# Division of Regulatory Power: Collaborative Regulation for Privacy-Preserving Blockchains

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***Abstract*—Decentralized anonymous payment schemes may be exploited for illicit activities, such as money laundering, bribery and blackmail. To address this issue, several regulatory-friendly decentralized anonymous payment schemes have been proposed. However, most of these solutions lack restrictions on the regulator's authority, which could potentially result in power abuse and privacy breaches. In this paper, we present a decentralized anonymous payment scheme with collaborative regulation (DAPCR). Unlike existing solutions, DAPCR reduces the risk of power abuse by distributing regulatory authority to two entities: Filter and Supervisor, neither of which can decode transactions to access transaction privacy without the assistance of the other one. Our scheme enjoys three major advantages over others: ① Universality, achieved by using zk-SNARK to extend privacy-preserving transactions for regulation. ② Collaborative regulation, attained by adding the ring signature with controllable linkability to the transaction. ③ Efficient aggregation of payment amounts, achieved through amount tags. As a key technology for realizing collaborative regulation in DAPCR, the ring signature with controllable linkability (CLRS) is proposed, where a user needs to specify a linker and an opener to generate a signature. The linker can extract pseudonyms from signatures and link signatures submitted by the same signer based on pseudonyms, without leaking the signer's identity. The opener can recover the signer's identity from a given pseudonym. The experimental results reflect the efficiency of DAPCR. The time overhead for transaction generation is $1231.2\,\mathrm{ms}$, representing an increase of less than $50\,\%$ compared to ZETH. Additionally, the time overhead for transaction verification is only $1.2\,\mathrm{ms}$.**

***Index Terms*—Ring Signature, Blockchain, Cryptocurrency, Regulation, Decentralized Finance.**

## I. Introduction

In recent years, blockchain technology has had a substantial economic and social impact on the real world. One of the most widely adopted applications is the decentralized payment system, also known as cryptocurrency. In 2021, the total volume of cryptocurrency transactions surged to $15.8 trillion. However, in contrast to traditional centralized payment mechanisms, decentralized payment systems such as Bitcoin [1] and Ethereum [2] lack support for the privacy preservation of user identities and payment amounts. To address this privacy concern, researchers have proposed decentralized anonymous payment (DAP) systems like Monero, Zerocash and Zether [3]–[5]. In these solutions, the addresses of traders and the specific payment amount for each transaction are kept confidential from other users.

However, providing unconditional privacy in DAP may lead to an increase in criminal activities. Cryptocurrencies could potentially be used for bribery, blackmail, terrorist financing, and money laundering. Chainalysis[1] pointed out that in 2021, cryptocurrency-related criminal cases increased by 79% compared to 2020. Although during the same period, the overall transaction volume grew by over 550%, indicating a decrease in the proportion of illegal activities in the total transactions, this does not imply that regulation is unnecessary. FATF[2] and APG[3] proposed that the absence of regulation has created significant loopholes for criminals, necessitating swift action to mitigate the risks of virtual assets being exploited by criminal and terrorist elements.

Numerous DAP schemes incorporating regulation have been proposed to combat illicit activities within decentralized payment systems. However, unrestricted regulation presents the risk of power abuse and privacy breaches. Therefore, our work focuses on achieving a delicate equilibrium between privacy preservation and regulation. Specifically, to mitigate the potential for regulatory power abuse, regulators should only have access to the sender's address for suspicious transactions, while ensuring that the privacy information of compliant transactions remains confidential to regulators. Furthermore, it is essential to monitor the total payment amounts conducted by individual users during a designated transaction period. This monitoring becomes necessary as traders might choose to execute multiple smaller transactions rather than a single large transaction when transferring assets.

### A. Advantages of DAPCR

In this paper, we propose a decentralized anonymous payment scheme with collaborative regulation (DAPCR). The advantages of DAPCR are as follows:

1. **Universality** means that DAPCR can be constructed based on any existing DAP scheme and is compatible with both

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[1]Chainalysis: https://www.chainalysis.com/

[2]Financial Action Task Force: https://www.fatf-gafi.org/

[3]Asia/Pacific Group on Money Laundering: https://apgml.org/

TABLE I: Properties of DAPCR and related works

| Scheme | Collaborative Regulation | Non-interaction | Amount Aggregation | Universality |
|---|---|---|---|---|
| DAPCR | ✓ | ✓ | ✓ | ✓ |
| [6] | × | ✓ | × | × |
| [7] | × | ✓ | × | × |
| [8] | ✓ | ✓ | × | × |
| [9] | × | × | × | × |
| [10] | × | × | × | × |

UTXO and account models. To achieve this universality, we extended the original transactions in a DAP scheme with zk-SNARK. By introducing an additional regulated field to a DAP transaction, Filter and Supervisor can extract regulatory data from the additional field using the same set of algorithms, independently of the original DAP transactions.

2. **Collaborative regulation** means that the regulatory authority in DAPCR is decentralized to Filter and Supervisor, similar to the separation of powers in governments, requiring their cooperation to regulate transactions. In particular, Filter is responsible for linking the transactions submitted by the same signer and extracting the amount tag from each transaction. Supervisor can recover the user's public key from a given pseudonym. Both regulators must collaborate to regulate transactions.
3. **Payment aggregation** means that Filter can aggregate payment amounts from the same user's transactions and get a total payment tag, which is used to determine whether the total payment amount exceeds the user's limit, with Filter do not open transactions during this process.

To clarify the advantages of our scheme, the properties of DAPCR and related works are shown in Table I.

### B. Paper Contributions

In summary, our contributions in this paper are as follows.

1. We propose the DAPCR scheme with the following advantages: ① Universality means that DAPCR can be constructed based on any existing DAP scheme and is compatible with both UTXO and account models. ② Collaborative regulation prevents abuse of power and privacy breaches. ③ Payment aggregation enables efficient screening of suspicious transactions. To our best knowledge, DAPCR is the first universal collaborative regulation scheme for DAP schemes.
2. We present security definitions of the DAPCR scheme and provide the security analysis for it.
3. We evaluate the performance of DAPCR on both local devices and the Fabric network. The time cost for transaction generation is about $1231.2\,\mathrm{ms}$ and that of transaction verification is about $12.5\,\mathrm{ms}$. These experimental results indicate the effectiveness of DAPCR.
4. We also propose the ring signature with controllable linkability (CLRS), which is a key technology for enabling collaborative regulation in DAPCR. It allows the designated user to link signatures from the same signer without revealing the signer's identity.

### C. Paper Outline

Section II presents an overview of DAPCR. In Section III, we review the background materials associated with our work. Section IV presents the system framework and security definitions of CLRS and DAPCR. Next, we propose an efficient construction of CLRS, which is the building block of DAPCR in Section V. In Section VI, we present an efficient and generic construction of DAPCR. We also provide the security analysis and performance analysis for DAPCR in Section VII and VIII. Section IX reviews the recent literature relevant to DAP and ring signatures. Section X concludes our work.

## II. Overview

To construct a decentralized anonymous payment scheme with regulation that satisfies universality, we extend the existing DAP scheme by adding additional regulated fields. Regulators can use a predefined set of algorithms to extract the necessary data from the regulated field, without considering the original structure of the DAP transaction. Furthermore, to ensure consistency between the regulatory data within a regulated field and the payment data within a DAP transaction, a user needs to generate a zero-knowledge proof to show this. As shown in Fig. 1, the regulated transaction $\mathsf{rtx}$ in our scheme consists of a DAP transaction $\mathsf{tx}$, a proof $\pi$, and an additional regulated field $\mathsf{reg}$.

To prevent the abuse of power and the leakage of privacy, we decentralize regulatory authority into two regulators: Filter and Supervisor. Both must cooperate to carry out the regulation effectively. Specifically, Filter can extract a pseudonym $nym$ and a tag $tag_v$ of payment amount from the regulated field in a transaction to assess whether the user with pseudonym $nym$ violates the transaction policy, as described in Section IV-A2. If a violation is detected, Filter will submit the user's suspicious transactions and pseudonym $nym$ to Supervisor, who can extract the sender's identity from $nym$.

As an example, Fig. 1 depicts the workflow of regulation. Initially, Filter retrieves transactions from the blockchain and extracts pseudonyms and amount labels from each transaction (Step ① in Fig. 1). To assess whether the user with pseudonym $nym_1$ complies with the transaction policy, Filter screens out all transactions from this user based on the pseudonym $nym_1$ and aggregates the amount labels of these transactions to obtain a total amount label $tag_{sum}$ (Step ②). Filter can then determine whether the user complies with the transaction policy according to $tag_v$. Then Filter submits the user's pseudonym $nym_1$ and transactions to Supervisor if the user violates the transaction policy (Step ③). Supervisor will recover the user's identity from $nym_1$ (Step ④).

Note that Filter cannot obtain identities or payment amounts from pseudonyms and labels, and Supervisor cannot obtain pseudonyms without Filter's assistance. Therefore, if Filter and Supervisor do not collude, payment privacy within compliant transactions will not be disclosed to either party. The threat model for our scheme is as follows.

*Threat model*: We assume that each user is selfish and potentially malicious, capable of launching active attacks such as double-spending attacks, malleability attacks, and over-spending attacks to steal assets or provide incorrect regulatory

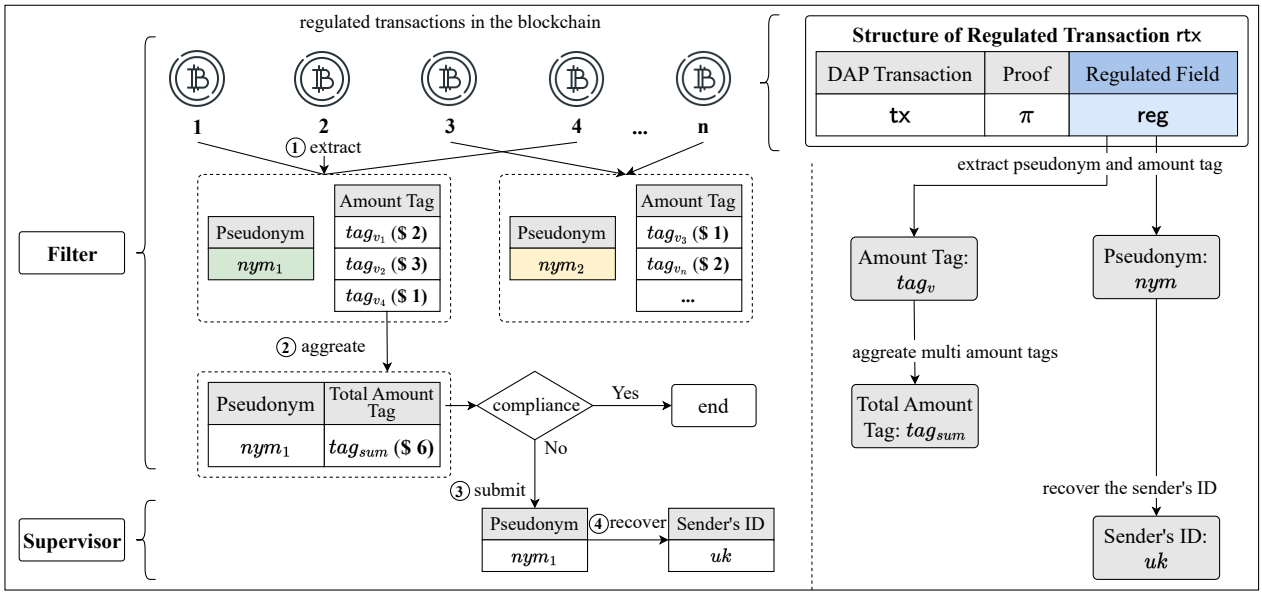



Fig. 1: Overview of DAPCR.

data to regulators. The malicious user also attempts to extract payment privacy within other users' transactions in the decentralized ledger. We also assume that Filter and Supervisor are honest-and-curious. They adhere to the DAPCR scheme and do not actively attack the system. Instead, they will attempt to obtain payment privacy within honest users' transactions. Furthermore, we also assume that Filter and Supervisor cannot collude. We believe this assumption holds in the consortium blockchain, as Filter and Supervisor can be two separate and independent entities participating in managing the consortium blockchain. In real-life scenarios, the law enforcement agency, such as the FBI, serves as Filter, while the judicial department, such as a court, serves as Supervisor. The law enforcement agency collects suspicious transactions from the decentralized ledger and submits them, along with evidence, to the judicial department. After verification, the judicial department removes the anonymity of these transactions and holds the relevant users accountable. It is important to note that these two entities operate independently and do not conspire together.

In the above threat model, an adversary cannot compromise both Filter and Supervisor. The DAPCR scheme prevents privacy leaks by distributing regulatory authority to Filter and Supervisor: ① If Filter is compromised, the adversary can extract users' pseudonyms from transactions but cannot recover public keys from these pseudonyms. ② If Supervisor is compromised, the adversary can only recover the senders' public keys from pseudonyms of suspicious transactions submitted by Filter, but cannot directly revoke the anonymity of transactions on the blockchain. Therefore, whether Filter or Supervisor is compromised, the privacy of transactions that adhere to the transaction policy will not be breached.

## III. Preliminaries

### A. Decentralized Anonymous Payments

A DAP scheme in the account model, such as Zether [5], can be highly simplified into algorithms as follows.

- $\mathsf{Setup}(1^\lambda) \to pp$. This setup algorithm takes a security parameter $\lambda$ as input and outputs a public parameter $pp$, which is an implicit input for other algorithms.
- $\mathsf{AddrGen}(pp) \to (addr, s)$. A user executes this algorithm to generate his address $addr$ and secret key $s$.
- $\mathsf{TxGen}(addr_S, addr_R, v, s, I_{\mathsf{pub}}, I_{\mathsf{pri}}) \to \mathsf{tx}$. A user executes this algorithm to generate a DAP transaction tx. The algorithm takes as input a sender's address $addr_S$, a receiver's address $addr_R$, the payment amount $v$, a secret key $s$, $I_{\mathsf{pub}}$ (which represents additional public inputs) and $I_{\mathsf{pri}}$ (which represents additional private inputs), and outputs a transaction tx.
- $\mathsf{TxVfy}(\mathsf{tx}, I_{\mathsf{pub}}) \to 0/1$. Anyone can execute this algorithm to verify the validity of a transaction tx. The algorithm takes as input a transaction tx and outputs 1 if tx is valid or 0 otherwise.

A secure DAP scheme generally satisfies indistinguishability, non-malleability and balance.

1. *Indistinguishability*. The ledger discloses no information to any adversary attempting to access information beyond what is publicly available.
2. *Non-malleability*. No adversary possesses the capability to modify the information contained within a valid transaction tx.
3. *Balance*. The amount paid by any adversary cannot exceed its balance.

The DAP scheme in the UTXO model, such as Zerocash [4], can also be represented using the above algorithms if only one-to-one transactions are allowed. In Appendix B, we discuss the minor differences between deploying DAPCR in the UTXO model and deploying it in the account model. In the construction of DAPCR, we overlook these minor differences and use the above algorithms to describe the DAP scheme in either model.

### B. NIZK & SoK

*1) NIZK Protocol:* We first present an NP-relation $\mathcal{R}$ defining the language $\mathcal{L}_{\mathcal{R}} = \{\phi | \exists \varpi : (\phi, \varpi) \in \mathcal{R}\}$ in which $\phi$ and $\varpi$ are considered as a statement and a witness. Non-interactive zero-knowledge protocol, also known as NIZK protocol, for the relation $\mathcal{R}$ is composed of three algorithms as follows [11].

