Easy-ABE: An Easy Ciphertext-Policy Attribute-Based Encryption

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Abstract. Attribute-Based Encryption is widely recognized as a leap forward in the field of public key encryption. It allows to enforce an access control on encrypted data. Decryption time in ABE schemes can be long depending on the number of attributes and pairing operations. This drawback hinders their adoption on a broader scale.

In this paper, we propose a non-monotone CP-ABE scheme that has no restrictions on the size of attribute sets and policies, allows fast decryption and is adaptively secure under the CBDH-3 assumption. To achieve this, we approached the problem from a new angle, namely using a set membership relation for access structure. We have implemented our scheme using the Java Pairing-Based Cryptography Library (JPBC) and the source code is available on GitHub.

Keywords: attribute-based encryption \cdot CBDH-3 assumption \cdot non-monotone \cdot random oracle model.

1 Introduction

Traditionally, access control was applied to protect unencrypted data stored on servers. But the problem with this system is that if a server is compromised, the attacker gains direct access to unencrypted data. A solution to this problem appeared in 2005 on the proposal of Amit Sahai and Brent Waters [33] called attribute-based encryption. It allows to enforce fine-grained and flexible access controls over encrypted data.

Attribute-Based Encryption (ABE) is widely recognized as a leap forward in the field of public key encryption. It has numerous applications, ranging from cloud services [25], internet of things [3], video streaming [30], to healthcare systems [15].

ABE offers the possibility to encrypt for multiple recipients at once. Those who wanted to access the plaintext from the ciphertext simply had to have the necessary attributes to satisfy the ciphertext's built-in access control. ABE comes in two flavors: Key-Policy Attribute-Based Encryption (KP-ABE) in which the access policy is embedded in the recipients' secret keys and Ciphertext-Policy Attribute-Based Encryption (CP-ABE) in which the access policy is embedded in the ciphertext. However, the design of such schemes faces many practical difficulties that hinder their wide adoption. A practical ABE scheme must have the following essential properties: [2, 34]:

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- 1. no restriction on the size of policies and attribute sets (unboundedness)
- 2. arbitrary string as an attribute (large universe);
- 3. based on the fast Type-3 pairings;
- 4. small number of pairings for decryption;
- 5. adaptive security under standard assumptions.

Many proposals have been made but few [2, 34] satisfy the properties mentioned above. We believe that a practical ABE scheme of simple structure and more efficient can be constructed.

In this paper, we propose a non-monotonic ciphertext-policy attribute-based encryption denoted Easy-ABE. Our scheme not only offers constant-size secret keys, but also adds the above five properties. Compared to FAME [2], the most efficient scheme in the literature (to our knowledge), our scheme performs much better. We have also implemented our scheme using the Java Pairing-Based Cryptography Library (JPBC) [13] and the source code is available on GitHub [22].

1.1 Related work

Attribute-Based Encryption is a natural extension of Identity Based Encryption [8, 7]. It comes in two flavors: Key-Policy ABE and Ciphertext-Policy ABE. The first KP-ABE scheme was presented by Goyal et al. [19]. Ciphertext-Policy ABE was first proposed by Bethencourt et al. [6] who prove its security under the Decisional Bilinear Diffie-Hellman (DBDH) assumption. It is followed by the work of Cheung et al. [12] in which access structures are AND gates on positive and negative attributes. They improve the security of their scheme by applying the Canetti-Halevi-Katz technique to obtain a chosen ciphertext attack (CCA) security.

Other ABE schemes have emerged focusing on constant-size ciphertexts [4, 11, 37, 16, 35, 14, 21, 36]. In [4], the private key size is linear in the number of attributes of the user. To address the efficiency problem that plagues many schemes due to high computational cost [35] also provides constant computational cost useful when computational and bandwidth issues are major concerns. [27, 20, 23] have designed constant-size secret key schemes. Besides the constant-size secret key, [27] provides low computational and storage overhead with an expressive AND-gate access structure.

Some schemes take scalability into account. They are called unbounded schemes. Unboundedness is an essential property for an ABE scheme because it allows adding new attributes without having to redeploy the scheme. Lewko and Waters [26] were the first to present such a scheme followed by [28, 10, 32, 15, 2, 34]. In [15] unboundedness is obtained by limiting the attribute elements in the ciphertexts to only those associated with the attribute group keys of the ciphertext attributes.

Most of the schemes in the literature are monotonous since it is natural to admit that a user having more attributes than required to access an information must have access to that information. But this could give rise to conflict of interest. Non-monotonic ABE schemes have also been proposed [34, 29, 28, 15]. But some of these proposals [29, 28] are inefficient when it comes to decryption and storage.

