Efficient Instances of Docked Double Decker With AES

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Abstract. A tweakable wide blockcipher is a construction which behaves in the same way as a tweakable blockcipher, with the difference that the actual block size is flexible. Due to this feature, a tweakable wide blockcipher can be directly used as a strong encryption scheme that provides full diffusion when encrypting plaintexts to ciphertexts and vice versa. Furthermore, it can be the basis of authenticated encryption schemes fulfilling the strongest security notions. In this paper, we present two instantiations of the docked double decker tweakable wide blockcipher: *ddd-AES* and *bbb-ddd-AES*. Both instances exclusively use similar building blocks as AES-GCM (AES and finite field multiplication), are designed for maximal parallelism, and hence, can make efficient use of existing hardware accelerators. Moreover, *bbb-ddd-AES* builds upon a novel beyond birthday bound secure pseudorandom function, a tweakable variant of the XOR of permutations, facilitating in the need to include a tweak in the AES evaluations without sacrificing flexibility in docked double decker.

Keywords: symmetric cryptography, tweakable wide blockcipher, docked double decker, tweakable XOR of permutations.

1 Introduction

1.1 Motivation

With modern Internet- and cloud-scale data creation and processing volumes being routinely measured in exabytes and approaching zettabytes, many existing ciphers become a bottleneck and sometimes even a security risk, because they were not designed to be used at such scale. As indicated in some cloud service provider (CSP) comments [15,17,27], the limitations of block size, nonce size and its uniqueness requirements, and corresponding birthday bounds, lead to many standardized mainstream ciphers and modes becoming too brittle when used for such large data sets. To protect that data while complying with cipher key/nonce

pair uniqueness requirements and data processing volume limitations, CSPs are forced to either employ inefficient techniques like frequent rekeying (every week or two, down to potentially mere seconds), or use tricks like having a static nonce and rekeying for every message.

A more efficient avenue is the development of a new cipher capable of dealing with larger data and constraints. Based on the CSP feedback [15,17,27], the following preferences can be formulated for encryption or authenticated encryption schemes intended to handle modern data volumes:

1. Processing more than 2^{64} blocks (e.g., blocks of 16 bytes in case of AES) while maintaining security. This means, for example, the need for a blockcipher operating on 256-bit block size, or using blockciphers with 128-bit block size in constructions enabling beyond birthday bound security. This allows us to overcome current data size limitations, which are the main problem;
2. Performance, obviously, as for such data volumes every percent matters;
3. An authenticated encryption scheme that can use large nonces to facilitate random generation with a negligible risk of reuse;
4. IV/nonce misuse resistance in case a counter cannot be maintained, or reliable random generation is hard or unavailable;
5. The ability to shorten authentication tags for some use cases, while keeping good enough security bounds (compared to the go-to AES-GCM [18,30]).

The US NIST (National Institute of Standards and Technology) has standardized a number of pure confidentiality modes of operation, like ECB, CBC, CFB, OFB, and CTR in NIST SP 800-38A [8], and XTS-AES in NIST SP 800-38E [9]. Although these modes see a wide-spread use in applications, they come with their own limitations. Most notably, none of the above-mentioned encryption methods provides full diffusion for encryption as well as decryption if the data to be encrypted exceeds a few blocks. This stands in sharp contrast with the fact that for modes that just provide confidentiality, full diffusion behavior is often a practical security benefit, since it limits the ability of an attacker to target specific fractions of the encrypted data [1].

1.2 Tweakable Wide Blockciphers

A very suitable solution, or building block for a solution to many of the aforementioned preferences, are tweakable wide blockciphers. Not surprisingly, in the recent third NIST workshop on blockcipher modes of operation in 2023, the organizers stated that "*NIST is particularly interested in discussing the possibility of standardizing a tweakable wide block encryption technique that could support a large range of input lengths.*" [21].

Indeed, a tweakable wide blockcipher extends the definition of a tweakable blockcipher [16] to arbitrarily large input and output size, this way allowing for flexibility in the block size and accommodating for the primary requirements above. Note that such a tweakable wide blockcipher also, unlike existing modes such as ECB, CBC, CFB, OFB, CTR, and XTS-AES, allows for full diffusion.

This way, it serves as viable drop-in replacement of these modes in many applications.

Furthermore, it can serve as the basis of an authenticated encryption scheme, or directly as authenticated encryption scheme, by either appending the nonce to the plaintext or putting the nonce in the tweak and appending zeros to the plaintext to strengthen authenticity [12]. The resulting construction essentially allows for flexibly sized tags and nonces, and has the potential to be misuse resistant and context committing.

The remaining boxes to be ticked are performance and beyond birthday bound security, and this brings us to our contribution.

1.3 Our Contributions

We present two tweakable wide blockciphers, *ddd-AES* and *bbb-ddd-AES*. Both are based on the same components as used in many NIST standardized schemes. Notably, both are based on the AES blockcipher [5, 6], as well as on operations in binary extension fields as used by GHash in AES-GCM [18, 30].

Both schemes are based on the docked double decker mode of Gunsing et al. [11] (see Figure 1). Docked double decker operates on top of a universal hash function H and a pseudorandom function F, and has the feature that it allows to provide beyond birthday bound security assuming it is not too often used with the same tweak. Both our instances *ddd-AES* and *bbb-ddd-AES* take *Polyval* [10] as universal hash function. The choice of pseudorandom function is different for the two constructions:

- In *ddd-AES*, the pseudorandom function is based on an *XE*-style [28] tweakable blockcipher, itself built on top of AES, evaluated in counter mode (see Section 4.2). The resulting construction achieves birthday bound security;
- In *bbb-ddd-AES*, to accommodate for the tweak, we wished to instantiate the pseudorandom function with a slightly compressing construction on top of AES that achieves beyond birthday bound security. To this end, we took the *XORP* construction as used in CENC [13], and extended it to include a tweak. In detail, this construction $\widetilde{XORP}$ extends *XORP* by including the tweak in an *XE*-style [28] manner (Section 4.3). We prove that $\widetilde{XORP}$ achieves around $2n/3$-bit security. We remark that this result — the introduction and security analysis of $\widetilde{XORP}$ as a "tweakable PRF" — is of independent interest.

1.4 Outline

We first discuss some preliminaries in Section 2. The docked double decker construction of Gunsing et al. [11] is recalled in Section 3. We specify *ddd-AES* and *bbb-ddd-AES* in Section 4, with the description of *Polyval* (as used in both *ddd-AES* and *bbb-ddd-AES*) in Section 4.1, the description of the pseudorandom function used in *ddd-AES* in Section 4.2, and the description of the pseudorandom function used in *bbb-ddd-AES* in Section 4.3. The security of *ddd-AES* and

bbb-ddd-AES is analyzed in Section 5, with the security proof of $\widetilde{XORP}$, which is of independent interest, in Section 6. We conclude in Section 7.

2 Preliminaries

For $n \in \mathbb{N}$, $\{0,1\}^n$ denotes the set of bit strings of length n, and $\{0,1\}^* = \cup_{n=0}^{\infty}\{0,1\}^n$ denotes the set of bit strings of arbitrary length. For a finite set $\mathcal{S}$, we denote by $s \xleftarrow{\$} \mathcal{S}$ the uniform random selection of s from $\mathcal{S}$. For $n, p \in \mathbb{N}$, we denote by $(n)_p = n(n-1)\cdots(n-p+1)$ the falling factorial.

2.1 Tweakable Wide Blockciphers

Our tweakable wide blockciphers will be parameterized by a value $n \in \mathbb{N}$. This will also be called the **block size**. They will require plaintexts of size at least $2n$ bits. Our tweakable wide blockciphers will also be parameterized by a key size $\kappa \in \mathbb{N}$ and a tweak size $w \in \mathbb{N}$. Finally, to formally argue security, we also limit the maximum size of an input plaintext or output ciphertext to some value $\ell_{\max} \in \mathbb{N}$ such that $\ell_{\max} \geq 2n$. We define the plaintext and ciphertext space to

$$\mathcal{S} := \bigcup_{i=2n}^{\ell_{\max}} \{0,1\}^i \,. \tag{1}$$

A tweakable wide blockcipher $TWBC : \{0,1\}^\kappa \times \{0,1\}^w \times \mathcal{S} \to \mathcal{S}$ is a family of permutations on $\mathcal{S}$ indexed by key $K \in \{0,1\}^\kappa$ and tweak $W \in \{0,1\}^w$. In other words, $TWBC$ satisfies the property that for fixed $K \in \{0,1\}^\kappa$ and $W \in \{0,1\}^w$,

$$TWBC_{K,W}(\cdot) := TWBC(K, W, \cdot)$$

is a length-preserving bijection. Its inverse for fixed K and W is denoted by $TWBC^{-1}_{K,W}$.

