

SQIASignHD: SQIsignHD Adaptor Signature

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Abstract

Adaptor signatures can be viewed as a generalized form of the standard digital signature schemes where a secret randomness is hidden within a signature. Adaptor signatures are a recent cryptographic primitive and are becoming an important tool for blockchain applications such as cryptocurrencies to reduce on-chain costs, improve fungibility, and contribute to off-chain forms of payment in payment-channel networks, payment-channel hubs, and atomic swaps. However, currently used adaptor signature constructions are vulnerable to quantum adversaries due to Shor's algorithm. In this work, we introduce **SQIASignHD**, a new quantum-resistant adaptor signature scheme based on isogenies of supersingular elliptic curves, using SQIsignHD - as the underlying signature scheme - and exploiting the idea of the artificial orientation on the supersingular isogeny Diffie-Hellman key exchange protocol, SIDH, as the underlying hard relation. We, furthermore, show that our scheme is secure in the Quantum Random Oracle Model (QROM).

Keywords: Post-quantum Cryptography, Blockchain, Isogeny-based Cryptography, Adaptor Signature, Payment Channel Network

1 Introduction

Blockchain technology, which has been sparking widespread attention since it was introduced anonymously in 2009 due to [1], proposes a novel payment paradigm in which financial transactions are maintained in a decentralized data structure. Each transaction on the blockchain can be treated as a scripting language that is validated by nodes through a decentralized consensus protocol. Cryptocurrencies such as Bitcoin and Ethereum are powered by blockchain technologies. Transactions on blockchains, however, can be quite costly since a user who wants to deploy and execute a transaction must pay a fee to the miners. The fee is calculated based on the storage and computational costs associated with running each transaction script. To address this issue, one way to reduce transaction size is to manage some transactions off-chain to lower the on-chain fee paid to nodes. In this context, Andrew Polestra proposed first the concept

of scriptless scripts [2], which was later formalized as adaptor signatures due to [3] and [4].

1.1 Adaptor Signature

Adaptor signature is a recent cryptographic primitive that can be viewed as a generalization of a standard digital signature and becoming an important tool for blockchain applications such as cryptocurrencies to reduce on-chain costs, improve fungibility, and contribute to off-chain forms of payment in payment-channel networks (PCNs), payment-channel hubs (PCHs), and atomic swaps [5]. Specifically, it conceals secret randomness within the signing process, which is exposed once the signature is created. The general procedure is to generate a pre-signature in the first phase; the pre-signature is then transformed into a signature using secret randomness; finally, secret randomness is derived from the signature via cryptographic processing.

Furthermore, the signature produced by an adaptor signature can be verified by using the underlying signature scheme’s verification algorithm. An adaptor signature also has specific features that ensure its security. A signer with a secret key can create a pre-signature on every message. This pre-signature can be converted into a full signature on a message if and only if the user has a witness to the statement. In addition, anyone who has access to the pre-signature and the corresponding full signature can extract the witness and reveal the hard relation.

1.2 Related Work and Our Contribution

As concrete instances, Aumayr et al. [3] give a formalization of adaptor signatures and their security, and apply it to ECDSA and Schnorr-based schemes. Malavolta et al. [5] analyze and design secure and privacy-preserving PCNs, identifying a new attack that affects major PCNs like the Lightning Network. They define Anonymous Multi-hop Locks (AMHLs), a cryptographic primitive, and show they can be constructed for PCNs with script languages using linear homomorphic one-way functions. Moreno-Sanchez et al. [6] demonstrate an instance of adaptor signature on the Monero’s linkable ring signature scheme to improve scalability and some other issues. Tairi et al. [7] introduce the PCHs protocol and a provably secure instantiation based on adaptor signatures. However, these constructions are vulnerable to quantum adversaries due to Shor’s algorithm [8]. More exactly, the security of blockchain technologies primarily is based on digital signature schemes, built on elliptic curve cryptography (ECC), to authenticate payment transactions. The security of ECC, in turn, depends on the intractability of computing the discrete logarithm problem which is secure against classical computers. Yet, the advent of Shor’s algorithm enabled quantum computers to efficiently compute discrete logarithms in polynomial running time. This situation makes the blockchain susceptible to quantum attacks. Such attacks involve forging signatures and modifying blocks (faking previous transactions). In the case of Bitcoin, for example, this means a person may spend more than they have or steal assets from other users. As a consequence of the drawbacks of public key cryptosystems, post-quantum cryptography began to attract more attention and became an active research area. Since public key cryptography depends on underlying hard mathematical problems, in order to make a cryptosystem

secure against quantum adversaries, the underlying problem must be intractable in the quantum setting.

In the context of post-quantum cryptography, the first established post-quantum adaptor signature is LAS [9], which is built on standard lattice assumptions such as Module-LWE and Module-SIS and has a simplified form of Dilithium [10] as its underlying signature. Applications that employ LAS in their structures require zero-knowledge proof to ensure that the extracted witness is of the desired norm and satisfies the hard relation. However, the most efficient variant of such a proof is 53KB [11] in size, resulting in significant off-chain communication costs. From a privacy standpoint, when LAS is utilized inside particular applications, such as establishing PCNs, it can leak non-trivial information, compromising the overall privacy of the architecture. It should also be noted that there was another attempt to design an adaptor signature, named SQI-AS, introduced in [12], using SQISign [13] as the underlying signature. The authors exploit the idea of SIDH [14] to apply the corresponding hard relation in their design. However, due to the devastating attacks [15, 16, 17] on SIDH, the SQI-AS lost its security. It is because of the fact that SQI-AS’s adapting algorithm benefits from SIDH-like operation, so in the pre-signature phase of the protocol, it is required to publish the image of the torsion points as auxiliary information to realize the adaptation phase while this SIDH-based additional information is one of the main ingredients in breaking the SIDH security, and consequently exposing the witness used in their construction.

The only secure isogeny-based adaptor signature scheme in the literature is IAS [18] using CSI-FiSh [19] as the underlying signature scheme which relies on the security of the key exchange protocol CSIDH [20]. IAS’s efficiency has some restrictions as its parameter sizes are based on CSI-FiSh. Specifically, CSI-FiSh operates on a maximum of CSIDH-512 parameters since knowledge of the class group structure is required to efficiently compute the class group action on uniformly random group elements. It is also noted that the CSIDH-512 is relatively slow and vulnerable to a quantum subexponential attack, and the concrete size of parameters required to provide a tangible security level has been a source of concern. Specifically, Bonnetian et al. [21] introduced a quantum algorithm for evaluating CSIDH-512 that uses fewer than 40,000 logical qubits. Peikert [22] utilized Kuperberg’s collimation sieve to attack CSIDH, combining classical memory and quantum random access techniques. These attempts demonstrate that the parameters given by the authors of CSIDH do not achieve the requisite quantum security and are subject to controversial debate. It is needed to mention that there exists a new isogeny-based group action of the class group of an imaginary quadratic order on a set of oriented supersingular curves, named SCALLOP proposed by De Feo et al. [23] that aims to solve the scaling problem with CSI-FiSh. Compared to CSIDH, the key advantage of SCALLOP is that it is simple to compute the class-group structure required to uniquely represent and efficiently act on random group elements, as needed in the CSI-FiSh signature scheme. However, SCALLOP requires more computations to execute the group action, making it slower than CSI-FiSh.

Contribution. Considering the aforementioned situations, this work, as a contribution, aims to construct and introduce a new post-quantum adaptor signature using SQIsignHD [24] as the underlying signature scheme. SQIsignHD is the most compact post-quantum signature and compared to the other isogeny-based signature schemes, it is generally

faster, flexible in its parameter sets, and also less storage space is needed when using it. In contrast to IAS, which requires more than 18,000 bytes, our approximated pre-signature takes ~ 200 bytes. Furthermore, our signature is 109 bytes in size, whereas the IAS signature may vary between 263 and 1880 bytes, depending on the security parameters employed. Thus, compared to IAS, SQIAsignHD significantly improves pre-signature and signature size. The main technical difficulties in constructing isogeny-based adaptor signatures emerge from the fact that not all post-quantum signatures, in particular SQIAsignHD, satisfy certain homomorphic properties. Due to [25], it was shown that signature schemes derived from identification (ID) schemes that also satisfy certain homomorphic features can be generically transformed into adaptor signature schemes. To tackle this problem, we apply the concept of "shifting signature by secret randomness" carefully by using several techniques in order to have SQIAsignHD adopt this feature.

1.3 Organization of the Paper

In Section 2, we give the necessary preliminaries required for Sections 3, and 4 which are the main sections. These preliminaries consist of two main parts. The former half is on the mathematical prerequisites needed for our construction, and the latter is on the cryptographic background as needed ingredients in the next two sections. In Section 3, as the main part of the paper, we introduce a new adaptor signature SQIAsignHD and examine it in detail. In Section 4, we analyze the security aspect of the SQIAsignHD and give formal proof to show its security in the quantum random oracle model.

2 Preliminaries

Notation. A *negligible* function $\text{negl} : \mathbb{N} \rightarrow \mathbb{R}$ is a function that, for every $k \in \mathbb{N}$, admits $\mathcal{O}(n^{-k})$ as its upper bound, i.e., there exists $n_0 \in \mathbb{N}$, such that for every $n \geq n_0$ it holds that $\text{negl}(n) \leq 1/n^k$. We denote the uniform sampling of the variable x from the set X by $x \xleftarrow{\$} X$. Moreover, we denote a probabilistic polynomial time (PPT) algorithm A on input y , outputs x by $x \leftarrow A(y)$. In case the algorithm A is a deterministic polynomial time (DPT), it is denoted by $x := A(y)$.

2.1 Elliptic Curves and Isogenies

Elliptic Curves. Let $k := \mathbb{F}_q$ be a finite field such that $q = p^n$, for some prime p and positive integer n , with $\text{char}(k) = p \neq 2, 3$. An *elliptic curve* E is a smooth projective variety of genus 1, defined over k , with distinguished rational point $\infty := [0 : 1 : 0]$. Since E is an elliptic curve, then its discriminant $\Delta(E)$ is nonzero, and its j -invariant $j(E)$ is uniquely defined up to $\overline{\mathbb{F}}_q$ -isomorphism. Let l be a positive integer, the l -*torsion subgroup* of an elliptic curve E is defined as $E[l] := \{P \in E(\overline{k}) \mid [l]P = \infty\}$. We say that an elliptic curve E is *supersingular* if there is no nontrivial p -torsion point over $\overline{\mathbb{F}}_p$, i.e., $E[p] = \{\infty\}$. In case of supersingularity of E , $\text{char}(k) = p$ divides $E(\mathbb{F}_q) - q - 1$.

Isogenies. An *isogeny* $\varphi : E_1 \rightarrow E_2$ is a surjective morphism that maps the point at infinity of E_1 to the point at infinity of E_2 . Two elliptic curves E_1, E_2 are *isogenous* over

\mathbb{F}_q in case there exists an isogeny between them over \mathbb{F}_q . Furthermore, Tate’s theorem [26] says that E_1 and E_2 are isogenous over \mathbb{F}_q if and only if $|E_1(\mathbb{F}_q)| = |E_2(\mathbb{F}_q)|$. The *degree* of isogeny φ is the degree of the field extension $[k(E_1) : \varphi^*(k(E_2))]$ where $k(E_i)$ is the function field of E_i , $i = 1, 2$, and φ^* is the pullback of φ defined as $\varphi^* : k(E_2) \rightarrow k(E_1)$ where for $f \in k(E_2)$, $\varphi^*(f) := f \circ \varphi$. The isogeny φ is called *separable* in case the field extension is separable. If $\gcd(\deg(\varphi), \text{char}(k)) = 1$, then the isogeny is necessarily separable. Since $\varphi(\infty_{E_1}) = \infty_{E_2}$, then $\varphi : E_1(k) \rightarrow E_2(k)$ is a group homomorphism. $|\ker(\varphi)| = \deg(\varphi)$ in case φ is separable. Therefore, in our context, an isogeny can be characterized by its kernel. In other words, there is a one-to-one correspondence between separable isogenies (up to an isomorphism of the target curve) and finite (normal) subgroups of $E_1(k)$. We can construct an isogeny from its kernel by using Vélu’s formulas [27]. The constructed isogeny is in the form $E \rightarrow E/G$ where G is a finite subgroup of E , and the kernel of the constructed isogeny. Since the degree of isogeny is multiplicative, i.e., for isogenies α and β , $\deg(\alpha \circ \beta) = \deg(\alpha)\deg(\beta)$, then for any isogeny ϕ of degree $l = \prod_{i=1}^n l_i$, ϕ can be factored as the composition of l_i -isogenies, $1 \leq i \leq n$ where integers l_i are not necessarily coprime. In case the l_i are pairwise coprime, then reordering of the l_i will produce a different set of isogenies because of the non-commutativity structure of isogenies of supersingular elliptic curves under composition. For $n = 2$, with some considerations, SQIsignHD benefits from this property in commutative type and introduces specific notations for the isogenies involved in the two possible decompositions, and calls them isogeny diagram as shown in Figure 1. More precisely, suppose that l_1, l_2 are two coprime integers and φ is a $l_1 l_2$ -isogeny. Then, φ can be decomposed in two ways, namely $\varphi = \psi_2 \circ \varphi_1 = \psi_1 \circ \varphi_2$. In this case, ψ_1 (respectively, ψ_2) is called the *push-forward* of φ_1 (respectively φ_2) through φ_2 (respectively, φ_1), denoted by $\psi_1 = [\varphi_2]_* \varphi_1$ (respectively, $\psi_2 = [\varphi_1]_* \varphi_2$). It can be shown that $\ker(\psi_1) = \varphi_2(\ker(\varphi_1))$, and $\ker(\psi_2) = \varphi_1(\ker(\varphi_2))$. Furthermore, φ_1 (respectively, φ_2) is called the *pull-back* of ψ_1 (respectively ψ_2) through φ_2 (respectively, φ_1), denoted by $\varphi_1 = [\varphi_2]^* \psi_1$ (respectively, $\varphi_2 = [\varphi_1]^* \psi_2$).

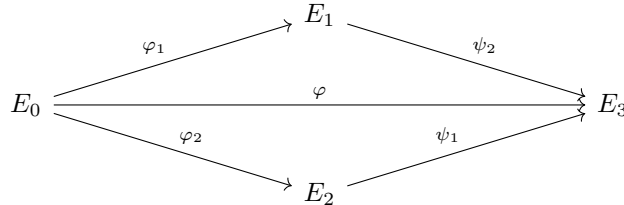


Figure 1: Commutative Isogeny Diagram.

