Pushing the Limits of Generic Side-Channel Attacks on LWE-based KEMs - Parallel PC Oracle Attacks on Kyber KEM and Beyond

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Abstract. In this work, we propose generic and novel adaptations to the binary Plaintext-Checking (PC) oracle based side-channel attacks for Kyber KEM. Binary PC oracle-based side-channel attacks are fairly generic and easy to mount on a given target, as the attacker requires very minimal information about the target device. However, these attacks have an inherent disadvantage of requiring a few thousand traces to perform full key recovery, as they only recover a single bit of information per trace. We propose novel *parallel PC oracle* based side-channel attacks, which are capable of recovering an arbitrary P number of bits of information about the secret key in a single trace. We experimentally validated our attacks on the fastest implementation of unprotected Kyber KEM in the *pqm4* library. Our experiments yielded improvements in the range of 2.89× and 7.65× in the number of queries, compared to state-of-the-art binary PC oracle attacks, while arbitrarily higher improvements are possible for a motivated attacker, given the generic nature of our attack. Finally, we also conduct a thorough study of the capability of our attack in different attack scenarios, based on the presence/absence of a clone device, and also partial key recovery. We also show that our proposed attacks are able to achieve the lowest number of queries for key recovery, even over implementations protected with low-cost countermeasures such as shuffling. Our work therefore, concretely demonstrates the power of PC oracle attacks on Kyber KEM, thereby stressing the need for concrete countermeasures such as masking.

Keywords: lattice-based cryptography · Side-Channel Analysis · Kyber · Plaintext-Checking Oracle · Chosen-Ciphertext Attack · Key Encapsulation Mechanism

1 Introduction

NIST very recently announced results for the third round of the Post-Quantum Cryptography (PQC) standardization process [AAC+22], in which CRYSTALS-Kyber [ABD+21] was selected as the sole candidate for standardization of Key Encapsulation Mechanisms (KEMs). The security of Kyber is based on the well known Module Learning With Errors (MLWE) problem, and served as one of the most promising candidates for KEMs in the NIST PQC process, owing to the confidence in its theoretical security guarantees, while also offering one of the best implementation performance compared to other PQC based KEMs [AASA+20]. Thus, one can expect Kyber KEM to be implemented and designed on a wide-variety of computational platforms and applications, in the coming years. Thus,

security of Kyber against physical attacks such as Side-Channel Attacks (SCA) naturally arises as an immediate concern, particularly for applications involving embedded devices. NIST had also particularly encouraged more research on analysing the security of PQC schemes against SCA [AASA+20].

In this respect, the cryptographic community has shown significant interest towards research on development of new attacks on several lattice-based schemes including Kyber KEM [RBRC21, RRCB20, BDH+21], as well as development of efficient side-channel protection techniques [HKL+22, BGR+21]. While there exists a wide variety of SCA particularly on Kyber KEM, we observe that they can be broadly classified into two main categories. The first category of attacks only require either a single trace or very few traces to perform key recovery or message recovery [NDGJ21, RBRC21, ACLZ20]. However, these attacks typically target very precise leakages from targeted operations within the scheme. This typically requires a fairly sophisticated setup, as well as a detailed knowledge of the target device. Moreover, exploitation of such leakages on different implementation platforms is not very straightforward, and at the very least requires significant modifications.

The second category of attacks are more generic, and exploit inherent vulnerabilities in the algorithm for key recovery, while remaining relatively somewhat agnostic to the target/implementation. These generic attacks typically work by instantiating a side-channel based *Plaintext-Checking (PC) oracle* for malformed chosen-ciphertexts, to perform key recovery. However, these side-channel assisted PC oracle attacks typically suffer from a disadvantage of requiring a few thousand queries for key recovery. Thus, we observe a clear trade-off for both the categories of attacks, based on the ease of mounting the attack versus number of traces for key recovery.

In this work, we attempt to *bridge this gap* through our proposal of *parallel* PC oracle attacks for LWE-based KEMs, with main focus on Kyber KEM. While all our attacks are demonstrated on Kyber KEM, our attack can be adapted to similar LWE/LWR-based KEMs such as Saber [DKR+20], FrodoKEM [ABD+20] etc. The main contributions of our work are as follows:

1. We propose generic and novel adaptations of side-channel assisted binary PC oracle-based attacks, referred to as P-way parallel PC oracle attack, which has the ability to simultaneously recover an arbitrary P number of bits of information about the secret key in a single trace, while state-of-the-art PC oracle attacks recover only a single bit at once.

2. We identify that existing approaches to construct chosen-ciphertext queries for optimal key recovery in the case of binary PC oracle attacks, are not optimal for our proposed attacks. We propose improved constructions of Binary Decision Trees (BDTs) to identify lower bounds for the number of queries for our proposed parallel PC oracle attacks.

3. We also propose generic improvements to the binary classifiers used in the binary PC oracle attack, resulting in multi-class classifiers for any 2^P number of classes for our proposed P-way oracle attack. We practically validated that our multi-class classifier is able to uniquely classify between 1024 classes (for $P = 10$) with 100% success rate, while arbitrarily higher values of P are also possible.

4. We experimentally validated our attacks on the fastest implementation of Kyber KEM in the *pqm4* library, a well known benchmarking and testing framework for PQC schemes on the ARM Cortex-M4 microcontroller. We practically validated improvements in the range of 2.89× and 7.65×, compared to state-of-the-art binary PC oracle attacks. However, we note that significant improvements are possible as shown later in the paper.

5. We also conduct a comprehensive analysis of the capabilities of our attack in different attack scenarios, based on (1) the presence/absence of a clone device and (2) partial key recovery for attackers with different capabilites for offline computation. We observe that our attack brings about arbitrarily high improvements in the number of traces, especially when the attacker can construct a very high number of templates on the clone device[1]

6. We also practically validated the applicability of our attacks to implementations protected with low-cost countermeasures such as shuffling. Our attack yields the lowest number of traces compared to existing state-of-the-art attacks targeting the shuffled implementation of Kyber KEM.

2 Preliminaries

2.1 Notation

We denote the ring of integers modulo $q \in \mathbb{Z}^+$ as $\mathbb{Z}_q$. Elements in $\mathbb{Z}_q$ are denoted using lower case letters (i.e.) $a \in \mathbb{Z}_q$, and the i^{th} bit of $a \in \mathbb{Z}_q$ is denoted as a_i. We use R_q to denote the polynomial ring $\mathbb{Z}_q[x]/(x^n+1)$ and polynomials in R_q are denoted using bold lower case letters (i.e.) $\mathbf{a} \in R_q$. The i^{th} coefficient of $\mathbf{a} \in R_q$ is denoted as $\mathbf{a}[i]$. A vector of polynomials in R_q (i.e.) R_q^k with $k \in \mathbb{Z}^+$ is denoted using bold lower case letters, while a matrix of polynomials in $R_q^{k \times \ell}$ with $(k, \ell) \in \mathbb{Z}^+$ are denoted using bold upper case letters. The i^{th} polynomial of $\mathbf{a} \in R_q^k$ is denoted as $\mathbf{a}_i$. Matrices and vectors of polynomials in R_q are together referred to as *modules*. The product of two polynomials $\mathbf{a}$ and $\mathbf{b}$ in R_q is denoted as $\mathbf{c} = \mathbf{a} \cdot \mathbf{b} \in R_q$, while pointwise multiplication using $\circ$ (i.e.) $\mathbf{c} = \mathbf{a} \circ \mathbf{b} \in R_q$. Byte arrays of length n are denoted as $\mathcal{B}^n$. The transpose of a matrix $\mathbf{A}$ is denoted as $\mathbf{A}^T$. The NTT representation of $\mathbf{a} \in R_q$ is denoted as $\hat{\mathbf{a}} \in R_q$.

2.2 Kyber KEM

Kyber is a chosen-ciphertext secure KEM (IND-CCA), built upon the hardness of the Module-LWE (MLWE) problem. It offers three parameter sets, hereby listed in increasing levels of security - (1) `Kyber-512` (NIST Security Level 1), (2) `Kyber-768` (Level 3) and (3) `Kyber-1024` (Level 5) with $k = 2, 3$ and 4 respectively. The CCA secure Kyber KEM is built upon a simpler chosen-plaintext secure PKE (IND-CPA), denoted as CPA.Kyber PKE. Refer to Algorithm 1 for a simplified description of the key-generation (CPA.KeyGen), encryption (CPA.Encrypt) and decryption procedures (CPA.Decrypt) of CPA.Kyber PKE. Sample_U is used to denote the operation sampling coefficients from a uniform distribution, Sample_B to denote sampling from a centered binomial distribution (CBD) and the function Expand, to denote expanding a small seed into a uniformly random matrix in $R_q^{k \times k}$. The function $\mathsf{Compress}(\mathbf{u}, d)$ lossily compresses $\mathbf{u} \in \mathbb{Z}_q$ into $v \in \mathbb{Z}_{2^d}$ with $q > 2^d$, while $\mathsf{Decompress}(\mathbf{v}, d)$ extrapolates $\mathbf{v} \in \mathbb{Z}_{2^d}$ into $u' \in \mathbb{Z}_q$.