Define by $\mathrm{perm}(w, 2n : \ell_{\max})$ the family of all length-preserving bijections on $\mathcal{S}$ of (1). The security of a tweakable wide blockcipher $TWBC$ is defined by how hard it is for an adversary A to distinguish $TWBC$ for a random and secret key $K \xleftarrow{\$} \{0,1\}^\kappa$ from a tweakable wide random permutation $TWRP \xleftarrow{\$} \mathrm{perm}(w, 2n : \ell_{\max})$:

$$\mathbf{Adv}^{\mathrm{twprp}}_{TWBC}(A) = \mathbf{Pr}\left(A^{TWBC_K} = 1\right) - \mathbf{Pr}\left(A^{TWRP} = 1\right) , \tag{2}$$

where the probabilities are taken over $K \xleftarrow{\$} \{0,1\}^\kappa$, $TWRP \xleftarrow{\$} \mathrm{perm}(w, 2n : \ell_{\max})$, and the random coins of A. The adversary is typically bounded by a certain number of queries q, and a total data complexity σ that counts the total amount of output data bits. Here, we remark that the amount of input data bits equals the amount of output data bits plus the tweak, the latter of which is of fixed size for each of the q queries. The adversary is also bounded by a certain amount of time in which it can make offline evaluations, but this time is not explicitly included.

2.2 Pseudorandom Permutations

A blockcipher $E : \{0,1\}^\kappa \times \{0,1\}^n \to \{0,1\}^n$ is a family of permutations on $\{0,1\}^n$ indexed by key $K \in \{0,1\}^\kappa$. We denote $E_K(\cdot) = E(K,\cdot)$, and its inverse for fixed K is denoted by E_K^{-1}.

Define by $\mathrm{perm}(n)$ the family of all bijections on $\{0,1\}^n$. The security of a blockcipher E is defined by how hard it is for an adversary A to distinguish E for a random and secret key $K \xleftarrow{\$} \{0,1\}^\kappa$ from a random permutation $RP \xleftarrow{\$} \mathrm{perm}(n)$:

$$\mathbf{Adv}_E^{\mathrm{prp}}(A) = \mathbf{Pr}\left(A^{E_K} = 1\right) - \mathbf{Pr}\left(A^{RP} = 1\right), \tag{3}$$

where the probabilities are taken over $K \xleftarrow{\$} \{0,1\}^\kappa$, $RP \xleftarrow{\$} \mathrm{perm}(n)$, and the random coins of A. The adversary is typically bounded by a certain number of queries q. Note that each query is of fixed size n bits.

2.3 Pseudorandom Functions

Let $a, b \in \mathbb{N} \cup \{*\}$. A pseudorandom function $F : \{0,1\}^\kappa \times \{0,1\}^a \to \{0,1\}^b$ is a family of functions from $\{0,1\}^a$ to $\{0,1\}^b$ indexed by key $K \in \{0,1\}^\kappa$. We denote $F_K(\cdot) = F(K,\cdot)$.

Define by $\mathrm{func}(a,b)$ the family of all functions from $\{0,1\}^a$ to $\{0,1\}^b$. The security of a pseudorandom function F is defined by how hard it is for an adversary A to distinguish F for a random and secret key $K \xleftarrow{\$} \{0,1\}^\kappa$ from a random function $RF \xleftarrow{\$} \mathrm{func}(a,b)$:

$$\mathbf{Adv}_F^{\mathrm{prf}}(A) = \mathbf{Pr}\left(A^{F_K} = 1\right) - \mathbf{Pr}\left(A^{RF} = 1\right), \tag{4}$$

where the probabilities are taken over $K \xleftarrow{\$} \{0,1\}^\kappa$, $RF \xleftarrow{\$} \mathrm{func}(a,b)$ (lazily-sampled), and the random coins of A. The adversary is typically bounded by a certain number of queries q, and a total output data complexity σ that counts the total amount of output data bits. Here, we remark that we will always use F on fixed input size and on varying output size.

In our case, the input to the function F may consist of a comma-separated list of multiple inputs. To be precise, we will use a function F that operates on a κ-bit key K, an n-bit input I, a domain separator nibble B, and a w-bit tweak that produces a variable length output O:

$$F(K, I, B, W) = O\,.$$

The function F internally concatenates I, B, and W.

2.4 Universal Hash Functions

Let $a, b \in \mathbb{N} \cup \{*\}$. Consider a family of hash functions $H : \{0,1\}^\kappa \times \{0,1\}^a \to \{0,1\}^b$. It is called ε-XOR-universal if for any two distinct $X, X' \in \{0,1\}^a$ and any $Y \in \{0,1\}^b$,

$$\mathbf{Pr}\left(H(K,X) \oplus H(K,X') = Y\right) \le \varepsilon\,,$$

where the probability is taken over $K \xleftarrow{\$} \{0,1\}^\kappa$.

3 Docked Double Decker

Let $\kappa, w, n, \ell_{\max}, m_{\max} \in \mathbb{N}$ such that $2n \leq \ell_{\max}$ and $m_{\max} = \lceil \ell_{\max}/n \rceil$. In this paper, we propose instantiations of the docked double decker (ddd) of Gunsing et al. [11]. The scheme is depicted in Figure 1. It gets as input two keys $K \in \{0,1\}^\kappa$, and $L \in \{0,1\}^n$, a tweak $W \in \{0,1\}^w$, and a plaintext $P \in \mathcal{S}$ of size at least $2n$ bits and at most $\ell_{\max}$ bits (see (1)). The plaintext P is parsed as $P = T\|U\|V$, where T and V are both n-bit long. Then, a four-round structure based on two independent instances of a pseudorandom function $F_K : \{0,1\}^{n+4+w} \to \{0,1\}^*$ and two instances of a universal hash function $H_L : \{0,1\}^* \to \{0,1\}^n$ is evaluated to obtain the ciphertext $C = X\|Y\|Z$, where X and Z are n-bit long and Y matches the size of U. We denote this as

$$ddd^{F,H}_{K,L}(W, T\|U\|V) = X\|Y\|Z\,. \tag{5}$$

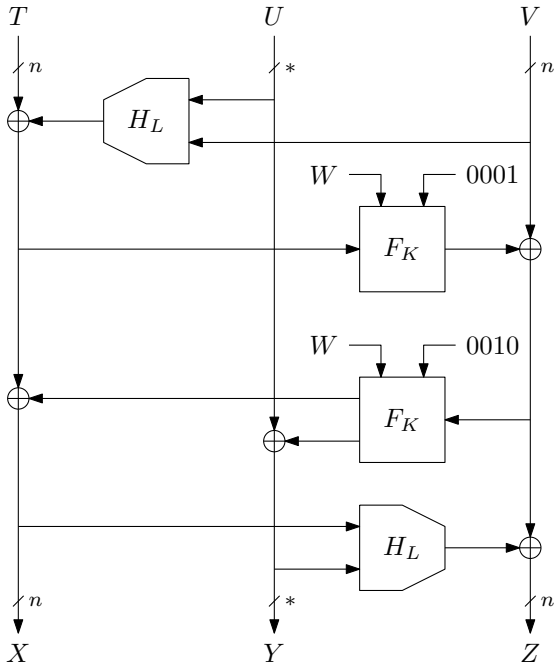

Fig. 1: The docked double decker construction.

We remark that we have slightly deviated from the specification of Gunsing et al. [11] in the sense that we do not use two different keys for F but rather use domain separation. However, their analysis directly carries over. In detail, Gunsing et al. [11] proved security under the assumption that the function F is a

pseudorandom function (PRF) and H a blinded keyed hash function. An XOR-universal hash function is a specific type of blinded keyed hash function, and we will adopt a simplification of their result to XOR-universal hash functions.

