Demystifying Just-in-Time (JIT) Liquidity Attacks on Uniswap V3

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***Abstract*—Uniswap is currently the most liquid Decentralized Exchange (DEX) on Ethereum. In May 2021, it upgraded to the third protocol version named Uniswap V3. The key feature update is "concentrated liquidity", which supports liquidity provision within custom price ranges. However, this design introduces a new type of Miner Extractable Value (MEV) source called Just-in-Time (JIT) liquidity attack, where the adversary mints and burns a liquidity position right before and after a sizable swap.**

We begin by formally defining the JIT liquidity attack and subsequently conduct empirical measurements on Ethereum. Over a span of 20 months, we identify $36{,}671$ such attacks, which have collectively generated profits of $7{,}498$ ETH. Our analysis suggests that the JIT liquidity attack essentially represents a whales' game, predominantly controlled by a select few bots. The most active bot, identified as 0xa57...6CF, has managed to amass 92% of the total profit. Furthermore, we find that this attack strategy poses significant entry barriers, as it necessitates adversaries to provide liquidity that is, on average, 269 times greater than the swap volume. In addition, our findings reveal that the JIT liquidity attack exhibits relatively poor profitability, with an average Return On Investment (ROI) of merely 0.007%. We also find this type of attack to be detrimental to existing Liquidity Providers (LPs) within the pool, as their shares of liquidity undergo an average dilution of 85%. On the contrary, this attack proves advantageous for liquidity takers, who secure execution prices that are, on average, 0.139% better than before. We further dissect the behaviors of the top MEV bots and evaluate their strategies through local simulation. Our observations reveal that the most active bot, 0xa57...6CF, conducted 27% of non-optimal attacks, thereby failing to capture at least $7{,}766$ ETH (equivalent to 16.1M USD) of the potential attack profit.

***Index Terms*—Decentralized Exchange, Blockchain, Decentralized Finance, Miner Extractable Value**

I. Introduction

Smart contracts enable building an ecosystem of financial products and services on top of permissionless blockchains, commonly referred to as Decentralized Finance (DeFi). DeFi is becoming increasingly popular, with the Total Value Locked (TVL) hitting an all-time-high of 178B USD on November 9th, 2021[1]. In addition to the fundamental functions drawn from traditional finance, DeFi also brings new innovative designs such as Automated Market Maker (AMM) DEX.

In contrast to centralized exchanges that rely on custodial infrastructures, DEXs allow users to trade cryptocurrencies directly in a non-custodial environment. AMM DEXs replace the traditional order matching system with smart-contract-enabled algorithmic models that determine the prices at which buyers and sellers can trade assets in liquidity pools. Constant Function Market Maker is a board class of AMMs that is widely adopted by most DEXs (e.g., Uniswap, SushiSwap, Curve). Uniswap is the most liquid DEX on Ethereum, with TVL of 3.85B USD and daily trading volume of 1.86B USD.

[1]https://defillama.com/, last accessed on April 14th, 2023.

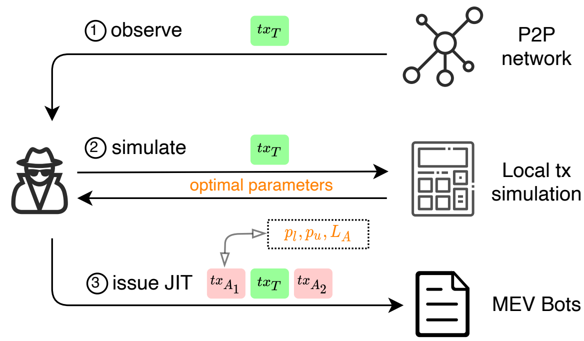


Fig. 1: Overview of JIT liquidity attack.

Adams *et al.* launched Uniswap V1 [1] in November 2018. Uniswap V2 [2] went alive in May 2020, and further upgraded to V3 [3] in May 2021. The Uniswap V1 contract supports only the ETH–ERC-20 liquidity pool, while V2 allows the swap between any ERC-20–ERC-20 token pairs. Besides, Uniswap V2 also introduces Wrapped Ether (WETH) into its core contract. However, Uniswap V2 requires the LPs to provide liquidity in the entire price range (e.g., from 0 to $+\infty$), which results in capital inefficiency because only a small fraction of assets are available at a given price [3]. To improve capital efficiency, Uniswap V3 introduces a new design called "concentrated liquidity", which allows LPs to concentrate their liquidity in custom price ranges smaller than $[0, +\infty]$ to supply more liquidity at targeted prices. Uniswap V3 also introduces the concept of "active liquidity". If the price of assets trading in the liquidity pool moves outside LP's specified price range $[p_l, p_u]$, the LP's position becomes inactive and stops earning fees. Therefore, a rational LP is incentivized to concentrate its liquidity in a profitable price range yet keep its position active.

Nevertheless, Uniswap V3's concentrated liquidity design introduces a new type of MEV source, known as "JIT liquidity attack", where the adversary mints and burns a position immediately before and after a sizable swap transaction (cf. Figure 1). More specifically, the adversary monitors the mempool via its spy node. Once observing a sizable pending swap, the adversary simulates the JIT liquidity attack locally with chosen parameters and launches the attack if profitable.

While the common MEV sources (e.g., arbitrage, liquidation and sandwich attack) have been comprehensively examined in recent academic literature, the mechanism underlying JIT liquidity attacks has received relatively scant attention. This study aims to provide a scientific formalization of JIT liquidity attack, understand the attack impact through empirical measurement, and evaluate the adversarial strategies via local simulation. The main contributions of this paper are summarized as follows:

Attack Formalization. We formalize the JIT liquidity attack and adversarial utility function. The price range $[p_l, p_u]$ and liquidity amount L_A are parameters that require optimization.

Empirical Measurement. We conduct empirical measurements of JIT liquidity attacks on Ethereum. During the course of 20 months, we identify 36,671 attacks with a total profit of 7,498 ETH. Our analysis reveals that the JIT liquidity attack indeed functions as a game for 'whales', overwhelmingly controlled by a handful of powerful bots. The most active bot 0xa57...6CF seizes 92% of the total attack profit. We detect an extremely high barrier to entry for this type of attack, as it necessitates the adversary to contribute liquidity that is, on average, 269 times greater than the swap volume. In addition, we observe that the profitability of the attack is relatively poor, with an average ROI ratio of only 0.007%. Furthermore, we find the attack detrimental to existing LPs in the pool, resulting in an average dilution of their liquidity shares by 85%. Conversely, it appears to favor liquidity takers, who benefit from improved execution prices by an average of 0.139%.

Comparision Study. We undertake a comparative analysis of JIT liquidity attacks and sandwich attacks. Our findings indicate that sandwich attacks present lower entry barriers and superior profitability performance. The initial capital required to launch a sandwich attack is merely 6 times the swap volume.

Strategy Analysis. We first analyze how top MEV bots choose attack parameters. We find that the top first bot 0xa57...6CF consistently deploys its entire token balance to add liquidity during each attack. We then assess the strategies of these top bots through local simulation. Interestingly, our simulation results indicate that 0xa57...6CF executed 27% of attacks suboptimally, thereby failing to seize at least 7,766 ETH (equivalent to 16.1M USD) of the potential attack profit.