- $\mathcal{G}(1^\lambda, \mathcal{R}) \rightarrow crs$. The setup algorithm takes a security parameter $\lambda$ and an NP-relation $\mathcal{R}$ as input, and outputs a common reference string $crs$.
- $\mathcal{P}(\phi, \varpi, crs) \rightarrow \pi$. The prover algorithm takes a statement $\phi$, a witness $\varpi$ and a common reference string $crs$ as input, and outputs a proof $\pi$.
- $\mathcal{V}(\phi, \pi, crs) \rightarrow 0/1$. The verifier algorithm takes a statement $\phi$, a proof $\pi$ and a common reference string $crs$ as input, and outputs 1 if $\pi$ is valid or 0 otherwise.

A NIZK scheme satisfies completeness, zero-knowledge and knowledge soundness.

1. *Completeness*. For any $(\phi, \varpi) \in \mathcal{R}$,
$$Pr\left[\begin{array}{c} crs \leftarrow \mathcal{G}(1^\lambda, \mathcal{R}); \pi \leftarrow \mathcal{P}(\phi, \varpi, crs) : \\ 1 \leftarrow \mathcal{V}(\phi, \varpi, crs) \end{array}\right] = 1.$$
2. *Zero-knowledge*. No information other than the truth of the statement is leaked. For any $(\phi, \varpi) \in \mathcal{R}$, any probabilistic polynomial-time (PPT) adversary $\mathcal{A}$ and a polynomial-time simulator $\mathcal{S} = (\mathcal{G}_{sim}, \mathcal{P}_{sim})$,
$$\left|Pr\left[\begin{array}{c} (crs, \tau) \leftarrow \mathcal{G}_{sim}(1^\lambda, \mathcal{R}); \pi' \leftarrow \mathcal{P}_{sim}(\phi, crs, \tau) : \\ \mathcal{A}(\pi', crs, \tau, \mathcal{R}) = 1 \end{array}\right] - Pr\left[\begin{array}{c} crs \leftarrow \mathcal{G}(1^\lambda, \mathcal{R}); \pi \leftarrow \mathcal{P}(\phi, \varpi, crs) : \\ \mathcal{A}(\pi, crs, \tau, \mathcal{R}) = 1 \end{array}\right]\right| \leq \mathsf{negl}(\lambda).$$
3. *Knowledge Soundness*. A secure NIZK scheme cannot prove a false statement. For any PPT adversary $\mathcal{A}$ and a PPT extractor $\mathcal{E}$,
$$Pr\left[\begin{array}{c} crs \leftarrow \mathcal{G}(1^\lambda, \mathcal{R}); \\ (\phi, \pi, \varpi) \leftarrow (\mathcal{A} \| \mathcal{E})(crs, \mathcal{R}) : \\ 1 \leftarrow \mathcal{V}(\phi, \pi, crs) \wedge (\phi, \varpi) \notin \mathcal{R} \end{array}\right] \leq \mathsf{negl}(\lambda).$$

Zk-SNARK, also known as the zero-knowledge succinct non-interactive argument of knowledge, is a special type of NIZK scheme. In addition to the above properties, zk-SNARK also satisfies succinctness: The runtime of a prover algorithm is polynomial in $|a| + \lambda$ and the size of $\pi$ output by the prover algorithm is polynomial in $\lambda$.

*2) SoK protocols:* Signature of knowledge [12], also known as SoK, allows one to issue signatures on behalf of any NP statement. A valid signature of knowledge implies that the signer possesses a witness $\varpi$ to the statement $\phi$. A SoK protocol $\Pi_{SoK}$ consists of three algorithms as follows.

- $\mathsf{SoK.Setup}(1^\lambda, \mathcal{R}) \rightarrow pp$. The setup algorithm takes a security parameter $\lambda$ and an NP-relation $\mathcal{R}$ as input, and outputs a public parameter $pp$.
- $\mathsf{SoK.Sign}(m, \phi, \varpi, pp) \rightarrow \pi$. The signing algorithm takes a message $m$, a statement $\phi$, a witness $\varpi$ and the public parameter $pp$ as input and outputs a signature of knowledge $\pi$ if $(\phi, \varpi) \in \mathcal{R}$ or $\perp$ otherwise.
- $\mathsf{SoK.Vfy}(m, \phi, \pi, pp) \rightarrow 0/1$. The verifying algorithm takes a message $m$, a statement $\phi$, a signature of knowledge $\pi$ and the public parameter $pp$ as input, and outputs 1 if $\pi$ is valid, or 0 otherwise.

A secure SoK satisfies correctness, simulatability and extractability.

1. *Correctness*. For any $(\phi, \varpi) \in \mathcal{R}$,
$$Pr\left[\begin{array}{c} pp \leftarrow \mathsf{Setup}(1^\lambda, \mathcal{R});\ \pi \leftarrow \mathsf{Sign}(m, \phi, \varpi, pp) : \\ \mathsf{Vfy}(m, \phi, \pi, pp) = 1 \end{array}\right] = 1 - \mathsf{negl}(\lambda).$$
2. *Simulatability*. There are two PPT algorithms SimSetup and SimSign where $\mathsf{SimSetup}(1^\lambda, \mathcal{R})$ outputs the public parameter $pp$ and a trapdoor $\tau$, and $\mathsf{SimSign}(m, \phi, \tau, pp)$ outputs a signature $\pi$, such that for any PPT adversary $\mathcal{A}$,
$$\left|Pr\left[(pp, \tau) \leftarrow \mathsf{SimSetup}(1^\lambda, \mathcal{R}) : \mathcal{A}^{\mathcal{O}_{sim}}(\mathcal{R}, pp) = 1\right] - Pr\left[pp \leftarrow \mathsf{Setup}(1^\lambda, \mathcal{R}) : \mathcal{A}^{\mathsf{Sign}}(\mathcal{R}, pp) = 1\right]\right| \leq \mathsf{negl}(\lambda),$$
where the oracle $\mathcal{O}_{sim}$ receives $(m, \phi, \varpi)$ as input and outputs $\pi \leftarrow \mathsf{SimSign}(m, \phi, \tau, pp)$ if $(\phi, \varpi) \in \mathcal{R}$ and $\perp$ otherwise.
3. *Extractability*. In addition to algorithms SimSetup and SimSign, there exists an extractor algorithm Ext such that for any PPT adversary $\mathcal{A}$,
$$Pr\left[\begin{array}{c} (pp, \tau) \leftarrow \mathsf{SimSetup}(1^\lambda, R); \\ (m, \phi, \pi) \leftarrow \mathcal{A}^{\mathcal{O}_{sim}}(\mathcal{R}, pp);\ \varpi \leftarrow \mathsf{Ext}(m, \phi, \pi, \tau, pp) : \\ (\phi, \varpi) \in \mathcal{R} \vee (m, \phi, \pi) \in L \vee\ \mathsf{Vfy}(m, \phi, \pi, pp) = 0 \end{array}\right] = 1 - \mathsf{negl}(\lambda),$$
where $L$ is a list of queries to SimSign.

Note that NIZK and SoK protocols in the random oracle model can be efficiently realized by applying the Fiat-Shamir transform [13] to $\Sigma$-protocols.

### C. Notations

Notations used in this paper and their descriptions are shown in Table II.

## IV. System Framework and Security Definitions

In this section, the system framework and security definitions of DAPCR are going to be introduced.

### A. DAP with Collaborative Regulation

*1) System Framework:* An entity that independently regulates the anonymous payment system may abuse regulatory power. To avoid the disadvantage, we divide the regulatory power into two authorities called Filter and Supervisor. Figure 2 depicts the system framework of workflow DAPCR, which consists of five entities as follows:

1. *Blockchain*. The DAPCR scheme is based on a permissioned blockchain, which is only accessed by users with permissions. Supervisor is responsible for registering users who are allowed to join the permissioned blockchain.
2. *User*. A new user needs permission from all consensus nodes to join the consortium blockchain [14]. Each user is assigned a payment limit $\mu$, and his total payment amount of anonymous transactions within each trading period should not exceed this limit.
3. *Consensus Nodes* are responsible for checking the validity of transactions and sealing the valid transactions into new blocks.

TABLE II: Notations and descriptions

| Notation | Description | Notation | Description |
|---|---|---|---|
| $\mathcal{F}$ | Filter | $\mathcal{S}$ | Supervisor |
| $\mathcal{U}$ | User | $(uk, sk)$ | user's public and private keys |
| $(pk_F, sk_F)$ | Filter's public and private keys | $(pk_O, sk_O)$ | Supervisor's public and private keys |
| tx | anonymous transaction in DAP | rtx | regulated transaction in DAPCR |
| $nym$ | user's pseudonym | $tag_\mu$ | tag of user's payment limit |
| $\mu$ | user's payment limit | $v_{sum}$ | user's total payment amount |
| reg | user's registration data | rpt | report of suspicious transactions |
| $(pk_L, sk_L)$ | linker's public and private keys | $(pk_O, sk_O)$ | opener's public and private keys |
| $R$ | set of public keys | $\sigma$ | signature in CLRS |

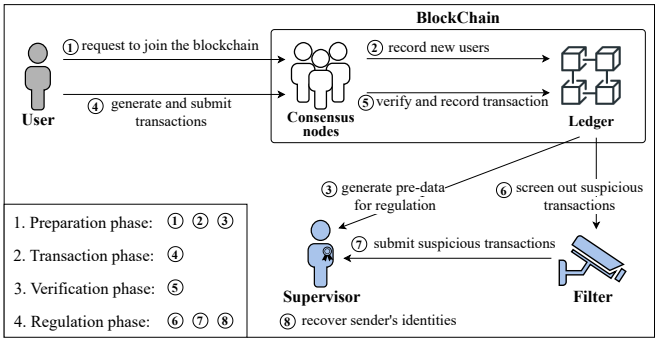



Fig. 2: System framework and workflow.

4. *Filter* has the ability to link transactions from the same sender and determine whether they comply with the transaction policy.
5. *Supervisor* is an entity responsible for registering users and identifying the sender's public key of suspicious transactions with the assistance of $\mathcal{S}$.

As shown in Fig. 2, the DAPCR scheme consists of four phases. In the preparation phase, the public parameter is published. Entities generate their public and private keys, and new users obtain permission from consensus nodes to join the blockchain. In the transaction phase, users generate regulated transactions to transfer their assets. In the verification phase, consensus nodes verify the validity of regulated transactions, and only valid transactions are sealed into blocks. In the regulation phase, Filter screens out suspicious transactions and submits the report on suspicious transactions to Supervisor. Once the report is received, Supervisor recovers the sender's public key of suspicious transactions.

*2) Transaction Policy:* Each user is assigned a payment limit, and his total payment amount of anonymous transactions within each trading period should not exceed the limit. To simplify the design and enhance regulatory efficiency, the DAPCR scheme requires that a user's total payment amount of anonymous transactions within a trading period equals his payment limit. If it does not match, Filter will mark these transactions as suspicious and include them in regulation. If the total payment amount does not reach the payment limit, the user can submit a transaction, of which the receiver is himself, with a payment amount equal to the shortfall, to make the total payment amount equal to the payment limit. With each anonymous transaction the user publishes, an equivalent amount is deducted from his payment limit. If the user exhausts his payment limit but still wants to transact, he can make a public transaction, which doesn't affect his payment limit.

*3) Formal Definition:* A DAPCR scheme is composed by the following PPT algorithms.

- $param \leftarrow \mathsf{Setup}(1^\lambda)$. This algorithm takes as input a security parameter $\lambda$ and outputs a public parameter $param$, which is implicit input to other algorithms.
- $(pk_F, sk_F) \leftarrow \mathsf{FInit}(param)$. $\mathcal{F}$ executes this algorithm to generate their public-private key pair $(pk_F, sk_F)$.
- $(pk_S, sk_S) \leftarrow \mathsf{SInit}(param)$. $\mathcal{S}$ executes this algorithm to generate their public-private key pair $(pk_S, sk_S)$.
- $(uk, sk) \leftarrow \mathsf{KeyGen}(pk_S)$. This algorithm takes $pk_S$ as input and outputs a public-private key pair $(uk, sk)$ for a user.
- $\mathsf{req} \leftarrow \mathsf{Join}(uk, \mu, pk_S)$. A new user executes this algorithm to generate a request req to join the blockchain for consensus nodes. This algorithm takes $uk$, $pk_S$, and the user's payment limit $\mu$ as input and outputs a request req. If the new user obtains permission from all nodes, req will be recorded on the blockchain.
- $(tag_\mu, nym) \leftarrow \mathsf{Reg}(\mathsf{req}, pk_S, sk_S)$. $\mathcal{S}$ reads req from the decentralized ledger and executes this algorithm to generate the data required for regulation. This algorithm takes req, $sk_S$ and $pk_S$ as input and outputs the user's limit tag $tag_\mu$ and his pseudonym $nym$.
- $\mathsf{rtx} \leftarrow \mathsf{RtxGen}(addr_S, addr_R, v, s, I_{\mathsf{pub}}, I_{\mathsf{pri}}, R, sk, pk_F)$. The algorithm takes as input a tuple $(addr_S, addr_R, v, s, I_{\mathsf{pub}}, I_{\mathsf{pri}}, R, sk, pk_F)$ and outputs a regulated transaction rtx, where $R = \{uk_0, uk_1, ..., uk_{n-1}\}$, $uk = uk_j \in R$ and $sk$ is a private key corresponding to $uk$.
- $0/1 \leftarrow \mathsf{Verify}(\mathsf{rtx}, pk_F, R, I_{\mathsf{pub}})$. This algorithm, which verify the validity of a regulated transaction rtx, takes rtx, $pk_F$, $R$ and $I_{\mathsf{pub}}$ as input, and outputs 1 if rtx is valid or 0 otherwise.
- $(tag_v, nym) \leftarrow \mathsf{Extract}(\mathsf{rtx}, sk_F)$. The algorithm takes rtx and $sk_F$ as input and extracts an amount tag $tag_v$ and the sender's pseudonym $nym$ from rtx. Since pseudonyms are uniquely associated with user identities, they can be used to link transactions from the same sender.
- $(S, \mathsf{susp}/\perp) \leftarrow \mathsf{Detect}(nym, tag_\mu, sk_F, \mathsf{ledger})$. This algorithm takes as input a pseudonym $nym$, a tag $tag_\mu$ of $nym$'s payment upper limit, Filter's private key $sk_F$ and a decentralized ledger ledger, which denoted all transactions sealed in blocks during a trading period, and outputs a set $S$ of all transactions submit by $nym$. The algorithm further outputs susp if $nym$'s total payment amount exceeds their upper limit, else outputs $\perp$.

TABLE III: Properties of CLRS and related signatures

| Scheme | Controllable Linkability | Irrevocable-iff-linked [1] | Collaborative Revocability [2] | No Issuer |
|---|---|---|---|---|
| CLRS | ✓ | ✓ | ✓ | ✓ |
| [15] | × | ✓ | × | ✓ |
| [16] | ✓ | ✓ | × | ✓ |
| [17], [18] | × | × | × | ✓ |
| [19], [20] | ✓ | × | × | ✓ |
| [21] | × | ✓ | × | ✓ |
| [22] | ✓ | ✓ | × | × |
| [23], [24] | × | × | × | ✓ |

[1] means that a linker cannot revoke the anonymity of signatures, even if these signatures are linked.
[2] means that the opener can only revoke the anonymity of signatures with the assistance of the linker.

- $\mathsf{rpt} \leftarrow \mathsf{Report}((S, \mathsf{susp}), nym, pk_F, sk_F)$. This algorithm takes as input a tuple $(S, \mathsf{susp})$, a pseudonym $nym$ and a public-private key pair $(pk_F, sk_F)$, and outputs a report $\mathsf{rpt}$ proving that $nym$ published suspicious transactions in a trading period.
- $(uk, v_{sum})/\perp \leftarrow \mathsf{Recover}(\mathsf{rpt}, pk_F, sk_S)$. This algorithm takes a report $\mathsf{rpt}$, Filter's public key $pk_F$ and Supervisor's private key $sk_S$ as input and check the validity of $\mathsf{rpt}$. If $\mathsf{rpt}$ is valid, it outputs the sender's public key $uk$ and the total payment amount $v_{sum}$ else outputs $\perp$.