Theorem 1 (Gunsing et al. [11, Theorem 1]). *Consider the docked double decker construction ddd on top of a pseudorandom function* $F : \{0,1\}^\kappa \times \{0,1\}^{n+4+w} \to \{0,1\}^*$ *and a universal hash function family* $H : \{0,1\}^n \times \{0,1\}^* \to \{0,1\}^n$*. For any adversary* A *making at most* q *queries, each of size at least* $2n$ *and at most* $\ell_{\max}$ *bits, and in total of size at most* σ *bits, we have*

$$\mathbf{Adv}^{\mathrm{twprp}}_{ddd}(A) \leq \mathbf{Adv}^{\mathrm{prf}}_{F}(A') + \sum_{W \in \{0,1\}^w} \binom{q_W}{2} \cdot \left(2\varepsilon + \frac{1}{2^{2n}}\right),$$

for some adversary A' *with a total query complexity* $q' = 2q$ *and a total data complexity* $\sigma' = \sigma$ *bits, and where* q_W *is the number of queries made for tweak* $W \in \{0,1\}^w$*.*

We remark that A' in fact makes q queries whose output is of size n bits, and q queries whose output is of arbitrary size but that add up to $\sigma - qn$ bits.

4 Specification of *ddd-AES* and *bbb-ddd-AES*

We will describe how we suggest to instantiate *ddd* using AES to obtain a birthday bound secure *ddd-AES* and a beyond birthday bound secure *bbb-ddd-AES*. For both of them, we suggest the same instantiation of H, as described in Section 4.1. The main bottleneck, however, will be the design of F, which gets an input of size $n + 4 + w$ bits and should operate on top of AES with a block size of $n = 128$ bits. We will assume that $4 + w \leq n$. The instantiation of F for *ddd-AES*, including rationale, is given in Section 4.2. The instantiation of F for *bbb-ddd-AES*, again including rationale, is given in Section 4.3.

4.1 Instantiation of *H*

Due to the addition of carry-less multiplication instructions on modern CPUs, instances for H_L based on polynomial evaluation are a viable option. Hence, we decided to instantiate H_L using *Polyval* [10]. On input of a key L and a list of s field elements I_i, all elements of $\mathbb{F}_{2^{128}}[x]/(x^{128} + x^{127} + x^{126} + x^{121} + 1)$, it is defined as

$$\mathit{Polyval}_L(I_1, I_2, \ldots, I_s) = \sum_{i=1}^{s} \left(L^{s-i+1} \cdot I_i \cdot x^{-128\cdot(s-i+1)}\right), \tag{6}$$

We will use it for arbitrary-length bit strings, always of length at most $\ell_{\max} - n$ bits. To process such string using $\mathit{Polyval}_L$, it is first 0-padded to the first multiple of n bits. Then, an n-bit string encoding the bit length of I is appended. The resulting bit string then represents $I_1 \| I_2 \| \ldots \| I_s$, noting that we can uniquely map elements from this field to bit strings in $\{0,1\}^{128}$. Particularly, in our case, $s \leq m_{\max}$, and for this case, *Polyval* is an ϵ-XOR-universal hash function with $\epsilon = m_{\max}/2^n$ [10, Lemma 3].

4.2 Instantiation of F for *ddd-AES*

We realize F by turning the AES-128 blockcipher E_K into an XE-style [28] tweakable blockcipher, where B and W function as tweak, and plugging this tweakable blockcipher into counter mode to obtain a keystream of arbitrary length. Note that the XE-style is sufficient as opposed to the XEX-style, as the primitive is never evaluated in inverse direction.

In detail, we define $F_K : \{0,1\}^{n+4+w} \to \{0,1\}^*$ as

$$F_K(I,B,W) = \left\lfloor E_K(I \oplus 2^0 S) \| E_K(I \oplus 2^1 S) \| \dots \| E_K(I \oplus 2^{m_{\max}-1} S) \right\rfloor_{\ell_{\max}} , \tag{7}$$

where $S = E_K(B\|W)$ serves as tweak-dependent subkey. In this case, we can support a tweak with a length of $w = 124$ bits. The keylength κ depends on the actual instance chosen for AES [5,6].

4.3 Instantiation of F for *bbb-ddd-AES*

To realize a function F that achieves beyond birthday bound security, we extend the $XORP[v]$ [14] that underlies CENC [13] to include a tweak.

Our tweak inclusion will be similar to the XE-style approach, albeit with counter included in the subkey. In detail, we assume F to have two keys instead of one, $K = K_1\|K_2 \in \{0,1\}^{2\kappa}$, and we consider the following approach for the subkey computation:

$$S_j = E_{K_2}(B\|W\|c\|j) , \tag{8}$$

where j will function as "inner counter" in the evaluation of F and c as "outer counter" for the mode employing F.

We subsequently define $\widetilde{XORP}[v]$ for $v \in \mathbb{N}$ on top of a blockcipher $E : \{0,1\}^\kappa \times \{0,1\}^n \to \{0,1\}^n$ as

$$\begin{aligned} \widetilde{XORP}[v]^E_K(I,B,W,c) = &\big(E_{K_1}(I \oplus S_0) \oplus E_{K_1}(I \oplus S_1)\big) \| \dots \\ &\dots \| \big(E_{K_1}(I \oplus S_0) \oplus E_{K_1}(I \oplus S_v)\big) . \end{aligned} \tag{9}$$

This construction is depicted in Figure 2. This approach leaves us with $n-4$ bits that can be distributed between the outer counter c, the inner counter j, and the tweak W. In case of AES, where $n = 128$, we suggest to use 28 bits split between the counters c and j, where j occupies $\lceil \log_2(v+1) \rceil \le 28$ bits and c gets $28 - \lceil \log_2(v+1) \rceil$ bits of space. This leaves room for a $(w = 96)$-bit tweak.

We finally define $F_K : \{0,1\}^{n+4+w} \to \{0,1\}^*$ as counter mode on top of $\widetilde{XORP}[v]$ truncated to the required length:

$$F_K(I,B,W) = \left\lfloor \widetilde{XORP}[v]^E_K(I,B,W,0) \| \dots \| \widetilde{XORP}[v]^E_K(I,B,W,\lceil m_{\max}/v \rceil) \right\rfloor_{\ell_{\max}} . \tag{10}$$

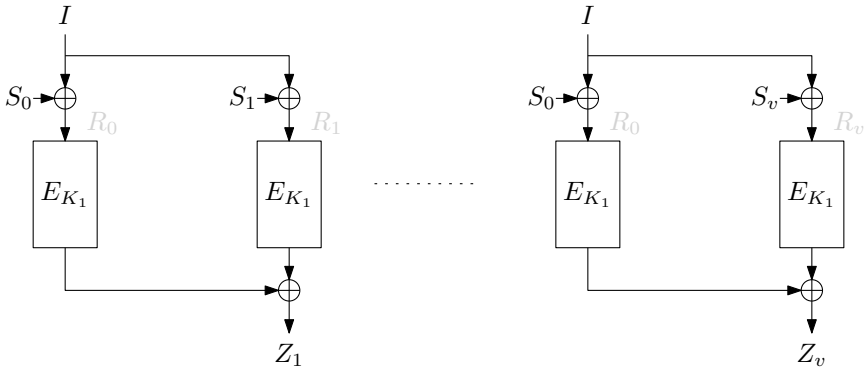

Fig. 2: The $\widetilde{XORP}[v]$ construction. Here, $S_j = E_{K_2}(B\|W\|c\|j)$ of (8). The parameters R_j will be used of the proof of Theorem 2 in Section 6.

5 Security of *ddd-AES* and *bbb-ddd-AES*

We will discuss the security of *ddd-AES* and *bbb-ddd-AES* in the security model of Section 2.1. Both security analyses have in common that they rely on the XOR-universality of H, which is already briefly stated in Section 4.1, but which we formally repeat here for convenience.

Lemma 1 (Gueron et al. [10, Lemma 3]). *The universal hash function Polyval of (6) is ϵ-XOR-universal with $\epsilon = m_{\max}/2^n$.*

Security of *ddd-AES* is now treated in Section 5.1 and security of *bbb-ddd-AES* in Section 5.2.

5.1 Security of *ddd-AES*

The *ddd-AES* scheme is based on the *XE* construction that operates on a blockcipher $E : \{0,1\}^\kappa \times \{0,1\}^n \to \{0,1\}^n$:

$$XE_K^E(I, B, W, j) = E_K(I \oplus 2^j E_K(B\|W))\,. \tag{11}$$

Rogaway [28] proved that this *XE* construction[4] behaves like a random tweakable permutation as long as the total number of evaluations q satisfies $4.5q^2/2^n$ and as long as E is PRP-secure after at most q queries. However, we will rather use the *XE* construction as a PRF, and looking at the proof of [28, Theorem 1], which can be found in the full version [29, Appendix B], it *first* proves *XE* to be PRF-secure and then as last step makes an RF-to-(T)RP switch at the cost of $0.5q^2/2^n$. We will require PRF-security of the *XE* construction, thus allowing us to use a slightly tighter bound.