II. Related Work

Heimbach *et al.* [4] empirically investigate Uniswap V3's resilience to unexpected price shocks and find that the price on Uniswap is inaccurate during UST–USDT stablecoin price shocks. Loesche *et al.* [5] studied the impermanent loss in Uniswap V3 by analyzing 17 liquidity pools. The result shows that providing liquidity in V3 pools makes around 50% of the LPs unprofitable compared to simply holding the assets. Heimbach *et al.* [6] examined the risks and returns of Uniswap V3 LPs and find that high returns can only be obtained by accepting high risks. Hashemseresht *et al.* [7] analyze the effect of concentrated liquidity on the return of the liquidity providers. Aigner *et al.* [8] discusses the impermanent loss and risk profile of Uniswap V2 and V3 LPs. Wan *et al.* [9] investigate the historical JITs on Uniswap V3 and discuss the microeconomic considerations. The principal differences distinguishing this study from [9] can be outlined as follows: Firstly, we offer a formal definition of the adversarial utility function and identify the parameters that require optimization. Secondly, we conduct a more comprehensive empirical analysis. Lastly, we dissect the behaviors of the top bots and evaluate their attack strategies.

III. Uniswap Liquidity Math

A. *Uniswap V2: Zero to Infinity*

For a simple X–Y liquidity pool, the V2 contracts require the token reserves to satisfy $x * y = k$, where k is a constant.

Liquidity Taker. Let p_0 be the marginal price of X (i.e., $p_0 = \frac{y_0}{x_0}$). Given the spot price p_0, for a liquidity taker who is willing to trade Δ_x amount of X for Δ_y amount of Y, the pool reserve should change to $x_1 = x_0 + \Delta_x$ and $y_1 = y_0 - \Delta_y$ such that $(x_0 + \Delta_x) * (y_0 - \Delta_y) = k$ holds when the swap ends.

Liquidity Provider. An LP who provides liquidity to the X–Y token pool must provide liquidity across the entire price range of $[0, +\infty]$. Such liquidity provision design results in capital inefficiency because most of the liquidity remains unused [3].

B. *Uniswap V3: Concentrated Liquidity*

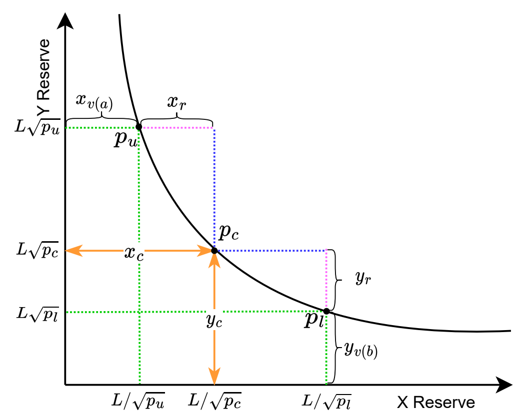


Fig. 2: Uniswap V3 price curve. Given the spot price p_c, an LP who wants to mint a position in the price range $[p_l, p_u]$ only needs to provide the real reserve of x_r and y_r.

1) Liquidity and Price: To improve capital efficiency, Uniswap V3 introduces a novel AMM design called "concentrated liquidity", which allows LPs to add liquidity in a customized price range of $[p_l, p_u]$. In Uniswap V3, a liquidity pool can be thought of as having *virtual liquidity* L and *virtual reserves* x and y such that $x * y = k = L^2$. The pool contract tracks `Liquidity` (L) and the marginal token price `sqrtPriceX96` ($\sqrt{P}$). As such, the relationship between price and liquidity can be expressed as follows:

$$L = \sqrt{xy}, \quad \sqrt{P} = \sqrt{\frac{y}{x}}$$
$$x = \frac{L}{\sqrt{P}}, \quad y = L\sqrt{P} \tag{1}$$

2) Positions and Ticks: When an LP adds liquidity in $[p_l, p_u]$, a new liquidity `Position` is minted, which is represented by a corresponding Non-fungible Token (NFT). Uniswap V3 uses discrete `ticks` to facilitate customized liquidity provision, with each tick representing a price where the contract's virtual liquidity can change. The price at tick i is 1.0001^i.

$$p(i) = 1.0001^i, \quad \sqrt{p(i)} = 1.0001^{\frac{i}{2}} \tag{2}$$

Every pool in Uniswap V3 is initialized with a given `tickSpacing`. Uniswap V3 supports fee tiers of $\{0.01\%$, 0.05%, 0.3%, $1\%\}$, which corresponds to the `tickSpacing` of $\{1, 10, 60, 200\}$. Tick i is initialized when liquidity is added to a range where tick i serves as the lower/upper bound and tick i is not referenced by any existing position. Note that only ticks that are divisible by `tickSpacing` can be initialized.

3) Liquidity Provision: Given spot price p_c, consider an LP who aims to provide liquidity in the price range $[p_l, p_u]$ ($p_l < p_c < p_u$) (cf. Figure 2). Recall that Uniswap v3 always ensures that the virtual reserves satisfy $x * y = L^2$. As such, the LP must provide x_r amount of X token and y_r amount of Y token, such that when the provided token amounts are added to the virtual reserves of X and Y in $[p_l, p_u]$, the resulting curve will behave according to the Constant Product Market Maker pricing curve. From Equation 1, the virtual reserves of X token at price p_u is $x_{v(u)} = \frac{L}{\sqrt{p_u}}$, and the virtual reserves of Y token at price p_l is $y_{v(l)} = L\sqrt{p_l}$. Hence, the x_r and y_r reserve provided by the LP must satisfy Equation 3 as follows:

$$(x_r + \frac{L}{\sqrt{p_u}})(y_r + L\sqrt{p_l}) = L^2 \tag{3}$$

From p_c, we know that the reserves are $x_c = \frac{L}{\sqrt{p_c}}$ and $y_c = L\sqrt{p_c}$. Hence, we can derive x_r and y_r by Equation 4:

$$x_r = \frac{L}{\sqrt{p_c}} - \frac{L}{\sqrt{p_u}}, \quad y_r = L\sqrt{p_c} - L\sqrt{p_l} \tag{4}$$

Note that this position only needs to hold enough amounts of token X and Y to support trading in the price range $[p_l, p_u]$. Particularly, when the marginal price P (i.e., the price of X in terms of Y) moves to the upper bound p_u, the reserve of X is fully depleted and the LP's position only holds Y token. Similarly, when the marginal price P moves to the lower bound p_l, the LP's position only holds X token. In other words, a position is active only if the pool's current price is within the specified price range. While Figure 2 describes the scenario where the current price is within $[p_l, p_u]$, it can be generalized to a broader format to resolve x_r and y_r (cf. Equation 5).

$$x_r = \begin{cases} \frac{L}{\sqrt{p_l}} - \frac{L}{\sqrt{p_u}} & p_c < p_l \\ \frac{L}{\sqrt{p_c}} - \frac{L}{\sqrt{p_u}} & p_l \leq p_c \leq p_u \\ 0 & p_u < p_c \end{cases}$$
$$y_r = \begin{cases} 0 & p_c < p_l \\ L\sqrt{p_c} - L\sqrt{p_l} & p_l \leq p_c \leq p_u \\ L\sqrt{p_u} - L\sqrt{p_l} & p_u < p_c \end{cases} \tag{5}$$

4) Transaction Fees: The LPs of a given liquidity pool earn fees when traders transact one asset for another. For instance, a trader who aims to swap Δx amount of USDC for ETH in the USDC–WETH pool with a fee tier of 0.05% pays $0.05\%\Delta x$ as the trading fee. The trading fees will be distributed among active LPs based on the liquidity they provide. For Uniswap V3, an LP receives fees only if the corresponding position is active. In case the price moves outside the specified range, the LP's position becomes inactive and thus stops earning fees. When the price reenters the range, the position becomes active and receives fees again. It is worth noting that while fees earned in Uniswap V2 are auto-compounded in the pool, they are stored separately in Uniswap V3 and held as tokens in Uniswap V3 [3]. Furthermore, there might be multiple V3 pools with different fee tiers for the same X–Y asset pairs.

