Trellis: Robust and Scalable Metadata-private Anonymous Broadcast

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Abstract

Trellis is a mix-net based anonymous broadcast system with cryptographic security guarantees. Trellis can be used to anonymously publish documents or communicate with other users, all while assuming full network surveillance. In Trellis, users send messages through a set of servers in successive rounds. The servers mix and post the messages to a public bulletin board, hiding which users sent which messages.

Trellis hides all network metadata, remains robust to changing network conditions, guarantees availability to honest users, and scales with the number of mix servers. Trellis provides three to five orders of magnitude faster performance and better network robustness compared to Atom, the state-of-the-art anonymous broadcast system with a comparable threat model.

In achieving these guarantees, Trellis contributes: (1) a simpler theoretical mixing analysis for a routing mix network constructed with a fraction of malicious servers, (2) anonymous routing tokens for verifiable random paths, and (3) lightweight blame protocols built on top of onion routing to identify and eliminate malicious parties.

We implement and evaluate Trellis in a networked deployment. With 128 servers, Trellis achieves a throughput of 320 bits per second. Trellis's throughput is only 100 to 1000× slower compared to Tor (which has 6,000 servers and 2 million daily users) and is potentially deployable at a smaller "enterprise" scale. Our implementation is open-source.

1 Introduction

Communication on the internet is prone to large-scale surveillance and censorship [15, 39, 44, 53, 75, 96]. People trying to hide their communication from powerful (e.g., state-sponsored) adversaries rely on anonymity tools such as Tor [33] or I2P [101] to hide their identities [102, 103, 108]. With Tor, users proxy their traffic through a chain of servers which breaks the link between the identity of the user and the traffic destination. With I2P, users proxy traffic through a chain of peer nodes. While these tools serve an important role, and help obfuscate the identity of the user, they can be insufficient against determined adversaries capable of observing large swaths of traffic (e.g., malicious ISPs). Such adversaries (e.g., ISPs or nation states) can easily identify users and the traffic content through metadata such as packet timing, packet size, and other identifying features—even when all traffic is encrypted [10, 14, 38, 47, 51, 61, 79, 84, 86, 99]. State-of-the-art attacks can deanonymize encrypted Tor traffic with upwards of 90% accuracy by analyzing the encrypted packet traffic [10, 47, 84].

This problem has motivated many systems [2, 20, 21, 36, 37, 63, 65, 80, 106, 112] for **anonymous broadcast**. Anonymous broadcast consists of anonymously posting a message to a public bulletin board and is the most general form of anonymous communication possible; it can be used for file sharing and to instantiate weaker primitives such as anonymous point-to-point communication [28, 64, 67, 68, 89, 95, 104, 107]. Anonymous broadcast provides *sender* anonymity: fully hiding which user sent which message. Most mix-net based systems, in contrast, only provide *relationship* anonymity [28, 64, 67, 68, 89, 95, 104, 107] (hiding who is communicating with whom). Other systems [20, 21, 80, 106, 112] exploit a *non-collusion* assumption (which inhibits scalability) to instantiate anonymous broadcast or just focus on recipient anonymity (without sender anonymity) [6].

As with prior work, Trellis is metadata-private, which guarantees that sender anonymity is preserved in the face of an adversary monitoring the entire network. Trellis also provides horizontal scalability—the ability to increase throughput with the number of (possibly untrusted) servers participating in the network. Horizontal scalability is key to real-world efficiency: Tor and I2P are only capable of handling millions of users [73] on a daily basis thanks to similar scalability guarantees. Moreover, horizontal scalability is important for real-world security: the more users an anonymity system can support, the more "plausible deniability" each user in the system has.

The state-of-the-art system for anonymous broadcast with horizontal scalability is Atom [63]. Unfortunately, Atom suffers from practical barriers to deployment even though it is *theoretically* optimal. (See Section 2.2 for overview of related work.) For example, Atom achieves a throughput of roughly 0.01 bits per second and is six to eight orders of magnitude slower than practical systems such as Tor [73].

All other systems either focus on weaker anonymity notions (e.g., point-to-point communication [3, 5, 16, 64, 67, 68, 95, 104, 107]) or sacrifice scalability by assuming non-colluding servers [2, 20, 21, 36, 37, 62, 80, 112].

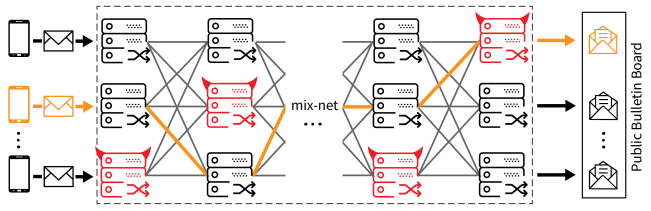

Figure 1: Trellis arranges the set of servers into "layers," (each server appears once per layer) to instantiate a mix-net. Users send messages through a random path which guarantees that all messages are mixed before reaching the public bulletin board, even when a subset of servers are malicious (colored in red).

Trellis is designed for *fast* anonymous broadcast with *large messages*. Trellis is inspired by a long line of work [17, 63, 65, 92] which augments mix-nets to instantiate anonymous broadcast. Trellis innovates on these works in three important ways: (1) Trellis decouples the use of expensive cryptography required for system setup from the broadcasting phase, (2) Trellis is robust to malicious parties and server churn, and (3) Trellis provides message delivery guarantees even when a *majority*[1] of servers and users are malicious (note that performance degrades with a larger proportion of adversarial servers; see Section 8). With these three key contributions, Trellis brings metadata-private anonymous broadcast a step closer to real-world deployment. However, we note that Trellis is still limited by challenges *outside* of the system design itself, such as requiring that all users remain online and contribute messages to ensure metadata privacy. These limitations will be present in any anonymous communication system design with strong anonymity guarantees [44]. Even so, in Section 8, we find that Trellis is only 100 to 1000× slower compared to systems like Tor and I2P, both of which sacrifice strong anonymity in favor of concrete performance.

Ideas, challenges, and contributions. We start by using a mix-net topology known as a random routing network [63, 65] (see Section 2 for an overview of mix-nets). In such mix-nets, servers are organized into L layers (with all servers appearing in each layer, see Figure 1). The user selects a random path through the servers and sends a message through the path for

[1]In the case of a dishonest majority of servers, Trellis reconfigures itself into a new network containing a dishonest *minority*; see Section 6.3.2.

mixing. Adding more servers increases the throughput, which achieves horizontal scalability. While this simple idea seems promising at first (and also proposed in prior work [63, 65]), there are several practical hurdles associated with it:

- Malicious users or servers might deviate from protocol, select non-random paths, or otherwise attack the system. The difficulty is detecting and blaming the responsible parties while preserving anonymity for honest users.
- Mix-nets that model permutation networks traditionally require that all servers are honest to achieve mixing guarantees. Atom [63] resolves this by emulating honest "servers" using random groups of servers. However, this technique is suboptimal and contributes to slow mixing times in practice (see Section 2).
- Real-world networks consisting of disparate servers are not perfect: server churn and elimination can occur frequently. Classic mix-nets are **not** designed to gracefully handle and adapt to changing network conditions on-the-fly.

In Trellis, we overcome the above challenges as follows.

Preventing deviation from protocol. To ensure availability and message delivery, we develop two new tools: anonymous routing tokens (Section 5.1) and boomerang encryption (Section 4.2). Routing tokens force all users to choose random paths through the mix-net. Boomerang encryption ensures that all servers along a path route messages correctly and enables efficient blaming of malicious servers when required. The combination of these two primitives allows us to port our mix-net to a setting with malicious users and servers actively deviating from protocol *without* compromising on anonymity.

Modeling a mix-net with malicious servers. Kwon [65] develops a novel theoretical mixing analysis for routing mix-nets constructed with some fraction of adversarial servers. In Trellis, we use a routing mix-net topology in conjunction with the analysis of Kwon [65] to avoid the inefficiencies of "emulating" honest servers (see Section 5). Additionally, we derive a more straightforward analysis and bound on the number of mix layers required. Our mixing analysis may be of independent interest to other systems in this area.

Efficient blame, server elimination, and network healing. We design Trellis so that honest servers can efficiently detect failures and deviations from protocol, assign blame, and eliminate responsible parties from the network. Our blame protocols (described in Section 6) handle malicious servers and users without ever deanonymizing honest users. Trellis gracefully handles network changes (e.g., offline or eliminated servers) using proactive secret sharing (explained in Section 6), which permits on-the-fly state recovery following server churn.

Contributions. In summary, this paper contributes:

1. the design of Trellis: a novel system for anonymous broadcast providing cryptographic anonymity guarantees for users, network robustness, and concrete efficiency,

2. a simpler theoretical mixing analysis for routing mix-nets instantiated with a subset of mix servers that are malicious and colluding with a network adversary,
3. anonymous routing tokens: a new tool for anonymously enforcing random path selection in mix-nets, with conceivable applications to other networking systems,
4. boomerang encryption: a spin on *onion* encryption that allows for efficient proofs of delivery and blame assignment,
5. and an open-source implementation which we evaluate on a networked deployment, achieving a throughput of 320 bits per second with 100,000 users and 10 kB messages.

Limitations. Trellis shares the primary limitation of other metadata-private systems [21, 28, 63–65, 67, 80, 104, 106, 107]: it provides anonymity only among honest online users and therefore requires all users to contribute a message in each round—even if they have nothing to send—so as to hide all network metadata [44]. (Users who stop participating correlate themselves with patterns or content of messages that stop being output [25, 26, 113].) Trellis does not, however, preclude the use of any external solutions to this problem (e.g., [113]).

2 Background and related work

We start by describing mix-nets and their guarantees in Section 2.1. We then describe related work and existing use of mix-nets for anonymous communication in Section 2.2.

2.1 Mix networks

In 1981, Chaum [17] developed the first mix network (mix-net) for the purpose of anonymous *email*. The idea behind Chaum's mix-net is simple. A set of servers is arranged in a sequence. Each server has a public key. Each user recursively encrypts their message under the sequence of server public keys. We call this recursive encryption a sequence of **envelopes** (a.k.a. onions [33]), with each envelope encrypted under all subsequent public keys in the sequence. All users send the first envelope to the first server in the sequence. Each server, in sequence, decrypts the envelopes it receives, randomly shuffles the decrypted envelopes, and forwards them to the next server in the sequence. This repeats for N layers. The use of onion encryption prevents intermediate servers from learning which messages they are routing (a server can only decrypt one layer of encryption). At the end of the sequence, the innermost envelope is decrypted to reveal the plaintext message. The mix-net guarantees that the ordering of the messages is uniformly random and independent from the input, which in turn guarantees unlinkability between each user and their submitted message. As such, classic mix-net architectures can be used for anonymous broadcast. Unfortunately, this architecture suffers from security and practicality challenges:

1. Mix-nets do not inherently guarantee metadata privacy: a network adversary observing all traffic can link messages to users through metadata, such as timing information.
2. Mix-nets are vulnerable to active attacks (e.g., dropped envelopes), which can be exploited by malicious servers to link users to their messages [72, 77, 81, 97, 111].
3. Sequential mix-nets do not provide scalability: adding more servers does **not** improve performance since *all* servers must process *all* envelopes.

Scalable mix-nets. Scalable systems built using mix-net architectures are designed around **parallel mix-nets** [46, 92], where users are assigned to a mix-net instantiated from a subset of servers in the network. Adding more servers thus proportionally increases the total capacity of the network. However, naïvely parallelizing mix-nets does not provide privacy, since a message may end up mixed with only a small subset of other users' messages. To avoid this, parallel mix-nets must model a *random permutation network* [23, 109] (a theoretical framework for modeling mixing in a communication network) to guarantee that all messages are mixed [46, 65, 92]. This is the strategy taken in Atom [63].

2.2 Related work

In this section, we focus on comparing Trellis to other systems that achieve anonymous broadcast. We pay particular attention to mix-net-based systems as other architectures have incomparable threat models.

Anonymous broadcast using mix-nets. Rackoff and Simon [92] put forth the idea of anonymous broadcast using parallel mix-nets based on permutation networks but do not build or evaluate their approach. Kwon et al. [63] are the first to use their ideas to build a prototype system called Atom. A problem with mix-nets instantiated as permutation networks is that they only guarantee mixing if all servers are honest. The insight in Atom is to use **anytrust groups**—sets of random servers such that at least one server in each set is honest with high probability—to emulate a trusted network from groups of possibly malicious servers. That is, each "server" in the mix-net is emulated by an anytrust group selected at random. Instantiating the mix-net in this way comes with a high concrete performance cost. The problem is that Atom requires expensive zero-knowledge proofs to ensure servers in each group behave correctly. These quickly become computational bottlenecks limiting the throughput and preventing scaling to large message sizes (Kwon et al. [63] only evaluate Atom on up to 160 byte messages). Another problem is that emulating honest servers with anytrust groups wastes resources and introduces large communication overheads. These performance barriers make Atom impractical to deploy.

Quark is a followup proposal for anonymous broadcast that appears in Kwons's PhD thesis [65] but is primarily of theoretical interest. Like Atom, Quark is designed on top of a random routing mix-net. However, Quark avoids emulating honest servers by developing a novel theoretical mixing analysis for a network containing malicious servers. Unfortunately, Quark does not prevent disruption attacks: any malicious user or server can cause the entire protocol to restart by dropping (or not sending) a message. Moreover, we find that Quark may not be secure: the blind Schnorr multi-signatures around which Quark is designed can be forged using parallel signing attacks [34, 82] (these attacks were discovered *after* Quark). In Trellis, we salvage the theoretical underpinning of Quark and use a routing mix-net as a starting foundation. We also contribute an improved mixing analysis for this routing mix-net when instantiated with malicious servers, inspired by the ideas of Kwon [65] (see Section 5.5 and Appendix C). Apart from the mix-net, all other components of Trellis are different.

Anonymous communication using mix-nets. Chaum [17], cMix [16], MCMix [5], Karaoke [67], Stadium [104], Vuvuzela [107], XRD [64], are all anonymous point-to-point communication systems built on top of mix-net architectures. Of these, Karaoke, Stadium, and XRD provide horizontal scalability. However, only XRD provides *cryptographic privacy*. Karaoke, Stadium, and Vuvuzela provide *differential privacy*, which introduces a host of practical challenges (see discussion of Kwon et al. [64, Section 1]). Other systems, such as Loopix [89], Streams [28], and Tor [33] are examples of *free-route* mix-nets [27, 41, 70, 78, 93], i.e., parallel mix-nets where users asynchronously route messages via a small subset of mix servers. Free-route mix-nets do not achieve metadata privacy in the face of network adversaries and are susceptible to traffic analysis and active attacks [10, 47, 84].

Other architectures. Other anonymous broadcast systems [2, 5, 8, 20, 21, 36, 74, 80, 112] are instantiated under different threat models. Dining cryptographer networks (DC-nets) and multi-party computation [2, 5, 36, 74] require two or more *non-colluding* servers to process messages and prevent malicious users from sending malformed content. Because of this, these techniques inherently do not provide horizontal scalability and impose high overheads on the servers. Other anonymous communication systems [4, 6, 18, 42, 62, 68, 69, 71, 95] focus on specific applications (e.g., group messaging [18] and voice-communication [4, 68, 69]) or focus on one-sided anonymity (e.g., only recipient anonymity [6]). We refer the reader to the excellent survey of systems in Piotrowska's PhD thesis [88].

3 System overview

In Section 3.1, we describe the core ideas and intuition underpinning the overall design of Trellis. In Section 3.2, we describe the threat model and guarantees achieved by Trellis.

3.1 Main ideas

Following other mix-net based systems [16, 28, 63, 65, 67, 68, 92, 95, 104, 107], Trellis is instantiated using a set of N servers and M users. The servers are organized into a network of L layers and users' messages are routed by servers from one layer to the next (see Figure 1). Trellis is organized in two stages. A one-time path establishment stage followed by a repeated message broadcasting stage. In both stages, Trellis resolves failures and attacks by malicious users and servers through on-demand blame protocols (see Section 6).

Do once: Path establishment. We reference the steps in Figure 2. The path establishment protocol proceeds layer-by-layer. ① Users obtain signed anonymous routing tokens (Section 5.1) from an anytrust group of signers, which guarantees the resulting path assigned to the user is uniformly random. Each token issued for a layer determines the server at the next layer and contains all the necessary routing information needed to route messages between the two layers (details in Sections 5 and 5.1). ② Users distribute the L tokens anonymously using boomerang encryption (Section 4.2). All servers obtain the necessary routing tokens without learning the full path.

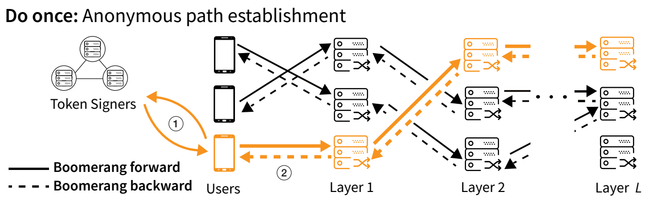

Figure 2: Users are assigned a server in each layer through a one-time setup protocol involving an anytrust group of signers.

Repeat: Anonymous broadcasting. We reference the steps in Figure 3. Anonymous broadcasting repeats indefinitely on the pre-established path. ① Each user sends their envelope (containing the user's message) through the server assigned by the routing token, for each of the L layers. ② Servers collect all the envelopes, decrypt one layer of encryption, and group the resulting envelopes based on their destination server in the next layer. Servers shuffle and pad each group with dummy envelopes as necessary. Each group is then sent to the destination server in the next layer. ③ Once envelopes reach the final layer, they are guaranteed to be shuffled and unlinkable to the users that sent them, at which point the final layer of encryption is removed to reveal the plaintext message.

On-demand: Blame, elimination, and recovery. It could be the case that a server goes offline or acts maliciously, both during path establishment and the broadcasting stages. To deal with such failures (intentional and otherwise) we develop on-demand blame and recovery protocols that automatically reassign affected paths. Importantly, failure resolution is handled *locally* on the link between two layers and does not

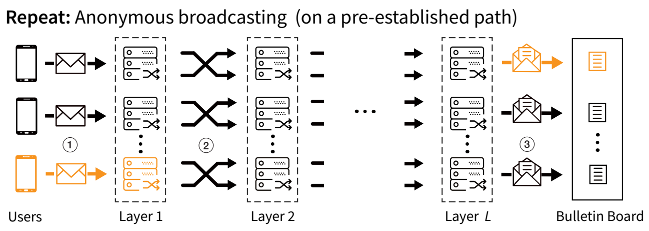

Figure 3: Users send messages through the layers, based on their paths. Users can repeat this process indefinitely to broadcast many different messages.

involve the users. (Failures caused by a malicious user cause the user to be eliminated by other means; see Section 6.) We follow the steps of Figure 4. Blame and recovery is invoked whenever an error is detected (e.g., wrong signature, dropped envelopes). ① A server detects an error in the received batch of envelopes in layer (i+2) and proceeds to blame the server in layer (i+1) that sent the batch. ② Servers evaluate evidence from both parties to decide which of the two is malicious. ③ Servers vote for the honest party by providing their secret share of the eliminated server's state. The state is used by the replacement server to take the place of the eliminated server.