[4] A small change is in the split of the nonce into B and W, and in the fact that the subkey $E_K(B\|W)$ is multiplied only by 2^j.

Lemma 2 (Rogaway [28, Theorem 1]). *Consider the construction XE of (11) on top of a pseudorandom permutation $E : \{0,1\}^\kappa \times \{0,1\}^n \to \{0,1\}^n$. For any adversary A making at most q queries, each of output size n bits, we have*

$$\mathbf{Adv}^{\mathrm{prf}}_{XE}(A) \leq \mathbf{Adv}^{\mathrm{prp}}_{E}(A') + \frac{4q^2}{2^n}\,,$$

for some adversary A' with a total query complexity $q' = 2q$.

The security of *ddd-AES* is now a direct corollary of Theorem 1, Lemma 1, and Lemma 2, the only work actually being the data complexity translation from bits queried in *ddd-AES* to actual evaluations of the underlying *AES*. To be precise, in *ddd* the underlying F is evaluated $2q$ times with a total output data complexity of σ bits. These amount to at most $\lceil \sigma/n \rceil$ evaluations of XE of (11).

Corollary 1. *Consider ddd-AES, the docked double decker construction ddd on top of Polyval $: \{0,1\}^\kappa \times \{0,1\}^* \to \{0,1\}^n$ and AES $: \{0,1\}^\kappa \times \{0,1\}^n \to \{0,1\}^n$ through XE of (11). For any adversary A making at most q queries, each of size at least $2n$ and at most $\ell_{\max}$ bits, and in total of size at most σ bits, we have*

$$\mathbf{Adv}^{\mathrm{twprp}}_{ddd\text{-}AES}(A) \leq \mathbf{Adv}^{\mathrm{prp}}_{E}(A') + \frac{4(\lceil \sigma/n \rceil)^2}{2^n} + \sum_{W \in \{0,1\}^w} \binom{q_W}{2} \cdot \left(\frac{2m_{\max}}{2^n} + \frac{1}{2^{2n}} \right)\,,$$

for some adversary A' with a total query complexity $q' = 2\lceil \sigma/n \rceil$, and where q_W is the number of queries made for tweak $W \in \{0,1\}^w$.

5.2 Security of *bbb-ddd-AES*

We will consider the security of the *bbb-ddd-AES* scheme. However, this analysis is not as simple as that of *ddd-AES* of Section 5.1. The reason for this is that *bbb-ddd-AES* is based on a new pseudorandom function design, namely $\widetilde{XORP}[v]$ of (9). Thus, we first have to analyze the PRF-security of $\widetilde{XORP}[v]$.

Theorem 2. *Let $v \in \mathbb{N}$. Consider the construction $\widetilde{XORP}[v]$ of (9) on top of a pseudorandom permutation $E : \{0,1\}^\kappa \times \{0,1\}^n \to \{0,1\}^n$. For any adversary A making at most q queries, each of output size vn bits, we have*

$$\mathbf{Adv}^{\mathrm{prf}}_{\widetilde{XORP}[v]}(A) \leq 2\mathbf{Adv}^{\mathrm{prp}}_{E}(A') + \frac{(v+1)^4 q^3}{5 \cdot 2^{2n}} + \frac{\binom{v+1}{2} q}{2^n} + \frac{2\binom{v+1}{2}^2 q^2}{2^{2n}}\,,$$

for some adversary A' with a total query complexity $q' = (v+1)q$, where we assume that $n(2v+1)^2 + (2v+1) \leq 2^{n/2}$ and $(2v+1)^2(v+1)q \leq 2^n/12$.

The proof of Theorem 2 is technically involved, and is given in Section 6.

The security of *bbb-ddd-AES* is now a direct corollary of Theorem 1, Lemma 1, and Theorem 2, the only work actually being the data complexity translation from bits queried in *bbb-ddd-AES* to actual evaluations of the underlying *AES*.

Consider a single evaluation of *bbb-ddd-AES* on input of ℓ_i bits and $m_i = \lceil \ell_i/n \rceil$ blocks. One evaluation of F is for 1 n-bit output block: it makes 1 evaluation of $\widetilde{XORP}[v]$ that costs 2 calls to each blockcipher. One evaluation of F is for $m_i - 1$ n-bit output blocks: it makes $\lceil (m_i - 1)/v \rceil$ evaluations of $\widetilde{XORP}[v]$ that cost at most $(v+1)\lceil (m_i-1)/v \rceil$ calls to each blockcipher. Summing over all q queries, *bbb-ddd-AES* incurs

$$\sum_{i=1}^{q} \left(\left\lceil \frac{m_i - 1}{v} \right\rceil + 1 \right) \le \frac{1}{v} \lceil \sigma/n \rceil + \frac{v+1}{v} q =: q_x \tag{12}$$

evaluations of $\widetilde{XORP}[v]$ with a total amount of at most

$$\sum_{i=1}^{q} \left((v+1) \left\lceil \frac{m_i - 1}{v} \right\rceil + 2 \right) \le \frac{v+1}{v} \lceil \sigma/n \rceil + \frac{3v+1}{v} q =: q_e \tag{13}$$

calls to each blockcipher, where we used that $\sum_{i=1}^{q} \ell_i \le \sigma$ and thus $\sum_{i=1}^{q} m_i \le \lceil \sigma/n \rceil + q$.

Corollary 2. *Consider bbb-ddd-AES, the docked double decker construction ddd on top of Polyval* $: \{0,1\}^\kappa \times \{0,1\}^* \to \{0,1\}^n$ *and AES* $: \{0,1\}^\kappa \times \{0,1\}^n \to \{0,1\}^n$ *through* $\widetilde{XORP}$ *of (9). Let* $v \in \mathbb{N}$ *and let* q_x *and* q_e *be as in (12) and (13). For any adversary A making at most q queries, each of size at least* $2n$ *and at most* $\ell_{\max}$ *bits (equivalent to* $m_{\max}$ n*-bit blocks), and in total of size at most* σ *bits, we have*

$$\mathbf{Adv}^{\mathrm{twprp}}_{\textit{bbb-ddd-AES}}(A) \le 2\mathbf{Adv}^{\mathrm{prp}}_{E}(A') + \frac{(v+1)^4 q_x^3}{5 \cdot 2^{2n}} + \frac{\binom{v+1}{2} q_x}{2^n} + \frac{2\binom{v+1}{2}^2 q_x^2}{2^{2n}} + \sum_{W \in \{0,1\}^w} \binom{q_W}{2} \cdot \left(\frac{2m_{\max}}{2^n} + \frac{1}{2^{2n}} \right),$$

for some adversary A' *with a total query complexity* $q' = q_e$*, where we assume that* $n(2v+1)^2 + (2v+1) \le 2^{n/2}$ *and* $(2v+1)^2(v+1)q_x \le 2^n/12$*, and where* q_W *is the number of queries made for tweak* $W \in \{0,1\}^w$*.*

6 Proof of Theorem 2

The $XORP[v]$ construction was introduced by Iwata [13] and proven to achieve $2n/3$-bit security. Later, Iwata et al. [14] demonstrated that $n - \log_2(w)$ security was achieved using the mirror theory [19, 20, 23, 25, 26], and Bhattacharya and Nandi [2] proved a similar bound using the χ^2 technique [7]. Very recently,

a concise proof of the mirror theory (for a very large limit on the maximum component size) was delivered [4] and the authors also applied it to $XORP[v]$. In fact, this mirror theory result considers sums of permutations, where each sum can be defined as an edge in a graph between two vertices, and where it is required that there is no circle in the graph and no too large tree. For $XORP[v]$ this is the case: each evaluation of $XORP[v]$ defines v edges over $v+1$ vertices that form a tree, basically even a star, and different evaluations of $XORP[v]$ are disconnected. Thus, $XORP[v]$ is a fairly simple application of this main mirror theory result.

It turns out that the exact same mirror theory result can *also* be used to argue security of $\widetilde{XORP}[v]$, but the application is a bit more subtle. The reason is that, in our case, again any evaluation of $\widetilde{XORP}[v]$ defines a star on v edges over $v+1$ vertices (basically as the masking values S_j of (1) are different for $j = 1, \ldots, v$) but any two different stars may collide and they may collide in $(v+1)^2$ ways. Excluding any such collision would force us into birthday bound security, but there is no need to exclude such collisions as any such collision merely implies a maximum tree size up to $2v+1$ elements. In general, as long as there is no too large tree of stars, the maximum component is still "small enough" for the mirror theory result of [4] to apply.