For a given isogeny $\alpha : E_1 \rightarrow E_2$ of degree d , its (unique) *dual* is an isogeny $\hat{\alpha} : E_2 \rightarrow E_1$ of degree d such that $\alpha \circ \hat{\alpha} = [d] : E_2 \rightarrow E_2$, and $\hat{\alpha} \circ \alpha = [d] : E_1 \rightarrow E_1$. An isogeny from an elliptic curve E to itself is called an *endomorphism*. For each $m \in \mathbb{Z}$, the *multiplication-by-m* map, i.e., $[m] : P \mapsto m \cdot P$, and the *Frobenius* map $\pi : (x, y) \mapsto (x^q, y^q)$ of an elliptic curve defined over E/\mathbb{F}_q are examples of endomorphisms. The set of all endomorphisms on E , denoted by $\text{End}(E)$, forms a ring under addition and composition which is called the *endomorphism ring* of E . Every supersingular elliptic curve in characteristic p is isomorphic to an elliptic curve defined over \mathbb{F}_{p^2} . It means

that each supersingular elliptic curve has an isomorphic representative defined over \mathbb{F}_{p^2} . The supersingular ℓ -isogeny graph is the graph whose vertices are the supersingular j -invariants in \mathbb{F}_{p^2} , and whose edges are the ℓ -isogenies between them. These graphs are connected [28], essentially undirected (since each ℓ -isogeny has a dual), $(\ell + 1)$ -regular (there are exactly $\ell + 1$ outgoing edges from each j -invariant), and Ramanujan [29].

2.2 Endomorphism Rings and Quaternion Orders

Quaternion Algebras. Let $a, b \in \mathbb{Q}^*$. A *quaternion algebra* \mathcal{B} over \mathbb{Q} is a four dimensional central simple \mathbb{Q} -algebra denoted by $\mathcal{B} := \left(\frac{a,b}{\mathbb{Q}}\right) = \mathbb{Q} + \mathbb{Q}i + \mathbb{Q}j + \mathbb{Q}k$ with basis $1, i, j, k$ such that $i^2 = a$, $j^2 = b$ and $k = ij = -ji$. Let l be a prime. The quaternion algebra $\mathcal{B}_l := \mathcal{B} \otimes_{\mathbb{Q}} \mathbb{Q}_l$ is obtained by extending the scalars of \mathcal{B} from \mathbb{Q} to \mathbb{Q}_l , where \mathbb{Q}_l is the set of l -adic numbers (fraction field of l -adic integers \mathbb{Z}_l which is the localization of \mathbb{Z} away from prime l). Also, we can define $\mathcal{B}_{\infty} := \mathcal{B} \otimes_{\mathbb{Q}} \mathbb{R}$. We say that \mathcal{B} is ramified at l (including $l = \infty$) if \mathcal{B}_l is a division algebra. We are only interested in $\mathcal{B}_{p,\infty}$ which is a quaternion algebra ramified at p and ∞ . A *fractional ideal* I is a \mathbb{Z} -lattice of rank four which can be written as $I = \mathbb{Z}\alpha_1 + \mathbb{Z}\alpha_2 + \mathbb{Z}\alpha_3 + \mathbb{Z}\alpha_4$ for a \mathbb{Q} basis $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ of \mathcal{B} .

Quaternionic Orders. An *order* is a fractional ideal that is also a subring of \mathcal{B} . An order \mathcal{O} is *maximal* in case for any other order \mathcal{O}' if $\mathcal{O} \subseteq \mathcal{O}'$, then $\mathcal{O} = \mathcal{O}'$. Let E be an elliptic curve defined over a field of characteristic p with no non-trivial p -torsion points, namely supersingular. Also, let $\mathcal{B}_{p,\infty}$ be a quaternion algebra \mathcal{B} over \mathbb{Q} ramified exactly at p and ∞ . The endomorphism algebra of such an elliptic curve is isomorphic to a quaternion algebra ramified at p and ∞ , and its endomorphism ring is isomorphic to a maximal order of the corresponding quaternion algebra, i.e., $\text{End}^0(E) := \text{End}(E) \otimes_{\mathbb{Z}} \mathbb{Q} \cong \mathcal{B}_{p,\infty}$, and $\text{End}(E) \cong \mathcal{O} \subseteq \mathcal{B}_{p,\infty}$. Conversely, for any maximal order in $\mathcal{B}_{p,\infty}$, there exists a supersingular elliptic curve over a field of characteristic p such that whose endomorphism ring is isomorphic to this maximal order. This, indeed, is a correspondence which is called *Deuring correspondence* studied in [30]. Generally, for a fixed maximal order $\mathcal{O}_0 \cong \text{End}(E_0)$, there exists an equivalence between the category of supersingular elliptic curves under isogenies and the category of left fractional \mathcal{O}_0 -ideals under homomorphisms of \mathcal{O}_0 -modules. Constructing a supersingular elliptic curve whose endomorphism ring is isomorphic to a given quaternion maximal order (one direction of the Deuring correspondence) is known to be polynomial-time over carefully selected base fields. Starting from a maximal order in a quaternion algebra, finding a supersingular elliptic curve with that maximal order as an endomorphism ring is called the *constructive Deuring correspondence*. Let $\mathcal{O} \subseteq \mathcal{B}_{p,\infty} \cong \text{End}^0(E)$ be a maximal order. Given I , an integral left \mathcal{O} -ideal, we define the set of I -torsion points of E as $E[I] := \{P \in E : \alpha(P) = 0 \text{ for all } \alpha \in I\}$ as the kernel of I . For the ideal I , we associate the isogeny φ_I with kernel $E[I]$ defined as $\varphi_I : E \rightarrow E_I := \frac{E}{E[I]}$.

2.3 Artificial Orientation

We now consider the concept of artificial orientation introduced in [31] to provide a new technique of securely computing SIDH-like operations against current SIDH attacks. For smooth, square-free, and relatively prime integers A and B , let p be a prime of the

$$\begin{array}{ccc}
 E & \xrightarrow{\phi} & E' \\
 \psi_1 \downarrow & & \downarrow \psi_2 \\
 E_1 & & E_2
 \end{array}$$

Figure 2: Parallel Isogenies

from $p = ABf - 1$, where f is a small cofactor specifying the p to be prime. Let E be a supersingular elliptic curve defined over \mathbb{F}_{p^2} . Then, an *artificial A -orientation* of E is a pair $\mathfrak{A} = (G_1, G_2)$, where G_1, G_2 are cyclic subgroups of $E[A]$, $|G_1| = |G_2| = A$, and $G_1 \cap G_2 = \{0\}$. In this case, the pair (E, \mathfrak{A}) is said to be an *artificially A -oriented* curve. For an artificially A -oriented curve (E, \mathfrak{A}) , a range of isogenies can be computed with kernels derived from $\mathfrak{A} = (G_1, G_2)$. Particularly, an isogeny ϕ is called *\mathfrak{A} -isogeny* in case its kernel can be written as the direct sum of a subgroup $H_1 \subseteq G_1$, and a subgroup $H_2 \subseteq G_2$, i.e., $\ker(\phi) = H_1 \oplus H_2$. In this case, ϕ can be decomposed into two relatively prime degrees isogenies ϕ_1, ϕ_2 , i.e., $\phi = \phi_2 \circ \phi_1$, where $\ker(\phi_1) = H_1 \subseteq G_1$, $\ker(\phi_2) = H_2 \subseteq \phi_1(G_2)$.

However, as described in [31], for an artificially A -oriented curve (E, \mathfrak{A}) , and a non-trivial \mathfrak{A} -isogeny $\phi : E \rightarrow E'$, the artificial A -orientation on E may not always be carried onto E' via the isogeny ϕ since at least one of the subgroups $\phi(G_1)$ and $\phi(G_2)$ of $E'[A]$ may have order smaller than A . To remedy this issue, the degree of the isogeny considered must be relatively prime to A . We, formally, have the following definition, as given in [31], as follows:

Definition 2.1. *For two artificially A -oriented curves (E, \mathfrak{A}) and (E', \mathfrak{A}') , and an integer B relatively prime to the A , the pairs is said to be B -isogenous in case there exists a B -isogeny $\phi : E \rightarrow E'$ such that*

$$\mathfrak{A}' = (G'_1, G'_2) = \phi(G_1, G_2) = \phi(\mathfrak{A}).$$

Assuming fixed generators $\langle P_1 \rangle = G_1$ and $\langle P_2 \rangle = G_2$, the subgroups G'_1 and G'_2 can be represented by $[\alpha]\phi(P_1)$ and $[\beta]\phi(P_2)$ respectively, for some $\alpha, \beta \in \mathbb{Z}/A\mathbb{Z}$. Therefore, for a supersingular curve, it is possible to define an artificial orientation on it. Artificial orientations, while not generating a commutative group action like standard orientations, as in [32], offer sufficient information for computing parallel isogenies. Concretely, for given two A -oriented curves (E, \mathfrak{A}) and (E', \mathfrak{A}') connected by a \mathfrak{B} -isogeny $\phi : E \rightarrow E'$, where $\mathfrak{A} = (G_1, G_2)$ and $\mathfrak{A}' = (G'_1, G'_2)$, the isogenies $\psi_1 : E \rightarrow E_1$, and $\psi_2 : E' \rightarrow E_2$, are parallel as shown in Figure 2, where $E_1 := E/\langle [A_1]G_1 + [A_2]G_2 \rangle$, and $E_2 := E'/\langle [A_1]G'_1 + [A_2]G'_2 \rangle$, i.e., we have $\ker(\psi_2) = \phi(\ker(\psi_1))$ and the codomain curves are also B -isogenous, connected by the isogeny ϕ' with $\ker(\phi') = \psi_1(\ker(\phi))$. Thus, the isogenies ψ_1 and ψ_2 are characterized by the decomposition of A as $A = A_1A_2$. We benefit from the properties of the artificial orientation to construct the adaptation and extraction phase of our protocol.

2.4 Computational Hardness Assumptions

Our hardness assumptions, which are derived from the generic hard problem of finding an isogeny between two isogenous elliptic curves defined over a field k , are given below and supposed to be computationally infeasible problems related to the hardness assumptions utilized in our scheme, applied in the pre-signing and adaptation phases of our protocol in Section 3.

Problem 2.2 (Supersingular Smooth Endomorphism Problem [13]). *Given a prime p and a supersingular elliptic curve E/\mathbb{F}_{p^2} , find a (non-trivial) cyclic endomorphism of E of smooth degree.*

Problem 2.3 (SSIP-A [31]). *Let (E, \mathfrak{B}) be an artificially B -oriented curve and let A be an integer coprime to B . Let $\phi : E \rightarrow E'$ be a cyclic isogeny of degree A and let $\mathfrak{B}' = \phi(\mathfrak{B})$. Given (E, \mathfrak{B}) and (E', \mathfrak{B}') and the degree A , compute ϕ .*

Problem 2.4 (SSIP-B [31]). *Let (E, \mathfrak{B}) be an artificially B -oriented curve and let A be an integer coprime to B . Let $\psi : E \rightarrow E'$ be a cyclic \mathfrak{B} -isogeny of degree B , with $A < B$. Let also P, Q be a basis of $E[A]$. Given (E, \mathfrak{B}) , together with the points P, Q , and the curve E' with the points $\psi(P)$ and $\psi(Q)$, compute ψ .*

2.5 Adaptor Signature Scheme

Hard Relation. Let us first recall the definition of a cryptographically hard relation:

Definition 2.5 (Hard Relation). *Let the subset $R \subseteq W \times S$ be a relation set of witness/statement pairs (w, s) . We define the language of R to be the set $\mathcal{L}_R := \{s \mid \exists w \text{ s.t. } (w, s) \in R\}$ of valid statements. The relation R is said to be a hard relation in case the following are satisfied:*

- There exists a PPT sampling algorithm $\text{GenR}(1^\lambda)$ taking the security parameter λ as input, and outputs a witness/statement pair $(w, s) \in R$.
- The validation of the relation is decidable in polynomial running time.
- For any PPT adversary \mathcal{A} , a negligible function negl exists such that:

$$\Pr \left[(w^*, s) \in R \mid \begin{array}{l} (w, s) \leftarrow \text{GenR}(1^\lambda) \\ w^* \leftarrow \mathcal{A}(s) \end{array} \right] \leq \text{Negl}(\lambda),$$

where the probability comes from the randomness of GenR and \mathcal{A} .

Non-interactive Proof System. Let $(w, s) \in R$ be cryptographically a hard relation, and \mathcal{H} be a random oracle. A *non-interactive proof system* is a pair (P, V) of two PPT oracle algorithms:

- $\pi_w / \perp \leftarrow P^{\mathcal{H}}(w, s)$: a prover P taking a pair $(w, s) \in R$ as input and outputting a proof π_w of the statement s with witness w . $P^{\mathcal{H}}(w, s) = \perp$ if $(w, s) \notin R$.

- $0/1 \leftarrow V^{\mathcal{H}}(s, \pi_w)$: a verifier V taking a pair (s, π_w) and outputting whether it accepts or rejects the proof π_w of s .

which satisfies the following conditions:

- i. Completeness: Let $(w, s) \in R$ and $\pi_w \leftarrow P^{\mathcal{H}}(w, s)$, then there exists a negligible function negl such that $\Pr[V^{\mathcal{H}} = 1] \geq 1 - \text{negl}(\lambda)$.
- ii. Zero-knowledge (NIZK): For a PPT algorithm \mathcal{S} , the zero-knowledge simulator, and for any pair (w, s) and PPT algorithm \mathcal{D} the following distributions are computationally indistinguishable:
 - $\pi_w \leftarrow P^{\mathcal{H}}(w, s)$ if $(w, s) \in R$ and $\pi_w \leftarrow \perp$ otherwise. Output $\mathcal{D}^{\mathcal{H}}(w, s, \pi_w)$.
 - $\pi_w \leftarrow \mathcal{S}(s, 1)$ if $(w, s) \in R$ and $\pi_w \leftarrow \mathcal{S}(s, 0)$ otherwise. Output $\mathcal{D}^{\mathcal{H}}(w, s, \pi_w)$.
- iii. Online-extractability: For a PPT algorithm \mathcal{E} , the online extractor, and for any algorithm A , let $(s, \pi_w) \leftarrow A^{\mathcal{H}}(\lambda)$ be the sequence of queries of A to \mathcal{H} and H_A be the \mathcal{H} 's answers. Let $w \leftarrow \mathcal{E}(s, \pi_w, H_A)$. Then it holds that

$$\Pr[(w, s) \notin R \wedge V^{\mathcal{H}}(s, \pi_w) = 1] \leq \text{negl}(\lambda).$$

Digital Signature Scheme. We recall the definition of a digital signature scheme and the properties that a signature scheme must satisfy to be called secure.