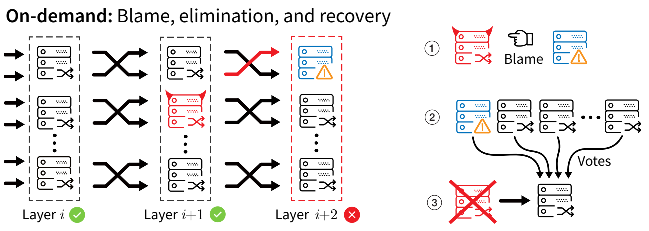

Figure 4: On-demand blame and recovery protocols are invoked by servers to automatically reassign affected paths and eliminate malicious servers from the network.

3.2 Threat model and assumptions

The threat model and assumptions underpinning Trellis mirror those of existing anonymous broadcast [63, 65] (and communication [5, 16, 64, 67, 68, 104, 107]) systems based on mix-nets that provide metadata-privacy guarantees.

Assumptions and adversarial model.

Observed network. We assume that all communication on the network is observed by a passive network adversary but that the communication *content* is encrypted using standard authenticated public key infrastructure (PKI)—e.g., TLS [94].

Malicious users and servers. We assume that a fixed fraction $0 \leq f < 1$ of servers and any number of users are malicious, may arbitrarily deviate from the protocol, and collude with the network adversary. Note: in the case of a dishonest majority ($f \geq 0.5$), our blame protocols in Section 6 cause successive network reconfigurations which converge to an *honest majority* (details in Section 6.3.2).

Cryptography. We make use of standard cryptographic assumptions. We require digital signatures, symmetric-key encryption, and the computational Diffie-Hellman (CDH) assumption [32] (and variants of the CDH assumption for *gap* groups [59]; see Appendix B.2). We also require pairings on elliptic curves [13, 55] and hash functions modeled as random oracles when constructing our routing tokens (Section 5.1).

Liveness. We require two liveness assumptions: synchronous communication and user liveness. Both assumptions are standard in all prior work on metadata-private anonymous communication [2, 5, 6, 20, 21, 36, 63–65, 80, 104, 107, 112].

- *Synchronous communication:* Servers are expected to be able to receive all envelopes for each layer before processing the next layer. This is equivalent to assuming all servers have access to a global clock [98].
- *User liveness:* We assume that all users submit a message for every round—even if they have nothing to send—to prevent the risk deanonymization via intersection attacks [25, 26, 113]. We do **not** assume server liveness.

Non-goals. Trellis does not aim to prevent denial-of-service (DoS) attacks on the network or handle infrastructure failures. However, Trellis is fully compatible with existing solutions [19, 22] for network availability and reliability.

Trellis guarantees. Under the above threat model and assumptions, Trellis provides the following guarantees.

Robustness and availability. No subset of malicious servers or colluding users can disrupt availability for honest users.

Message delivery. All messages submitted by honest users are guaranteed to be delivered and result in a "receipt" that can be efficiently verified by each user.

Unlinkability. Fix any small $\epsilon > 0$ and any subset of honest users. After mixing, no message (or set of messages) can be linked with any honest user (or set of honest users) with probability ϵ over random guessing by the adversary.

Metadata privacy. The unlinkability guarantee holds even when all network communication is visible to the adversary who is controlling the malicious users and servers.

4 Building blocks

In this section, we describe the cryptographic tools and concepts that we use to instantiate Trellis in Section 5.

4.1 Cryptographic foundations

We briefly describe classic cryptographic primitives that we use as building blocks in Trellis. We point to excellent textbooks for more details on these standard primitives [12, 45, 57].

Diffie-Hellman key agreement. The Diffie-Hellman key agreement protocol [32] allows two parties to establish a shared secret key over an authenticated communication channel. Let $\mathbb{G}$ be a suitable group of prime order p with generator g.

```
DHKeyGen($\mathbb{G}, g$) → $(A, a)$:
  1: Sample $a \in \mathbb{Z}_p$.
  2: Output $(A := g^a, a)$.

DHKeyAgree$(A, b)$ → sk:
  1: Output $\mathsf{sk} := (A)^b$.
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Observe that for all (A, a) and (B, b) sampled according to $\mathsf{DHKeyGen}(\mathbb{G}, g)$, $\mathsf{DHKeyAgree}(A, b) = \mathsf{DHKeyAgree}(B, a)$. Thus, two parties derive the same secret key sk.

Digital signatures. A digital signature (DS) scheme consists of three algorithms (KeyGen, Sign, Verify) satisfying *correctness* and *unforgeability*. Informally, correctness requires Verify to output yes on all valid signatures and unforgeability states that no efficient adversary should be able to produce a forged signature under vk that is accepted by Verify.

Symmetric key encryption. An encryption scheme (e.g., AES [24, 83]) consists of three algorithms (KeyGen, Enc, Dec) satisfying correctness and IND-CCA security. Informally, correctness requires that $\mathsf{Dec}_{\mathsf{sk}}(\mathsf{Enc}_{\mathsf{sk}}(m)) = m$ for all messages m and secret keys sk sampled according to KeyGen. IND-CCA security requires the distribution of ciphertexts induced by any pair of messages to be computationally indistinguishable *and* authenticated (see Boneh and Shoup [12] for definitions).

Anytrust groups. Let κ be a statistical security parameter. Given a set of servers where some fraction $0 \le f < 1$ are malicious, there exists an integer s such that the probability that any group of s random servers contains *at least* one honest server is $1 - 2^{-\kappa}$ (proof: it is enough to pick s such that $f^s < 2^{-\kappa}$) [63].

4.2 New tool: Boomerang encryption

A core challenge in a mix-net is ensuring that all envelopes get to the appropriate server on the path. A user complaining of protocol deviation (e.g., dropped messages) runs the risk of deanonymization. Boomerang encryption—a simple extension of onion encryption [17]—solves this problem by providing "receipts" that can be used by servers to *anonymously* blame on a user's behalf. Boomerang encryption is designed to (1) ensure that all servers on a path receive messages addressed to them and (2) allow honest servers to trace the source of active attacks and assign blame accordingly.

Onion encryption and routing. Onion encryption [17, 33] involves recursively encrypting a message under a sequence of keys. Only the corresponding secret-key holders can recover the message by "peeling off" the layers of encryption one by one. Formally, given any symmetric-key encryption scheme with algorithm Enc, onion encryption is defined as:

$$\overrightarrow{\mathsf{onion}}(\mathbf{sk}, m) := \mathsf{Enc}_{\mathsf{sk}_1}(\mathsf{Enc}_{\mathsf{sk}_2}(\ldots \mathsf{Enc}_{\mathsf{sk}_\ell}(m)\ldots)),$$

where $\mathbf{sk} := (\mathsf{sk}_1, \ldots, \mathsf{sk}_\ell)$ and m is the message.

Onion routing. Onion encryption is the backbone of onion routing, where a chain of servers route onion-encrypted messages, removing one layer of encryption at a time. Onion routing prevents any intermediate server from learning the plaintext message. We will use the term *envelope* to describe the intermediate ciphertexts of an onion-encrypted message.

Boomerang encryption and routing. Onion routing works in one "direction:" routing envelopes *forward* through the chain of servers such that only the destination server learns the plaintext message. In contrast, boomerang routing is bidirectional: it applies onion encryption *twice*, once in the forward direction and once in reverse (hence the name, *boomerang*). If we define reverse-onion encryption as:

$$\overleftarrow{\mathsf{onion}}(\mathbf{sk}, m) := \mathsf{Enc}_{\mathsf{sk}_\ell}(\mathsf{Enc}_{\mathsf{sk}_{\ell-1}}(\ldots \mathsf{Enc}_{\mathsf{sk}_1}(m)\ldots)),$$

then boomerang encryption is defined as:

$$\overleftrightarrow{\mathsf{boomerang}}(\mathbf{sk}, \overrightarrow{m}, \overleftarrow{m}) := \overrightarrow{\mathsf{onion}}(\mathbf{sk}, \overrightarrow{m} \,\|\, \overleftarrow{\mathsf{onion}}(\mathbf{sk}, \overleftarrow{m})),$$

where $\overrightarrow{m}$ is the forward message and $\overleftarrow{m}$ is the returned message. We note that the $\overleftarrow{m}$ cannot be returned without first decrypting all envelopes in the forward onion chain.

Boomerang decryption as an anonymous proof-of-delivery. We make the following somewhat surprising observation: a successful boomerang *decryption* can be turned into a proof of message delivery when applied to onion routing.[2] Specifically, letting $\overleftarrow{m}$ (the returned message) be a random nonce acts as a message delivery receipt: it guarantees that every server along the path decrypted the correct onion layer and forwarded it to the next server. To see why this holds, observe that in order to successfully recover $\overleftarrow{m}$, all envelopes in the forward onion must be decrypted to recover $\overrightarrow{m} \,\|\, \overleftarrow{\mathsf{onion}}(\mathbf{sk}, \overleftarrow{m})$. Therefore, message $\overrightarrow{m}$ must have been delivered to the server at the end of the path (i.e., the last envelope in the forward onion chain must have been decrypted). We formalize this in Appendix A.3.

5 The Trellis system

In this section, we provide further details on the path establishment and broadcasting protocols (overviewed in Section 3). The formal protocol descriptions are in Appendix D. We start by describing anonymous routing tokens (Section 5.1), which are integral to the design of Trellis.

[2]Note that proof-of-delivery requires the underlying encryption scheme to be IND-CCA secure (such as Encrypt-then-MAC [12] with AES).

5.1 New tool: Anonymous routing tokens

Anonymous routing tokens (ARTs) are used in Trellis to determine and verify routing paths. Our construction of ARTs is inspired by anonymous tokens based on blind signatures [29, 35, 59], but has two key differences: namely, ARTs are signed by a *group* of signers and are publicly verifiable. Specifically, tokens are signed by an anytrust group. This guarantees that at least one honest signer is present when signing each token and forms the basis for integrity in Trellis. The blind signature serves two purposes: ensuring that (1) the payload encapsulated by the ART comes from an (anonymous) user, and (2) the path through the mix-net, as determined by the ART, is uniformly random (even for malicious users).

How our routing tokens work. Routing tokens are generated by users and signed by an anytrust group. Each token encapsulates the necessary information a server requires to route envelopes on a link in the mix-net. Each signer (a server in an anytrust group) holds a partial signing key x_i associated with a global public signature verification key tvk_ℓ for the ℓth layer. The user has each group member provide a partial signature for the blinded token. The user then unblinds and recovers the multi-signature with respect to tvk_ℓ by combining the partial signatures. If at least one signer is honest, then the signature is unforgeable and cannot be traced to the tokens seen during signing. The randomness of each token signature—which is uniform and independent provided at least one signer is honest—determines how envelopes are routed through the network.

Definition 1 (Anonymous Routing Tokens; ARTs)**.** Fix a security parameter λ, integers $s, N > 1$, a common reference string crs generated using a trusted Setup process, and a set $\mathbb{S}$ of N server identifiers. An ART scheme consists of algorithms KeyGen, $\mathsf{NextServer}$, Verify, and an interactive protocol Sign instantiated between a user and s signers.

- $\mathsf{KeyGen}(\mathsf{crs}) \to (\mathsf{tvk}, x_1, \dots, x_s)$: takes as input the crs; outputs a public key tvk and s partial signing keys.
- $\mathsf{Sign}\langle\mathsf{User}(\mathsf{tvk}, \mathsf{payload}), \mathsf{Signers}(x_1, \dots, x_s)\rangle \to \langle \mathsf{t}, \perp\rangle$. The user inputs a payload consisting of the server identity $S \in \mathbb{S}$, a **new** Diffie-Hellman message and a **new** digital signature verifcation key. The ith signer provides as input the partial signing key x_i. The user obtains a (signed) routing token t committing to the $\mathsf{payload}$. The signers obtain no output ($\perp$). See Figure 5 for an overview.
- $\mathsf{NextServer}(\mathsf{tvk}_\ell, \mathsf{t}_\ell) \to (S_{\ell+1})$: takes as input the layer public key tvk_ℓ, the token t_ℓ; outputs the identity of the next server $S_{\ell+1}$ deterministically chosen based on t_ℓ.
- $\mathsf{Verify}(\mathsf{tvk}_\ell, S_{\ell+1}, \mathsf{payload}_\ell, \mathsf{t}_\ell) \to$ yes or no: takes as input the public key for layer ℓ, the payload $\mathsf{payload}_\ell$, identity of the next server $S_{\ell+1}$, and signed token t_ℓ; outputs yes if ($S_{\ell+1}$ and $\mathsf{payload}_\ell$ are valid under t_ℓ and no otherwise.

The above functionality must satisfy the standard properties of blind digital signatures [90] (and anonymous tokens [29, 59]): *completeness*, *unforgeability*, *unlinkability*, in addition to a new property we call *unpredictability*. Unforgeability ensures that if the ART verifies under the public key tvk, then the encapsulated payload came from some user and was not tampered with. Unlinkability for ARTs guarantees that the encapsulated payload is random and independent of any user (thus the Diffie-Hellman message and a signature verification key are sampled randomly). Finally, unpredictability requires that $S_{\ell+1}$ (as output by $\mathsf{NextServer}$) be unpredictable given the payload. We formalize these properties in Appendix B.1.

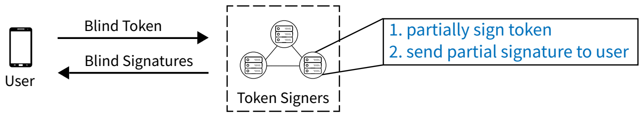


Figure 5: Overview of the ART signing protocol (Sign).

5.1.1 ART construction

We realize an ART scheme satisfying Definition 1 following the blueprint of BLS signatures [13] and anonymous tokens [29, 59]. In Appendix B.2, we prove security of our construction under a variant of the computational Diffie-Hellman (CDH) assumption in gap groups [11, 13, 59]. Let $\mathbb{G}$ and $\mathbb{G}_T$ be groups of prime order p with generator g and equipped with an efficiently computable non-degenerate bilinear pairing $e : \mathbb{G} \times \mathbb{G} \to \mathbb{G}_T$ [12]. We let $\mathsf{crs} := (\mathbb{Z}_p, \mathbb{G}, g) \leftarrow \mathsf{Setup}(1^\lambda)$ for a security parameter λ.

```
KeyGen(crs):
1: $x_1, \dots, x_s \leftarrow_R \mathbb{Z}_p$
2: $x \leftarrow \sum_{i=1}^{s} x_i$
3: $\mathsf{tvk} \leftarrow (g^{x_1}, \dots, g^{x_s}, g^x)$
4: return $(\mathsf{tvk}, x_1, \dots, x_s)$
```

```
NextServer(tvk_ℓ, t_ℓ):
1: $S_{\ell+1} := \mathsf{Hash}_{\mathbb{Z}_N}(\mathsf{tvk}_\ell || \mathsf{t}_\ell)$
2: return $S_{\ell+1}$
```

```
Verify(tvk_ℓ, S_{ℓ+1}, payload_ℓ, t_ℓ):
1: parse $\mathsf{tvk}_\ell = (g^{x_1}, \dots, g^{x_s}, g^x)$
2: $y \leftarrow \mathsf{Hash}_{\mathbb{G}}(\mathsf{payload}_\ell)$
3: $S'_{\ell+1} \leftarrow \mathsf{NextServer}(\mathsf{tvk}_\ell, \mathsf{t}_\ell)$
4: return $e(\mathsf{t}_\ell, g) = e(y, g^x)$
       and $S'_{\ell+1} = S_{\ell+1}$
```

Token signing protocol. We instantiate the token signing protocol in one round between the user and each signer. The user begins by generating a fresh Diffie-Hellman key exchange message that will later be used to derive the encryption key for its ℓth link. The user also generates a fresh digital signature key pair, which will be used to verify integrity of its encryptions on the ℓth link.

Step 1 (user):

```
1: $(A_\ell, a_\ell) \leftarrow \mathsf{DHKeyGen}(\mathbb{G}, g)$;  $(\mathsf{vk}_\ell, \mathsf{sk}_\ell) \leftarrow \mathsf{DS.KeyGen}(1^\lambda)$.
2: $r \leftarrow_R \mathbb{Z}_p$;  $T_\ell \leftarrow \mathsf{Hash}_{\mathbb{G}}(\underbrace{S_\ell || A_\ell || \mathsf{vk}_\ell}_{\mathsf{payload}_\ell})^{1/r}$.
3: send $T_\ell$ to all signers.
```

Step 2 (ith signer): Upon receiving T_ℓ, the ith signer partially signs the blind hash using their share x_i of the signing key:

```
1: $W_{\ell,i} \leftarrow T_\ell^{x_i}$.
2: send $W_{\ell,i}$ to the user.
```

Step 3 (user): The user combines the partial signatures and outputs an unblinded token:

```
1: $\mathsf{t} \leftarrow \left(\prod_{i=1}^{s} W_i\right)^r$.
```

The user then checks that t_ℓ was signed correctly:

```
2: $S_{\ell+1} \leftarrow \mathsf{NextServer}(\mathsf{tvk}_\ell, \mathsf{t}_\ell)$.
3: $\mathsf{Verify}(\mathsf{tvk}_\ell, S_{\ell+1}, \mathsf{payload}_\ell, \mathsf{t}_\ell) \overset{?}{=} \mathsf{yes}$.
```

If Verify outputs no, then the user discards the token and outputs $\perp$. Otherwise, the user stores $(\mathsf{payload}_\ell, \mathsf{t}_\ell, \mathsf{sk}_\ell, a_\ell)$.

Generalization. We remark that our construction of ARTs can be generalized to support more generic anonymous tokens such as the ones described in Kreuter et al. [59] and Davidson et al. [29] but instantiated with a *set* of signers rather than just one signer as in previous work [29, 59, 105]. Additionally, unlike prior constructions, ARTs are *publicly* verifiable. These features make ARTs potentially of independent interest. For simplicity, we restrict our definition of ARTs to Trellis.

5.2 Putting things together

We decouple the expensive cryptographic operations from the lightweight ones (similarly to cMix [16]). During key generation and path establishment, all necessary cryptographic key material is distributed via the ARTs to servers on the path. During the mixing phase, messages are routed through the established paths. This is a lightweight procedure: each server only needs to decrypt and shuffle envelopes. The mixing phase can be repeated indefinitely (with many different messages) and does not involve the setup overhead of the path establishment phase.

Key generation. Each server uses DHKeyGen and publishes the Diffie-Hellman public key ("A") to the PKI. Each server uses Feldman's verifiable secret sharing (VSS) scheme [40] (described in Appendix D.2 for completeness) to share the Diffie-Hellman secret, for state recovery purposes (Section 6.3.1). In addition, we use proactive secret sharing [50, 76] to generate shares of a common ART blind signature key for each anytrust group. With proactive secret sharing, multiple *independent* sets of shares are generated for the *same* secret. This ensures that each anytrust group holds shares of the secret key while preventing subsets of colluding servers across *different* groups from recovering the secret key. We provide details on proactive secret sharing in Appendix D.1 for completeness.

ART generation. Each user is issued one ART per layer by a random anytrust group. Each user can only send (boomerang encrypted) messages through the network according to the path dictated by the ARTs. All invalid or duplicate envelopes are discarded by the honest servers. Servers that improperly route envelopes are blamed by the next honest server on the path (blame protocols are described in Section 6).

Routing tokens and path selection. During the path establishment protocol (described in further detail in Section 5.3), each server (anonymously) receives ARTs for the links it is responsible for. Each token determines the next server on a link (and can be checked with ART.NextServer), which allows each server to (1) determine where to send envelopes in the subsequent layer and (2) anonymously verify path adherence (without knowledge of the full path, only the local link).

Boomerang routing on a path. We use boomerang encryption to route envelopes through the mix-net. We assume that the boomerang encryption scheme is instantiated using IND-CCA secure encryption (where the shared encryption key is derived from the Diffie-Hellman message in the ART payload). Each intermediate layer ciphertext (i.e., envelope) is signed and can be verified by the server using the signature verification key given in the corresponding ART payload. See Protocol 1 in Appendix D for details. This gives us *authenticated* boomerang routing which ensures that: (1) each server receives every envelope for each link and (2) all envelopes on a link are valid authenticated encryptions under the associated link public keys. Any failure of these two properties (e.g., failed decryption, bad signatures, or missing envelopes) triggers a blame protocol (described in Section 6.1.2).

Dummy envelopes. To prevent leaking how many messages are passed between servers in each layer (required to avoid leaking parts of the mixing permutation applied on the messages to the network adversary), each server batches the envelopes (intermediate boomerang ciphertexts) it sends to the destination server(s) in the next layer, padding the batch with "dummy" envelopes as needed. The dummy envelopes are then discarded by the receiving server(s).

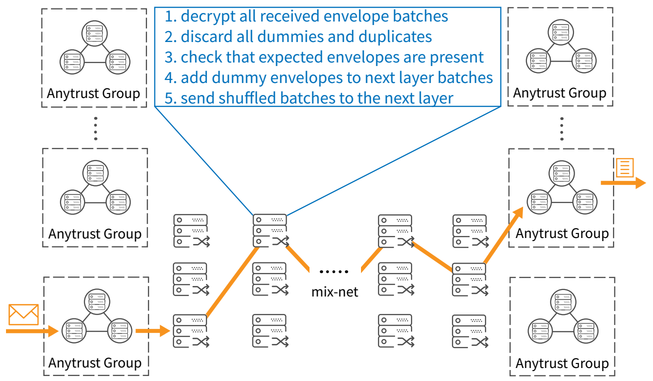
Assigning blame. All communication between parties is additionally signed using a digital signature (DS) scheme for lightweight blaming of malicious parties. See Section 6.

5.3 Do once: Path establishment

We now describe how we resolve the first challenge: anonymously establishing random paths. The difficulty is that paths must be created in a way that (1) does not reveal the path to any party other than the user and (2) guarantees that all paths are uniformly random, even when generated by malicious users. Each server on the path must receive an ART associated with the path link but must not learn which user provided it. We solve this with an incremental approach: we inductively extend an existing path from one server to another while simultaneously forcing all users to choose random paths. Each user starts with a random path of length one, with the choice of this first server assigned by the first ART. This base case does not yet provide anonymity since the first server knows the user's identity. We then proceed by induction, as follows. Given a path of length ℓ ending at server S_ℓ, the user extends the path to length $\ell+1$ using boomerang routing through $S_1 \to S_2 \to \cdots \to S_\ell$ to communicate with the new server $S_{\ell+1}$. The user sends the routing information $S_{\ell+1}$ needs: the ART t_ℓ for the link $(S_\ell \to S_{\ell+1})$, the ART $\mathsf{t}_{\ell+1}$ for the link $(S_{\ell+1} \to S_{\ell+2})$, and the associated Diffie-Hellman messages and signature verification keys. (The user also sends a signature on the ART $\mathsf{t}_{\ell+1}$ under the verification key t_ℓ to prove that t_ℓ and $\mathsf{t}_{\ell+1}$ are part of the same path.) Because the user communicates exclusively through the existing path and because ARTs are unlinkable, the path extension procedure reveals no information to any server. After receiving and verifying the tokens, $S_{\ell+1}$ routes the reverse boomerang envelopes backwards along the path. Each server verifies the authenticated encryption, which implicitly ensures that the user and all servers $S_\ell \to S_{\ell-1} \to \cdots \to S_1$ know that $S_{\ell+1}$ received (and validated) the tokens. This process is repeated L times until the full path to S_L is established.

Anytrust group bookends. The Trellis mix-net consists of L mix-net layers "bookended" between two layers of anytrust groups (see Figure 6). The anytrust groups at the entry and exit layers ensure that the start and end of a mix-net path are always processed by an honest server, which prevents dropped messages and denial-of-service attacks. The anytrust group at the entry layer verifies the boomerang receipts on behalf of the user: when the boomerang receipt comes back, each member of the group can check the attached signature and ensure that the receipt was correctly decrypted. Additionally, the entry group stores the envelopes it is given so as to prevent denial-of-service: if the first k servers in the mix-net are malicious and drop messages, the entry group can resend the envelopes after the adversarial servers are eliminated. The anytrust group at the exit layer outputs the messages to the world (the "broadcast" part of anonymous broadcast), ensuring that all messages make it out. During path establishment, these exit groups are at the "peak" of the boomerang and use a group decryption protocol to decrypt the Lth boomerang envelopes. In this protocol, each group verifies the ART for layer L and retains the included signature verification key. (Note that the verification keys are unlinkable to the start of the path because of the mixing guarantees provided by the mix-net.) Then, during the repeated broadcast rounds, these exit groups check that one message is received for each of the verification keys they have, ensuring delivery of all messages. See Protocol 2 in Appendix D for details.

Why path establishment works. Both the incremental approach and boomerang message receipts are required to prevent dropped messages. The incremental approach to path establishment enables immediate blame assignment and prevents errors from propagating. The message receipt is designed to allow servers on the existing path to assign blame on a user's behalf if the new server deviates from protocol. Specifically, each server expects to receive an envelope for each established key, in both the forward and reverse direction. This gives rise to a simple blame protocol in the case of a dropped



1. decrypt all received envelope batches
2. discard all dummies and duplicates
3. check that expected envelopes are present
4. add dummy envelopes to next layer batches
5. send shuffled batches to the next layer
Anytrust Group
Anytrust Group
Anytrust Group
Anytrust Group
Anytrust Group
Anytrust Group
mix-net

Figure 6: Detailed overview of the Trellis mix-net "bookended" by anytrust groups to prevent denial-of-service attacks at the entry and exit points. Users send boomerang-encrypted messages through a pre-selected random path in the network. The anytrust group at the last layer ensures that all messages get published to the bulletin board.

envelope: blame the previous server. The previous server, if innocent, should have reported the envelope missing during the last synchronous layer; because it did not do so, it must be malicious (see Section 6 for details).

5.4 Repeat: Mixing rounds

The repeated broadcasting rounds use the established paths to mix (different) message anonymously and quickly. Messages in Trellis are of fixed size (e.g., 10kB), and all users must submit a message of that size, padding if necessary. Because mixing rounds can be repeated many times, users with larger messages can split their broadcasts across several rounds. The random routes guarantee that all messages are mixed before reaching the final layer (see mixing analysis in the next section).

5.5 Number of layers required

We now turn to analyzing the number of layers required to achieve a random permutation on the delivered messages. This analysis forms the crux of our security argument in Section 7. Previous theoretical work [48, 92] performs similar analyses in the context of *permutation networks*. Unfortunately, such analyses are only applicable to networks consisting exclusively of *honest* servers. The analysis of Kwon [65] is the first to consider networks containing a fraction of malicious servers. We improve the mixing analysis of Kwon and give a simpler analysis of mixing guarantees to bound the number of layers required in a random routing mix-net.

Modeling the network. We consider an information-theoretic view of an adversary observing the network and learning a fraction f of the permutation applied to the messages in each layer (obtained from the servers that collude with

	Number of users				
	100 K	**500 K**	**1 M**	**5 M**	**10 M**
$f = 0.1$	56	58	58	60	60
$f = 0.2$	86	88	89	92	93
$f = 0.3$	124	128	129	133	134
$f = 0.4$	179	184	186	191	194
$f = 0.5$	266	273	277	284	288
$f = 0.6$	419	431	436	448	453
$f = 0.7$	736	757	767	787	796
$f = 0.8$	1603	1649	1668	1715	1734
$f = 0.9$	6069	6244	6319	6494	6569

Table 1: Number of layers required for mixing as a function of the malicious server fraction f and the number of users. See Theorem 7 of Appendix C for derivation.

the adversary in Trellis). We construct a Markov chain where each state consists of a list of which server each message is in and also a deck of M cards, with each card uniquely corresponding to a message. We make one step in the Markov chain with each layer-to-layer transition. The servers are chosen randomly, which corresponds to the random path assignment dictated by the ARTs. We shuffle the deck according to the hidden part of the permutation, summarized by the following observation: only messages sent between two *honest* servers (in adjacent layers) are mixed, meaning that messages passing between two *malicious* servers are not mixed.

We model the adversary's view as a probability distribution over all possible permutations of the deck and message locations. At the start, the adversary knows who submitted each message, and can choose the starting state. We determine the probability distribution for subsequent layers through the Markov process. The above process operates on the entire group of permutations, and so eventually converges to the uniform distribution where each deck permutation is equally likely (see [9, Lemma 15.1]). This corresponds to a uniformly random permutation of the messages.

Determining the number of layers. To determine the number of layers required, we must find the "time" to convergence. Computing this value requires additional analysis. This analysis, while straightforward, is somewhat tedious to derive (and is similar in nature to the analysis of Kwon [65]). Therefore, we defer the calculation to Appendix C. We report the number of layers as a function of the corruption fraction f in Table 1. We note that the number of layers is logarithmic in M (the number of users) and is not dependent on N (the number of servers). This is because the probability of being routed through an honest server only depends on f.

6 Blame, adversary removal, and recovery

If all parties behave honestly, then all envelopes are present and well-formed. In this case, boomerang routing and mixing analysis (Section 5.5) ensure that all messages are delivered and mixed (see security analysis; Section 7). However, an adversarial user or server may deviate from the protocol in arbitrary ways. To prevent system disruption, or worse, user deanonymization following active attacks, each (honest) server is expected to verify all envelopes it receives for layer ℓ *before* sending the shuffled batch of envelopes to layer ℓ+1. If any of the checks do not pass, the server must notify (all) the other servers before the protocol finishes layer ℓ (note that this server will not send messages for ℓ+1 until the error is resolved, and so layer ℓ should not complete). This prevents errors from propagating through honest servers; honest servers blame immediately in the layer where a check fails and recovery protocols are initiated to eliminate the blamed server(s) and find suitable replacements.

Blaming without deanonymization. We make an important observation which allows us to maintain the anonymity of honest users when blaming malicious servers: if we can show that at least one of the two servers on a link misbehaved, then we can reveal the envelopes along that link *without* compromising user anonymity. We formalize this in Claim 1.

Claim 1 (Malicious links provide no anonymity)**.** Links between two servers, where at least one of the servers is an adversary, can be revealed without compromising anonymity.

Proof. Consider the view of an adversarial server on a link consisting of at least one malicious server. The adversary knows all encrypted envelopes on the link. Revealing any of this information (to all of the servers) does not further the adversary's knowledge. Our analysis of Section 5.5 relies only on links between pairs of *honest* servers for this reason. □

6.1 Blame protocols

In order to illustrate our blame protocols, we need to provide more details about what exactly is sent between servers (which we fully specify in Appendix D). First, all signatures sign the round, layer, source server (which may be different from the signer), destination server, protocol, length, and direction of travel. This will allow anyone to publicly verify which step of which protocol a message corresponds to. Second, servers sign the batch of envelopes and corresponding ART keys. Signing the concatenation of the batch ensures that no server can equivocate on the number of envelopes it sent for a given round and layer. These signatures can use any standard public-key signature scheme [12, 57].

We assume that all blame messages are sent to all servers (e.g., using a standard gossip protocol [66, 85]). If we assume an honest majority of servers ($f < 0.5$; see Section 6.3.2 for why this assumption is without loss of generality), the majority vote assigns blame and selects a replacement server. In Section 7, we cover how blame protocols support our delivery guarantees. In short, we ensure that no honest server or honest user can ever be blamed through a blame protocol, and thus only adversaries are blamed. The blame protocols ensure that the tokens, signatures, and envelopes are present and well-formed, which will imply successful delivery.

6.1.1 Duplicate envelopes for the same ART

There should be exactly one message per ART per round and layer. In the case of duplicates, a server can blame a user by producing as evidence two unique signed envelopes with the same round and layer. This evidence is submitted to all of the servers, who vote to remove the user if the evidence is valid (or to remove the server if the evidence is invalid). We set $N(1-f)$ as the minimum threshold of votes required to remove a server, as described in Section 6.3.1. All of the protocols below follow this format: a party will produce some publicly verifiable evidence, which the servers will check and vote on accordingly.

6.1.2 Missing envelope

Since their is one message per ART per round and layer, each server expects to receive the corresponding envelope and blames if it is not received. In the event that an envelope is missing, $S_{\ell+1}$ blames server S_ℓ for not sending the envelope (as S_ℓ did not report the envelope missing during the previous layer). We have two cases to consider:

- $S_{\ell+1}$ is maliciously invoking the blame protocol, or
- S_ℓ is malicious and dropped an envelope.

By Claim 1, $S_{\ell+1}$ can reveal the signed concatenation of the envelopes on link $(S_\ell \rightarrow S_{\ell+1})$. Specifically, $S_{\ell+1}$ provides the signed batch (with the missing envelope) that it claims to have received from S_ℓ in addition to a signed batch from a previous round showing that S_ℓ sent an envelope with the ART in question. We can then apply the protocol described in Section 6.3.1 to remove the offending server.

Remark 1 (An edge case)**.** At the peak of the boomerang during path establishment, note the pattern of sending $(S_\ell \rightarrow S_{\ell+1} \rightarrow S_\ell)$. Because $S_{\ell+1}$ does not know which tokens it is supposed to receive yet, it cannot blame S_ℓ for not sending them. Therefore, we allow $S_{\ell+1}$ to respond to claims from S_ℓ (who is the party responsible for blaming if $S_{\ell+1}$ drops an receipt envelope) by complaining that S_ℓ never sent the ART in the first place. It does this by presenting the signed concatenation from $(S_\ell \rightarrow S_{\ell+1})$ which is secure by Claim 1.

6.1.3 Envelope decryption failure

If an envelope fails to decrypt, then server $S_{\ell+1}$ can request that the previous server S_ℓ be blamed. This means that we are in one of three cases:

1. a malicious user sent an ill-formed envelope,
2. S_ℓ tampered with the envelope, or
3. $S_{\ell+1}$ is maliciously invoking the blame protocol.

Since the potentially malicious user, S_ℓ, and $S_{\ell+1}$ all know both the envelope and the verifying ART token, $S_{\ell+1}$ can reveal these to prove the envelope is ill-formed (in a similar argument to Claim 1). In response, S_ℓ may claim it never sent the ill-formed envelope and token. It then requests that $S_{\ell+1}$ reveal the signed concatenation of envelopes on the link to prove that the ill-formed envelope and ART were sent by S_ℓ. In this case, either S_ℓ or $S_{\ell+1}$ is blamed.

Otherwise, S_ℓ agrees that the envelope was ill-formed with respect to the corresponding ART, and we won't blame $S_{\ell+1}$. We still need to decide if the user sent the ill-formed envelope, or S_ℓ tampered with it. If S_ℓ is honest, then it will be able to present a proof that it acted correctly. This proof involves revealing each step taken by S_ℓ and proving that each step was executed correctly. To prove this, S_ℓ first reveals the ART t_ℓ and the encapsulated payload (containing a partial Diffie-Hellman message and verification key). Using a discrete log equivalence proof [29, 52, 59], S_ℓ proves correct decryption (without revealing its long-term secret key). Finally, it provides the user's signature on $\mathsf{t}_{\ell+1}$ under the key from t_ℓ to show that $\mathsf{t}_{\ell+1}$ is the correct next token. With these elements the other servers can verify the following:

- S_ℓ received an envelope that was well-formed (signed), and therefore S_ℓ was not required to blame earlier.
- The decryption of said envelope was computed correctly, but the output of the decryption was ill-formed.
- The keys are correct and came from the user who created both of the ARTs, as t_ℓ signs the payload.

With this information, it can only be that the user is to blame, and we apply the protocol in Section 6.3.3 to deanonymize and remove the malicious user. This well-formed envelope is signed (anonymously) by the user under some key vk_ℓ, and so could only have been created by them. Similarly, the token $\mathsf{t}_{\ell+1}$ is signed by vk_ℓ and so is the correct next token, and the correct $\mathsf{vk}_{\ell+1}$. Therefore, the server has produced a proof that the decryption was performed correctly, but the decryption output is ill-formed.

6.1.4 Arbitration protocol

Our final blame protocol deals with a missing or incorrect server signature of the concatenated envelopes. The arbitration protocol begins when a server $S_{\ell+1}$ reports that the signature given by S_ℓ is missing (including if no envelopes were sent at all) or incorrect. In this case, we have either that S_ℓ is adversarial or $S_{\ell+1}$ is maliciously initiating the blame protocol. An anytrust group randomly chooses an arbitrator from the other servers (the honest member will contribute true randomness and ensure the selection is random). We then require S_ℓ to send messages to the arbitrator, denoted $A_{\ell,\ell+1}$, who forwards them to $S_{\ell+1}$. The arbitrator also checks that the signature on the batch is correct (using S_ℓ's public verification key). If messages are delivered successfully (from S_ℓ to $S_{\ell+1}$) through $A_{\ell,\ell+1}$, then $A_{\ell,\ell+1}$ continues to arbitrate for future rounds. The performance of Trellis is only minimally impacted by arbitration, as it simply adds an extra forwarding and signature check for one layer.

If the arbitrator itself has a dispute with S_ℓ or $S_{\ell+1}$, then it can determine which server is honest from its perspective. When the arbitrator is honest, it will never have a dispute with the honest server. Once an arbitrator decides who the malicious server is (because it has a dispute), a new arbitrator is randomly selected by an anytrust group. If there is an honest majority of servers, then there will be more disputes involving the dishonest server than there are involving the honest server. For a dishonest majority, we can track all server disputes with a trust graph [110]: a graph in which each edge represents if two servers trust each other. Although the initial disagreement breaks only one trust graph link, we can choose arbitrators who trust both parties. Then, each dispute between the arbitrator and a party breaks another trust graph link. Wan et al. [110] prove that such an algorithm eventually converges to the honest clique, who can then reform without the troublesome members.

Because of the synchronous communication assumption, messages are always delivered between honest parties in a timely manner, and so the arbitration protocol is only invoked with adversaries. Arbitration can introduce a maximum overhead of a factor of 2×, as each message is forwarded one extra time.

6.2 Malicious anytrust group member

First, if one of the signers in an anytrust group incorrectly signs the ART during the signing protocol described in Section 5.1, the user receives an invalid signature (which it locally determines is invalid using $\mathsf{ART.Verify}$). In this case, the user can request that each signer (publicly) proves it computed the partial signature correctly. In our construction, each signer can do so efficiently by providing a zero-knowledge proof of discrete log equivalence [29, 52, 59], proving that $W_{\ell,i}$ was calculated correctly with respect to g^{x_i}. Second, during path establishment, we require the anytrust group to compute a group decryption at the last layer. If decryption fails, each signer can prove it computed the partial decryption correctly also with a zero-knowledge discrete log equivalence proof. Any signer that does not comply is eliminated using the protocol of Section 6.3.1.

6.3 Removal and recovery protocols

Once we have identified the adversarial party, we can proceed to remove them from the system. To remove adversarial servers, we ensure that each server's secret keys are t-out-of-n shared, using a verifiable secret sharing [91] (VSS) scheme, with $t > Nf$ at setup time (see Appendix D.2 for details). Blame decisions are made as a group, using the t-out-of-n secret key shares as votes. Using the shares allows the server receiving t (or more) shares (i.e., votes) to recover the offending server's secrets and replace it.

6.3.1 Server removal and state recovery

When servers vote to blame a malicious server, a replacement server is chosen randomly from among the remaining servers by an anytrust group (as the arbitrator was in Section 6.1.4). Each server sends its share of the blamed server's secret key sk to the replacement server, who can use the VSS scheme to reconstruct sk if at least t servers agree that the server is at fault. Next, to reconstruct the routing information, the ARTs and corresponding information can be resent by the other servers. We note that the replacement server may additionally need to show that it did not receive an ART during this replay and pass blame for a missing message accordingly, which it can do using the signature of the replay batch. With the keys, the replacement can decrypt and forward the messages encrypted to the old server. Note that the replacement server does not recover the batch signature key. Instead, it signs batches with a new signing key which lets other servers know that envelopes came from the replacement rather than the eliminated server.[3]

Remark 2 (Server churn)**.** In the event of server churn, where an (honest or adversarial) server leaves the network and stops responding, we can also apply the state recovery protocol to select a replacement and continue.

6.3.2 Dishonest majority blame and recovery