2.2.1 IND-CCA Security

The IND-CPA secure PKE is transformed into an IND-CCA secure KEM using the well-known Fujisaki-Okamoto transformation [FO99]. It involves the use of two hash functions $(\mathcal{H}, \mathcal{G})$ and a key-derivation function KDF, forming a wrapper denoted as encapsulation (CCA.Encaps) and decapsulation (CCA.Decaps) procedures of CCA.Kyber KEM (Refer Alg.2).

[1] $\approx 25\times$ improvement for an arbitrarily strong attacker capable of constructing 2^{32} templates on the clone device

Algorithm 1 CPA Secure Kyber PKE (Simplified)

1: **procedure** CPA.KeyGen
2: $(seed_A, seed_B) \in \mathcal{B}^* \leftarrow \mathsf{Sample}_U()$ ▷ Generate uniform seeds $seed_A, seed_B$
3: $\hat{\mathbf{A}} \in R_q^{k \times k} \leftarrow \mathsf{Expand}(seed_A)$ ▷ Expand $seed_A$ into $\hat{\mathbf{A}}$
4: $\mathbf{s}, \mathbf{e} \in (R_q^k \times R_q) \leftarrow \mathsf{Sample}_B(seed_B, coins)$ ▷ Sample $\mathbf{s}, \mathbf{e}$
5: $\hat{\mathbf{s}} \in R_q^k \leftarrow \mathsf{NTT}(\mathbf{s})$; $\hat{\mathbf{e}} \in R_q^k \leftarrow \mathsf{NTT}(\mathbf{e})$ ▷ $\mathsf{NTT}(\mathbf{s})$, $\mathsf{NTT}(\mathbf{e})$
6: $\hat{\mathbf{t}} = \hat{\mathbf{A}} \circ \hat{\mathbf{s}} + \hat{\mathbf{e}}$ ▷ $\mathbf{t} = \mathbf{A} \cdot \mathbf{s} + \mathbf{e}$ in NTT domain
7: Return $(pk = (seed_A, \hat{\mathbf{t}}), sk = (\hat{\mathbf{s}}))$
8: **end procedure**

9: **procedure** CPA.Encrypt$(pk, m \in \{0,1\}^{256}, seed_R \in \{0,1\}^{256})$
10: $\hat{\mathbf{A}} \in R_q^{k \times k} \leftarrow \mathsf{Expand}(seed_A)$
11: $\mathbf{r}, \mathbf{e_1}, \mathbf{e_2} \in (R_q^k \times R_q^k \times R_q) \leftarrow \mathsf{Sample}_B(seed_R)$ ▷ Sample $\mathbf{r}, \mathbf{e_1}, \mathbf{e_2}$
12: $\hat{\mathbf{r}} \in R_q^k \leftarrow \mathsf{NTT}(\mathbf{r})$ ▷ $\mathsf{NTT}(\mathbf{r})$
13: $\mathbf{u} \in R_q^k \leftarrow \mathsf{INTT}(\mathbf{A}^T \circ \hat{\mathbf{r}}) + \mathbf{e_1}$ ▷ $\mathbf{u} = \mathbf{A}^T \cdot \mathbf{r} + \mathbf{e_1}$
14: $\mathbf{v}' \in R_q \leftarrow \mathsf{INTT}(\hat{t}^T \circ \hat{\mathbf{r}}) + \mathbf{e_2}$ ▷ $\mathbf{v}' = \mathbf{t}^T \cdot \mathbf{r} + \mathbf{e_2}$
15: $\mathbf{v} \in R_q \leftarrow \mathbf{v} + \mathsf{Decompress}(m, 1)$ ▷ $\mathbf{v} = \mathbf{v} + \mathsf{Encode}(m)$
16: Return $ct = \mathsf{Compress}(\mathbf{u}, d_1), \mathsf{Compress}(v, d_2)$
17: **end procedure**

18: **procedure** CPA.Decrypt(sk, ct)
19: $(\mathbf{u}, \mathbf{v}) \leftarrow \mathsf{Decompress}(ct, d_1, d_2)$
20: $\hat{\mathbf{u}} = \mathsf{NTT}(\mathbf{u})$
21: $\mathbf{m} = \mathbf{v} - \mathsf{INTT}(\hat{\mathbf{u}} \circ \mathbf{s})$ ▷ $\mathbf{m} = \mathbf{v} - \mathbf{u} \cdot \mathbf{s}$
22: $m \in R_q \leftarrow \mathsf{Compress}(\mathbf{m}, 1)$ ▷ Decoding $\mathbf{m} \in R_q$ into $m \in \mathcal{B}^{32}$
23: Return m
24: **end procedure**

Within this framework, the encryption procedure is deterministic and depends solely on the message m for a given public key pk. This is done by ensuring that the seed input r' to the encryption procedure is derived by hashing m with pk (Line 4-5 in CCA.Encaps). In the decapsulation procedure (CCA.Decaps), the decrypted message m is *re-encrypted* (Line 12-13) to generate the ciphertext ct'. The received ciphertext ct is compared with ct', and the valid shared key is generated (Line 17) only if $ct = ct'$, denoting a valid ciphertext, else a pseudo-random key is generated (invalid ciphertext). This enables to detect invalid/malicious ciphertexts, thereby offering concrete protection against chosen-ciphertext attacks. We refer the reader to [ABD+21] for more details on CCA secure Kyber KEM.

2.3 Prior Side-Channel Attacks and Motivation

LWE/LWR-based KEMs including Kyber KEM have been subjected to a wide variety of side-channel attacks whose primary target is the long-term secret key $\mathbf{s}$ used in the decapulation procedure. Recovery of a single secret key $\mathbf{s}$ leads to compromise of all the corresponding session keys K, that were derived using $\mathbf{s}$.

In this respect, we can broadly classify existing attacks on Kyber KEM into two categories: (1) Target Operation Independent attacks (TO_Indep) and (2) Target Operation Dependent (TO_Dep) attacks. TO_Dep attacks are those that exploit side-channel leakage from a specific targeted operation within the decapsulation procedure [XPR+21, ACLZ20, PPM17]. On the other hand, TO_Indep attacks are not limited to any single operation, but can collectively exploit leakage from several operations within the decapsulation procedure [RRCB20, DTVV19, GJN20]. Moreover, TO_Indep attacks are generic and to a

Algorithm 2 CCA secure Kyber KEM

procedure CCA.ENCAPS(pk)
  $m \leftarrow \{0,1\}^{256}$
  $m' = \mathcal{H}(m)$
  $(\bar{K}', r') = \mathcal{G}(m' \| \mathcal{H}(pk))$ ▷ Generate $\bar{K}'$ and r' using m and pk
  $ct = \mathsf{CPA.Encrypt}(pk, m', r')$
  $K = \mathsf{KDF}(\bar{K}' \| \mathcal{H}(ct))$
  Return (ct, K)
end procedure

procedure CCA.DECAPS(sk, ct)
  $(pk, \mathcal{H}(pk), z) \leftarrow \mathsf{UnpackSK}(sk)$
  $m = \mathsf{CPA.Decrypt}(sk, ct)$
  $(\bar{K}, r) = \mathcal{G}(m, \mathcal{H}(pk))$
  $ct' = \mathsf{CPA.Encrypt}(pk, m, r)$ ▷ $\mathsf{Re\_Encrypt}(m, pk)$
  if $(ct' == ct)$ **then**
    Return $K = \mathsf{KDF}(\bar{K} \| \mathcal{H}(ct')$ ▷ Ciphertext Comparison Success
  else
    Return $K = \mathsf{KDF}(z \| \mathcal{H}(ct')$ ▷ Ciphertext Comparison Failure
  end if
end procedure

certain degree, agnostic to the target implementation, while TO_Dep attacks are more specific to the target device/implementation.

2.3.1 Target Operation Independent Attacks (TO_Indep)

Several works have shown that an adversary with the ability to query the decapsulation device with chosen-ciphertexts can amplify side-channel leakage related to the secret key [RRCB20, DTVV19, GJN20]. The modus operandi of such attacks is as follows: The attacker submits malformed ciphertexts ct to the decapsulation device, whose corresponding decrypted message m has a close relation to the secet key $\mathbf{s}$. An attacker who can exploit side-channel leakage to recover information about m, in effect instantiates an *oracle* which leads to recovery of $\mathbf{s}$. These attacks can be further classified into two categories:

Plaintext-Checking (PC) Oracle-based SCA: D'Anvers *et al.* [DTVV19] demonstrated that chosen-ciphertexts can be constructed to restrict the decrypted message to a only two possible values (i.e.) $m = 0$ or $m = 1$, with the value of m depending on targeted single coefficients of $\mathbf{s}$. They utilized the timing side-channel from variable time error correcting codes, to recover m (i.e.) $\mathsf{Time(ECC\_Decode}(0)) ! = \mathsf{Time(ECC\_Decode}(1))$, thereby instantiating a Plaintext-Checking (PC) oracle. Ravi *et al.* [RRCB20] subsequently generalized the attack to multiple LWE/LWR-based KEMs including Kyber KEM through the EM side-channel. They showed that a single bit change between $m = 0$ and $m = 1$ results in vastly different computations in the re-encryption procedure, which can be easily distinguished in a single trace. Every chosen-ciphertext query only enables recovery of a single bit information about m, thus full key recovery is only possible in a few thousand queries ($\approx 2k - 3k$).