Definition 2.6 (Digital Signature Scheme). *A digital signature is a triple scheme $\Sigma = (\text{KeyGen}, \text{Sig}, \text{Ver})$ consisting of three polynomial-time algorithms:*

- $(\text{sk}, \text{pk}) \leftarrow \text{KeyGen}(1^\lambda)$: a PPT key pairs generating algorithm that takes security parameter λ as its input, and outputs a key pair (sk, pk) ;
- $\sigma \leftarrow \text{Sig}(\text{sk}, m)$: a PPT signing algorithm that takes a secret key sk and message $m \in \{0, 1\}^*$ as input, and outputs a signature σ ;
- $0/1 \leftarrow \text{Ver}(\text{pk}, m, \sigma)$: a DPT verifying algorithm that takes a public key pk , message $m \in \{0, 1\}^*$ and signature σ as input, and outputs a bit $b \in \{0, 1\}$.

The first property that each signature scheme must satisfy, to guarantee the correctness of the scheme, is *signature correctness*, i.e., for any security parameter $\lambda \in \mathbb{N}$, and a message $m \in \{0, 1\}^*$:

$$\Pr \left[\text{Ver}(\text{pk}, m, \text{Sig}(\text{sk}, m)) = 1 \mid (\text{sk}, \text{pk}) \leftarrow \text{KeyGen}(1^\lambda) \right] = 1.$$

There are several definitions of security requirements for a signature scheme. One of the most common of those properties is *existential unforgeability under chosen message attacks*, abbreviated as EUF-CMA. Satisfying this property basically means forging a verifiable signature on a message m without knowing the private key sk is infeasible even in case the PPT adversary has access to many previously produced valid signatures on messages of his choice but message m . The formal definition of this property is as follows:

Definition 2.7 (EUF-CMA Security). *A signature scheme Σ is EUF-CMA secure if for every PPT adversary \mathcal{A} , there exists a negligible function negl such that*

$$\Pr[\text{SigForge}_{\mathcal{A},\Sigma}(\lambda) = 1] \leq \text{negl}(\lambda),$$

where the experiment $\text{SigForge}_{\mathcal{A},\Sigma}$ is defined as follows:

$\text{SigForge}_{\mathcal{A},\Sigma}(\lambda)$	$\mathcal{O}_S(m)$
1: $\mathcal{Q} \leftarrow \emptyset$	1: $\sigma \leftarrow \text{Sig}(\text{sk}, m)$
2: $(\text{sk}, \text{pk}) \leftarrow \text{KeyGen}(1^\lambda)$	2: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$
3: $(m, \sigma) \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot)}(\text{pk})$	3: return σ
4: return $(m \notin \mathcal{Q} \wedge \text{Ver}(\text{pk}, m, \sigma))$	

Furthermore, we have a stronger definition indicating the difficulty of transforming a valid signature on a message m into another valid signature on m , namely *strong existential unforgeability under chosen message attacks*, abbreviated as **SUF-CMA**. This property guarantees that the adversary is not able even to produce a new signature for a previously signed message, i.e., assume that an adversary obtains a message/signature pair (m, σ) together with some message/signature pairs of his choice, the signature scheme is called **SUF-CMA** secure in case the adversary cannot produce a new signature σ^* for the message m . Formally, we have the following definition:

Definition 2.8 (SUF-CMA Security). *A signature scheme Σ is SUF-CMA secure if for every PPT adversary \mathcal{A} , there exists a negligible function negl such that*

$$\Pr[\text{StrongSigForge}_{\mathcal{A},\Sigma}(\lambda) = 1] \leq \text{negl}(\lambda),$$

where the experiment $\text{StrongSigForge}_{\mathcal{A},\Sigma}$ is defined as follows:

$\text{StrongSigForge}_{\mathcal{A},\Sigma}(\lambda)$	$\mathcal{O}_S(m)$
1: $\mathcal{Q} \leftarrow \emptyset$	1: $\sigma \leftarrow \text{Sig}(\text{sk}, m)$
2: $(\text{sk}, \text{pk}) \leftarrow \text{KeyGen}(1^\lambda)$	2: $\mathcal{Q} := \mathcal{Q} \cup \{m, \sigma\}$
3: $(m, \sigma) \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot)}(\text{pk})$	3: return σ
4: return $((m, \sigma) \notin \mathcal{Q} \wedge \text{Ver}(\text{pk}, m, \sigma))$	

Adaptor Signature Scheme. An adaptor signature is a cryptographic primitive that can be treated as a generalization of an ordinary digital signature. More precisely, it hides secret randomness within the signature so that the secret randomness is revealed once the signature is generated. The general procedure is that a pre-signature is generated in the first step. Then, the pre-signature is shifted by secret randomness and adapted into a signature. Finally, the secret randomness is extracted from the signature based on cryptographic processing. Furthermore, the signature produced by an adaptor signature is verifiable by the verification algorithm of the underlying signature scheme.

An adaptor signature also has some properties to guarantee its security. For any statement $s \in \mathcal{L}_R$, a signer holding a secret key \mathbf{sk} can produce a pre-signature $\tilde{\sigma}$ on any message m . This pre-signature can be adapted into a full signature σ on m if and only if a user has a witness w to the statement s , i.e., $(w, s) \in R$. Additionally, anyone with access to the pre-signature $\tilde{\sigma}$, full signature σ , and witness s can extract the witness w and thus reveal the hard relation.

The formal definition of an adaptor signature and its properties are given as follows:

Definition 2.9 (Adaptor Signature Scheme). *An adaptor signature scheme with respect to a hard relation R and a signature scheme $\Sigma = (\text{KeyGen}, \text{Sig}, \text{Ver})$ is a quadruple $\Xi_{R, \Sigma} = (\text{PreSig}, \text{Adapt}, \text{PreVer}, \text{Ext})$ defined as:*

- $\tilde{\sigma} \leftarrow \text{PreSig}(\mathbf{sk}, m, s)$: a PPT algorithm that takes a secret key \mathbf{sk} , message $m \in \{0, 1\}^*$, and statement $s \in \mathcal{L}_R$, and produces a pre-signature $\tilde{\sigma}$.
- $0/1 \leftarrow \text{PreVer}(\mathbf{pk}, m, s, \tilde{\sigma})$: a DPT algorithm that takes a public key \mathbf{pk} , a message $m \in \{0, 1\}^*$, a statement $s \in \mathcal{L}_R$, and a pre-signature $\tilde{\sigma}$, and produces a bit $b \in \{0, 1\}$.
- $\sigma \leftarrow \text{Adapt}(\tilde{\sigma}, w)$: a DPT algorithm that takes a valid pre-signature $\tilde{\sigma}$, and a witness w , and produces a signature σ .
- $w/\perp \leftarrow \text{Ext}(\sigma, \tilde{\sigma}, s)$: a DPT algorithm that takes a pre-signature $\tilde{\sigma}$, a corresponding signature σ , and a statement $s \in \mathcal{L}_R$, and produces a witness w (to the statement s) such that $(w, s) \in R$, or \perp .

For an adaptor signature, KeyGen and Ver are inherited from the underlying signature scheme Σ , and GenR is based on the underlying hard relation to generate secret/statement pair $(w, s) \in R$.

As mentioned before, some properties are required to guarantee the security of an adaptor signature scheme. The first property is *pre-signature correctness* ensuring that an honestly generated pre-signature can be adapted to a signature.

Definition 2.10 (Pre-signature Correctness). *An adaptor signature scheme $\Xi_{R, \Sigma}$ satisfies pre-signature correctness if for any $\lambda \in \mathbb{N}$, any message $m \in \{0, 1\}^*$, and any witness/statement pair (w, s) , the following holds:*

$$\Pr \left[\begin{array}{l} \text{PreVer}(\mathbf{pk}, m, s, \tilde{\sigma}) = 1 \\ \text{Ver}(\mathbf{pk}, m, \sigma) = 1 \\ (w', s) \in R \end{array} \middle| \begin{array}{l} (\mathbf{sk}, \mathbf{pk}) \leftarrow \text{KeyGenR}(1^\lambda) \\ \tilde{\sigma} \leftarrow \text{PreSig}(\mathbf{sk}, m, s) \\ \sigma := \text{Adapt}(\tilde{\sigma}, w) \\ w' := \text{Ext}(\sigma, \tilde{\sigma}, s) \end{array} \right] = 1.$$

The second property required for an adaptor signature is *pre-signature adaptability*. It states that any valid (but not necessarily honestly generated) pre-signature with respect to a statement s can be adapted into a valid signature using the witness w such that $(w, s) \in R$.

Definition 2.11 (Pre-signature Adaptability). *An adaptor signature scheme $\Xi_{\mathbb{R},\Sigma}$ satisfies pre-signature adaptability if for any $\lambda \in \mathbb{N}$, message $m \in \{0,1\}^*$, witness/statement pair $(w, s) \in \mathbb{R}$, key pair $(sk, pk) \leftarrow \text{KeyGen}(1^\lambda)$, and pre-signature $\tilde{\sigma} \leftarrow \{0,1\}^*$ such that $\text{PreVer}(pk, m, s, \tilde{\sigma}) = 1$, the following holds:*

$$\Pr[\text{Ver}(pk, m, \text{Adapt}(\tilde{\sigma}, w)) = 1] = 1.$$

There exists another property that is about the unforgeability of an adaptor signature called *existential unforgeability under chosen message attack*, abbreviated as **aEUF-CMA**. It states that even in the presence of a pre-signature on a message m with respect to a random statement $s \in \mathcal{L}_{\mathbb{R}}$, forging a valid signature σ for m is computationally infeasible for an adversary.

Definition 2.12. [**aEUF – CMA Security**] *An adaptor signature scheme $\Xi_{\mathbb{R},\Sigma}$ is aEUF – CMA secure if for any PPT adversary \mathcal{A} , there exists a negligible function negl such that*

$$\Pr[\text{aSigForge}_{\mathcal{A},\Xi_{\mathbb{R},\Sigma}}(\lambda) = 1] \leq \text{negl}(\lambda),$$

where the experiment $\text{aSigForge}_{\mathcal{A},\Xi_{\mathbb{R},\Sigma}}$ is defined as follows:

$\text{aSigForge}_{\mathcal{A},\Xi_{\mathbb{R},\Sigma}}(\lambda)$	$\mathcal{O}_S(m)$
1: $\mathcal{Q} := \emptyset$	1: $\sigma \leftarrow \text{Sig}(sk, m)$
2: $(sk, pk) \leftarrow \text{KeyGen}(1^\lambda)$	2: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$
3: $m \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot)}(pk)$	3: return σ
4: $(w, s) \leftarrow \text{GenR}(1^\lambda)$	$\mathcal{O}_{pS}(m, s)$
5: $\tilde{\sigma} \leftarrow \text{PreSig}(sk, m, s)$	1: $\tilde{\sigma} \leftarrow \text{PreSig}(sk, m, s)$
6: $\sigma \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot)}(\tilde{\sigma}, s)$	2: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$
7: return $m \notin \mathcal{Q} \wedge \text{Ver}(pk, m, \sigma)$	3: return $\tilde{\sigma}$

The fourth and last property is called *witness extractability* stating that any valid pre-signature/signature pair on a message m with respect to a statement s , suffice to extract the corresponding witness w such that $(w, s) \in \mathbb{R}$.

Definition 2.13 (Witness Extractability). *An adaptor signature scheme $\Xi_{\mathbb{R},\Sigma}$ is witness extractable if for any PPT adversary \mathcal{A} , there exists a negligible function negl such that the following holds:*

$$\Pr[\text{aWitExt}_{\mathcal{A},\Xi_{\mathbb{R},\Sigma}}(\lambda) = 1] \leq \text{negl}(\lambda),$$

where the experiment $\text{aWitExt}_{\mathcal{A},\Xi_{\mathbb{R},\Sigma}}$ is defined as follows:

$\text{aWitExt}_{\mathcal{A}, \Xi_{\mathbb{R}, \Sigma}}(\lambda)$	$\mathcal{O}_S(m)$
1: $\mathcal{Q} := \emptyset$	1: $\sigma \leftarrow \text{Sig}(\text{sk}, m)$
2: $(\text{sk}, \text{pk}) \leftarrow \text{KeyGen}(1^\lambda)$	2: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$
3: $(m, \mathbf{s}) \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(\text{pk})$	3: return σ
4: $\tilde{\sigma} \leftarrow \text{PreSig}(\text{sk}, m, \mathbf{s})$	$\mathcal{O}_{pS}(m, \mathbf{s})$
5: $\sigma \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(\tilde{\sigma})$	1: $\tilde{\sigma} \leftarrow \text{PreSig}(\text{sk}, m, \mathbf{s})$
6: $\mathbf{w}' := \text{Ext}(\sigma, \tilde{\sigma}, \mathbf{s})$	2: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$
7: return $(m \notin \mathcal{Q} \wedge (\mathbf{w}', \mathbf{s}) \notin \mathbb{R} \wedge \text{Ver}(\text{pk}, m, \sigma))$	3: return $\tilde{\sigma}$

As it can be seen, the witness extractability experiment aWitExt is analogous to the experiment aSigForge . Still, an important difference exists in the sense that the adversary can choose the forgery statement \mathbf{s} . Therefore, we can think of this situation as if the adversary knows a witness for \mathbf{s} and can generate a valid signature on the forgery message m . But note that this is not sufficient to win the experiment. The adversary wins only in case the valid signature does not reveal a witness for \mathbf{s} .

In the light of the above properties on the adaptor signature scheme, we have the following definition:

Definition 2.14 (Secure Adaptor Signature Scheme). *An adaptor signature scheme $\Xi_{\mathbb{R}, \Sigma}$ is secure, if it is aEUF-CMA secure, pre-signature adaptable, and witness extractable.*

2.6 SQIsignHD

SQIsignHD [24] is a post-quantum digital signature scheme inspired by SQISign [13] that uses the algorithmic breakthrough from the attacks [15], [16], and [17] on SIDH to efficiently represent isogenies of arbitrary degrees. It scales well to high-security levels, is simpler and more efficient, and has smaller signature sizes than SQISign. The SQIsignHD protocol is as follows:

Let $D_\varphi := \prod_{i=1}^n \ell_i^{e_i}$ be a smooth number and $\mu(D_\varphi) := \prod_{i=0}^n \ell_i^{e_i-1}(\ell_i + 1)$. Also, let $\Phi_{D_\varphi}(E, h)$ be an arbitrary function that maps an integer $h \in [1, \mu(D_\varphi)]$ to a non-backtracking isogeny of degree D_φ starting at E . Consider a hash function $\mathbb{H} : \{0, 1\}^* \rightarrow [1, \mu(D_\varphi)]$ which is cryptographically secure.

