Dory: Asynchronous BFT with Reduced Communication and Improved Efficiency

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***Abstract*—Asynchronous Byzantine fault-tolerant (BFT) protocols have received increasing attention, as they are particularly robust against timing and performance attacks. This paper designs and implements Dory, an asynchronous BFT protocol with reduced communication and improved efficiency compared to existing systems. In particular, Dory reduces the communication both *asymptotically* and *concretely* and gains in improved performance. To achieve the goal, we have devised a novel primitive called *asynchronous vector data dissemination*, and we have developed the idea of *supplemental consensus* originally used in DispersedLedger for higher throughput and fairness without using threshold encryption.**

We have implemented and deployed our system using up to 151 replicas on Amazon EC2. We demonstrate that even without using the technique of separating data transmission from agreement, Dory has up to 5x the throughput of Speeding Dumbo (sDumbo), while lowering the communication cost for different batch sizes.

I. Introduction

Purely asynchronous Byzantine fault-tolerant (BFT) protocols assuming no timing assumptions are more robust than the partially synchronous BFT protocols and thus have recently received renewed attention. State-of-the-art asynchronous BFT protocols (with implementations) can be roughly divided into three categories: 1) parallel ABA (asynchronous binary Byzantine agreement) based [1]–[6]; 2) MVBA (multi-valued Byzantine agreement) based [7]–[9]; and 3) DAG (directed acyclic graph) based [10], [11]. All of the instantiations derived from the three paradigms enjoy unique features and are suitable for certain applications.

This paper focuses on the MVBA based protocols, where the state-of-the-art protocols are Speeding Dumbo (sDumbo) [9] and Dumbo-NG [8]. In particular, Dumbo-NG separates data transmission from agreement and achieves roughly 2x the throughput of sDumbo (when $n = 64$). We design and implement Dory with two distinguishing features: 1) reduced communication (both asymptotically and concretely), and 2) improved performance (4.7x the throughput of sDumbo for $n = 64$, even without using the technique of separating transmission from agreement).

Communication cost barriers. For MVBA based BFT protocols, the communication bound that one could theoretically hope for (so far) is $O(n^2|m| + \lambda n^2)$ [12]. However, known instantiations fall short of the expectation, due to multiple bottleneck components. Take sDumbo [9], a dedicated MVBA based BFT, as an example (Fig. 1 where PB stands for provable broadcast, i.e., a variant of consistent broadcast). In sDumbo, there are three communication bottleneck ingredients: the MVBA used in sDumbo (called sMVBA), the erasure coding recovery phase, and the threshold decryption phase incurs $O(\lambda n^3)$, $O(n^2|m| + \lambda n^3 \log n)$, and $O(\lambda n^3)$ communication, respectively.

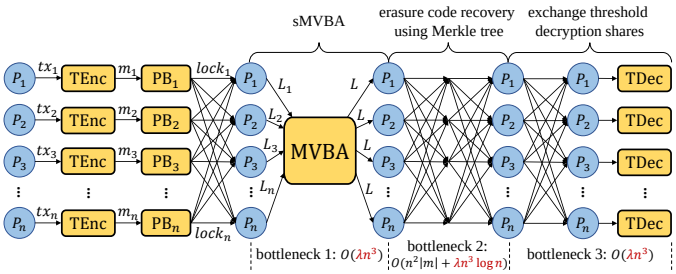

Fig. 1. sDumbo phases and its communication bottleneck.

Wasted proposals and censorship resilience. Existing MVBA based BFT constructions order proposed transactions from $n - f$ replicas, although in the normal case, all replicas may have proposed and successfully transmitted transactions. Namely, the computation and the bandwidth involving up to f proposals could be "wasted."

A related issue is that if directly allowing replicas to propose transactions in parallel, an adversarial network scheduler can censor transactions in the MVBA phase, thereby attacking censorship resilience (aka liveness). To tackle the issue, one would use:

- threshold encryption [3], which leads to increased computation and λn^3 communication;
- *inter-node-linking* [6] (also called *supplemental consensus* in this paper), which is only possible if using more expensive reliable broadcast instead of consistent broadcast—resulting in $O(n^3)$ messages;

- pipelined certificates [8], which is incompatible with supplemental consensus, impacts blockchain quality—in case one chain led by a faulty replica being extended (unbounded) fast, and sacrifices both $O(1)$ time and state transfer time, as discussed [13].

Our approach. In designing Dory, we have two goals in mind: 1) saving the communication; 2) improving the performance (while maintaining censorship resilience).

First, to push the communication cost to the known bound of $O(n^2|m| + \lambda n^2)$ [12], we have carefully designed and implemented the building blocks for Dory, including the first implementation of MVBA achieving $O(n|m| + \lambda n^2 \log n)$ communication, and a new design of asynchronous data dissemination (ADD) protocol [15] that can be triggered only on request and reduces the communication concretely (compared to the protocol running ADD in parallel).

Also, we integrate the technique of supplemental consensus stemming from DispersedLedger [6] in our MVBA framework to achieve high throughput as well as censorship resilience. The supplemental consensus idea works only for data transmission with totality (from reliable broadcast [16], [17]) and inherently incurs $O(n^3)$ messages for the protocol, but we are able to enable it with $O(n^2)$ communication only.

A. Our Contributions

We summarize our contributions in the following.

- We propose Dory, a scalable and bandwidth-efficient asynchronous BFT protocol achieving $O(n^2|m| + \lambda n^2 \log n + n^3 \log n)$ communication complexity, the first one close to the known bound [12].
- We propose a new primitive called *asynchronous vector data dissemination* (AVDD) that concretely reduces the communication compared to the one running ADD in parallel.
- We have developed a new technique that simultaneously enables supplemental consensus and censorship resilience in the MVBA framework.
- We implement Dory and sDumbo [9] in Golang, and evaluate them on up to 151 Amazon EC2 instances distributed in 10 regions. Our experimental results demonstrate that Dory has significantly lower communication cost compared with sDumbo, preserves low latency (less than 8s) even for a large network (151 replicas), and achieves high throughput (135k tx/s for 16 replicas and 57k tx/s for 151 replicas), which is 2-5× the throughput of sDumbo. Our library on Dory and sDumbo in Golang has been open-sourced.

Integrating techniques in Dumbo-NG. According to Dumbo-NG [8], Dumbo-NG is about 2x the throughput of sDumbo (the largest network evaluated is with $n = 64$ replicas). The performance improvement is mainly due to the separation of transmission and agreement and the transmission pipelining. In contrast, Dory without using the separation and the pipelining techniques is 4.7x the throughput of sDumbo for $n = 64$. That said, Dory can use the separation and the pipelining techniques used in Dumbo-NG to further accelerate the performance.

II. System and Threat Model

Let $[n]$ denote the set of integers $\{1, 2, ..., n\}$. We consider distributed computing protocols, where f out of n replicas ($\{P_i\}_{i\in[n]}$) may fail arbitrarily (Byzantine failures). The protocols we consider in this work (BFT, MVBA, and AVDD) assume $f \leq \lfloor \frac{n-1}{3} \rfloor$, which is optimal. We consider completely asynchronous systems making no timing assumptions on message processing or transmission delays. A (Byzantine) *quorum* is a set of $\lceil \frac{n+f+1}{2} \rceil$ replicas. For simplicity, we may assume $n = 3f + 1$ and a quorum size of $2f + 1$. In our protocols, we may associate each protocol instance with a unique session identifier ID, tagging each message in the protocol with ID; we may omit these identifiers when no ambiguity arises.

This paper studies BFT protocols. Our implementations tolerate static corruption, where the adversary needs to choose the set of faulty replicas before the execution of the protocol. We also assume trusted setup for the threshold cryptosystems.

Syntactically, in BFT, a replica *outputs* (atomically deliver) *transactions*, each *being input* by some client. The client computes a final response to its submitted transaction from its responses from replicas. The correctness of a BFT protocol is specified as follows:

- *Agreement.* If any correct replica outputs a transaction m, then every correct replica outputs m.
- *Total order.* If a correct replica outputs a transaction m before outputting m', then no correct replica outputs a transaction m' without first outputting m.
- *Liveness.* If a correct replica inputs a transaction m, then every correct replica eventually outputs m.

III. Building Blocks

Multi-valued validated Byzantine agreement (MVBA). MVBA allows each replica that has an input to agree on a value, which satisfies a global and polynomial-time computable $\mathcal{Q}$ known by all replicas [7]. More formally, an MVBA protocol satisfies the following properties:

- *Agreement.* If any correct replica outputs m, then every correct replica outputs m.
- *External Validity.* If a correct replica outputs a value m, then m is valid, i.e., $\mathcal{Q}(m) = 1$.
- *Termination.* If $n - f$ correct replicas have an input, then every correct replica eventually gets an output.