### B. Ring Signature with Controllable Linkability

We propose a ring signature with controllable linkability, which is the building block of DAPCR. When signing a message, the signer needs to specify both a linker and an opener.

The CLRS scheme offers controllable linkability and collaborative revocability. The former means that only a designated linker can link signatures from the same sender. The latter means that the designated opener can revoke the anonymity of a signature with the assistance of the linker, but cannot do so independently, preventing the abuse of revocation authority and protecting privacy. Specifically, the linker can extract the pseudonym from each signature, and signatures with the same pseudonym come from the same signer. The opener can recover the signer's identity from any pseudonym but cannot directly open signatures. Table III compares the CLRS scheme with related works to show the advantage of our scheme.

The linker and opener are two semi-honest entities, which means they adhere to the CLRS protocol, do not actively sign messages, but can access ring signatures published in the system and attempt to identify the signers. We also assume that linker and opener do not collude.

A CLRS scheme consists of the following algorithms:

- $pp \leftarrow \mathsf{Setup}(1^\lambda)$. The algorithm takes a security parameter $\lambda$ as input, and generates a public parameter $pp$ which is implicitly input to other algorithms.
- $(pk_L, sk_L) \leftarrow \mathsf{LKGen}(pp)$. The algorithm outputs a public-private key pair $(pk_L, sk_L)$ for a linker.
- $(pk_O, sk_O) \leftarrow \mathsf{OKGen}(pp)$. The algorithm outputs a public-private key pair $(pk_O, sk_O)$ for an opener.
- $(uk, sk) \leftarrow \mathsf{UKGen}(pk_O)$. The algorithm takes an opener's public key $pk_O$ as input and outputs a user's public-private key pair $(uk, sk)$.
- $\sigma/\perp \leftarrow \mathsf{Sign}(R, m, pk_L, sk)$. The algorithm takes a ring $R = \{uk_0, uk_1, ..., uk_{n-1}\}$, a message $m$, a linker's public key $pk_L$ and a user's private key $sk$ as input. If $sk$ is the private key corresponding to $uk_j \in R$ and $j \in \{0, 1, ..., n-1\}$, the algorithm outputs a signature $\sigma$ of $(R, m)$ else outputs $\perp$.
- $0/1 \leftarrow \mathsf{Vfy}(R, m, \sigma, pk_L)$. The algorithm takes a ring $R$, a message $m$, a linker's public key $pk_L$ and a signature $\sigma$ as input and outputs a bit $b$. If $b = 1$, $\sigma$ is valid otherwise is invalid.
- $nym \leftarrow \mathsf{Ext}(\sigma, sk_L)$. The algorithm takes a valid signature $\sigma$ and a linker's private key $sk_L$ as input and outputs the signer's pseudonym $nym$.
- $\mathsf{link}/\mathsf{unlink} \leftarrow \mathsf{Link}(\sigma_0, \sigma_1, sk_L)$. A linker executes the algorithm CLRS.Ext to extract pseudonyms $nym_0$ and $nym_1$ from $\sigma_0$ and $\sigma_1$. If $nym_0 = nym_1$, this algorithm outputs $\mathsf{link}$ else outputs $\mathsf{unlink}$.
- $uk \leftarrow \mathsf{Open}(nym, sk_O)$. The algorithm takes a pseudonym $nym$ and an opener's private key $sk_O$ as input and outputs the user's public key $uk$.

In addition, a linker can prove that the same pseudonyms $nym$ are extracted from multiple signatures $\sigma_i|_{i=0}^{n-1}$. In other words, these signatures are signed by the same user with pseudonym $nym$.

- $\pi \leftarrow \mathsf{Prove}(\sigma_i|_{i=0}^{n-1}, nym, pk_L, sk_L)$. This algorithm takes several signatures $\sigma_i|_{i=0}^{n-1}$, a pseudonym $nym$ and a linker's public-private key pair $(pk_L, sk_L)$ as input, and outputs a proof $\pi$ of correct extraction.
- $0/1 \leftarrow \mathsf{Judge}(\sigma_i|_{i=0}^{n-1}, nym, \pi, pk_L)$. This algorithm takes several signatures $\sigma_i|_{i=0}^{n-1}$, a pseudonym $nym$, a proof $\pi$ and a linker's public key $pk_L$ as input, and outputs a bit $b$. If $b = 1$, $\pi$ is valid else is invalid.

### C. Security Definition

*1) Security Definition of CLRS:* First, we introduce two adversaries: $\mathcal{A}_1$, who has compromised the linker and obtained the linking key $sk_L$, and $\mathcal{A}_2$, who has compromised the opener and obtained the opening key $sk_O$. However, we assume that an adversary cannot compromise both the opener and the linker at the same time, because with $sk_L$ and $sk_O$, an adversary would be able to trace the signer of any valid signature. We believe that this assumption is realistic.

**Definition 1.** *We say that a CLRS scheme is secure if it satisfies correctness, unforgeability, anonymity, nym-extractability and nym-soundness.*

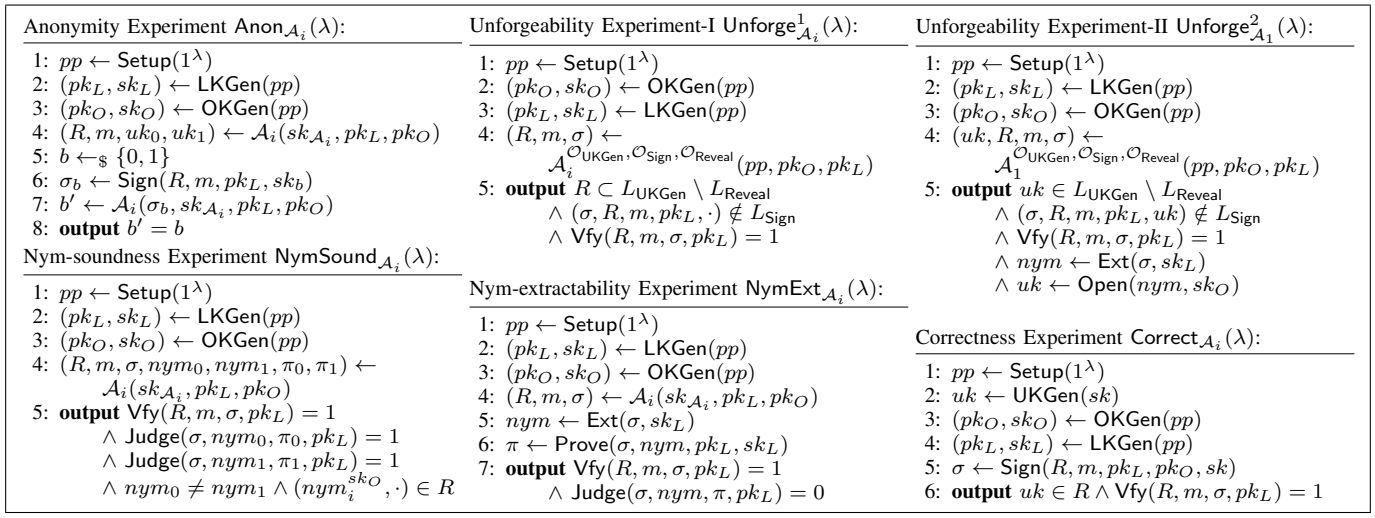



Fig. 3: Security experiments for CLRS.

1. *Correctness*. CLRS satisfies correctness if $\mathsf{Correct}_{\mathcal{A}_i}(\lambda) \leq \mathsf{negl}(\lambda)$.
2. *Unforgeability*. For $i \in \{1, 2\}$, $\mathcal{A}_i$ without any ring member's private key cannot forge a ring signature on behalf of the ring. In addition, $\mathcal{A}_1$ cannot forge a signature from which the pseudonym extracted is associated with an honest user. A CLRS scheme satisfies unforgeability if for any PPT adversary $\mathcal{A}_i$, $\mathsf{Unforge}^1_{\mathcal{A}_i}(\lambda) \leq \mathsf{negl}(\lambda)$, and for any PPT adversary $\mathcal{A}_1$, $\mathsf{Unforge}^2_{\mathcal{A}_1}(\lambda) \leq \mathsf{negl}(\lambda)$.
   In these two experiments,
   - $\mathcal{O}_{\mathsf{UKGen}}$ is an oracle that runs $(uk, sk) \leftarrow \mathsf{UKGen}(pk_O)$. $L_{\mathsf{UKGen}}$ is a list of users' public keys generated by this oracle;
   - $\mathcal{O}_{\mathsf{Reveal}}$ is an oracle which returns the corresponding secret key when queried on a user's public key $uk$. $L_{\mathsf{Reveal}}$ is a list of public keys of which the secret key has been revealed;
   - $\mathcal{O}_{\mathsf{Sign}}$ is an oracle that on query $(R, m, pk_L, uk)$ returns $\sigma \leftarrow \mathsf{Sign}(R, m, pk_L, sk)$. $L_{\mathsf{Sign}}$ is a list of the queries and responses $(\sigma, R, m, pk_L, uk)$.
3. *Anonymity*. CLRS satisfies anonymity if for any PPT adversary $\mathcal{A}_i$, $\mathsf{Anon}_{\mathcal{A}_i}(\lambda) \leq \mathsf{negl}(\lambda)$.
4. *Nym-extractability.* A linker can always extract the signer's pseudonym from a signature and generate a proof for correct extraction. CLRS satisfies nym-extractability if for any PPT adversary $\mathcal{A}_i$, $\mathsf{NymExt}_{\mathcal{A}_i}(\lambda) \leq \mathsf{negl}(\lambda)$ where $sk_{\mathcal{A}_1} = sk_L$ and $sk_{\mathcal{A}_2} = sk_O$.
5. *Nym-soundness.* Nym-soundness ensures that a linker cannot extract pseudonyms of two different signers from a signature, even if users in the ring are fully corrupt. CLRS satisfies nym-soundness if for any PPT adversary $\mathcal{A}_i$, $\mathsf{NymSound}_{\mathcal{A}_i}(\lambda) \leq \mathsf{negl}(\lambda)$.

Security experiments for CLRS are presented in Fig. 3.

*2) Security Definition of DAPCR:* We also present two adversaries for DAPCR: $\mathcal{A}'_1$ is an adversary that compromises Filter, while $\mathcal{A}'_2$ is an adversary that compromises Supervisor. However, we assume that adversaries cannot simultaneously compromise both Filter and Supervisor because they could collaborate to obtain private information from suspicious transactions. We believe this assumption is reasonable.

**Definition 2.** *We say that a DAPCR scheme is secure if it satisfies indistinguishability, $\mathcal{F}$-extractability, $\mathcal{F}$-soundness, balance and non-malleability.*

1. *Indistinguishability.* For $\mathcal{A}'_2$, the regulated transaction reveals no information about transaction privacy. For $\mathcal{A}'_1$, they can link regulated transactions from the same user but cannot obtain any additional information. We say that DAPCR satisfies indistinguishability if for any PPT adversary $\mathcal{A}'_i$, $\mathsf{Ind}_{\mathcal{A}'_i}(\lambda) \leq \mathsf{negl}(\lambda)$.
2. *$\mathcal{F}$-extractability*. Filter can extract the sender's pseudonym and the tag of payment amount from any valid transaction. DAPCR satisfies $\mathcal{F}$-extractability if for any PPT adversary $\mathcal{A}'_i$, $\mathsf{FExt}_{\mathcal{A}'_i}(\lambda) \leq \mathsf{negl}(\lambda)$ where $\mathsf{amount}(\mathsf{rtx})$ is the transaction rtx's payment amount, and $\mathsf{nym}(uk)$ is the pseudonym of the public key $uk$.
3. *$\mathcal{F}$-soundness*. For a valid regulated transaction, $\mathcal{F}$ cannot extract two different pseudonyms or tags from the transaction. DAPCR satisfies $\mathcal{F}$-soundness if for any PPT adversary $\mathcal{A}'_i$, $\mathsf{FSound}_{\mathcal{A}'_i}(\lambda) \leq \mathsf{negl}(\lambda)$.
4. *Balance.* The amount paid by any PPT adversaries cannot exceed their balance.
5. *Non-malleability.* No PPT adversary possesses the capability to modify the information contained within a regulated transaction rtx.

Security experiments for DAPCR are presented in Fig. 4.

## V. Ring Signature with Controllable Linkability

First, we introduce three NIZK protocols $\Pi_{enc}$, $\Pi_{dec}$ and $\Pi_{mem}$ that are used to construct CLRS.

### A. NIZK Protocols in CLRS

1. $\Pi_{mem} = (\mathcal{G}_{mem}, \mathcal{P}_{mem}, \mathcal{V}_{mem})$ represents a NIZK protocol for the relation

$$\mathcal{R}_{mem} = \left\{ \begin{array}{c} ((c_0, c_1, ..., c_{n-1}), (l, r), (g, h)) : \\ \forall i, c_i \in \mathbb{G} \wedge c_l = g^0 h^r \wedge \\ l \in \{0, 1, ..., n-1\} \wedge r \in \mathbb{Z}_q^* \end{array} \right\}.$$

Indistinguishability Experiment $\mathsf{Ind}_{\mathcal{A}'_i}(\lambda)$:

$param \leftarrow \mathsf{Setup}(1^\lambda)$
$(pk_F, sk_F) \leftarrow \mathsf{FInit}(param)$
$(pk_S, sk_S) \leftarrow \mathsf{SInit}(param)$
$(T_0, T_1, R) \leftarrow \mathcal{A}'_i(sk_{\mathcal{A}'_i}, pk_S, pk_F)$
  where $T_j = (addr_{S_j}, addr_{R_j}, v_j, uk_j) \wedge uk_0, uk_1 \in R$
$\mathsf{rtx}_b \leftarrow$
  $\mathsf{RtxGen}(addr_{S_b}, addr_{R_b}, v_b, uk_b, sk_b, R, \cdot)$
$b' \leftarrow \mathcal{A}'_i(\mathsf{rtx}_b, sk_{\mathcal{A}'_i}, pk_S, pk_F)$
**output** $b' = b$

$\mathcal{F}$-extractability Experiment $\mathsf{FExt}_{\mathcal{A}'_i}(\lambda)$:

$param \leftarrow \mathsf{Setup}(1^\lambda)$
$(pk_F, sk_F) \leftarrow \mathsf{FInit}(param)$
$(pk_S, sk_S) \leftarrow \mathsf{SInit}(param)$
$(R, \mathsf{rtx}, v, r) \leftarrow \mathcal{A}'_i(sk_{\mathcal{A}'_i}, pk_S, pk_F)$
$(nym, tag_v) \leftarrow \mathsf{Extract}(\mathsf{rtx}, sk_F)$
**output** $v \neq \mathsf{amount}(\mathsf{rtx})$
  $\vee\ (\forall\ uk \in R, nym \neq \mathsf{nym}(uk))$
  $\wedge\ \mathsf{Verify}(\mathsf{rtx}, pk_F, R, I_{\mathsf{pub}}) = 1$

$\mathcal{F}$-soundness Experiment $\mathsf{FSound}_{\mathcal{A}'_i}(\lambda)$:

$param \leftarrow \mathsf{Setup}(1^\lambda)$
$(pk_F, sk_F) \leftarrow \mathsf{FInit}(param)$
$(pk_S, sk_S) \leftarrow \mathsf{SInit}(param)$
$(R, \mathsf{rtx}, T_0, T_1) \leftarrow \mathcal{A}'_i(sk_{\mathcal{A}'_i}, pk_S, pk_F)$
  where $T_j = (nym_j, tag_j, \pi_j^{nym}, \pi_j^{tag})$
**output** $V_{nym}(\mathsf{rtx}, nym_0, \pi_0^{nym}) = 1$
  $\vee\ V_{nym}(\mathsf{rtx}, nym_1, \pi_1^{nym}) = 1$
  $\vee\ V_{tag}(\mathsf{rtx}, tag_0, \pi_0^{tag}) = 1$
  $\vee\ V_{tag}(\mathsf{rtx}, tag_1, \pi_1^{tag}) = 1$
  $\wedge\ \mathsf{Verify}(\mathsf{rtx}, pk_F, R, I_{\mathsf{pub}}) = 1$

Fig. 4: Security experiments for DAPCR.