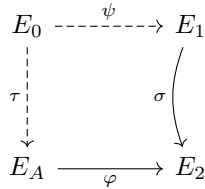


Figure 3: SQIAsignHD Protocol

Setup. Choose a prime p and supersingular elliptic curve E_0/\mathbb{F}_{p^2} with known endomorphism ring $\mathcal{O}_0 \cong \text{End}(E_0)$ such that E_0 has smooth torsion defined over a small extension of \mathbb{F}_{p^2} of degree 1 or 2.

KeyGen. Generate a random secret isogeny $\tau : E_0 \rightarrow E_A$ of fixed smooth degree D_τ . The secret/public key pair is $(\text{sk}, \text{pk}) = (\tau, E_A)$.

Sign. Generate a random (secret) commitment isogeny $\psi : E_0 \rightarrow E_1$. Then, for signing a message m , build the isogeny $\Phi_{D_\varphi}(E_A, h) = \varphi : E_A \rightarrow E_2$, where $h = H(j(E_1), m)$. Finally, from the knowledge of the secret key τ , and isogenies φ, ψ , construct an efficient representation $R = (\sigma(P_1), \sigma(P_2), q)$ given by the image of torsion points by a response isogeny $\sigma : E_1 \rightarrow E_2$ and return the pair $\Sigma := (E_1, R)$ as a signature.

Verify. Upon receiving a signature $\Sigma = (E_1, R)$ associated with the message m and public key E_A , a verifier first recovers $h = H(j(E_1), m)$ and then $\varphi = \Phi(E_A, h) : E_A \rightarrow E_2$, finally checks that R represents correctly an isogeny $\sigma : E_1 \rightarrow E_2$ by computing a higher dimensional isogeny, as described in SQIsignHD.

As shown in the protocol, the signature is a data $(E_1, q, \sigma(P_1), \sigma(P_2))$, with $q < \ell^f$, $\sigma : E_1 \rightarrow E_2$ a q -isogeny and (P_1, P_2) a basis of $E_1[\ell^f]$. This data is based upon the following definition:

Definition 2.15 ([24]). *Suppose that A is an algorithm and $\varphi : E \rightarrow E'$ is an \mathbb{F}_q -rational isogeny. Then, an efficient representation of isogeny φ (with respect to A) is some data $D \in \{0, 1\}^*$ such that:*

1. D has polynomial size in $\log(\deg(\varphi))$ and $\log(q)$.
2. On input D and $P \in E(\mathbb{F}_{q^k})$, A returns $\varphi(P)$ in polynomial time in $k \log(q)$ and $\log(\deg(\varphi))$.

The public parameters for SQIsignHD are easy to generate. Specifically, the underlying prime field needs only be of the form $p = c\ell^f\ell'^{f'} - 1$ where $\ell \neq \ell'$ are two primes (which in practice $\ell = 2$ and $\ell' = 3$), $c \in \mathbb{N}^*$ is a small cofactor, and $\ell^f \approx \ell'^{f'} \approx \sqrt{p}$, which is made to ensure sufficient accessible torsion for the isogeny computations. Because of this high flexibility property of the underlying prime p , we are able to replace $\ell'^{f'}$ with an odd integer $B = \prod_{i=1}^t \ell_i$ where ℓ_i 's are small distinct primes coprime to ℓ in order to provide a suitable setting to apply the notion of artificial orientation to our protocol. More exactly, in our adaptor signature construction, the pre-signature $\tilde{\sigma}$ (full signature σ , respectively) isogeny is of degree \tilde{q} (of degree q , respectively) which is a power of ℓ , where $\tilde{q} \approx \sqrt{p}$, $q \approx \sqrt[4]{p^3}$, and isogenies φ, τ and ψ are of degree coprime to ℓ .

3 New Adaptor Signature Construction

In this section, we introduce a new post-quantum adaptor signature scheme using SQIsignHD [24] as the underlying signature scheme and exploiting the idea of the artificial orientation, on the supersingular isogeny Diffie-Hellman key exchange protocol

(SIDH), introduced as $\text{binSIDH}^{\text{hyb}}$ variant in [31], to apply the corresponding hard relation.

At the moment, the only secure post-quantum isogeny-based adaptor signature is IAS introduced in [18] which is constructed upon CSI-FiSh [19]. IAS has restrictions in terms of efficiency due to its parameter sizes relying on CSI-FiSh. More precisely, CSI-FiSh works on at most the CSIDH-512 parameters since knowledge about the class group structure is required to efficiently compute the class group action on uniformly random group elements. We, now, introduce our post-quantum adaptor signature scheme in detail and illustrate our scheme's protocol in Algorithm 1.

3.1 Public Parameters

To deploy our protocol, we need to set some initial parameters. These public parameters are inspired by those used in $\text{binSIDH}^{\text{hyb}}$ and SQIsignHD. Therefore, the setup of our scheme is as follows.

We set a prime p of the form $p = cAB - 1$ such that $A = \ell^f$, and $B = \prod_{i=1}^t \ell_i$ are coprime integers, ℓ and ℓ_i 's are distinct small primes, $A \approx B \approx \sqrt{p}$, and integer c is some (small) cofactor. Let E_0/\mathbb{F}_{p^2} be a supersingular elliptic curve with known endomorphism ring $\text{End}(E_0) \cong \mathcal{O}_0 \subset \mathcal{B}_{p,\infty}$, and $|E_0(\mathbb{F}_{p^2})| = (p+1)^2$. Additionally, we assume that $\mathfrak{B} = (G_1, G_2)$ is an artificial B -orientation on E_0 , and set $\langle P, Q \rangle = E_0[\ell^{f'}]$, where $2f' \approx f$, and $\ell^{f'} \approx \sqrt[p]{p}$. We, furthermore, pick a secure hash function $\mathbf{H} : \{0, 1\} \rightarrow [1, \mu(D_\varphi)]$ similar to that given in SQIsignHD.

3.2 Key Generation & Hard Relation

The key generation step is identical to the generic procedure in SQIsignHD. More precisely, a random secret isogeny $\tau : E_0 \rightarrow E_\tau$ is chosen, thereby the secret/public pair is set as $(\text{sk}, \text{pk}) = (\tau, E_\tau)$.

To define the hard relation in our scheme, we set the witness/statement pairs as follows:

$$\mathbf{R}_{\mathfrak{A}} := \left\{ \left(w, (E_w, w(\mathfrak{B})) \right) \left| \begin{array}{l} w : E_0 \rightarrow E_w := E_0/\langle P + [\alpha]Q \rangle, \\ \text{where } \langle P, Q \rangle = E_0[\ell^{f'}], \alpha \in \mathbb{Z}/\ell^{f'}\mathbb{Z}. \\ (E_0, \mathfrak{B}) \text{ is artificially } B\text{-oriented.} \end{array} \right. \right\},$$

where w is the witness secret isogeny $w : E_0 \rightarrow E_w$ with the artificially B -oriented curve (E_0, \mathfrak{B}) as its domain, and the pair $(E_w, w(\mathfrak{B}))$ is the statement consisting of the target elliptic curve E_w , and the image the artificially B -orientation $\mathfrak{B} = (G_1, G_2)$ under witness isogeny w .

3.3 Pre-signature

The procedure of the pre-signing algorithm carries some similarities to that described in the SQIsignHD protocol, however, it differs slightly in producing the commitment isogeny (curve, accordingly), as well as some additional ingredients that are required during the adaption phase.

In some sense (unlike SQIsignHD), in the pre-signature phase, we have two (secret) commitment isogenies: one is used in the pre-signature phase, and the other, which is

generated from involving of the statement curve, required for the adaption phase. Let us examine them more closely.

Commitment ψ . The first commitment isogeny ψ is a \mathfrak{B} -oriented isogeny $\psi : E_0 \rightarrow E_\psi$ generated by sampling uniformly at random a vector \vec{b} from $\{1, 2\}^t$ to compute $\ker(\psi) := \langle G_{b_1}^1, G_{b_2}^2, \dots, G_{b_t}^t \rangle$, where $G_1 := \langle G_1^1, G_1^2, \dots, G_1^t \rangle$ and $G_2 := \langle G_2^2, G_2^2, \dots, G_2^t \rangle$ and $|G_1^i| = |G_2^i| = \ell_i$, for $1 \leq i \leq t$. Moreover, by using isogeny ψ , the image of public parameters P and Q is determined. We set these images as $S := (\psi(P), \psi(Q))$.

Commitment ψ' . The second commitment isogeny ψ' is obtained via push-forward of the first commitment ψ through the witness $w : E_0 \rightarrow E_w$ with the help of the component $w(\mathfrak{B})$ of the public statement that is the image of the artificially B -orientation \mathfrak{B} under the witness isogeny w , i.e. $\psi' := [w]_*\psi : E_w \rightarrow E_1$. This way, we obtain the second commitment curve E_1 whose j -invariant is used to compute the challenge isogeny.

Now, the challenge and pre-signature isogenies are produced as follows:

Challenge φ . To produce a challenge isogeny, the j -invariant of the second commitment curve E_1 , which is obtained from implicitly involving the statement curve E_w , along with a message m induce an isogeny starting at the public key E_τ . Specifically, for $h := \mathbf{H}(j(E_1), m)$, let the challenge isogeny be defined as an isogeny $\varphi := \Phi(E_\tau, h) : E_\tau \rightarrow E_2$.

Pre-signature $\tilde{\sigma}$. In order to complete the pre-signing phase of a message m with a secret key isogeny $\tau : E_0 \rightarrow E_\tau$, from the knowledge of isogenies τ, φ and ψ , an efficient representation $\mathcal{R}_{\tilde{\sigma}} = (\tilde{\sigma}(R_1), \tilde{\sigma}(R_2), \deg(\tilde{\sigma}))$ is constructed by the image of a canonically determined basis (R_1, R_2) of $E_\psi[\ell^f]$ under a pre-signature isogeny $\tilde{\sigma} : E_\psi \rightarrow E_2$. Hence, the pre-signature tuple is defined as $\tilde{\Sigma} := (E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}})$, thereby the pre-signing algorithm is defined as follows:

$$\tilde{\Sigma} = (E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}}) \leftarrow \text{PreSig}(\text{sk}, m, s) = \text{PreSig}(\tau, m, (E_w, w(\mathfrak{B}))).$$

3.4 Pre-verification

To pre-verify the pre-signature, first, upon receiving the pairs $S = (\psi(P), \psi(Q))$ from the pre-signature step, the equality $e_{\ell^{f'}}(\psi(P), \psi(Q)) = e_{\ell^{f'}}(P, Q)^B$ of Weil pairings is checked whether the image of the points P, Q is correctly computed under the isogeny ψ . Additionally, by using S , it is checked if the target elliptic curve obtained from computing parallel isogeny $[\psi]_*w$ is isomorphic to the commitment isogeny E_1 . Finally, after computing $h = \mathbf{H}(j(E_1), m)$ and recovering the challenge isogeny $\varphi = \Phi(E_\tau, h) : E_\tau \rightarrow E_2$, it is checked that $\mathcal{R}_{\tilde{\sigma}}$ represents correctly an isogeny $\tilde{\sigma} : E_\psi \rightarrow E_2$ by computing a higher dimensional isogeny, as explained in SQISignHD. In case the aforementioned conditions are not met, it aborts. Thus, the pre-verification algorithm is defined as follows:

$$0/1 \leftarrow \text{PreVer}(\text{pk}, m, s, \tilde{\Sigma}) = \text{PreVer}(E_A, m, (E_w, w(\mathfrak{B})), (E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}})).$$

3.5 Adaptation

To adapt the pre-signature into a (full) signature, first the parallel isogeny w' to the witness isogeny w is computed by using the additional information $S = (\psi(P), \psi(Q))$

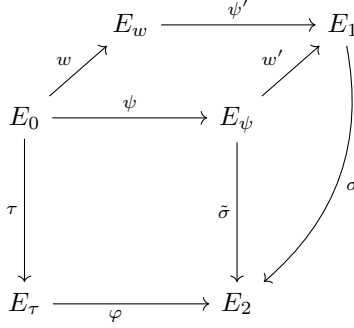


Figure 4: SQIAsignHD Protocol

which is necessarily led to coincide the second commitment curve E_1 , i.e., $w' := [\psi]_* w : E_\psi \rightarrow E_1$. Next, an efficient representation data is constructed via the image of torsion points by the (full) signature isogeny $\sigma := \tilde{\sigma} \circ \hat{w}' : E_1 \rightarrow E_2$ as follows:

1. Determine a canonical basis $\langle P_0, Q_0 \rangle = E_1[\ell^{f+f'}]$.
2. Compute $\hat{w}'(P_0)$ and $\hat{w}'(Q_0)$ by explicit description of isogeny \hat{w}' .
3. Compute $A(\mathcal{R}_{\tilde{\sigma}}, \hat{w}'(P_0)) =: \sigma(P_0)$ and $A(\mathcal{R}_{\tilde{\sigma}}, \hat{w}'(Q_0)) =: \sigma(Q_0)$.
4. Generate an efficient representation

$$\mathcal{R}_\sigma := (\sigma(P_0), \sigma(Q_0), \deg(\sigma))$$

of the isogeny $\sigma : E_1 \rightarrow E_2$.

The signature is defined as $\Sigma := (E_1, \mathcal{R}_\sigma)$. Thus, the adapting algorithm is designed as follows:

$$\Sigma := (E_1, \mathcal{R}_\sigma) \leftarrow \text{Adapt}(\tilde{\Sigma}, w) = \text{Adapt}((E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}}), w).$$

3.6 Extraction

Now, in the last phase of our scheme, in order to extract the secret witness isogeny w , by using publicly known pre-signature $\tilde{\Sigma}$ and signature Σ , we exploit two computing approach: one is computing the discrete logarithm (of modulus a sufficiently smooth integer), denoted by \mathcal{A}_{DLP} , and the other is the attack for key recovery of an isogeny satisfying $n^2 > 4d$ via SIDH attack [15], denoted by $\mathcal{A}_{\text{SIDH}}$, where d is the degree of the isogeny and n is the order of the given torsion points information. We, furthermore, make use of an algorithm, which is denoted by A , in the sense of the Definition 2.15. Therefore, the extraction process follows the following steps:

1. Determine a canonical basis $\langle P_1, Q_1 \rangle = E_1[N]$ satisfying $4\ell^{f'} < N^2$.
2. Set $P' := A(\mathcal{R}_\sigma, P_1)$, $Q' := A(\mathcal{R}_\sigma, Q_1)$, where $P', Q' \in E_2[N]$.

