$ and $t \leftarrow \mathbb{Z}_q$, and compute $G := g^t, H \leftarrow h^t$.
    - Send $(\text{PROGRAM}, \mathsf{sid}, `R'||\mathsf{seed}_2, (G,H))$ to $\mathcal{G}_{\mathsf{rpRO1}}$.

* Select $x \leftarrow \mathbb{Z}_q$, compute $B_1 := g^x$ and $H_2 := h^x$, and $(\mathsf{seed}_1, B_1, B_2)$ to $S^*$.

– *Case 5*: If $S^*$ and $R^*$ are corrupted, $\mathcal{S}$ ends the simulation.

- Local Computation:
  - *Case 1,2*: If $S$ and $R$ are honest, $\mathcal{S}$ performs nothing.
  - *Case 3*: If $R^*$ is corrupted and $S$ is honest, $\mathcal{S}$ performs the following:
    * Invoke $\mathcal{G}_{\mathsf{rpRO1}}$ on $(\text{IsProgramed}, \mathsf{sid}, `S'||\mathsf{seed}_2)$ and abort if it returns $(\text{IsProgramed}, \mathsf{sid}, 1)$.
    * Compute $(G, H) := \mathcal{G}_{\mathsf{rpRO1}}(\mathsf{sid}, `R'||\mathsf{seed}_2)$.
    * If $B_2 = B_1^\alpha$, set $b := 0$; else if $\frac{B_2}{H} = (\frac{B_1}{G})^\alpha$, set $b := 1$; else, set $b := \perp$.
    * Compute $m_0, m_1$ honestly. If $b = \perp$, do nothing; else, send $(\text{Receive}, \mathsf{sid}, S, R^*, b)$ to $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted receiver, and return $(\text{Receive}, \mathsf{sid}, S, R^*, b, m_b)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\text{Receive}, \mathsf{sid}, S, R^*)$.
  - *Case 4*: If $S^*$ is corrupted and $R$ is honest, $\mathcal{S}$ performs the following:
    * Return $(\text{IsProgramed}, \mathsf{sid}, 0)$ when $S^*$ invokes $\mathcal{G}_{\mathsf{rpRO1}}$ on $(\text{IsProgramed}, \mathsf{sid}, `R'||\mathsf{seed}_2)$.
    * Compute $m_0 := \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||z^x), m_1 := \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||z^{x-t})$, sends $(\text{Send}, \mathsf{sid}, S^*, R)$ to $\mathcal{F}_{\mathsf{EOT}}$ on behalf of the dummy corrupted sender, and returns $(\text{Send}, \mathsf{sid}, S^*, R, m_0, m_1)$ to $\mathcal{F}_{\mathsf{EOT}}$ when $\mathcal{F}_{\mathsf{EOT}}$ sends $(\text{Receive}, \mathsf{sid}, S^*, R)$.

- Post-Execution Corruption:
  - *Case 1,2*: If $S$ and $R$ are honest, $\mathcal{S}$ performs the following based on the sequence of corruption:
    * *Case 1*: If $R$ is honest and $S$ gets corrupted first, $\mathcal{S}$ obtains $(m_0, m_1)$ from $\mathcal{F}_{\mathsf{EOT}}$ and programs $\mathcal{G}_{\mathsf{rpRO2}}$ such that $m_0 = \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||B_1^r B_2^s)$, $m_1 = \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||(\frac{B_1}{G})^r(\frac{B_2}{H})^s)$. The simulator $\mathcal{S}$ then reveals $(r, s)$ as the honest sender's internal states. When $R$ gets corrupted, $\mathcal{S}$ obtains $b$ and provides $x' := x - b \cdot t$ as the honest receiver's internal state.
    * *Case 2*: If $S$ is honest and $R$ gets corrupted first, $\mathcal{S}$ obtains $(b, m_b)$ from $\mathcal{F}_{\mathsf{EOT}}$ and programs $\mathcal{G}_{\mathsf{rpRO2}}$ such that $m_b = \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||B_1^r B_2^s (G^r H^s)^{-b})$. The simulator $\mathcal{S}$ then reveals $x' := x - b \cdot t$ as the honest receiver's internal state. When the sender $S$ gets corrupted, $\mathcal{S}$ obtains $(m_0, m_1)$ from $\mathcal{F}_{\mathsf{EOT}}$, programs $\mathcal{G}_{\mathsf{rpRO2}}$ such that $m_{1-b} = \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||B_1^r B_2^s (G^r H^s)^{b-1})$ and provides $r, s$ as the honest sender's internal state.
  - *Case 3*: If $R^*$ is corrupted and $S$ gets corrupted, $\mathcal{S}$ obtains $(m_0, m_1)$ from $\mathcal{F}_{\mathsf{EOT}}$, programs $\mathcal{G}_{\mathsf{rpRO2}}$ such that $m_{1-b} = \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||B_1^r B_2^s (G_1^r H_2^s)^{b-1})$ and provides $r, s$ as the honest sender's internal state.
  - *Case 4*: If $S^*$ is corrupted and $R$ gets corrupted, $\mathcal{S}$ obtains $b$ and provides $x' := x - b \cdot t$ as the honest receiver's internal state.