$\Sigma$-protocols for the above relation called *one-out-of-many proofs* [25] that can be used to prove that one out of many commitments can be opened to 0 without requiring the prover to possess knowledge of the openings of the other commitments. Applying the Fiat-Shamir transform to this $\Sigma$-protocol yields the NIZK protocol $\Pi_{mem}$. Moreover, the protocol $\Pi_{mem}$ requires no trusted setup, and the proof size is logarithmic in the number of all commitments.

2. $\Pi_{enc} = (\mathcal{G}_{enc}, \mathcal{P}_{enc}, \mathcal{V}_{enc})$ represents a NIZK protocol for the relation

$$\mathcal{R}_{enc} = \left\{ \begin{array}{c} ((c,u),(\alpha,\beta),(g,h,pk)) : \\ c = g^\alpha h^\beta \wedge u = pk^\beta \wedge pk, g, h \in \mathbb{G} \wedge \alpha, \beta \in \mathbb{Z}_q^* \end{array} \right\}.$$

The protocol allows one to prove the correctness of an ElGamal ciphertext $ct = (c, u)$ given a specific public key $pk$.

3. $\Pi_{dec} = (\mathcal{G}_{dec}, \mathcal{P}_{dec}, \mathcal{V}_{dec})$ represents a NIZK protocol for the relation

$$\mathcal{R}_{dec} = \left\{ \begin{array}{c} ((ct_i = (c_i, u_i)|_{i=0}^{n-1}, m, pk), sk, h) : \\ \forall i, c_i = m \cdot u_i^{\frac{1}{sk}} \wedge pk = h^{sk} \wedge \\ m, pk, h, c_i, u_i \in \mathbb{G} \wedge sk \in \mathbb{Z}_q^* \end{array} \right\}$$

where $ct_i|_{i=0}^{n-1}$ are $n$ ElGamal ciphertexts. This protocol can prove that the decryption result of multiple ciphertexts is the same plaintext $m$.

Similar to the protocol $\Pi_{mem}$, $\Pi_{enc}$ and $\Pi_{dec}$ are respectively transformed from interactive protocols $\Sigma_{enc}$ and $\Sigma_{dec}$, of which the details are presented in Appendix A.

### B. Construction of CLRS

An efficient construction of CLRS consists of the following algorithms:

- Setup. Consider two big prime numbers $p$ and $q$. Elliptic curve cryptography (ECC) is based on the use of non-singular elliptic curves $E$ on $F_p$. $g$ is a generator of group $\mathbb{G}$, which is a cyclic group with order $q$. ECC can be given by a tuple $(\mathbb{G}, g, p, q)$. The public parameter used in our construction is denoted by $pp = (g, h, \mathcal{H})$ where $g, h \in \mathbb{G}$ and the hash function $\mathcal{H} : \{0,1\}^* \rightarrow \mathbb{Z}_q^*$. $pp$ is implicitly input to other algorithms.
- LKGen. A linker randomly samples private key $sk_L \in \mathbb{Z}_q^*$ and computes $pk_L = h^{sk_L}$.
- OKGen. An opener randomly samples private key $sk_O \in \mathbb{Z}_q^*$ and computes $pk_O = g^{sk_O}$.
- UKGen. A user randomly samples private key $sk \in \mathbb{Z}_q^*$ and computes $pk = pk_O^{sk}$. Then the user randomly samples $r \in \mathbb{Z}_q^*$ and computes $c = g^{sk} h^r$. Lastly, the user generates a proof $\pi_{enc} \leftarrow \mathcal{P}_{enc}((c, pk), (r, sk), (h, g, pk_O))$. The user outputs $uk = (pk, c, \pi_{enc})$.
  One calculates $b \leftarrow \mathcal{V}_{enc}((c, pk), \pi_{enc}, (h, g, pk_O))$ to verify the validity of $uk$. If $b = 1$, $uk$ is valid else is invalid. When implementing this scheme in the blockchain, a smart contract can be deployed as a bulletin board, which is responsible for verifying the validity of public keys and recording valid public keys.
- Sign. To generate a signature $\sigma$ for $m$, a user with the key pair $(uk, sk)$ chooses a ring $R = \{uk_0, ..., uk_{n-1}\}$ that satisfies $uk = uk_j \in R$. Initially, the user randomly samples $k \in \mathbb{Z}_q^*$ and computes $com = g^{sk} h^k$ and $K = pk_L^k$. Subsequently, the user extracts $c_i$ from each public key $uk_i \in R$, computes $c'_i = c_i / com$ and gets a new ring $R' = \{c'_0, c'_1, ..., c'_{n-1}\}$ where $c'_j = g^0 h^{r-k}$. The user also calculates a proof $\pi_{mem} \leftarrow \mathcal{P}_{mem}(R', (j, r-k), (g, h))$. Then the user randomly chooses $x_1, x_2 \in \mathbb{Z}_q^*$ and computes

$$\begin{aligned} com' &= g^{x_1} h^{x_2}, \\ K' &= pk_L^{x_2}, \\ e &= \mathcal{H}(R|m|com|com'|K|K'), \\ y_1 &= x_1 + e \cdot sk, \\ y_2 &= x_2 + e \cdot k. \end{aligned} \tag{1}$$

  $\pi = (com', K', y_1, y_2)$ is a signature of knowledge proving $((com, K), (sk, k)) \in \mathcal{R}_{enc}$. The formula 1 is a SoK scheme transformed from the interactive protocol $\Sigma_{enc}$ presented in Appendix A-A. Finally, the user outputs $\sigma = (com, K, \pi_{mem}, \pi)$.
- Vfy. Given a signature $\sigma$ of $(R, m)$, an opener's public key $pk_O$ and a linker's public key $pk_L$, a verifier checks the validity of $\sigma$. Initially, the verifier computes $R'$ and $b \leftarrow \mathcal{V}_{mem}(R', \pi_{mem}, (g, h))$. If $b = 0$, $\sigma$ is invalid. Otherwise, the verifier computes $e = \mathcal{H}(R|m|com|com'|K|K')$ checks whether the following equations hold.

$$\begin{aligned} g^{y_1} h^{y_2} &\stackrel{?}{=} com' com^e \\ pk_L^{y_2} &\stackrel{?}{=} K' K^e \end{aligned}$$

If both of the above equations hold, $\sigma$ is valid else is invalid.

- Ext. Given a valid signature $\sigma = (com, K, \pi_{mem}, \pi)$ and a linker's private key $sk_L$, the linker computes the pseudonym $nym = K^{-\frac{1}{sk_L}} com = g^{sk}$ of the signer.
- Link. Given two signatures $\sigma_0$, $\sigma_1$ and a linker's private key $sk_L$, a linker executes the algorithm CLRS.Ext to extract $nym_0 = g^{sk_0}$ and $nym_1 = g^{sk_1}$ from $\sigma_0$ and $\sigma_1$. If $nym_0 = nym_1$ the algorithm outputs link else outputs unlink.
- Open. Given a pseudonym $nym$ and an opener's private key $sk_O$, an opener recovers $pk = nym^{sk_O} = pk_O^{sk}$ from the pseudonym and outputs $uk = (pk, \cdot)$.

In addition, a linker can execute the algorithm CLRS.Prove to prove that a pseudonym $nym$ is extracted from $\sigma$ without revealing the private key $sk_L$. One can verify whether $nym$ is extracted from $\sigma$.

- Prove. Given several signatures $\sigma_i = (com_i, K_i, \cdot)$ for $i \in \{0, 1, ..., n-1\}$, a pseudonym $nym$ and a linker's private key $sk_L$, the linker computes

$$\pi_{dec} \leftarrow \mathcal{P}_{dec}(((com_i, K_i)|_{i=0}^{n-1}, nym, pk_L), sk_L, h)$$

to prove correct decryption of $(com_i, K_i)|_{i=0}^{n-1}$ and that the decryption result of these ciphertexts is a same message $nym$. In other words, $\pi_{dec}$ is a proof for that several signatures are published by the same signer with pseudonym $nym$.
- Judge. Given several signatures $\sigma_i = (com_i, K_i, \cdot)$ for $i \in \{0, 1, ..., n-1\}$, a pseudonym $nym$, a linker's public key $pk_L$ and a proof $\pi_{dec}$, one can compute

$$b \leftarrow \mathcal{V}_{dec}(((com_i, K_i)|_{i=0}^{n-1}, nym, pk_L), \pi_{dec}, h).$$

If $b = 1$, $\pi_{dec}$ is valid else is invalid.

**Theorem 1.** *The proposed CLRS scheme satisfies correctness.*

PROOF. For a ring $R = \{uk_0, uk_1, ..., uk_{n-1}\}$, the user with public key $uk_j = (pk_j, c_j, \pi_{enc}) = (pk_O^{sk}, g^{sk}h^r, \pi_{enc})$ calculates $\sigma = (com, K, \pi_{mem}, \pi)$ of $(R, m)$ that satisfies $\sigma \leftarrow \mathsf{CLRS.Sign}(R, m, pk_L, sk)$.

To verify the validity of $\sigma$, the verifier first computes $c_i' = c_i/com$ for $i \in \{0, 1, ..., n-1\}$ and gets $R' = \{c_0', c_1', ..., c_{n-1}'\}$. If the NIZK protocol $\Pi_{mem}$ satisfies completeness and the SoK protocol for the relation $\mathcal{R}_{enc}$ satisfies correctness, both $\pi_{mem}$ and $\pi$ can be successfully verified. The signature $\sigma$ will also pass the verification as a result.

Therefore, if the NIZK protocol used in the CLRS scheme satisfies completeness, and the SoK protocol satisfies correctness, then the CLRS scheme achieves correctness. □

## VI. DAP WITH COLLABORATIVE REGULATION

In this section, we present an efficient DAPCR scheme based on a CLRS scheme and several NIZK protocols.

### A. NIZK Protocols in DAPCR

First, we introduce two NIZK protocols $\Pi_{log}$ and $\Pi_v$ that are used to construct the DAPCR scheme.

1. $\Pi_{log} = (\mathcal{G}_{log}, \mathcal{P}_{log}, \mathcal{V}_{log})$ represents the NIZK protocol for the relation

$$\mathcal{R}_{log} = \{(A, \alpha, g) : A = g^{\alpha} \wedge g \in \mathbb{G} \wedge \alpha \in \mathbb{Z}_q^*\}.$$

The protocol can prove the knowledge of a discrete logarithm while ensuring the confidentiality of its actual value.

2. $\Pi_v = (\mathcal{G}_v, \mathcal{P}_v, \mathcal{V}_v)$ represents the NIZK protocol for the relation

$$\mathcal{R}_v = \left\{ \begin{array}{c} ((\mathsf{tx}, c, I_{\mathsf{pub}}, \hbar, g, h), (addr_S, addr_R, I_{\mathsf{pri}}, v, s, r)) : \\ \mathsf{tx} \leftarrow \mathsf{TxGen}(addr_S, addr_R, v, s, I_{\mathsf{pub}}, I_{\mathsf{pri}}) \wedge \\ c = g^v h^r \wedge g, h \in \mathbb{G} \wedge v, r \in \mathbb{Z}_q^* \end{array} \right\}.$$

The protocol can prove that a committed value in $c$ is the payment amount $v$ of a privacy-preserving transaction tx. The details of this protocol are introduced in Appendix A.

### B. Construction of DAPCR

An efficient DAPCR scheme consists of four phases: the preparation phase, the transaction phase, the verification phase, and the regulation phase.

*1) Preparation Phase:* The public parameter is published. Entities generate their public and private keys, and new users obtain permission from consensus nodes to join the blockchain.

**Setup.** Consensus nodes execute

$$(g, h, \mathcal{H}) \leftarrow \mathsf{CLRS.Setup}(1^{\lambda}),$$
$$crs \leftarrow \mathcal{G}_v(1^{\lambda}, \mathcal{R}_v),$$

and output $param = (g, h, \mathcal{H}, crs)$ that is implicit input to other algorithms.

**FInit.** $\mathcal{F}$ executes the algorithm CLRS.LKGen to get the public-private key pair $(pk_L, sk_L)$ and publishes their public key $pk_F = pk_L$.

**SInit.** $\mathcal{S}$ executes the algorithm CLRS.OKGen to get the public-private key pair $(pk_O, sk_O)$ and publishes their public key $pk_S = pk_O$.

**KeyGen.** A user computes $(uk, sk) \leftarrow \mathsf{CLRS.UKGen}(pk_O)$ and publishes their public key $uk$.

**Join.** A new user $\mathcal{U}$ randomly samples $\beta \in \mathbb{Z}_q^*$, and computes $B = g^{\beta}$ and $w = \mathcal{H}(pk_O^{\beta}|uk)$. Then $\mathcal{U}$ sends a request $\mathsf{req} = (uk, B, \mu)$, where $\mu$ is his payment limit within each trading period, to consensus nodes. To join the blockchain, $\mathcal{U}$ must obtain permission from all nodes. If successful, $(uk, B, \mu)$ will be recorded on the chain.

**Register.** If a new user with public key $uk$ has joined the blockchain, $\mathcal{S}$ reads $\mathsf{req} = (uk, B, \mu)$ from the decentralized ledger, and computes $w = \mathcal{H}(B^{sk_O}|uk)$, $tag_{\mu} = g^{\mu}h^w$ and $nym = pk^{\frac{1}{sk_O}}$. Then, $\mathcal{S}$ adds $(uk, \mu, nym, \mathsf{tag}_{\mu})$ to the list $L_{\mathcal{S}}$. Moreover, $\mathcal{S}$ sends $(nym, tag_{\mu})$ to $\mathcal{F}$. Once $(nym, tag_{\mu})$ is received, $\mathcal{F}$ adds it to the list $L_{\mathcal{F}}$.

*2) Transaction Phase:* Users are allowed to submit two types of transactions: regulated transactions with privacy preservation and public transactions. In a trading period, the total amount paid by a user through regulated transactions cannot exceed their privacy payment limit.

**RtxGen.** Suppose that a user $\mathcal{U}$ intends to submit $n$ regulated transactions within a trading period. To submit the $i$-th regulated transaction $\mathsf{rtx}_i$, for $i \in \{0, 1, ..., n-2\}$, $\mathcal{U}$ randomly samples $z_i, w_i \in \mathbb{Z}_q^*$ and computes

$$\mathsf{tx}_i \leftarrow \mathsf{DAP.TxGen}(addr_S, addr_R, v_i, s, I_{\mathsf{pub}}, I_{\mathsf{pri}}),$$
$$\sigma_i \leftarrow \mathsf{CLRS.Sign}(R_i, ct_i, pk_L, sk),$$
$$ct_i = (g^{v_i}h^{z_i}, pk_L^{z_i - w_i}) = (c_i, u_i),$$

where $ct_i$ is a ElGamal ciphertext of a pedersen commitment $g^{v_i}h^{w_i}$. Then $\mathcal{U}$ computes

$$\pi_i^{log} \leftarrow \mathcal{P}_{log}(u_i, z_i - w_i, pk_L),$$
$$\pi_i^{v} \leftarrow \mathcal{P}_{v}((\mathsf{tx}_i, c_i, I_{\mathsf{pub}}, \hbar_i, g, h), (addr_S, addr_R, I_{\mathsf{pri}}, v_i, s, z_i), crs),$$

where $\hbar_i = \mathcal{H}(\sigma_i)$, and sets $\mathsf{rtx}_i = (\mathsf{tx}_i, ct_i, \pi_i^v, \pi_i^{log}, \sigma_i)$.