Decryption-Failure (DF) Oracle-based SCA: These attacks work by submitting perturbed ciphertexts $ct' = ct_{\mathsf{valid}} + \epsilon$ to the target, which induce a *decryption failure* depending on the value of the secret key $\mathbf{s}$. An attacker who can detect a decryption failure (i.e.) $m = m_{\mathsf{valid}}$ or $m = m_{\mathsf{invalid}}$ can construct *linear hints* about the secret key, enabling full key recovery in a few thousand queries ($5k - 6k$).

Guo *et al.* [GJN20] exploited variable time implementations of the ciphertext comparison operation in the decapsulation procedure (Line 14 in CCA.Decaps of Alg.2) to instantiate a DF oracle, while subsequent works [BDH+21, DHP+22] demonstrated exploitation of the the EM side-channel for key recovery. While these attacks specifically targeted the ciphertext comparison operation for leakage, these attack can also exploit leakage from the entire re-encryption procedure of an unprotected implementation, to easily distinguish between $\mathsf{Re-Encrypt}(m_{\mathsf{valid}}, pk)$ and $\mathsf{Re-Encrypt}(m_{\mathsf{invalid}}, pk)$.

In essence, all the aforementioned attacks recover upto a single bit information about the secret key sk, thereby instantiating a *binary* oracle, which requires a few thousand queries for key recovery. These attacks are particularly attractive for their inherent *simplicity* in performing key recovery, where the attacker requires very minimal information about the underlying target.

2.3.2 Target Operation Dependent Attacks (TO_Dep)

These attacks target leakage from specific leaky operations in the decapsulation procedure for key recovery. We discuss two major types of attacks.

Targeting Message Encoding/Decoding Operation: Several attacks have targeted the message encoding (Line 15 of CPA.Encrypt in Alg.1) and message decoding (Line 22 of CPA.Decrypt) operations, enabling recovery of the entire message m in a single trace [ACLZ20, SKL+20, RBRC21, NDJ21]. These attacks mainly exploit very fine leakages from manipulations of single bits of the sensitive message m, enabling full message recovery. Xu *et al.* [XPR+21] showed that the aforementioned leakage can be exploited to instantiate a *Full Decryption* (FD) oracle in a chosen-ciphertext setting. This enabled full key recovery in only 6 queries for Kyber512. Adaptations of the same attack has also been demonstrated on masked and shuffled implementations of Kyber KEM [NDJ21, NDGJ21].

While these attacks consume very few queries for full key recovery, they suffer from inherent disadvantages. Firstly, they exploit very fine leakages from single bit manipulations of the message. Thus, leakage is limited to single clock cycles or very few samples for each message bit, thereby naturally being sensitive to acquisition noise (SNR), as shown in [RBRC21]. Moreover, it is not clear if similar leakages can be exploited on complex devices with features such as heavy parallelism, deep pipelining and inherent jitter, especially given the sensitivity of these attacks to noise.

Targeting NTT Operation: Several attacks have targeted the Number Theoretic Transform (NTT), used for polynomial multiplication in Kyber KEM [PPM17, PP19]. These attacks enable full key recovery in a single trace or very few traces, exploiting advanced algebraic side-channel techniques. However, they also suffer from the same disadvantages of exploiting fine leakages from multiple targeted instructions, while also requiring a sophisticated setup or a powerful side-channel adversary for key recovery. While improved attacks to counter noise have been demonstrated [HHP+21], the attacks still are relatively hard to implement. Moreover, the threat of the same attack on more sophisticated platforms is not clear.

2.3.3 Trade-off In TO_Indep and TO_Dep Attacks

We obseve a very clear trade-off between the ease of attack and the number of traces/queries to perform full key recovery. While TO_Dep attacks only require a handful of traces for key recovery, these attacks rely on very delicate leakages from targeted operations/instructions for key recovery. However, TO_Indep attacks fall on the other side of the spectrum, with respect to ease of key recovery. The attacks can exploit leakage from practically the entire re-encryption procedure, but require traces/queries ranging in the few thousands for key

recovery. The number of queries is particularly important since refreshing the key pair is a common strategy to reduce exposure of the key to possible classical/side-channel attacks [RBRC21]. Thus, a natural question that arises is "*whether it is possible to construct attacks that can obtain the best of both worlds?*" (i.e.) attacks that can exploit leakage independent of the target operation, as well as enable efficient key recovery in very few queries.

In this work, we answer this question positively by proposing generic and novel *parallel* Plaintext-Checking (PC) oracle based attacks, bringing about significant improvement in number of queries in key recovery, compared to the binary PC oracle attacks [RRCB20]. We lay main focus on improving the efficiency of simple and generic side-channel attacks on unprotected implementations. Thus, masking is naturally out of scope of this paper. Our work is motivated from the view point of a designer, contemplating the decision to use heavy countermeasures such as masking for SCA protection of Kyber [HKL+22,BGR+21]. In this respect, our work attempts to answer the question: "*what is the simplest and most efficient attack on an unprotected implementation, with a basic SCA setup and very limited knowledge about the target?*. While our main focus is on unprotected implementations, our proposed attacks also perform better than existing attacks on *lightly protected* implementations using countermeasures such as shuffling [RBRC21]. Refer to Fig.1 for an illustration of a qualitative comparaison of reported SCA applicable to Kyber, with respect to target dependency and number of traces.

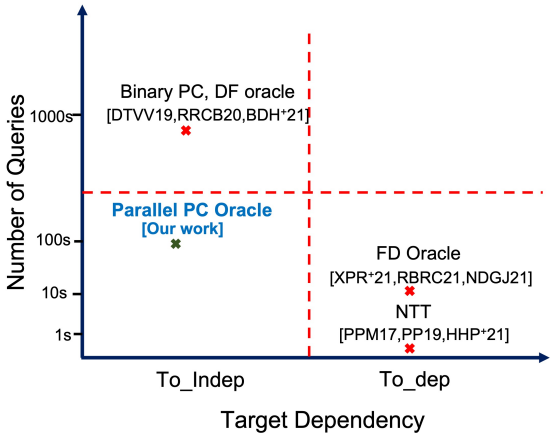


Figure 1: Qualitative comparison of reported SCA applicable to Kyber, with respect to target dependency and number of traces[2]

3 Improved PC Oracle-based CCA

3.1 Attacker Model

We assume that the attacker has physical access to the target device implementing the decapsulation procedure of Kyber KEM. The attacker has the ability to query the target device with chosen-ciphertexts *ct* of his/her choice. Moreover, prior knowledge of the secret key of the DUT or detailed knowledge about the underlying implementation such as the source code or compiled executable is not required. The attacker also does not require the ability to profile the side-channel leakage of the DUT with known keys.

In the following, we explain the binary PC oracle attack of Ravi *et al.* [RRCB20] on Kyber KEM, which serve as the basis of our improved attacks.

3.2 Binary PC Oracle-based CCA

The attack works by recovering the secret key one coefficient at a time. We therefore demonstrate recovery of a single coefficient $\mathbf{s}[0]$, while other coefficients can be recovered in a similar manner. For simplicity, we also assume that all the components are only polynomials in R_q, however the same technique can be extended to higher dimensions (i.e.) R_q^k.