If there is a dishonest majority, then the vote described in Section 6.3.1 can deadlock, with neither side meeting the threshold required to find a replacement. However, note that all honest servers will vote together, and only adversaries will vote incorrectly. If we partition into a new instance along the voting lines, then the partition with the honest servers (who all voted together) will have a higher fraction of honest servers than before. If we let $h := 1 - f$ be the fraction of honest servers, then we will require no more than $\lfloor 1/h \rfloor - 1$ restarts after voting deadlocks in order to achieve an honest majority. For example, if $1/2 < f < 2/3$, we will need at most 1 restart. This is because in order to deadlock the vote, at least $1/3$ of the servers must vote incorrectly (all servers that vote incorrectly are adversaries), who will form one partition. The other partition will then consist of at most $2/3$ of the servers, including all of the (at least $1/3$) honest servers, giving it an honest majority. Since the evidence is publicly verifiable, it can also be verified by the users, who can decide which partition to join. The honest users will evaluate the evidence

[3]We note that VSS-based key recovery is only necessary to finish the current round of the protocol without restarting. In future rounds, the replaced servers can post new public keys, which users can use for authenticated encryption.

the same as the honest servers, and so they will choose to send messages to the honest partition.

6.3.3 Blocking malicious users

Once a user has been blamed, we can revoke their keys by having each server on the user's path reveal the user's ARTs. Specifically, a server can produce both of the user's ARTs and the signature binding them together. Then, we can repeat this for each server on the path (by following the identities committed in the ARTs), to remove all of the user's tokens from the servers' lists. We assume that the set of users is fixed, as users who join late are inherently hard to anonymize due to statistical disclosure attacks (see Section 3.2). Users that are late must wait until the next restart of the protocol to join.

7 Security analysis

In this section, we analyze the security properties of Trellis. We show that (1) messages are mixed and no metadata is leaked in the process, (2) messages are guaranteed to be delivered, and (3) blame protocols eliminate adversarial parties—users or servers—without compromising anonymity.

Claim 2 (Path establishment)**.** All honest users establish a random path through the layers and each server on the path obtains a decryption key and a signature verification key.

Proof. Boomerang encryption verifies that the anonymous routing tokens are delivered to each server along the path (see Claim 8 in Appendix A.3). The blame and recovery protocols (Section 6) ensure this process completes. In particular, note that the user establishes a key with the final anytrust group. □

Claim 3 (Path adherence)**.** All messages submitted by honest users are routed through all honest servers along the chosen path determined by the ARTs.

Proof. Each honest user follows the ART signing protocol and obtains valid ARTs for each link. Suppose, towards contradiction, that the envelope sent by the user does **not** travel through an honest server, say S_ℓ, as dictated by the ART on the ℓth layer. The presence of a correctly signed message in the final anytrust group at the last layer implies that all preceding onion layers were decrypted correctly (see Lemma 1 in Appendix A.3). In turn, this means that S_ℓ decrypted a layer, which is a contradiction. □

Claim 4 (Metadata-private shuffling)**.** All envelopes sent between two honest servers on a link appear indistinguishable to the network adversary.

Proof. The dummy messages ensure that each batch of envelopes exchanged between two honest servers is of fixed size B (and TLS prevents the network adversary from seeing the contents of the batch). Crucially, no malicious server or user can overload an honest-honest link such that there are more than B envelopes. This holds because (1) the envelopes expected by each server are uniquely associated with ARTs provided during path establishment and (2) no honest server would forward any envelope not uniquely associated with an ART. By (1) and (2), the size of the batch sent to the second honest server is always B (padded with dummies as needed). □

Claim 5 (User anonymity)**.** All messages contributed by honest users are mixed, resulting in a random permutation of all messages being output to the public bulletin board.

Proof. We combine Claim 3 and Claim 4. Since all honest messages travel through all honest servers, they travel through all honest-honest links. Since the messages travel along honest-honest links, these messages are shuffled. Then, by Section 5.5 we have that all honest messages become unlinkable with their senders as the total variation distance of the applied permutation is ϵ-close to a truly random permutation of all messages. □

Claim 6. An honest user is never blamed as a result of any blame protocol described in Section 6, except with negligible probability in the computational security parameter λ.

Proof. A user is only blamed if either (1) there exist two envelopes for the same round and layer (Section 6) or (2) if there is an ill-formed envelope at some layer ℓ (Section 6.1.3). Because an honest user's envelopes are always well-formed, (1) and (2) do not apply to honest users. Furthermore, a user cannot be blamed by a malicious invocation of Section 6.3.3 (user removal) because each ART used to trace back the path to the user is signed by the verification key in the next ART, and an honest user would never sign a different user's ART. Hence, if an honest user is blamed then a malicious party was capable of forging the honest user's signature, which can only happen with negligible probability. □

Claim 7. An honest server is never blamed as a result of a blame protocol described in Section 6, except with negligible probability in the computational security parameter λ.

Proof. A server is only blamed if (1) there is a (signed) batch of envelopes showing deviation from the protocol or (2) the server refuses to produce a valid proof of correct decryption. (1) and (2) do not apply to honest servers who follow the protocol. Hence, if an honest server is blamed then a malicious party was capable of forging the honest server's signature, which can only happen with negligible probability. □

8 Implementation and evaluation

Implementation. We implement Trellis in Go 1.17. Our implementation is open source [1] and consists of approximately

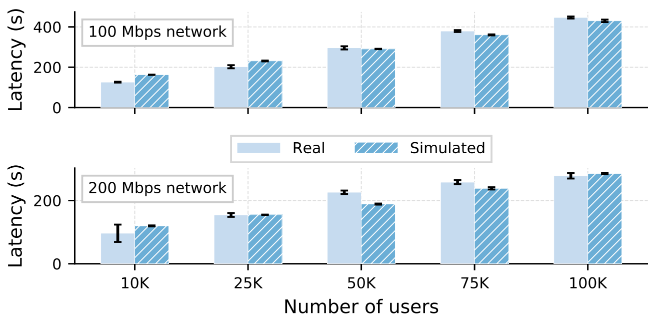


Figure 7: Comparison between broadcasting round latency on an international (Oregon, Virginia, Frankfurt, and Stockholm) network and our simulated network. **Top**: 100 Mbps bandwidth connections. **Bottom**: 200 Mbps bandwidth connections.

11,000 lines of code. We implement distributed key generation [56] and proactive secret-sharing [31] using the Kyber library [30]. We use the BLS12-381 [13, 115] elliptic curve to implement ARTs. Symmetric-key encryption is instantiated using AES [24, 83] and digital signatures using EdDSA [54]. Diffie-Hellman key agreement is performed over the ed25519 elliptic curve to match EdDSA. For our empirical comparison to Atom, we use their open source implementation [60].

Environment. We distribute the servers evenly among four geographic regions: Oregon, Virginia, Frankfurt, and Stockholm, to roughly match the distribution of Tor servers [73]. We use the general purpose `m5.xlarge` instances (quad-core; 16 GiB of RAM). We measured median round trip ping times of approximately 150 ms, with rare latency spikes of up to one second.

Due to high international networking costs, we run most of our evaluations on a simulated network environment informed from our real network deployment. In Figure 7, we report the latency of Trellis (broadcast round) under a 100 Mbps and 200 Mbps real network and compare latency to our simulated network deployment. We ensure that the simulated network accurately approximates the real network by evaluating the broadcasting rounds (on pre-established paths). We observe modest differences in latency (less than 20% in either direction) between the real and simulated networks.

We use the Linux NetEm [49] tool to model latency and packet loss rates for each geographic region. We apply a Pareto distribution to simulate the infrequent, but large, latency spikes observed in the real network. Properly simulating these latency spikes is important because the all-to-all nature of the mixnet in Trellis is sensitive to the *highest* latency observed across all links. As such, tail latencies can significantly impact performance and lead to high variance in overall broadcast latency.

Systems optimizations. We describe a few basic system-level and network optimizations that we incorporate into our implementation in Appendix E.

Parameters. We set $\epsilon := 2^{-64}$ for the total variational distance of the mixing (see Appendix C). (We also set the size of each anytrust group so that the probability that *any* anytrust group has all adversaries is at most 2^{-64}.) Note that both of these parameters set the *statistical* security of Trellis. To simplify our evaluation, we fix the message size to 10 kB (over 60× larger compared to the 160 B messages evaluated in Atom [63]). We vary the number of users M, number of servers N, and the fraction of malicious servers f (which influences the number of layers L and anytrust group size s).

8.1 Evaluation

The goals of our evaluation are to:

- determine the processing overheads of Trellis on the user and on the servers through microbenchmarks,
- measure the network overhead as a function of the number of users and fraction of malicious servers in the mix-net,
- evaluate the overhead of dummy envelopes between layers,
- evaluate how Trellis scales with additional servers, and
- compare Trellis to Atom—the state-of-the-art mix-net based system for metadata-private anonymous broadcast.

Computational and network overheads. The computational overhead of the broadcasting round is minimal given the relatively lightweight cryptography required: AES-decryption of envelopes and digital signature verification. All the computationally expensive cryptographic operations (e.g., pairings and token generation) are precomputed during path establishment. These computational workloads scale with the number of servers, as we show in Figure 9. However, the network latency incurred by routing through L layers does not scale with the number of servers and thus the scalability (for a given number of users) eventually plateaus. Furthermore, as N^2 approaches M, the overhead of dummy envelopes increases, as we must always send at least one dummy envelope between each pair of servers. This results in a limit on the horizontal scalability of Trellis. To illustrate this point further, in Figure 10, we compare a broadcast round in Trellis on a real network deployment with and without dummy envelopes. We find that dummy envelopes account for roughly 2.5× in latency overhead.

Path establishment. Path establishment can be seen as running consecutive broadcasting rounds with $\ell = 1, 2, \ldots, L$ layers in each consecutive round. Therefore, the asymptotic time complexity for path establishment is $O(L^2)$, and the concrete time it takes follows accordingly (see Figure 8). Path establishment can take one to five hours, depending on parameters. We note that path establishment depends only on the number of users, and not on the message size or number of subsequent broadcast rounds. We report two micro-benchmarks:

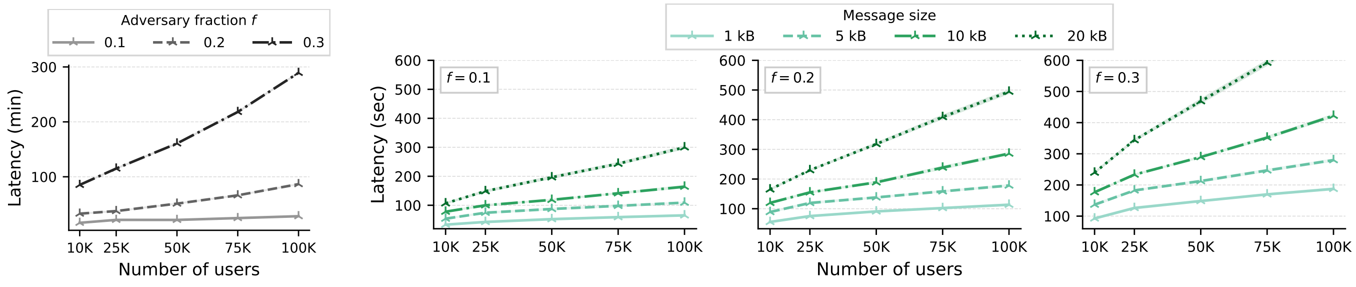


Figure 8: Left: Path establishment with different fractions of malicious servers (f). **Right:** Broadcast round scaling with number of users M (messages), message sizes, and different adversary fractions. Both experiments are run on a simulated 200 Mbps network configuration with $N = 128$ servers. Shaded regions represent a 95% confidence interval (mostly invisible).

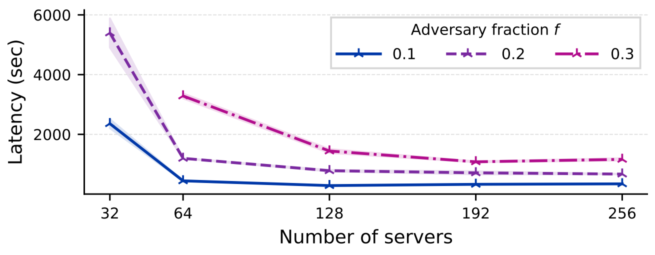


Figure 9: Latency decreases with the number of servers added to the network (until reaching a constant overhead) with 200 Mbps bandwidth cap. Number of messages is $M = 2,000,000$ and message size is set to 1 kB. Shaded regions represent a 95% confidence interval.

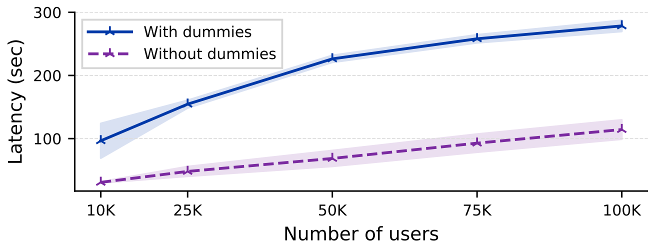


Figure 10: Dummy envelopes between layers incur a large overhead due to increased bandwidth requirements. Broadcast round (with and without dummy envelopes) evaluated on the real network with 200Mbps bandwidth cap, $f = 0.2$, and message size set to 10 kB. Shaded regions represent a 95% confidence interval.

- ART.PartSign: 0.08 ms (per token). // Signer overhead.
- ART.Verify: 1.09 ms (per token). // Token verification.

Broadcasting rounds. Once the path establishment completes, broadcasting rounds can be run indefinitely to broadcast messages (even following server churn and elimination). In Figure 8, we report the latency of the broadcasting rounds on four different message sizes (ranging between 1 kB and 20 kB) and fraction of adversarial servers. The broadcasting rounds are 10-120× faster compared to a path establishment round. With $f = 0.2$ and 100,000 users, broadcasting a 10 kB message incurs approximately four minutes of latency. We report micro-benchmarks for the decryption and signing (performed by servers on each link):

- AES.Dec: 16 µs for 10 kB. // Envelope decryption.
- DS.Verify: 71 µs for 10 kB. // Signature verification.

Blame protocols. The primary computational overhead of blame protocols is generating a proof-of-decryption. In Trellis, this is done using a discrete-log equivalence proof (DLEQ), as explained in Section 6. We run a microbenchmark to determine the proving (and verification) time of each DLEQ:

- DLEQ.Prove: 152 µs. // Proving signing and correct decryption.
- DLEQ.Verify: 310 µs. // Proof verification by server and user.

The low overheads of the DLEQ proofs make our blame protocols lightweight. Similarly, arbitration (in case of a dispute) incurs one extra hop, which translates to an overall latency increase of 150 ms (in our network configuration).

Throughput. In Table 2, we report the throughput (in bits per second) with 32, 64, and 128 servers and 100,000 users. The throughput decreases with more users and increases with more servers but eventually plateaus (see Figure 9 for scaling). For comparison, Tor achieves around 400,000 bits per second [73].

Trellis Throughput (bits/s)

	32	**64**	**128**
$f = 0.1$	234 ± 9	435 ± 6	534 ± 15
$f = 0.2$	169 ± 3	220 ± 4	324 ± 8
$f = 0.3$	84 ± 1	168 ± 2	222 ± 5

Table 2: Throughput of Trellis's broadcast as a function of the adversary fraction f and number of servers for 20 kB messages. Includes 95% confidence interval.

8.2 Comparison to Atom

To the best of our knowledge, Atom [63] is the state-of-the-art system for metadata-private anonymous broadcast. Compared to Atom, Trellis achieves four orders of magnitude lower latency (and proportionally higher throughput). Trellis scales easily to support large message sizes (e.g., in the kilobytes); in contrast, Atom does not, due to the zero-knowledge proofs and message passing within each anytrust group. We evaluate Atom on 32 B messages and multiply out to obtain the estimate time required to broadcast a 1 MB file (Kwon et al. [63] explicitly state that latency grows linearly with larger messages). We report the result for varying number of users and match Atom's $f = 0.2$ in Figure 11.

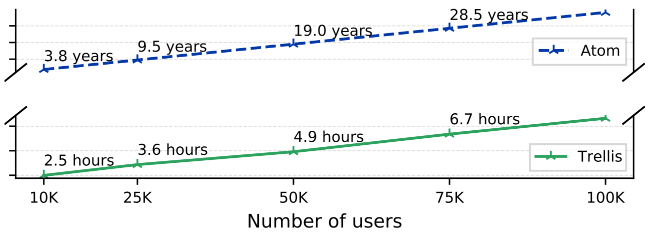


Figure 11: Latency of Trellis vs. Atom to broadcast a 1 MB file. We vary the number of users and set $f = 0.2$ for both systems (we run Atom on 32 B messages and extrapolate) with 200 Mbps bandwidth cap.

8.3 Discussion and practical considerations

By decoupling path establishment and expensive operations from the message size, Trellis handles large messages better than all existing horizontally-scalalble metadata-private anonymous broadcast systems, especially as the number of users increases. While Trellis is targeted to asynchronous applications (e.g., email, document uploads, surveys, etc.) due to concrete performance limitations (notably, high latency), further reducing latency and bandwidth overheads would require sacrificing security guarantees. If then adversary has a full view of the entire network, then user anonymity can only be guaranteed by providing metadata privacy. However, metadata privacy comes at the high cost of requiring extra bandwidth. Therefore, all metadata-private systems, including Trellis, will inherently have to pay this cost so as to maintain a uniform view of all network traffic.

In Trellis, to achieve metadata privacy, we must introduce dummy envelopes between servers and fix the set of participating users. Our evaluation in Section 8 demonstrates that Trellis pushes metadata-private anonymous broadcast to its limit by being bandwidth—rather than computationally—bottlenecked. As a consequence, any further improvements in performance (beyond minor optimizations) will require sacrificing metadata privacy. Specifically, there are three possible security compromises that can be made: (1) removing dummy envelopes between layers, (2) allowing user churn, and (3) reducing the number of layers in the mixnet. All three of these compromises are made by systems like Tor so as to scale to millions of daily users and achieve *real-time* (synchronous) communication (e.g., web browsing). We briefly elaborate on the consequences of applying (1) and (2) on the practical security guarantees of Trellis. Applying (3) would result in messages not being mixed and therefore voids all guarantees.

- Not using dummy envelopes would maintain all the guarantees of Section 3.2, except for metadata privacy. In return, however, removing dummy envelopes would reduce the bandwidth overhead on the servers and improve the overall scalability of Trellis (see Figure 10). Of course, in practice, it may be reasonable to assume a weaker adversary that is only capable of corrupting a subset of the servers and without a view of the entire network. Under this (weaker) threat model, the metadata privacy guarantee of Trellis can be sacrificed in favor of performance (without impacting the cryptographic anonymity guarantees of Trellis).
- Choosing to let users come and go makes Trellis (and all anonymous communication systems) vulnerable to intersection attacks [25, 26, 113]. Online (or offline) users correlate themselves with the message output patterns, making it possible to link messages back to the set of users that sent them. In practice, however, with sufficiently many users and a high churn rate (e.g., 50% of users churn in each round) the effectiveness of such attacks can be mitigated. Other methods, such as buddy sets [113], can also be applied to reduce the effectiveness of intersection attacks.

Future work. The theoretical mixing analysis of Kwon [65] and our analysis in Section 5.5 assumes that the same number of envelopes are exchanged on each link, hence servers need to pad each envelope batch with dummy envelopes. One direction for improving the performance of Trellis would be to incorporate *different sized* batches into the mixing analysis and avoid adding dummy envelopes in Trellis (at the cost of increasing the number of layers). Doing so may reduce latency and improve the horizontal scalability of Trellis, as illustrated in Figure 10. Another avenue could be proving a tighter bound on the number of layers required for mixing. Additionally, eliminating the need for digital signatures (currently required for our blame protocols) would concretely improve efficiency of mixing by a small factor, which may have a cumulative impact when the number of layers becomes very large. For example, it is conceivable that message authentication codes (MACs) [12] could be suitable replacements to our digital signatures and result in smaller boomerang envelopes.

9 Conclusions

Scalability and support for large messages plays a key role in many applications. For example, web browsing requires megabytes of data transfer (the median web page in 2021 was 2MB in size [100]). Additionally, in the real world, scalability and concrete efficiency play an important role in *anonymity* as well: the more users there are, the larger the anonymity set, and the more plausible deniability each user obtains.

Trellis is the first system to support large messages and horizontal scalability, advancing the state-of-the-art for anonymous broadcast. Our prototype of Trellis is potentially efficient enough to deploy at a small "enterprise" scale with a few hundred thousand users. In such a deployment, Trellis would incur a few hours of overhead to broadcast megabyte-sized files (e.g., PDF documents). There is still an efficiency gap to bridge in terms of performance when compared to systems like Tor and I2P, which provide weaker anonymity guarantees but better performance. However, Trellis is a step toward closing this gap.

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A Onion and boomerang encryption

A.1 Onion encryption properties

Remark 3. We require that the onion encryption (and later boomerang encryption) provide Indistinguishability under Chosen-Ciphertext Attacks (IND-CCA security) [12]. While it is conceivable to require weaker security guarantees from the encryption (e.g., Indistinguishability under Chosen-*Plaintext* Attacks; IND-CPA security), our use of these primitives in Trellis requires IND-CCA security for efficient blame assignment.

Let $\mathcal{E} := (\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ be any symmetric-key encryption scheme and $\ell \geq 2$. For convenience, we define:

$$\begin{aligned}
\overrightarrow{\mathsf{onion}}.\mathsf{Enc}(\mathbf{sk}, m) &:= \mathsf{Enc}_{\mathsf{sk}_1}(\mathsf{Enc}_{\mathsf{sk}_2}(\ldots \mathsf{Enc}_{\mathsf{sk}_\ell}(m) \ldots)),\\
\overrightarrow{\mathsf{onion}}.\mathsf{Dec}(\mathbf{sk}, c) &:= \mathsf{Dec}_{\mathsf{sk}_1}(\mathsf{Dec}_{\mathsf{sk}_2}(\ldots \mathsf{Dec}_{\mathsf{sk}_\ell}(c) \ldots)),\\
\overleftarrow{\mathsf{onion}}.\mathsf{Enc}(\mathbf{sk}, m) &:= \mathsf{Enc}_{\mathsf{sk}_\ell}(\mathsf{Enc}_{\mathsf{sk}_{\ell-1}}(\ldots \mathsf{Enc}_{\mathsf{sk}_1}(m) \ldots)),\\
\overleftarrow{\mathsf{onion}}.\mathsf{Dec}(\mathbf{sk}, c) &:= \mathsf{Dec}_{\mathsf{sk}_\ell}(\mathsf{Dec}_{\mathsf{sk}_{\ell-1}}(\ldots \mathsf{Dec}_{\mathsf{sk}_1}(c) \ldots)),
\end{aligned}$$

where $\mathbf{sk} := (\mathsf{sk}_1, \ldots, \mathsf{sk}_\ell)$ are secret keys sampled i.i.d. according to KeyGen.

Onion encryption must satisfy *correctness* and *IND-CCA security*, defined as follows.

Correctness. For all security parameters λ, integers $\ell \geq 2$, and messages m in the message space,

$$\Pr\left[\begin{array}{l}\mathbf{sk} := (\mathsf{sk}_1, \ldots, \mathsf{sk}_\ell) \text{ where } \mathsf{sk}_i \leftarrow \mathcal{E}.\mathsf{KeyGen}(1^\lambda)\ \forall i \in \{1, \ldots, \ell\} :\\ \overrightarrow{\mathsf{onion}}.\mathsf{Dec}(\mathbf{sk}, \overrightarrow{\mathsf{onion}}.\mathsf{Enc}(\mathbf{sk}, m)) = \overleftarrow{\mathsf{onion}}.\mathsf{Dec}(\mathbf{sk}, \overleftarrow{\mathsf{onion}}.\mathsf{Enc}(\mathbf{sk}, m)) = m\end{array}\right] = 1,$$

where the probability is over the randomness of $\mathcal{E}.\mathsf{KeyGen}$, $\overrightarrow{\mathsf{onion}}.\mathsf{Enc}$, and $\overleftarrow{\mathsf{onion}}.\mathsf{Enc}$.

IND-CCA security. Let $\mathcal{E} := (\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ be an IND-CCA secure symmetric-key encryption scheme. For all integers $\ell \geq 2$, there exists a negligible function negl and security parameter λ such that for all PPT adversaries $\mathcal{A}$ it holds that:

$$\Pr\left[\ \textsc{OnionCCA}_{\mathsf{onion}, \mathcal{E}, \ell, \mathcal{A}}(\lambda) = \mathsf{yes}\ \right] \leq \frac{1}{2} + \mathsf{negl}(\lambda),$$

where $\textsc{OnionCCA}_{\mathsf{onion}, \mathcal{E}, \ell, \mathcal{A}}(\lambda)$ is defined in Figure 12 and $\mathsf{onion} \in \left\{\overrightarrow{\mathsf{onion}}, \overleftarrow{\mathsf{onion}}\right\}$. In words, the adversary can corrupt an arbitrary subset of keys in an attempt to win the CCA game. Additionally, when given the challenge ciphertext c_b, the adversary can continue to query for arbitrary decryptions provided it does not query for any envelopes present in the challenge c_b.

Theorem 1 (Correctness and security of onion encryption [17, 33] (informal))**.** Fix integer $\ell \geq 2$. If $\mathcal{E} := (\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ is a correct and IND-CCA-secure symmetric-key encryption scheme, then $\mathsf{onion} \in \left\{\overrightarrow{\mathsf{onion}}, \overleftarrow{\mathsf{onion}}\right\}$ is correct and IND-CCA secure (as defined in Figure 12).

Proof (sketch). Correctness follows immediately from the correctness of $\mathcal{E}$. The proof of IND-CCA security for onion encryption is folklore and follows from a straightforward reduction to the IND-CCA security of $\mathcal{E}$. If a PPT $\mathcal{A}$ wins the $\textsc{OnionCCA}_{\mathsf{onion}, \mathcal{E}, \ell, \mathcal{A}}(\lambda)$ game, then there exists a PPT $\mathcal{B}$ that wins the IND-CCA game for the underlying encryption scheme by simulating all but one of the encryption keys and answering queries issued by $\mathcal{A}$ by querying its own IND-CCA oracles for the one key it does not have. $\mathcal{B}$ outputs exactly as $\mathcal{A}$ does. Therefore, if the underlying encryption scheme is IND-CCA secure, then the IND-CCA security of onion encryption follows. □

A.2 Boomerang encryption

In this section, we formalize the properties required of boomerang encryption. Let $\overrightarrow{\mathsf{onion}}$ and $\overleftarrow{\mathsf{onion}}$ be as defined in Appendix A.1. For convenience, we define $\mathbf{sk} = (\overrightarrow{\mathbf{sk}}, \overleftarrow{\mathbf{sk}})$ and

$$\begin{aligned}
\mathsf{boomerang}.\mathsf{Enc}(\mathbf{sk}, \overrightarrow{m}, \overleftarrow{m}) &:= \overrightarrow{\mathsf{onion}}(\overrightarrow{\mathbf{sk}}, \overrightarrow{m} \parallel \overleftarrow{\mathsf{onion}}(\overleftarrow{\mathbf{sk}}, \overleftarrow{m})),\\
\mathsf{boomerang}.\mathsf{Dec}(\mathbf{sk}, c) &:= \overrightarrow{\mathsf{onion}}.\mathsf{Dec}(\overrightarrow{\mathbf{sk}}, c)[0] \parallel \overleftarrow{\mathsf{onion}}.\mathsf{Dec}(\overleftarrow{\mathbf{sk}}, \overrightarrow{\mathsf{onion}}.\mathsf{Dec}(\overrightarrow{\mathbf{sk}}, c)[1]),
\end{aligned}$$

where $\overrightarrow{\mathsf{onion}}.\mathsf{Dec}(\overrightarrow{\mathbf{sk}}, c)$ is parsed as a tuple $(\overrightarrow{m}, \overleftarrow{\mathsf{onion}}(\overleftarrow{\mathbf{sk}}, \overleftarrow{m}))$ so that $\overrightarrow{\mathsf{onion}}.\mathsf{Dec}(\overrightarrow{\mathbf{sk}}, c)[0] = \overrightarrow{m}$ and $\overrightarrow{\mathsf{onion}}.\mathsf{Dec}(\overrightarrow{\mathbf{sk}}, c)[1] = \overleftarrow{\mathsf{onion}}(\overleftarrow{\mathbf{sk}}, \overleftarrow{m})$.