IV. Models

A. System Model

We consider a blockchain peer-to-peer (P2P) network and assume the existence of the following participants in the system:
Trader $\boldsymbol{T}$: who initiates a swap tx_T on Uniswap V3;
Adversary $\boldsymbol{A}$: who observes the pending transaction tx_T through its own spy node (e.g., a custom Ethereum client) and launches a JIT liquidity attack over the targeted transaction;
LPs $\boldsymbol{L}$: who served liquidity to the liquidity pool implied by tx_T, supporting the swaps in the pool and earning fees;
Miners and Validators $\mathcal{M}$: a set of miners (in a Proof-of-Work blockchain) or validators (in a Proof-of-Stake blockchain) who have the capacity to manipulate the order of transactions.

B. Threat Model

We consider an economically rational adversary $\boldsymbol{A}$ who is well-connected to the P2P network and is able to monitor unconfirmed transactions in the mempool through its spy node. We also assume that $\boldsymbol{A}$ holds a sufficient balance of assets required by the corresponding JIT liquidity attack to issue transactions. In addition, $\boldsymbol{A}$ can participate in auctions for priority transaction inclusion, in an attempt to extract the MEV contained in the JIT liquidity attack. More specifically, $\boldsymbol{A}$ can submit transactions to miners/validators either *(i)* publicly through the P2P network or *(ii)* privately through a centralized Front-running as a Service (e.g., Flashbots). To compete with other MEV searchers, $\boldsymbol{A}$ can bribe the miners/validators by offering high gas prices or through direct Coinbase transfer.

C. Attack Model

1) Pool Mechanism: We consider the following pool actions.
`Mint`: An LP add liquidity ΔL to Pool(X,Y,ϕ) by specifiying a price range $[p_l, p_u]$ and the amount (ΔX_d, ΔY_d) desired to be added (cf. Equation 6). A unique NFT is minted accordingly to represent this position. The periphery contract computes ΔL automatically (cf. Equation 7). It first calculates ΔL according to the amount (ΔX_d, ΔY_d), and then computes the actual amounts (ΔX_a, ΔY_a) added to the pool based on ΔL.

$$(L, \sqrt{P}) \xrightarrow[\Delta X \in R+,\ \Delta Y \in R+]{\texttt{Mint}(p_l, p_u, \Delta X_d, \Delta Y_d)} (L + \Delta L, \sqrt{P}) \tag{6}$$

$$L=\begin{cases}\Delta X_d(\frac{1}{\sqrt{p_l}}-\frac{1}{\sqrt{p_u}}) & p_c<p_l\\ \min\{\Delta X_d(\frac{1}{\sqrt{p_c}}-\frac{1}{\sqrt{p_u}}),\Delta Y_d\frac{1}{\sqrt{p_c}-\sqrt{p_l}}\} & p_l\le p_c\le p_u\\ \Delta Y_d\frac{1}{\sqrt{p_u}-\sqrt{p_l}} & p_u<p_c\end{cases}\tag{7}$$

`Burn`: An LP burns its position by removing liquidity ΔL from Pool(X,Y,ϕ). The LP will collect the remaining X, Y reserves, and the swap fees (held separately in tokens) up to a maximum amount of fees owed to its position to the recipient.

$$(L,\sqrt{P})\xrightarrow[\text{NFTID}\in R+]{\texttt{Burn}(\text{NFTID})}(L-\Delta L,\sqrt{P})\tag{8}$$

`Swap`: A liquidity taker can swap one token for another. `ZeroForOne` is a boolean variable and is true when swapping X for Y. `AmountSpecified` configures the swap as exact input when it is positive, and exact output when it is negative. While a `Swap` changes $\sqrt{P}$, it does not necessarily change L.

$$(L,\sqrt{P})\xrightarrow[\text{ZeroForOne}\in\{0,1\},\ \text{AmountSpecified}\in R]{\texttt{Swap}(\text{ZeroForOne, AmountSpecified})}(L',\sqrt{P'})\tag{9}$$

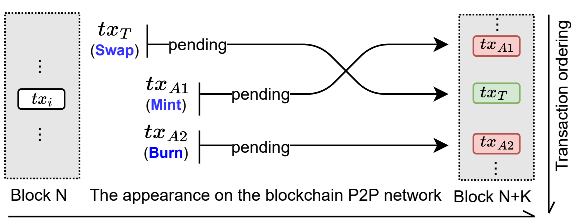


Fig. 3: JIT liquidity attack mechanism.

2) Attack Mechanism: ❶ The adversary $\boldsymbol{A}$ observes a sizable pending (zero-confirmation) swap tx_T in the mempool via its spy node; ❷ $\boldsymbol{A}$ predicts the price range $[p_l,p_u]$ into which the tx_T will be in range for, and issues tx_{A1} right before tx_T to mint a new position by adding liquidity in $[p_l,p_u]$. ❸ $\boldsymbol{A}$ issues tx_{A2} immediately after tx_T to burn its position by removing liquidity in $[p_l,p_u]$ and collect the earned fees .

3) Utility Formalization: By launching a JIT liquidity attack, a rational adversary $\boldsymbol{A}$ aims to maximize its financial gain.

Revenue. The adversarial revenue comes from two sources: *(i)* the *swap fee* taken unfairly from other LPs; and *(ii)* the *portfolio value change* caused by price impact of the trade.

$$\text{Revenue}=\text{Fee}+\Delta\text{Value}\tag{10}$$

The liquidity taker is responsible for paying fees for swap execution. The adversary earns swap fees proportional to how much liquidity it contributes compared to the total liquidity during the swap interval (cf. Equation 11). If the swap is executed crossing ticks, $\boldsymbol{A}$ earns swap fees only for the swap range covered by the price range in which $\boldsymbol{A}$ supplies liquidity.

$$\text{Fee}(L_A)=\begin{cases}\Delta x_{\text{in}}\cdot\phi\cdot L_A/L_T\cdot P_{(ETH/X)} & \text{ZeroForOne}\\ \Delta y_{\text{in}}\cdot\phi\cdot L_A/L_T\cdot P_{(ETH/Y)} & \text{!ZeroForOne}\end{cases}\tag{11}$$

$\boldsymbol{A}$ may also benefit from the change in its portfolio value. The marginal price of asset X (i.e., `sqrtPriceX96`) changes because of the swap execution, thus affecting the adversarial portfolio value. Equation 12 calculates the adversarial portfolio value change (in ETH) as the difference between the value after $(p_1\cdot x_1+y_1)$ and before $(p_0\cdot x_0+y_0)$ swap, where P_0 and P_1 denote X's marginal prices before and after respectively.

$$\Delta\text{Value}(p_1,x_1,y_1)=[(p_1\cdot x_1+y_1)-(p_0\cdot x_0+y_0)]\cdot P_{(ETH/Y)}\tag{12}$$