Many ABE schemes [12, 15, 16, 19, 20, 27, 29, 32, 35, 37] offering various attractive properties (such as constant-size ciphertexts, constant-size secret key, scalability, unboundedness) have been shown to be secure only in the selective security model which is weaker than the adaptive security enjoyed by our scheme. The problem with selective security is that for the deployment of the scheme one adversary have to declare the access structure he wants to attack. Which is very unlikely to happen in reality.

Building efficient schemes has been the goal of [2,34]. Their schemes are based on the fast Type-3 pairings, have simultaneously unboundedness, large universe, fast decryption and are adaptively secure under standard assumptions. However, [2] is more efficient than [34] but is less expressive since it does not support negation and multi-use of attributes like [34].

1.2 Organization

This paper is structured as follows: after the introduction, section 2 presents the notations, terminologies and tools necessary for the formal description of our proposal in section 3. The proof of security of our scheme is done in section 4. In section 5, we compare the performance of Easy-ABE to other schemes available in the literature. Section 6 concludes this work.

2 Preliminaries

In this section, we present the notations, terminologies and tools necessary for the presentation of Easy-ABE.

2.1 Access structures

We denote by $\mathcal{U} = \{A_1, A_2, \cdots, A_l\}$ the universe of attributes where $A_{i,i=1,..,l}$ are attributes. In our scheme, a set of user attributes $\mathcal{S} \subseteq \mathcal{U}$ is mapped to an $|\mathcal{U}|$ -bit string $\omega = b_l \cdots b_1$ where $l = |\mathcal{U}|$ and

$$b_i = \begin{cases} 0 & \text{if } A_i \notin \mathcal{S} \\ 1 & \text{otherwise} \end{cases}$$

In the rest of the paper, we call ω a user attribute string. For example, if $S = \{A_2, A_4, A_5\}$, the user attribute string ω will be $0 \cdots 011010 \in \{0, 1\}^l$.

To generate the user's secret key, ω will be taken as a binary number in \mathbb{Z}_p^* and then mapped to a group element. Representing ω in reverse order (of

the indexes) makes our schema scalable since by expanding the universe of attributes (addition of new attributes) the keys generated before the expansion remain valid and no re-encryption of data is needed. For example, if \mathcal{U} expands to $\{A_1, A_2, \dots, A_l, \dots, A_n\}$, the user attribute set $\mathcal{S} = \{A_2, A_4, A_5\}$ will be represented by $0 \cdots 011010 \in \{0, 1\}^n$ and will remain unchanged when considered as a binary number. This representation of the set of user attributes guarantees unboundedness and large universe (since any arbitrary string can be used as an attribute) to our scheme.

Definition 1 (Access Structure). We say that an access structure $\mathbb{A} \subseteq \{0, 1\}^n$ is the set of authorized user attribute strings. That is a user attribute string ω is authorized if and only if $\omega \in \mathbb{A}$.

From our definition of the access structure, it is clear that our scheme is nonmonotonic since a monotonic access structure is defined as follows:

Definition 2 (Monotonic Access Structure [2]). If \mathcal{U} denotes the universe of attributes, then an access structure \mathbb{A} is a collection of non-empty subsets of \mathcal{U} , i.e., $\mathbb{A} \in 2^{\mathcal{U}} \setminus \emptyset$. It is called monotone if for every $B, C \subseteq \mathcal{U}$ such that $B \subseteq C, B \in \mathbb{A} \Rightarrow C \in \mathbb{A}$.

Although it is non-monotonic, Easy-ABE becomes monotonic if it is accepted for a user to query for a secret key associated to a subset of her/his set of attributes.

The access structure can also be defined without the use of a universe of attributes by considering directly the set of binary strings representing authorized users identities. For example, informations on citizen id card can be hashed to serve as user attribute string.

2.2 Ciphertext-Policy ABE

A Ciphertext-Policy Attribute-Based Encryption consists of four algorithms (adapted from [6]):

- Setup(λ): The algorithm takes a security parameter λ and outputs the system parameters params, a master public key mpk and a master secret key msk.
- Encrypt(mpk, A, m): The algorithm takes the master public key mpk, a set of authorized user attribute strings A and a message m then outputs a ciphertext ct.
- **KeyGen**(mpk, msk, ω): The algorithm takes the master public key mpk, the master secret key msk and a user attribute string ω then outputs a secret key sk.
- $\mathbf{Decrypt}(\mathsf{ct},\mathsf{sk})$: The algorithm takes a ciphertext ct and a secret key sk then outputs the plaintext m or \bot .