This will also be the main proof strategy: in a nutshell, we will demonstrate that (i) there is no too large tree of stars except with a small probability, (ii) there is no cycle of stars except with a small probability, and (iii) the mirror theory of [4] can be applied akin to the example of [4, Section 4.2], with the maximum component size roughly v times the largest tree of stars.

To do this rigorously, we first need to introduce additional notation in Section 6.1. A proof overview is given in Section 6.2, with the definition of bad transcripts in Section 6.3, and probability analyses in Section 6.4 and Section 6.5. The proof is concluded in Section 6.6.

6.1 Additional Notation

Patarin's H-Coefficient Technique. Consider any two oracles $\mathcal{O}$ and $\mathcal{P}$, and a deterministic adversary A that has query access to either of these oracles, and write

$$\mathbf{Adv}(A) = \mathbf{Pr}\left(A^{\mathcal{O}} = 1\right) - \mathbf{Pr}\left(A^{\mathcal{P}} = 1\right) . \tag{14}$$

The adversary can make q queries, and its communication with its oracle is recorded in a transcript τ. Denote by $X_{\mathcal{O}}$ the probability distribution of transcripts in interaction with $\mathcal{O}$, and similarly $X_{\mathcal{P}}$ the probability distribution of transcripts in interaction with $\mathcal{P}$. A transcript τ is called attainable if $\mathbf{Pr}\left(X_{\mathcal{P}} = \tau\right) > 0$, and we denote by $\mathcal{T}$ the set of all attainable transcripts.

Patarin's H-coefficient technique [3, 22, 24] states the following:

Theorem 3 (H-coefficient technique). *Let $\delta, \varepsilon \in [0,1]$. Consider a partition $\mathcal{T} = \mathcal{T}_{\text{bad}} \cup \mathcal{T}_{\text{good}}$ of the set of attainable transcripts such that*

– $\mathbf{Pr}\left(X_{\mathcal{P}} \in \mathcal{T}_{\text{bad}}\right) \leq \delta$,
– *for all* $\tau \in \mathcal{T}_{\text{good}}$, $\dfrac{\mathbf{Pr}\left(X_{\mathcal{O}} = \tau\right)}{\mathbf{Pr}\left(X_{\mathcal{P}} = \tau\right)} \geq 1 - \varepsilon$.

Then, the distinguishing advantage of (14) satisfies $\mathbf{Adv}(A) \leq \delta + \varepsilon$.

Mirror Theory. Patarin's mirror theory [19, 20, 23, 25, 26] can be used to prove close to optimal security of constructions that can be described as the sum of permutations, or bijections. We adopt the notation and result of Cogliati et al. [4], be it in their graph representation rather than in their matrix representation.

Let $m, p \in \mathbb{N}$. Consider p *distinct* n-bit unknowns $\{X_1, \ldots, X_p\}$. A system of m difference equations over these unknowns is defined as

$$\begin{cases} X_{a_1} \oplus X_{b_1} = \lambda_1\,, \\ \vdots \\ X_{a_m} \oplus X_{b_m} = \lambda_m\,, \end{cases} \tag{15}$$

where $a_i, b_i \in \{1, \ldots, p\}$ ($a_i \neq b_i$ for all i) and $\lambda_i \in \{0,1\}^n$ for $i = 1, \ldots, m$. We associate a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ to this system of equations, where the unknowns are represented by vertices $\mathcal{V} = \{X_1, \ldots, X_p\}$ and equations by edges $\mathcal{E}$, where $X_a \overset{\lambda}{\longleftrightarrow} X_b$ if $(a, b, \lambda) = (a_i, b_i, \lambda_i)$ for some $i \in \{1, \ldots, m\}$.

The graph is called p.d.-consistent (pairwise distinct consistent) if there is no path whose labels λ_i sum to 0. In addition, the graph is called acyclic if it is cycle-free. Finally, for a graph $\mathcal{G}$, we define the maximum component size, i.e., the size of the largest component, by $\xi_{\max}$ vertices. The mirror theory result of Cogliati et al. [4] states the following:

Theorem 4 (Mirror theory). *Consider a graph* $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ *that is p.d.-consistent, acyclic, and whose largest component is at most of size* $\xi_{\max}$. *As long as* $n\xi_{\max}^2 + \xi_{\max} \leq 2^{n/2}$ *and* $p \leq 2^n/(12\xi_{\max}^2)$, *the number of solutions for* $\mathcal{V}$ *such that the equations of* $\mathcal{E}$ *are satisfied is at least*

$$\frac{(2^n)_p}{2^{nm}}\,.$$

6.2 Proof Overview

Let $K = K_1 \| K_2 \xleftarrow{\$} \{0,1\}^{2\kappa}$. We consider an adversary A that makes q queries to either $\widetilde{XORP}[v]_K^E$ of (9) on top of a pseudorandom permutation $E : \{0,1\}^\kappa \times \{0,1\}^n \to \{0,1\}^n$, or to a random function RF with the same domain and range of $\widetilde{XORP}[v]_K^E$, and it aims to distinguish them:

$$\mathbf{Adv}^{\text{prf}}_{\widetilde{XORP}[v]}(A) = \mathbf{Pr}\left(A^{\widetilde{XORP}[v]_K^E} = 1\right) - \mathbf{Pr}\left(A^{RF} = 1\right)\,. \tag{16}$$

As a first step, we replace the blockcipher evaluations E_{K_1}, E_{K_2} by random permutations $\pi_1, \pi_2 \xleftarrow{\$} \mathrm{perm}(n)$, respectively. These serve as key in the construction, and we abuse notation and denote it by $\widetilde{XORP}[v]_\pi$ for $\boldsymbol{\pi} = (\pi_1, \pi_2)$. We have

$$\mathbf{Adv}^{\mathrm{prf}}_{\widetilde{XORP}[v]}(A) \leq \left(\mathbf{Pr}\left(A^{\widetilde{XORP}[v]_{\boldsymbol{\pi}}} = 1\right) - \mathbf{Pr}\left(A^{RF} = 1\right)\right) + 2\mathbf{Adv}^{\mathrm{prp}}_{E}(A')\,, \tag{17}$$

for some adversary A' with a total query complexity $q' = (v+1)q$.

Transcripts. The adversary A makes q queries to its construction (either $\widetilde{XORP}[v]_{\boldsymbol{\pi}}$ or RF) and these are summarized in a transcript

$$\tau = \{(I_i, B_i, W_i, c_i, Z_{i,1} \| \cdots \| Z_{i,v})\}_{i=1}^{q}\,.$$

Without loss of generality, we assume that $(I_i, B_i, W_i, c_i) \neq (I_{i'}, B_{i'}, W_{i'}, c_{i'})$ whenever $i \neq i'$.

Note that in the real world, there are additional values related to the evaluation of $\widetilde{XORP}[v]_{\boldsymbol{\pi}}$, namely

$$S_{i,j} = \pi_2(B_i \| W_i \| c_i \| j) \tag{18}$$

for $i \in \{1, \ldots, q\}$ and $j \in \{0, 1, \ldots, v\}$. We extend the transcript by adding those values:

$$\tau_{\mathrm{ext}} = \{(I_i, B_i, W_i, c_i, S_{i,0} \| \cdots \| S_{i,v}, Z_{i,1} \| \cdots \| Z_{i,v})\}_{i=1}^{q}\,. \tag{19}$$

In the ideal world, the values $S_{i,j}$ will be *dummy* values sampled uniformly without replacement whenever the value $B_i \| W_i \| c_i \| j$ is different (simply said, in the ideal world we will also use π_2 to draw those values $S_{i,j}$).

Finally, we write for $i \in \{1, \ldots, q\}$ and $j \in \{0, 1, \ldots, w\}$:

$$R_{i,j} = I_i \oplus S_{i,j}\,. \tag{20}$$

These values are implicit in the extended transcript τ_{ext}.

Meaning of Transcripts. In the real world, each transcript tuple $(I_i, B_i, W_i, c_i, S_{i,0} \| \cdots \| S_{i,v}, Z_{i,1} \| \cdots \| Z_{i,v}) \in \tau_{\mathrm{ext}}$ basically consists of two portions.

Firstly, there are the $v+1$ distinct evaluations of π_2 of the form (18):

$$\begin{cases} \pi_2(B_i \| W_i \| c_i \| 0) = S_{i,0}\,, \\ \vdots \\ \pi_2(B_i \| W_i \| c_i \| v) = S_{i,v}\,. \end{cases} \tag{21}$$

If two queries $i, i' \in \{1, \ldots, q\}$ are made for the same tweak $B_i \| W_i \| c_i = B_{i'} \| W_{i'} \| c_{i'}$, these $v+1$ evaluations coincide; otherwise they are all distinct.