We prove the indistinguishability through the following hybrid experiments:

- $\mathcal{H}_0$: This is the real world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}}, \mathcal{F}_{\mathsf{Auth}}}_{\Pi_{\mathsf{EOT\text{-}GrpRO}}, \mathcal{A}, \mathcal{Z}}$.

- $\mathcal{H}_1$: Same as $\mathcal{H}_0$, except if $S$ is honest before the first OT message is sent, the simulator $\mathcal{S}$ programs $\mathcal{G}_{\mathsf{rpRO1}}$ on $\mathsf{seed}_1$, so $\mathcal{S}$ knows $\alpha$ s.t. $h = g^\alpha$. Indistinguishability has been proven in the static security proof. This happens in *Case 1-3* of Figure 31.

- $\mathcal{H}_2$: Same as $\mathcal{H}_1$, except if $S$ and $R$ are honest before the second OT message is sent, the simulator $\mathcal{S}$ programs $\mathcal{G}_{\mathsf{rpRO1}}$ on $\mathsf{seed}_2$, so $\mathcal{S}$ knows $t$ s.t. $G = g^t, H = h^t$. Indistinguishability follows from the DDH assumption as explained in Lemma 17. This happens in *Case 1,2* of Figure 31.

- $\mathcal{H}_3$: Same as $\mathcal{H}_2$, except if $S$ and $R$ are honest during the protocol and $S$ gets corrupted first in post-execution and then $R$ gets corrupted, in this case the simulator $\mathcal{S}$ obtains $(m_0, m_1)$ from $\mathcal{F}_{\mathsf{EOT}}$ and programs $\mathcal{G}_{\mathsf{rpRO2}}$ such that $m_0 = \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||B_1^r B_2^s)$, $m_1 = \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||(\frac{B_1}{G})^r(\frac{B_2}{H})^s)$. The simulator $\mathcal{S}$ then reveals $(r, s)$ as the honest sender's internal states. When $R$ gets corrupted, $\mathcal{S}$ obtains $b$ and provides $x' := x - b \cdot t$ as the honest receiver's internal state. This completes *Case 1* of Figure 31.

  **Lemma 18.** *Assume the CDH assumption holds in $\mathbb{G}$, then $\mathcal{H}_3$ is computationally indistinguishable from $\mathcal{H}_2$.*

  *Proof.* Note that, the simulator $\mathcal{S}$ runs the honest sender's algorithm to produce the sender's message. Therefore, the simulator $\mathcal{S}$'s revealed internal states $r, s$ have the same distribution with the honest sender's internal states. On the other hand, due to the trapdoor property of Pedersen commitment, no PPT adversary can distinguish from the revealed internal state $x'$ from the honest receiver's internal state.