Let $V_0=p_0\cdot x_0+y_0$ and $L_T=L_A+L_0$, Equation 13 represents ΔValue as the function of L_A, p_l and p_u (cf. Appendix A).

$$\Delta\text{Value}(L_A,p_l,p_u)=\begin{cases}[L_A(\sqrt{p_u}-\sqrt{p_l})-V_0]\cdot P_{(ETH/Y)}\\ \qquad\text{if } p_u<p_1\text{, which implies !ZeroForOne;}\\ [L_A(\frac{1}{\sqrt{p_l}}-\frac{1}{\sqrt{p_u}})\frac{(L_A+L_O)\sqrt{p_0}}{(L_A+L_O)+\sqrt{p_0}\Delta x_{\text{in}}}-V_0]\cdot P_{(ETH/Y)}\\ \qquad\text{if } p_1<p_l\text{, which implies ZeroForOne;}\\ [L_A(\sqrt{p_u}-\sqrt{p_l}-\frac{1}{\sqrt{p_u}}(\frac{(L_A+L_O)\sqrt{p_0}}{(L_A+L_O)+\sqrt{p_0}\Delta x_{\text{in}}}-\sqrt{p_u})^2)-V_0]\cdot P_{(ETH/Y)}\\ \qquad\text{if } p_l\le p_1\le p_u\text{ and ZeroForOne;}\\ [L_A(\sqrt{p_u}-\sqrt{p_l}-\frac{1}{\sqrt{p_u}}((\sqrt{p_0}+\frac{\Delta y_{\text{in}}}{L_A+L_O})-\sqrt{p_u})^2)-V_0]\cdot P_{(ETH/Y)}\\ \qquad\text{if } p_l\le p_1\le p_u\text{ and !ZeroForOne;}\end{cases}\tag{13}$$

Cost. The adversarial cost comes from two sources: *(i)* the gas cost of the mint (tx_{A1}) and burn (tx_{A2}) transaction; and *(ii)* the amount of ETH transferred to miners/validators for priority inclusion. $\boldsymbol{A}$ may participate in auctions for the priority transaction ordering in the block. As such, in addition to the gas cost, $\boldsymbol{A}$ may also bribe the miners/validators via direct Coinbase transfer through smart contracts.

$$\text{Cost}=\text{gas}(tx_{A1})+\text{gas}(tx_{A2})+\text{transfer}\tag{14}$$

Profit. $\boldsymbol{A}$ aims to maximize its profit by choosing optimal values (cf. Equation 15) for the following parameters: *(i)* the upper and lower bound for the price range $[p_l,p_u]$ in which $\boldsymbol{A}$ adds liquidity; and *(ii)* the amount liquidity L_A to add. Without loss of generality, we assume that the cost is invariant to the parameter values. Hence, maximizing profit is equivalent to maximizing revenue (cf. Equation 15).

$$(L_A,p_l,p_u)=\underset{(L_A,p_l,p_u)}{\operatorname{argmax}}\ (\text{Fee}+\Delta\text{Value})\tag{15}$$

V. Empirical Measurement

This section presents empirical measurement on JIT liquidity attacks based on real-world data of Uniswap V3. We provide the overall statistics of JIT liquidity attacks, analyze the adversarial revenue and cost, and dissect the adversarial attack strategies.

A. Data Collection

We conduct our empirical measurement for JIT liquidity attacks on Ethereum from blocks 12545219 (June 1st, 2021) to 16530247 (Jan 31st, 2023), during the course of 20 months. We apply the following heuristic to identify JIT liquidity attacks.

Heuristic 1 (JIT Liquidity Attack Heuristics). $\boldsymbol{A}$ mints (tx_{A1}) and burns (tx_{A2}) an LP position in the same block.

The target transaction (tx_T) is a swap in the same pool. The issuer of tx_{A1} and tx_{A2} is different from the issuer of tx_T. The transaction indices of tx_{A1}, tx_T and tx_{A2} are consecutive.

B. Overall Statistics

By applying Heuristic 1, we have successfully identified 36,671 JIT liquidity attacks on Ethereum. We calculate the distance between the tick before and after the swap as $|i_1 - i_0|/\texttt{tickSpacing}$. We discover that 97.84% the price change is less than or equal to one `tickSpacing` (cf. Table I).

$\lvert i_1 - i_0\rvert/\texttt{tickSpacing}$	≤1	(1, 2]	(2, 3]	(3, 4]	>4
Number of Attacks	35878	1210	218	78	77
Percentage of Attacks	97.84%	1.65%	0.30%	0.11%	0.11%

TABLE I: Summarization of the distance between i_0 and i_1.

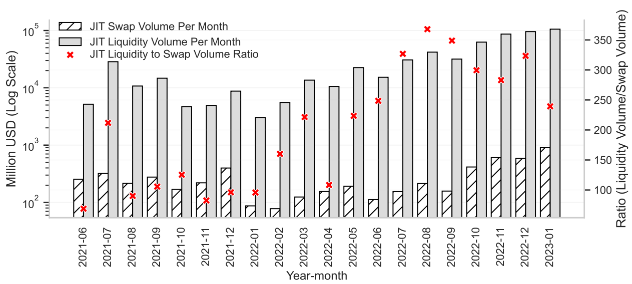


Fig. 4: The average JIT liquidity to swap volume ratio is 269×.

> **Insight 1.** The JIT liquidity attacks pose high entry barriers, necessitating participants to hold substantial initial funds.

Our data show that the total swap volume impacted by JIT liquidity attacks is 5.64B USD, and the average swap volume is 153.8K USD. Hence, we can infer that $\boldsymbol{A}$ only targets sizable swaps in the mempool to maximize its utility. Interestingly, we discover that the total liquidity provided by adversaries amounts to 602.3B USD, and the average liquidity volume exceeds 16.4M USD. This result indicates that to initiate a JIT liquidity attack against tx_T, an adversary needs to provide liquidity that is on average 269 times higher than the swap volume of tx_T (cf. Figure 4). Hence, we conclude that the JIT liquidity attack presents extremely high barriers to entry.

Upon examining the pool statistics, we discover that the USDC–WETH–0.03% (0x88e...640) is the most frequently targeted liquidity pool for JIT liquidity attacks. We find that the USDC–WETH-0.03% pool attracts 47% of the attacks (17,368 / 36,671) and 60% of the revenue (6,123 / 10,125 ETH). The USDC–WETH–0.03% takes up most of the JIT liquidity volume (72%) and swap volume (61%). Besides, while comparing different fee tiers, we observe that the 0.05% fee tier captures most of the JIT liquidity volume (51%) and swap volume (33%). Clearly, adversaries tend to avoid stable pools with a 0.01% fee tier. This could be attributed to these pools exhibiting minimal volatility and limited financial prospects.

C. Profitability Analysis

While the accumulative attack profit is growing, the profit-making potential of various JIT attackers varies. In the following, we provide a detailed analysis of adversarial profitability.

1) Revenue: We detect that the total attack revenue amounts to 10,125 ETH, and the average attack revenue is 0.276 ETH.

> **Insight 2.** JIT liquidity attacks significantly impact existing LPs, diluting their liquidity shares by an average of 85% and thereby substantially affecting their financial interests.