Secondly, there is the relation between the values $R_{i,j}$ (implicitly defined by the transcript as (20)) and the values $Z_{i,j}$, which corresponds to v equations over $v+1$ unknowns (note, here, the outputs of the function π_1 are regarded as unknowns):

$$\begin{cases} \pi_1(R_{i,0}) \oplus \pi_1(R_{i,1}) = Z_{i,1}\,, \\ \vdots \\ \pi_1(R_{i,0}) \oplus \pi_1(R_{i,v}) = Z_{i,v}\,. \end{cases} \tag{22}$$

In graph-speak, these form a star with v edges, as $R_{i,j} \neq R_{i,j'}$ whenever $j \neq j'$. As a matter of fact, if we were not considering $\widetilde{XORP}[v]$ but rather $XORP[v]$, the q tuples in τ_{ext} together form a forest of q stars with v edges. In the case of $\widetilde{XORP}[v]$, however, cross-star collisions may occur, turning two or more stars into a tree or even a cycle. See also the explanation in Figure 3. We will thus define a neat ensemble of bad events to avoid cycles and too large trees.

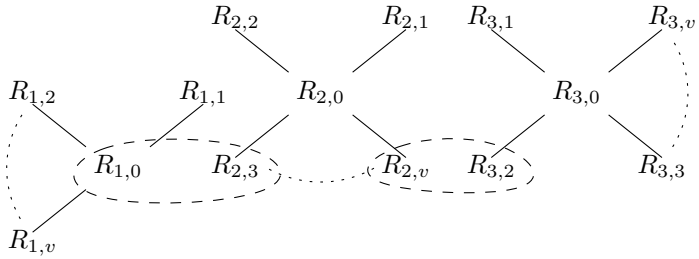

Fig. 3: Graph structure representing the evaluations of $\widetilde{XORP}[v]$. Here, each evaluation of $\widetilde{XORP}[v]$ defines a star (solid edges), but these stars may be connected to each other in case, e.g., $R_{1,0} = R_{2,3}$ and $R_{2,v} = R_{3,2}$ (dashed circle around them, meaning that they represent a single vertex).

6.3 Bad Transcripts

We will define bad events that would make the mirror theory inapplicable. Intuitively, we have to assure that (i) within stars, the system of difference equations is p.d.-consistent and acyclic, and (ii) among stars, the system of difference equations is p.d.-consistent and acyclic too. In addition, (iii) we require that there is no too large tree of stars, the reason being that any tree of μ stars basically results in a component in the graph of size exactly $\mu(v+1) - \mu + 1 = \mu v + 1$ vertices (assuming no cycles, of course).

In detail, for the case of problems within isolated stars, case (i) of above paragraph, the mirror theory is inapplicable if a transcript in τ_{ext} satisfies one of the following events:

$\mathsf{BAD}^{\star}_{\mathsf{pdinc}}$ There exist $i \in \{1,\ldots,q\}$ and $j \in \{1,\ldots,v\}$, such that $Z_{i,j} = 0^n$, or $i \in \{1,\ldots,q\}$ and distinct $j, j' \in \{1,\ldots,v\}$, such that $Z_{i,j} = Z_{i,j'}$;

$\mathsf{BAD}^{\star}_{\mathsf{cycle}}$ There exist $i \in \{1,\ldots,q\}$ and distinct $j, j' \in \{0,\ldots,v\}$, such that $R_{i,j} = R_{i,j'}$.

We note that the index sets for j, j' are *not* a typo: for $Z_{i,j}$, j, j' run from 1 to v, whereas for $R_{i,j}$, j, j' run from 0 to v. Note that any star contains paths of length 1 and length 2 only, and $\mathsf{BAD}^{\star}_{\mathsf{pdinc}}$ covers p.d.-inconsistencies over any of those paths. Event $\mathsf{BAD}^{\star}_{\mathsf{cycle}}$ will be used to excludes cycles, both of length 1 (if j or j' equals 0) and of length 2 (if both j and j' are unequal to 0).

For the case of problems among stars, case (ii) of above paragraph, these two events generalize as follows:

$\mathsf{BAD}^{\star\star}_{\mathsf{pdinc}}$ There exist $\ell \geq 2$, distinct $i_1,\ldots,i_\ell \in \{1,\ldots,q\}$, and distinct $j_\alpha, k_\alpha \in \{0,\ldots,v\}$ for each $\alpha \in \{1,\ldots,\ell\}$, such that

$$\forall_{\alpha=1}^{\ell-1} \,:\, R_{i_\alpha,j_\alpha} = R_{i_{\alpha+1},k_{\alpha+1}}\,,$$

and

$$\sum_{\alpha=1}^{\ell} \left(Z_{i_\alpha,j_\alpha} \oplus Z_{i_\alpha,k_\alpha} \right) = 0\,,$$

where $Z_{i,0} = 0^n$ for all i by definition;

$\mathsf{BAD}^{\star\star}_{\mathsf{cycle}}$ There exist $\ell \geq 2$, distinct $i_1,\ldots,i_\ell \in \{1,\ldots,q\}$, and distinct $j_\alpha, k_\alpha \in \{0,\ldots,v\}$ for each $\alpha \in \{1,\ldots,\ell\}$, such that

$$\forall_{\alpha=1}^{\ell} \,:\, R_{i_\alpha,j_\alpha} = R_{i_{\alpha+1},k_{\alpha+1}}\,,$$

where $(i_{\ell+1}, k_{\ell+1}) = (i_1, k_1)$ by definition.

Event $\mathsf{BAD}^{\star\star}_{\mathsf{pdinc}}$ considers the case that there is a path of ℓ distinct stars and considers all vertex paths that are included within this path of stars. Note that for any such path, for any individual inner star (so $\alpha = 2,\ldots,\ell-1$) the vertex path cannot traverse freely but has to traverse from R_{i_α,j_α} to R_{i_α,k_α}, adding exactly $Z_{i_\alpha,j_\alpha} \oplus Z_{i_\alpha,k_\alpha}$ to the checksum, noting that $Z_{i,0} = 0^n$ by definition. For the outer stars, so $\alpha = 1, \ell$, it may or may not traverse further to any R_{i_1,k_1} or R_{i_α,j_α} respectively, again adding exactly $Z_{i_\alpha,j_\alpha} \oplus Z_{i_\alpha,k_\alpha}$ to the checksum. Likewise, event $\mathsf{BAD}^{\star\star}_{\mathsf{cycle}}$ considers the case that there is a cycle over ℓ distinct stars. Note that for both events, the condition that $j_\alpha \neq k_\alpha$ is reasonable to make: in case of equality, there would have been a shorter path or cycle without equality at the αth indices; in case of equality for all indices, both bad events would become meaningless.

Finally, there is the case of a too large tree of stars, case (iii) of above paragraph. Basically, we have to define any threshold $\mu \in \mathbb{N}$ and state the event that there is a tree that connects $\mu+1$ stars. This is quite cumbersome to define. On the other hand, looking ahead, we will only be able to bound the probability of this event to occur for $\mu = 2$. In this case, the event is more straightforward to define (as a tree of 3 stars is necessarily a path of 3 stars):

$\mathsf{BAD}^{\star\star}_{\mathsf{tree}}$ There exist distinct $i_1, i_2, i_3 \in \{1, \ldots, q\}$ and $j_1, j_2, k_2, k_3 \in \{0, \ldots, v\}$ (with no further distinctness condition), such that

$$R_{i_1,j_1} = R_{i_2,k_2}\,,$$
$$R_{i_2,j_2} = R_{i_3,k_3}\,.$$

Bad event $\mathsf{BAD}^{\star\star}_{\mathsf{tree}}$ differs from $\mathsf{BAD}^{\star\star}_{\mathsf{pdinc}}$ and $\mathsf{BAD}^{\star\star}_{\mathsf{cycle}}$ in that there is no distinctness condition on the values j_α, k_α. After all, $\mathsf{BAD}^{\star\star}_{\mathsf{tree}}$ is meant to capture, basically to upper bound, the size of the largest component in the graph. To derive this bound, all that matters is to figure out the maximum number of stars that are connected, and it is irrelevant *how* they are connected.