```
Game $\textsc{OnionCCA}_{\mathsf{onion},\mathcal{E},\ell,\mathcal{A}}(\lambda)$
  for $i \in \{1,\dots,\ell\}$ do
      $\mathsf{sk}_i \leftarrow \mathcal{E}.\mathsf{KeyGen}(1^\lambda)$
  $\mathbf{sk} := (\mathsf{sk}_1,\dots,\mathsf{sk}_\ell)$
  $C := \{\}$
  $(\mathsf{st}, m_0, m_1) \leftarrow \mathcal{A}^{\mathsf{Corrupt},\mathsf{Enc},\mathsf{Dec}}(1^\lambda)$
  $c_0 \leftarrow \mathsf{onion.Enc}(\mathbf{sk}, m_0)$;  $c_1 \leftarrow \mathsf{onion.Enc}(\mathbf{sk}, m_1)$
  $b \leftarrow_R \{0,1\}$
  $b' \leftarrow \mathcal{A}^{\mathsf{Corrupt},\mathsf{Enc},\mathsf{Dec}}(\mathsf{st}, c_b)$
  return $(\{1,\dots,\ell\} \setminus C \neq \emptyset$ and $b' = b)$
```

```
Oracle $\mathsf{Corrupt}(j)$
  $C := C \cup \{j\}$
  return $\mathsf{sk}_j$
```

```
Oracle $\mathsf{Enc}(j, m; r)$
  // encrypt with randomness $r$
  $c \leftarrow \mathcal{E}.\mathsf{Enc}_{\mathsf{sk}_j}(m; r)$
  return $c$
```

```
Oracle $\mathsf{Dec}(j, c)$
  $c_0 := c_b$  // challenge ciphertext
  for $i \in \{0,\dots,\ell-1\}$ do
      if $\mathsf{onion} = \overrightarrow{\mathsf{onion}}$ then
          $c_{i+1} \leftarrow \mathcal{E}.\mathsf{Dec}_{\mathsf{sk}_{i+1}}(c_i)$
      else
          $c_{i+1} \leftarrow \mathcal{E}.\mathsf{Dec}_{\mathsf{sk}_{\ell-i}}(c_i)$
  if $c \in \{c_0,\dots,c_\ell\}$ then return $\perp$
  else return $\mathcal{E}.\mathsf{Dec}_{\mathsf{sk}_j}(c)$
```

Figure 12: Onion encryption IND-CCA security game.

Remark 4. For notational convenience, in our proofs we will denote $\mathbf{sk}$ as a vector of 2ℓ keys $(\mathsf{sk}_1,\dots,\mathsf{sk}_{2\ell})$ instead of $(\overrightarrow{\mathbf{sk}}, \overleftarrow{\mathbf{sk}})$.

Boomerang encryption must satisfy *correctness* and IND-CCA security, defined as follows.

Correctness. Let $\mathcal{E} := (\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ be a IND-CCA secure symmetric key encryption scheme. For all security parameters λ, integers $\ell \geq 2$, and pairs of messages $(\overrightarrow{m}, \overleftarrow{m})$ in the message space,

$$\Pr\left[\begin{array}{l} \mathbf{sk} := (\mathsf{sk}_1,\dots,\mathsf{sk}_{2\ell}) \text{ where } \mathsf{sk}_i \leftarrow \mathcal{E}.\mathsf{KeyGen}(1^\lambda)\ \forall i \in \{1,\dots,2\ell\}: \\ \mathsf{boomerang.Dec}(\mathbf{sk}, \mathsf{boomerang.Enc}(\mathbf{sk}, \overrightarrow{m}, \overleftarrow{m})) = \overrightarrow{m} \| \overleftarrow{m} \end{array}\right] = 1,$$

where the probability is over the randomness of KeyGen and $\mathsf{boomerang.Enc}$.

IND-CCA security. Let $\mathcal{E} := (\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ be a IND-CCA secure encryption scheme. For all integers $\ell \geq 2$, there exists a negligible function negl and security parameter λ such that for all PPT adversaries $\mathcal{A}$ it holds that:

$$\Pr\left[\ \textsc{BoomerangCCA}_{\mathsf{boomerang},\mathcal{E},\ell,\mathcal{A}}(\lambda) = \mathsf{yes}\ \right] \leq \frac{1}{2} + \mathsf{negl}(\lambda),$$

where $\textsc{BoomerangCCA}_{\mathsf{boomerang},\mathcal{E},\ell,\mathcal{A}}(\lambda)$ is defined in Figure 13.

Theorem 2 (Security of boomerang encryption). Fix integer $\ell \geq 2$. If $\mathsf{onion} \in \left\{\overrightarrow{\mathsf{onion}}, \overleftarrow{\mathsf{onion}}\right\}$ meets the correctness and IND-CCA security properties of onion encryption, then the boomerang encryption scheme is correct and IND-CCA secure (as defined in Figure 13).

Proof (sketch). Suppose, towards contradiction, that there exists a PPT $\mathcal{A}$ that wins the IND-CCA security game of boomerang encryption with non-negligible advantage $\delta(\lambda)$. Construct a PPT $\mathcal{B}$ that wins the onion IND-CCA security game with the same advantage. On input (1^λ), $\mathcal{B}$ samples ℓ keys according to KeyGen and either sets these keys as the keys for $\overrightarrow{\mathsf{onion}}$ or $\overleftarrow{\mathsf{onion}}$ (either is without loss of generality). At this point, $\mathsf{boomerang}$ is defined over 2ℓ keys, of which $\mathcal{B}$ only knows ℓ. $\mathcal{B}$ runs $\mathcal{A}$ on (1^λ) and answers all $\mathsf{Corrupt}$, Enc, and Dec queries as follows. If queried on an index for which it knows the secret key sk_j, $\mathcal{B}$ answers the query accordingly; otherwise, it queries the $\mathsf{Corrupt}$, Enc, or Dec oracle for onion encryption and responds as they do. Upon receiving $(\mathsf{st}, \overrightarrow{m}_0, \overrightarrow{m}_1, \overleftarrow{m}_0, \overleftarrow{m}_1)$, $\mathcal{B}$ outputs $(\mathsf{st}, \overrightarrow{m}_0 \parallel \overleftarrow{\mathsf{onion}}(\mathbf{sk}, \overleftarrow{m}_0), \overrightarrow{m}_1 \parallel \overleftarrow{\mathsf{onion}}(\mathbf{sk}, \overleftarrow{m}_1))$ (by querying the Enc oracle as needed