JIT adversaries have taken a tremendous amount of swap fees from existing LPs in the liquidity pool. Our data show that the total revenue of 10,125 ETH earned by JIT adversaries consists of 7,111 ETH of portfolio change revenue and 3,014 ETH of swap fee revenue taken from other LPs. For each attack, an average of 85% swap fees are exploited by adversaries.

Additionally, we observe that the change in portfolio value is not consistently advantageous for adversaries. From November 2022, the main source of JIT revenue is portfolio value change, with up to 83% of the total contribution (c.f. Figure 5). However, before November 2022, the swap fee and portfolio revenue dominate the revenue generation alternatively. We detect that the portfolio value change results in a positive contribution only in 48.4% of JIT liquidity attacks ($17,743/36,671$), indicating that the adversary must carefully consider the impact of portfolio value change before executing such attacks.

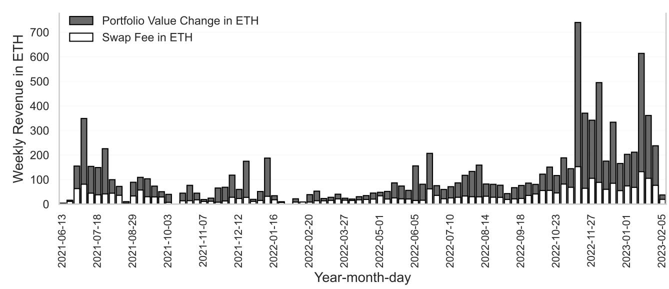


Fig. 5: Weekly portfolio change and swap fee revenue.

2) Cost: We calculate adversarial cost using Equation 13.

> **Insight 3.** Instead of the gas fee, the direct transfer incurs most of the cost. Over 99% of the detected JIT liquidity attacks do not use the gas fee to reward miners/validators.

The primary source of JIT cost is the direct Coinbase transfer used to bribe miners/validators. We find that the average attack cost is 0.072 ETH, with a total cost amounting to 2,027 ETH, consisting of 539 ETH for gas cost and 2,088 ETH for direct transfer to miners/validators. We further perform a detailed breakdown of JIT cost (c.f. Figure 6) and observe that the direct Coinbase transfer increasingly serves as the main cost component, accounting for 81% of total cost in January 2023. This indicates a rising level of competition in the MEV game. Moreover, on average, JIT adversaries are willing to sacrifice $43.7\% \pm 25.5\%$ of their revenue to bribe the miners/validators.

Interestingly, in most of the attacks, adversaries do not use gas fees to bribe miners/validators. Before EIP-1559, the

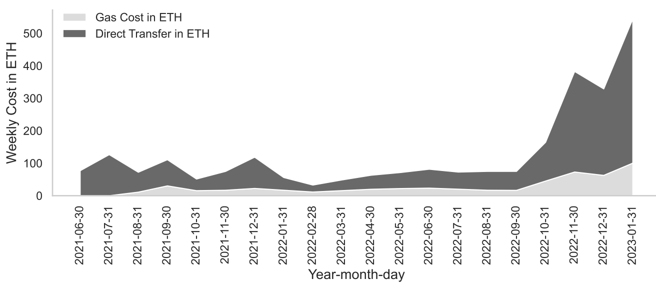


Fig. 6: Gas cost and direct transfer to miners/validators.

adversary could set the gas price of a transaction to zero, and bribe the miners only via direct transfer. We detect $2{,}464$ (99.91%) such JIT attacks. After EIP-1559, the adversary pays both the base fee and priority fee as the gas cost. The gas price cannot be set as zero because the adversary must pay the base fee. However, the adversary can set the priority fee to zero and only use direct transfer as the reward to miners. We detect $34{,}048$ (99.54%) attacks with zero priority fee.

3) Profit: We calculate adversarial profit using Equation 15.

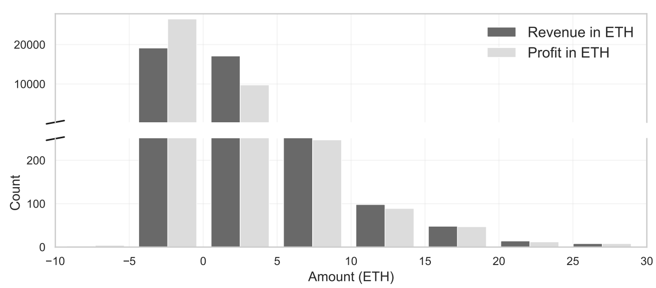


Fig. 7: Histogram of JIT revenue and profit.

> **Insight 4.** JIT liquidity attacks exhibit subpar profitability due to their notably low ROI ratio.

The adversaries obtain a total profit of $7{,}498$ ETH, and an average profit of 0.204 ETH. Surprisingly, only $20{,}068$ (54.7%) JIT liquidity attacks are profitable. Figure 7 shows the distribution of adversarial revenue and profit, which is right-skewed with a tail in the positive region. For unprofitable attacks, the adversary's loss concentrates in $[-0.1, 0]$ ETH. The adversary earns more when the attack is profitable.

Besides, we observe that JIT liquidity attacks demonstrate extremely low ROI. ROI is a metric to evaluate the performance of an investment. It is calculated as the profit over the invested capital. The average ROI for JIT liquidity attacks is only 0.007%, indicating poor profitability performance.

4) Top MEV Bots: We proceed to analyze the strategy and behavior of top MEV bots who have the highest profitability (i.e., 0xa57...6CF, 0x57C...c94 and 0xCD9...5C4).

> **Insight 5.** The JIT liquidity attack is predominantly controlled by "whales", making it inaccessible to retail users.

Interestingly, the $36{,}671$ detected JIT liquidity attacks were related only to 18 MEV bots. By tracing the coin flow of the top first MEV-bot 0xa57...6CF, we detect that it uses 308 Externally-Owned Accounts to launch attacks, but the earned funds are all transferred to the same receiver 0x561...Bf9 (cf. Figure 8). Surprisingly, 0xa57...6CF has issued $27{,}983$ (76%) attacks, consumed 435 ETH (81%) as the gas fee, transferred $1{,}628$ (78%) ETH to bribe miners/validators, and siphons $6{,}900$ ETH (92%) as attack profit (cf. Table III). It is evident that 0xa57...6CF has dominated the entire JIT game, leaving few opportunities for retail users. Does the top first bot 0xa57...6CF surpass in profitability? Indeed, 0xa57...6CF garners an average profit of 0.247 ETH, exceeding the general average. However, it's worth noting that 0xa57...6CF's average ROI stands at a mere 0.013%. This further underscores that JIT liquidity attacks, while sometimes profitable for top players, typically represent low-yield investments with underwhelming profitability.

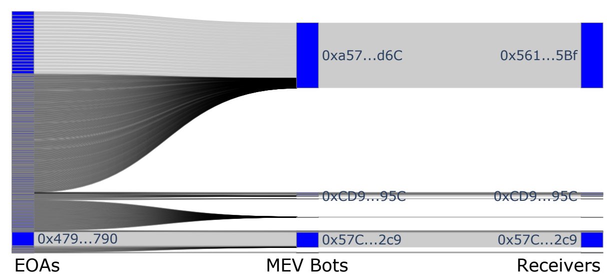


Fig. 8: The coin flow of JIT MEV bots. The attack revenue of 0xa57...6CF always flows to 0x56...Bf9.

D. Impact on Liquidity Takers

How do JIT liquidity attacks affect liquidity takers (i.e., the issuer of tx_T)? In order to answer the question, for every JIT liquidity attack, we simulate the pool state for the same swap tx_T as if the attack (i.e., tx_{A1} and tx_{A2}) had never happened.