For the final regulated transaction $\mathsf{rtx}_{n-1}$ within a trading period, $\mathcal{U}$ calculates $w_{n-1} = w - \sum_{i=0}^{n-2} w_i$ instead of sampling a random number. All other operations remain unchanged. Therefore, if the total payment amount within the trading period equals $\mathcal{U}$'s privacy payment limit $\mu$, the equation $\prod_{i=0}^{n-1} g^{v_i}h^{w_i} = tag_\mu$ holds.

If $\mathcal{U}$ exhausts his payment limit $\mu$ but still needs to transact, he can submit a public trans-action, which doesn't affect his payment limit. To generate a public transaction $\mathsf{tx}_{pub}$, $\mathcal{U}$ calculates a DAP transaction $\mathsf{tx}$, attaches $(addr_S, addr_R, v)$ to it, and generates a proof $\pi_{pub}$ that confirms $addr_S$ as the sender's address, $addr_R$ as the receiver's address, and $v$ as the payment amount of $\mathsf{tx}$. Finally, $\mathcal{U}$ gets a public transaction $\mathsf{tx}_{pub} = (\mathsf{tx}, addr_S, addr_R, v, \pi_{pub})$. As the public transactions is not a primary focus of this paper, we will refrain from delving into their specifics.

*3) Verification Phase:* Consensus nodes (or smart contracts) verify the validity of regulated transactions, and only valid transactions are sealed into blocks.

**Verify.** For a regulated transaction $\mathsf{rtx} = \{\mathsf{tx}, ct, \pi_v, \pi_{log}, \sigma\}$, one executes the following steps to verify the validity of $\mathsf{rtx}$:

$$b_1 \leftarrow \mathsf{DAP.TxVfy}(\mathsf{tx}, I_{\mathsf{pub}}),$$
$$b_2 \leftarrow \mathsf{CLRS.Vfy}(R, ct, \sigma, pk_L),$$
$$b_3 \leftarrow \mathcal{V}_v((\mathsf{tx}, c, I_{\mathsf{pub}}, \hbar, g, h), \pi_v, crs),$$
$$b_4 \leftarrow \mathcal{V}_{log}(u, \pi_{log}, pk_L),$$

where $\hbar = \mathcal{H}(\sigma)$. If all validations pass, $\mathsf{rtx}$ is valid and the valid transaction is sealed into a new block.

*4) Regulation Phase:* $\mathcal{F}$ screens out suspicious transactions and submits the report on suspicious transactions to $\mathcal{S}$. Once the report is received, $\mathcal{S}$ obtains the sender's public key and payment amounts of suspicious transactions.

**Extract.** For a valid transaction $\mathsf{rtx}_i = (ct_i, \sigma_i, \cdot)$, $\mathcal{F}$ extracts the signer's pseudonym and the amount tag from $\mathsf{rtx}_i$.

$$nym_i \leftarrow \mathsf{CLRS.Ext}(\sigma_i, sk_L),$$
$$tag_{v_i} = c_i u_i^{-\frac{1}{sk_L}} = g^{v_i}h^{w_i}.$$

**Detect.** Let $S = \{\mathsf{rtx}_0, \mathsf{rtx}_1, ..., \mathsf{rtx}_{n-1}\}$ be the set of all $n$ transactions submitted by some user during a trading period. $\mathcal{F}$ can link these transactions according to the user's pseudonym $nym$.

To determine if the transaction behavior of a user with pseudonym $nym$ complies with the transaction rules, $\mathcal{F}$ searches for $tag_\mu$ corresponding to $nym$ in $L_{\mathcal{F}}$ and computes $tag_{sum} = \prod_{i=0}^{n-1} tag_{v_i}$ where $tag_{v_i}|_{i=0}^{n-1}$ are extracted from all transactions published by the user with pseudonym $nym$ in a period. Considering that $tag_\mu = g^\mu h^w$ and $tag_{sum} = g^{\sum_{i=0}^{n-1} v_i} h^{\sum_{i=0}^{n-1} w_i} = g^{\sum_{i=0}^{n-1} v_i} h^w$, the total payment volume is equal to the upper limit if $tag_\mu = tag_{sum}$. Otherwise, $\mathcal{F}$ flags these transactions in $S$ as suspicious.

To increase supervision efficiency, $\mathcal{F}$ can aggregate ciphertexts and then extract $tag_{sum}$:

$$C = \prod_{i=0}^{n-1} c_i = g^{\sum_{i=0}^{n-1} v_i} h^{\sum_{i=0}^{n-1} z_i},$$
$$U = \prod_{i=0}^{n-1} u_i = pk_L^{\sum_{i=0}^{n-1}(z_i - w_i)},$$
$$tag_{sum} = CU^{-\frac{1}{sk_L}}.$$

**Report.** For a set $S = \{\mathsf{rtx}_0, \mathsf{rtx}_1, ..., \mathsf{rtx}_{n-1}\}$ of suspicious transactions, $\mathcal{F}$ proves that these transactions are published by the user with pseudonym $nym$ and $tag_{sum}$ is formed by aggregating the amount tags extracted from these transactions. $\mathcal{F}$ first generates a proof $\pi_{dec}$:

$$\pi_{dec} \leftarrow \mathcal{P}_{dec}((ct_i|_{i=0}^{n-1}, nym, pk_L), sk_L, h).$$

Then $\mathcal{F}$ generates a proof $\pi_{sum}$ for that $tag_{sum}$ is extracted from $(C, U)$:

$$\pi_{sum} \leftarrow \mathcal{P}_{dec}((C, U, tag_{sum}, pk_L), sk_L, h).$$

Finally, $\mathcal{F}$ sends $\mathsf{rpt} = (S, nym, \pi_{dec}, tag_{sum}, \pi_{sum})$ to $\mathcal{S}$.

**Recover.** Once a report $\mathsf{rpt} = (S, nym, \pi_{dec}, tag_{sum}, \pi_{sum})$ is received, $\mathcal{S}$ checks whether the transactions in $S$ are submitted by the same user with pseudonym $nym$:

$$b_1 \leftarrow \mathcal{V}_{dec}((ct_i|_{i=0}^{n-1}, nym, pk_L), \pi_{dec}, h).$$

Then $\mathcal{S}$ checks the validity of $tag_{sum}$:

$$b_2 \leftarrow \mathcal{V}_{dec}((\prod_{i=0}^{n-1} c_i, \prod_{i=0}^{n-1} u_i, tag_{sum}, pk_L), \pi_{sum}, h).$$

If $b_1 = 0 \vee b_2 = 0$, $\mathsf{rpt}$ is invalid else is valid. For a valid report $\mathsf{rpt}$, $\mathcal{S}$ computes $uk \leftarrow \mathsf{CLRS.Open}(nym, sk_O)$ and then requires the user with public key $uk$ to submit $v_{sum}$ and $w'$ satisfies $tag_{sum} = g^{v_{sum}}h^{w'}$.

**Theorem 2.** *The proposed DAPCR scheme satisfies correctness.*

PROOF. For any transaction $\mathsf{rtx}_i = (\mathsf{tx}_i, ct_i, \pi_i^v, \pi_i^{log}, \sigma_i)$ generated according to the DAPCR.RtxGen algorithm, a verifier employs the DAPCR.Verify algorithm to ascertain the validity of $\mathsf{rtx}_i$. This verification process involves verifying $\mathsf{tx}_i$, $\pi_i^v$, $\pi_i^{log}$ and $\sigma_i$. Only if all of these components pass validation, the transaction $\mathsf{rtx}_i$ is deemed valid.

NIZK protocols $\Pi_v$ and $\Pi_{log}$ used to construct the DAPCR scheme satisfies completeness. Additionally, both the DAP scheme and the CLRS scheme satisfy correctness. Hence, each of $\mathsf{tx}_i$, $\pi_i^v$, $\pi_i^{log}$ and $\sigma_i$ can successfully pass the validation, meaning that the transaction $\mathsf{rtx}_i$ is validity. □

## VII. SECURITY ANALYSIS

In this section, we analyze the security of our scheme.

### A. Security Analysis of CLRS

**Theorem 3.** *No PPT adversary can break the anonymity of CLRS with non-negligible advantage.*

PROOF. We use $\mathsf{G}^{\mathsf{Ano}}_{\mathsf{real}}$ to denote the anonymity experiment $\mathsf{exp}^{\mathsf{Ano}}_{\mathcal{A}_i}(\lambda)$ executed in the real world. To analyze the security of CLRS, we design a simulation $\mathsf{G}^{\mathsf{Ano}}_{\mathsf{sim}}$. In $\mathsf{G}^{\mathsf{Ano}}_{\mathsf{sim}}$, the challenger $\mathcal{C}$ interacts with $\mathcal{A}_1$ (or $\mathcal{A}_2$) as in $\mathsf{G}^{\mathsf{Ano}}_{\mathsf{real}}$. The only modification is that $\mathcal{C}$ outputs a signature $\sigma_{sim} = (com_{sim}, K, \pi^{mem}_{sim}, \pi_{sim})$ of $(R, m)$ in which $com_{sim}$, $K$, $\pi^{mem}_{sim}$ and $\pi_{sim}$ are independent of $uk_0$ and $uk_1$.

**Game** $\mathsf{G}^{\mathsf{Ano}}_{\mathsf{real}}$. With the purpose of initializing this experiment, $C$ first computes

$$crs_{mem} \leftarrow \mathcal{G}_{mem}(1^\lambda, \mathcal{R}_{mem}),$$
$$pp_{enc} \leftarrow \mathsf{SoK.Setup}(1^\lambda, \mathcal{R}_{enc}),$$

and other public parameters. $C$ further computes public keys of honest users, and $R_1$ represents the set of these public keys.

Next, $\mathcal{A}_i$ generates a ring $R_2$, a message $m$ and two public keys $uk_0, uk_1$ satisfies $uk_0, uk_1 \in R_1 \wedge R_2$. $\mathcal{A}_i$ sends $(R_2, m, uk_0, uk_1)$ to $C$.

Once $(R_2, m, uk_0, uk_1)$ is received, $C$ randomly chooses $b \in \{0, 1\}$ and computes

$$\sigma_b \leftarrow \mathsf{CLRS.Sign}(R_2, m, pk_L, sk_b)$$

where $\sigma_b = (com, K, \pi_{mem}, \pi) = (g^{sk_b} h^k, pk^k_L, \pi_{mem}, \pi)$. $C$ sends $\sigma_b$ to $\mathcal{A}_i$.

After receiving $\sigma_b$, $\mathcal{A}_i$ attempts to determine which public key was used to compute the signature. $\mathcal{A}_1$ can query an oracle $\mathcal{Q}_{\mathsf{Ext}}$ to extract pseudonyms from signatures. $\mathcal{A}_2$ can query an oracle $\mathcal{Q}_{\mathsf{Open}}$ to obtain a public key from a given pseudonym. Finally, $\mathcal{A}_i$ makes a guess $b'$ for the value of $b$, and wins the experiment if and only if $b = b'$.

**Game** $\mathsf{G}^{\mathsf{Ano}}_{\mathsf{sim}}$. During the initialization phase, $C$ computes the public parameters and public keys of honest users, denoted by $R_1$. The only difference is that trapdoors are generated alongside the computation of parameters $crs_{mem}$ and $pp_{enc}$.

$$(crs_{mem}, \tau_{mem}) \leftarrow \mathcal{G}^{mem}_{sim}(1^\lambda, \mathcal{R}_{mem})$$
$$(pp_{enc}, \tau_{enc}) \leftarrow \mathsf{SoK.Setup}_{sim}(1^\lambda, \mathcal{R}_{enc})$$

Similar to the case in $\mathsf{G}^{\mathsf{Ano}}_{\mathsf{real}}$, $\mathcal{A}_i$ generates a ring $R_2$, a message $m$ and two public keys $uk_0, uk_1$ satisfies $uk_0, uk_1 \in R_1 \wedge R_2$. $\mathcal{A}_i$ sends $(R_2, m, uk_0, uk_1)$ to $C$.

Once $(R_2, m, uk_0, uk_1)$ is received, $C$ randomly chooses $b \in \{0, 1\}$ and $com_{sim} \in \mathbb{G}^*$. Then $C$ computes

$$\pi^{mem}_{sim} \leftarrow \mathcal{P}^{mem}_{sim}(R'_2, \tau_{mem}, crs_{mem}),$$
$$\pi_{sim} \leftarrow \mathsf{SoK.Sign}_{sim}((com_{sim}, K, pk_L), \tau_{enc}, pp_{enc}),$$

and sends $\sigma_{sim} = \{com_{sim}, K, \pi^{mem}_{sim}, \pi_{sim}\}$ to $\mathcal{A}_i$. Since $\sigma_{sim}$ is independent of $uk_0$ and $uk_1$, the advantage of $\mathcal{A}_i$ in winning $\mathsf{G}^{\mathsf{Ano}}_{\mathsf{sim}}$ is negligible.

Based on the simulatability of a SoK protocol and the zero-knowledge of a NIZK protocol, it can be deduced that the distribution of $(com, \pi_{mem}, \pi)$ is equivalent to that of $(com_{sim}, \pi^{mem}_{sim}, \pi_{sim})$. While $\mathcal{A}_1$ can query $\mathcal{Q}_{\mathsf{Ext}}$ to extracted $nym$ from $\sigma_b$ and $nym'$ from $\sigma_{sim}$, $nym = pk_b^{1/sk_O}$ and $nym'$ are indistinguishable since $sk_O$ is unknown to $\mathcal{A}_1$. Therefore, $\sigma_{sim}$ in $\mathsf{G}^{\mathsf{Ano}}_{\mathsf{sim}}$ and $\sigma_b$ in $\mathsf{G}^{\mathsf{Ano}}_{\mathsf{real}}$ are indistinguishable. Considering the negligible advantage of $\mathcal{A}_i$ in winning $\mathsf{G}^{\mathsf{Ano}}_{\mathsf{sim}}$ and the indistinguishability between $\mathsf{G}^{\mathsf{Ano}}_{\mathsf{real}}$ and $\mathsf{G}^{\mathsf{Ano}}_{\mathsf{sim}}$, the advantage of $\mathcal{A}_i$ in winning $\mathsf{G}^{\mathsf{Ano}}_{\mathsf{real}}$ is also negligible. □

**Theorem 4.** *No PPT adversary can break the unforgeability of CLRS with non-negligible advantage.*

PROOF. For a discrete logarithm problem $(g, A = g^a)$ where $g \in \mathbb{G}$ and $a \in \mathbb{Z}^*_q$, if $\mathcal{A}_i$ breaks the unforgeability of CLRS, $C$ can make use of $\mathcal{A}_i$ to solve the discrete logarithm problem. Adversaries can win the experiments in two ways:

**Case-1**: For an honest ring, i.e. all members in the ring are honest, $\mathcal{A}_i$ generates a valid signature.

To initialize this experiment, $C$ first executes

$$(crs_{enc}, \tau_{enc}) \leftarrow \mathcal{G}^{enc}_{sim}(1^\lambda, \mathcal{R}_{enc}).$$

For $j \in \{0, 1, ..., l\}$, $C$ randomly chooses $r_j \in \mathbb{Z}^*_q$ and $pk_j \in \mathbb{G}^*$, and computes

$$\pi'_j \leftarrow \mathcal{P}^{enc}_{sim}((h^{r_j} A, pk_j), \tau_{enc}, crs_{enc}).$$

Then $C$ sets $c_j = h^{r_j} A$ and publishes $uk_j = (pk_j, c_j, \pi'_j)$. $L_\tau$ represents the set of public keys generated based on the trapdoor. $\mathcal{A}_i$ can query $C$ for user's secret keys and signatures. If $\mathcal{A}_i$ queries $C$ for the secret key corresponding to $uk$, the experiment aborts.