Threshold signature. Threshold signature allows any t replicas to produce a valid signature, while any replicas less than t cannot [18], [19]. It consists of the following five algorithms:

- *Key generation:* $\{pk, sk\} \leftarrow \mathsf{KeyGen}(\lambda, n, t)$. Given a security parameter λ, the total number of replicas n and a threshold t, the algorithm outputs a public key pk, and a vector of secret keys $sk = (sk_1, sk_2, \ldots, sk_n)$. For simplicity, pk is dropped for the following algorithms.
- *Signing:* $\rho_i \leftarrow \mathsf{Sign}_t(sk_i, m)$. Given a secret key sk_i, a message m, the algorithm outputs a signature share ρ_i.
- *Share verification:* $0/1 \leftarrow \mathsf{VerifyShare}_t(m, (i, \rho_i))$. Given a message m, an index i and a signature share ρ_i, the algorithm

TABLE I
COMPARISON FOR PERFORMANCE METRICS

Protocol	Message Complexity	Communication Complexity	Communication Cost in the Optimistic Case†	Time Complexity
HoneyBadger [3]	$O(n^3)$	$O(n^2\|m\| + \lambda n^3 \log n)$	$3n^2\|m\| + O(\lambda n^3 \log n)$	$O(\log n)$
DispersedLedger [6]	$O(n^3)$	$O(n^2\|m\| + \lambda n^3 \log n)$	$3n^2\|m\| + O(\lambda n^3 \log n)$	$O(\log n)$
Dumbo [14]	$O(n^3)$	$O(n^2\|m\| + \lambda n^3 \log n)$	$3n^2\|m\| + O(\lambda n^3 \log n)$	$O(1)$
sDumbo [9]	$O(n^2)$	$O(n^2\|m\| + \lambda n^3 \log n)$	$n^2\|m\| + O(\lambda n^3)$	$O(1)$
Dumbo-NG [8]	$O(n^2)$	$O(n^2\|m\| + \lambda n^3 \log n)$	$n^2\|m\| + O(\lambda n^3)$	$O(1)$
Dory (this work)	$O(n^2)$	$O(n^2\|m\| + \lambda n^2 \log n + n^3 \log n)$	$n^2\|m\| + O(\lambda n^2 \log n + n^3)$	$O(1)$

†The "Optimistic Case" means that all replicas are correct and there is a fair network scheduler that never reorders messages.

outputs 1 if and only if ρ_i is a valid signature share computed by replica P_i for m.

- *Combining:* $\sigma/\perp \leftarrow \mathsf{Combine}_t(m, \{(i,\rho_i)\}_{i\in S})$. Given a set of pairs $\{(i,\rho_i)\}_{i\in S}$, where $S \subset [n]$ and $|S| = t$, the algorithm outputs a signature σ if and only if all shares in S are valid.
- *Signature verification:* $0/1 \leftarrow \mathsf{Verify}_t(m,\sigma)$. Given a message m and a signature σ, the algorithm outputs 1 if σ is a valid signature for m; otherwise, it outputs 0.

A (n,t) threshold signature scheme should satisfy the conventional robustness and unforgeability properties.

Hash. We use a collision-resistant hash function $\mathcal{H}$.

Provable broadcast (PB). PB is a consistent broadcast protocol among n replicas, where a designed replica (also called sender) with ID consistently multicasts some m [7], [9], [20], [21]. Additionally, the sender will also output a tuple (h,σ), where h is the hash of m and σ is a threshold signature for h and ID. Formally, a PB protocol with an identifier ID satisfies the following properties:

- *Provability.* If the sender outputs any two tuples (h,σ) and (h',σ') s.t. $\mathsf{Verify}_{n-f}(\langle \mathsf{ID}, h\rangle, \sigma) = 1$ and $\mathsf{Verify}_{n-f}(\langle \mathsf{ID}, h'\rangle, \sigma') = 1$, then $h = h'$ and at least $f+1$ correct replica output m s.t. $\mathcal{H}(m) = h$.
- *Termination.* If the sender is correct and inputs a value m, then all correct replicas will output m. In addition, the sender will output (h,σ) satisfying $\mathcal{H}(m) = h$ and $\mathsf{Verify}_{n-f}(\langle \mathsf{ID}, h\rangle, \sigma) = 1$.

The PB protocol can be easily instantiated using a $(n, n-f)$ threshold signature, and achieving $O(n)$ messages and $O(n|m| + \lambda n)$ communication.

Error correcting code. Error correcting code enables correcting errors or recovering missing fragments of the encoded data. It consists of the following algorithms:

- *Encode:* $\{d_1, d_2, \ldots, d_n\} \leftarrow \mathsf{Encode}(m,n,t)$. Given a data block m, which is split into t coefficients of a polynomial $p(\cdot)$ in a Galois Field $\mathbb{F}$, the algorithm encodes m to n fragments $\{d_1, d_2, \ldots, d_n\}$, where $d_i \in \mathbb{F}$ for $i \in [n]$.
- *Decode:* $m' \leftarrow \mathsf{Decode}(T,t,r)$. Given a set of fragments of T, some of which may be incorrect, the algorithm outputs a $t-1$ degree polynomial, i.e., a data block m', by correcting up to r errors in T.

It is well-known that the decode algorithm can successfully output the original data block provided $|T| \geq t+2r$ [22] (e.g., the Berlekamp-Welch algorithm [23], Gao's algorithm [24]).

IV. ASYNCHRONOUS VECTOR DATA DISSEMINATION

As discussed in Fig. 1 in the introduction, one bottleneck is the erasure coding recovery phase which has a $n^2|m| + \lambda n^3 \log n$ overhead. To lower the communication, one intuitive idea is to run the *asynchronous data dissemination* (ADD) protocol proposed recently by Das, Xiang, and Ren [15]. Running ADD-based protocol for n parallel proposals incurs $O(n^2|m| + n^3 \log n)$ communication. However, the ADD protocol is concretely bandwidth-expensive, with a hidden constant of 6 (i.e., $6n^2|m|$).

More critically, the ADD protocol requires replicas to disseminate fragments of the data block to all replicas, no matter whether they need. However, in sDumbo and Dumbo-NG, the fragments are sent only when replicas do not have the corresponding data blocks. We find that adapting the same idea to this on-request mode in parallel ADD is technically challenging, so we define *asynchronous vector data determination* as a first-class primitive in the following.

A. AVDD

Asynchronous vector data dissemination (AVDD). In an AVDD protocol with n replicas, suppose that we have a global ℓ-dimensional vector $M = (m_1, \ldots, m_k, \ldots, m_\ell)$ and for any $k \in [\ell]$, at least $f+1$ correct replicas hold the same m_k and confirm its correctness. The goal of AVDD is to allow every correct replica to output the common M. Formally, the AVDD protocol for an ℓ-dimensional vector M satisfies the following correctness property:

- *Correctness.* For any $k \in [\ell]$, if at least $f+1$ correct replicas input the same m_k and other correct replicas input $\perp$, then every correct replica outputs the same vector M.

Our AVDD protocol is based on the Reed-Solomon error correcting code. We show the pseudocode of the protocol for P_i in Algorithm 1. Our protocol consists of three phases: request, dispersal, and an optional confirm phase, which we describe below.