3.2.1 Construction of Chosen-Ciphertexts

The attacker constructs chosen-ciphertexts $ct = (\mathbf{u}, \mathbf{v}) \in (R_q \times R_q)$ as $\mathbf{u} = k_u \cdot x^0$ and $\mathbf{v} = k_v \cdot x^i$ where $(k_u, k_v) \in \mathbb{Z}_q$. The corresponding decrypted message m is given as:

$$m_i = \begin{cases} \mathsf{Decode}(\mathbf{v} - \mathbf{u} \cdot \mathbf{s}[0]) & \text{for} \quad i = 0 \\ \mathsf{Decode}(-\mathbf{u} \cdot \mathbf{s}[i]) & \text{for} \quad i \in \{0, n-1\} \end{cases} \tag{1}$$

for $i \in [0, n-1]$. Thus, every bit m_i of the decrypted message is only dependent on the corresponding secret coefficient $\mathbf{s}[i]$. Now, the attacker can choose values for (k_u, k_v) such that:

$$m_i = \begin{cases} \mathcal{F}(\mathbf{s}[0]), & \text{if } i = 0 \\ 0, & \text{for } 1 \leq i \leq n-1 \end{cases} \tag{2}$$

where m can only take two possible values (i.e.) $m = 0$ and $m = 1$ (all bits except LSB have a value of 0). Moreover, $m = 0/1$ for a given ct, solely depends upon the value of $\mathbf{s}[0]$. This makes it possible to build a binary distinguisher for $\mathbf{s}[0]$ based on $m = 0/1$. Thus, an attacker who can instantiate a binary PC oracle through *side-channels* can uniquely recover the secret coefficient $\mathbf{s}[0]$. Similarly, other coefficients can be recovered by exploiting the rotational property of polynomial multiplication in the ring R_q. Multiplying $\mathbf{r} \in R_q$ with x^p rotates $\mathbf{r}$ by p positions in an anti-cyclic fashion [RRCB20]. Thus, for the chosen-ciphertext $\mathbf{u} = k_u \cdot x^p$ and $\mathbf{v} = k_v \cdot x^0$ with $p \in \mathbb{Z}^+$, the first message bit m_0 is given as:

$$m_0 = \begin{cases} k_v - k_u \cdot \mathbf{s}[0], & \text{if } p = 0 \\ k_v - k_u \cdot (-\mathbf{s}[n-p]), & \text{for } 1 \leq p \leq n-1 \end{cases} \tag{3}$$

while the other bits are fixed to a value of 0. Thus, changing the parameter p ensures that m_0 depends upon different secret coefficients of $\mathbf{s}$, which can also be recovered in the same manner as $\mathbf{s}[0]$.

3.2.2 Instantiating Binary PC Oracle through SCA

A close observation of the decapsulation procedure ($\mathsf{CCA.Decaps}$ in Alg.2) reveals that the decrypted message m is hashed with the public-key ($\mathcal{G}$ in Line 12), and its result r' along with the message m' is fed into the re-encryption procedure (Line 13). For brevity, we denote these operations together as $\mathsf{Re\_Encrypt}(m, pk)$. This re-encryption procedure is deterministic and solely depends upon m for a given public key pk. Thus, a single bit difference between $m = 0$ and $m = 1$ induces very different computations throughout the re-encryption procedure. This amounts to a few thousand leaky Points of Interest (PoI) which can be used to easily distinguish between $m = 0$ and $m = 1$, thereby instantiating a binary PC oracle.

3.3 Optimizing the Number of Queries for Key-Recovery

Ravi *et al.* [RRCB20] targeted the Round 2 specification of Kyber512 with secret coefficients in $[-2, 2]$. They utilized a *non-adaptive* approach to query the decapsulation device with chosen-ciphertexts, thereby using 5 ciphertexts to recover a single coefficient in $[-2, 2]$, which is clearly suboptimal. Thus, subsequent works [BDHD+19, HDV20] proposed improved adaptive attacks to reduce the number of queries for key recovery. The most recent work of Qin *et al.* [QCZ+21] proposed a systematic approach to find the lower bounds for the number of queries. They propose to build a Binary Decision Tree (BDT), which can be traversed based on the oracle's responses $m = 0/1$ to efficiently recover the correct coefficient.

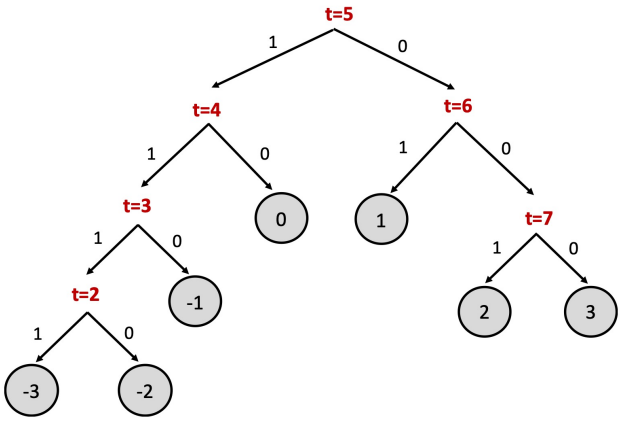


Figure 2: Optimal BDT for Kyber512 to minimise $\mathcal{Q}_{bin}$ for binary PC oracle-based CCA

3.3.1 Construction of an Optimal BDT

The core idea to construct an optimal BDT is based on the observation that the secret coefficients of Kyber follow a *non-uniform* CBD distribution. Thus, the optimal minimum for queries can be attained by constructing a non-uniform distinguisher with the following strategy: *higher the frequency of a secret coefficient candidate, lower should be the number of queries for unique distinguishability.* Thus, the number of queries to uniquely recover a candidate is inversely proportional to the probability of its occurrence.

Let q_x denote the number of queries to uniquely distinguish $\mathbf{s}[i] = x$, and $\mathsf{Pr}(x)$ denote the probability that a secret coefficient $\mathbf{s}[i] = x$. Then, the objective is to build a BDT which yields a minimum for $\mathcal{Q}_{bin}$ which is given as:

$$\mathcal{Q}_{bin} = \sum_{i=-\eta}^{i=\eta} q_x \cdot \mathsf{Pr}(x) \tag{4}$$

We adopted the technique of Qin *et al.* [QCZ+21] to construct BDTs for all parameter sets of Kyber. For our chosen-ciphertexts, we choose $(k_u, k_v) = (208, 208 \cdot t)$ where $t \in \mathbb{Z}^+$. Refer to Figure.2 for the corresponding optimal BDT for Kyber512 with $\mathcal{Q}_{bin} = 2.5625$, distinguishing every candidate in $[-3, 3]$ in not more than 4 queries. A node, edge and leaf of the BDT denotes a chosen-ciphetext query, oracle response and a recovered secret coefficient respectively. The optimal BDT for Kyber768 and Kyber1024 (with coefficients in $[-2, 2]$) is shown in Figure.3, with $\mathcal{Q}_{bin} = 2.3125$. Thus, the average number of queries

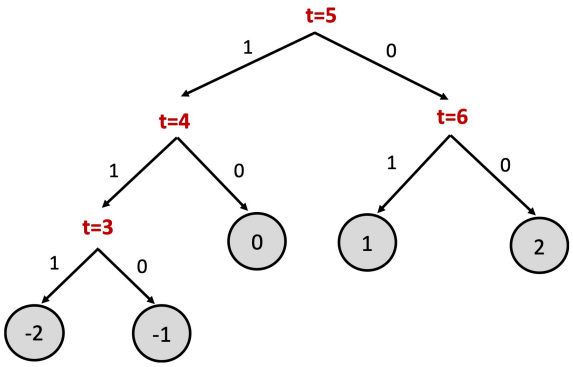


Figure 3: Optimal BDT for Kyber768, Kyber1024 to minimise $\mathcal{Q}_{bin}$ for binary PC oracle-based CCA

for full key recovery is given as:

$$\mathcal{Q}_{attack} = 2^8 \cdot k \cdot \mathcal{Q}_{bin} \tag{5}$$

Thus, $\mathcal{Q}_{attack}$ amounts to 1312, 1776 and 2368 for Kyber512, Kyber768 and Kyber1024 respectively. We refer to the constructed trees as $\mathsf{BDT}_{\mathsf{min\_ent}}$, since the minimum average number of queries is very similar to computation of a certain Shannon entropy.

3.3.2 Critical Observations on the Binary PC Oracle-based CCA

We make two critical observations on the binary PC oracle CCA.

1. Observation-1: The decrypted message m for the chosen-ciphertexts only contains a single secret dependent message bit (i.e.) $m_0 = 0/1$, while all other bits $m_i = 0$ $\forall\, i = \{1, n-1\}$.
2. Observation-2: Leakage from the entire re-encryption procedure has only been exploited to recover a single bit of m, and therefore the secret key $\mathbf{s}$, especially when there are a few thousand leakage points [RRCB20].

This motivates us to investigate if it is possible to recover multiple bits of information about the secret key, exploiting leakage from the re-encryption procedure in a generic manner. In the following, we propose novel extensions of the binary PC oracle attack, which parallelize secret key recovery in a generic and configurable manner. This yields significant improvements in the possible lower bounds achievable for key recovery.