```
Game $\textsc{BoomerangCCA}_{\mathsf{boomerang},\mathcal{E},\ell,\mathcal{A}}(\lambda)$
  for $i \in \{1,\ldots,2\ell\}$ do
    $\mathsf{sk}_i \leftarrow \mathcal{E}.\mathsf{KeyGen}(1^\lambda)$
  $\mathbf{sk} := (\mathsf{sk}_1,\ldots,\mathsf{sk}_{2\ell})$
  $C := \{\}$
  $(\mathsf{st},\overrightarrow{m}_0,\overrightarrow{m}_1,\overleftarrow{m}_0,\overleftarrow{m}_1) \leftarrow \mathcal{A}^{\mathsf{Corrupt},\mathsf{Enc},\mathsf{Dec}}(1^\lambda)$
  $c_0 \leftarrow \mathsf{boomerang.Enc}(\mathbf{sk},\overrightarrow{m}_0,\overleftarrow{m}_0)$
  $c_1 \leftarrow \mathsf{boomerang.Enc}(\mathbf{sk},\overrightarrow{m}_1,\overleftarrow{m}_1)$
  $b \leftarrow_R \{0,1\}$
  $b' \leftarrow \mathcal{A}^{\mathsf{Corrupt},\mathsf{Enc},\mathsf{Dec}}(\mathsf{st},c_b)$
  if $\ell \in C$ and $\overrightarrow{m}_0 \neq \overrightarrow{m}_1$ then
    return no // trivial edge case 1
  if $2\ell \in C$ and $\overleftarrow{m}_0 \neq \overleftarrow{m}_1$ then
    return no // trivial edge case 2
  return $(\{1,\ldots,2\ell\} \setminus C \neq \emptyset$ and $b' = b)$
```

```
Oracle $\mathsf{Corrupt}(j)$
  $C := C \cup \{j\}$
  return $\mathsf{sk}_j$

Oracle $\mathsf{Enc}(j,m;r)$
  // encrypt with randomness $r$
  $c \leftarrow \mathcal{E}.\mathsf{Enc}_{\mathsf{sk}_j}(m;r)$
  return $c$

Oracle $\mathsf{Dec}(j,c)$
  $c_0 := c_b$ // challenge ciphertext
  for $i \in \{0,\ldots,\ell-1\}$ do
    $c_{i+1} \leftarrow \mathcal{E}.\mathsf{Dec}_{\mathsf{sk}_{i+1}}(c_i)$
  for $i \in \{0,\ldots,\ell-1\}$ do
    $c_{\ell+i+1} \leftarrow \mathcal{E}.\mathsf{Dec}_{\mathsf{sk}_{\ell-1}}(c_{\ell+i})$
  if $c \in \{c_0,\ldots,c_{2\ell}\}$ then
    return $\perp$
  else return $\mathcal{E}.\mathsf{Dec}_{\mathsf{sk}_j}(c)$
```

Figure 13: Boomerang encryption IND-CCA security game.

to generate the onion encryption). Upon receiving challenge (st, c_b), $\mathcal{B}$ runs $\mathcal{A}$ on input (st, c_b). $\mathcal{B}$ answers Enc and Dec queries as before, but outputs $\perp$ when queried on any challenge ciphertext ($\mathcal{B}$ can check the necessary condition by using the secret keys it knows and querying the Dec oracle). Finally, $\mathcal{B}$ outputs as $\mathcal{A}$ does. It is easy to check that $\mathcal{B}$ succeeds whenever $\mathcal{A}$ succeeds, contradicting the assumption that $\mathsf{onion} \in \left\{\overrightarrow{\mathsf{onion}}, \overleftarrow{\mathsf{onion}}\right\}$ meets the IND-CCA security property of onion encryption. □

A.3 Boomerang routing and proof-of-delivery

In Trellis, boomerang encryption is used to route a messages through a chain of ℓ servers (and back). Here, we formalize the key property of boomerang routing that we used in Trellis: proof of delivery.

Lemma 1 (Boomerang decryption requires all keys)**.** *If the* boomerang *scheme satisfies the correctness and IND-CCA security definitions of boomerang encryption, then a successful boomerang decryption of a ciphertext implies that each of the* 2ℓ *secret keys were used to decrypt the boomerang ciphertext.*

Proof. If a secret key was *not* used in the decryption process, then it is possible to construct an adversary $\mathcal{A}$ that can decrypt a layer in the nested ciphertext *without* the secret key for that layer. This immediately gives rise to an adversary that wins the $\textsc{BoomerangCCA}_{\mathsf{boomerang},\mathcal{E},\ell,\mathcal{A}}(\lambda)$ game (Figure 13), contradicting the IND-CCA security of the boomerang encryption scheme. □

Claim 8 (Boomerang decryption as proof-of-delivery)**.** Let $r \in \{0,1\}^\kappa$ be a random secret nonce known only to the encryptor where κ is a statistical security parameter. A successful boomerang decryption consisting of $\overleftarrow{m} = r$ given a boomerang encryption of $(\overrightarrow{m} \in \{0,1\}^*,\ \overleftarrow{m} = r)$ implies that $\overrightarrow{m}$ was decrypted using secret key sk_ℓ. In boomerang routing, this implies that the ℓth server decrypted (and thus received) $\overrightarrow{m}$, with probability at least $1 - 2^{-\kappa}$.

Proof. Suppose the claim is false. Then, either (1) the secret key sk_ℓ was not used which contradicts Lemma 1 or (2) a *different* (potentially shorter) boomerang encryption of r was generated, without using sk_ℓ. However, because r is random and kept secret by the decryptor, this can only happen with probability $2^{-\kappa}$. □

B Anonymous routing tokens

B.1 ART properties

We present the syntax of ARTs in Definition 1. Here, we present the formal properties we require from ARTs.

Notation. We define the Setup algorithm to output a $\mathsf{crs} := (\mathbb{Z}_p, \mathbb{G}, g)$ where $\mathbb{G}$ is a group of prime order p with generator g; we assume that the prime p is proportional to the security parameter λ. We let $s \geq 1$ be the number of signers holding partial token signing keys. Finally, for a fixed integer $N \geq 2$ we let $\mathbb{S}$ be the set of N server identifiers.

Modeling the signing protocol. To simplify our analysis, we will model the signing protocol as a triple of (non-interactive) algorithms. This model captures one-round protocols instantiated between the user and the signers and corresponds to the three steps in the construction described in Section 5.1. For convenience, we let $\mathcal{P}$ be a distribution over $(S_\ell, A_\ell, \mathsf{vk}_\ell)$ where S_ℓ is the ℓth server identity, A_ℓ is a random Diffie-Hellman message, and vk_ℓ is a signature verification key. We denote by $\mathsf{payload}$ a random sample from $\mathcal{P}$.

- $\mathsf{User}_0(\mathsf{tvk}, \mathsf{payload}_\ell) \to (\mathsf{st}_\ell, T_\ell)$: takes as input a token payload $\mathsf{payload}_\ell$; outputs secret user state st and blind token T_ℓ.
- $\mathsf{PartSign}(\mathsf{tvk}, x_i, T_\ell) \to W_{\ell,i}$: takes as input a partial signing key x_i and blind token T_ℓ; outputs a partial signature $W_{\ell,i}$.
- $\mathsf{User}_1(\mathsf{st}_\ell, W_{\ell,1}, \ldots, W_{\ell,s}) \to \mathsf{t}_\ell$: takes as input the secret user state st_ℓ and s partial signatures; outputs signed token t_ℓ.

Completeness. An honest user engaging with honest signers obtains a valid token which makes Verify output yes. Formally, for all security parameters λ and all payloads $\mathsf{payload}_\ell$, it holds that:

$$\Pr\left[\begin{array}{l} \mathsf{crs} \leftarrow \mathsf{Setup}(1^\lambda); \\ (\mathsf{tvk}, x_1, \ldots, x_s) \leftarrow \mathsf{KeyGen}(\mathsf{crs}); \\ (\mathsf{st}_\ell, T_\ell) \leftarrow \mathsf{User}_0(\mathsf{tvk}, \mathsf{payload}_\ell); \\ W_{\ell,i} \leftarrow \mathsf{PartSign}(\mathsf{tvk}, x_i, T_\ell)\ \forall i \in \{1, \ldots, s\}; \\ \mathsf{t}_\ell \leftarrow \mathsf{User}_1(\mathsf{st}_\ell, W_{\ell,1}, \ldots, W_{\ell,s}); \\ S_{\ell+1} \leftarrow \mathsf{NextServer}(\mathsf{tvk}, \mathsf{t}_\ell) \end{array} : \mathsf{Verify}(\mathsf{tvk}, S_{\ell+1}, \mathsf{payload}_\ell, \mathsf{t}_\ell) = \mathsf{yes}\right] = 1,$$

where the probability is over the randomness of Setup, KeyGen, User_0, $\mathsf{PartSign}$, and User_1.

Unforgeability. No (strict) subset of signers and/or user should be able to generate valid tokens that are accepted by Verify, even after seeing $\ell - 1$ valid tokens. Formally, an ART scheme is said to be *one-more unforgeable* if there exists a negligible function negl such that for all PPT adversaries $\mathcal{A}$, and any $\ell \geq 1$:

$$\Pr\left[\ \textsc{Omu}_{\mathsf{ART},\mathcal{A},\ell}(\lambda) = \mathsf{yes}\ \right] \leq \mathsf{negl}(\lambda),$$

where $\textsc{Omu}_{\mathsf{ART},\mathcal{A},\ell}(\lambda)$ is defined in Figure 14. In words, the adversary's task is to forge a fresh token after seeing $\ell - 1$ signed tokens of its choosing, without corrupting all the signing keys.

```
Game $\textsc{Omu}_{\mathsf{ART},\mathcal{A},\ell}(\lambda)$
  $\mathsf{crs} \leftarrow \mathsf{Setup}(1^\lambda)$
  $(\mathsf{tvk}, x_1, \ldots, x_s) \leftarrow \mathsf{ART.KeyGen}(\mathsf{crs})$
  $q_1, \ldots, q_s := 0;\ C := \{\}$
  $\{(\_, \mathsf{payload}_i, \mathsf{t}_i) \mid 1 \leq i \leq \ell\} \leftarrow \mathcal{A}^{\mathsf{Corrupt},\mathsf{PartSign},\mathsf{Verify}}(\mathsf{crs}, \mathsf{tvk})$
  for $i \in \{1, \ldots, \ell\}$ do
      $S_{i+1} \leftarrow \mathsf{ART.NextServer}(\mathsf{t}_i)$
  $\tilde{C} := \{1, \ldots, s\} \setminus C$
  return ($\tilde{C} \neq \emptyset$ and $\forall i \in \tilde{C}\ \ q_i \leq \ell - 1$ and
              $\forall i \neq j \in \{1, \ldots, \ell\}\ \ \mathsf{t}_i \neq \mathsf{t}_j$ and
              $\forall i \in \{1, \ldots, \ell\}\ \ \mathsf{ART.Verify}(\mathsf{tvk}, S_{i+1}, \mathsf{payload}_i, \mathsf{t}_i) = \mathsf{yes}$)
```

```
Oracle $\mathsf{Corrupt}(j)$
  $C := C \cup \{j\}$
  return $x_j$
```

```
Oracle $\mathsf{PartSign}(j, T_i)$
  $q_j := q_j + 1$
  $W_{i,j} \leftarrow \mathsf{ART.PartSign}(\mathsf{tvk}, x_j, T_i)$
  return $W_{i,j}$
```

```
Oracle $\mathsf{Verify}(S_{j+1}, \mathsf{payload}_j, \mathsf{t}_j)$
  return $\mathsf{ART.Verify}(\mathsf{tvk}, S_{j+1}, \mathsf{payload}_j, \mathsf{t}_j)$
```

Figure 14: One-more unforgeability game for anonymous routing tokens (ARTs).

Unlinkability. Each token produced through the Sign protocol should be unlinkable to the inputs given to the signers (even if all signers collude, which we model as the adversary choosing secret keys after being given the crs). Formally, an ART scheme is said to be unlinkable if there exists a negligible function negl such that for all PPT adversaries $\mathcal{A}$, and any $\ell \geq 2$:

$$\Pr\left[\ \textsc{Unlink}_{\mathsf{ART},\mathcal{P},\mathcal{A},\ell(\lambda)} = \mathsf{yes}\ \right] \leq \frac{1}{\ell} + \mathsf{negl}(\lambda),$$

where $\textsc{Unlink}_{\mathsf{ART},\mathcal{P},\mathcal{A},\ell(\lambda)}$ is defined in Figure 15.

```
Game $\textsc{Unlink}_{\mathsf{ART},\mathcal{P},\mathcal{A},\ell(\lambda)}$
 1 :  $\mathsf{crs} \leftarrow \mathsf{ART.Setup}(1^\lambda)$
 2 :  $(\mathsf{st}, \mathsf{tvk}, x_1, \ldots, x_s) \leftarrow \mathcal{A}(\mathsf{crs})$
 3 :  $q_0 := 0;\quad q_1 := 0;\quad Q := \{\}$
 4 :  $(\mathsf{st}, T_1, \ldots, T_\ell) \leftarrow \mathcal{A}^{\mathsf{User}_0, \mathsf{User}_1}(\mathsf{st})$
 5 :  if $Q = \emptyset$ then return $\mathsf{no}$
 6 :  for $i \in \{1, \ldots, \ell\}$ do
 7 :    for $j \in \{1, \ldots, s\}$ do
 8 :      $W_{i,j} \leftarrow \mathsf{ART.PartSign}(x_j, T_j)$
 9 :    $\mathsf{t}_i \leftarrow \mathsf{ART.User}_1(\hat{\mathsf{st}}_i, W_{i,1}, \ldots, W_{i,s})$
10 :  $j \leftarrow_R Q$
11 :  $j' \leftarrow \mathcal{A}(\mathsf{st}, (\mathsf{payload}_j, \mathsf{t}_j))$
12 :  return $j' = j$
```

```
Oracle $\mathsf{User}_0()$
 1 :  $q_0 := q_0 + 1$
 2 :  $\mathsf{payload}_{q_0} \leftarrow_R \mathcal{P}$
 3 :  $(\hat{\mathsf{st}}_{q_0}, T_{q_0}) \leftarrow \mathsf{ART.User}_0(\mathsf{tvk}, \mathsf{payload}_{q_0})$
 4 :  $Q := Q \cup \{q_0\}$
 5 :  return $T_{q_0}$
```

```
Oracle $\mathsf{User}_1(j, W_1, \ldots, W_s)$
 1 :  if $j \notin Q$ then return $\perp$
 2 :  $\mathsf{t}_j \leftarrow \mathsf{ART.User}_1(\hat{\mathsf{st}}_j, W_1, \ldots, W_s)$
 3 :  $S_{j+1} \leftarrow \mathsf{ART.NextServer}(\mathsf{t}_j)$
 4 :  if $\mathsf{Verify}(\mathsf{tvk}_j, S_{j+1}, \mathsf{payload}_j, \mathsf{t}_j) = \mathsf{yes}$ then
 5 :    $Q := Q \setminus \{j\}$
 6 :    $q_1 := q_1 + 1$
 7 :  return $(\mathsf{payload}_j, \mathsf{t}_j)$
```

Figure 15: Unlinkability game for the anonymous routing tokens (ARTs).

Unpredictability. The output of $\mathsf{NextServer}$ should be unpredictable given only the payload $\mathsf{payload}_\ell$. Formally, the ART scheme is said to generate unpredictable links if there exists a negligible function negl such that for all $S_\ell \in \mathbb{S}$ and all $S^* \in \mathbb{S}$,

$$\Pr\left[\begin{array}{l} \mathsf{crs} \leftarrow \mathsf{Setup}(1^\lambda); \\ (\mathsf{tvk}, x_1, \ldots, x_s) \leftarrow \mathsf{KeyGen}(\mathsf{crs}); \\ (\mathsf{st}_\ell, T_\ell) \leftarrow \mathsf{User}_0(\mathsf{tvk}, S_\ell); \\ W_{\ell,i} \leftarrow \mathsf{PartSign}(\mathsf{tvk}, x_i, T_\ell)\ \forall i \in \{1, \ldots, s\}; \\ \mathsf{t}_\ell \leftarrow \mathsf{User}_1(\mathsf{st}_\ell, W_{\ell,1}, \ldots, W_{\ell,s}); \\ S_{\ell+1} \leftarrow \mathsf{NextServer}(\mathsf{tvk}, \mathsf{t}_\ell) \end{array} : S_{\ell+1} = S^* \right] = \frac{1}{|\mathbb{S}|} - \mathsf{negl}(\lambda),$$

where the probability is over the randomness of Setup, KeyGen, User_0, $\mathsf{PartSign}$, and User_1.

B.2 ART security analysis

Chosen-target Gap Diffie-Hellman (CTGDH). We prove the *unforgeability* property of our ART tokens under the CTGDH assumption, following prior work on anonymous tokens [59]. Informally, the assumption is related to the computational Diffie-Hellman (CDH) assumption in *gap* groups where *decisional* Diffie-Hellman (DDH) is easy but the CDH assumption is assumed to be computationally intractable. Specifically, certain bilinear groups (such as the ones we use to construct ARTs) have an efficiently computable pairing, which makes the decisional variant of Diffie-Hellman easy [13]. In these groups, the gap Diffie-Hellman (GDH) assumption is the bilinear counterpart to the classic CDH assumption. The chosen-target variant of GDH, is introduced by Kreuter et al. [59] to capture a setting where the adversary can compute $\ell - 1$ instances of CDH with the help of an oracle *and* an oracle for DDH. The adversary is tasked with generating ℓ CDH instances (one more than it queried for, hence *one-more unforgeability*). This assumption is rather natural (see Kreuter et al. [59, Section 2.1 & Appendix A.2] for details and how it relates to prior work) and makes our analysis straightforward. We describe the CTGDH security game in Figure 16 and refer the reader to Kreuter et al. [59] for additional information.

Assumption 1 (CTGDH [59])**.** Let Setup be a group generator algorithm that, given a security parameter 1^λ, outputs a group $\mathbb{G}$ of prime order p where $\lambda = \lceil \log_2 p \rceil$ and a nothing-up-my-sleeve generator[4] g of $\mathbb{G}$. CTGDH is said to hold if there exists a security parameter λ and negligible function negl such that for all PPT $\mathcal{A}$ and any $\ell \geq 1$:

$$\Pr\left[\ \textsc{ctgdh}_{\mathsf{Setup},\mathcal{A},\ell}(\lambda) = \mathsf{yes}\ \right] \leq \mathsf{negl}(\lambda),$$

where $\textsc{ctgdh}_{\mathsf{Setup},\mathcal{A},\ell}(\lambda)$ is defined in Figure 16.