Every replica P_i begins with an input M_i, which consists of a vector of ℓ elements, i.e., $m_1, \ldots, m_\ell$. Depending on the protocol that triggers the AVDD protocol, some elements might be $\perp$. Replica P_i initializes several global parameters: a set S_i tracking the set of elements that need to be reconstructed; a set of values d^*_{ki} for $k \in [\ell]$, where each d^*_{ki} is used to store

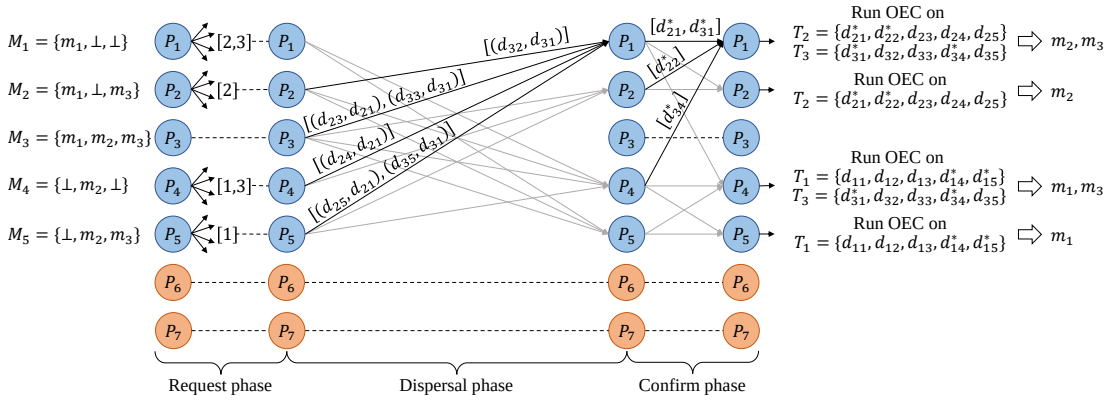

Fig. 2. An example of the AVDD protocol with 7 replicas among which P_6, P_7 are faulty. In the request phase, P_1 requests its missing elements. After the dispersal phase, P_1 will receive $f+1=3$ consistent fragments for m_2, m_3 respectively, and hence set d^*_{21} and d^*_{31}. Similarly, P_2 will set d^*_{22}, P_4 will set d^*_{14}, d^*_{34}, and P_5 will set d^*_{15}. In the confirm phase, P_1 will receive d^*_{21}, d^*_{22} for m_2 and d^*_{31}, d^*_{34} for m_3. Thus, P_1 is able to collect $2f+1=5$ correct fragments for m_2, m_3 and successfully reconstruct them. Similarly, other replicas also reconstruct their missing elements.

Algorithm 1 AVDD protocol with identifier ID for P_i

1: **Initialization:** $S_i \leftarrow \{\}$; $ReadyFlag \leftarrow false$; **for** $k \in [\ell]$ **do**, $d^*_{ki} \leftarrow \perp$; $T_i \leftarrow \{\}$; $A_i \leftarrow \{\}$
2: **upon** receiving input $M_i = (m_1, m_2, \ldots, m_\ell)$ **do**
3: **for** $1 \le k \le \ell$ **do**
4: **if** $m_k = \perp$ **then**
5: $S_i \leftarrow S_i \cup \{k\}$
6: **if** S_i is not empty **then**
7: **broadcast** $(\textsc{Request}, \mathsf{ID}, S_i)$ ▷ Request phase
8: $ReadyFlag \leftarrow true$

9: **wait** until $ReadyFlag = true$
10: **upon** receiving $(\textsc{Request}, \mathsf{ID}, S_j)$ from P_j **do**
11: $S_D \leftarrow \{\}$, $S_C \leftarrow \{\}$
12: **for** any $k \in S_j$ and $m_k \neq \perp$ **do** ▷ Disperse phase
13: $(d_{k1}, d_{k2}, \ldots, d_{kn}) \leftarrow \mathsf{Encode}(m_k, n, f+1)$
14: $S_D \leftarrow S_D \cup (k, d_{ki}, d_{kj})$
15: **send** $(\textsc{Disperse}, \mathsf{ID}, S_D)$ to P_j
16: **for** any $k \in S_j$ and $m_k = \perp$ **do**
17: **wait** until $d^*_{ki} \neq \perp$ ▷ Updated in ln 26
18: $S_C \leftarrow S_C \cup (k, d^*_{ki})$
19: **if** S_C is not empty **then**
20: **send** $(\textsc{Confirm}, \mathsf{ID}, S_C)$ to P_j ▷ Confirm phase
21: **upon** receiving $(\textsc{Disperse}, \mathsf{ID}, S_D)$ from P_j **do**
22: **for** any $(k, d_{kj}, d_{ki}) \in S_D$ **do**
23: $T_i[k] \leftarrow T_i[k] \cup \{(j, d_{kj})\}$
24: $A_i[k] \leftarrow A_i[k] \cup \{(j, d_{ki})\}$
25: **if** there are $f+1$ consistent $(\cdot, d_{ki})$ in $A_i[k]$ **then**
26: $d^*_{ki} \leftarrow d_{ki}$
27: **upon** receiving $(\textsc{Confirm}, \mathsf{ID}, S_C)$ from P_j **do**
28: **for** any $(j, d_{kj}) \in S_C$ **do**
29: $T_i[k] \leftarrow T_i[k] \cup \{(j, d_{kj})\}$

30: **for** any $k \in S_i$ **do**
31: **upon** $|T_i[k]| \ge 2f+1$ **do** ▷ Trigger OEC for m_k
32: **for** $0 \le r \le f$ **do**
33: **wait** until $|T_i[k]| \ge 2f+r+1$
34: $p_k(\cdot) \leftarrow \mathsf{Decode}(T_i[k], f+1, r)$
35: **if** $2f+1$ $(j, y) \in T_i[k]$ satisfy $p_k(j) = y$ **then**
36: $m_k \leftarrow$ coefficients of $p_k(\cdot)$

37: **wait** until no element of M_i is $\perp$
38: **output** M_i

the data fragments only for the case that $m_k = \perp$; two maps T_i and A_i for storing the received data fragments.

Request. At the beginning of the protocol, P_i first checks M_i and adds k to a set S_i if $m_k = \perp$ for some k such that $1 \le k \le \ell$. If S_i is not empty, P_i broadcasts a $(\textsc{Request}, \mathsf{ID}, S_i)$ message to all replicas (ln 2-8) and then waits for all the elements in M_i to become non-empty (ln 37).

Dispersal. If P_i receives an incoming $\textsc{Request}$ message from replica P_j, it checks M_i and initializes two sets, S_D and S_C. S_D is used to store a set of fragments for any $k \in S_j$ and m_k is not $\perp$. S_C is used to store a set of fragments for $k \in S_j$ and m_k is $\perp$. Note that P_i can directly update S_D and send a set of fragments to P_j, but it does not hold any fragments for $m_k = \perp$. In our AVDD protocol, the update of S_C might be deferred but will eventually be completed.

Specifically, we distinguish two cases for each replica P_i:

- Case 1: for any $k \in S_j$ that $m_k \neq \perp$, P_i encodes m_k and obtains the i^{th} and j^{th} data fragments and adds a tuple (k, d_{ki}, d_{kj}) to S_D. After S_D is updated for all such $k \in S_j$, P_i sends a $(\textsc{Disperse}, \mathsf{ID}, S_D)$ message to P_j (ln 12-15). Meanwhile, for the elements P_i requests, upon receiving a $(\textsc{Disperse}, \mathsf{ID}, S_D)$ message from P_j, P_i adds the data fragments to the T_i and A_i sets. If there are $f+1$ matching fragments d_{ki}, P_i sets d^*_{ki} as d_{ki}. As we show in our proof, for every element P_i requests in S_i, it will receive at least $f+1$ matching d_{ki} from other replicas (ln 21-26).
- Case 2 **(optional confirm phase)**: for any $k \in S_j$ such that $m_k = \perp$, P_i waits until d^*_{ki} is updated in the dispersal phase (case 1) for the messages it requests based on its own S_i. After all such data fragments are updated, P_i adds them to S_C and sends a $\textsc{Confirm}$ message to P_j (ln 16-20).

If case 2 is triggered, the replica that requests the corresponding element may receive a $\textsc{Confirm}$ message from other replicas. If this is the case, P_i adds the received data fragments to T_i (ln 27-29).

Upon collecting $2f+1$ fragments for any $k \in S_i$, P_i triggers the *online error correcting* (OEC) algorithm [1] to reconstruct m_k. Concretely, each execution of the OEC algorithm performs up to f trials of reconstruction. The number of required fragments increases with the number of trials. As the f^{th} trial

satisfies $|T_i[k]| \geq 3f+1$, P_i eventually reconstruct m_k, as mentioned in Section III (ln 30-36).

Finally, P_i waits until it reconstructs all the elements such that there is no $\perp$ in M_i. Then P_i outputs M_i.

B. Discussion, Complexity, and Comparison

Building AVDD in an on-request manner with $O(n^2)$ messages is not easy. Consider the example shown in Fig. 2 where $n=7$, $f=2$, and replicas P_6 and P_7 are faulty. P_1 misses m_2 and m_3 and thus requests them via a REQUEST message. Each replica then processes the request from other replicas. A naive approach is that each replica processes each requested element in the REQUEST message one by one. In this particular, as P_4 only holds m_2 but not m_3, it can simply send the data fragment of m_2 to P_1. After P_4 receives $f+1$ data fragments for m_3, it sends the fragment of m_3 to P_1. It is then not difficult to see that such an approach incurs $O(\ell n^2)$ message complexity, as each replica needs to broadcast up to ℓ messages.