3.4 Parallel PC Oracle-based CCA

The core idea of our attack lies in constructing ciphertexts, such that multiple targeted bits of the message m (i.e.) m_i for $i \in \{0, P-1\}$ ($P \in \mathbb{Z}^+$) depend upon the P corresponding coefficients of the secret key. To achieve the same, we choose $\mathbf{u} = 208 \cdot x^0$ and $\mathbf{v} = 208 \cdot t \cdot (1+x)$ where $t \in \mathbb{Z}^+$. Thus, the decrypted message m is given as:

$$m_i = \begin{cases} \mathsf{Decode}(208 \cdot t - 208 \cdot \mathbf{s}[i]), & \text{if } i \in [0, P-1] \\ \mathsf{Decode}(208 \cdot \mathbf{s}[i]), & \text{for } i \in [P, n-1] \end{cases} \tag{6}$$

For the same values of t used to build the optimal BDT (Fig.2-3), the decrypted message m is given as:

$$m_i = \begin{cases} \mathcal{F}(\mathbf{s}[i]), & \text{if } i \in [0, P-1] \\ 0, & \text{for } i \in [P, n-1] \end{cases} \tag{7}$$

Thus, the first P bits of m are now dependent on the corresponding coefficients of $\mathbf{s}$, while all the other bits are fixed to 0. This technique is subtly different from attacks exploiting the *Full-Decryption* (FD) oracle [XPR+21, RBRC21], where all the bits of m are dependent on the corresponding coefficients of $\mathbf{s}$. However, our technique allows us to control the number and position of the secret dependent message bits. As seen later in Sec.4, this nuanced difference in approach allows to exploit leakage from the re-encryption procedure, in a parallel as well as generic manner.

An adversary able to recover the correct value of the message (i.e.) $m \in [0, 2^P - 1]$ can simultaneously recover a configurable P bits of information about $\mathbf{s}$. While the binary PC oracle attack traverses one BDT in a single query (Fig.2-3), our attack allows us to simultaneously traverse P distinct BDTs (BDT_i for $i \in [0, P-1]$) in a single query, where each BDT_i is traversed using the corresponding message bit m_i. Thus, a generic number of P secret coefficients can be simultaneously recovered in not more than 4 queries for Kyber512 using $\mathsf{BDT}_{\mathsf{min\_ent}}$ (Figure.2). Similarly, P secret coefficients can be simultaneously recovered in not more than 3 queries for Kyber768, Kyber1024 using $\mathsf{BDT}_{\mathsf{min\_ent}}$ (Figure.3).

Similar to the binary attack, the rotational property of polynomial multiplication can be used to recover different P coefficients at a time, thereby recovering the full key. We refer to P as the parallelization factor and our attack as the P-way parallel PC oracle attack.

3.5 Optimal Key Recovery for P-way Parallel Attack

While the approach of Qin *et al.* [QCZ+21] yields the lower bound for queries for the binary attack, it is not clear if it is optimal in the P-way parallel attack. Our intuition is based on the following observation. If we utilize $\mathsf{BDT}_{\mathsf{min\_ent}}$ (Figure.2) for a 2-way parallel attack on Kyber512, then recovering two coefficients with a value of $(0, 0)$ simultaneously, only requires two queries. However, pairs with values of $(-2, 0)$ or $(-3, 0)$, can only be recovered in 4 queries, since -2 and -3 occur at a higher depth in the BDT. Thus, the number of queries to recover a given set of P random coefficients depends upon that coefficient in the set, with the maximum depth in the BDT. Moreover, increasing the parallelization factor P, only increases the probability of observing coefficients with a higher depth (e.g.) $-2, -3$, leading to increase in average number of queries to recover a given set of P coefficients.

3.5.1 More efficient BDTs for the P-way Parallel Attack

This leads us to hypothesize if BDTs with a lower depth yield lower average number of queries for the P-way parallel attack, in particular for higher values of P. We refer to Eqn.6 to construct our chosen-ciphertexts. After careful consideration of all possible values of t and corresponding m_i, we construct a BDT with a depth of 3 (Figure.4), which is the lowest achievable depth for unique distinguishability of coefficients in $[-3, 3]$. We also verified that it is not possible to build a BDT, with lower entropy and a depth of 3. We refer to this alternate tree as $\mathsf{BDT}_{\mathsf{min\_depth}}$.

We now derive an expression to compute the average number of queries to recover a set of P random coefficients for a generic BDT. We introduce some notation to explain our analysis. Refer to Figure.5 for the corresponding illustration of the same. We use *subtree* to denote a tree with smaller depth starting from the root. We use the notation st_d to denote a subtree with depth d where the depth of the root node is 0. So, for any BDT with maximum depth d, there are $d+1$ such subtrees (i.e.) $\{st_0, st_1, \ldots, st_d\}$. We denote the set of subtrees that contain atleast a single leaf (recovered coefficient) as $\mathcal{V}$. The set of leaves in a given sub-tree st_i is denoted as $\mathcal{L}_{st_i}$ and specifically the leaves in the last layer of the subtree are denoted as $\mathcal{M}_{st_i}$. The average number of queries required to recover all

P coefficients using the tree BDT denoted as $\mathcal{Q}_{set}$ is therefore given as:

$$\mathcal{Q}_{set} = \sum_{\forall st_i \in \mathcal{V}} i \cdot \mathsf{R}_i \tag{8}$$

where R_i denotes the probability that a set of random P coefficients belong to $\mathcal{L}_{st_i}$, with at least one coefficient in the set $\mathcal{M}_{st_i}$. With knowledge about the apriori distribution of the secret coefficients, it is possible to compute $\mathcal{Q}_{set}$ for any given BDT.

Refer to Figure.6 for the plot of $\mathcal{Q}_{set}$ for different values of P for Kyber512, for both $\mathsf{BDT}_{\mathsf{min\_ent}}$ (Figure.2) and $\mathsf{BDT}_{\mathsf{min\_depth}}$ (Figure.4). We can clearly see that $\mathcal{Q}_{set}$ for our proposed $\mathsf{BDT}_{\mathsf{min\_depth}}$ graph is lower than that of $\mathsf{BDT}_{\mathsf{min\_ent}}$ for $P \geq 3$, thereby confirming our hypothesis. Thus, our proposed BDT (i.e.) $\mathsf{BDT}_{\mathsf{min\_depth}}$ clearly yields lesser number of queries for Kyber512, compared to $\mathsf{BDT}_{\mathsf{min\_ent}}$ for $P \geq 3$. However, in the case of Kyber768 and Kyber1024, $\mathsf{BDT}_{\mathsf{min\_ent}}$ already has the minimum possible achievable depth of 3. Since the BDT already has the lowest entropy and depth, $\mathsf{BDT}_{\mathsf{min\_ent}}$ yields the lowest number of queries for the P-way parallel attack, for Kyber768 and Kyber1024. Putting it all together, if an adversary has access to a perfect P-way PC oracle, then the average number of queries for full key recovery, denoted as $\mathcal{Q}_{attack}$ is given as:

$$\mathcal{Q}_{attack} = \lceil \frac{2^8}{P} \rceil \cdot k \cdot \mathcal{Q}_{set} \tag{9}$$

Refer to Figure.7 for $\mathcal{Q}_{attack}$ versus the parallelization factor P, for all parameter sets of Kyber. Our experimental simulations assuming a perfect P-way parallel PC oracle

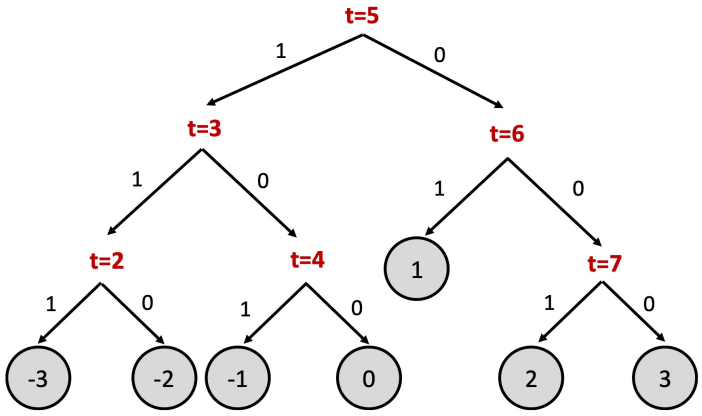


Figure 4: Optimal BDT ($\mathsf{BDT}_{\mathsf{min\_depth}}$) for P-way parallel oracle attack on Kyber512

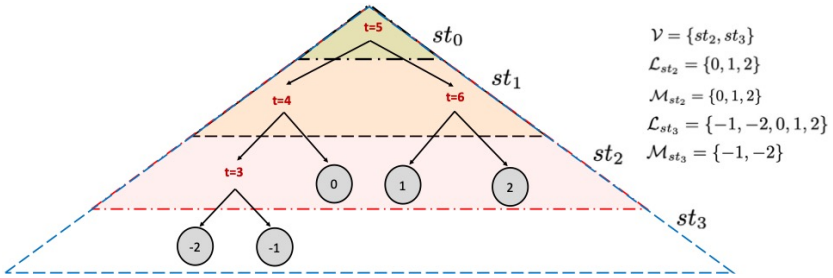


Figure 5: Illustration for calculation of $\mathcal{Q}_{set}$ (i.e.) average number of queries to recover P coefficients for a given BDT

yielded a 100% success rate in recovering the secret key for a generic value of P. In the following, we demonstrate that an attacker can realize a very efficient and practical P-way parallel PC oracle through exploitation of side-channel leakage from the re-encryption procedure.