2.3 Security model

In this paper, we are interested in indistinguishability under chosen plaintext attack (IND-CPA) modelled by the following IND-CPA game between a challenger C and an adversary A:

- Setup. C runs the Setup algorithm of a CP-ABE scheme denoted Π with the security parameter λ as input and gives params and mpk to \mathcal{A} .
- Phase 1. \mathcal{A} can make repeated (at will) secret key queries for user attribute strings ω and receives from \mathcal{C} their corresponding secret keys.
- Challenge. \mathcal{A} submits a set of user attribute strings \mathbb{A} and two messages $\mathsf{m}_0, \mathsf{m}_1$ of the same length. One restriction should be noted: the intersection of \mathbb{A} and the set of user attribute strings used for secret key queries must be empty. \mathcal{C} randomly selects $b \in \{0, 1\}$, encrypts the message m_b and sends the result to the \mathcal{A} .
- Phase 2. Similar to phase 1 with the restriction that the intersection of A and the set of user attribute strings used for secret key queries must be empty.
- **Guess**. The adversary outputs a guess b' of b. We say that \mathcal{A} succeeded if b' = b.

The advantage of an adversary \mathcal{A} in the IND-CPA game is defined as

$$\mathsf{Adv}_{\mathsf{\Pi}}^{\mathcal{A}}(\lambda) = \mathsf{Pr}[b'=b] - rac{1}{2}$$

Definition 3. A CP-ABE scheme Π is fully or adaptively IND-CPA secure if for any polynomial time adversary \mathcal{A} , $\mathsf{Adv}_{\Pi}^{\mathcal{A}}$ is negligible, that is to say $\mathsf{Adv}_{\Pi}^{\mathcal{A}}$ is smaller than the inverse of any polynomial, for all large enough values of λ .

2.4 Bilinear maps and Diffie-Hellman assumption

Let \mathbb{G}_1 , \mathbb{G}_2 and \mathbb{G}_T be three cyclic groups of prime order p. A bilinear pairing is a map $e: \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ with the following properties [17, 9]:

- 1. Bilinearity: $e(g_1g'_1, g_2g'_2) = e(g_1, g_2)e(g_1, g'_2)e(g'_1, g_2)e(g'_1, g'_2)$ for all $g_1, g'_1 \in \mathbb{G}_1, g_2, g'_2 \in \mathbb{G}_2$.
- 2. Non-degeneracy: for any $g_1 \in \mathbb{G}_1$, if $e(g_1, g_2) = 1$ for all $g_2 \in \mathbb{G}_2$, then $g_1 = 1$ (and similarly with \mathbb{G}_1 , \mathbb{G}_2 reversed).
- 3. Computability: The map e is efficiently computable.

The pairing is asymmetric when $\mathbb{G}_1 \neq \mathbb{G}_2$ and of Type-3 when no efficientlycomputable isomorphism is known from \mathbb{G}_2 to \mathbb{G}_1 (or from \mathbb{G}_1 to \mathbb{G}_2).

Definition 4 (Bilinear group generator [17]). For the purpose of simplicity we say that an asymmetric bilinear group generator is an algorithm \mathcal{G} that takes as input a security parameter λ and outputs a description of three groups \mathbb{G}_1 , \mathbb{G}_2 and \mathbb{G}_T of prime order p. We assume that this description permits efficient (polynomial-time in λ) group operations and random sampling in each group. The algorithm also outputs an efficiently computable map $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ and generators g_1 and g_2 for \mathbb{G}_1 and \mathbb{G}_2 , respectively.

Definition 5 (Computational Bilinear Diffie–Hellman Problem in Type-3 (CBDH-3) [9]). Given $g_1^{\alpha}, g_1^{\beta}, g_1^{\gamma} \in \mathbb{G}_1$ and $g_2^{\beta}, g_2^{\gamma} \in \mathbb{G}_2$ for $\alpha, \beta, \gamma \in_R \mathbb{Z}_p^*$, the CBDH-3 problem is to compute the Type-3 pairing value $e(g_1, g_2)^{\alpha\beta\gamma}$. The CBDH-3 assumption asserts that CBDH-3 problem is hard. That is to say for all PPT adversaries \mathcal{A} the advantage:

$$\mathsf{Adv}^{\mathcal{A}}_{\mathsf{CBDH-3}}(\lambda) = \mathsf{Pr} \begin{bmatrix} pair-grp, \\ g_1^{\alpha}, g_1^{\beta}, g_1^{\gamma} \in \mathbb{G}_1, \\ g_2^{\beta}, g_2^{\gamma} \in \mathbb{G}_2 \end{bmatrix} = e(g_1, g_2)^{\alpha\beta\gamma} \begin{vmatrix} pair-grp \leftarrow \mathcal{G}(\lambda), \\ \alpha, \beta, \gamma \in_R \mathbb{Z}_p^* \end{vmatrix}$$

is negligible in λ , where pair-grp = $(p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, g_1, g_2, e)$.