3. Set $X := \hat{w}'(P_1)$ and $Y := \hat{w}'(Q_1)$ as unknowns for which we look for the value. Then, X and Y can be written as

$$X = [a]P_\psi + [b]Q_\psi, \quad Y = [c]P_\psi + [d]Q_\psi,$$

for some unknown values $a, b, c, d \in \mathbb{Z}/N\mathbb{Z}$, where $\langle P_\psi, Q_\psi \rangle = E_\psi[N]$.

4. From the action of isogeny $\tilde{\sigma}$ on X and Y , that is,

$$\tilde{\sigma}(X) = \tilde{\sigma}([a]P_\psi + [b]Q_\psi) = [a]\tilde{\sigma}(P_\psi) + [b]\tilde{\sigma}(Q_\psi),$$

$$\tilde{\sigma}(Y) = \tilde{\sigma}([c]P_\psi + [d]Q_\psi) = [c]\tilde{\sigma}(P_\psi) + [d]\tilde{\sigma}(Q_\psi),$$

we form the following equations:

$$[a]\tilde{\sigma}(P_\psi) + [b]\tilde{\sigma}(Q_\psi) = P',$$

$$[c]\tilde{\sigma}(P_\psi) + [d]\tilde{\sigma}(Q_\psi) = Q',$$

where P', Q' are obtained from step 2.

5. Set initial values for a and c , (we let $a = c = 1$). Using Discrete Logarithm (DL) algorithm, \mathcal{A}_{DLP} , the values of b and d can be found. This way, the action of \hat{w}' on P_1 and Q_1 is determined.
6. Exploit the SIDH attack, $\mathcal{A}_{\text{SIDH}}$, to find the kernel of the isogeny \hat{w}' . Then, compute dual of \hat{w}' , that is the isogeny $w' : E_\psi \rightarrow E_1$. Let $\ker(w') = \langle [\alpha_1]P'_\psi + [\alpha_2]Q'_\psi \rangle$, for some $\alpha_1, \alpha_2 \in \mathbb{Z}/\ell^{f'}\mathbb{Z}$, and $\langle P'_\psi, Q'_\psi \rangle = E_\psi[\ell^{f'}]$.
7. Recompute α_1, α_2 by making change of basis $\langle P'_\psi, Q'_\psi \rangle$ into $\langle \psi(P), \psi(Q) \rangle$, obtain $\alpha \in \mathbb{Z}/\ell^{f'}\mathbb{Z}$ for which $\ker(w') = \langle \psi(P) + [\alpha]\psi(Q) \rangle$.
8. Compute pull-back of the isogeny w' through ψ using public parameters P, Q , to extract the witness isogeny $w : E_0 \rightarrow E_w := E_0/\langle P + [\alpha]Q \rangle$.

Thus, the extracting algorithm is defined as follows:

$$w / \perp \leftarrow \text{Ext}(\Sigma, \tilde{\Sigma}, \mathfrak{s}) = \text{Ext}\left((E_1, \mathcal{R}_\sigma), (E_1, E_\psi, S, R_{\tilde{\sigma}}), (E_w, w(\mathfrak{B}))\right).$$

Algorithm 1 SQIAsignHD : Adaptor Signature $\Xi_{\mathcal{R}_{\mathfrak{A}}, \Sigma_{\text{SQIsignHD}}}$

-
- 1: **Public Parameters.** A prime $p = c\ell^f \prod_{i=1}^n \ell_i - 1$, where ℓ, ℓ_i 's are coprime, and $\ell^f \approx \prod_{i=1}^n \ell_i \approx \sqrt{p}$. A supersingular elliptic curve E_0/\mathbb{F}_{p^2} with $\text{End}(E_0) \cong \mathcal{O}_0 \subset \mathcal{B}_{p, \infty}$, and $|E_0(\mathbb{F}_{p^2})| = (p+1)^2$. An artificial B -orientation $\mathfrak{B} = (G_1, G_2)$ on E_0 . A torsion basis $\langle P, Q \rangle = E_0[\ell^{f'}]$, where $\ell^{f'} \approx \sqrt[3]{p}$.
 - 2: **Procedure PreSign**(sk, m, s)
 - 3: Compute a secret isogeny $\psi : E_0 \rightarrow E_\psi$.
 - 4: Compute the image of P, Q under ψ , and set $S := (\psi(P), \psi(Q))$.
 - 5: Compute the push-forward $\psi' := [w]_* \psi : E_w \rightarrow E_1$ via $w(\mathfrak{B})$.
 - 6: Compute $\varphi := \Phi(E_\tau, h) : E_\tau \rightarrow E_2$, where $h := H(j(E_1), m)$.
 - 7: Compute representation $\mathcal{R}_{\tilde{\sigma}} := (\tilde{\sigma}(R_1), \tilde{\sigma}(R_2), \tilde{q})$ where $\tilde{\sigma} : E_\psi \rightarrow E_2$.
 - 8: Return $\tilde{\Sigma} := (E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}})$
 - 9: **Procedure PreVer**(pk, m, s, $\tilde{\Sigma}$)
 - 10: Parse $\tilde{\Sigma}$ as $(E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}})$.
 - 11: Check that $e_{\ell^{f'}}(\psi(P), \psi(Q)) = e_{\ell^{f'}}(P, Q)^B$.
 - 12: Compute $h := H(j(E_1), m)$ and recover $\varphi := \Phi(E_\tau, h) : E_\tau \rightarrow E_2$.
 - 13: Check that $\mathcal{R}_{\tilde{\sigma}}$ represent $\tilde{\sigma} : E_\psi \rightarrow E_2$.
 - 14: Return 0/1.
 - 15: **Procedure Adapt**($\tilde{\Sigma}, w$)
 - 16: Compute push-forward $w' := [\psi]_* w : E_\psi \rightarrow E_1$ via S .
 - 17: Determine a canonical basis $\langle P_0, Q_0 \rangle = E_1[\ell^{f+f'}]$.
 - 18: Compute $\sigma(P_0) := A(\mathcal{R}_{\tilde{\sigma}}, \hat{w}'(P_0))$, and $\sigma(Q_0) := A(\mathcal{R}_{\tilde{\sigma}}, \hat{w}'(Q_0))$.
 - 19: Set $\mathcal{R}_\sigma := (\sigma(P_0), \sigma(Q_0), q)$ where $\sigma : E_1 \rightarrow E_2$, and $q = \deg(\sigma)$.
 - 20: Return $\Sigma := (E_1, \mathcal{R}_\sigma)$
 - 21: **Procedure Ext**($\tilde{\Sigma}, \Sigma, s$)
 - 22: Parse Σ as $(E_1, \mathcal{R}_\sigma)$.
 - 23: Recover $\hat{w}' : E_1 \rightarrow E_\psi$ via \mathcal{A}_{DLP} and $\mathcal{A}_{\text{SIDH}}$.
 - 24: Compute $w' : E_\psi \rightarrow E_\psi / \langle [\alpha_1]P'_\psi + [\alpha_2]Q'_\psi \rangle$ where $\langle P'_\psi, Q'_\psi \rangle = E_\psi[\ell^{f'}]$.
 - 25: Recompute α_1, α_2 by changing basis $\langle P'_\psi, Q'_\psi \rangle$ into $\langle \psi(P), \psi(Q) \rangle$.
 - 26: Extract w by pulling w' back through ψ via public points P, Q .
 - 27: Return \perp / w
-

4 Security Proof

In this section, we analyze and formally prove the security of the new adaptor signature $\Xi_{\mathcal{R}_{\mathfrak{A}}, \Sigma_{\text{SQIsignHD}}}$. We show that $\Xi_{\mathcal{R}_{\mathfrak{A}}, \Sigma_{\text{SQIsignHD}}}$ satisfies pre-signature correctness, pre-signature adaptability, aEUF-CMA, and witness extractability properties. Verifying these properties suffices to prove the Theorem 4.11.

Lemma 4.1. *The adaptor signature $\Xi_{\mathcal{R}_{\mathfrak{A}}, \Sigma_{\text{SQIsignHD}}}$ is pre-signature correct.*

Proof. First, we let $(\mathbf{w}, \mathbf{s}) := (w, (E_w, w(\mathfrak{B})))$ be a fixed witness/statement pair of the defined hard relation $\mathcal{R}_{\mathfrak{A}}$, generated by the GenR algorithm, where w is an isogeny from E_0 to the target elliptic curve E_w , and $w(\mathfrak{B})$ is the image of B -orientation \mathfrak{B} under the witness isogeny w . Moreover, suppose that $(\mathbf{sk}, \mathbf{pk}) := (\tau, E_\tau)$ is a fixed secret/public key pair generated by the KeyGen algorithm.

Assume that for a message $m \in \{0, 1\}^*$, the pre-signature $\tilde{\Sigma} = (E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}})$ is generated via PreSig algorithm, i.e., $\tilde{\Sigma} \leftarrow \text{PreSig}(\tau, m, (E_w, w(\mathfrak{B})))$. In this case, we necessarily obtain $1 \leftarrow \text{PreVer}(E_\tau, m, E_w, \tilde{\Sigma})$ meaning that $\mathcal{R}_{\tilde{\sigma}}$ is an efficient representation of an isogeny $\tilde{\sigma}$ from E_ψ to E_2 , where E_2 is the target curve of isogeny φ depending on the message m and the commitment curve E_1 which is implicitly obtained from the statement $(E_w, w(\mathfrak{B}))$.

Next, the (full) signature $\Sigma = (E_1, \mathcal{R}_\sigma)$ is produced by adaptation algorithm, that is $\Sigma \leftarrow \text{Adapt}(\tilde{\Sigma}, w)$. Here, \mathcal{R}_σ is an efficient representation of the signature isogeny σ with domain E_1 and codomain E_2 obtained from the composition of \hat{w}' (the dual of the push-forward of the witness w through ψ using S), i.e., $\sigma := \tilde{\sigma} \circ \widehat{[\psi]_* w} = \tilde{\sigma} \circ \hat{w}' : E_1 \rightarrow E_2$. Hence, the verification of the signature can be done by SQIsignHD verifying algorithm Ver necessarily yielding $1 \leftarrow \text{Ver}(E_\tau, m, \text{Adapt}(\tilde{\Sigma}, w))$. It means that the data \mathcal{R}_σ represent correctly the signature isogeny σ which is a map from the curve E_1 to E_2 . From the knowledge of the pre-signature $\tilde{\Sigma} = (E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}})$ and signature $\Sigma = (E_1, \mathcal{R}_\sigma)$, and using discrete logarithm \mathcal{A}_{DLP} and SIDH attack $\mathcal{A}_{\text{SIDH}}$, we can extract the $w' : E_\psi \rightarrow E_1$ revealing the witness w via its pull-back through the secret isogeny ψ using points $P, Q, \psi(P)$, and $\psi(Q)$. This means that $w \leftarrow \text{Ext}(\Sigma, \tilde{\Sigma}, (E_w, w(\mathfrak{B})))$ is successfully performed to obtain the witness w . Thus, the adaptor signature $\Xi_{\mathcal{R}_{\mathfrak{A}}, \Sigma_{\text{SQIsignHD}}}$ satisfies the pre-signature correctness property. \square

Lemma 4.2. *The adaptor signature $\Xi_{\mathcal{R}_{\mathfrak{A}}, \Sigma_{\text{SQIsignHD}}}$ is pre-signature adaptable.*

Proof. Let define a fixed witness/statement pair $(\mathbf{w}, \mathbf{s}) := (w, (E_w, w(\mathfrak{B}))) \in \mathcal{R}_{\mathfrak{A}}$, a fixed public key $\mathbf{pk} = E_\tau$, and a pre-signature $\tilde{\Sigma}$, and a message $m \in \{0, 1\}^*$ as in the previous Lemma.

We want to prove that any verifiably valid (but, not necessarily honestly generated) pre-signature $\tilde{\Sigma} = (E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}})$ passing PreVer algorithm, can be adapted into a valid (full) signature Σ . To show this, let $\text{PreVer}(E_\tau, m, E_w, \tilde{\Sigma}) = 1$ meaning that $\mathcal{R}_{\tilde{\sigma}}$ is a data representing an isogeny from E_ψ to E_2 , where E_2 is a target curve of φ generated by the message m and commitment curve E_1 . In this case, by using the correctness property as shown in the previous Lemma, the adapting algorithm Adapt necessarily yields a full signature Σ which is verifiable by the verifying algorithm Ver of the $\Sigma_{\text{SQIsignHD}}$. Finally,

the witness $w = w$ can be extracted by using the valid pre-signature/signature pair and the corresponding statement $s = (E_w, w(\mathfrak{B}))$, i.e., $w \leftarrow \text{Ext}(\Sigma, \tilde{\Sigma}, s)$. Therefore, the adaptor signature $\Xi_{R_{\mathfrak{A}}, \Sigma_{\text{SQIsignHD}}}$ satisfies the pre-signature adaptability property. \square

Lemma 4.3. *Let the SQIsignHD signature scheme $\Sigma_{\text{SQIsignHD}}$ be SUF-CMA secure, and let $R_{\mathfrak{A}}$ be a hard relation. Then, the adaptor signature scheme $\Xi_{R_{\mathfrak{A}}, \Sigma_{\text{SQIsignHD}}}$ is aEUF-CMA secure in the quantum random oracle model.*

Proof. We start our proof by reducing the SQIAsignHD adaptor signature’s unforgeability to the SQIsignHD signature scheme’s strong unforgeability. Starting with the aSigForge, we play a series of games with adversary \mathcal{A} in which we can respond to all of \mathcal{A} ’s query calls, up until the last game in the series. Our initial focus is how to provide \mathcal{A} with the signing and pre-signing queries. If we are successful in responding to these calls, we will be able to leverage its forgery to win our aSigForge game. In order to achieve this, we construct a simulator \mathcal{S} that employs \mathcal{A} ’s forgery in aSigForge to win its strong unforgeability experiment for the SQIsignHD signature scheme. In this case, \mathcal{S} has access to both signing oracle $\text{Sig}^{\text{SQIsignHD}}$ and the random oracle $\mathcal{H}^{\text{SQIsignHD}}$, and it utilizes them to simulate oracle queries for \mathcal{A} , namely random oracle \mathcal{H} , signing queries \mathcal{O}_S , and pre-signing queries \mathcal{O}_{pS} .