4 Realizing a Side-Channel based P-way Parallel PC Oracle

4.1 Experimental Setup

Our DUT is the STM32F407VG microcontroller, mounted on the STM32F4DISCOVERY evaluation board. We target the fastest implementation of Kyber KEM (m4speed version), taken from the public *pqm4* library [KRSS19], a benchmarking and testing framework for PQC schemes on the 32-bit ARM Cortex-M4 microcontroller. The target is clocked at 24 MHz. We utilize the Electromagnetic Emanation (EM) side-channel for our experiments. We obtain EM leakage using a near-field EM probe mounted on top of the chip, with the

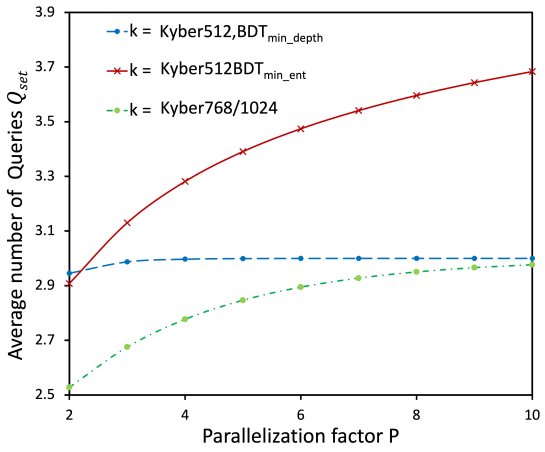


Figure 6: Average number of queries to recover P coefficints (i.e.) $\mathcal{Q}_{set}$ versus the parallelization factor p, for all parameter sets of Kyber

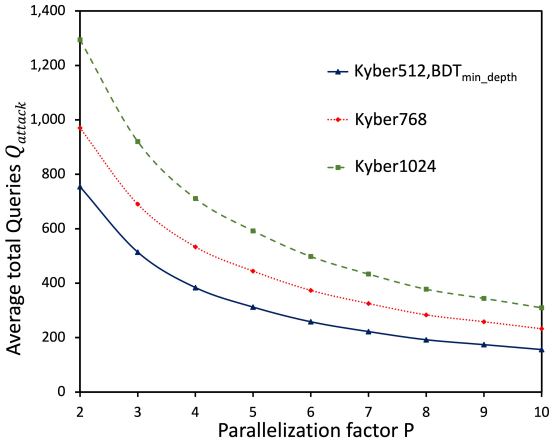


Figure 7: Average number of queries $\mathcal{Q}_{attack}$ for full key recovery, versus parallelization factor P for all parameter sets of Kyber

measurements collected on a Lecroy HD6104 oscilloscope, using a sampling rate of 250 MSam/sec, amplified 30dB with a pre-amplifier.

4.2 Side-Channel Methodology

Our task is to build a side-channel classifier for $m \in [0, 2^P - 1]$, using leakage from the re-encryption procedure (i.e.) $\mathsf{Re\_Encrypt}(m, pk)$. While prior works [RRCB20, QCZ+21] utilized the same leakage to distinguish between $m = 0$ and $m = 1$, we demonstrate that it is possible to classify an arbitrary number of values for m with a very high accuracy.

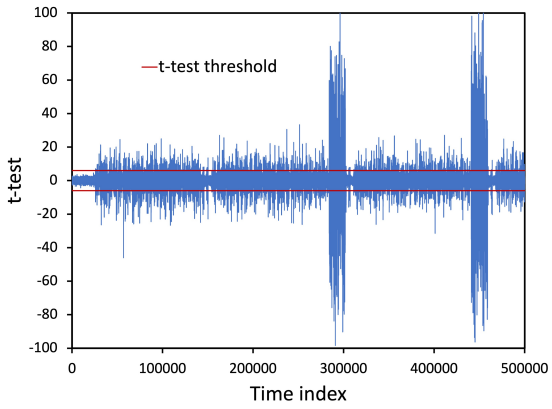


Figure 8: Welch's t-test plot computed for Tr_0 and Tr_1 for Kyber768

4.2.1 Building a Multi-Class Side-Channel Classifier

Our approach for multi-class classification, builds upon the binary classification approach using the well-known Welch's t-test [RRCB20]. We will briefly explain the binary classification method, and subsequently explain our generic extensions to arbitrary 2^P number of classes.

The binary classification is done in two phases: (1) Pre-Processing Phase and (2) Classification Phase. The pre-processing phase involves construction of side-channel templates for each class 0 and 1. The subsequent classification phase uses the templates to classify a given side-channel trace into one of the 2 classes. At no point during any of the two phases, does the attacker require to operate the target device with known secret keys.

Pre-Processing Phase: The adversary obtains T repeated measurements corresponding to $\mathsf{Re\_Encrypt}(m, pk)$ for both $m = 0$ and $m = 1$. This is done by repeatedly querying the decapsulation device with valid ciphertexts for $m = 0$ and $m = 1$ (T times each). We denote the trace set for $m = i$ as Tr_i, and the complete trace set as $\mathsf{Tr} = \cup_{i=0}^{i=1} \mathsf{Tr}_i$. The number of repeated measurements T is a parameter of the experimental setup.

- Every trace tr_j in Tr is normalized by removing the mean and dividing by its standard deviation to obtain t'_j.
- The Welch's t-test is computed between Tr_0 and Tr_1 to detect univariate leakage, based on Equation (10).

$$\mathsf{t\text{-}value} = \frac{\mu_1 - \mu_2}{\sqrt{\frac{\sigma_1^2}{T} + \frac{\sigma_2^2}{T}}} \tag{10}$$

where μ_i and σ_i are the mean and standard deviation of trace set Tr_i.

Refer to Figure.8 for the t-test plot between Tr_0 and Tr_1 for Kyber768 ($T = 20$ traces). The plot shows several peaks about the t-test threshold of ± 5, clearly indicating significant difference between the two computations. In particular, there are two distinct peaks (over multiple samples) with very high t-test values. Upon inspection, we identified it to be the sampling of polynomials of $\mathbf{r}$ from the CBD distribution (Line 11 of CPA.Encrypt in Alg.1). The attacker however does not require this information to perform the attack.

- Those features/points whose *absolute t-test value* is greater than a chosen threshold $\mathsf{Th}_{\mathsf{PoI}}$ are selected as the Points of Interest (PoI) set, denoted as $\mathcal{P}$. The threshold value $\mathsf{Th}_{\mathsf{PoI}}$ is also a parameter of the experimental setup and is empirically determined.
- The set $\mathcal{P}$ is used to derive a reduced trace set for each class, which we denote as Tr'_i for $i \in \{0, 1\}$, and the mean of the reduced trace set Tr'_i is the reduced template $m_{(i,\mathcal{P})}$ for class i with $i \in \{0, 1\}$.

Thus, the reduced templates $m_{(i,\mathcal{P})}$ for $i \in \{0, 1\}$ are the output of the pre-processing phase. Since the target operation $\mathsf{Re\_Encrypt}(m, pk)$ depends upon both the message m and the public key pk, the pre-processing phase is not *one-time*, and therefore has to be carried out for every new public key.

Classification Phase: The reduced templates obtained from the pre-processing phase are now used to classify a given trace tr for a chosen-ciphertext, into either $m = 0/1$. The trace tr is first normalized, and the reduced trace $t'_{\mathcal{P}}$ is obtained. Then, the sum-of-squared difference Γ_* is computed with the reduced template of each class $m_{(i,\mathcal{P})}$ for $i \in \{0, 1\}$ as follows:

$$\Gamma_0 = (t'_{\mathcal{P}} - m_{0,\mathcal{P}})^\top \cdot (t'_{\mathcal{P}} - m_{0,\mathcal{P}}) \text{ and } \Gamma_1 = (t'_{\mathcal{P}} - m_{1,\mathcal{P}})^\top \cdot (t'_{\mathcal{P}} - m_{1,\mathcal{P}}). \tag{11}$$

The trace tr belongs to the class with the least sum-of-squared difference (i.e.) $\mathsf{Class}(tr) = 0$ if $\Gamma_0 < \Gamma_1$, else $\mathsf{Class}(tr) = 1$. Thus, a single side-channel trace can be used to distinguish between the two classes $m = 0$ and $m = 1$, thereby instantiating a binary PC oracle. Refer to Figure.9 which shows clear distinguishability of a sample trace tr into either of the two classes. We were able to obtain a 100% success rate for the binary classification $m = 0/1$, as also shown in Figure.9.