```
Game $\textsc{ctgdh}_{\mathsf{Setup},\mathcal{A},\ell}(\lambda)$
1:  $(\mathbb{G}, p, g) \leftarrow \mathsf{Setup}(1^\lambda)$
2:  $x \leftarrow_R \mathbb{Z}_p$;  $\mathsf{pk} \leftarrow g^x$
3:  $q := 0$;  $Q := []$
4:  $\{(w_i, z_i) \mid 1 \leq i \leq \ell\} \leftarrow \mathcal{A}^{\mathsf{Target},\mathsf{Help},\mathsf{DDH}}((\mathbb{G}, p, g), \mathsf{pk})$
5:  for $i \in \{1, \ldots, \ell\}$ do
6:    if $w_i \notin Q$ then return no
7:    $y_i := Q[w_i]$
8:  return ($q \leq \ell - 1$ and
9:          $\forall i \neq j \in \{1, \ldots, \ell\}$  $w_i \neq w_j$ and
10:         $\forall i \in \{1, \ldots, \ell\}$  $(y_i)^x = z_i$)
```

```
Oracle Target($w$)
1:  if $w \in Q$ then
2:    $y := Q[w]$
3:  else
4:    $y \leftarrow_R \mathbb{G}$
5:    $Q[w] := y$
6:  return $y$
```

```
Oracle Help($y$)
1:  $q := q + 1$
2:  return $y^x$
```

```
Oracle DDH($y, z$)
1:  return $y = z^x$
```

Figure 16: The Chosen-target gap Diffie–Hellman game [59].

Theorem 3 (ART Completeness)**.** The ART construction presented in Section 5.1 satisfies the completeness property of Appendix B.1.

Proof. Consider any $S_\ell \in \mathbb{S}$. The output of User_0 is $T_\ell := \mathsf{Hash}_{\mathbb{G}}(\mathsf{payload})^{1/r}$ where r is random in $\mathbb{Z}_p$. The output of $\mathsf{PartSign}$ is then $W_{\ell,i} := T_\ell^{x_i}$ and so it follows that $\mathsf{t}_\ell = \prod_{i=1}^{s} W_{\ell,i} = \prod_{i=1}^{s} T_i^{x_i} = \left(\prod_{i=1}^{s} T_i\right)^x = \mathsf{Hash}_{\mathbb{G}}(\mathsf{payload})^{x/r}$. Given this, the output of User_1 is t_ℓ. By the above, we have that $\mathsf{t}_\ell = \left(\mathsf{Hash}_{\mathbb{G}}(\mathsf{payload})^{x/r}\right)^r = \mathsf{Hash}_{\mathbb{G}}(\mathsf{payload})^x$. It follows that $e(\mathsf{t}_\ell, g) = \mathsf{Hash}_{\mathbb{G}}(\mathsf{payload})^x \in \mathbb{G}_T$ and $e(y_\ell, g^x) = \mathsf{Hash}_{\mathbb{G}}(\mathsf{payload})^x \in \mathbb{G}_T$, only when $y_\ell := \mathsf{Hash}_{\mathbb{G}}(\mathsf{payload})$. As such, we have that the following two conditions are satisfied:

(1) $e(\mathsf{t}_\ell, g) = e(y_\ell, g^x)$ and
(2) $S'_{\ell+1} = \mathsf{Hash}_{\mathbb{Z}_p}(\mathsf{t}) = S_{\ell+1}$.

Together, (1) and (2) imply that Verify outputs yes, as required. □

Theorem 4 (ART Unforgeability)**.** The ART construction presented in Section 5.1 satisfies the unforgeability property of Appendix B.1 under the CTGDH assumption.

Proof. Suppose that $\mathcal{A}$ wins the $\textsc{Omu}_{\mathsf{ART},\mathcal{A},\ell}(\lambda)$ game with probability $\delta(\lambda)$, for some non-negligible function δ. Construct an adversary $\mathcal{B}$ that wins the $\textsc{ctgdh}_{\mathsf{Setup},\mathcal{A},\ell}(\lambda)$ with probability $\frac{\delta(\lambda)}{s}$, where s is the number of signers. $\mathcal{B}$ works as follows.

1. On input $(\mathbb{Z}_p, \mathbb{G}, g)$ and $\mathsf{pk} := g^x$, set $\mathsf{crs} := (\mathbb{Z}_p, \mathbb{G}, g)$.
2. Sample $j^* \leftarrow_R \{1, \ldots, s\}, x_1, \ldots, x_s \leftarrow_R \mathbb{Z}_p$ and set $\mathsf{pk}' = g^x \cdot \prod_{i=1, i \neq j^*}^{s} g^{x_i}$.
3. Run $\mathcal{A}(\mathsf{crs}, \mathsf{pk}')$ and answer $\mathsf{Hash}_{\mathbb{G}}$, $\mathsf{Corrupt}$, $\mathsf{PartSign}$, and Verify queries as follows.
   - For each query to $\mathsf{Hash}_{\mathbb{G}}$, query the Target oracle and respond as it does.
   - For each $\mathsf{Corrupt}$ query of the form j:
     - If $j = j^*$ then **abort**,
     - Else respond with x_j.

[4]In practice, this simply means that Setup must be computed by a trusted setup process to avoid having any party unfairly choosing the generator.

- For each PartSign query of the form (j, T_i):
  - If $j = j^*$ then query the Help oracle on input T_i to get $W_{i,j^*} := (T_i)^x$,
  - Else set $W_{i,j} := (T_i)^{x_j}$.
  - Respond with $W_{i,j}$.
- For each Verify query of the form $(S_{j+1}, \mathsf{payload}_j, \mathsf{t}_j)$:
  - Query the Target oracle on input $\mathsf{payload}_j$ to get y.
  - Query the DDH oracle on input (t_j, g) and (y, g^x) to get answers a_0 and a_1, respectively.
  - If $a_0 \neq a_1$ return no,
  - Else query the random oracle on input t_j to get S'_{j+1}.
  - If $S'_{j+1} \neq S_{j+1}$ return no.
  - Else return yes.

4. Obtain from $\mathcal{A}$ the output $(\_, \mathsf{payload}_i, \mathsf{t}_i)$ for $i \in \{1, \ldots, \ell\}$.
5. Set $w_i := \mathsf{payload}_i$ for $i \in \{1, \ldots, \ell\}$.
6. Compute $\Delta := \sum_{i=1, i \neq j^*}^{s} x_i$.
7. Set $z_i := \mathsf{t}_i g^{-\Delta}$.
8. Output $\{(w_i, z_i) \mid 1 \leq i \leq \ell\}$.

We now analyze the reduction and argue why $\mathcal{B}$ succeeds with probability $\frac{\delta(\lambda)}{s}$. First, observe that pk' is a uniformly random element of $\mathbb{G}$ and hence matches the distribution expected by $\mathcal{A}$. Similarly, each partial signing key, x_i is a uniformly random element of $\mathbb{Z}_p$ and hence also distributed identically. The responses to the PartSign and Verify queries are distributed identically to the distribution expected by $\mathcal{A}$ in the $\textsc{Omu}_{\mathsf{ART},\mathcal{A},\ell}(\lambda)$ game. As such, if $\mathcal{A}$ wins the $\textsc{Omu}_{\mathsf{ART},\mathcal{A},\ell}(\lambda)$ game, it must output ℓ tuples of the form $(\_, \mathsf{payload}_i, \mathsf{t}_i)$ such that:

1. the total queries to PartSign for $j = j^*$ was at most $\ell - 1$,
2. the total unique queries to Corrupt was at most $s - 1$,
3. $\forall i \neq j \;\; \mathsf{t}_i \neq \mathsf{t}_j$, and
4. $\forall i \;\; \mathsf{ART.Verify}(\mathsf{tvk}, S_{j+1}, \mathsf{payload}_i, \mathsf{t}_i) = \mathsf{yes}$.

Because the total number of queries to PartSign for $j = j^*$ was at most $\ell - 1$, $\mathcal{B}$ queries the Help oracle at most $\ell - 1$ times. Because the total unique queries to Corrupt was at most $s - 1$, the probability that $\mathcal{A}$ doesn't query j^* is at least $\frac{1}{s}$. By the last two properties (and the construction of ART.Verify), it holds that each t_i is unique and, moreover, $\mathsf{t}_i = \mathsf{Hash}_{\mathbb{G}}(\mathsf{payload}_i)^x$. It follows that $z_i = (y_i)^x$, where $y_i = Q[w_i] = Q[\mathsf{payload}]$ is as defined in Figure 16. In sum, we get that $\forall i \neq j \;\; w_i \neq t_j$ and $\forall i \;\; (y_i)^x = z_i$. These conditions make $\mathcal{B}$ win with the same probability as $\mathcal{A}$, *provided that $\mathcal{B}$ doesn't abort*. The probability that $\mathcal{B}$ aborts is entirely conditioned on $\mathcal{A}$ querying Corrupt on index j^* (for which $\mathcal{B}$ doesn't have the secret key). Because j^* is chosen at random, the probability that $\mathcal{B}$ aborts is at most $\frac{1}{s}$ ($\mathcal{A}$ queries at most $s - 1$ keys). As such, the probability that $\mathcal{B}$ wins the $\textsc{ctgdh}_{\mathsf{Setup},\mathcal{A},\ell}(\lambda)$ game is $\frac{\delta(\lambda)}{s}$, concluding the proof.

□

Theorem 5 (ART Unlinkability)**.** The ART construction presented in Section 5.1 unconditionally satisfies the unlinkability property described above.

Proof. Each blind token T_i output by User_0 is of the form $\mathsf{Hash}_{\mathbb{G}}(\mathsf{payload})^{\frac{1}{r}}$. Because $r \in \mathbb{Z}_p$ is random and $\mathsf{Hash}_{\mathbb{G}}$ is a random oracle, it follows that T_i is uniformly random in $\mathbb{G}$. The output of User_1 is t_i where $\mathsf{t}_i = ((T_i)^x)^r = ((T_i)^r)^x = \mathsf{Hash}_{\mathbb{G}}(\mathsf{payload})^x$ (recall that the user verifies that the token is computed correctly with respect to tvk using Verify, which prevents "tagging" the token with a maliciously computed $x^* \neq x$ [29]). As such, t_i is independent of T_i and well-formed if the user accepts. Finally, the payload $\mathsf{payload}_i$ is chosen independently of r and hence is also independent of T_i. This means that the outputs of User_0 and User_1 are also independent of each other. It then follows that because $\mathcal{A}$ does not query User_1 on at least one index j in $\textsc{Unlink}_{\mathsf{ART},\mathcal{P},\mathcal{A},\ell(\lambda)}$ (otherwise $Q = \emptyset$) and because $(\mathsf{payload}_j, \mathsf{t}_j)$ is only revealed by the User_1 oracle (which we know $\mathcal{A}$ did not query for the chosen challenge), $\mathcal{A}$ has no advantage in correctly guessing j. Thus, unlinkability holds unconditionally. □

Theorem 6 (ART Unpredictability)**.** If $\mathsf{Hash}_{\mathbb{G}}$ and $\mathsf{Hash}_{\mathbb{Z}_N}$ are hash functions modeled as random oracles, then the ART construction presented in Section 5.1 satisfies the unpredictability property described above.

Proof. Each signature t_i is uniformly random due to $\mathsf{Hash}_{\mathbb{G}}$ and unpredictable due to the signing process (without knowledge of x it is not possible to generate t_i by the one-more unforgeability property). In turn, $S_{i+1} := \mathsf{Hash}_{\mathbb{Z}_N}(\mathsf{t}_i)$ is uniformly random over $\mathbb{Z}_N$ and cannot be predicted by any subset of colluding signers or the user. □

C Full analysis for the number of layers required

In this section, we provide the full analysis for the number of layers required to achieve a random permutation on the messages with variation distance at most ϵ, for any ϵ (e.g., $\epsilon < 2^{-64}$).

Theorem 7. Let M be the number of messages being shuffled (equivalent to the number of users) and $N \geq 2$ be the number of servers in the network. Fix $0 \leq f < 1$ to be a fraction of corrupted servers and let $\epsilon < 1$ be a statistical security parameter. If each message is routed through a random server L times and it holds that:

$$L \geq \left\lceil \frac{-\log\epsilon + \log M + \log(2\sqrt{4f-3f^2}) + \log(2-f+\sqrt{4f-3f^2})}{\log(\frac{2}{f+\sqrt{4f-3f^2}})} \right\rceil + 1,$$

then the distrubtion of messages output by servers in the Lth layer is "ϵ-close" to a uniformly random permutation of all messages (the output distribution has variation distance at most ϵ to the distribution of random permutations on all messages).

Proof. Without loss of generality, we assume that (1) the servers are numbered $1, 2, \ldots N$ with the honest servers first and that (2) each message is represented as a card labeled from 1 to M.

We construct a deck of cards which maps to the ordering given from the first message in server 1 to the last message in server N, and then we consider how this deck is shuffled as the messages transition between the servers. Each state of the network can be thought of as a $M \times 2$ matrix. In this matrix, each row corresponds to the card labeled i. The first column indicates the current position of the ith message in the deck. The second column indicates the server currently holding the ith message. Messages move between the servers at random[5] and we track how the deck of cards (messages) is shuffled from the adversary's view. Crucially, we must consider how this view changes when cards (messages) are located in adversarial servers. We model this process as follows.

1. Choose the next server for each message uniformly at random. This corresponds to a vector of length M with entries chosen uniformly at random from $\mathbb{Z}_N$.
2. All messages that start and end in an honest server in one time step are shuffled together. We represent this by selecting the corresponding cards out of the deck, shuffling them, and placing them back on the top of the card deck. We use this to model the following behaviors:
   - Cards that travel between honest servers are shuffled in the view of the adversary, and placed on the top of the deck (the top of the deck consists of the honest servers).
   - The remaining cards are pushed down the deck, but do not change ordering. The only shuffling we consider is during honest-to-honest transitions. All of the remaining cards stay in the same relative order.
   - Some cards are pushed from the honest section of the deck to the adversarial, modeling honest-to-adversary transitions.
   - Some cards remain in order in the adversary section of the deck, and correspond to adversary-to-adversary transitions.
   - Finally, there are adversary-to-honest transitions, which we ignore for simplicity. [6]
3. Finally, we output the new state, consisting of the new arrangement of cards in the deck.

We now define a Markov chain coupling (see the excellent exposition of Klappenecker [58]) by using the randomness in step one to run two Markov chains in parallel [9]. We select the next server position for each message in both chains to be the same, so we will transition from honest to honest servers at the same time (after the first step). We choose the shuffles in step two so that each of the shuffled cards (messages) we place at the top of the deck are given the same position in both chains (all permutations of the selected set are equally likely). We consider cards that are at the same server and position in the deck at a given timestep to be *coupled*. Coupled cards have the same state row in both chains, and will continue to have the same state indefinitely given that they are either (1) pushed down the stack together or (2) both selected and shuffled to the same position.

We start one of the chains in the uniform distribution of permutations, and the other chain in an arbitrary starting distribution chosen by the adversary. We know, however, that the total variation distance between the two distributions must decrease over time given that when all cards are coupled, both chains become identical and transition in the same way (the two chains have the same distribution once all cards are coupled). In particular, the uniform distribution is stationary for this chain, so any distribution will eventually become uniform.

The time to coupling (and hence the number of layers required) can be derived as follows. For each card, the probability the card does **not** couple in the first ℓ steps is the probability it is not in consecutive honest servers. The probability of being in an

[5]In Trellis this is guaranteed by the verifiable randomness of the ART tokens, which chooses the next server uniformly at random.

[6]Shuffling within an honest server does not significantly decrease the number of layers required (and greatly increases the complexity of our proof) which is why we can ignore the details of this transition.

honest server is $1-f$ (recall that we select the next server uniformly among all of the servers). This can be modeled as flipping a coin which comes up heads with probability $(1-f)$ a total of ℓ times and asking what the probability is of **not** coming up heads twice in a row. We can solve for this probability using a linear recurrence relation. Then, we union bound the probability that *any* message has not coupled, and require this to be less than $\epsilon = 2^{-\kappa}$. The dominant term of the solution[7] yields at least ℓ layers, where

$$\ell := \frac{\kappa \log 2 + \log M + \log(2\sqrt{4f-3f^2}) + \log(2-f+\sqrt{4f-3f^2})}{\log(\frac{2}{f+\sqrt{4f-3f^2}})}.$$

Accounting for the first step, the number of layers required is $L \geq \lceil \ell + 1 \rceil$. □

D Formal protocols

D.1 Anytrust key generation

ARTs signed by different groups must be indistinguishable. Therefore, each group needs to have the *same* public key to prevent leaking information (using different signing keys would leak which anytrust group signed the token). One simple (but flawed!) way to achieve this would be to give the same secret shares to all of the anytrust groups. Unfortunately, the failure of this approach is that an adversary controlling different servers across different groups can learn the entire set of (secret) signing keys. (The adversary can corrupt different members in each group to obtain all s signing keys.)

Fortunately, this is the exact use case for *proactive* secret sharing [50, 76] (and redistributable secret sharing [114]). The beautiful idea behind proactive secret sharing is that is is possible to generate *secret shares* of secret shares. Doing so generates a fresh set of secret shares encoding the *same* secret. The end result is that the anytrust groups have (1) a common public key but (2) *different secret shares* of the secret signing key x.

Generating ART signing keys in Trellis. One anytrust group, which we call the *main group*, runs a distributed key generation protocol [43, 56, 87] to generate secret shares of the signing key x corresponding to a public verification key tvk. The shares of x, denoted x_i where the ith group member holds x_i satisfy the property that $x = \sum_{i=1}^{s} x_i$. The challenge, as described above, is to efficiently generate shares x_i', for each i, such that $x = \sum_{i=1}^{s} x_i'$ (specifically, we want to avoid *inefficient* multi-party computation or re-running distributed key generation again). Using the idea of proactive secret sharing, each group member uses a *verifiable* secret sharing scheme [91] to generate fresh secret shares of their share x_i, which we will call *sub-shares* of x_i and denote by $(x_i^{(1)}, \ldots, x_i^{(s)})$. Each set of sub-shares sums to one of the original shares, so $x_i = \sum_{j=1}^{s} x_i^{(j)}$. Then, one sub-share from each set is distributed to each member of the new group. This group member can sum the sub-shares to get a share of the original secret, since the sum of all of the sub-shares is equal to the sum of the shares. Specifically, the ith member in the new group computes: $x_i' = \sum_{j=1}^{s} x_j^{(i)}$. Observe that $x_1' \ldots x_s'$ held by the members in the new group satisfy $x = \sum_{i=1}^{s} x_i'$, and each share is random given that one of the original group members is honest. This can be seen by flipping the order of summation: we have $\sum_{i=1}^{s} x_i' = \sum_{i=1}^{s} \left(\sum_{j=1}^{s} x_j^{(i)} \right) = \sum_{i=1}^{s} (x_i) = x$.

D.2 Verifiable secret sharing for Diffie-Hellman messages

A t-out-of-n verifiable secret sharing scheme (VSS) allows any subset of t shares to reconstruct the secret. Furthermore, every share is verifiable, meaning the share can be verified for consistency against the secret. In particular, we use the Feldman scheme [12, 40, 57]: Each server will act as the dealer for a VSS instance and create a random polynomial $P(x) = a_0 + a_1 x + \ldots a_{t-1} x^{t-1}$, where a_0 is the Diffie-Hellman secret. For each coefficient, the dealer publishes a binding commitment g^{a_i}, and note that g^{a_0} is the Diffie-Hellman public message. This dealer gives to each of the other servers a share $s_i = \alpha_i, P(\alpha_i)$, where α_i is a point on the polynomial. To avoid duplicate shares, it is standard to choose $\alpha_i = i$ for $i = 1 \ldots n$, for some public ordering of the servers. Anyone can verify a share is valid by checking that

$$(g^{a_0})(g^{a_1})^{\alpha_i}(g^{a_2})^{\alpha_i^2} \ldots (g^{a_{t-1}})^{\alpha_i^{t-1}} \stackrel{?}{=} g^{P(\alpha_i)}.$$

In particular, all servers will verify their share from the dealer, and blame the dealer by producing a signed message containing an incorrect share (or signed messages with equivocating commitments, etc.). In our case, the first commitment is the Diffie-Hellman

[7]See https://puzzling.stackexchange.com/questions/29256/no-two-heads-in-a-row for how to solve the linear recurrence relation (with unbiased coins).

public message g^{a_0}. Or, in other words, the commitment value is used as the Diffie-Hellman public message for encryption, and the dealer will be unable to decrypt messages if it uses a different value.

If the dealer is to be replaced in blame protocols, servers vote using their secret shares for the corresponding VSS instance. These shares are sent to the (randomly) chosen replacement, who tallies the votes by verifying each share it is given. With t (valid) shares, the replacement can interpolate the coefficients of the polynomial and recover the Diffie-Hellman secret, a_0. For any given α_i, the value $g^{P(\alpha_i)}$ can be computed using only the public commitments and is itself a binding commitment to $P(\alpha_i)$. There is only one possible share value, $P(\alpha_i)$, which is the discrete log of the publicly computable value $g^{P(\alpha_i)} = \prod_{j=0 \ldots t-1} (g^{a_j})^{\alpha_i}$. Since there is only one value that matches the verification equation, it is not possible for a server to provide a fake share.

D.3 Onion-routing on a pre-established path

A common building block that we will use repeatedly is to route envelopes on a pre-established path, formalized in Protocol 1.

Protocol 1: Onion routing on a pre-established path

This protocol is used by servers to route envelopes on paths consisting of pre-established links through ART tokens. The servers verify that envelopes are signed and on routes matching the tokens, and that all envelopes are accounted for.

Server state:

- List of digital signature public keys, one for each server, and the secret signing key for this server,
- list of ART tokens and corresponding information $(\mathsf{t}_\ell, A_\ell, \mathsf{vk}_\ell, \mathsf{vk}_{\ell+1})$ for each layer $\ell \in \{1, \ldots, L\}$,
- Diffie-Hellman key agreement secret b,
- and set of redeemed tokens $T := \{\}$.

Server S_k input: Signed batches of envelopes $(\mathcal{B}_{j,k} := \{E_{i,j} \mid 1 \le i \le B\}, \sigma_{j,k,\ell})$ from each server $S_j \in \mathbb{S}$ for layer ℓ.
Server S_k output: Signed batches of envelopes $(\mathcal{B}_{k,j} := \{E_{i,j} \mid 1 \le i \le B\}, \sigma_{k,j,\ell+1})$ for each server $S_j \in \mathbb{S}$ for layer $\ell + 1$.

Procedure: Run once for each layer ℓ