2.5 Some cryptographic primitives

In this section, we briefly present some cryptographic primitives used in our scheme.

Symmetric encryption scheme. A symmetric encryption scheme (SYM) is a pair of probabilistic polynomial-time algorithms (Enc, Dec) such that:

- 1. The encryption algorithm Enc takes as input a key $k \in \{0,1\}^n$ (*n* is related to a security parameter) and a plaintext message $m \in \{0,1\}^*$, and returns a ciphertext $c \in \{0,1\}^*$.
- 2. The decryption algorithm Dec takes as input a key k and a ciphertext c, and returns a message m.

It is required that for all k and m, $Dec_k(Enc_k(m)) = m$.

The symmetric encryption scheme is said to be secure in the sense of INDistinguishability under Chosen-Plaintext Attacks (IND-CPA) if from the encryption of one of its two messages, the adversary cannot tell which one has been encrypted even if it has knowledge of encryptions of many other messages of its choice.

Message authentication codes. A message authentication code (MAC) is a pair of probabilistic polynomial-time algorithms (Mac, Vrfy) such that:

1. The tag-generation algorithm Mac takes as input a key $k \in \{0,1\}^n$ (*n* is related to a security parameter) and a message $m \in \{0,1\}^*$, and returns a tag *t*.

2. The verification algorithm Vrfy takes as input a key k, a message m and a tag t. It returns 1 indicating that t is valid, thus m is authentic and 0 indicating that t is invalid, thus m is unauthentic.

It is required that for all k and m, $Vrfy_k(m, Mac_k(m)) = 1$.

The message authentication code is said to satisfy Strong Unforgeability under Chosen-Message Attacks (SUF-CMA) it is computationally infeasible for the adversary to provide a new tag t for a message m even if it has knowledge of many other tags for messages of its choice [5].

Key derivation function. Formally, a key derivation function (KDF) is define in [24] as an algorithm that takes as input four arguments: a value σ sampled from a source of keying material, a value l indicating the length of the secret key to return, and two additional arguments, a salt value r defined over a set of possible salt values and a context variable c, both of which are optional, i.e., can be set to the null string or to a constant.

Informally, the key derivation function is said to be secure if its output distribution is computationally indistinguishable from the uniform distribution over $\{0, 1\}^l$.

In our scheme, the source of keying material is an algebraic group.

2.6 Diffie-Hellman Integrated Encryption Scheme

Abdalla et al [1] suggested a method for encrypting strings using the Diffie-Hellman assumption. The method is called Diffie-Hellman Integrated Encryption Scheme (DHIES) and is secure against chosen-ciphertext attack. The version [18] we describe here uses a symmetric encryption scheme SYM = (Enc, Dec) a message authentication code MAC = (Mac, Vrfy) and a key derivation function KDF.

Let \mathbb{G} be a finite cyclic group of order p generated by g. Let $a \in_R \mathbb{Z}_p^*$ and $h = g^a$. The public key is (\mathbb{G}, g, h) and the private key is a.

Encrypt(m, h): To encrypt $m \in \{0, 1\}^*$, do the following:

- 1. Choose a random $k \in \mathbb{Z}_p^*$ and set $c_1 = g^k$
- 2. Set $\kappa = \mathsf{KDF}(h^k, l_1 + l_2)$ and parse κ as $\kappa_1 || \kappa_2$ where κ_1 and κ_2 are l_1 and l_2 bit binary strings respectively.
- 3. Set $c_2 = \mathsf{Enc}_{\kappa_1}(\mathsf{m})$ and $c_3 = \mathsf{Mac}_{\kappa_2}(c_2)$.
- 4. Transmit the ciphertext (c_1, c_2, c_3) .

 $Decrypt(c_1, c_2, c_3, a)$:

- 1. Compute $\kappa = \mathsf{KDF}(c_1^a, l_1 + l_2)$ and parse it as $\kappa_1 || \kappa_2$ where κ_1 and κ_2 are l_1 and l_2 bit binary strings respectively.
- 2. Check whether $Vrfy_{\kappa_2}(c_3, Mac_{\kappa_2}(c_2)) = 1$ (if not then return \perp and halt).
- 3. Return $\mathsf{m} = \mathsf{Dec}_{\kappa_1}(c_2)$.