Alternatively, each replica can wait until it has the corresponding data fragments for all the requested elements in a REQUEST message. In this way, the message complexity is $O(n^2)$. Such an approach, however, may have a deadlock issue. For instance, upon receiving a request from P_1, although P_4 holds m_2, it waits until receiving the correct fragments of m_1 from P_1, P_2, and P_3. Accordingly, both P_1 and P_4 wait for the replies from each other, creating a deadlock.

In contrast, our AVDD protocol solves it in an elegant way using a dispersal phase and a confirm phase. In particular, every replica only sends the fragments it holds to the replica that requests the corresponding elements. For the elements that each replica does not hold at the beginning of the protocol, the replica may delay the process until it receives the fragments it requests and then sends them in the confirm phase. Our protocol then achieves $O(n^2)$ messages.

Let $|m|$ be the size of each element in the ℓ-dimensional vector M, the communication complexity is $O(\ell n|m|+\ell n^2 \log n)$. The concrete communication is 0 in the optimistic case (where none of the elements in the M vector is $\perp$ for all replicas), and $4\ell n|m| + O(\ell n^2 \log n)$ in the "worst" case (as we will prove shortly).

The data dissemination problem of a ℓ-dimensional vector can be alternatively solved by ℓ parallel ADD [15] instances—hereinafter abbreviated as ℓ-ADD. As shown in Table II, running ℓ-ADD incurs $6\ell n|m|+O(\ell n^2 \log n)$ communication, significantly higher than AVDD.

TABLE II
COMPARISON BETWEEN AVDD AND ℓ-ADD

Protocol	Communication Cost	
	Optimistic Case†	Worst Case‡
AVDD	0	$4\ell n\|m\| + O(\ell n^2 \log n)$
ℓ-ADD [15]	$6\ell n\|m\| + O(\ell n^2 \log n)$	

†The "Optimistic Case" means that all replicas are correct and have the complete vector at the beginning.
‡The "Worst Case" means that there are f faulty replicas and each element in the vector is held by only $f+1$ correct replicas at the beginning.

C. Analysis

Our AVDD protocol has clearly $O(n^2)$ messages. We now give an analysis of Algorithm 1. Recall that its goal is to make every correct replica output a common ℓ-dimensional vector $M=(m_1,\ldots,m_k,\ldots,m_\ell)$. We first prove the correctness, i.e, provided that for any $k \in [\ell]$, at least $f+1$ correct replicas input the same m_k and other correct replicas input $\perp$, every correct replica will output the same vector M.

Lemma 1 After broadcasting a REQUEST message, each replica P_i will hold the correct i^{th} fragments of all its missing elements.

Proof: We assume that P_i broadcasts a REQUEST message carrying an index set $S_i \subseteq [\ell]$. For every $k \in S_i$, P_i sets d^*_{ki} only if it receives $f+1$ consistent fragments through DISPERSE messages from different replicas, at least one of which is correct. We know that no correct replica will send an incorrect fragment, so it is certain that d^*_{ki} is correct. Besides, since every element in M is held by at least $f+1$ correct replicas, P_i can always receive $f+1$ consistent fragments for every $k \in S_i$. Thus, each replica P_i will hold the correct i^{th} fragments of all its missing elements. ■

Lemma 2 At the end of the protocol, every correct replica outputs the same M. Thus, correctness is satisfied.

Proof: We assume that P_i broadcasts a REQUEST message carrying an index set $S_i \subseteq [\ell]$. From Lemma 1, each replica P_j will hold either a full data or a j^{th} fragment for every element in M. Thus, upon receiving a REQUEST message from P_i, P_j will eventually return the j^{th} fragments of all elements in S_i through DISPERSE and CONFIRM messages. Then, for every element in S_i, P_i will receive $2f+1$ fragments to trigger the OEC algorithm. Indeed, there may be up to f error fragments from faulty replicas. However, P_i will eventually receive $2f+1$ correct fragments from all correct replicas, so the OEC algorithm will eventually succeed and output the same element as the one that correct replicas input at the beginning. Therefore, every correct replica will output the same M. ■

Lemma 3 The concrete communication cost of the AVDD protocol is bounded by $4\ell n|m| + O(\ell n^2 \log n)$.

Proof: We assume that the number of elements requested by replica P_i is no more than $\ell_i (\ell_i \leq \ell)$ for any $i \in [n]$. Then each REQUEST message carries a set of indices of missing elements which is no more than ℓ_i. Thus, the communication cost of the request phase is at most $n\sum_{i\in[n]} \ell_i$. During the dispersal and confirm phases, each replica P_i receives at most n messages carrying at most $2\ell_i$ Reed-Solomon code fragments of $\max(\frac{|m|}{f}, \log n) < \frac{|m|}{f} + \log n$ bits and ℓ_i indices, so the total communication cost of the AVDD protocol is at

most

$$n\sum_{i\in[n]}\ell_i+\sum_{i\in[n]}n(2\ell_i(\frac{|m|}{f}+\log n)+O(\ell_i)) \\ =2n\frac{|m|}{f}\sum_{i\in[n]}\ell_i+O(n\log n\sum_{i\in[n]}\ell_i). \tag{1}$$

Moreover, each element in M has been held by at least $f+1$ correct replicas at the beginning, and thus may be requested by at most $2f$ replicas, i.e., $\sum_{i\in[n]}\ell_i\leq\ell(2f)=2\ell f$. Therefore, the communication cost of our AVDD protocol is bounded by:

$$2n\frac{|m|}{f}\sum_{i\in[n]}\ell_i+O(n\log n\sum_{i\in[n]}\ell_i) \\ \leq 4\ell n|m|+O(\ell n^2\log n). \tag{2}$$

■

Theorem 1 In an asynchronous network of $n=3f+1$, Algorithm 1 solves the data dissemination problem of a ℓ-dimensional vector with at most $4\ell n|m|+O(\ell n^2\log n)$ communication.

Proof: From Lemma 2 and Lemma 3, it is immediate that our AVDD protocol satisfies correctness and the communication cost is at most $4\ell n|m|+O(\ell n^2\log n)$. ■

D. From AVDD to BFT

We can directly use AVDD to build an asynchronous BFT with lower communication compared to existing ones. For instance, as shown in Fig. 3, we can replace the recovery phase of sDumbo with AVDD (setting $\ell=n-f$) to obtain a BFT with lower communication, i.e., $O(n^2|m|+\lambda n^3+n^3\log n)$, where the λn^3 term is due to the use of sMVBA and threshold encryption. We will show how to further reduce it in Section V.

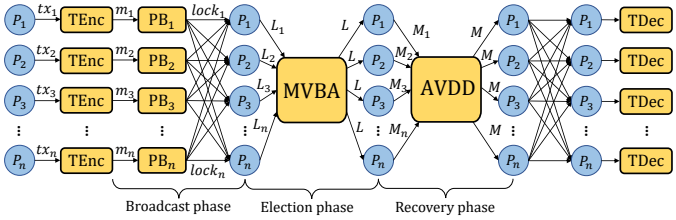

Fig. 3. BFT using AVDD (in sDumbo).

V. The Dory Protocol

This section presents the design of Dory. We begin with the challenges of reducing the communication and then present our protocol in detail.

A. Building Practical BFT with Lower Communication

The censorship resilience challenge. To cope with the censorship resilience challenge [3], most prior works use a threshold encryption scheme to prevent transactions from being censored by the adversary [4], [9], [14]. The use of threshold encryption scheme incurs a minimum of $O(\lambda n^3)$ communication. In particular, since there are $O(n)$ proposals in each epoch, every replica needs to broadcast $O(n)$ decryption shares of λ size, so the communication is $O(\lambda n^3)$. This term is in general not the bottleneck for prior approaches as the communication of the BFT protocols is higher (e.g., HoneyBadger, BEAT, Dumbo, sDumbo all have $O(n^2|m|+\lambda n^3\log n)$ communication so the $O(\lambda n^3)$ term is not the bottleneck any more). In this work, we aim to do better and overcome this communication bottleneck.

Using supplemental consensus to enhance the performance. The *supplemental consensus* mechanism, originally used in DispersedLedger [6], provides an efficient approach to utilize the un-delivered but received proposals from prior epochs to enhance the system performance. The core idea is that for the proposals that are not delivered in the prior epoch, instead of discarding them directly, replicas can still propose them in the current epoch. Accordingly, replicas reach a supplementary consensus of them while reaching an agreement on the proposals of the current epoch. In addition to enhancing the system performance, this approach naturally solves the censorship resilience issue, as the un-delivered proposals can still be included in the proposals in newer epochs. This approach, however, still incurs $O(\lambda n^3\log n)$ communication that we seek to overcome.