Game₀. This game corresponds to the aSigForge experiment given in Definition 2.12, where the adversary \mathcal{A} has access to a random oracle \mathcal{H} , in the random oracle model, and many previously produced valid pre-signatures and signatures through pre-signing \mathcal{O}_{pS} and signing oracles \mathcal{O}_S on messages of its choice but a message m , and forges a verifiable signature Σ^* on the message m . Hence, in this setting, it follows that

$$\Pr[\text{Game}_0 = 1] = \Pr[\text{aSigForge}_{\mathcal{A}, \Xi_{R_{\mathfrak{A}}, \Sigma_{\text{SQIsignHD}}}}(\lambda) = 1].$$

Game₁. This game is analogous to the game Game₀. The only difference is that if a valid signature Σ^* , forged by the adversary \mathcal{A} , is the same as the output of adaptation of the pre-signature into a signature with the help of the corresponding witness, then the game aborts.

Game ₀	$\mathcal{H}(x)$
1: $\mathcal{Q} := \emptyset$	1: if $H[x] = \perp$
2: $H := [\perp]$	2: $H[x] \leftarrow \mathcal{H}^{\text{SQIsignHD}}(x)$
3: $(\tau, E_\tau) \leftarrow \text{KeyGen}(1^\lambda)$	3: return $H[x]$
4: $m \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(E_\tau)$	$\mathcal{O}_{pS}(m, (E_w, w(\mathfrak{B})))$
5: $(w, (E_w, w(\mathfrak{B}))) \leftarrow \text{GenR}(1^\lambda)$	1: $\tilde{\Sigma} \leftarrow \text{PreSig}(\tau, m, (E_w, w(\mathfrak{B})))$
6: $\tilde{\Sigma} \leftarrow \text{PreSig}(\tau, m, (E_w, w(\mathfrak{B})))$	2: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$
7: $\Sigma^* \leftarrow \mathcal{A}(\tilde{\Sigma}, (E_w, w(\mathfrak{B})))$	3: return $\tilde{\Sigma}$
8: $b := \text{Ver}(E_\tau, m, \Sigma^*)$	
9: return $m \notin \mathcal{Q} \wedge b$	
$\mathcal{O}_S(m)$	
1: $\Sigma \leftarrow \text{Sig}(\tau, m)$	
2: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$	
3: return Σ	

Game ₁	$\mathcal{H}(x)$
1: $\mathcal{Q} := \emptyset$	1: if $H[x] = \perp$
2: $H := [\perp]$	2: $H[x] \leftarrow \mathcal{H}^{\text{SQIsignHD}}(x)$
3: $(\tau, E_\tau) \leftarrow \text{KeyGen}(1^\lambda)$	3: return $H[x]$
4: $m^* \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(E_\tau)$	$\mathcal{O}_{pS}(m, (E_w, w(\mathfrak{B})))$
5: $(w, (E_w, w(\mathfrak{B}))) \leftarrow \text{GenR}(1^\lambda)$	1: $\tilde{\Sigma} \leftarrow \text{PreSig}(\tau, m, (E_w, w(\mathfrak{B})))$
6: $\tilde{\Sigma} \leftarrow \text{PreSig}(\tau, m^*, (E_w, w(\mathfrak{B})))$	2: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$
7: $\Sigma^* \leftarrow \mathcal{A}(\tilde{\Sigma}, (E_w, w(\mathfrak{B})))$	3: return $\tilde{\Sigma}$
8: if $\text{Adapt}(\tilde{\Sigma}, w) = \Sigma^*$	
9: abort	
10: $b := \text{Ver}(E_\tau, m^*, \Sigma^*)$	
11: return $m^* \notin \mathcal{Q} \wedge b$	
$\mathcal{O}_S(m)$	
1: $\Sigma \leftarrow \text{Sig}(\tau, m)$	
2: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$	
3: return Σ	

Claim 4.4. *If Bad_1 is the event that Game_1 aborts, then we claim that for a negligible function negl in λ , $\Pr[\text{Bad}_1] \leq \text{negl}(\lambda)$.*

Proof. The claim is proven by a reduction to the hardness of the relation $\text{R}_{\mathfrak{R}}$. To do this, we construct a simulator \mathcal{S} breaking the hardness of $\text{R}_{\mathfrak{R}}$ and assuming that it has access to an adversary \mathcal{A} that causes Game_1 to abort with the non-negligible probability. The simulator receives a challenge $\mathfrak{s}^* = (E_w, w(\mathfrak{B}))^*$, and generates a secret/public key pair $(\tau, E_\tau) \leftarrow \text{KeyGen}(1^\lambda)$ to simulate \mathcal{A} 's queries to the oracles \mathcal{H} , \mathcal{O}_{pS} and \mathcal{O}_S . The functionalities of the simulated oracles are as described in Game_1 . Based on receiving the challenge message m^* from \mathcal{A} , \mathcal{S} computes a pre-signature $\tilde{\Sigma} \leftarrow \text{PreSig}(\tau, m^*, (E_w, w(\mathfrak{B}))^*)$ and returns the pair $(\tilde{\Sigma}, (E_w, w(\mathfrak{B}))^*)$ to the adversary who forges a signature by using the returned pair. Assuming that Bad_1 happened (i.e., $\text{Adapt}(\tilde{\Sigma}, w) = \Sigma^*$), since the $\Xi_{\text{R}_{\mathfrak{R}}, \Sigma_{\text{SQIsignHD}}}$ is pre-signature correct as in Definition 2.10, the simulator can extract w^* via $\text{Ext}(\Sigma^*, \tilde{\Sigma}, (E_w, w(\mathfrak{B}))^*)$ to obtain a valid witness/statement pair such that $(w^*, (E_w, w(\mathfrak{B}))^*) \in \text{R}_{\mathfrak{R}}$, thereby \mathcal{S} breaks the security of the relation $\text{R}_{\mathfrak{R}}$. We note that the view of \mathcal{A} in this simulation and Game_1 are indistinguishable since the challenge $(E_w, w(\mathfrak{B}))^*$ is an instance of the hard relation $\text{R}_{\mathfrak{R}}$ and has the same distribution to the public output of GenR . Therefore, the probability that \mathcal{S} breaks the hardness of $\text{R}_{\mathfrak{R}}$ is equal to the probability that the event Bad_1 happens that is non-negligible by assumption. This contradicts the hardness of $\text{R}_{\mathfrak{R}}$. Since Game_1 and Game_0 are equivalent except in case of happening the event Bad_1 , it follows that

$$\Pr[\text{Game}_1 = 1] \leq \Pr[\text{Game}_0 = 1] + \text{negl}(\lambda).$$

□

Game_2 . This game is analogous to the previous game. The only difference is a modification in the pre-signing oracle \mathcal{O}_{pS} . That is, in this game we apply the online extractor algorithm \mathcal{E} taking the statement $(E_w, w(\mathfrak{B}))$, and the list of random oracle queries \mathcal{H} as input to extract a witness w through the \mathcal{O}_{pS} queries. The game aborts in case $(w, (E_w, w(\mathfrak{B}))) \notin \text{R}_{\mathfrak{R}}$ for the extracted witness w .

Claim 4.5. *If Bad_2 is the event that Game_2 aborts during an \mathcal{O}_S execution, then $\Pr[\text{Bad}_2] \leq \text{negl}(\lambda)$ for a negligible function negl in λ .*

Proof. In the quantum random oracle model, the oracle can extract the witness using its online extractor algorithm \mathcal{E} . More precisely, there is a non-negligible probability that $(w, (E_w, w(\mathfrak{B}))) \in \text{R}_{\mathfrak{R}}$, where $w := \mathcal{E}(E_w, w(\mathfrak{B}), H)$. □

Since games Game_2 and Game_1 are equivalent except in case Bad_2 happens, it follows that

$$\Pr[\text{Game}_2 = 1] \leq \Pr[\text{Game}_1 = 1] + \text{negl}(\lambda).$$

Game ₂	$\mathcal{H}(x)$
1: $\mathcal{Q} := \emptyset$ 2: $H := [\perp]$ 3: $(\tau, E_\tau) \leftarrow \text{KeyGen}(1^\lambda)$ 4: $m^* \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(E_\tau)$ 5: $(w, (E_w, w(\mathfrak{B}))) \leftarrow \text{GenR}(1^\lambda)$ 6: $\tilde{\Sigma} \leftarrow \text{PreSig}(\tau, m^*, (E_w, w(\mathfrak{B})))$ 7: $\Sigma^* \leftarrow \mathcal{A}(\tilde{\Sigma}, (E_w, w(\mathfrak{B})))$ 8: if $\text{Adapt}(\tilde{\Sigma}, w) = \Sigma^*$ 9: abort 10: $b := \text{Ver}(E_\tau, m^*, \Sigma^*)$ 11: return $m^* \notin \mathcal{Q} \wedge b$	1: if $H[x] = \perp$ 2: $H[x] \leftarrow \mathcal{H}^{\text{SQIsignHD}}(x)$ 3: return $H[x]$ <hr/> $\mathcal{O}_{pS}(m, (E_w, w(\mathfrak{B})))$ 1: $w^* := \mathcal{E}(E_w, w(\mathfrak{B}), H)$ 2: if $(w^*, (E_w, w(\mathfrak{B}))) \notin \mathcal{R}_\mathfrak{A}$ 3: abort 4: $\tilde{\Sigma} \leftarrow \text{PreSig}(\tau, m, (E_w, w(\mathfrak{B})))$ 5: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$ 6: return $\tilde{\Sigma}$
<hr/> $\mathcal{O}_S(m)$ 1: $\Sigma \leftarrow \text{Sig}(\tau, m)$ 2: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$ 3: return Σ	

Game ₃	$\mathcal{H}(x)$
1: $\mathcal{Q} := \emptyset$ 2: $H := [\perp]$ 3: $(\tau, E_\tau) \leftarrow \text{KeyGen}(1^\lambda)$ 4: $m^* \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(E_\tau)$ 5: $(w, (E_w, w(\mathfrak{B}))) \leftarrow \text{GenR}(1^\lambda)$ 6: $\tilde{\Sigma} \leftarrow \text{PreSig}(\tau, m^*, (E_w, w(\mathfrak{B})))$ 7: $\Sigma^* \leftarrow \mathcal{A}(\tilde{\Sigma}, (E_w, w(\mathfrak{B})))$ 8: if $\text{Adapt}(\tilde{\Sigma}, w) = \Sigma^*$ 9: abort 10: $b := \text{Ver}(E_\tau, m^*, \Sigma^*)$ 11: return $m^* \notin \mathcal{Q} \wedge b$	1: if $H[x] = \perp$ 2: $H[x] \leftarrow \mathcal{H}^{\text{SQIsignHD}}(x)$ 3: return $H[x]$ <hr/> $\mathcal{O}_{pS}(m, (E_w, w(\mathfrak{B})))$ 1: $w^* := \mathcal{E}(E_w, w(\mathfrak{B}), H)$ 2: if $(w^*, (E_w, w(\mathfrak{B}))) \notin \mathcal{R}_\mathfrak{A}$ 3: abort 4: $\Sigma \leftarrow \text{Sig}(\tau, m)$ 5: Parse Σ as $(E_1, \mathcal{R}_\sigma)$ 6: Extract $\mathcal{R}_{\tilde{\sigma}}$ by 7: \mathcal{A}_{DLP} and $\mathcal{A}_{\text{SIDH}}$ 8: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$ 9: return $\tilde{\Sigma} := (E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}})$
<hr/> $\mathcal{O}_S(m)$ 1: $\Sigma \leftarrow \text{Sig}(\tau, m)$ 2: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$ 3: return Σ	

Game₃. In this game, we apply further modifications to the pre-signing oracle \mathcal{O}_{pS} to create a correct pre-signature $\tilde{\Sigma}$. First, by executing the **Sig** algorithm, the signature Σ is produced. Then, with the help of the signature Σ and the witness w which has been already extracted from the online extractor \mathcal{E} , the pre-signature is created. We see that this game is indistinguishable from the previous game, and it follows that

$$Pr[\text{Game}_3 = 1] \leq Pr[\text{Game}_2 = 1] + \text{negl}(\lambda).$$

Game₄. In this game, after receiving the challenge message m^* from \mathcal{A} , as in the previous game during the \mathcal{O}_{pS} execution, the game generates a signature Σ by running the **Sig** algorithm and converting the resulting signature into a valid pre-signature. Therefore, in this game as well, the same indistinguishability argument that held in the previous game holds. Thus, it follows that

$$Pr[\text{Game}_4 = 1] \leq Pr[\text{Game}_3 = 1] + \text{negl}(\lambda).$$

Game ₄	$\mathcal{H}(x)$
1 : $\mathcal{Q} := \emptyset$	1 : if $H[x] = \perp$
2 : $H := [\perp]$	2 : $H[x] \leftarrow \mathcal{H}^{\text{SQIsignHD}}(x)$
3 : $(\tau, E_\tau) \leftarrow \text{KeyGen}(1^\lambda)$	3 : return $H[x]$
4 : $m^* \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(E_\tau)$	$\mathcal{O}_{pS}(m, (E_w, w(\mathfrak{B})))$
5 : $(w, (E_w, w(\mathfrak{B}))) \leftarrow \text{GenR}(1^\lambda)$	1 : $w^* := \mathcal{E}(E_w, w(\mathfrak{B}), H)$
6 : $\Sigma \leftarrow \text{Sig}(\tau, m^*)$	2 : if $(w^*, (E_w, w(\mathfrak{B}))) \notin \mathcal{R}_{\mathfrak{M}}$
7 : Parse Σ as $(E_1, \mathcal{R}_\sigma)$	3 : abort
8 : Extract $\mathcal{R}_{\tilde{\sigma}}$ by	4 : $\Sigma \leftarrow \text{Sig}(\tau, m)$
9 : \mathcal{A}_{DLP} and $\mathcal{A}_{\text{SIDH}}$	5 : Parse Σ as $(E_1, \mathcal{R}_\sigma)$
10 : $\tilde{\Sigma} := (E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}})$	6 : Extract $\mathcal{R}_{\tilde{\sigma}}$ by
11 : $\Sigma^* \leftarrow \mathcal{A}(\tilde{\Sigma}, (E_w, w(\mathfrak{B})))$	7 : \mathcal{A}_{DLP} and $\mathcal{A}_{\text{SIDH}}$
12 : if $\text{Adapt}(\tilde{\Sigma}, w) = \Sigma^*$	8 : $\mathcal{Q} := \mathcal{Q} \cup \{m\}$
13 : abort	9 : return $\tilde{\Sigma} := (E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}})$
14 : $b := \text{Ver}(E_\tau, m^*, \Sigma^*)$	
15 : return $m^* \notin \mathcal{Q} \wedge b$	
<hr/>	
$\mathcal{O}_S(m)$	
1 : $\Sigma \leftarrow \text{Sig}(\tau, m)$	
2 : $\mathcal{Q} := \mathcal{Q} \cup \{m\}$	
3 : return Σ	

As it can be seen, the transformation of the **aSigForge** game into game **Game₄** is indistinguishable. Thus, the original **aSigForge** game has now been reduced to **Game₄**, a game

in which we are able to respond to \mathcal{A} 's query calls. More precisely, if the adversary \mathcal{A} queries the signing oracle \mathcal{O}_S , the simulator \mathcal{S} queries the $\text{Sig}^{\text{SQIsignHD}}$ signing oracle $\text{Sig}^{\text{SQIsignHD}}$ and returns its response to \mathcal{A} . In case \mathcal{A} queries the pre-signing oracle, the simulator, first, extracts w using the online extractability of NIZK, then queries the SQIsignHD signing oracle to get the signature, finally uses the signature and the resulting witness to create a valid pre-signature. Moreover, based on \mathcal{A} querying the oracle \mathcal{H} on input x , in case $H[x] = \perp$, the \mathcal{S} queries $\mathcal{H}^{\text{SQIsignHD}}(x)$, otherwise the simulator outputs $H[x]$. Thus, adversary \mathcal{A} is able to make any queries to the oracles it requires, thereby generates a forgery. The only thing remaining to show is that there exists a simulator that simulates Game_4 and utilizes the resulting forgery due to \mathcal{A} to win the $\text{SQIsignHD SigForge}$ game or the StrongSigForge game.