> **Insight 6.** Liquidity takers gain advantages from JIT liquidity attacks, obtaining more favorable swap prices.

We compare the liquidity taker's price slippage with/without JIT liquidity attack. Interestingly, we find that liquidity takers obtain better execution prices with JIT liquidity attacks. The average price improvement is 0.139%. This is reasonable, since after the adversary mints its position, the pool's liquidity increases and the price slippage decreases. We further compare the price improvement across pools with different fee tiers (cf. Table II). We detect that liquidity takers in 1% fee tier pools obtain the highest average price improvement of 0.699%.

E. Comparison Study

This section compares JIT liquidity attacks with sandwich attacks by analyzing their attack mechanisms and statistics.

1) Attack Mechanism: A JIT liquidity attacker $\boldsymbol{A}$ attempts to mint and burn a concentrated position immediately before and after the target swap transaction. In contrast, upon observing a target pending swap tx_T (e.g, swap X for Y), a sandwich

	Quantiles (%)				Statistics (%)			
Fee	10%	25%	50%	75%	min	max	mean	std
1%	0.135	0.232	0.466	0.911	0.014	12.611	0.699	0.737
0.3%	0.022	0.046	0.110	0.252	0.119	22.752	0.194	0.341
0.05%	0.008	0.014	0.029	0.052	0.000	0.734	0.040	0.039
0.01%	0.000	0.000	0.000	0.001	0.000	0.662	0.009	0.066

TABLE II: Price improvement statistics for different fee tiers.

attacker $\boldsymbol{A}'$ attempts to front-run tx_T by purchasing asset Y and back-run tx_T by purchasing asset X. As such, the liquidity taker bears higher slippage than anticipated.

2) Attack Statistics: We proceed to compare the JIT and sandwich attack statistics on Uniswap V3. We conduct our empirical measurement on Ethereum from blocks 12545219 (June 1st, 2021) to 16530247 (Jan 31st, 2023). We apply the heuristics mentioned in [10] to identify sandwich attacks.

> **Insight 7.** Compared with JIT liquidity attacks, sandwich attacks have lower entry barriers, offer greater profitability, and are more accessible to retail users.

Overall Statistics. we identify in total 208,149 sandwich attacks, which is $5.7\times$ the number of JIT liquidity attacks. We observe that the sandwich attacks present lower entry barriers. In contrast to JIT liquidity attacks which require the adversarial to hold a significant amount of initial funds that is on average 269 times higher than the swap volume, a sandwich attack requires an initial capital of 8.37 ETH, which is on average 6 times higher than the swap volume, indicating that the sandwich game is more accessible to retail MEV searchers.

Profitability. Our data show that sandwich attackers earn a total and an average profit of 12,242 ETH and 0.059 ETH respectively. The average ROI for sandwich attacks is 1.629%, implying a better profitability performance than JIT liquidity attacks. In addition, we observe that on average, sandwich attackers are willing to sacrifice $11.5\% \pm 25.9\%$ of their attack revenues to bribe the miners/validators, which is much lower than that of JIT liquidity attacks.

Top MEV Bots Statistics. We detect 143 bots participating in the sandwich game. The top sandwich MEV bots by profits are 0x000...B40, 0x000...e7D and 0x000...94e, who siphon 38%, 18% and 16% of the total profit respectively. This result indicates that the sandwich game is less monopolized than the JIT game. Surprisingly, while the top first JIT bot 0xa57...6CF only achieves an average ROI of 0.013%, the top first sandwich bot 0x000...B40 manages to achieve an average ROI of 1.642%.

Impact on Liquidity Takers. While JIT liquidity attacks improve the liquidity takers' execution price by an average of 0.139%, our simulation results show that sandwich liquidity takers' execution prices are worsened by an average of -5.93%.

VI. JIT Strategy Analysis

A. Attack Stratgy

As shown in Figure 1, we observe that the existing JIT attackers all adopt the following strategies in practice:

① The adversary $\boldsymbol{A}$ observes a sizable pending and potentially profitable swap transaction tx_T in the network via its spy node;
② $\boldsymbol{A}$ simulates tx_T locally to predict the pool price after the execution of tx_T. In this step, $\boldsymbol{A}$ searches the optimal values for price range$[p_l, p_u]$ and the amount liquidity L_A to add.
③ $\boldsymbol{A}$ issues the mint transaction tx_{A_1} and the burn transaction tx_{A_2} by calling the MEV bot contract. The input data of tx_{A_1} includes the optimal parameters generated in Step ②.

Note that Step ② is performed off-chain and the adversary searches the optimal values privately. However, we can leverage the on-chain public data to decode tx_{A1}'s input and execution process, which can help analyze how the adversary chooses the liquidity amount L_A and the price range $[p_l, p_u]$.

1) Liquidity Amount L_A Choice: We observe two methods to choose the liquidity amount L_A: *(i)* For all of the 7,932 (22%) JIT attacks launched by the bots 0x57C...c94 and 0x596...87E, L_A is explicitly specified in the add transaction input. *(ii)* For add transactions issued by other JIT bots, e.g., 0xa57...6CF, L_A is not specified in the transaction input data. Instead, the bots input ΔX_d and ΔY_d that they desire to add to the pool.

We take 0xa57...6CF and 0x57C...c94 as examples to compare their adding token amounts with the corresponding pool liquidity and their balances. We observe that, although the bot 0x57C...c94 does not always choose to supply all its token balance to add liquidity to the pool (cf. Figure 9a), the average liquidity share occupied by 0x57C...c94 is larger than that occupied by 0xa57...6CF (cf. Figures 9b and 9c).

2) Price Range $[p_l, p_u]$ Choice: We fetch the tick ranges $[i_l, i_u]$ by decoding the adversarial mint transactions, and compare them with the i_0 (tick before the swap) and the tick i_1 (tick after the swap). We observe that 18,766 (51%) swap transactions do not trigger the tick update in the liquidity pool (i.e., $i_0 = i_1$), and 17,112 (47%) swap transactions only trigger the tick update less than one tick space (i.e., $|i_1 - i_0| \leq 1\times$ `tickSpacing`, cf. Table I and IV). This can well explain why almost all (more than 99.99%) JIT attacks only cover one single tick space when setting the tick range $[i_l, i_u]$.