To win the experiment, $\mathcal{A}_i$ forges a signature $\sigma = (com, K, \pi_{mem}, \pi)$ of $(R, m)$ where $R \subset L_{\mathsf{UKGen}} \setminus L_{\mathsf{Reveal}}$ and $(\sigma, R, m, pk_L, \cdot) \notin L_{\mathsf{Sign}}$. The probability that $R \subset L_\tau$ is $\frac{1}{\eta_1(\lambda)}$ where $\eta_1(\lambda)$ is a polynomial. By the extractability of a SoK protocol, $C$ can extract a valid witness $\varpi = (a, k)$ to the statement $\phi = (com, K)$ from $\pi$. Therefore, $C$ can make use of $\mathcal{A}_i$ to obtain $a$ such that $A = g^a$.

**Case-2**: There exists an honest user with $uk = (pk, c, \pi_{enc})$ whose private key $sk$ is unknown to $\mathcal{A}_1$. $\mathcal{A}_1$ calculates a signature $\sigma$ of $(R, m)$ and a proof $\pi_{dec}$ for that $nym$ is correctly extracted from $\sigma$. If $nym$ is the pseudonym of $uk$, and $\sigma$ and $\pi_{dec}$ are valid, $\mathcal{A}_1$ successfully breaks the unforgeability of CLRS.

To initialize this experiment, $C$ first executes

$$(crs_{enc}, \tau_{enc}) \leftarrow \mathcal{G}^{enc}_{sim}(1^\lambda, \mathcal{R}_{enc}).$$

Then $C$ randomly chooses $r \in \mathbb{Z}^*_q$ and computes

$$\pi^{enc}_{sim} \leftarrow \mathcal{P}^{enc}_{sim}((h^r A, A^{sk_O}), \tau_{enc}, crs_{enc}).$$

Finally, $C$ publishes $uk = (A^{sk_O}, h^r A, \pi^{enc}_{sim}) = (pk, c, \pi^{enc}_{sim})$.

To win the experiment, $\mathcal{A}_1$ randomly selects an honest user and forges a signature $\sigma = (com, K, \pi_{mem}, \pi)$ of $(R, m)$ where $uk \notin L_{\mathsf{UKGen}} \setminus L_{\mathsf{Reveal}}$ and $(\sigma, R, m, pk_L, uk) \notin L_{\mathsf{Sign}}$. Assuming $\mathcal{A}_1$ selects $uk$ with a probability $\frac{1}{\eta_2(\lambda)}$, where $\eta_2(\lambda)$ represents an upper bound on the number of honest users. In the experiment, $C$ aborts if $\mathcal{A}_1$ queries for the private key $sk$ of the public key $uk$. $\mathcal{A}_1$ also generates a proof for that $nym$ is the result of correctly decrypting the ciphertext $(com, K)$. If $nym$ is a pseudonym of $uk$, i.e. $nym = g^a$, the knowledge-soundness of NIZK protocols implies that $\phi_{dec} = (com, K, nym, pk_L)$ is a valid statement such that $(\phi_{dec}, sk_L) \in \mathcal{R}_{dec}$. Furthermore, if $\pi$ is a valid signature of

knowledge, the extractability of a SoK protocol ensures that $C$ can extract a valid witness $\varpi_{enc} = (a, k)$ to the statement $\phi_{enc} = (com, K)$ from $\pi$. Therefore, $C$ can make use of $\mathcal{A}_1$ to obtain $a$ such that $A = g^a$.

In conclusion, if $\mathcal{A}_i$ can break the unforgeability of CLRS with a non-negligible probability, $C$ can solve the DL problem using $\mathcal{A}_i$'s successful attack. □

**Theorem 5.** *No PPT adversary can break the nym-extractability of CLRS with non-negligible advantage.*

PROOF. For a valid public key $uk = (pk, c, \pi_{enc})$, the completeness of the protocol $\Pi_{enc}$ implies that $pk = pk_O^{sk}$ and $c = g^{sk}h^r$. For a valid signature $\sigma = (com, K, \pi_{mem}, \pi)$ of $(R, m)$, correctness of the protocol $\Pi_{mem}$ implies that $c' = c/com = g^0 h^{r'}$. Thus, $com = g^{sk}h^k$ and $k = r - r'$. The extractability of the SoK protocol implies that a valid witness $\varpi = (sk, k)$ to the statement $\phi = (com, K)$ can be extracted from $\pi$. The witness satisfies $uk = (pk_O^{sk}, \cdot) \in R$ and $(com, K) = (g^{sk}h^k, pk_L^k)$.

Since $(com, K)$ can be viewed as an ElGamal encryption under the public key $pk_L$, correctness of the ElGamal encryption scheme ensures that decrypting $(com, K)$ with the private key $sk_L$ yields $nym = g^{sk}$, which is the output of $\mathsf{CLRS.Ext}$. In addition, based on the completeness of the protocol $\Pi_{dec}$, the proof $\pi_{dec}$ for that $nym$ is correctly extracted from $\sigma$ will be verified correctly. Therefore, $\mathsf{CLRS.Judge}$ takes $\sigma$, $nym$ and $\pi_{dec}$ as input and outputs 1. □

**Theorem 6.** *No PPT adversary can break the nym-soundness of CLRS with non-negligible advantage.*

PROOF. As analyzed in the proof of Theorem 5, the completeness of the protocol $\Pi_{enc}$ implies that $pk = pk_O^{sk}$ and $c = g^{sk}h^r$ if a public key $uk = (pk, c, \pi_{enc})$ is valid. In addition, the correctness of the protocol $\Pi_{mem}$ implies that $c'_j = g^0 h^{r'}$ if $\sigma = (com, K, \pi_{mem}, \pi)$ is valid. Thus, $com = g^{sk}h^k$ and $k = r - r'$. The extractability of the SoK protocol implies that a valid witness $\varpi = (sk, k)$ to the statement $\phi = (com, K)$ can be extracted from $\pi$. The witness satisfies $uk = (pk_O^{sk}, \cdot) \in R$ and $(com, K) = (g^{sk}h^k, pk_L^k)$.

If the proof $\pi_{dec}$ is valid, the soundness of the protocol $\Pi_{dec}$ ensures that $nym$ is the result of decrypting $(com, K)$ with a private key $sk_L$. Considering the perfect correctness of the ElGamal encryption scheme, the result of decrypting the ciphertext using a given private key is unique. If this is not the case, $\mathcal{A}_i$ can be employed to construct another adversary capable of compromising the perfect correctness of the ElGamal encryption scheme.

Therefore, the pseudonym extracted from a signature is unique. □

### *B. Security Analysis of DAPCR*

**Theorem 7.** *No PPT adversary can break the indistinguishability of the DAPCR scheme with non-negligible advantage.*

PROOF. We use $\mathsf{G}^{\mathsf{ind}}_{\mathsf{real}}$ to denote the experiment $\mathsf{exp}^{\mathsf{ind}}_{\mathcal{A}'_i}(\lambda)$ executed in the real world. To analyze the security of DAPCR, we design a simulation $\mathsf{G}^{\mathsf{ind}}_{\mathsf{sim}}$. In $\mathsf{G}^{\mathsf{ind}}_{\mathsf{sim}}$, the challenger $\mathcal{C}$ interacts with $\mathcal{A}'_i$ as in $\mathsf{G}^{\mathsf{ind}}_{\mathsf{real}}$. The only modification is that $\mathcal{C}$ outputs a challenge transaction $\mathsf{rtx}_{sim} = \{\mathsf{tx}_{sim}, ct_{sim}, \pi^v_{sim}, \pi^{log}_{sim}, \sigma\}$ where $\mathsf{tx}_{sim}$, $ct_{sim}$, $\pi^v_{sim}$ and $\pi^{log}_{sim}$ are independent of $T_b$.

**Game** $\mathsf{G}^{\mathsf{ind}}_{\mathsf{real}}$. $C$ first calculates

$$param \leftarrow \mathsf{DAPCR.Setup}(1^\lambda),$$
$$(pk_F, sk_F) \leftarrow \mathsf{DAPCR.FInit}(param),$$
$$(pk_S, sk_S) \leftarrow \mathsf{DAPCR.SInit}(param),$$

publishes $(param, pk_F, pk_S)$ and sends $sk_F$ to $\mathcal{A}'_1$ (or $sk_S$ to $\mathcal{A}'_2$). $C$ also generates public keys of honest users, and $R_1$ represents the set of these public keys.

Next, $\mathcal{A}'_i$ generates a ring $R_2$ and two tuples $T_0$ and $T_1$ where $tuple_j = (addr_{S_j}, addr_{R_j}, v_j, uk_j)$ and $uk_0, uk_1 \in R_1 \wedge R_2$. $\mathcal{A}'_i$ sends $(T_0, T_1, R)$ to $C$.

After receiving $(T_0, T_1, R)$, $C$ randomly chooses $b \in \{0, 1\}$ and computes

$$\mathsf{rtx}_b \leftarrow \mathsf{RtxGen}(addr_{S_b}, addr_{R_b}, v_b, R, sk_b, \cdot)$$

where $sk_b$ is the private key associated with $uk_b$. $C$ sends $\mathsf{rtx}_b = (\mathsf{tx}, ct, \pi_v, \pi_{log}, \sigma)$ to $\mathcal{A}'_i$.

Once $\mathsf{rtx}_b$ is received, $\mathcal{A}'_i$ makes a guess $b'$ for the value of $b$, and wins the experiment if and only if $b = b'$.

**Game** $\mathsf{G}^{\mathsf{ind}}_{\mathsf{mid}}$. The experiment in $\mathsf{G}^{\mathsf{ind}}_{\mathsf{mid}}$ is similar to that in $\mathsf{G}^{\mathsf{ind}}_{\mathsf{real}}$, but the only modification is that $\mathcal{C}$ replaces $\mathsf{tx}$ with $\mathsf{tx}_{sim}$. To initialize the experiments, $C$ makes use of simulators to generate trapdoors alongside the computation of common reference strings. After receiving $(T_0, T_1, R)$, $C$ generates a transaction $\mathsf{tx}_{sim}$ that is independent of $T_0$ and $T_1$, and makes use of a trapdoor to calculate a proof $\pi^v_{mid}$. $C$ sends $\mathsf{rtx}_{mid} = (\mathsf{tx}_{sim}, ct, \pi^v_{mid}, \pi_{log}, \sigma)$ to $\mathcal{A}'_i$.

If the DAP scheme used to construct DAPCR is secure, $\mathsf{tx}$ and $\mathsf{tx}_{sim}$ are indistinguishable. Then zero-knowledge of NIZK protocols implies that the distribution of $\pi_v$ is identical to that of $\pi^v_{mid}$. Thus, the absolute value of the difference between the advantage of $\mathcal{A}'_i$ in winning $\mathsf{G}^{\mathsf{ind}}_{\mathsf{mid}}$ and that in winning $\mathsf{G}^{\mathsf{ind}}_{\mathsf{real}}$ is negligible.

**Game** $\mathsf{G}^{\mathsf{ind}}_{\mathsf{sim}}$. $\mathsf{G}^{\mathsf{ind}}_{\mathsf{sim}}$ is similar to $\mathsf{G}^{\mathsf{ind}}_{\mathsf{mid}}$, but the only modification is that $\mathcal{C}$ replaces $ct$ with $ct_{sim}$. To initialize the experiments, $C$ makes use of simulators to generate trapdoors alongside the computation of common reference strings. After receiving $(T_0, T_1, R)$, $C$ randomly samples $ct_{sim} = (c_{sim}, u_{sim}) \in \mathbb{G}^2$ that is independent of $T_0$ and $T_1$, and makes use of a trapdoor to calculate proofs $\pi^v_{sim}$ and $\pi^{log}_{sim}$. $C$ sends $\mathsf{rtx}_{sim} = (\mathsf{tx}_{sim}, ct_{sim}, \pi^v_{sim}, \pi^{log}_{sim}, \sigma)$ to $\mathcal{A}'_i$.

For $\mathcal{A}'_2$, the indistinguishability of the ElGamal encryption ensures that the distribution of $ct_{sim}$ is identical to that of $ct$. The zero-knowledge of NIZK protocols implies that the distribution of $(\pi^v_{sim}, \pi^{log}_{sim})$ is identical to that of $(\pi^v_{mid}, \pi_{log})$.

For $\mathcal{A}'_1$, the amount tag can be extracted from a transaction. In particular, $\mathcal{A}'_1$ calculates $tag_{sim} = c_{sim}u_{sim}^{-1/sk_L}$ and $tag_v = cu^{-1/sk_L} = g^v h^{w_i}$, both of which have the same distribution. Similar to the case of $\mathcal{A}'_2$, the distribution of $(ct_{sim}, \pi^v_{sim}, \pi^{log}_{sim})$ is identical to that of $(ct, \pi^v_{mid}, \pi_{log})$.

Thus, the absolute value of the difference between the advantage of $\mathcal{A}'_i$ in winning $\mathsf{G}^{\mathsf{ind}}_{\mathsf{sim}}$ and that in winning $\mathsf{G}^{\mathsf{ind}}_{\mathsf{mid}}$ is negligible. Since $\mathsf{tx}_{sim}$, $ct_{sim}$, $\pi^v_{sim}$ and $\pi^{log}_{sim}$ are independent of $uk_0$ and $uk_1$, the advantage of $\mathcal{A}'_i$ in winning $\mathsf{G}^{\mathsf{ind}}_{\mathsf{sim}}$ is equal to the advantage of $\mathcal{A}'_i$ in breaking the anonymity of CLRS.

Therefore, the advantage of $\mathcal{A}_i'$ in winning $\mathsf{G}_{\mathsf{real}}^{\mathsf{ind}}$ is negligible. The proposed DAPCR scheme satisfies indistinguishability if the DAP scheme, the NIZK protocols, the ElGamal encryption and the CLRS scheme making up it are secure. □

**Theorem 8.** *No PPT adversary can break the $\mathcal{F}$-extractability of the DAPCR scheme with non-negligible advantage.*

PROOF. For a valid transaction $\mathsf{rtx} = (\mathsf{tx}, ct, \pi_v, \pi_{log}, \sigma)$ where $ct = (c, u)$, the completeness of the protocol $\Pi_v$ implies that $c = g^v h^z$ where $v$ is the payment amount of $\mathsf{tx}$, and the completeness of the protocol $\Pi_{log}$ ensures that $u = pk_L^r$ where $r \in \mathbb{Z}_q^*$. Since $ct = (g^v h^z, pk_L^r)$ can be viewed as a well-formed ciphertext of a pedersen commitment $g^v h^{z-r}$, the correctness of the encryption scheme implies that decrypting $ct$ with the private key $sk_F = sk_O$ yields $tag_v = g^v h^{z-r}$.

Since the CLRS scheme satisfies nym-extractability, $\mathcal{F}$ can extract the signer's pseudonym from $\sigma$. Thus, the pseudonym $nym$ and the amount tag $tag_v$ can be extracted from $\mathsf{rtx}$ by $\mathcal{F}$. □

**Theorem 9.** *No PPT adversary can break the $\mathcal{F}$-soundness of the DAPCR scheme with non-negligible advantage.*

PROOF. For a valid transaction $\mathsf{rtx} = (\mathsf{tx}, ct, \pi_v, \pi_{log}, \sigma)$ where $ct = (c, u)$, the completeness of the protocol $\Pi_v$ implies that $c = g^v h^z$ where $v$ is the payment amount of $\mathsf{tx}$, and the completeness of the protocol $\Pi_{log}$ ensures that $u = pk_L^r$ where $r \in \mathbb{Z}_q^*$. Since $ct = (g^v h^z, pk_L^r)$ can be viewed as a well-formed ciphertext of a pedersen commitment $g^v h^{z-r}$, the perfect correctness of the encryption scheme implies that the result of decrypting the ciphertext $ct$ using a given private key $sk_F = sk_O$ is unique. If this is not the case, $\mathcal{A}_i'$ can be employed to construct another adversary capable of compromising the perfect correctness of the encryption scheme.