We write

$$\mathsf{BAD} = \mathsf{BAD}^{\star}_{\mathsf{pdinc}} \vee \mathsf{BAD}^{\star}_{\mathsf{cycle}} \vee \mathsf{BAD}^{\star\star}_{\mathsf{pdinc}} \vee \mathsf{BAD}^{\star\star}_{\mathsf{cycle}} \vee \mathsf{BAD}^{\star\star}_{\mathsf{tree}}\,. \tag{23}$$

6.4 Probability of Bad Transcripts

Following Theorem 3, we have to upper bound the probability that a bad transcript occurs in the ideal world, i.e., for RF. By basic probability theory,

$$\begin{aligned}\mathbf{Pr}\left(X_{RF} \in \mathcal{T}_{\mathrm{bad}}\right) \le\ & \mathbf{Pr}\left(\mathsf{BAD}^{\star}_{\mathsf{pdinc}}\right) + \mathbf{Pr}\left(\mathsf{BAD}^{\star}_{\mathsf{cycle}}\right) + \mathbf{Pr}\left(\mathsf{BAD}^{\star\star}_{\mathsf{tree}}\right) \\ & + \mathbf{Pr}\left(\mathsf{BAD}^{\star\star}_{\mathsf{pdinc}} \mid \neg\mathsf{BAD}^{\star\star}_{\mathsf{tree}}\right) + \mathbf{Pr}\left(\mathsf{BAD}^{\star\star}_{\mathsf{cycle}} \mid \neg\mathsf{BAD}^{\star\star}_{\mathsf{tree}}\right)\,.\end{aligned} \tag{24}$$

We investigate the probabilities separately.

$\mathbf{Pr}\left(\mathsf{BAD}^{\star}_{\mathsf{pdinc}}\right)$. The event is set whenever $Z_{i,j} = 0$ for some i, j (vq choices) or whenever $Z_{i,j} = Z_{i,j'}$ for some i, j, j' with $j \neq j'$ ($\binom{v}{2}q$ choices). As the values $Z_{i,j}$ are uniformly randomly generated, this bad event happens with probability at most

$$\frac{(v + \binom{v}{2})q}{2^n} = \frac{\binom{v+1}{2}q}{2^n}\,.$$

(As a matter of fact, the derivation and bound are identical to that of [4, Section 4.2] with the difference that they bound $\binom{v}{2}$ to $v^2/2$.)

$\mathbf{Pr}\left(\mathsf{BAD}^{\star}_{\mathsf{cycle}}\right)$. The event is set whenever $R_{i,j} = R_{i,j'}$ for some i, j, j' with $j \neq j'$. However, from (20), we see that this happens whenever

$$I_i \oplus S_{i,j} = I_i \oplus S_{i,j'}\,,$$

i.e., whenever $S_{i,j} = S_{i,j'}$. As in the ideal world, the dummy values $S_{i,j}$ and $S_{i,j'}$ are drawn randomly without replacement, the event happens with probability 0.

$\mathbf{Pr}\left(\mathsf{BAD}^{\star\star}_{\mathsf{tree}}\right)$. Recall that we will perform the analysis for $\mu = 2$. Consider any of the $\binom{q}{3}$ choices for i_1, i_2, i_3 and any of the $(v+1)^4$ choices for j_1, j_2, k_2, k_3. The event is set if

$$S_{i_1,j_1} \oplus S_{i_2,k_2} = I_{i_1} \oplus I_{i_2}\,,$$
$$S_{i_2,j_2} \oplus S_{i_3,k_3} = I_{i_2} \oplus I_{i_3}\,.$$

As the three queries are distinct, and the adversary never repeats queries, we necessarily have $(I_{i_1}, B_{i_1}, W_{i_1}, c_{i_1}) \neq (I_{i_2}, B_{i_2}, W_{i_2}, c_{i_2})$, which implies that the first equation can only be satisfied if $B_{i_1}\|W_{i_1}\|c_{i_1}\|j_1 \neq B_{i_2}\|W_{i_2}\|c_{i_2}\|k_2$. This means that, necessarily, $S_{i_1,j_1} \neq S_{i_2,k_2}$ and the two sources of randomness in the first equation do not cancel each other out. Likewise, the second equation can only be satisfied if $B_{i_2}\|W_{i_2}\|c_{i_2}\|j_2 \neq B_{i_3}\|W_{i_3}\|c_{i_3}\|k_3$, and the two sources of randomness S_{i_2,j_2} and S_{i_3,k_3} do not cancel each other out.

Finally, we have to argue that both equations are sufficiently independent, i.e., that neither

- $S_{i_1,j_1} = S_{i_2,j_2}$ and $S_{i_2,k_2} = S_{i_3,k_3}$, nor
- $S_{i_1,j_1} = S_{i_3,k_3}$ and $S_{i_2,k_2} = S_{i_2,j_2}$.

However, to the contrary, suppose one of these two conditions hold. The condition particularly implies that $(B_{i_1}, W_{i_1}, c_{i_1}) = (B_{i_3}, W_{i_3}, c_{i_3})$. The condition also implies, by addition of the two equations of the events, that $I_{i_1} = I_{i_3}$. This contradicts with the condition that the queries are distinct.

Thus, there are at least three sources of randomness in the two equations (note that, if $j_2 = k_2$, $S_{i_2,k_2} = S_{i_2,j_2}$). The values $S_{i,j}$ are drawn uniformly randomly from a set of size at least $2^n - (v+1)q$ elements, and thus, the two equations are satisfied with probability at most $\left(\frac{1}{2^n-(v+1)q}\right)^2$.

In conclusion, the bad event is set with probability at most

$$\binom{q}{3}(v+1)^4\left(\frac{1}{2^n-(v+1)q}\right)^2 \leq \frac{(v+1)^4 q^3}{5\cdot 2^{2n}}\,,$$

using that $(v+1)q \leq 2^{n-6}$ for the inequality. (We remark that this condition is more stringent than the "usual" $\leq 2^{n-1}$, but this more stringent condition is in fact implied by a condition that we will need for the application of the mirror theory anyway.)

$\mathbf{Pr}\left(\mathsf{BAD}^{\star\star}_{\mathsf{pdinc}} \mid \neg\mathsf{BAD}^{\star\star}_{\mathsf{tree}}\right)$. We have to consider any $\ell \geq 2$, but w.l.o.g., $\ell \leq \mu = 2$ by negation of $\mathsf{BAD}^{\star\star}_{\mathsf{tree}}$. Consider any of the $\binom{q}{2}$ choices for i_1, i_2 and any of the $\binom{v+1}{2}^2$ choices for j_α, k_α for $\alpha = 1, 2$. The event is set if

$$S_{i_1,j_1} \oplus S_{i_2,k_2} = I_{i_1} \oplus I_{i_2}\,,$$
$$Z_{i_1,j_1} \oplus Z_{i_1,k_1} \oplus Z_{i_2,j_2} \oplus Z_{i_2,k_2} = 0\,,$$

where we recall that $Z_{i,0} = 0^n$ for all i by definition.

As in the case of $\mathsf{BAD}^{\star\star}_{\mathsf{tree}}$ above, the two sources of randomness S_{i_1,j_1} and S_{i_2,k_2} in the first equation do not cancel each other out, as the adversary never repeats queries. Thus, this equation is satisfied with probability at most $\frac{1}{2^n-(v+1)q}$. For the second equation, which is independent of the first one, note that at least one of the values j_1, k_1, j_2, k_2 is non-zero, meaning that for this index, we can rely on the random drawing of the Z-value. The equation is set with probability at most $1/2^n$.

In conclusion, the bad event is set with probability at most

$$\binom{q}{2}\binom{v+1}{2}^2 \frac{1}{2^n-(v+1)q}\frac{1}{2^n} \le \frac{\binom{v+1}{2}^2 q^2}{2^{2n}}\,,$$

using that $(v+1)q \le 2^{n-1}$ for the inequality.

$\mathbf{Pr}\left(\mathsf{BAD}^{\star\star}_{\mathsf{cycle}} \mid \neg\mathsf{BAD}^{\star\star}_{\mathsf{tree}}\right)$. We have to consider any $\ell \ge 2$, but w.l.o.g., $\ell \le \mu = 2$ by negation of $\mathsf{BAD}^{\star\star}_{\mathsf{tree}}$. Consider any of the $\binom{q}{2}$ choices for i_1, i_2 and any of the $\binom{v+1}{2}^2$ choices for j_α, k_α for $\alpha = 1, 2$. The event is set if

$$S_{i_1,j_1} \oplus S_{i_2,k_2} = I_{i_1} \oplus I_{i_2}\,,$$
$$S_{i_2,j_2} \oplus S_{i_1,k_1} = I_{i_1} \oplus I_{i_2}\,.$$

As in the case of $\mathsf{BAD}^{\star\star}_{\mathsf{tree}}$ above, the two sources of randomness S_{i_1,j_1} and S_{i_2,k_2} in the first equation do not cancel each other out, as the adversary never repeats queries. The same holds for S_{i_2,j_2} and S_{i_1,k_1} in the second equation.