$\mathcal{S}^{\text{Sig}^{\text{SQIsignHD}}, \mathcal{H}^{\text{SQIsignHD}}}(E_\tau)$	$\mathcal{H}(x)$
1 : $Q := \emptyset$	1 : if $H[x] = \perp$
2 : $H := [\perp]$	2 : $H[x] \leftarrow \mathcal{H}^{\text{SQIsignHD}}(x)$
3 : $(\tau, E_\tau) \leftarrow \text{KeyGen}(1^\lambda)$	3 : return $H[x]$
4 : $m^* \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot)}(E_\tau)$	$\mathcal{O}_{pS}(m, (E_w, w(\mathfrak{B})))$
5 : $(w, (E_w, w(\mathfrak{B}))) \leftarrow \text{GenR}(1^\lambda)$	1 : $w^* := \mathcal{E}(E_w, \pi_w, H)$
6 : $\Sigma \leftarrow \text{Sig}^{\text{SQIsignHD}}(m^*)$	2 : if $(w^*, (E_w, w(\mathfrak{B}))) \notin \text{R}_{\mathfrak{A}}$
7 : Parse Σ as $(E_1, \mathcal{R}_\sigma)$	3 : abort
8 : Extract $\mathcal{R}_{\tilde{\sigma}}$ by	4 : $\tilde{\Sigma} \leftarrow \text{Sig}^{\text{SQIsignHD}}(m)$
9 : \mathcal{A}_{DLP} and $\mathcal{A}_{\text{SIDH}}$	5 : Parse $\tilde{\Sigma}$ as $(E_1, \mathcal{R}_\sigma)$
10 : $\tilde{\Sigma} := (E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}})$	6 : Extract $\mathcal{R}_{\tilde{\sigma}}$ by
11 : $\Sigma^* \leftarrow \mathcal{A}(\tilde{\Sigma}, (E_w, w(\mathfrak{B})))$	7 : \mathcal{A}_{DLP} and $\mathcal{A}_{\text{SIDH}}$
12 : return (m^*, Σ^*)	8 : $Q := Q \cup \{m\}$
	9 : return $\tilde{\Sigma} := (E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}})$
$\mathcal{O}_S(m)$	
1 : $\Sigma \leftarrow \text{Sig}^{\text{SQIsignHD}}(m)$	
2 : $Q := Q \cup \{m\}$	
3 : return Σ	

Claim 4.6. (m^*, Σ^*) constitutes a valid forgery in the StrongSigForge game.

Proof. To prove this claim, we must show that the pair (m^*, Σ^*) has not been output by the oracle $\text{Sig}^{\text{SQIsignHD}}$ before. Note that the adversary \mathcal{A} has not previously made a query on the challenge message m^* to either \mathcal{O}_S or \mathcal{O}_{pS} . Therefore, $\text{Sig}^{\text{SQIsignHD}}$ is only queried on m^* during the challenge phase. As shown in the game Game_1 , the adversary outputs a forgery Σ^* which is equal to the signature Σ output by $\text{Sig}^{\text{SQIsignHD}}$ during the challenge phase only with negligible probability. Hence, oracle $\text{Sig}^{\text{SQIsignHD}}$ has never output Σ^* on query m^* before, and thus (m^*, Σ^*) is a valid forgery for the StrongSigForge game. \square

From the game Game_0 to the game Game_4 , we have that

$$\Pr[\text{Game}_0 = 1] \leq \Pr[\text{Game}_4 = 1] + \text{negl}(\lambda).$$

Since the simulator \mathcal{S} provides a perfect simulation of game Game_4 , we obtain

$$\text{Adv}_{\mathcal{A}}^{\text{aSigForge}} = \Pr[\text{Game}_0 = 1] \leq \Pr[\text{Game}_4 = 1] + \text{negl}(\lambda) \leq \text{Adv}_{\mathcal{S}}^{\text{StrongSigForge}} + \text{negl}(\lambda).$$

By assumption, as SQIsignHD is secure in QROM with $\mathcal{H}^{\text{SQIsignHD}}$ programmed as a quantum random oracle, it follows that our adaptor signature, SQIAsignHD, is aEUF-CMA secure in QROM. \square

Lemma 4.7. *Let the SQIsignHD signature scheme $\Sigma_{\text{SQIsignHD}}$ be SUF-CMA, and $R_{\mathfrak{A}}$ be a hard relation. Then, the adaptor signature $\Xi_{R_{\mathfrak{A}}, \Sigma_{\text{SQIsignHD}}}$ is witness extractable in the quantum random oracle model.*

Proof. The proof of this lemma is almost identical to the proof of Lemma 4.3. We prove this lemma by reducing the witness extractability of $\Xi_{R_{\mathfrak{A}}, \Sigma_{\text{SQIsignHD}}}$ to the strong unforgeability of the SQIsignHD signature scheme. More precisely, let \mathcal{A} be a PPT adversary who wins the aWitExt game, then we build another PPT adversary \mathcal{S} so that it wins the StrongSigForge game.

Analogous to the proof of the previous Lemma, the main challenge comes from the simulation of pre-signing queries. The difference now from the previous Lemma is that in aWitExt, the adversary \mathcal{A} outputs the statement $(E_w, w(\mathfrak{B}))$ for the relation $R_{\mathfrak{A}}$ along with the challenge message m^* . This means that the pair $(w, (E_w, w(\mathfrak{B})))$ is not chosen by the game. Consequently, \mathcal{S} is unable to convert a valid signature into a pre-signature as it does not have access to the witness w . However, w can be extracted by the online extractor \mathcal{E} since we are in the QROM. Once w is extracted, then \mathcal{S} can simulate the pre-signing queries as in the previous Lemma. We, now, begin with designing a series of games required for the proof.

Game₀. This game is the aWitExt game given in Definition 2.13. For a given pre-signature $\tilde{\Sigma}$ and witness/statement pair $(w, (E_w, w(\mathfrak{B})))$, the adversary \mathcal{A} who has access to oracles \mathcal{H} , \mathcal{O}_{pS} and \mathcal{O}_S , needs to generate a valid signature Σ for a message m of its choice such that $(w^*, (E_w, w(\mathfrak{B}))) \notin R_{\mathfrak{A}}$, where $w^* = \text{Ext}(\tilde{\Sigma}, \Sigma, (E_w, w(\mathfrak{B})))$. Since Game_0 is exactly the aWitExt game, then we have

$$\Pr[\text{Game}_0 = 1] = \Pr[\text{aWitExt}_{\mathcal{A}, \Xi_{R_{\mathfrak{A}}, \Sigma_{\text{SQIsignHD}}}}(\lambda) = 1].$$

Game ₀	$\mathcal{H}(x)$
1: $\mathcal{Q} := \emptyset$ 2: $H := [\perp]$ 3: $(\tau, E_\tau) \leftarrow \text{KeyGen}(1^\lambda)$ 4: $(m, (E_w, w(\mathfrak{B}))) \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(E_\tau)$ 5: $\tilde{\Sigma} \leftarrow \text{PreSig}(\tau, m, (E_w, w(\mathfrak{B})))$ 6: $\Sigma \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(\tilde{\Sigma})$ 7: $w^* := \text{Ext}(\tilde{\Sigma}, \Sigma, (E_w, w(\mathfrak{B})))$ 8: $b_1 := \text{Ver}(E_\tau, m, \Sigma)$ 9: $b_2 := m \notin \mathcal{Q}$ 10: $b_3 := (w^*, (E_w, w(\mathfrak{B}))) \notin \mathbb{R}_{\mathfrak{A}}$ 11: return $b_1 \wedge b_2 \wedge b_3$	1: if $H[x] = \perp$ 2: $H[x] \leftarrow \mathcal{H}^{\text{SQIsignHD}}(x)$ 3: return $H[x]$
$\mathcal{O}_S(m)$	$\mathcal{O}_{pS}(m, (E_w, w(\mathfrak{B})))$
1: $\Sigma \leftarrow \text{Sig}(\tau, m)$ 2: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$ 3: return Σ	1: $\tilde{\Sigma} \leftarrow \text{PreSig}(\tau, m, (E_w, w(\mathfrak{B})))$ 2: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$ 3: return $\tilde{\Sigma}$

Game ₁	$\mathcal{H}(x)$
1: $\mathcal{Q} := \emptyset$ 2: $H := [\perp]$ 3: $(\tau, E_\tau) \leftarrow \text{KeyGen}(1^\lambda)$ 4: $(m^*, (E_w, w(\mathfrak{B}))) \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(E_\tau)$ 5: $\tilde{\Sigma} \leftarrow \text{PreSig}(\text{sk}, m^*, (E_w, w(\mathfrak{B})))$ 6: $\Sigma^* \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(\tilde{\Sigma})$ 7: $w^* := \text{Ext}(\tilde{\Sigma}, \Sigma^*, (E_w, w(\mathfrak{B})))$ 8: $b_1 := \text{Ver}(E_\tau, m^*, \Sigma^*)$ 9: $b_2 := m^* \notin \mathcal{Q}$ 10: $b_3 := (w^*, (E_w, w(\mathfrak{B}))) \notin \mathbb{R}_{\mathfrak{A}}$ 11: return $b_1 \wedge b_2 \wedge b_3$	1: if $H[x] = \perp$ 2: $H[x] \leftarrow \mathcal{H}^{\text{SQIsignHD}}(x)$ 3: return $H[x]$
$\mathcal{O}_S(m)$	$\mathcal{O}_{pS}(m, (E_w, w(\mathfrak{B})))$
1: $\Sigma \leftarrow \text{Sig}(\tau, m)$ 2: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$ 3: return Σ	1: $w := \mathcal{E}(E_w, w(\mathfrak{B}), H)$ 2: if $(w, (E_w, w(\mathfrak{B}))) \notin \mathbb{R}_{\mathfrak{A}}$ 3: abort 4: $\tilde{\Sigma} \leftarrow \text{PreSig}(\tau, m, (E_w, w(\mathfrak{B})))$ 5: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$ 6: return $\tilde{\Sigma}$

Game₁. This game is the same as **Game₀** except that some changes is applied to the pre-signing oracle \mathcal{O}_{pS} . More precisely, during the \mathcal{O}_{pS} queries, this game extracts a witness w by executing the online extractor algorithm \mathcal{E} on inputs which are the statement $(E_w, w(\mathfrak{B}))$ and the list of random oracle queries H . The game aborts in case for the extracted witness w , $(w, (E_w, w(\mathfrak{B}))) \in \mathcal{R}_{\mathfrak{A}}$ is not satisfied.

Claim 4.8. *If Bad_1 is the event that **Game₁** aborts while the execution of \mathcal{O}_{pS} , then $\Pr[\text{Bad}_1] \leq \text{negl}(\lambda)$.*

Proof. From the online extractor property of NIZK, the witness w can be extracted via the extractor \mathcal{E} for which $(w, (E_w, w(\mathfrak{B}))) \in \mathcal{R}_{\mathfrak{A}}$ is satisfied except with negligible probability. \square

It follows that **Game₁** and **Game₀** are equivalent except for the case that the event Bad_1 happens. Thus, we get that

$$\Pr[\text{Game}_0 = 1] \leq \Pr[\text{Game}_1 = 1] + \text{negl}(\lambda).$$

Game₂. We apply further modifications to the \mathcal{O}_{pS} oracle from the previous game. In this game, first a valid full signature Σ is created by executing the **Sig** algorithm and converted Σ into a pre-signature using the extracted witness w obtained from the online extractor \mathcal{E} . We see that this game is indistinguishable from the previous game, and it follows that

$$\Pr[\text{Game}_1 = 1] \leq \Pr[\text{Game}_2 = 1] + \text{negl}(\lambda).$$

Game ₂	$\mathcal{H}(x)$
1: $\mathcal{Q} := \emptyset$	1: if $H[x] = \perp$
2: $H := [\perp]$	2: $H[x] \leftarrow \mathcal{H}^{\text{SQIsignHD}}(x)$
3: $(\tau, E_\tau) \leftarrow \text{KeyGen}(1^\lambda)$	3: return $H[x]$
4: $(m^*, (E_w, w(\mathfrak{B}))) \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(E_\tau)$	$\mathcal{O}_{pS}(m, (E_w, w(\mathfrak{B})))$
5: $\tilde{\Sigma} \leftarrow \text{PreSig}(\tau, m^*, (E_w, w(\mathfrak{B})))$	1: $w := \mathcal{E}(E_w, w(\mathfrak{B}), H)$
6: $\Sigma^* \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(\tilde{\Sigma})$	2: if $(w, (E_w, w(\mathfrak{B}))) \notin \mathcal{R}_{\mathfrak{A}}$
7: $w^* := \text{Ext}(\tilde{\Sigma}, \Sigma^*, (E_w, w(\mathfrak{B})))$	3: abort
8: $b_1 := \text{Ver}(E_\tau, m^*, \Sigma^*)$	4: $\Sigma \leftarrow \text{Sig}(\tau, m)$
9: $b_2 := m^* \notin \mathcal{Q}$	5: Parse Σ as $(E_1, \mathcal{R}_\sigma)$
10: $b_3 := (w^*, (E_w, w(\mathfrak{B}))) \notin \mathcal{R}_{\mathfrak{A}}$	6: Extract $\mathcal{R}_{\tilde{\sigma}}$ by
11: return $b_1 \wedge b_2 \wedge b_3$	7: \mathcal{A}_{DLP} and $\mathcal{A}_{\text{SIDH}}$
$\mathcal{O}_S(m)$	8: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$
1: $\Sigma \leftarrow \text{Sig}(\tau, m)$	9: return $\tilde{\Sigma} = (E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}})$
2: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$	
3: return Σ	