Integrating supplemental consensus with AVDD for lower communication. We attempt to integrate supplemental consensus with our proposed approach in Fig. 3 to build a protocol with $O(n^2)$ messages and lower communication—without threshold encryption. The workflow is briefly summarized below.

- First, each replica P_i keeps track of all PB instances and maintains a view vector V_i to keep track of the received proposals. The view vector only stores the instance identifiers instead of the proposals. In particularly, if P_i stores all P_j's proposals up to epoch e, and receives the corresponding $lock$ proofs, P_i will update V_i and set $V_i[j]$ as e.
- At the beginning of an epoch $e'(e'>e)$, each replica P_i includes V_i in its proposal for epoch e'. After the election phase, every replica will decide a common subset consisting of $n-f$ proposals. Every replica first uses an AVDD instance to reconstruct these proposals, where each proposal includes a view vector V_i. Then, based on the $n-f$ view vectors, each replica computes a common view vector V according to Equation (3) shown below. The agreement on the view vectors is called a supplemental consensus.
- Finally, an additional AVDD instance is used to obtain the proposals indexed in V. The union of the transactions from the proposals created in epoch e' and those indexed in V will be delivered.

$$V=\{\max_{f+1}(V_1[1],V_2[1],\ldots,V_{n-f}[1]), \\ \max_{f+1}(V_1[2],V_2[2],\ldots,V_{n-f}[2]), \\ \ldots \\ \max_{f+1}(V_1[n],V_2[n],\ldots,V_{n-f}[n])\} \tag{3}$$

Analysis. Unfortunately, the solution above fails to achieve the agreement property for the proposals indexed in the view vectors. In particular, this is because the common view vector

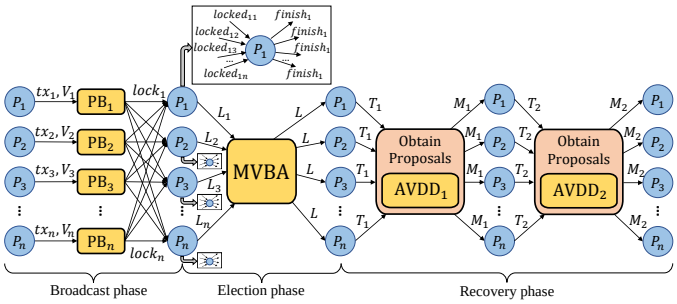

Fig. 4. The workflow of Dory.

V only includes the instance identifiers and if some proposal m is indexed in V, we can only guarantee that at least one correct replica receives the corresponding $lock$ proof for m. In this case, only one correct replica is able to input m in the additional AVDD instance. However, for AVDD to successfully reconstruct each proposal, we need to guarantee that at least $f+1$ correct replicas input m at the beginning of the AVDD protocol. In fact, the fundamental reason that this solution fails is that the PB primitive cannot achieve the totality property, in contrast to the AVID-M [6] primitive used in DispersedLedger.

In Dory, we use two steps, lock and finish, in addition to each PB instance, to bypass this barrier. The finish step can be executed concurrently with MVBA, as it is useful only for the supplementary consensus.

Moreover, we implement a more bandwidth-efficient MVBA protocol called dMVBA achieving $O(n|m| + \lambda n^2 \log n)$ communication. Based on these changes, we can obtain a secure BFT protocol with $O(n^2|m| + \lambda n^2 \log n + n^3 \log n)$ communication.

B. Dory

We are now ready to present the Dory protocol. As illustrated in Fig. 4, the protocol consists of three phases: broadcast, election, and recovery. Briefly speaking, the workflow of an epoch proceeds as follows. At the beginning, each replica P_i includes its view vector in the proposal. The broadcast phase executes n parallel PB instances where each replica P_i starts an instance to send its proposal to all replicas. After each PB instance completes, the sender P_i broadcasts a $lock$ proof via a LOCK message. After each replica receives at least $n-f$ $lock$ proofs, it provides the $lock$ proofs as input to MVBA and starts the election phase. Meanwhile, to achieve agreement for the supplementary consensus, we need the finish step that can be executed concurrently with MVBA. In particular, upon receiving a $lock$ proof for some proposal, each replica replies with a LOCKED message carrying its signature share to the sender. After receiving $n-f$ valid signature shares, P_i combines the signature shares and sends the signature as a $finish$ proof via a FINISH message. After MVBA outputs, replicas enter the recovery phase that involves two AVDD instances to reconstruct the proposals.

We now present in detail the workflow. According to the progress of the replicas, the status of each proposal m_j^e (created by P_j in epoch e) maintained by replica P_i can be one of the following: $locked$, $finished$, and $committed$, as shown below.

- $locked$. If P_i has received a proposal m_j^e from P_j and receives a $lock$ proof (h, σ) where h is a hash for m_j^e and σ is a valid signature for $\langle e, j, h\rangle$ (i.e., $\mathcal{H}(m_j^e) = h$ and $\mathsf{Verify}_{n-f}(\langle e, j, h\rangle, \sigma) = 1$), then the status is $locked$ and P_i sets $lock^e[j]$ as 1.
- $finished$. If P_i receives a proof σ' in a FINISH message for the proposal m_j^e (i.e., $\mathsf{Verify}_{n-f}(\langle e, j, locked\rangle, \sigma') = 1$), then the status is $finished$ and P_i sets $finish^e[j]$ as 1.
- $committed$. If the proposal m_j^e is delivered, then the status is $committed$ and P_i sets $commit^e[j]$ as 1.

The status is useful for each replica to track the undelivered proposals and for our protocol to achieve its security properties. If the status is $locked$, at least $f+1$ correct replica has already received the proposal. If the status is $finished$, at least $f+1$ correct replicas have received the proposal and known its correctness, which is useful for supplementary consensus: A proposal un-delivered in prior epochs can be delivered iff its status is $finished$.

The pseudocode of Dory is shown in Algorithm 3 and the utility functions are shown in Algorithm 2.

Broadcast. The broadcast phase involves n parallel PB instances. At the beginning of each epoch e, each replica P_i first updates V_i by querying the $\mathrm{UpdateView}(e)$ function. The function returns a n-dimensional vector V_i, where each component stores the latest epoch number, up to which the proposals of P_j are set as $finished$ by P_i. Then, P_i includes a batch of transactions tx_i and V_i as the proposal for the current epoch and starts the i^{th} PB instance, denoted as $\mathrm{PB}[\langle e, i\rangle]$.

Algorithm 2 Utility functions of Dory. Code shown for P_i.

```
1: procedure UpdateView(e):
2:     initialize a |n|-dimensional vector V_i
3:     for any j ∈ [n] do
4:         V_i[j] ← the latest epoch e' s.t. e' < e and finish^{e''}[j] = 1 for
   all 1 ≤ e'' ≤ e'
5:     return V_i

6: procedure ObtainProposals(ID, T):
7:     initialize a |T|-dimensional vector M_i, set k ← 0
8:     for any (e, j) ∈ T do
9:         if lock^e[j] = 1 then
10:            M_i[k] = m_j^e
11:        else
12:            M_i[k] = ⊥
13:        k ← k + 1
14:    invoke AVDD[ID] with input M_i
15:    wait until AVDD[ID] outputs M
16:        for (e, j) ∈ T do
17:            lock^e[j] ← 1, finish^e[j] ← 1, commit^e[j] ← 1
18:        return M

19: procedure CheckViews(views, T):
20:    initialize a n-dimensional vector V
21:    for any j ∈ [n] do
22:        V[j] ← the (f+1)^th largest value among {V_k[j]|V_k ∈ views}
23:    for any j ∈ [n] and 1 ≤ e ≤ V[j] do
24:        if commit^e[j] = 0 then
25:            T ← T ∪ {(e, j)}
```

Algorithm 3 The Dory protocol. Code shown for P_i.

let the $\mathcal{Q}$ of MVBA[ID] be the following predicate: $\mathcal{Q}_{\mathsf{ID}}(\{(j_1, h_{j_1}, \sigma_{j_1}), \ldots, (j_{n-f}, h_{j_{n-f}}, \sigma_{j_{n-f}})\}) \equiv$ (for any $k \in [n-f]$, $\mathsf{Verify}_{n-f}(\langle \mathsf{ID}, j_k, h_{j_k} \rangle, \sigma_{j_k}) = 1$)