Since the proposed CLRS scheme satisfies nym-soundness, $\mathcal{F}$ cannot two different valid pseudonyms from $\sigma$. Thus, the pseudonym and the amount tag extracted from $\mathsf{rtx}$ are unique. □

The proposed DAPCR scheme, which is based on DAP, adds additional regulated fields to privacy-preserving transactions. If an adversary generates a regulated transaction $\mathsf{rtx} = (\mathsf{tx}, ct, \pi_v, \pi_{log}, \sigma)$ with a payment exceeding their balance, they also gets a transaction $\mathsf{tx}$ breaking the balance of DAP. If an adversary modifies the data stored within a regulated transaction $\mathsf{rtx}$, they also obtains a modified transaction $\mathsf{tx}$ breaking the non-malleability of DAP, or forges a proof breaking the security of zk-SNARK. Thus, the proposed DAPCR scheme satisfies balance and non-malleability if the DAP scheme and the zk-SNARK scheme making up it is secure.

### C. Attacks

In this section, we take the example of DAPCR constructed based on ZETH or Zether to analyze how the scheme resists the following malicious attacks.

*1) Sybil attack:* The DAPCR scheme is designed for consortium blockchains, making it effective against certain malicious attacks targeting public blockchains, such as Sybil attacks. Consortium chains have strict controls on user admissions. Each user in the system undergoes identity authentication. If any user is found engaging in malicious behavior, they will be removed from the blockchain.

*2) One-time account attack:* One-time account attack means that an adversary controls multiple one-time accounts for transactions, making it difficult for Filter to link transactions from this adversary and thus evade regulation. Since the DAPCR scheme is deployed on a consortium blockchain, a new account needs permission from all consensus nodes to join the consortium chain, and the user's account address and real-world identity are transparent to consensus nodes. Therefore, a user cannot simultaneously control multiple accounts and conduct one-time account attacks.

*3) Double-spending attack:* Double-spending attack means that an adversary spends the same amount multiple times. In the DAPCR scheme based on ZETH, it is necessary to publish the corresponding serial number to spend the balance in a coin, and the same serial number cannot be used twice. If consensus nodes detect that a serial number has already been revealed, the transaction will be rejected. In the Zether-based scheme, each user is limited to publishing a single transaction in each epoch. This transaction contains proof that the nonce is generated based on the user's private key and the epoch number. The associated smart contract accumulates these nonces and, crucially, rejects any transaction bearing a duplicate nonce. In simple terms, if the anonymous payment mechanism that constitutes DAPCR can resist double-spending attacks, then the DAPCR scheme itself can resist such attacks.

*4) Over-spending attack:* Over-spending attack means that an adversary spends more than they possess. In the DAPCR scheme based on ZETH or Zether, a user should attach a proof to his transaction, which can show that the amount spent does not exceed the balance in his coin or account. The security of zero-knowledge proof protocols ensures that the probability of any adversary forging a valid proof for over-spending is negligible. Therefore, if the anonymous payment mechanism that constitutes DAPCR can resist over-spending attacks, then the DAPCR scheme itself can resist such attacks.

*5) Malleability attack:* Malleability attack means that an adversary modifies the data stored within a transaction. In the ZETH-based scheme, the honest user generates a transaction $\mathsf{rtx} = (\mathsf{tx}, ct, \pi_v, \pi_{log}, \sigma)$. The payment data is stored within $\mathsf{tx}$, which can be considered as a transaction in ZETH, and the regulatory data is stored within the regulated field $\mathsf{reg} = (ct, \sigma)$. Due to the non-malleability of ZETH, any adversary cannot modify the payment data in $\mathsf{tx}$. To modify the regulatory data within $\mathsf{reg}$, the adversary needs to forge a proof $\pi_v'$ to replace $\pi_v$. The security of zk-SNARK ensures that the probability of any adversary forging a valid proof is negligible.

## VIII. PERFORMANCE EVALUATION

In this section, we present a performance evaluation of the DAPCR scheme proposed in Section VI.

We design four experiments to assess the performance of DAPCR, all of which are executed on a local device with

an 8-core Intel(R) Core(TM) i7-10700 @ 2.90GHz CPU, 8 GB RAM and Ubuntu 20.04 LTS OS. DAPCR is constructed based on ZETH [26] which is an adaptive version of Zerocash designed for deployment on public or consortium blockchains with smart contracts. In addition, we utilize the zk-SNARK algorithm Groth16 [11], along with the SNARK-friendly elliptic curve BabayJubjub [27] and the MiMC hash algorithm to implement the DAPCR scheme. Moreover, the experimental implementation makes use of the programming language Golang[4] and the zero-knowledge proof tool Snarkjs[5].

In Experiment-I, we evaluate the computational and communication overhead of CLRS, which is shown in Fig. 5a. First, the time cost for user key generation is $0.7\,\mathrm{ms}$, independent of the ring-size. Then, we evaluate the time overhead for signature generation and verification under various ring-sizes. It is evident that the time costs of signature generation and verification are linearly related to the ring-size. When the ring-size is 16 which is similar to the ring-size employed in Monero, the time costs for signature generation and verification are $13.3\,\mathrm{ms}$ and $6.9\,\mathrm{ms}$, respectively. Additionally, we analyze the signature-length, which exhibits a logarithmic relationship with the ring-size. When the ring-size is set to 16, the signature-length is $2.1\,\mathrm{KB}$. Hence, we consider the CLRS scheme to be efficient.

In Experiment-II, we evaluate the time cost of transaction generation in DAPCR, which is shown in Fig. 5b. The time cost of transaction generation is linear to the number of transactions. It takes about $1231.2\,\mathrm{ms}$ to generate one transaction and less than $250\,\mathrm{s}$ to generate 200 transactions in DAPCR. We also compare the time cost of transaction generation in DAPCR and that in XLN22 [28], which is also a DAP scheme with regulation based on Zerocash. In XLN22, the average time required to generate a transaction is $11.52\,\mathrm{s}$, while in DAPCR, it is only $1.23\,\mathrm{s}$. Compared to Zerocash, XLN22 introduces an additional computational cost of $10.70\,\mathrm{s}$ and a communication cost of over $75\,\mathrm{KB}$ for a transaction, while DAPCR only introduces an additional computational cost of $0.41\,\mathrm{s}$ and a communication cost of $2.1\,\mathrm{KB}$. Hence, we consider DAPCR to be efficient in the transaction phase.

In Experiment-II, we also evaluate the computational overhead of transaction verification in DAPCR and compared it with that in ZETH. Fig. 5c presents the time cost of transaction verification in a consensus node, which is linear to the number of transactions. It takes about only $12.5\,\mathrm{ms}$ to verify one transaction and about $2.5\,\mathrm{s}$ to verify 200 transactions in DAPCR. Although the time cost of transaction verification in DAPCR is approximately three times higher compared to ZETH, it still remains at a relatively low level. Furthermore, we deploy the DAPCR scheme in Fabric[6], where the time interval between a user submitting a transaction and receiving a response indicating successful verification is about $2.0\,\mathrm{s}$. Hence, we consider DAPCR to be efficient in the verification phase.

[4]https://go.dev/

[5]https://github.com/iden3/snarkjs

[6]We deploy a Fabric (v2.4.9) network on a local device, which consists of two peer nodes and one order node.

In Experiment-III, we evaluate the time cost of Filter in screening out suspicious transactions, which is shown in Fig. 5d. We use *Extract Nym* to denote the time cost of extracting pseudonyms from transactions, which is linear to the number of transactions. If there are 10,000 transactions in the DAPCR system, *Extract Nym* is less than $2.5\,\mathrm{s}$. Assuming that transactions from Alice account for 2% (5%, 10%) of all transactions, the time cost of obtaining the total amount tag of Alice is shown in Fig. 5d. Compared to *Extract Nym*, the time cost of extracting the total amount tag is negligible. The result of Experiment-III indicates that DAPCR is effective in the regulation phase.

## IX. Related Work

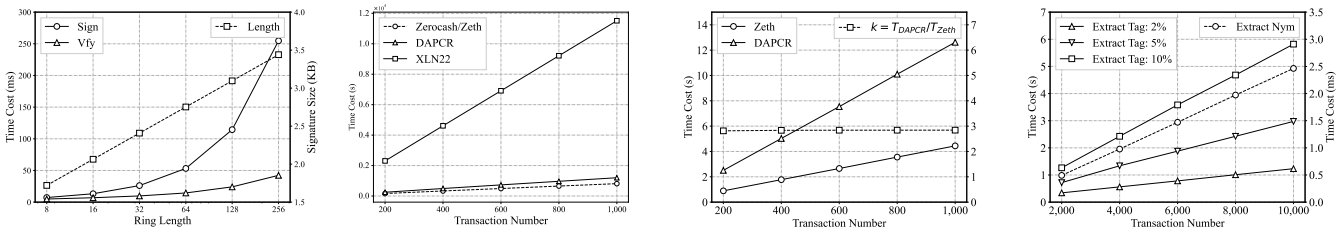
### A. DAP with Regulation

Wang et al. [6] propose a decentralized anonymous payment scheme with supervision (DAPS) based on zk-SNARK and the elliptic curve cryptography. A transaction in DAPS contains a ciphertext encrypted with the public key of the regulator. The regulator can decrypt ciphertexts in transactions with the private key to obtain the privacy. Faced with numerous transactions, it is inefficient for the regulator needs to decrypt the ciphertext in each transaction. Lin et al. [7] present a secure and efficient decentralized conditional anonymous payment system (DCAP) based on signatures of knowledge. A transaction in DCAP contains anonymous addresses of the sender and the recipient. The regulator can trace the long-term address for an anonymous address, but the payment amount of each transaction is public in DCAP. Wang et al. [6] and Lin et al. [7] neglect to restrict the regulatory power, which may be abused.

Some solutions recognize the importance of restricted regulation, but still have some shortcomings. Garman et al. [8] design a DAP scheme based on Zerocash that forces users to comply with specific policies and grants regulators the power of coin tracing and user tracing. [8] restricts the power of the regulator who is asked to provide an accountable record of the power being used. However, the regulator can still obtain privacy from a transaction. PRCash [29] is a new blockchain currency with privacy preservation and regulation, in which the sender's identity is also encrypted with the regulator's public key and included in the transaction. UTT [30] is a decentralized e-cash system with accountable privacy. In UTT each user needs to get *budget coins*, which are used to limit the total sum of payments, from the auditor per month.

zkLedger [10] and miniLedger [9] are two decentralized payment systems that achieve privacy preservation and verifiability auditing. These solutions realize rich auditing functions, but require auditors to interact with users. However, users may not always be online, and a malicious user may ignore the auditor's queries, which leads to delays in the audit.

Platypus [31] and PEReDi [32] are two central bank digital currencies with privacy preservation and regulation. In Platypus each user needs to encrypt his privacy with the regulator's public key and the ciphertext is included in the transaction. The regulator can decrypt the ciphertext for transaction privacy. In PEReDi several authorities form a committee to revoke privacy




(a) Time cost of signing and verifying (b) Time cost of transaction generation (c) Time cost of transaction verification (d) Time cost of transaction regulation

Fig. 5: Performance experiments for DAPCR and CLRS

or trace transactions from some user. The committee revokes the privacy of a transaction by decrypting the ciphertext saved in the ledger. It is inefficient to decrypt the ciphertext of each transaction for its privacy when auditing numerous transactions.

### *B. Ring Signature with Privacy Preservation and Regulation*

The ring signature [33] allows a user in the ring to sign a message on behalf of all members with unconditional anonymity, which poses a potential risk of signing malicious messages. To address this issue, the linkable ring signature has been proposed. In linkable ring signatures [15], anyone can link signatures signed by the same user without opening these signatures. In the ring signature with escrowed linkability [16], only a trusted authority can link signatures, which maintains anonymity for regular users. In the traceable ring signature [17], [18], also known as the revocable-iff-linked ring signature, if a user generate two or more signatures with the same tag, the anonymity of these signatures can be revoked.

There are some schemes that attempt to achieve the balance between anonymity and accountability. The accountable ring signature [19] and the revocable ring signature [20] allow the designated user or trusted authority to revoke the anonymity of signatures. Additionally, in the revocable and linkable ring signature [21], anyone can link signatures signed by the same user, and the trusted authority can revoke the anonymity of signatures. However, if the designated user or trusted authority is compromised, it can lead to a severe security crisis.

In the ring signature with user-controlled linkability [23] and the claimable ring signature [24], a signer can link their signatures and provide a publicly verifiable linking proof. These schemes are not suitable for anonymous payment mechanisms because only the signer can link their signatures. In group signatures with controlled linkability [22], only entities authorized by a trusted center can link signatures. As an inherent characteristic of group signatures, the trusted center can revoke the anonymity of the signatures. In addition, a new user needs to engage in an interactive protocol with the issuer to join the group signature system.

Compared to group/traceable signatures [34], users in CLRS generate public and private keys locally, avoiding centralized key generation and distribution. Furthermore, in group/traceable signatures, the central authority can unconditionally revoke the anonymity of a signature, whereas in the CLRS scheme, the opener must cooperate with the linker to determine the identity of the signer, preventing the abuse of power and privacy leakage.

## X. Conclusion

In this paper, we propose a decentralized anonymous payment scheme with collaborative regulation, which achieves universality, collaborative regulation and efficient aggregation of transaction amounts. Regulation in our scheme relies on the functioning of Filter at the end of each trading period, which is a potential target for malicious attackers. However, transaction validation is handled by consensus nodes, so even if Filter is compromised, validated transactions can still be added to the decentralized ledger. In future work, we can optimize the regulatory process to prevent single points of failure, and modify the DAPCR scheme to achieve UC security.

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## APPENDIX A
## NIZK PROTOCOLS FOR CLRS AND DAPCR

### A. $\Sigma$-protocol for the relation $\mathcal{R}_{enc}$

For a ElGamal ciphertext $ct = (c, u) = (g^{\alpha}h^{\beta}, pk^{\beta})$, a prover interacts with a verifier to prove $((c, u), (\alpha, \beta)) \in \mathcal{R}_{enc}$.

1. The prover randomly chooses $x_1, x_2 \in \mathbb{Z}_q^*$, computes $c' = g^{x_1}h^{x_2}$ and $u' = pk^{x_2}$ and sends $(c', u')$ to the verifier.
2. Once receiving $(c', u')$, the verifier randomly chooses $e \in \mathbb{Z}_q^*$ and sends it to the prover.
3. Once receiving $e$, the prover computes $y_1 = x_1 + e \cdot \alpha$, $y_2 = x_2 + e \cdot \beta$ and sends them to the verifier.
4. Finally, the verifier determines whether equations $pk^{y_2} \stackrel{?}{=} u'u^e$ and $g^{y_1}h^{y_2} \stackrel{?}{=} c'c^e$ hold. If both of the equations hold, the proof is valid else is invalid.

### B. $\Sigma$-protocol for the relation $\mathcal{R}_{dec}$

For several ciphertexts $ct_i = (c_i, u_i)$, $i \in \{0, 1, ..., n-1\}$, a message $m$ and a public key $pk = h^{sk}$, a prover interacts with a verifier to prove $((ct_i|_{i=0}^{n-1}, m, pk), sk) \in \mathcal{R}_{dec}$.

1. The prover randomly chooses $x \in \mathbb{Z}_q^*$, computes $A = h^x$ and $B_i = (c_i/m)^x$ for $i \in \{0, 1, ..., n-1\}$, and sends $(A, B_i|_{i=0}^{n-1})$ to the verifier.
2. Once receiving $(A, B_i|_{i=0}^{n-1})$, the verifier randomly chooses $e \in \mathbb{Z}_q^*$ and sends it to the prover.
3. Once receiving $e$, the prover computes $y = x + e \cdot sk$ and sends it to the verifier.
4. Finally, the verifier determines whether the equations $h^y \stackrel{?}{=} pk^e A$ and
$$(\prod_{i=0}^{n-1} \frac{c_i}{m})^y \stackrel{?}{=} (\prod_{i=0}^{n-1} u_i)^e \prod_{i=0}^{n-1} B_i$$
hold. If both of the equations hold, the proof is valid else is invalid.