```
1: for each incoming batch, from server $S_j$
    1: Verify $\sigma_{j,k,\ell}$ signs $\mathcal{B}_{j,k}$ with server $S_j$'s public key.
    2: for each received non-dummy envelope $E_{i,j} \in \mathcal{B}_{j,k}$
        2.1: Parse $E_{i,j} = (\mathsf{vk}_\ell, c_\ell)$.
        2.2: Find the token and corresponding information $(\mathsf{t}_\ell, A_\ell, \mathsf{vk}_\ell, \mathsf{vk}_{\ell+1})$ in the server state that has $(\mathsf{vk}_\ell)$.
            • If the token with $\mathsf{vk}_\ell$ does not exist, or does not have as the previous server $S_j$, blame with the protocol in Section 6.1.3.
        2.3: $\mathsf{sk}_i \leftarrow \mathsf{DHKeyAgree}(A_\ell, b)$.
        2.4: $\mathsf{Verify}(\mathsf{vk}_\ell, c_\ell)$.
            • If verification fails, blame with the protocol in Section 6.1.3.
        2.5: $c_{\ell+1} \leftarrow \mathsf{Dec}_{\mathsf{sk}_i}(c_\ell)$.
        2.6: $S_{\ell+1} \leftarrow \mathsf{NextServer}(\mathsf{tvk}_\ell, \mathsf{t}_\ell)$.
        2.7: Add $(\mathsf{vk}_{\ell+1}, c_{\ell+1})$ to the batch $\mathcal{B}_{k,z}$ to send to $S_{\ell+1}$.
        2.8: Add $\mathsf{t}_\ell$ to the list of redeemed tokens.
2: Check that there are no duplicates in $T$. If there exist duplicates, blame with the protocol in Section 6.1.1.
3: Check that $|T|$ is equal to the number of ARTs for layer $\ell$. If not, blame with the protocol in Section 6.1.2.
4: for each server $S_j \in \mathbb{S}$
    1: Pad batch $\mathcal{B}_{k,j}$ with dummy envelopes to length $B$.
    2: Randomly permute $\mathcal{B}_{k,j}$.
    3: Compute $\sigma_{k,j,\ell+1}$ on $\mathcal{B}_{k,j}$ using $\mathsf{sk}_k$.
    4: Send each batch and signature $(\mathcal{B}_{k,j}, \sigma_{k,j,\ell+1})$ to server $S_j$.
```

D.4 Path establishment protocol

In Protocol 2, we formalize path establishment, and we describe how a user can (anonymously) establish a path through the layers through an inductive path establishment procedure. First, a user constructs ARTs with an anytrust group. The ART for the first layer determines which entry group the user should send the (boomerang encrypted) message to. We can then view this as a path of length one ($\ell = 1$), and call it the *current path*. Then, the user uses the current path to communicate with a new server at layer $\ell + 1$ using the same ideas as in Protocol 1. The only difference is that the new server, at layer $\ell + 1$, checks and records the routing information it receives through boomerang encryption from the server at layer ℓ. Then, Protocol 1 is used again, but in the reverse direction for the return onion in the boomerang encryption. The roles of ℓ and $\ell + 1$ will be swapped: server S_ℓ now needs to decrypt (whereas $S_{\ell+1}$ was decrypting before). Specifically, the shared key is computed with S_ℓ's public Diffie-Hellman message (rather than $S_{\ell+1}$'s) and the Diffie-Hellman message from the payload of $\mathsf{t}_{\ell+1}$. Each server on the reverse direction checks that each $\mathsf{t}_{\ell+1}$ appears exactly once (rather than t_ℓ), and the identity of the next server in reverse order can be found in the payload of the corresponding t_ℓ.

D.5 Broadcast round protocol

The last protocol combines authenticated onion routing on a pre-established path with a simple check by the anytrust groups that all messages are present at the end of the mix net.

Protocol 3: Broadcast round

Round $\mathbf{r}$, where a user has message m. Same assumptions as in Protocol 2.

User
```
1: $\sigma_{L+1} = \mathsf{Sign}_{\mathsf{sk}_{L+1}}(\mathbf{r}||m)$.
2: $c_0 \leftarrow \overrightarrow{\mathsf{onion}}(\mathsf{sk}_{1\ldots L}, m||\sigma_{L+1})$.
3: Send $c_0$ to $S_1$.
```
Servers
```
1: Servers apply authenticated onion routing (Protocol 1) using the pre-established paths to route and decrypt $c_0$ to $S_L$.
2: The last server, $S_L$, sends each $(\mathsf{vk}_{L+1}, c_{L+1})$ to each member of the anytrust group handling $\mathsf{vk}_{L+1}$.
```
Anytrust group members
```
1: for each received message $(\mathsf{vk}_{L+1}, c_{L+1})$:
    1: Parse $m, \sigma_{L+1} = c_{L+1}$.
    2: Check $\mathsf{VerifySignature}_{\mathsf{vk}_{L+1}}(\mathbf{r}||m, \sigma_{L+1})$. If the signature is incorrect, blame with the protocol in Section 6.1.3.
2: Check that each verification key $\mathsf{vk}_{L+1}$ was used exactly once. If not, blame with the protocol in Section 6.1.2.
3: Forward messages to the final destination.
```

E Optimizations and extensions

In this section, we briefly highlight some practical optimizations we can exploit to improve concrete performance on Trellis in a deployed environment (we use these straightforward optimizations in our implementation).

- **Reduced authentication overhead:** Since each AES ciphertext is signed, we do not need to include MACs to verify integrity.
- **Synchronizing rounds:** After each server receives and checks the signed message from the other servers for layer ℓ, it can start processing layer $\ell + 1$, provided it avoids the computational side channel of revealing the time to process the last received message. With this optimization, the servers do not need to wait for the worst-case latency, and each server can adapt to the current network conditions on-the-fly.
- **Latency ordering:** Signing and then sending the messages along the longest latency connections first enables the signing and verification computation latency to occur in parallel with the networking latency.
- **Path establishment receipt latency:** The key feature of the boomerang receipt is that it must pass through an honest server to detect that the message was dropped. However, we can compute the number of servers it must pass through to have passed through one honest server with cryptographically high probability (e.g., with probability greater than $1 - 2^{-64}$) in the same manner as we do for computing the anytrust group size. Then, the backward onion can be truncated at $L' < L$ layers.
- **Message overhead:** The key corresponding to each envelope can be appended by the server from the stored data instead of being included as part of the ciphertext. This reduces the overhead added for each layer. The first non-token containing an envelope along a link should include the key within the ciphertext, but the key can be omitted for later envelopes on that link (so envelopes only contain the next onion ciphertext). The block public signature signs the keys so servers can prove which ones they were told to decrypt with. During traceback, a server produces and proves correct decryption of the first envelope to show a key decrypts into another and is the correct key to send with later envelopes.

- **Path establishment forwarding latency:** We need to wait for the receipt to be processed before we extend the path, but we do not need to wait for the first part of the path that has already been established. Therefore, the forward sending can take place beforehand, in parallel. We can then further reduce overhead by combining the forward envelopes with the same key together.
- The overhead of dummy envelopes is significant, especially when setting the statistical security to 2^{-64}. To reduce this overhead, we first bound the probability that more envelopes are sent on any link than the dummy cover traffic allows for to 2^{-8}. This ensures that the probability that any 8 links are observable to the network adversary is less than 2^{-64}. To compensate, we increase the number of layers to $L+8$, in order to have L layers with unobservable mixing guarantees, as required by Section 5.5. We calculate a theoretical bound by modeling each link as a binomial distribution and bound accordingly [7]. In practice, we observe that the overhead of dummy messages is typically between 100% and 200% and decreases as M increases.

Protocol 2: Path extension

This protocol is used by the user and servers to extend an existing path of length ℓ to a new path of length $\ell+1$. This protocol is run simultaneously by all users in synchronous rounds. We assume that all messages to and from anytrust groups during ART signing are authenticated using a digital signature. Let gpk be an anytrust group public key generated as in Appendix D.1 for the DH group.

One-time setup for servers.
Each server generates and posts the public part of a Diffie-Hellman messages:

```
1: $B, b \leftarrow \mathsf{DHKeyGen}(\mathbb{G}, g)$.
2: Publish $B$ to a publicly accessible bulletin board (or send to all users).
```

User-side ART generation and path initialization.
Each user generates ART tokens for all L layers. The ℓth token encapsulates a new Diffie-Hellman message and new signature verification key for the ℓth layer.

```
1: for $\ell = 1 \ldots (L+1)$:
    1.1: $(\mathsf{vk}_\ell, \mathsf{ssk}_\ell) \leftarrow \mathsf{DS.KeyGen}(1^\lambda)$; $(A_\ell, a_\ell) \leftarrow \mathsf{DHKeyGen}(\mathbb{G}, g)$; $\mathsf{payload}_\ell := (S_\ell, A_\ell, \mathsf{vk}_\ell)$. // $S_1 := \perp$
    1.2: $(\mathsf{st}_\ell, T_\ell) \leftarrow \mathsf{User}_0(\mathsf{tvk}_\ell, \mathsf{payload}_\ell)$.
    1.3: Send $T_\ell$ to an anytrust group for signing; receive in return $W_{\ell,i}$ from the $i$th group member, for $i \in \{1, \ldots, s\}$.
    1.4: $\mathsf{t}_\ell \leftarrow \mathsf{ART.User}_1(\mathsf{st}_\ell, W_{\ell,1}, \ldots, W_{\ell,s})$; $S_{\ell+1} \leftarrow \mathsf{ART.NextServer}(\mathsf{tvk}_\ell, \mathsf{t}_\ell)$.
    1.5: Check the signature using $\mathsf{ART.Verify}(\mathsf{tvk}_\ell, S_{\ell+1}, \mathsf{payload}_\ell, \mathsf{t}_\ell)$. // If Verify outputs no, blame with the protocol in Section 6.2).
    1.6: $B_\ell \leftarrow$ the public Diffie-Hellman message published by server $S_\ell$.
    1.7: $\mathsf{sk}_\ell \leftarrow \mathsf{DHKeyAgree}(B_\ell, a_\ell)$ // (key for forward boomerang); $\mathsf{sk}'_\ell \leftarrow \mathsf{DHKeyAgree}(B_{\ell-1}, a_\ell)$. // (key for reverse boomerang).
2: $\mathbf{sk} := (\mathsf{sk}_1, \ldots, \mathsf{sk}_\ell, \mathsf{sk}'_\ell, \ldots, \mathsf{sk}'_1)$. // Boomerang encryption keys
```

```
1: for $\ell = 1 \ldots L$:
    1.1: $r \leftarrow_R \{0,1\}^\kappa$. // Random nonce for receipt of delivery
    1.2: $\sigma \leftarrow \mathsf{DS.Sign}(\mathsf{ssk}_\ell, \mathsf{t}_\ell || \mathsf{t}_{\ell+1})$.
    1.3: $c_{\ell,0} \leftarrow \mathsf{boomerang.Enc}(\mathbf{sk}, \mathsf{t}_\ell || \mathsf{t}_{\ell+1} || A_\ell || \mathsf{vk}_\ell || A_{\ell+1} || \mathsf{vk}_{\ell+1} || \sigma, r)$ and each intermediate envelope is signed against the corresponding $\mathsf{vk}_\ell$. If $\ell$ is $L$, then additionally encrypt the reverse onion with $\mathsf{DHKeyAgree}(\mathsf{gpk}, a_{L+1})$.
    1.4: Submit $c_{\ell,0}$ to $S_1$. When all users have submitted, servers run path establishment for layer $\ell$ (below).
    1.5: Verify that the returned message is properly formed. If not, blame with the protocol in Section 6.1.3.
```

Path establishment for layer ℓ.
Servers apply authenticated onion routing (Protocol 1) to route and decrypt $\overleftarrow{\mathsf{onion}}$ to S_ℓ (the path is pre-established through $\ell - 1$).
Each server (denoted by S_ℓ):

```
1: Run Protocol 1 with the following modifications:
    • After step 2.5,
        1: $\overrightarrow{m} \leftarrow \mathsf{Dec}_{\mathsf{sk}_i}(c_\ell)$ and parse $\overrightarrow{m} = \mathsf{t}_\ell || \mathsf{t}_{\ell+1} || A_\ell || \mathsf{vk}_\ell || A_{\ell+1} || \mathsf{vk}_{\ell+1} || \sigma || c'_\ell$.
        2: Check the following signatures:
            – $\mathsf{ART.Verify}(\mathsf{tvk}_\ell, S_j, S_\ell, (A_\ell, \mathsf{vk}_\ell), \mathsf{t}_\ell)$,
            – $\mathsf{ART.Verify}(\mathsf{tvk}_{\ell+1}, S_\ell, S_{\ell+1}, (A_{\ell+1}, \mathsf{vk}_{\ell+1}), \mathsf{t}_{\ell+1})$, and
            – $\mathsf{Verify}(\mathsf{vk}_\ell, \mathsf{t}_\ell || \mathsf{t}_{\ell+1})$.
           If any check fails, blame with the protocol in Section 6.1.3.
        3: Store $(\mathsf{t}_\ell, A_\ell, \mathsf{vk}_\ell, \mathsf{vk}_{\ell+1})$ in the server state list for layer $\ell$.
    • For step 2.7, instead add $(\mathsf{vk}_\ell, c'_\ell)$ to the batch for $S_j$, the server that the envelope came from, rather than the next server.
    • If $\ell = L$, then also:
        1: Send $(A_{L+1}, \mathsf{vk}_{L+1}, \mathsf{t}_{L+1})$ to each member of an anytrust group. Each group member:
            – Check $\mathsf{ART.Verify}(\mathsf{tvk}_{L+1}, S_{L+1}, (S_L, A_{L+1}, \mathsf{vk}_{L+1}), \mathsf{t}_{L+1}) = \mathsf{yes}$. // If Verify outputs no, blame with the protocol in Section 6.1.3.
            – Store $\mathsf{vk}_{L+1}$ for future use.
            – Output $(A_{L+1})^{x_j}$ where $x_j$ is the member's share of the anytrust group secret key.
        2: Compute the product $\prod_j (A_{L+1})^{x_j} = \mathsf{DHKeyAgree}(\mathsf{gpk}, \mathsf{sk}_{L+1})$ to decrypt one layer of $c'_\ell$.
        3: If any decryption fails, blame with the protocol in Section 6.2.
2: All servers: Apply authenticated onion routing (Protocol 1) in reverse to route and decrypt $c'_\ell$ back to the user.
```