  Now the only thing left is to argue that the simulator $\mathcal{S}$ can program $\mathcal{G}_{\mathsf{rpRO2}}$ successfully, i.e., the external adversary $\mathcal{A}$ cannot query $B_1^r B_2^s$ or $(\frac{B_1}{G})^r(\frac{B_2}{H})^s$ to $\mathcal{G}_{\mathsf{rpRO2}}$ without corrupting any party and without knowing $(r, s)$ or $x$. Here we only discuss the case where the PPT adversary $\mathcal{A}$ cannot query $B_1^r B_2^s$ to $\mathcal{G}_{\mathsf{rpRO2}}$, the case concerning $(\frac{B_1}{G})^r(\frac{B_2}{H})^s$ is similar. Formally, we observe that if such adversary $\mathcal{A}$ exists, then we can build a reduction $\mathcal{B}$ which breaks the CDH assumption. The reduction $\mathcal{B}$ interacts with the CDH game challenger $\mathcal{C}$ and receives $(A_1, A_2, A_3)$ from $\mathcal{C}$, and we assume that $A_1 := g, A_2 := g^r, A_3 := g^x$. Then $\mathcal{B}$ simulates $\mathcal{G}_{\mathsf{rpRO1}}, \mathcal{G}_{\mathsf{rpRO2}}$ and starts the protocol $\Pi_{\mathsf{EOT\text{-}GrpRO}}$ with $\mathcal{A}$ by running $\mathcal{A}$ internally as a black-box. Note that $\mathcal{B}$ has full control over $\mathcal{G}_{\mathsf{rpRO1}}, \mathcal{G}_{\mathsf{rpRO2}}$. In round 1, $\mathcal{B}$ samples $s, \mathsf{seed}_1, \alpha$ randomly, computes $h := g^\alpha$ and programs the $\mathcal{G}_{\mathsf{rpRO1}}$ such that it can output $(g, h)$ on input $\mathsf{seed}_1$. Finally it sends $z := A_2 \cdot h^s$. In round 2, $\mathcal{B}$ generates $\mathsf{seed}_2, G, H$ honestly and sends $B_1 := A_3, B_2 := A_3^\alpha$. Finally, $\mathcal{B}$ randomly selects a queries $q$ made to $\mathcal{G}_{\mathsf{rpRO2}}$ by $\mathcal{A}$ and sends $A_4 := \frac{q}{A_3^{s\alpha}}$ to $\mathcal{C}$. If there are $Q$ queries made to $\mathcal{G}_{\mathsf{rpRO2}}$ by $\mathcal{A}$ in total, then $A_4 = \frac{q}{A_3^{s\alpha}} = \frac{B_1^r B_2^s}{B_2^s} = B_1^r = g^{rx}$ happens at probability $\frac{1}{Q}$. Therefore, $\mathcal{B}$ wins the CDH game at probability $\frac{1}{Q}$ which is non-negligible. In conclusion, if the CDH assumption holds, then the simulator $\mathcal{S}$ can program $\mathcal{G}_{\mathsf{rpRO2}}$ successfully. Therefore, we prove that $\mathcal{H}_3$ is computationally indistinguishable from $\mathcal{H}_2$. □

- $\mathcal{H}_4$: Same as $\mathcal{H}_3$, except if $S$ and $R$ are honest during the protocol and $R$ gets corrupted first in post-execution and then $S$ gets corrupted, in this case the simulator $\mathcal{S}$ obtains $(b, m_b)$ from $\mathcal{F}_{\mathsf{EOT}}$ and programs $\mathcal{G}_{\mathsf{rpRO2}}$ such that $m_b = \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||B_1^r B_2^s (G^r H^s)^{-b})$. The simulator $\mathcal{S}$ then reveals $x' := x - b \cdot t$ as the honest receiver's internal state. When the sender $S$ gets corrupted, $\mathcal{S}$ obtains $(m_0, m_1)$ from $\mathcal{F}_{\mathsf{EOT}}$, programs $\mathcal{G}_{\mathsf{rpRO2}}$ such that $m_{1-b} = \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||B_1^r B_2^s (G^r H^s)^{b-1})$ and provides $r, s$ as the honest sender's internal state. This completes *Case 2* of Figure 31.

  **Lemma 19.** *Assume the CDH assumption holds in $\mathbb{G}$, then $\mathcal{H}_4$ is computationally indistinguishable from $\mathcal{H}_3$.*

  *Proof.* Here we only argue that the simulator can program the $\mathcal{G}_{\mathsf{rpRO2}}$ such that $m_{1-b} = \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||B_1^r B_2^s (G^r H^s)^{b-1})$, i.e., the adversary $\mathcal{A}$ cannot query $B_1^r B_2^s (G^r H^s)^{b-1}$ to $\mathcal{G}_{\mathsf{rpRO2}}$ even after corrupting the receiver $R$. The rest is analogously to the Lemma 18.