1: **upon** invocation of epoch e **do**
2: **Initialization:** $lock^e \leftarrow (0_1, \ldots, 0_n)$; $finish^e \leftarrow (0_1, \ldots, 0_n)$; $commit^e \leftarrow (0_1, \ldots, 0_n)$; $S_i \leftarrow \{\}$; $L_i \leftarrow \{\}$; $T_1 \leftarrow \{\}$; $T_2 \leftarrow \{\}$.
3: **upon** receiving transactions tx_i to be proposed in epoch e **do**
4: $V_i \leftarrow \text{UpdateView}(e)$ ▷ Broadcast phase
5: **let** $m_i^e = (tx_i, V_i)$ be the proposal of epoch e
6: **invoke** PB$[\langle e, i \rangle]$ with input m_i^e
7: **upon** receiving (h_i, σ_i) from PB$[\langle e, i \rangle]$ **do**
8: **broadcast** (LOCK, e, h_i, σ_i)
9: **upon** receiving m_j^e from PB$[\langle e, j \rangle]$ **do**
10: **store** m_j^e
11: **upon** receiving (LOCK, e, h_j, σ_j) from P_j **do**
12: **wait** until $m_j^e \neq \perp$
13: **if** $\mathcal{H}(m_j^e) = h_j$ and $\mathsf{Verify}_{n-f}(\langle e, j, h_j \rangle, \sigma_j) = 1$ **then**
14: $lock^e[j] \leftarrow 1$ ▷ Locked
15: $\rho_i \leftarrow \mathsf{Sign}_{n-f}(sk_i, \langle e, j, locked \rangle)$
16: $L_i \leftarrow L_i \cup \{j, h_j, \sigma_j\}$
17: **send** (LOCKED, e, ρ_i) to P_j
18: **upon** receiving (LOCKED, e, ρ_j) from P_j **do**
19: **if** $\mathsf{VerifyShare}_{n-f}(\langle e, i, locked \rangle, (j, \rho_j)) = 1$ **then**
20: $S_i \leftarrow S_i \cup \{j, \rho_j\}$
21: **if** $|S_i| = n - f$ **then**
22: $\sigma_i' \leftarrow \mathsf{Combine}_{n-f}(\langle e, i, locked \rangle, S_i)$
23: **broadcast** (FINISH, e, σ_i')
24: **upon** receiving (FINISH, e, σ_j') from P_j **do**
25: **if** $\mathsf{Verify}_{n-f}(\langle e, j, locked \rangle, \sigma_j') = 1$ **then**
26: $finish^e[j] \leftarrow 1$ ▷ Finished
27: **upon** $|L_i| = n - f$ **then** ▷ Election phase
28: **invoke** MVBA$[e]$ with input L_i
29: **upon** receiving $L = \{(j_k, h_{j_k}, \sigma_{j_k})\}_{k \in [n-f]}$ from MVBA$[e]$ **do**
30: **for** any $(j_k, h_{j_k}, \sigma_{j_k}) \in L$ **do** ▷ Recovery phase
31: **if** $m_{j_k}^e \neq \perp$ and $\mathcal{H}(m_{j_k}^e) = h_{j_k}$ **then**
32: $lock^e[j_k] \leftarrow 1$ ▷ Locked
33: $T_1 \leftarrow T_1 \cup \{(e, j_k)\}$
34: $M_1 \leftarrow \text{ObtainProposals}(\langle e, 1 \rangle, T_1)$ ▷ 1st AVDD
35: **for** any $m_j^e \in M_1$ **do**
36: **decompose** m_j^e into transactions tx_j^e and view V_j
37: CheckViews$(\{V_j | m_j^e \in M_1\}, T_2)$
38: $M_2 \leftarrow \text{ObtainProposals}(\langle e, 2 \rangle, T_2)$ ▷ 2nd AVDD
39: **for** any $m_{j'}^{e'} \in M_2$ **do**
40: **extract** transactions $tx_{j'}^{e'}$ from $m_{j'}^{e'}$
41: **output** $\{tx_j^e | m_j^e \in M_1\} \cup \{tx_{j'}^{e'} | m_{j'}^{e'} \in M_2\}$

After PB$[\langle e, i \rangle]$ completes, (h_i, σ_i) is returned, where h_i is the hash of m_i^e and σ_i is a signature for $\langle e, i, h_i \rangle$. Then P_i broadcasts a LOCK message (ln 3-8). Meanwhile, if P_i receives the proposal m_j^e from P_j in PB$[\langle e, j \rangle]$, it stores m_j^e. If P_i receives a valid LOCK message for PB$[\langle e, j \rangle]$, it creates a signature share for $\langle e, j, locked \rangle$ and sends a LOCKED message to P_j. Finally, if P_i receives $n-f$ signature shares from the LOCKED messages, it combines the signature shares into a signature σ_i' and then broadcasts a (FINISH, e, σ_i') message (ln 9-26). Each replica P_i keeps track of all the proposals and updates the $lock^e$, $finish^e$, and $commit^e$ parameters according to the description mentioned above.

Election. After the status of $n-f$ proposals of epoch e become $locked$, P_i invokes MVBA$[e]$ providing the $lock$ proofs as input (ln 27-28).

As sMVBA is another communication bottleneck, we implement a more efficient MVBA protocol called dMVBA as shown in Fig. 5. Specifically, we apply the APDB protocol [12] on sMVBA to reduce the communication from $O(n^2|m| + \lambda n^2)$ to $O(n|m| + \lambda n^2 \log n)$ (where $|m|$ is the length of the input for MVBA). At the beginning, P_i encodes its input into fragments and disperses them with Merkle tree witnesses via ECHO messages. Then, upon a receiving valid fragment, each replica returns a signature share for the Merkle root rt_i via a READY message. After collecting $n-f$ signature shares, P_i combines them into a signature σ_i and triggers sMVBA with rt_i and σ_i. Finally, sMVBA will output a tuple (rt_s, σ_s) and replicas will reconstruct the corresponding input as the output of dMVBA. The security of dMVBA follows from [12].

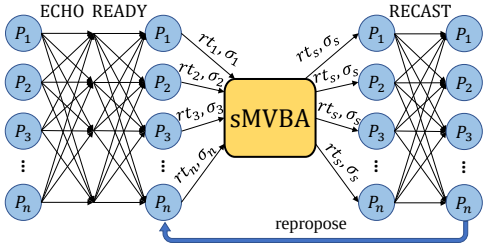

Fig. 5. The workflow of dMVBA.

Recovery. After MVBA$[e]$ outputs L, P_i starts the recovery phase. There are two AVDD instances, one for recovering the proposals created in the current epoch, and one for recovering the proposals indexed in the view vectors. In particular, for every $(j_k, h_{j_k}, \sigma_{j_k})$ in L, if P_i has stored m_j^e but the status is not $locked$, P_i sets the status as $locked$. Then, P_i starts the first AVDD instance by querying the ObtainProposals$(\langle e, 1 \rangle, T_1)$ function. After a vector of proposals M_1 is obtained from AVDD, each replica obtains the transactions included in the proposals (ln 29-36). Additionally, P_i further extracts the view vectors and combines them into a common vector V, by querying the CheckViews$(\{V_j | m_j^e \in M_1\}, T_2)$ function. Then P_i starts the second AVDD instance to reconstruct the proposals indexed in V, also by querying the ObtainProposals() function. A set of transactions are obtained (ln 36-40). Finally, P_i takes a union of the transactions included in the output of two AVDD instances, and deliver them according to a predefined deterministic order (ln 41).

C. Efficiency Analysis

Complexity. The Dory protocol only involves all-to-all communication so the message complexity is $O(n^2)$. We now analyze the communication complexity. In the broadcast phase, the input of each PB instance includes a proposal and a n-dimensional view vector, where the size of transactions is $|m|$ and the size of the view vector is $O(n)$ considering epoch number is a constant. As the broadcast phase involves n parallel PB instances, the communication complexity is $O(n^2|m| + \lambda n^2 + n^3)$. The election phase has one dMVBA instance and each replica's input includes $O(n)$ hashes, and $O(n)$ signatures. The communication complexity is thus $O(\lambda n^2 \log n)$. We then focus on the recovery phase. Recall that in every

epoch, any replica will not invoke MVBA until the status of $n-f$ proposals is $locked$. Accordingly, none of correct replicas will request more than f proposals with the same epoch number in any AVDD instance. Therefore, on average, the expected number of proposals requested by each replica in each epoch does not exceed f. As we show in Equation (1), the amortized communication cost of the recovery phase is no more than $2n\frac{|m|}{f}\cdot nf+O(n\log n\cdot nf) \leq 2n^2|m|+O(n^3\log n)$, and thus the $O(n^2|m|+n^3\log n)$ communication complexity.