Easy-ABE: Our CP-ABE scheme 3

In this section, we give a formal description of the four algorithms (Setup, Encrypt, KeyGen and Decrypt) that characterise Easy-ABE:

Setup(λ): To produce the system parameters params, the master public key mpk and the master secret key msk, the algorithm performs the following steps:

- (1) Run $\mathcal{G}(\lambda)$ to obtain $(p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, g_1, g_2, e)$.
- (2) Choose two cryptographic hash functions:

$$H_1: \{0,1\}^n \to \mathbb{G}_1$$
 and $H_2: \mathbb{G}_T \to \mathbb{Z}_p^*$

- (3) Choose two random exponents α and $\beta \in \mathbb{Z}_p^*$.
- (4) Since our scheme uses a Diffie-Hellman Integrated Encryption Scheme (DHIES) [1], choose an IND-CPA secure symmetric encryption scheme SYM = (Enc, Dec)a SUF-CMA secure message authentication code MAC = (Mac, Vrfy) and a secure key derivation function KDF.
- (5) Return the system parameters, the master public key and the master secret key:

$$\begin{aligned} \mathsf{params} &= (H_1, H_2, \mathsf{SYM}, \mathsf{MAC}, \mathsf{KDF}) \\ \mathsf{mpk} &= (g_1, g_1^\alpha, g_1^\beta, g_2, g_2^\beta) \\ \mathsf{msk} &= (\alpha, \beta) \end{aligned}$$

KeyGen(mpk, msk, ω): To produce the user secret key for the user attribute string ω , the algorithm performs the following steps:

- (1) Compute $h_{\omega} = H_1(\omega) \in \mathbb{G}_1$.
- (2) Pick a random $r \in \mathbb{Z}_p^*$ and return the secret key

$$\mathsf{sk} = (g_1^{\alpha\beta} h_\omega^r, g_2^r)$$

Encrypt(mpk, A, m): To encrypt a message $m \in \{0,1\}^*$ under the set of authorized user attribute strings \mathbb{A} , the algorithm performs the following steps:

- (1) Pick a random exponent $s \in \mathbb{Z}_p^*$. (2) Compute $h_{\omega} = H_1(\omega) \in \mathbb{G}_1$, $\forall \omega \in \mathbb{A}$.
- (3) Compute $\sigma = H_2(e(g_1^{\alpha}, g_2^{\beta})^s)$
- (4) Perform a DHIES [1].

 - (a) Pick a random k ∈ Z_p^{*}, set c₁ = g₁^βg₁^k.
 (b) Compute κ = KDF(c₁^σ, l₁ + l₂) and split the binary string κ in two substrings κ_1 and κ_2 of length l_1 and l_2 respectively ($\kappa = \kappa_1 || \kappa_2$). l_1 and l_2 must match the key lengths of the underlying SYM and MAC schemes respectively.
 - (c) Compute $c_2 = \mathsf{Enc}_{\kappa_1}(m)$ and $c_3 = \mathsf{Mac}_{\kappa_2}(c_2)$.
- (5) Return the ciphertext:

$$\mathsf{ct} = (c_1, c_2, c_3, g_2^s, \{h_{\omega}^s\}_{\omega \in \mathbb{A}})$$

Decrypt(ct, sk): To decrypt the ciphertext **ct** with the user secret key **sk**, the algorithm performs the following steps:

- (1) Find h^s_{ω} in ct with the same index ω as in $g_1^{\alpha\beta}h^r_{\omega}$ in sk. If the search fails then return \perp and halt.
- (2) Otherwise, let h^s_{ω} be the returned value from the search then compute:

$$\rho = e(g_1^{\alpha\beta}h_{\omega}^r, g_2^s)/e(h_{\omega}^s, g_2^r)
= e(g_1, g_2)^{\alpha\beta s} e(h_{\omega}, g_2)^{rs}/e(h_{\omega}, g_2)^{sr}
= e(g_1, g_2)^{\alpha\beta s}$$
(1)

- (3) Compute $\sigma = H_2(\rho)$.
- (4) Recover m with DHIES [1].
 - (a) Compute $\kappa = \mathsf{KDF}(c_1^{\sigma}, l_1 + l_2)$ and split the binary string κ in two substrings κ_1 and κ_2 of length l_1 and l_2 respectively ($\kappa = \kappa_1 || \kappa_2$). l_1 and l_2 must match the key lengths of the underlying SYM and MAC schemes respectively.
 - (b) Compute $b = \mathsf{Vrfy}_{\kappa_2}(c_3, \mathsf{Mac}_{\kappa_2}(c_2))$, if b = 0 then return \perp and halt.
 - (c) Otherwise, compute $\mathbf{m} = \mathsf{Dec}_{\kappa_1}(c_2)$.
- (5) Return the plaintext m.