  We observe that if such adversary $\mathcal{A}$ exists, then we can build a reduction $\mathcal{B}$ which breaks the CDH assumption. The reduction $\mathcal{B}$ interacts with the CDH game challenger $\mathcal{C}$ and receives $(A_1, A_2, A_3)$ from $\mathcal{C}$, and we assume that $A_1 := g, A_2 := g^r, A_3 := g^t$. Then $\mathcal{B}$ simulates $\mathcal{G}_{\mathsf{rpRO1}}, \mathcal{G}_{\mathsf{rpRO2}}$ and starts the protocol $\Pi_{\mathsf{EOT\text{-}GrpRO}}$ with $\mathcal{A}$ by running $\mathcal{A}$ internally as a black-box. Note that $\mathcal{B}$ has full control

over $\mathcal{G}_{\mathsf{rpRO1}}, \mathcal{G}_{\mathsf{rpRO2}}$. In round 1, $\mathcal{B}$ samples $s, \mathsf{seed}_1, \alpha$ randomly, computes $h := g^{\alpha}$ and programs the $\mathcal{G}_{\mathsf{rpRO1}}$ such that it can output $(g, h)$ on input $\mathsf{seed}_1$. Finally it sends $z := A_2 \cdot h^s$. In round 2, $\mathcal{B}$ samples $\mathsf{seed}_2, x$ randomly and programs the $\mathcal{G}_{\mathsf{rpRO1}}$ such that it can output $(G := A_3, H := A_3^{\alpha})$ on input $\mathsf{seed}_2$. It then sends $B_1 := g^x h^b, B_2 := G^x H^b$. When $R$ gets corrupted, $\mathcal{B}$ obtains $b$ and reveals $x' := x - b \cdot \alpha$ as the internal states. Finally, $\mathcal{B}$ randomly selects two queries $q_1, q_2$ made to $\mathcal{G}_{\mathsf{rpRO2}}$ by $\mathcal{A}$ and sends $A_4 := \frac{q_1}{q_2 \cdot H^s}$ to $\mathcal{C}$. If there are $Q$ queries made to $\mathcal{G}_{\mathsf{rpRO2}}$ by $\mathcal{A}$ in total, then $A_4 = \frac{B_1^r B_2^s}{B_1^r B_2^s (G^r H^s)^{-1} \cdot H^s} = \frac{G^r H^s}{H^s} = G^r = g^{rt}$ happens at probability $\frac{1}{2 \cdot \binom{Q}{2}}$. Therefore, $\mathcal{B}$ wins the CDH game at probability $\frac{1}{2 \cdot \binom{Q}{2}}$ which is non-negligible. In conclusion, if the CDH assumption holds, then the adversary $\mathcal{A}$ cannot query $B_1^r B_2^s (G^r H^s)^{b-1}$ to $\mathcal{G}_{\mathsf{rpRO2}}$ even after corrupting the receiver $R$. Therefore, we prove that $\mathcal{H}_4$ is computationally indistinguishable from $\mathcal{H}_3$. □

- $\mathcal{H}_5$: Same as $\mathcal{H}_4$, except if $S$ is honest and $R^*$ is corrupted before the second OT message is sent, the simulator $\mathcal{S}$ extracts the choice bit $b$ of $R^*$ and finishes the rest of the simulation. Indistinguishability has been proven in the static security proof. This happens in *Case 3* of Figure 31.

- $\mathcal{H}_6$: Same as $\mathcal{H}_5$, except if $S$ is honest and $R^*$ is corrupted before the second OT message is sent and finally $S$ gets corrupted post-execution, the simulator $\mathcal{S}$ obtains $(m_0, m_1)$ from $\mathcal{F}_{\mathsf{EOT}}$, programs $\mathcal{G}_{\mathsf{rpRO2}}$ such that $m_{1-b} = \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||B_1^r B_2^s (G_1^r H_2^s)^{b-1})$ and provides $r, s$ as the honest sender's internal state. Indistinguishability follows from the DDH assumption (as explained in Lemma 16). This completes *Case 3* of Figure 31.

- $\mathcal{H}_7$: Same as $\mathcal{H}_6$, except if $S^*$ is statically corrupted and $R$ is honest at the time of sending the second OT message, the simulator $\mathcal{S}$ programs $\mathcal{G}_{\mathsf{rpRO1}}$ on $\mathsf{seed}_2$, so $\mathcal{S}$ knows $t$ s.t. $G = g^t, H = h^t$, and computes $B_1 := g^x, B_2 := h^x$; furthermore, the simulator $\mathcal{S}$ would return $(\text{IsProgramed}, \mathsf{sid}, 0)$ when $S^*$ invokes $\mathcal{G}_{\mathsf{rpRO1}}$ on $(\text{IsProgramed}, \mathsf{sid}, `R'||\mathsf{seed}_2)$. Indistinguishability follows from the DDH assumption as explained in Lemma 17. This happens in *Case 4* of Figure 31.