The Fiat-Shamir transform can convert the above $\Sigma$-protocol into a NIZK protocol or a SoK protocol for the same relation.

### C. NIZK protocol for the relation $\mathcal{R}_v$

To ensure the general applicability of our scheme, we adopt the zk-SNARK protocol to generate a proof for the relation $\mathcal{R}_v$. Before calculating proofs, an arithmetic circuit $C$ needs to be constructed based on the relation $\mathcal{R}_v$.

Public inputs to $C$ are as follows.

1. privacy-preserving transaction $\mathsf{tx}$

2. pedersen commitment $c$
3. additional public inputs $I_{\mathsf{pub}}$ used to calculate $\mathsf{tx}$
4. hash $\hbar$ of a signature $\sigma$
4. $g, h \in \mathbb{G}$
5. public parameter $pp_{\mathsf{tx}}$ in the DAP scheme

Private inputs to $C$ are as follows.

1. sender's address $addr_S$ and receiver's address $addr_R$
2. transaction amount $v$
3. sender's private key $s$
4. additional private inputs $I_{\mathsf{pri}}$ used to calculate $\mathsf{tx}$
5. random number $r \in \mathbb{Z}_q^*$

$C$ imposes the following constraints on public inputs and private inputs.

1. $\mathsf{tx}$ is generated by the algorithm $\mathsf{DAP.TxGen}(\cdot)$ with inputs $(addr_S, addr_R, v, s, I_{\mathsf{pub}}, I_{\mathsf{pri}})$.
2. $c$ is a pedersen commitment of $(v, r)$, i.e. $c = g^v h^r$.

Based on the arithmetic circuit $C$, a trusted center can execute the algorithm $\mathcal{G}_v$ to generate $crs_v$. However, if the process of parameter generation is compromised, an adversary can generate forged proofs. To address these issues, Bowe et al. [35] proposed a secure multiparty computation protocol among $n$ nodes, which ensures that no one can generate forged proofs if at least one node is honest. Thus, we can deploy multiple nodes to execute the MPC protocol for initializing the DAPCR system.

## APPENDIX B
## DISCUSSION OF DEPLOYING DAPCR IN DIFFERENT MODELS

A regulated transaction $\mathsf{rtx} = (\mathsf{tx}, ct, \pi_v, \pi_{log}, \sigma)$ consists of three parts: a DAP transaction $\mathsf{tx}$, a regulated additional field $\mathsf{reg} = \{ct, \sigma\}$, and the zero-knowledge proof $\pi = \{\pi_v, \pi_{log}\}$. Whether the DAPCR scheme is deployed in the UTXO model or the account model, the algorithm for generating regulated fields is the same. The difference lies in the zero-knowledge proof component.

1) *Account Model*: In the account model, a user's balance is stored in their account. A user publishes a transaction to deduct a certain amount from his account and add an equivalent amount to the recipient's account. To transform the DAP transaction $\mathsf{tx}$ into a regulated one, the regulated field $\mathsf{reg} = \{ct, \sigma\}$ should be attached to $\mathsf{tx}$. The sender also needs to provide a proof $\pi = \{\pi_v, \pi_{log}\}$ to show the consistency of regulatory data within $\mathsf{reg}$ and payment data within $\mathsf{tx}$.

2) *UTXO Model*: In the UTXO model, a user's balance is stored in unspent transaction outputs (UTXOs). Users need to combine or split UTXOs to make payments of arbitrary amounts. Taking Zerocash in the UTXO model as an example, a transaction takes two old coins $c_1^{old}$ and $c_2^{old}$ as inputs and outputs two new coins $c_1^{new}$ and $c_2^{new}$. To realize effective regulation, we restrict transactions to be one-to-one. In other words, coins $c_1^{old}$, $c_2^{old}$ and $c_1^{new}$ must belong to the same address, with $c_1^{new}$ considered as change and its value can be 0. The actual payment amount in the transaction is the value of $c_2^{new}$. This restriction can be realized using zk-SNARKs: the sender generates an additional proof when constructing the transaction to show that $c_1^{old}$, $c_2^{old}$ and $c_1^{new}$ belong to the same address. Hence, the DAP scheme in the UTXO model can also be represented using algorithms in Section III-A. Specifically, the transaction generation algorithm is $\mathsf{TxGen}(addr_S, addr_R, v, s, I_{\mathsf{pub}}, I_{\mathsf{pri}}) \rightarrow \mathsf{tx}$, where $addr_S$ is the address to which coins $c_1^{old}$, $c_2^{old}$, and $c_1^{new}$ belong, $addr_R$ is the address to which $c_2^{new}$ belongs, $v$ represents the value of $c_2^{new}$, and $(c_1^{old}, c_2^{old}, c_1^{new}, c_2^{new})$ are included in the transaction $\mathsf{tx}$. To transform the DAP transaction $\mathsf{tx}$ in the UTXO model into a regulated one, the regulated fields $\mathsf{reg} = \{ct, \sigma\}$ should be attached to $\mathsf{tx}$. The zero-knowledge proof $\pi'$ should not only prove the consistency of regulatory data within $\mathsf{reg}$ and payment data within $\mathsf{tx}$ but also show that coins $c_1^{old}$, $c_2^{old}$, and $c_1^{new}$ belong to the same address. This is the distinction between deploying DAPCR in the UTXO model and deploying it in the account model.

## APPENDIX C
## ALGORITHMS OF DAPCR SCHEME

For convenience, we summarize the main algorithms of DAPCR as follows.

**An Efficient Construction of DAPCR**

The workflow of DAPCR is divided into four phases: the preparation phase, the transaction phase, the verification phase, and the regulation phase.

**I-Preparation Phase**

In the preparation phase, the public parameter is published. Entities generate their public and private keys, and new users obtain permission from consensus nodes to join the blockchain.

Consensus nodes:

DAPCR.Setup:

- Inputs: security parameter $\lambda$
- Outputs: public parameter $param$
- Consensus nodes execute the following steps to generate the public parameter:
  1. compute $(g, h, \mathcal{H}) \leftarrow \mathsf{CLRS.Setup}(1^\lambda)$
  2. compute $crs \leftarrow \mathcal{G}_v(1^\lambda, \mathcal{R}_v)$
- Consensus nodes publish $param = (g, h, \mathcal{H}, crs)$.

Filter:

DAPCR.FInit:

- Inputs: public parameter $param$
- Outputs: Filter's public-private key pair $(pk_F, sk_F)$
- Filter executes the following steps to generate their public-private key pair:
  1. compute $(pk_L, sk_L) \leftarrow \mathsf{CLRS.LKGen}(pp)$
  2. set $sk_F = sk_L$ and $pk_F = pk_L$
- Filter publishes their public key $pk_F$.

Supervisor:

DAPCR.SInit:

- Inputs: public parameter $param$
- Outputs: Supervisor's public-private key pair $(pk_S, sk_S)$
- Supervisor executes the following steps to generate their public-private key pair:
  1. compute $(pk_O, sk_O) \leftarrow \mathsf{CLRS.OKGen}(pp)$
  2. set $sk_S = sk_O$ and $pk_S = pk_O$
- Supervisor publishes their public key $pk_O$.

User:

DAPCR.KeyGen:

- Inputs: Supervisor's public key $pk_S$
- Outputs: user's public-private key pair $(uk, sk)$
- A user executes the algorithm $\mathsf{CLRS.UKGen}$ to generate their public-private key pair: $(uk, sk) \leftarrow \mathsf{CLRS.UKGen}(pk_O)$.
- The user publishes their public key $uk$.

User $\rightarrow$ Consensus nodes:

DAPCR.Join:

- User:
  1. randomly sample $\beta \in \mathbb{Z}_q^*$ and compute $B = g^\beta$
  2. compute $w = \mathcal{H}(pk_O^\beta | uk)$
  3. send $\mathsf{req} = (uk, B, \mu)$ to consensus nodes
- To join the blockchain, the user must obtain permission from all nodes. If successful, req will be recorded on the blockchain.

Supervisor:

DAPCR.Register:

- Supervisor:
  1. read $\mathsf{req} = (uk, B, \mu)$ from the decentralized ledger
  2. compute $w = \mathcal{H}(B^{sk_O} | uk)$, $tag_\mu = g^\mu h^w$ and $nym = pk^{1/sk_O}$
  3. add $(uk, \mu, nym, \mathsf{tag}_\mu)$ to the list $L_\mathcal{S}$
- Supervisor also sends $(nym, tag_\mu)$ to Filter. Once $(nym, tag_\mu)$ is received, Filter adds it to the list $L_\mathcal{F}$.

**II-Transaction Phase**

In the transaction phase, users generates regulated transactions to transfer their assets.

User:

DAPCR.RtxGen:

- Inputs:
  1. sender's address $addr_S$
  2. receiver's address $addr_R$
  3. payment amount $v_i$
  4. sender's secret key $s$
  5. additional public inputs $I_{\mathsf{pub}}$ and private inputs $I_{\mathsf{pri}}$
  6. Filter's public key $pk_F$
  7. user's private key $sk$
  8. ring $R_i$
- Outputs: regulated transaction $\mathsf{rtx}_i$
- A user executes the following steps to generate a regulated transaction:
  1. compute $\mathsf{tx}_i \leftarrow \mathsf{DAP.TxGen}(addr_S, addr_R, v_i, s, I_{\mathsf{pub}}, I_{\mathsf{pri}})$
  2. sample $z_i, w_i \in \mathbb{Z}_q^*$ and compute $ct_i = (g^{v_i} h^{z_i}, pk_L^{z_i - w_i})$
  3. compute $\sigma_i \leftarrow \mathsf{CLRS.Sign}(R_i, ct_i, pk_L, sk)$ and $\hbar_i = \mathcal{H}(\sigma_i)$
  4. compute $\pi_i^v \leftarrow \mathcal{P}_v((\mathsf{tx}_i, c_i, I_{\mathsf{pub}}, \hbar_i, g, h), (addr_S, addr_R, I_{\mathsf{pri}}, v_i, s, z_i), crs)$
  5. compute $\pi_i^{log} \leftarrow \mathcal{P}_{log}(u_i, z_i - w_i, pk_L)$
  6. set $\mathsf{rtx}_i = (\mathsf{tx}_i, ct_i, \pi_i^v, \pi_i^{log}, \sigma_i)$
- The user submits the regulated transaction $\mathsf{rtx}_i$.

**III-Verification Phase**

In the verification phase, consensus nodes (or smart contracts) verify the validity of regulated transactions, and only valid transactions are sealed into blocks.

Consensus nodes:

DAPCR.Verify:

- Inputs:
  1. regulated transaction rtx
  2. Filter's public key $pk_F$
  3. ring $R$
  4. additional public inputs $I_{\mathsf{pub}}$
- Outputs: $0/1$
- Consensus nodes execute the following steps to verify a transaction:
  1. compute $\hbar_i = \mathcal{H}(\sigma_i)$
  2. compute $b_1 \leftarrow \mathsf{DAP.TxVfy}(\mathsf{tx}, I_{\mathsf{pub}})$
  3. compute $b_2 \leftarrow \mathsf{CLRS.Vfy}(R, ct, \sigma, pk_L)$
  4. compute $b_3 \leftarrow \mathcal{V}_v((\mathsf{tx}, c, I_{\mathsf{pub}}, \hbar, g, h), \pi_v, crs)$
  5. compute $b_4 \leftarrow \mathcal{V}_{log}(u, \pi_{log}, pk_L)$
  6. if $b_1 \wedge b_2 \wedge b_3 \wedge b_4 = 1$ output 1 else output 0
- Consensus nodes seal valid transactions into blocks.

**IV-Regulation Phase**

In the regulation phase, Filter screens out suspicious transactions and submits the report on suspicious transactions to Supervisor. Once the report is received, Supervisor obtains the sender's public key and payment amounts of suspicious transactions.

Filter:

DAPCR.Extract:

- Inputs:
  1. regulated transaction $\mathsf{rtx}_i$
  2. Filter's private key $sk_F$
- Outputs:
  1. pseudonym $nym_i$
  2. amount tag $tag_{v_i}$
- Filter extracts the pseudonym and the amount tag from a transaction:
  1. compute $nym_i \leftarrow \mathsf{CLRS.Ext}(\sigma_i, sk_L)$
  2. compute $tag_{v_i} = c_i u_i^{-1/sk_L} = g^{v_i} h^{w_i}$

DAPCR.Detect:

- Inputs:
  1. pseudonym $nym$ and its corresponding upper limit tag $tag_\mu$
  2. Filter's private key $sk_F = sk_L$
  3. ledger which denoted all transactions sealed in blocks during a trading period
- Outputs:
  1. a set $S = \{\mathsf{rtx}_0, \mathsf{rtx}_1, ..., \mathsf{rtx}_{n-1}\}$ of transactions submitted by $nym$
  2. $\mathsf{susp}/\perp$
- Filter executes the following steps to determine if the transaction behavior of a user with pseudonym $nym$ complies with the transaction rules:
  1. extract pseudonyms from all transactions in ledger
  2. link transactions submitted by $nym$ and get a set $S = \{\mathsf{rtx}_0, \mathsf{rtx}_1, ..., \mathsf{rtx}_{n-1}\}$
  2. extract amount tags $tag_{v_i}|_{i=0}^{n-1}$ from all transactions in $S$
  3. compute $tag_{sum} = \prod_{i=0}^{n-1} tag_{v_i}$
- If $tag_\mu = tag_{sum}$, Filter outputs $(S, \perp)$ else outputs $(S, \mathsf{susp})$.

DAPCR.Report:

- Inputs:
  1. $(S, \mathsf{susp})$ where $S$ is a set of transactions
  2. pseudonym $nym$
  3. Filter's public-private key pair $(pk_F, sk_F)$
- Outputs: report rpt of suspicious transactions
- Filter executes the following steps to generate a report:
  1. compute $\pi_{dec} \leftarrow \mathcal{P}_{dec}((ct_i|_{i=0}^{n-1}, nym, pk_L), sk_L, h)$
  2. compute $\pi_{sum} \leftarrow \mathcal{P}_{dec}((\prod_{i=0}^{n-1} c_i, \prod_{i=0}^{n-1} u_i, tag_{sum}, pk_L), sk_L, h)$
  3. set $\mathsf{rpt} = (S, nym, \pi_{dec}, tag_{sum}, \pi_{sum})$ to $\mathcal{S}$
- Filter sends rpt to Supervisor.

Supervisor:

DAPCR.Recover:

- Inputs:
  1. report rpt of suspicious transactions
  2. Filter's public key $pk_F$
  3. Supervisor's private key $sk_S$
- Outputs:
  1. sender's public key $uk$
  2. sender's total payment amount $v_{sum}$
- Supervisor executes the following steps to verify the report:
  1. compute $b_1 \leftarrow \mathcal{V}_{dec}((ct_i|_{i=0}^{n-1}, nym, pk_L), \pi_{dec}, h)$
  2. compute $b_2 \leftarrow \mathcal{V}_{dec}((\prod_{i=0}^{n-1} c_i, \prod_{i=0}^{n-1} u_i, tag_{sum}, pk_L), \pi_{sum}, h)$
  3. if $b_1 \wedge b_2 = 1$, the report is valid else is invalid
- If the report is valid, $\mathcal{S}$ computes $uk \leftarrow \mathsf{CLRS.Open}(nym, sk_O)$ and then requires the user with public key $uk$ to submit $v_{sum}$ and $w'$ satisfies $tag_{sum} = g^{v_{sum}} h^{w'}$.