4 Security analysis

The security of Easy-ABE is based on the hardness of the Computational Bilinear Diffie–Hellman Problem in Type-3 (CBDH-3). Assuming that the underlying primitives (SYM, MAC and KDF) are secure, we show that the proposed scheme has ciphertext indistinguishability against chosen plaintext attack (IND-CPA) in the random oracle model.

Theorem 1. Let H_1 and H_2 be two random oracles. Let SYM be a symmetric encryption scheme, let MAC be a message authentication code and let KDF be a key derivation function. If there exists an IND-CPA adversary \mathcal{A} that has advantage $\epsilon(\lambda)$ against Easy-ABE by making q secret key queries and a challenge set of size m of user attribute strings, then there exists a simulator that solves the CBDH-3 problem with advantage at least $m\epsilon(\lambda)/q$ and a running time $\mathcal{O}(time(\mathcal{A}))$.

Proof. The simulator is given a random instance of the CBDH-3 problem: g_1 , g_1^a , g_1^b , $g_1^c \in \mathbb{G}_1$ and g_2 , g_2^b , $g_2^c \in \mathbb{G}_2$ for a, b and c randomly choosen from \mathbb{Z}_p^* . The simulator must output the solution of the CBDH-3 problem that is $e(g_1, g_2)^{abc} \in \mathbb{G}_T$. The groups \mathbb{G}_1 , \mathbb{G}_2 and \mathbb{G}_T are of prime order p, g_1 and g_2 are generators of \mathbb{G}_1 and \mathbb{G}_2 respectively.

Now we describe how the simulator uses adversary \mathcal{A} to solve the CBDH-3 problem in the following IND-CPA security game:

Setup: The simulator runs the Setup algorithm of Easy-ABE and sends to \mathcal{A} the system parameters and the master public key:

$$\mathsf{params} = (H_1, H_2, \mathsf{SYM}, \mathsf{MAC}, \mathsf{KDF})$$

 $\mathsf{mpk} = (g_1, g_1^a, g_1^b, g_2, g_2^b)$

Note that the two hash functions H_1 and H_2 are random oracles controlled by the simulator as described below.

 H_1 -queries: The adversary \mathcal{A} can send an H_1 query at any time. Therefore the simulator maintains a list initially empty of the form $(\omega, t, h_{\omega}) \in \{0, 1\}^n \times \mathbb{Z}_p^* \times \mathbb{G}_1$. This list is denoted H_1 -list. When a query $H_1(\omega)$ is received, the simulator checks if the tuple (ω, t, h_{ω}) is in the list. If yes responds with $H_1(\omega) = h_{\omega}$. Otherwise it picks a random $t \in \mathbb{Z}_p^*$, computes $h_{\omega} = g_1^a g_1^t \in \mathbb{G}_1$, adds the tuple (ω, t, h_{ω}) to the list and responds with $H_1(\omega) = h_{\omega}$.

 H_2 -queries: The adversary \mathcal{A} can send an H_2 query at any time. Therefore the simulator maintains a list initially empty of the form $(x, y) \in \mathbb{G}_T \times \mathbb{Z}_p^*$. This list is denoted H_2 -list. When a query $H_2(x)$ is received, the simulator checks if the tuple (x, y) is in the list. If yes responds with $H_2(x) = y$. Otherwise, it picks a random $y \in \mathbb{Z}_p^*$, adds the tuple (x, y) to the list and responds with $H_2(x) = y$.

Phase 1: The adversary \mathcal{A} can send secret key queries for user attribute strings $\{\omega_i\}_{i=1}$... l of its choice. Therefore, the simulator maintains a list initially empty of the form $(\omega_i, \mathsf{sk}_i = (\mu_i, \nu_i))$. This list is denoted sk -list. When a secret key query for $\omega_i \in \{0, 1\}^n$ is received, the simulator runs the H_1 -queries algorithm to obtain the tuple $(\omega_i, t_i, h_{\omega_i})$ corresponding to ω_i from the H_1 -list, picks a random $r \in \mathbb{Z}_p^*$ and responds to \mathcal{A} with

$$\mathsf{sk}_i = \left((g_1^b)^{-t_i} h_{\omega_i}^r, g_2^r / g_2^b \right) \tag{2}$$

The simulator adds the tuple $(\omega_i, \mathsf{sk}_i)$ to the sk-list. One can see that sk_i is a valid secret key because

$$(g_1^b)^{-t_i}h_{\omega_i}^r = g_1^{ab}g_1^{-ab}g_1^{-bt_i}(g_1^ag_1^{t_i})^r = g_1^{ab}(g_1^ag_1^{t_i})^{(r-b)} = g_1^{ab}h_{\omega_i}^{(r-b)} = g_1^{ab}h_{\omega_i}^{\tilde{r}}$$

which shows that (2) satisfies $(g_1^{ab}h_{\omega_i}^{\tilde{r}}, g_2^{\tilde{r}})$ matching the definition of a secret key in the **KeyGen** algorithm of Easy-ABE.