- $\mathcal{H}_8$: Same as $\mathcal{H}_7$, except if $S^*$ is statically corrupted and $R$ is honest at the local computation phase, the simulator $\mathcal{S}$ computes $m_0 := \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||z^x)$, $m_1 := \mathcal{G}_{\mathsf{rpRO2}}(\mathsf{sid}, `S'||z^{x-t})$. Indistinguishability has been proven in the static security proof. This happens in *Case 4* of Figure 31.

- $\mathcal{H}_8$: Same as $\mathcal{H}_7$, except if $S^*$ is statically corrupted and $R$ get corrupted post-execution, the simulator obtains $b$ and provides $x' := x - b \cdot t$ as the honest receiver's internal state. Indistinguishability follows from the trapdoor property of the Pedersen commitment. This completes *Case 4* of Figure 31.

- $\mathcal{H}_9$: Same as $\mathcal{H}_8$, except if $S^*$ is statically corrupted and $R^*$ is corrupted before the second OT message is sent, the simulator $\mathcal{S}$ simply halts. Indistinguishability follows from the fact that the distribution is identical in both hybrids since both two parties are controlled by the adversary before sending anything. This completes *Case 5* of Figure 31.

Hybrid $\mathcal{H}_2$ is identical to the ideal world execution $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}}}_{\mathcal{F}_{\mathsf{EOT}}, \mathcal{S}, \mathcal{Z}}$. In conclusion, when the sender or the receiver gets corrupted adaptively, $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}}, \mathcal{F}_{\mathsf{Auth}}}_{\Pi_{\mathsf{EOT\text{-}GrpRO}}, \mathcal{A}, \mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}}}_{\mathcal{F}_{\mathsf{EOT}}, \mathcal{S}, \mathcal{Z}}$ holds.

The proof of adaptive security is completed. □

# D A Lower Bound on Round Complexity of OT in the GrpRO Model

As depicted in Theorem 11, we find it impossible to construct 2-round GUC-secure OT protocols in the $\mathcal{G}_{\mathsf{rpRO}}$ model even with static security. In the following, we recall the Theorem 11 and prove it formally.

**Theorem 11.** *There exists no terminating 2-round protocol* $\Pi$ *that GUC-realizes* $\mathcal{F}_{\mathsf{OT}}$ *depicted in Figure 4 with static security in the* $\{\mathcal{G}_{\mathsf{rpRO}}, \mathcal{F}_{\mathsf{Auth}}\}$*-hybrid world.*

*Proof.* We proceed by contradiction. Suppose there exists such a protocol $\Pi$ that GUC-realizes $\mathcal{F}_{\mathsf{OT}}$ in the $\{\mathcal{G}_{\mathsf{rpRO}}, \mathcal{F}_{\mathsf{Auth}}\}$-hybrid world. Then there must exist a PPT simulator $\mathcal{S}$ such that $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}}}_{\mathcal{F}_{\mathsf{OT}},\mathcal{S},\mathcal{Z}} \overset{c}{\approx} \mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}},\mathcal{F}_{\mathsf{Auth}}}_{\Pi,\mathcal{A},\mathcal{Z}}$ for any PPT adversary $\mathcal{A}$ and any PPT $\mathcal{G}_{\mathsf{rpRO}}$-externally constrained environment $\mathcal{Z}$. For simplicity, we let $\mathcal{A}$ be a dummy adversary that simply forwards protocol flows between the corrupt party and $\mathcal{Z}$.

In the following, we show two cases: (i) the sender sends its message first; (ii) the receiver sends its message first. We will prove that there is a contradiction in both cases, hence we prove the theorem.