Algorithm 1: Attack Simulation for 0xa57...6CF

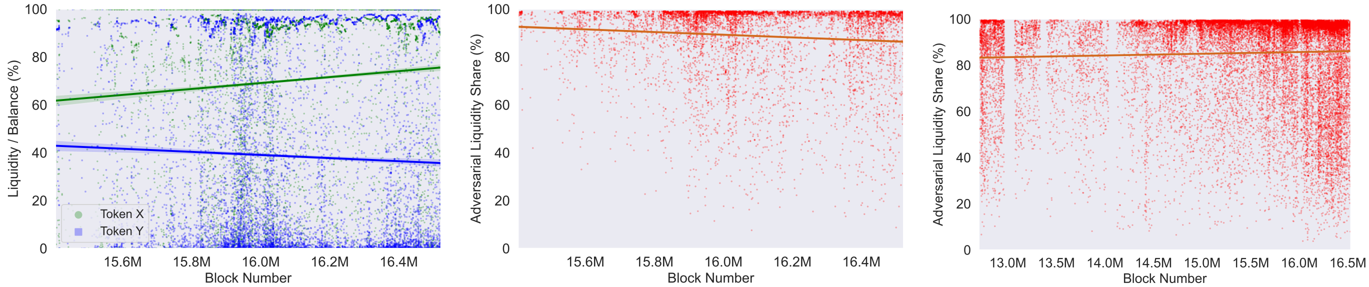
Data: Original JIT transactions tx_{A_1}, tx_T and tx_{A_2}
Result: optimal lower tick $i_l^{optimal}$, maximum profit P_{max}

i_l^{actual}, `tickSpacing` $\leftarrow$ decode tx_{A_1} input data ;
$i_l^{optimal} \leftarrow i_l^{actual}$;
$P_{max} \leftarrow 0$;
while $-4 \leq \delta \leq 4$ **do**
  $\hat{tx}_{A_1} \leftarrow$ modify tx_{A_1} by setting the lower tick provided in the input data as $i_l^{actual} + \delta \cdot$ `tickSpacing`;
  $P \leftarrow$ execute $\hat{tx}_{A_1}$, tx_T and tx_{A_2} sequentially;
  if $P > P_{max}$ **then**
    $P_{max} \leftarrow P$;
    $i_l^{optimal} \leftarrow i_l^{actual} + \delta \cdot$ `tickSpacing`;
  $\delta \leftarrow \delta + 1$;

3) Simulating the Attacks of 0xa57...6CF: Our analysis in Section VI-A indicates that bot 0xa57...6CF adopts the following strategy: *(i)* providing the maximum amount of

	num. Attacks	Revenue	Avg. Rev.	Gas Fee	Avg. Gas	Transfer	Avg. Trans.	Profit	Avg. Pro.	Liq. Share	Bribe Ratio	ROI
All	36,671	10,125	0.276	539	0.015	2,088	0.057	7498	0.204	85.42%	23.94%	0.007%
0xa57	27,983 (76%)	8,963 (88%)	0.320	435 (81%)	0.016	1628 (78%)	0.058	6900 (92%)	0.247	85.27%	23.39%	0.013%
0x57C	7,808 (21%)	981 (10%)	0.126	94 (17%)	0.012	403 (19%)	0.052	484 (6%)	0.062	88.85%	26.30%	0.005%
0xCD9	454 (1%)	84 (1%)	0.186	1 (0%)	0.002	30 (1%)	0.066	53 (1%)	0.117	45.16%	21.78%	0.033%
Others	426 (1%)	95 (1%)	0.225	10 (2%)	0.022	26 (1%)	0.062	60 (1%)	0.141	75.32%	18.81%	-0.325%

TABLE III: Overview of top MEV bots statistics. The revenue, gas fee, Coinbase transfer, and profit are measured in ETH.



(a) The liquidity/balance ratio for 0x57C...c94. (b) Adversarial liquidity share for 0x57C...c94. (c) Adversarial liquidity share for 0xa57...6CF.

Fig. 9: Statistics of top MEV bots behaviors.

Distribution			# Attacks
$d = \texttt{ts}$	$i_0 < i_1$	$i_1 < i_l$	2
		$i_0 < i_l, i_l \leq i_1 \leq i_u$	2,727
		$i_l \leq i_0 \leq i_u, i_l \leq i_1 \leq i_u$	5,441
		$i_l \leq i_0 \leq i_u, i_1 > i_u$	83
	$i_0 = i_1$	$i_l \leq i_0 = i_1 \leq i_u$	18,766
	$i_0 > i_1$	$i_1 < i_l, i_l \leq i_0 \leq i_u$	27
		$i_l \leq i_1 \leq i_u, i_l \leq i_0 \leq i_u$	5,980
		$i_l \leq i_1 \leq i_u, i_0 > i_u$	3,614
		$i_1 > i_u$	6
$d > \texttt{ts}$			5

TABLE IV: Distribution of JIT adversaries' choices of tick parameters i_l and i_u. i_0 and i_1 denote the tick before and after swap respectively, and $\texttt{ds}$ denotes $\texttt{tickSpacing}$. $d = |i_u - i_l|$ represents the distance between i_l and i_u.

available token balance to add liquidity in each JIT, and *(ii)* setting the lower tick i_l and the upper tick i_u with the constraints of $|i_u - i_l| = 1\times$ $\texttt{tickSpacing}$. Through analyzing the input data of 0xa57...6CF's mint transactions, we find that the adversary only explicitly specifies the distance between the lower tick and the current tick in the transaction data. Because the current tick is obtained by calling the Uniswap V3 pool smart contract, upon receiving the transaction data, the bot contract can execute the JIT liquidity attack with the corresponding parameters chosen by the adversary.

As shown in Algorithm 1, we simulate the JIT liquidity attacks issued by 0xa57...6CF with modified parameter values, in an attempt to test whether the modified attacks yield higher profits. For each JIT, we first decode the input data of the mint transaction to extract to actual lower tick i_l^{actual} chosen by the adversary. We then modify the add transaction data by changing the lower tick to $i_l^{actual} + \delta \cdot \texttt{tickSpacing}$, where $-4 \leq \delta \leq 4$. We finally output the optimal parameter $i_l^{optimal}$

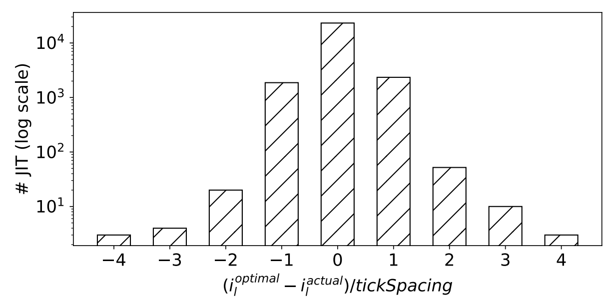


Fig. 10: Distribution of optimal tick ranges after simulating JITs of 0xa57...6CF. $i_l^{optimal}$ is the optimal lower tick obtained via simulation, and i_l^{actual} is the actual lower tick. 0xa57...6CF always ensures $|i_u - i_l| = 1\times$ $\texttt{tickSpacing}$.

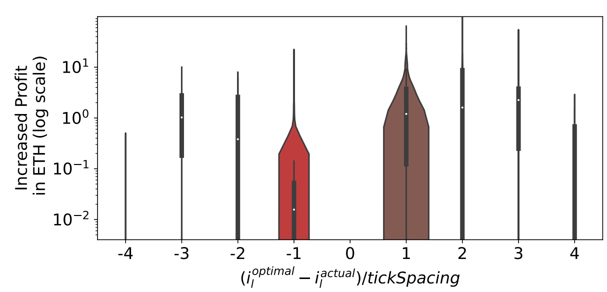


Fig. 11: Distribution of the increased profit through simulation.

which yields the highest attack profit P_{max}.

> **Insight 8.** The most active bot 0xa57...6CF does not always choose the optimal parameters to launch attacks, thus failing to capture at least 7,766 ETH attack profit.

After simulating the attacks issued by bot 0xa57...6CF, we observe that 73% of them indeed achieve the maximum attack profit, i.e., $i_l^{actual} = i_l^{optimal}$ (cf. Figure 10). For the remaining 27% JITs, we can whether a different lower tick value in the range of $[i_l^{actual} - 4 \cdot \texttt{tickSpacing}, i_l^{actual} + 4 \cdot \texttt{tickSpacing}]$ could achieve higher attack profits. While the original profit for these non-optimal attacks is 6,900 ETH, we detect that the bot could have achieved a total profit 14,667 ETH if it had chosen the optimized lower tick values. As a result, 0xa57...6CF fails to capture at least 7,766 ETH (equivalent to 16.1M USD) of the potential attack profit (cf. Figure 11).