Challenge: When the adversary feels ready for the challenge, it submits a set of user attribute strings \mathbb{A} and two messages $\mathbf{m}_0, \mathbf{m}_1 \in \{0, 1\}^*$ of the same length. If there is an ω present at the same time in both \mathbb{A} and sk-list then the simulator terminates its interactions with \mathcal{A} and outputs abort. Otherwise the simulator selects $\mathbf{m}_{b,b\in\{0,1\}}$, picks two random $c_2, c_3 \in \{0,1\}^*$ (with the appropriate lengths related to the output-length of Enc and Mac), m random $s_i \in \mathbb{Z}_p^*$ where $m = |\mathbb{A}|$ the number of elements in \mathbb{A} . The simulator responds to \mathcal{A} with the ciphertext

$$\mathsf{ct} = (g_1^c, c_2, c_3, g_2^c, \{(g_1^c)^{s_i}\}_{i=1 \dots m}) = (g_1^c, c_2, c_3, g_2^c, \{(g_1^{s_i})^c\}_{i=1 \dots m})$$

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Note that the tuple (g_1^c, c_2, c_3) is indistinguishable from the encryption of a random message m as in the real ciphertext since the underlying primitives (SYM, MAC and KDF) are secure [31]. Also, note that from the H_1 -queries algorithm each $\omega \in \mathbb{A}$ is mapped to a random element $h_{\omega} \in \mathbb{G}_1$ as the $g_1^{s_i}$ for $i = 1 \dots m$ are random in \mathbb{G}_1 . Therefore, ct is a valid ciphertext.

One can see that with a secret key of the form $\mathsf{sk}_i = (g_1^{ab}g_1^{s_ir}, g_2^r)$ for $i \in [1 \dots m]$ the solution $e(g_1, g_2)^{abc}$ of the instance of the CBDH-3 problem given to the simulator appears in the **Challenge** phase.

Phase 2: \mathcal{A} sends more secret key queries for user attribute strings $\{\omega_i\}_{i=l+1}$... q of its choice as in **Phase 1** with the restriction that none of the ω_i is in the set of user attribute strings \mathcal{A} used in the **Challenge** phase.

Guess: Adversary \mathcal{A} outputs its guess $b' \in \{0, 1\}$. The simulator selects a random tuple $(\omega_j, \mathsf{sk}_j = (\mu_j, \nu_j))$ from the sk-list, computes $\rho = e(\mu_j, g_2^c)/e(g_1^{s_ic}, \nu_j)$ for an $i \in [1 ... m]$ and outputs ρ as the solution to the given instance of CBDH-3 problem.

The fact that the secret keys and the ciphertext sent to \mathcal{A} from queries are valid justifies that the view of \mathcal{A} when used by the simulator is distributed identically to \mathcal{A} 's view in a real attack against Easy-ABE.

The simulator outputs the correct solution when b' = b and there is a tuple in the H_1 -list of the form $(*, *, g^{s_i})_{i \in [1 \dots m]}$ produced when the **Phase 1** algorithm is run. Let H_1 -sublist be the subset of H_1 -list containing tuples produced by executions of the **Phase 1** algorithm and let \mathcal{E} be the event that the simulator's output is correct. Let \mathcal{B} be the event that there exist an $i \in [1 \dots m]$ such that the tuple $(*, *, g^{s_i})$ appears in H_1 -sublist. We have:

$$\Pr[b' = b] = \Pr[b' = b \mid \mathcal{B}] \cdot \Pr[\mathcal{B}] + \Pr[b' = b \mid \overline{\mathcal{B}}] \cdot \Pr[\overline{\mathcal{B}}]$$

$$\leq \Pr[b' = b \mid \mathcal{B}] + \Pr[b' = b \mid \overline{\mathcal{B}}] \cdot \Pr[\overline{\mathcal{B}}]$$
(3)