**Case 1: The sender sends its message first.** Let $\mathcal{Z}$ corrupt the receiver $R^*$ at first and feed the honest sender $S$ with two messages $m_0, m_1 \leftarrow \{0,1\}^\lambda$. Then $\mathcal{Z}$ waits for $S$ to send its message $\pi_S$. Note that, once the message $\pi_S$ is given to $\mathcal{F}_{\mathsf{SYN}}$, $\mathcal{Z}$ is allowed to see it. After $\mathcal{Z}$ obtaining $\pi_S$, $\mathcal{Z}$ chooses a random bit $b \leftarrow \{0,1\}$ and instructs $R^*$ to execute the honest receiver's algorithm, i.e., generates and sends receiver's message $\pi_R$. Finally, $\mathcal{Z}$ performs the local computation to obtain $m_{b'}$. If $m_b = m_{b'}$, $\mathcal{Z}$ outputs 1; otherwise, $\mathcal{Z}$ outputs 0.

In order to make the GUC experiment above remain indistinguishable, the simulator $\mathcal{S}$ needs to determine $m_b$ before sending $\pi_S$. However, the adversary instructs $R^*$ to execute its algorithm until $\pi_S$ is obtained. There are only two ways for $\mathcal{S}$ to obtain $m_b$: (i) obtain the $m_b$ from $\mathcal{F}_{\mathsf{OT}}$ by guessing the choice bit $b$ of the receiver correctly, which would happen at probability $\frac{1}{2}$; (ii) guess the message $m_b$ chosen by $\mathcal{Z}$ correctly, which would happen at probability $2^{-\lambda}$. Therefore, in the ideal world, our $\mathcal{Z}$ outputs 1 with probability $2^{-1-\lambda}$; in the real world, our $\mathcal{Z}$ always outputs 1. In other words, our $\mathcal{Z}$ can distinguish the real world and the ideal world with non-negligible probability, contradicting our assumption that $\Pi$ is GUC-secure.

**Case 2: The receiver sends its message first.** Let us first consider the session with SID $\mathsf{sid}_1$. Our $\mathcal{Z}$ proceeds by corrupting the receiver $R^*$ at first. Then $\mathcal{Z}$ chooses a random bit $b \leftarrow \{0,1\}$ and instructs $R^*$ to perform the honest receiver algorithm on input $b$ and send message $\pi_R$ to the honest sender $S$. After that, $\mathcal{Z}$ chooses two random messages $m_0, m_1 \leftarrow \{0,1\}^\lambda$ and gives them as input to the honest sender $S$. Next, $\mathcal{Z}$ waits for $S$ to send the message $\pi_S$ and executes the honest receiver algorithm to obtain $m_{b'}$ from $\pi_S$. If $m_b = m_{b'}$, $\mathcal{Z}$ outputs 1; otherwise, $\mathcal{Z}$ outputs 0.

In order to make the GUC experiment above remain indistinguishable, the simulator $\mathcal{S}$ needs to perform the following strategy. First, $\mathcal{S}$ extracts the choice bit $b$ of the receiver from message $\pi_R$. Then $\mathcal{S}$ sends $(\textsc{Receive}, \mathsf{sid}, b)$ to $\mathcal{F}_{\mathsf{OT}}$ on behalf of the dummy receiver, and receives $(\mathsf{sid}, m_b)$ from $\mathcal{F}_{\mathsf{OT}}$. Finally, $\mathcal{S}$ picks a random $m_{1-b} \leftarrow \{0,1\}^\lambda$ and invokes the honest sender algorithm on input $m_b, m_{1-b}$ to output $\pi_S$. It is easy to see that the crucial part lies in the extraction of the choice bit $b$ of the receiver. Recall that, the main advantage of $\mathcal{S}$ is that it can program the $\mathcal{G}_{\mathsf{rpRO}}$ on unqueried points without being detected, but the simulator is external to the $\mathcal{G}_{\mathsf{rpRO}}$ and it can not know in real time what queries other parties are sending to $\mathcal{G}_{\mathsf{rpRO}}$. For notation convenience, we denote by $\mathsf{Prog}_{\mathsf{sid}_1}$ the queries programmed by $\mathcal{S}$. If the adversary happens to use the points that belongs to $\mathsf{Prog}_{\mathsf{sid}_1}$, then $\mathcal{S}$ has the chance of extracting the private information of the malicious party. We also note that, the simulator $\mathcal{S}$ also can query $\mathcal{G}_{\mathsf{rpRO}}$ just like other parties. In order to describe the process of querying to $\mathcal{G}_{\mathsf{rpRO}}$, we denote by $\mathcal{G}^*_{\mathsf{rpRO}}$ the simplified version of the $\mathcal{G}_{\mathsf{rpRO}}$, i.e., the $\mathcal{G}_{\mathsf{rpRO}}$ without the $\textsc{Program}$ interface. We write $\mathcal{S}^{\mathcal{G}^*_{\mathsf{rpRO}}}$ to denote the event that $\mathcal{S}$ is given query access to $\mathcal{G}_{\mathsf{rpRO}}$ and can continuously query to $\mathcal{G}_{\mathsf{rpRO}}$. With notations above, we denote by $b \leftarrow \mathcal{S}^{\mathcal{G}^*_{\mathsf{rpRO}}}(\mathsf{sid}_1, \pi_R, \mathsf{Prog}_{\mathsf{sid}_1})$ the event where $\mathcal{S}$ extracts the choice bit $b$ from $\pi_R$ using $\mathsf{Prog}_{\mathsf{sid}_1}$. Recall that, (i) the simulator $\mathcal{S}$ should be able to handle any PPT adversary $\mathcal{A}$ and any PPT environment $\mathcal{Z}$; (ii) the (corrupted) receiver sends its message first in this case. Hence we can consider the following case: the environment $\mathcal{Z}$ queries $\mathcal{G}_{\mathsf{rpRO}}$ everything that will be needed to compute the receiver's message $\pi_R$ in advance (we denote these queries as $\mathcal{Q}_{\mathsf{sid}_1,\mathcal{Z}}$), then $\mathcal{Z}$ starts the protocol $\Pi$ and instructs $R^*$ to run the receiver algorithm on those previously queried points. In this case, we have