To summarize, Dory has an $O(n^2)$ message complexity and an $O(n^2|m|+\lambda n^2\log n+n^3\log n)$ communication complexity. The time complexity is $O(1)$ as we follow the classic MVBA-based paradigm.

Communication cost in the optimistic case. Dory has a fast path that enjoys even lower communication in the optimistic case. In particular, if all replicas have already received all the proposals, none of the replicas will trigger the recovery phase. In this case, the communication cost in the optimistic case of Dory is only $n^2|m|+O(\lambda n^2\log n+n^3)$.

D. Security Analysis

Lemma 4 In epoch e, every correct replica will invoke the MVBA instance and get some output L from it.

Proof: Due to the termination property of PB, all correct replicas will complete a PB instance as the sender and broadcast the corresponding $lock$ proof. It means that each replica P_i will store m_j^e and receive valid (h_j,σ_j) from at least $n-f$ correct replicas. Thus, P_i will invoke the MVBA instance using a valid L_i as input. Due to the termination of MVBA, after all correct replicas invoke the MVBA instance with valid inputs, they will get an output L from it. ■

Lemma 5 In epoch e, every correct replica will get the same L, such that $\mathsf{Verify}_{n-f}(\langle e,j,h_j\rangle,\sigma_j)=1$ for any tuple $(j,h_j,\sigma_j)\in L$.

Proof: Due to Lemma 4 and the agreement of MVBA, every correct replica will get the same output L. Moreover, due to the external validity of MVBA, every tuple (j,h_j,σ_j) in L satisfies $\mathsf{Verify}_{n-f}(\langle e,j,h_j\rangle,\sigma_j)=1$. ■

Lemma 6 For any PB instance $\mathrm{PB}[\langle e,k\rangle]$, if any two correct replicas P_i and P_j set the status of the corresponding proposal as $locked$ and have stored $(m_k^e)^i$ and $(m_k^e)^j$ respectively, then $(m_k^e)^i=(m_k^e)^j$.

Proof: Suppose $(m_k^e)^i\neq(m_k^e)^j$, then P_i and P_j must have received different $lock$ proofs, i.e., (h,σ) and (h',σ') where $h=\mathcal{H}((m_k^e)^i)$ and $h'=\mathcal{H}((m_k^e)^j)$. It violates the provability property of PB. Thus, P_i and P_j must have stored the same proposal from $\mathrm{PB}[\langle e,k\rangle]$. ■

By extending Lemma 6, we get that if a replica P_i sets $lock^e[j]$ as 1, then it must be able to know that it has stored the correct m_j^e.

Lemma 7 In each epoch, every correct replica will set T_1 to the same value, and get the same corresponding proposals included in M_1.

Proof: T_1 is determined by MVBA's output L. Due to Lemma 4 and Lemma 5, every correct replica will get the same L and thus decide the same T_1 by deterministic algorithms. Each proposal indexed in T_1 has been stored by at least $f+1$ correct replicas, and these replicas will set its status as $locked$ because they are all able to see the corresponding $lock$ proof due to the *Agreement* of MVBA. Then due to Lemma 6, the AVDD condition for the vector of these proposals is satisfied and every correct replica will get the same M_1 including them. ■

Lemma 8 In each epoch, every correct replica will set T_2 to the same value, and get the same corresponding proposals included in M_2.

Proof: T_2 is determined by the view vectors included in M_1. Due to Lemma 7, every correct replica will get the same M_1 and thus decide the same T_2 by deterministic algorithms. In the $\mathrm{CheckViews}()$ function (Algorithm 2), the common view vector V is computed by taking the $(f+1)^{th}$ largest value among $n-f$ view vectors for each component in V. Therefore, for any $i\in[n]$, $V[i]$ is no larger than at least one $V_j[i]$ from a correct replica. Namely, for each proposal indexed in T_2, at least one correct replica has set it as $finished$. Thus, at least $f+1$ correct replicas have set it as $locked$. Then due to Lemma 6, the AVDD condition for the vector of these proposals is satisfied and every correct replica will get the same M_2 including them. ■

Lemma 9 For any proposal m_i^e created by a correct replica P_i in epoch e, if it is not delivered in epoch e, then it will eventually be delivered in a later epoch e'.

Proof: At the beginning of epoch e, P_i inputs m_i^e to $\mathrm{PB}[\langle e,k\rangle]$. Due to *Termination* of PB, P_i is able to get the corresponding $lock$ proof. Then, since P_i is correct, it will broadcast the $lock$ proof to all replicas, collect $n-f$ signature shares in LOCKED from correct replicas at least, and then broadcast a $finish$ proof. Therefore, every correct replicas will see the $finish$ proof and set $finish^e[i]$ as 1. Later, at the beginning of some epoch $e'(e'>e)$, every correct replicas will index m_j^e in the view vector associated with its proposal. In the recovery phase of epoch e', the first AVDD instance will output a vector M_1 containing $n-f$ view vectors, at least $f+1$ of which are from correct replicas. As the common view vector V is computed by taking the $(f+1)^{th}$ largest value among these view vectors for each component of V, m_i^e must be indexed in V and thus delivered through the second AVDD instance. ■

Theorem 2 Dory achieves agreement, total order, and liveness.

Proof: Agreement follows from Lemma 7 and Lemma 8. Then, due to the agreement, every replica outputs the same transactions in the same epoch. Since the transactions in a single epoch are delivered according to a pre-defined deterministic order and Dory is invoked sequentially according to monotonically increasing epoch numbers, it is straightfor-

ward that Dory achieves total order. Liveness follows from Lemma 9. Namely, if some transactions are not delivered in the epoch that they are proposed, they are still able to deliver through the supplemental consensus. ■

VI. Implementation and Evaluation

We implement[1] Dory and sDumbo in Golang (both open-sourced), and evaluate their communication cost and performance in WAN settings. Our evaluation results show that 1) Dory concretely saves the communication cost compared with sDumbo, 2) Dory achieves low latency—less than 8s even for $n = 151$ replicas, and 3) Dory achieves high throughput (135k tx/s for 16 replicas and 57k tx/s for 151 replicas) which is 2-5$\times$ that of sDumbo.

Implementation. We implement Dory and sDumbo in Golang using the same underlying modules, libraries and security parameters for a fair comparison. For the network connection, we use TCP sockets to realize reliable point-to-point channels, while running n message sending goroutines and one message receiving goroutine at each replica. For threshold signature and coin-tossing, we use Boldyreva's pairing-based threshold scheme [18] implemented in kyber[2]. For Reed-Solomon error correcting code, we use an open-source implementation in infectious[3] that can easily process transactions at a speed of gigabits per second even for $n = 100$.

Experiment setup. We deploy Dory and sDumbo on Amazon EC2 using 151 instances where the instances are evenly distributed in up to 10 regions (Singapore, Mumbai, Stockholm, Paris, Frankfurt, St. Paulo, California, Virginia and Canada). Each replica runs on a t3.medium instance with two virtual CPUs and 4GB memory. We assume that each transaction is a random string of 250 bytes which matches the size of basic Bitcoin transactions. The batch size represents the number of transactions input by all replicas in a single epoch. Following the prior works [3], [14], we define the latency as the time interval between the time the first replica starts a new epoch and the time when the $(n-f)^{th}$ replica finishes this epoch. We also evaluate the basic latency which denotes the latency in contention-free scenarios. We simply let each replica input one transaction, i.e., the batch size is n. In all experiments, we run both Dory and sDumbo for ten epochs and report the average value as the result.