4.2.2 Towards Multi-Class Classification

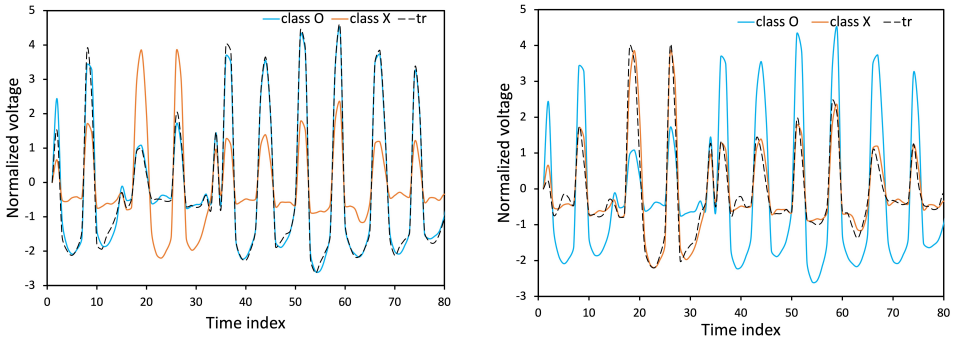
Our approach towards multi-class classification is based on the observation that it is possible to classify any two random values of m in the same manner, as $m = 0/1$. This is rendered possible due to the diffusion property of hash functions used in the re-encryption procedure. For an illustration, refer to Figure.11 for the t-test based binary classification between $m = 330$ and $m = 559$, as illustration. It is well known that unique identification of a particular candidate within a group is possible, if there exists a pairwise classifier for every possible pair of candidates [KU02]. For a P-way parallel PC oracle attack, there are 2^P possible classes for m. Thus, if we are able to classify between any two pairs of m with $m \in [0, 2^P - 1]$, it is also possible to uniquely identify the value of m. This applies for any generic value of P. Thus, the P-way parallel PC oracle can be realized in two phases in

the following manner.

Pre-Processing Phase: The adverary collects T repeated measurements corresponding to $\mathsf{Re\_Encrypt}(m, pk)$ for all 2^P values of $m \in [0, 2^P - 1]$. The complete trace set for all classes is denoted as $\mathsf{Tr} = \cup_{i=0}^{i=2^P-1} \mathsf{Tr}_i$.

Classification Phase: Given an attack trace tr, the adversary uses pairwise binary classification similar to a knock-out tournament with 2^P players. The correct class (resp. winner) is selected after $(2^P - 1)$ pairwise classifications (resp. matches). A match in this context is nothing but binary classification of a given trace tr between two classes $m = x$ and $m = y$, which is denoted as $\mathsf{Classify}(x, y)$. Refer to Figure.10 for an illustration for 8 classes (i.e.) $m = [0, 7]$, where the attack trace tr corresponds to $m = 6$.

This is an optimal approach which only requires $h - 1$ pairwise binary classifications for h classes. There is another costlier approach, of doing pairwise classification of all possible pairs of classes and adopting a majority voting approach to select the correct class. However, this yields a much higher h^2 binary classifications for h classes. Thus, we adopt the former and more efficient approach for classification. The aforementioned technique yields the correct candidate as long as the correct candidate for m is correctly classified, when paired with any other value of $m \in [0, 2^P - 1]$. This is similar to the case



(a) $\mathsf{Class}(tr) = 0$ (b) $\mathsf{Class}(tr) = 1$

Figure 9: Matching the reduced attack trace tr' with the reduced templates of the two classes $m = 0$ and $m = 1$

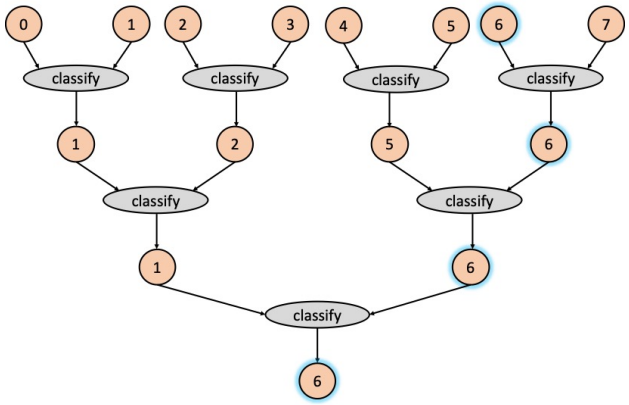
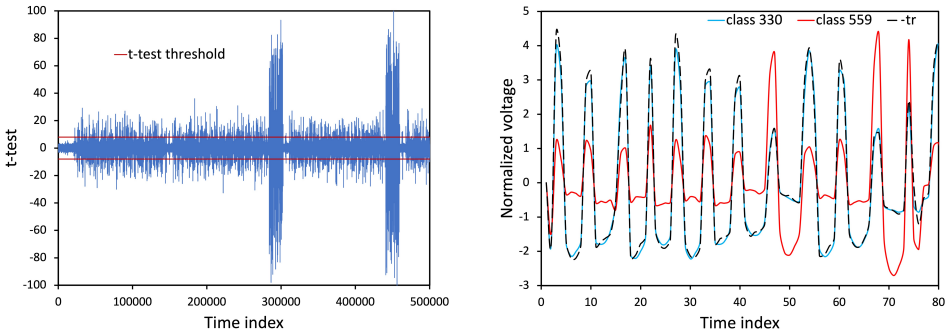


Figure 10: Illustration to classify the attack trace tr among 8 classes, m = [0,7]



(a) t-test (b) Class$(tr) = 330$

Figure 11: t-test plot and matching a given reduced attack trace tr' correponding to class 330, against reduced templates for classes 330 and 559

of having a player, who is capable of winning against any other player in the tournament, and therefore emerges as the winner.

4.2.3 Experimental Validation

We validated our proposed attack on the speed optimized implementation of Kyber768 from the *pqm4* library (Refer Sec.4.1 for the experimental setup). We were able to achieve full key recovery with 100% success rate for a parallelization factor of $P = 10$ (1024 classes). We utilize $T = 5$ traces to build templates for each class, which amounts to 5520 traces. While this is the maximum value of P used for our experiments, it is possible to increase P to any arbitrary value. In this respect, we also verified the success of binary classification of several pairs of messages with $P = 12$ (i.e.) which amounts to 4096 classes. We were able to classify all the collected pairs correctly with 100% accuracy.

We believe this is sufficient evidence that the success rate for key recovery does not deteriorate with increase in the parallelization factor P. This is also due to the fact that there are several hundred PoIs for classification, to distinguish any pair of values for m. We also believe that an upper limit for P for perfect classification if exists, is challenging to determine through experiments.

While prior works [RRCB20] underutilized leakage from the re-encryption procedure by only recovering a single bit, we show that the same leakage can be exploited to recover an arbitrary P number of secret dependent message bits. Moreover, we can see that our proposed attack is generic and clearly agnostic to the target implementation, and requires almost no information about the design of the target.

5 Evaluating Total Cost for Key Recovery

From an attacker's perspective, the number of queries is the primary cost of the attack, as he/she looks for key recovery with minimum possible interaction with the target device. The cost of key recovery, therefore includes the number of queries for the pre-processing phase denoted as $\mathcal{Q}_{template}$, as well as for the classification phase, denoted as $\mathcal{Q}_{attack}$. The pre-processing phase requires T queries for each of the 2^P classes, which amounts to $2^P \cdot T$ queries. The number of queries in the classification phase is nothing but the total number of chosen-ciphertext queries for key recovery (Refer Eqn.8 and Fig.7 in Sec.3.4). Thus, the

Table 1: Number of coefficients to be recovered, for scenarios considering attackers with different offline computational capabilities

	Full_Recovery	Partial_Recovery_2^{32}	Partial_Recovery_2^{64}
Kyber512	512	354	184
Kyber768	768	667	463
Kyber1024	1024	1010	782

total number of queries for key recovery denoted as $\mathcal{Q}_{total}$ is given as:

$$\mathcal{Q}_{total} = \mathcal{Q}_{template} + \mathcal{Q}_{attack} \tag{12}$$

$$= 2^P \cdot T + \lceil \frac{2^8}{P} \rceil \cdot k \cdot \mathcal{Q}_{set} \tag{13}$$

5.1 Analysis for Partial Key Recovery

In a bid to reduce the number of traces, an attacker can also resort to recovering $m < (k \cdot n)$ coefficients of Kyber, and recovering the remaining coefficients using suitable lattice-based solvers in an offline manner. In this respect, we consider three possible cases for an attacker with differing capabilities to perform offline computations:

1. Full_Recovery - Full Key Recovery with 0 remaining offline computations.
2. Partial_Recovery_2^{32} - Partial Key Recovery with 2^{32} remaining offline computations.
3. Partial_Recovery_2^{64} - Partial Key Recovery with 2^{64} remaining offline computations.

We utilized the leaky LWE estimator developed by Dachman-Soled *et al.* [DSDGR20] to estimate the number of coefficients to be recovered, to reduce the security strength of Kyber to 2^{32} and 2^{64} respectively. The tool allows us to include exact or approximate hints and estimate the remaining cost to recover the secret. Refer to Table.1 for the exact number of coefficients to be recovered for the aforementioned attacker scenarios.