Finally, we have to argue that both equations are sufficiently independent, i.e., that neither

- $S_{i_1,j_1} = S_{i_2,j_2}$ and $S_{i_2,k_2} = S_{i_1,k_1}$, nor
- $S_{i_1,j_1} = S_{i_1,k_1}$ and $S_{i_2,k_2} = S_{i_2,j_2}$.

The second case does not hold as $j_1 \neq k_1$ and $j_2 \neq k_2$. The first case would, again contradict with the fact that the two queries are distinct.

Thus, there are four sources of randomness in the two equations. The values $S_{i,j}$ are drawn uniformly randomly from a set of size at least $2^n - (v+1)q$ elements, and thus, the two equations are satisfied with probability at most $\left(\frac{1}{2^n-(v+1)q}\right)^2$.

In conclusion, the bad event is set with probability at most

$$\binom{q}{2}\binom{v+1}{2}^2 \left(\frac{1}{2^n-(v+1)q}\right)^2 \le \frac{\binom{v+1}{2}^2 q^2}{2^{2n}}\,,$$

using that $(v+1)q \le 2^{n-2}$ for the inequality.

Conclusion. We obtain from (24) and the individual bounds that

$$\mathbf{Pr}\left(X_{RF} \in \mathcal{T}_{\mathrm{bad}}\right) \le \frac{(v+1)^4 q^3}{5\cdot 2^{2n}} + \frac{\binom{v+1}{2} q}{2^n} + \frac{2\binom{v+1}{2}^2 q^2}{2^{2n}}\,, \tag{25}$$

provided $(v+1)q \le 2^{n-6}$. We set δ equal to this value.

6.5 Probability Ratio for Good Transcripts

Consider any good transcript τ_{ext}. Following Theorem 3, we have to compute a lower bound on the fraction $\frac{\mathbf{Pr}\left(X_{\widetilde{XORP}[v]_{\boldsymbol{\pi}}}=\tau_{\text{ext}}\right)}{\mathbf{Pr}(X_{RF}=\tau_{\text{ext}})}$. We will actually derive a lower bound on the probability in the numerator and the actual value for the probability in the denominator, and then combine them.

For the derivation of each of the two probabilities, below, consider any good transcript $\tau_{\text{ext}} = \{(I_i, B_i, W_i, c_i, S_{i,0}\| \cdots \|S_{i,v}, Z_{i,1}\| \cdots \|Z_{i,v})\}_{i=1}^{q}$. For any $B\|W\|c \in \{0,1\}^{n-\lceil \log_2(v+1)\rceil}$, let q_{BWc} denote the number of tuples in τ_{ext} such that $B_i\|W_i\|c_i = B\|W\|c$. Let q' denote the number of strings $B\|W\|c$ for which $q_{BWc} > 0$ (i.e., q' denotes the number of different domain separator and tweak combinations).

$\mathbf{Pr}\left(X_{\widetilde{XORP}[v]_{\boldsymbol{\pi}}} = \tau_{\text{ext}}\right)$. For the computation of this probability, we have to compute the probability that $\boldsymbol{\pi} = (\pi_1, \pi_2) \xleftarrow{\$} \text{perm}(n)^2$ could have resulted in the transcript. The transcript consists of two portions, namely

$$\tau^2_{\text{ext}} = \{(B_i, W_i, c_i, S_{i,0}\| \cdots \|S_{i,v})\}_{i=1}^{q}$$

corresponding to the evaluation of π_2, and

$$\tau^1_{\text{ext}} = \{(R_{i,0}\| \cdots \|R_{i,v}, Z_{i,1}\| \cdots \|Z_{i,v})\}_{i=1}^{q}$$

corresponding to the evaluation of π_1, where we recall that $R_{i,j}$ of (20) is implicit in the transcript. As for τ^2_{ext}, this sub-transcript defines exactly vq' input-output tuples for π_2, namely (21) for all q' different domain separator and tweak combinations that occur in the transcript. There are exactly $(2^n - vq')!$ permutations π_2 that could have yielded this sub-transcript. As for τ^1_{ext}, as the transcript is good, this sub-transcript defines a graph on $m := vq$ equations and $p \leq (v+1)q$ unknowns (we do not need an exact value of p) that is p.d.-consistent, acyclic, and whose largest component is of size $\xi_{\max} := \mu v + 1 = 2v + 1$. We can thus apply Theorem 4 and obtain that, provided $n\xi^2_{\max} + \xi_{\max} \leq 2^{n/2}$ and $p \leq 2^n/(12\xi^2_{\max})$, there are at least

$$\frac{(2^n)_p}{2^{nvq}}$$

solutions to the p unknowns. For any of these solutions, we have exactly $(2^n - p)!$ permutations π_1 that could have yielded any of these solutions.

We obtain that

$$\mathbf{Pr}\left(X_{\widetilde{XORP}[v]_{\boldsymbol{\pi}}} = \tau_{\text{ext}}\right) \geq \frac{\frac{(2^n)_p}{2^{nvq}}(2^n - vq')!(2^n - p)!}{2^n!2^n!} = \frac{1}{(2^n)_{vq'}(2^n)^{vq}}\,.$$

$\mathbf{Pr}\left(X_{RF} = \tau_{\text{ext}}\right)$. For the computation of this probability, we can likewise split the transcript into the two portions τ^2_{ext} and τ^1_{ext}, with the difference that, now,

τ^2_{ext} is generated by randomly selecting dummy variables $S_{i,j}$ and τ^1_{ext} is generated through RF. The probability that the random world yields τ^2_{ext} equals $(2^n)_{vq'}$ by definition of how the dummy values $S_{i,j}$ are generated, and the probability that RF yields τ^1_{ext} equals $1/(2^n)^{vq}$.

We obtain that

$$\mathbf{Pr}\left(X_{RF} = \tau_{\text{ext}}\right) = \frac{1}{(2^n)_{vq'}(2^n)^{vq}}\,.$$

Conclusion. We obtain from the individual bounds that

$$\frac{\mathbf{Pr}\left(X_{\widetilde{XORP}[v]_{\boldsymbol{\pi}}} = \tau_{\text{ext}}\right)}{\mathbf{Pr}\left(X_{RF} = \tau_{\text{ext}}\right)} \geq \frac{\frac{1}{(2^n)_{vq'}(2^n)^{vq}}}{\frac{1}{(2^n)_{vq'}(2^n)^{vq}}} = 1\,. \tag{26}$$

We set $\varepsilon = 0$.

6.6 Conclusion

From the H-coefficient technique of Theorem 3, the initial steps (16) and (17) of the proof, and from the values δ obtained in (25) and ε obtained in (26), we obtain

$$\mathbf{Adv}^{\text{prf}}_{\widetilde{XORP}[v]}(A) \leq \frac{(v+1)^4 q^3}{5 \cdot 2^{2n}} + \frac{\binom{v+1}{2} q}{2^n} + \frac{2\binom{v+1}{2}^2 q^2}{2^{2n}} + 2\mathbf{Adv}^{\text{prp}}_E(A')\,,$$

assuming that $(v+1)q \leq 2^{n-6}$, and $n\xi^2_{\max} + \xi_{\max} \leq 2^{n/2}$ and $(v+1)q \leq 2^n/(12\xi^2_{\max})$ for $\xi_{\max} := 2v+1$. These three conditions simplify to $n(2v+1)^2 + (2v+1) \leq 2^{n/2}$ and $(2v+1)^2(v+1)q \leq 2^n/12$.

7 Conclusion

In this paper, we explored instances of the docked double decker construction that can make efficient use of already existing hardware that speeds up the execution of AES and GHash. We did this, so that the resulting tweakable wide blockcipher is essentially just a mode of operation for the AES blockcipher. In the process of designing the beyond birthday bound secure tweakable wide blockcipher *bbb-ddd-AES*, we also designed an efficient beyond birthday bound secure blockcipher based PRF called $\widetilde{XORP}$, which is able to process up to $2n$-bit inputs. We proved that this construction achieves around $2n/3$-bit security. Since we do not have an attack matching the bound of $\widetilde{XORP}$, it remains future work to see if such an attack can be found, or if the bound can be improved. Furthermore, we think that it is also possible to show that $\widetilde{XORP}$ is secure when the blockcipher call generating the masks is replaced with a finite field multiplication of input and a key, or more generally a universal hash function.

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