Communication cost. We first evaluate the communication cost of Dory and compare it with sDumbo. We measure the total communication bytes for all the messages sent by each replica while running the protocols. We consider the *ideal cost* as $n|m|$, as this is the minimum communication cost one could expect for atomic broadcast at a single replica; note that the ideal cost also equals a multiplication of the size of each transaction and the batch size. We define *redundant communication cost* is the communication cost minus the ideal cost. As shown in Fig. 6a, though both Dory and sDumbo's

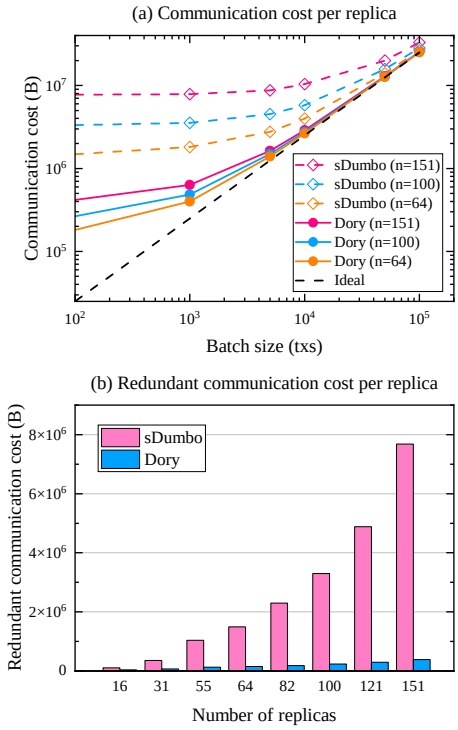

Fig. 6. Communication cost of Dory and sDumbo.

communication cost increase as the number of replicas scales, Dory keeps a tighter distance with the ideal cost. For example, when $n = 151$ and the batch size reaches 10,000, Dory costs about 2.75MB per replica, which is only 15% higher than the ideal, while sDumbo costs about 9.91MB per replica which is $4\times$ that of the ideal cost. Only when the batch size becomes much larger (e.g. $> 10^5$), the communication cost of Dory and sDumbo becomes closer to the ideal cost. This is because the $n^2|m|$ term dominates the communication for large batches.

We also visualize the redundant communication cost of the two BFT protocols in Fig. 6b, which helps understand the performance difference between the two protocols. When the number of replicas increases from 16 to 151, the redundant communication cost of Dory increases from 35KB to just 375KB, which is in sharp contrast to that of sDumbo.

Performance. Fig. 7 shows batch size vs. throughput and throughput vs. latency of Dory and sDumbo for different network sizes and batch sizes. For both throughput and latency, Dory consistently outperforms sDumbo. In particular, when $n = 151$, the throughput of Dory is more than $5\times$ that of sDumbo for *all* batch sizes. We report the latency vs. throughput in Fig. 7b. For all settings, Dory has shown consistently and significantly better performance than sDumbo.

We also report the peak throughput of Dory and sDumbo. As shown in Fig. 8, the throughput of Dory is 2-5$\times$ of sDumbo. The highest peak throughput Dory achieves in our experiments is 135k tx/s for $n = 16$. In particular, when $n = 64$, the throughput of Dory is 4.7x that of sDumbo, where in contrast, the throughput of Dumbo-NG is only about 2x that of sDumbo.

[1]https://github.com/xygdys/Consensus
[2]https://github.com/dedis/kyber
[3]https://github.com/vivint/infectious

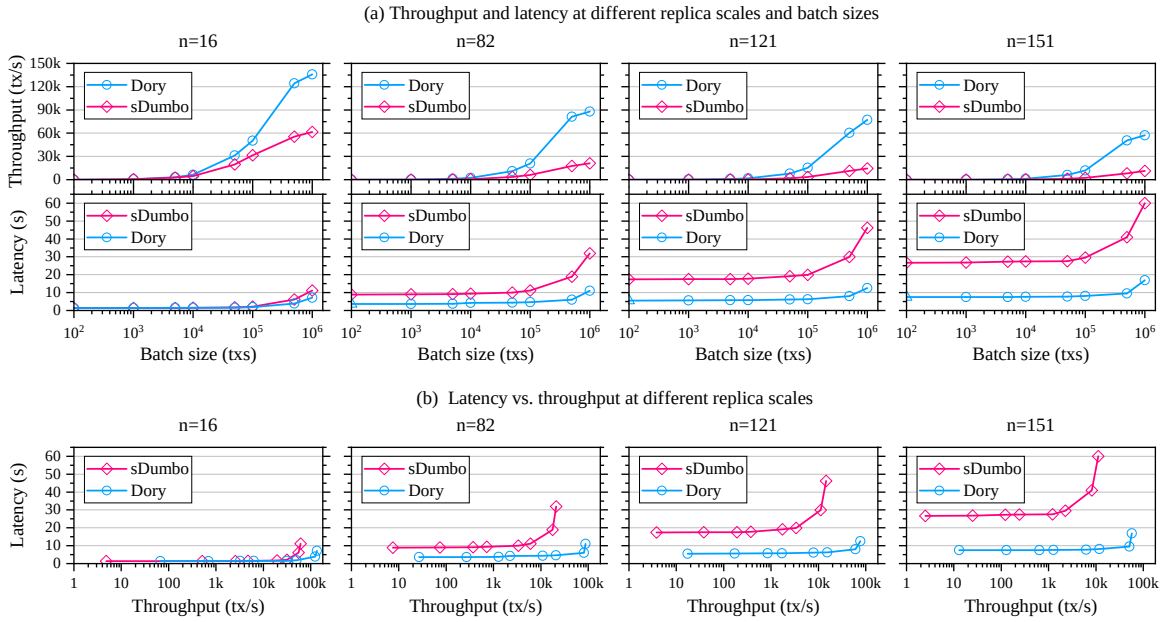

Fig. 7. Throughput and latency of Dory and sDumbo at different replica scales and batch sizes.

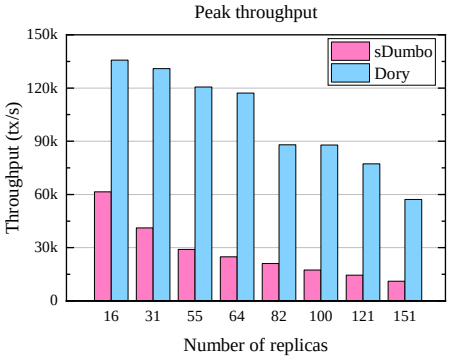

Fig. 8. Peak throughput of Dory and sDumbo.

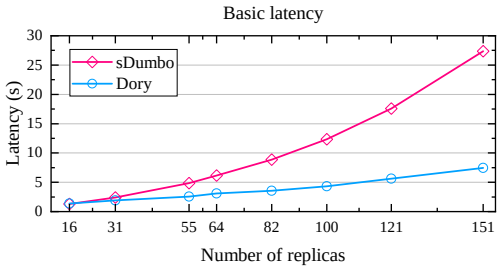

Fig. 9. Basic latency cost of Dory and sDumbo.

We report the basic latency for Dory and sDumbo for different network sizes. As shown in Fig. 9, the basic latency of Dory is much lower than that of sDumbo for all the experiments we have conducted. When $n = 151$, the basic latency of Dory is only 7.5s which is only 27% of that of sDumbo.

All experiments conducted have shown the scalability of Dory.

VII. Additional Related Work

Much related work has been discussed in the course of the paper; here we discuss additional related work.

The Byzantine agreement problem was first introduced by Lamport, Shostak and Pease [25]; since then, various Byzantine-resilient primitives have been studied. The BFT protocols studied in this paper are Byzantine fault-tolerant state machine replication protocols that distinguish clients from servers.

This paper focuses on asynchronous BFT protocols with implementations. In addition to the three types of asynchronous BFT protocols mentioned in the introduction, we also have BFT protocols from locally generated coins [26]–[28].

Our technique of enabling supplemental consensus in our framework—namely using an additional round of linear communication with threshold signatures—may be viewed as applying the technique of HotStuff [21], [29].

The MVBA primitive was introduced by Cachin, Kursawe, Petzold, and Shoup [7]. Abraham, Malkhi, and Spiegelman proposed a MVBA protocol [21] that attain $O(n^2|m| + \lambda n^2)$ communication (with optimal word complexity) and additionally achieves a quality property. Lu et al. [12] reduced the communication from $O(n^2|m| + \lambda n^2)$ to $O(n|m| + \lambda n^2)$ by using vector commitments. The recent work from Guo et al. [9] and Gelashvili et al. [30] focused on how to reduce the expected number of rounds while achieving $O(n^2|m| + \lambda n^2)$. Our dMVBA protocol uses the framework of Dumbo-MVBA by Lu et al. [12] but uses sMVBA to save steps. We did not use pairing-based constant-size vector commitments (e.g., KZG commitments [31]) but chose to use log-size Merkle trees for efficiency; this is also the reason why our dMVBA has

$O(n|m| + \lambda n^2 \log n)$ communication (with an additional $\log n$ factor).

VIII. Conclusion

This paper designs and implements an efficient and scalable asynchronous BFT protocol called Dory with reduced communication and improved efficiency compared to existing protocols. We designed a novel primitive called asynchronous vector data dissemination, and we developed the idea of supplemental consensus. We have implemented and deployed Dory using 151 Amazon EC2 instances evenly distributed in 10 regions. We show that even without using the technique of separating data transmission from agreement, Dory has lower communication and up to 5x the throughput of sDumbo.

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