5.2 On the Presence of Clone Device

A close observation of Eqn.13 to calculate the total number of queries $\mathcal{Q}_{total}$ reveals that the cost of pre-processing phase to generate templates cannot be ignored, especially given that the pre-processing phase is required to be done for every new public key. In this respect, we identify two possible scenarios, with respect to whether or not the adversary has access to a clone device.

5.2.1 With Clone Device

In this scenario, the adversary has access to a clone device. Thus, he/she can generate templates for Re_Encrypt(m, pk) from the clone device, since the computations only depend upon known values to the attacker (i.e.) m and pk. Thus, the pre-processing phase is completely taken offline. In this case, the number of queries to the target, denoted as $\mathcal{Q}_{target}$ is nothing but:

$$\mathcal{Q}_{target} = \mathcal{Q}_{attack} \tag{14}$$

$$= \lceil \frac{2^8}{P} \rceil \cdot k \cdot \mathcal{Q}_{set} \tag{15}$$

Thus, $\mathcal{Q}_{target}$ simply scales inversely with the parallelization factor P, with the lower bound for $\mathcal{Q}_{target}$ only limited by the number of templates, built by the attacker on the clone device. Refer to Figure.13(a) for the plot of number of queries to the target versus P for Kyber768, considering both full key recovery and partial key recovery. We can clearly

see that the number of queries scales inversely with increase in P. For the experimentally verified case of $P = 10$, the attacker requires ≈ 232 queries for full key recovery, which improves over the state-of-the-art binary PC oracle attack [QCZ+21] by a factor of $\approx 7.6\times$.

((Arbitarily) Best Case Scenario: We also consider an arbitarily strong attacker capable of building 2^{32} templates on the clone device. In this case, full key recovery is possible in just 72 queries, which is an improvement by a factor of ≈ 24.6 compared to the binary PC oracle attack. Naturally, we also observe better improvements for the case of partial key recovery as seen in Table.2.

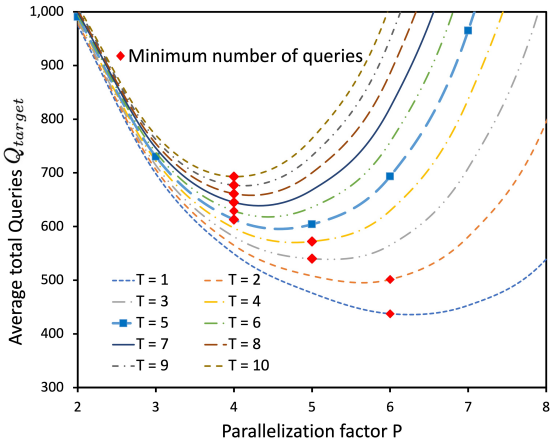


Figure 12: Total number of queries required for full key recovery for Kyber768 in the Scenario_Without_Clone versus the parallelization factor P, for different values of T, where T is the number of traces per template

5.2.2 Without Clone Device

In this scenario, the adversary does not have access to a clone device. Recall that our attack is possible even without knowledge of the key in pre-processing phase. Thus, both the pre-processing as well as classification phase has to be carried out directly on the target device. Here $\mathcal{Q}_{target}$ is nothing but:

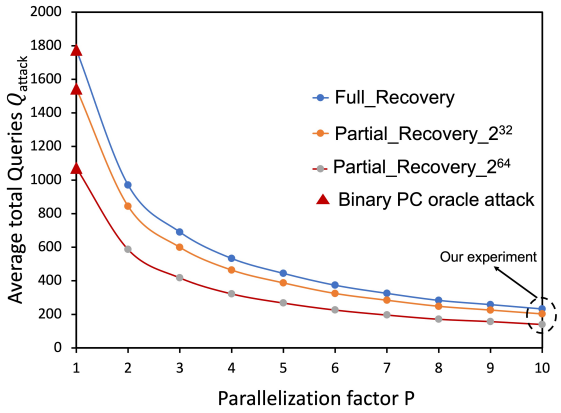
$$\mathcal{Q}_{target} = \mathcal{Q}_{template} + \mathcal{Q}_{attack} \tag{16}$$

$$= 2^P \cdot T + \lceil \frac{2^8}{P} \rceil \cdot k \cdot \mathcal{Q}_{set} \tag{17}$$

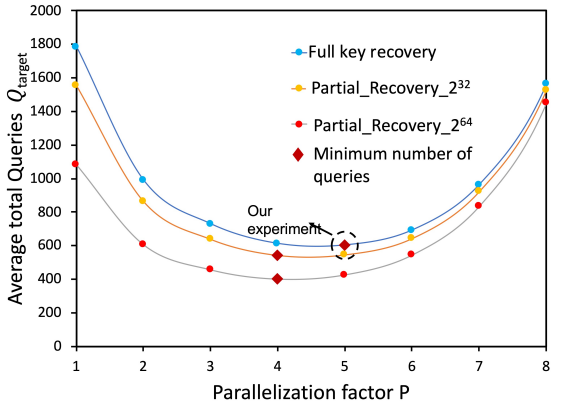
We can observe that $\mathcal{Q}_{template}$ scales exponentially with P (i.e.) 2^P and also increases linearly with T (number of traces per template), while $\mathcal{Q}_{attack}$ scales inversely with P. Thus, there exists a fine trade-off between the cost of pre-processing and classification phase. It is not possible to arbitrarily increase P to improve $\mathcal{Q}_{target}$, as the cost of the pre-processing phase outweighs the cost of the classification phase for higher values of P, unlike when attacker has access to clone device. Refer to Figure.12 for the plot of number of queries versus the parallelization factor P, for different values of T. As expected, we observe a certain minima for number of queries for each value of $T \in [1, 10]$.

We experimentally verified that full key recovery is possible with $T = 5$. For $T = 5$, the parallelization factor $P = 4$ yields the lowest number of queries (i.e.) 613. This is an

improvement by a factor of $\approx 2.89\times$ compared to the binary PC oracle attack. However, a lower value for T could be achieved with a better experimental setup with low acquisition noise. For the best possible scenario of $T = 1$ (single trace per template), the parallelization factor $P = 6$ yields the lowest number of queries (i.e.) 437, an improvement factor of $\approx 4\times$ compared to the binary PC oracle attack. Refer to Figure.13(b) for the number of queries versus P, for $T = 5$, for both full key recovery and partial key recovery.



(a)



(b)

Figure 13: Estimates for the total number of queries to the target versus the parallelization factor P for Kyber768 (a) With clone device and (b) Without clone device, and also considering partial key recovery and full key recovery

5.3 Extensions to Lightly Protected Implementations

Given the heavy performance penatly of masking countermeasures for LWE/LWR-based KEMs with upto 3.1× in runtime as shown in [BGR+21], there is significant interest in low-cost countermeasures to offer protection against known side-channel attacks. One such approach is the shuffling countermeasure, which was proposed to protect the message encoding procedure against single trace message recovery attacks [ACLZ20, SKL+20]. Shuffling ensures that the attacker can still recover all the 256 bits of the message, but not its correct order, thereby offering concrete protection. However, Ravi *et al.* [RBRC21]

Table 2: Tabulation of the total number of queries for key recovery for Kyber768, considering attack scenarios with respect to clone device, as well as the attacker's offline computational capability.

	Parallelization Factor P					
	With Clone				Without Clone	
	1	10	12	32	4 (T=5)	6 (T =1)
Full_Recovery	1776	232	197	72	613	437
Partial_Recovery_2^{32}	1545	202	170	63	544	388
Partial_Recovery_2^{64}	1073	140	120	45	402	290

demonstrated a novel attack on the shuffling countermeasure in a chosen-ciphertext setting, which can recover 1 targeted message bit per query. While shuffling does not offer concrete protection, it atleast prevents single trace message recovery, and reduces the attacker's capability to only recover a single bit per query, which is equivalent to a binary PC oracle attack. This is the best known attack on such a lightly protected shuffled implementation of Kyber KEM.

Since our proposed parallel PC oracle attack is independent of leakage from the shuffled encoding operation, we hypothesize that our attack can defeat the lightly protected implementation in the same manner as an unprotected implementation. We therefore mounted our parallel PC oracle attack on the decapsulation procedure of Kyber KEM, with a shuffled message encoding procedure. Confirming our hypothesis, we were able to successfully recover the secret key in the same manner as the unprotected implementation. Thus, our parallel PC oracle attack also serves as the best attack on a lightly protected implementation.

6 Conclusion

In this work, we propose novel *parallel PC oracle* based side-channel attacks, which are capable of recovering an arbitrary P number of bits of information about the secret key in a single trace. We experimentally validated our attacks on the fastest implementation of unprotected Kyber KEM in the *pqm4* library. Our experiments yielded improvements in the range of 2.89× and 7.65× in the number of queries, compared to state-of-the-art binary PC oracle attacks, while arbitrarily high improvements are possible given the generic nature of the attack. We also show that our proposed attacks are able to achieve the lowest number of queries for key recovery, even over implementations protected with low-cost countermeasures such as shuffling.

Acknowledgment

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