VII. Discussion

In this study, we provide an overview of JIT liquidity attacks on Uniswap V3. Based on the findings of our empirical measurement, we suggest the following potential future research avenues. First, it is worth investigating the existence of analytic solutions for maximizing adversary utility. While it is difficult to solve Equation 15, the adversarial strategies could be evaluated more effectively if an analytical solution could be found. Second, the protocol should consider how to defend JIT liquidity attacks for existing LPs. A potential solution could be to prevent minting and burning in the same block. However, this may negatively impact the traders who have rebalancing needs (e.g., 0x9ec...832, 0x2b0...dd3, 0x8a6...e2d). Consequently, it's crucial for academic researchers to devise robust metrics that evaluate the net impact of JIT liquidity attacks on market efficiency, market fairness, and blockchain security. Such insights could guide protocol developers in formulating elegant countermeasures while taking the welfare economics of various market participants into account.

VIII. Conclusion

This paper provides an empirical analysis of JIT liquidity attacks, a new type of MEV source introduced by the concentrated liquidity design of Uniswap V3. We find that the attack poses notably high entry barriers, necessitating a significant initial capital. Moreover, the JIT game remains out of reach for retail users, as a handful of bots overwhelmingly dominate the landscape. In addition, we find that such attacks represent low-yield investments, with an average ROI ratio of only 0.007%. Additionally, we detect such attacks detrimental to existing LPs, but beneficial to liquidity takers. We further dissect the JIT strategies of top MEV bots and evaluate their strategies through local simulation. We find that 27% of the attacks issued by 0xa57...6CF are non-optimal, thus failing to capture at least 7,766 ETH of the potential attack profit. Based on our measurement result, we advise academic researchers to devise robust metrics to assess the net social impact of JIT liquidity attacks. Furthermore, we recommend that protocol developers implement appropriate features to address potential concerns.

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Appendix A
Derivation of Equation 13

Equation 12 can be written as $\Delta\text{Value}(p_1, x_1, y_1) = [(p_1 \cdot x_1 + y_1) - V_0] \cdot P_{(ETH/Y)}$. Therefore, we only need to focus on $p_1 \cdot x_1 + y_1$. To represents ΔValue as the function of L_A, p_l and p_u, we discuss the following scenarios.

Scenario 1: $\mathbf{p_u < p_1}$. In this scenario, tx_T swaps Y for X (i.e., !ZeroForOne). According to Equation 5, when $p_u < p_1$, $\boldsymbol{A}$'s portfolio after swap consists of asset Y only. Plugging $x_1 = 0$ and $y_1 = L_A(\sqrt{p_u} - \sqrt{p_l})$ into Equation 12, we have:.

$$\Delta\text{Value} = [L_A(\sqrt{p_u} - \sqrt{p_l}) - V_0] \cdot P_{(ETH/Y)} \tag{16}$$

Scenario 2: $\mathbf{p_1 < p_l}$. In this scenario, tx_T swaps X for Y (i.e., ZeroForOne). According to Equation 5, when $p_1 < p_l$, $\boldsymbol{A}$'s portfolio after swap consists of asset X only. Plugging $x_1 = L_A(\frac{1}{\sqrt{p_l}} - \frac{1}{\sqrt{p_u}})$ and $y_1 = 0$ into Equation 12, we have:

$$\Delta\text{Value} = [L_A(\frac{1}{\sqrt{p_l}} - \frac{1}{\sqrt{p_u}})p_1 - V_0] \cdot P_{(ETH/Y)} \tag{17}$$

When swapping X for Y, Equation 18 holds.

$$\Delta x_{\text{in}} = \Delta\frac{1}{\sqrt{p}}L_T = (\frac{1}{\sqrt{p_1}} - \frac{1}{\sqrt{p_0}})L_T \tag{18}$$

Hence, p_1 can be derived by Equation 19:

$$\sqrt{p_1} = \frac{L_T\sqrt{p_0}}{L_T + \sqrt{p_0}\Delta x_{\text{in}}} = \frac{(L_A + L_O)\sqrt{p_0}}{(L_A + L_O) + \sqrt{p_0}\Delta x_{\text{in}}}, \text{ if ZeroForOne} \tag{19}$$

Plugging Equation 19 into Equation 17 we have:

$$\Delta\text{Value} = [L_A(\frac{1}{\sqrt{p_l}} - \frac{1}{\sqrt{p_u}})\frac{(L_A + L_O)\sqrt{p_0}}{(L_A + L_O) + \sqrt{p_0}\Delta x_{\text{in}}} - V_0] \cdot P_{(ETH/Y)} \tag{20}$$

Scenario 3: $\mathbf{p_l \le p_1 \le p_u}$. In this scenario, $\boldsymbol{A}$'s portfolio after swap consists of both asset X and Y. Plugging $x_1 = L_A(\frac{1}{\sqrt{p_1}} - \frac{1}{\sqrt{p_u}})$ and $y_1 = L_A(\sqrt{p_1} - \sqrt{p_l})$ into Equation 12:

$$\Delta\text{Value} = [L_A \cdot (\sqrt{p_u} - \sqrt{p_l} - \frac{1}{\sqrt{p_u}}(\sqrt{p_1} - \sqrt{p_u})^2) - V_0] \cdot P_{(ETH/Y)} \tag{21}$$

Since the swap direction (i.e., the value of ZeroForOne) is unknown, we should further discuss Scenario 3(a) and 3(b).

Scenario 3(a): $\mathbf{p_l \leq p_1 \leq p_u}$ and ZeroForOne. When swapping X for Y, we can derive p_1 using Equation 19.

$$\Delta\text{Value} = [L_A(\sqrt{p_u} - \sqrt{p_l} - \frac{1}{\sqrt{p_u}}(\frac{(L_A + L_O)\sqrt{p_0}}{(L_A + L_O) + \sqrt{p_0}\Delta x_{\text{in}}} - \sqrt{p_u})^2) - V_0] \cdot P_{(ETH/Y)} \tag{22}$$

Scenario 3(b): $\mathbf{p_l \leq p_1 \leq p_u}$ and !ZeroForOne. When swapping Y for X, Equation 23 holds.

$$\Delta y_{\text{in}} = \Delta\sqrt{p}L_T = (\sqrt{p_1} - \sqrt{p_0})L_T \tag{23}$$

Hence, p_1 can be derived by Equation 24:

$$\sqrt{p_1} = \sqrt{p_0} + \frac{\Delta y_{\text{in}}}{L_T} = \sqrt{p_0} + \frac{\Delta y_{\text{in}}}{L_A + L_O}, \text{ if !ZeroForOne} \tag{24}$$

Plugging Equation 24 into Equation 21 we have:

$$\Delta\text{Value} = [L_A(\sqrt{p_u} - \sqrt{p_l} - \frac{1}{\sqrt{p_u}}((\sqrt{p_0} + \frac{\Delta y_{\text{in}}}{L_A + L_O}) - \sqrt{p_u})^2) - V_0] \cdot P_{(ETH/Y)} \tag{25}$$