$\Pr[\mathsf{Prog}_{\mathsf{sid}_1} \cap \mathcal{Q}_{\mathsf{sid}_1,\mathcal{Z}} = \emptyset] = 0$, where $\mathcal{Q}_{\mathsf{sid}_1,\mathcal{Z}}$ is the queries used for generating the receiver's message $\pi_R$. However, $\mathcal{S}$ should still be able to extract the choice bit; otherwise, $\mathcal{Z}$ will distinguish the ideal world from the real world. In other words, the algorithm $b \leftarrow \mathcal{S}^{\mathcal{G}^*_{\mathsf{rpRO}}}(\mathsf{sid}_1, \pi_R, \emptyset)$ should work even if we replace the programmed queries $\mathsf{Prog}_{\mathsf{sid}_1}$ with an empty set $\emptyset$. We note that $\mathcal{S}^{\mathcal{G}^*_{\mathsf{rpRO}}}(\mathsf{sid}_1, \pi_R, \emptyset)$ works as long as the appropriate inputs are provided, even if we switch to the session with a different SID.

Next, we show that the existence of the simulator $\mathcal{S}$ above contradicts the security of $\Pi$ against static corruptions, by creating a particular $\mathcal{Z}'$ which distinguishes $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}}}_{\mathcal{F}_{\mathsf{OT}},\mathcal{S}',\mathcal{Z}'}$ from $\mathsf{EXEC}^{\mathcal{G}_{\mathsf{rpRO}},\mathcal{F}_{\mathsf{Auth}}}_{\Pi,\mathcal{A}',\mathcal{Z}'}$ after a static corruption operation for any PPT simulator $\mathcal{S}'$. Let us consider the session with SID $\mathsf{sid}_2$. We let $\mathcal{Z}'$ corrupt the sender $S^*$ at first. Then $\mathcal{Z}'$ feeds the honest receiver $R$ with a randomly selected bit $b$, and waits for the arrival of message $\pi_R$. Note that, $b$ is the hidden input of the honest receiver, and the receiver sends $b$ only to $\mathcal{F}_{\mathsf{OT}}$ which hides it from $\mathcal{S}$ information theoretically. Therefore, the entire computation of $\pi_R$ is totally independent of $b$. Now $\mathcal{Z}'$ instructs $S^*$ to invoke $b' \leftarrow \mathcal{S}^{\mathcal{G}^*_{\mathsf{rpRO}}}(\mathsf{sid}_2, \pi_R, \emptyset)$. In the real world, we always have $b' = b$. In the ideal world, we have $b' = b$ with probability at most $\frac{1}{2}$ since $\pi_R$ is totally independent of $b$. Therefore, $\mathcal{Z}'$ can distinguish between the real world and the ideal world with a non-negligible probability, contradicting our assumption that $\Pi$ is GUC-secure. □