If event \mathcal{B} does not occur, then the view of \mathcal{A} in its interactions with the simulator is independent of $e(g_1, g_2)^{abc}$. Therefore $\Pr[b' = b \mid \overline{\mathcal{B}}] = 1/2$ and (3) becomes:

$$\Pr[b'=b] \le \Pr[b'=b \mid \mathcal{B}] + \frac{1}{2}\Pr[\overline{\mathcal{B}}] \le \Pr[b'=b \mid \mathcal{B}] + \frac{1}{2}$$
(4)

which leads to

$$\Pr[b' = b \mid \mathcal{B}] \ge \Pr[b' = b] - \frac{1}{2} = \mathsf{Adv}_{\Pi}^{\mathcal{A}}(\lambda) = \epsilon$$
(5)

To complete the proof of the theorem, we have:

$$\begin{aligned} \mathsf{Pr}[\mathcal{E}] &= \mathsf{Pr}[b' = b \land \mathcal{B}] \\ &= \mathsf{Pr}[b' = b \mid \mathcal{B}] \cdot \mathsf{Pr}[\mathcal{B}] \\ &= \mathsf{Pr}[b' = b \mid \mathcal{B}] \cdot \mathsf{Pr}[\exists i \text{ s.t. } (*, *, g^{s_i}) \in H_1\text{-sublist}] \\ &\geq \epsilon \cdot \sum_{i=1}^m \mathsf{Pr}[(*, *, g^{s_i}) \in H_1\text{-sublist}] = m\epsilon/q \end{aligned}$$

5 Theoretical Comparison

We compare Easy-ABE with FAME by Agrawal and Chase [2]. FAME satisfies the five essential properties: based on the fast Type-3 pairings, adaptive security, unboundedness, large universe, fast decryption.

In this comparison, we are interested in two metrics that are the computational cost and storage cost. For the computational cost, we compare the number of multiplications, exponentiations, hash function calls and pairings in key-generation, encryption and decryption algorithms, see tables 1 to 3. For the storage cost we compare the number of group elements in secret keys and ciphertexts, see table 4.

Table 1. Comparison of the number of operations for Key generation. T denotes the number of attributes.

	Key generation					
Scheme	\mathbb{G}_1			\mathbb{G}_2		
	Mul	Exp	Hash	Mul	Exp	Hash
Our	1	2	1	_	1	—
FAME [2]	8T + 9	9T + 9	6(T+1)	_	3	—

Table 2. Comparison of the number of operations for encryption. m is the size of access structure \mathbb{A} and n_1 , n_2 are the dimensions of the monotone span programs (MSP).

	Encryption					
Scheme	\mathbb{G}_1			\mathbb{G}_2		
	Mul	Exp	Hash	Mul	Exp	Hash
Our	1	m+2	m	_	1	—
FAME [2]	$12n_1n_2 + 6n_1$	$6n_1$	$6(n_1 + n_2)$	-	3	-

Table 3. Comparison of the number of operations for decryption. I is the number of attributes used in decryption.

	Decryption				
Scheme	neme Multiplication			Pairing	
	\mathbb{G}_1	\mathbb{G}_2	\mathbb{G}_T		
Our	—	—	1	2	
FAME [2]	6I + 3	_	6	6	

Table 4. Comparison of the storage cost of secret key and ciphertext. m is the size of access structure \mathbb{A} , T denotes the number of attributes; and n_1 , n_2 are the dimensions of the monotone span programs (MSP).

	Storage cost				
Scheme	Key	size	Ciphertext size		
	\mathbb{G}_1	\mathbb{G}_2	\mathbb{G}_1	\mathbb{G}_2	
Our	1	1	m + 1	1	
FAME [2]	3(T+1)	3	$3n_1$	3	

From these tables, it is clear that our scheme is more efficient than FAME [2] in terms of computational cost and storage cost. FAME in turn works slightly better than [34]. However, the latter is more expressive since it can deal with natural negation and multi-use of attributes. It should be noted that in the comparison we did not take into account the running time of the symmetric encryption scheme, the running time of the message authentication code and the complexity of the search algorithm used in the first step of our decryption algorithm.

6 Conclusion

In this paper, we have proposed a ciphertext-policy attribute-based encryption scheme denoted Easy-ABE that is efficient in terms of computationnal and storage cost. Our scheme is non-monotonic but can be monotonic when it is accepted for a user to query for a secret key associated to a subset of her/his set of attributes. Easy-ABE has the five essential properties required for a practical ABE scheme: it is based on the fast Type-3 pairings, is adaptively secure under CBDH-3 assumption, has unboundedness, large universe and fast decryption.

With the use of DHIES, we believe that our scheme has indistinguishable encryptions under a chosen-ciphertext attack, but this remains to be proven. It would be interesting to prove it without resorting to the random oracle heuristic.

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