Game ₃	$\mathcal{H}(x)$
1: $\mathcal{Q} := \emptyset$	1: if $H[x] = \perp$
2: $H := [\perp]$	2: $H[x] \leftarrow \mathcal{H}^{\text{SQIsignHD}}(x)$
3: $(\tau, E_\tau) \leftarrow \text{KeyGen}(1^\lambda)$	3: return $H[x]$
4: $(m^*, (E_w, w(\mathfrak{B}))) \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(E_\tau)$	$\mathcal{O}_{pS}(m, (E_w, w(\mathfrak{B})))$
5: $w := \mathcal{E}(E_w, w(\mathfrak{B}), H)$	1: $w := \mathcal{E}(E_w, w(\mathfrak{B}), H)$
6: if $(w, (E_w, w(\mathfrak{B}))) \notin \mathcal{R}_{\mathfrak{A}}$	2: if $(w, (E_w, w(\mathfrak{B}))) \notin \mathcal{R}_{\mathfrak{A}}$
7: abort	3: abort
8: $\tilde{\Sigma} \leftarrow \text{PreSig}(\tau, m^*, (E_w, w(\mathfrak{B})))$	4: $\Sigma \leftarrow \text{Sig}(\tau, m)$
9: $\Sigma^* \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(\tilde{\Sigma})$	5: Parse Σ as $(E_1, \mathcal{R}_\sigma)$
10: $w^* := \text{Ext}(\tilde{\Sigma}, \Sigma^*, (E_w, w(\mathfrak{B})))$	6: Extract $\mathcal{R}_{\tilde{\sigma}}$ by
11: $b_1 := \text{Ver}(E_\tau, m^*, \Sigma^*)$	7: \mathcal{A}_{DLP} and $\mathcal{A}_{\text{SIDH}}$
12: $b_2 := m^* \notin \mathcal{Q}$	8: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$
13: $b_3 := (w^*, (E_w, w(\mathfrak{B}))) \notin \mathcal{R}_{\mathfrak{A}}$	9: return $\tilde{\Sigma} = (E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}})$
14: return $b_1 \wedge b_2 \wedge b_3$	
$\mathcal{O}_S(m)$	
1: $\Sigma \leftarrow \text{Sig}(\tau, m)$	
2: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$	
3: return Σ	

Game₃. In this game, for the challenge phase, we apply the identical modifications implemented in **Game₁**'s \mathcal{O}_{pS} oracle. In the challenge phase, a witness w is extracted by the online extractor algorithm \mathcal{E} taking the statement $(E_w, w(\mathfrak{B}))$, and the list of random oracle queries H as inputs. In case for the extracted witness w , the relation $(w, (E_w, w(\mathfrak{B}))) \in \mathcal{R}_{\mathfrak{A}}$ is not satisfied, then the game aborts.

Claim 4.9. *If Bad_2 is the event that **Game₃** aborts during the challenge phase, then $\Pr[\text{Bad}_2] \leq \text{negl}(\lambda)$.*

Proof. The same arguments in **Claim 4.8** hold for proving this claim. \square

Hence, **Game₃** and **Game₂** are equivalent except for the case that the event Bad_2 happens. Thus, we have

$$\Pr[\text{Game}_2 = 1] \leq \Pr[\text{Game}_3 = 1] + \text{negl}(\lambda).$$

Game₄. The challenge phase of this game uses the similar modifications implemented in **Game₂** for the \mathcal{O}_{pS} oracle. That is, using the extracted witness w , this game first uses the Sig algorithm to construct a valid full signature Σ , which it then transforms into

a pre-signature. Hence, this game is indistinguishable from the previous game, and it follows that

$$\Pr[\text{Game}_3 = 1] \leq \Pr[\text{Game}_4 = 1] + \text{negl}(\lambda).$$

After demonstrating that the transformation of original aWitExt game into Game Game_4 is indistinguishable, it is necessary to show that there exists a simulator that accurately simulates Game_4 and utilizes the adversary \mathcal{A} to win the StrongSigForge game.

Game ₄	$\mathcal{H}(x)$
1: $Q := \emptyset$	1: if $H[x] = \perp$
2: $H := [\perp]$	2: $H[x] \leftarrow \mathcal{H}^{\text{SQIsignHD}}(x)$
3: $(\tau, E_\tau) \leftarrow \text{KeyGen}(1^\lambda)$	3: return $H[x]$
4: $(m^*, (E_w, w(\mathfrak{B}))) \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(E_\tau)$	$\mathcal{O}_{pS}(m, (E_w, w(\mathfrak{B})))$
5: $w := \mathcal{E}(E_w, w(\mathfrak{B}), H)$	1: $w := \mathcal{E}(E_w, w(\mathfrak{B}), H)$
6: if $(w, (E_w, w(\mathfrak{B}))) \notin \mathcal{R}_{\mathfrak{A}}$	2: if $(w, (E_w, w(\mathfrak{B}))) \notin \mathcal{R}_{\mathfrak{A}}$
7: abort	3: abort
8: $\Sigma \leftarrow \text{Sig}(\tau, m)$	4: $\Sigma \leftarrow \text{Sig}(\tau, m)$
9: Parse Σ as $(E_1, \mathcal{R}_\sigma)$	5: Parse Σ as $(E_1, \mathcal{R}_\sigma)$
10: Extract $\mathcal{R}_{\tilde{\sigma}}$ by	6: Extract $\mathcal{R}_{\tilde{\sigma}}$ by
11: \mathcal{A}_{DLP} and $\mathcal{A}_{\text{SIDH}}$	7: \mathcal{A}_{DLP} and $\mathcal{A}_{\text{SIDH}}$
12: $\tilde{\Sigma} = (E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}})$	8: $Q := Q \cup \{m\}$
13: $\Sigma^* \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(\tilde{\Sigma})$	9: return $\tilde{\Sigma} = (E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}})$
14: $w^* := \text{Ext}(\tilde{\Sigma}, \Sigma^*, (E_w, w(\mathfrak{B})))$	
15: $b_1 := \text{Ver}(E_\tau, m^*, \Sigma^*)$	
16: $b_2 := m^* \notin Q$	
17: $b_3 := (w^*, (E_w, w(\mathfrak{B}))) \notin \mathcal{R}_{\mathfrak{A}}$	
18: return $b_1 \wedge b_2 \wedge b_3$	
$\mathcal{O}_S(m)$	
1: $\Sigma \leftarrow \text{Sig}(\tau, m)$	
2: $Q := Q \cup \{m\}$	
3: return Σ	

In the internal mechanism of the simulator S , in case the adversary \mathcal{A} queries the signing oracle \mathcal{O}_S on input m , then the simulator S will query the SQIsignHD signing oracle $\text{Sig}^{\text{SQIsignHD}}$ and returns its response to the adversary \mathcal{A} . In case \mathcal{A} queries the pre-signing oracle on input $(m, (E_w, w(\mathfrak{B})))$, first the simulator extracts witness w using the extractability property of NIZK, then queries the SQIsignHD signing oracle on input m to get the signature, finally uses the signature and the corresponding witness to construct a valid pre-signature. Moreover, upon \mathcal{A} querying the oracle \mathcal{H} on input x , in case $H[x] = \perp$, then S will query $\mathcal{H}^{\text{SQIsignHD}}(x)$, otherwise the simulator outputs $H[x]$.

$\mathcal{S}\text{Sig}^{\text{SQIsignHD}}, \mathcal{H}^{\text{SQIsignHD}}(E_\tau)$	$\mathcal{H}(x)$
1: $\mathcal{Q} := \emptyset$	1: if $H[x] = \perp$
2: $H := [\perp]$	2: $H[x] \leftarrow \mathcal{H}^{\text{SQIsignHD}}(x)$
3: $(\tau, E_\tau) \leftarrow \text{KeyGen}(1^\lambda)$	3: return $H[x]$
4: $(m^*, (E_w, w(\mathfrak{B}))) \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(E_\tau)$	$\mathcal{O}_{pS}(m, (E_w, w(\mathfrak{B})))$
5: $w := \mathcal{E}(E_w, w(\mathfrak{B}), H)$	1: $w := \mathcal{E}(E_w, w(\mathfrak{B}), H)$
6: if $(w, (E_w, w(\mathfrak{B}))) \notin \mathcal{R}_{\mathfrak{A}}$	2: if $(w, (E_w, w(\mathfrak{B}))) \notin \mathcal{R}_{\mathfrak{A}}$
7: abort	3: abort
8: $\Sigma \leftarrow \text{Sig}^{\text{SQIsignHD}}(m^*)$	4: $\Sigma \leftarrow \text{Sig}^{\text{SQIsignHD}}(m)$
9: Parse Σ as $(E_1, \mathcal{R}_\sigma)$	5: Parse Σ as $(E_1, \mathcal{R}_\sigma)$
10: Extract $\mathcal{R}_{\tilde{\sigma}}$ by	6: Extract $\mathcal{R}_{\tilde{\sigma}}$ by
11: \mathcal{A}_{DLP} and $\mathcal{A}_{\text{SIDH}}$	7: \mathcal{A}_{DLP} and $\mathcal{A}_{\text{SIDH}}$
12: $\tilde{\Sigma} = (E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}})$	8: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$
13: $\Sigma^* \leftarrow \mathcal{A}^{\mathcal{O}_S(\cdot), \mathcal{O}_{pS}(\cdot, \cdot)}(\tilde{\Sigma})$	9: return $\tilde{\Sigma} = (E_1, E_\psi, S, \mathcal{R}_{\tilde{\sigma}})$
14: return (m^*, Σ^*)	
$\mathcal{O}_S(m)$	
1: $\Sigma \leftarrow \text{Sig}^{\text{SQIsignHD}}(m)$	
2: $\mathcal{Q} := \mathcal{Q} \cup \{m\}$	
3: return Σ	

Therefore, adversary \mathcal{A} can make any queries to the oracles it needs during forgery. In the challenge phase, after \mathcal{A} creates the message $(m, (E_w, w(\mathfrak{B})))$ as the challenge message, the \mathcal{S} uses NIZK's extractability to extract witness w , and sends the message m to the Oracle $\text{Sig}^{\text{SQIsignHD}}$ and receives the resulting signature Σ to transform it into a pre-signature. Ultimately, based on forgery made by \mathcal{A} , the simulator \mathcal{S} outputs the forgery (m^*, Σ^*) .

We end up the proof by showing that there exists a simulator that simulates Game_4 and utilizes the resulting forgery made by \mathcal{A} to win the StrongSigForge game.

Claim 4.10. (m^*, Σ^*) constitutes a valid forgery in the StrongSigForge game.

Proof. It is enough to show that the pair (m^*, Σ^*) has not been created by the oracle $\text{Sig}^{\text{SQIsignHD}}$ before. We note that neither \mathcal{O}_{pS} nor \mathcal{O}_S has received a query from adversary \mathcal{A} regarding the challenge message m^* . $\text{Sig}^{\text{SQIsignHD}}$ is therefore only queried on m^* during the challenge phase. In case the adversary \mathcal{A} creates a forgery Σ^* equal to the signature Σ due to $\text{Sig}^{\text{SQIsignHD}}$ during the challenge phase, then the extracted w would be in relation with the corresponding statement $(E_w, w(\mathfrak{B}))$. Hence, Σ^* on query m^* has been never output by the $\text{Sig}^{\text{SQIsignHD}}$ before. Thus, (m^*, Σ^*) constitutes a valid forgery for the StrongSigForge game. \square

From the Game_0 to the Game_4 , we get that

$$\Pr[\text{Game}_0 = 1] \leq \Pr[\text{Game}_4 = 1] + \text{negl}(\lambda).$$

Since \mathcal{S} provides a perfect simulation of Game_4 , we obtain

$$\text{Adv}_{\mathcal{A}}^{\text{aWitExt}} = \Pr[\text{Game}_0 = 1] \leq \Pr[\text{Game}_4 = 1] + \text{negl}(\lambda) \leq \text{Adv}_{\mathcal{S}}^{\text{StrongSigForge}} + \text{negl}(\lambda).$$

Since SQIsignHD is secure in QROM with $\mathcal{H}^{\text{SQIsignHD}}$ modeled as a quantum random oracle, this implies that the SQIAsignHD adaptor signature scheme $\Xi_{R_{\mathcal{Q}}, \Sigma_{\text{SQIsignHD}}}$ achieves witness extractability even against quantum adversaries. \square

Theorem 4.11. *If the SQIsignHD signature scheme, $\Sigma_{\text{SQIsignHD}}$, is SUF-CMA, and $R_{\mathcal{Q}}$ is a hard relation, then the SQIAsignHD adaptor signature scheme is secure in quantum random oracle model (QROM).*

Proof. Due to the previous Lemmas of this section, we have shown that the adaptor signature $\Xi_{R_{\mathcal{Q}}, \Sigma_{\text{SQIsignHD}}}$ satisfies pre-signature correctness, pre-signature adaptability, aEUF-CMA, and witness extractability properties. Verifying these properties completes the proof of the theorem. \square

Conclusion

Adaptor signatures, which are a generalization of standard digital signatures, are a crucial cryptographic primitive for blockchain applications in reducing costs, improving fungibility, and supporting off-chain payment in payment-channel networks and hubs. In the present work, we have introduced SQIAsignHD, a new adaptor signature scheme with quantum-resistant security based on isogenies of supersingular elliptic curves. Thereby, it provides security and privacy concepts relevant to off-chain applications built on it. In SQIAsignHD, we use SQIsignHD as the underlying signature scheme and make use of the idea of artificial orientation, on the supersingular isogeny Diffie-Hellman key exchange protocol, to apply the hard relation. In contrast to the only isogeny-based adaptor signature construction, IAS [18], which requires more than 18,000 bytes, our approximated pre-signature takes ~ 200 bytes. Furthermore, our signature is 109 bytes in size, whereas the IAS signature may vary between 263 and 1880 bytes, depending on the security parameters employed. Thus, compared to IAS and LAS [9], our construction significantly improves pre-signature and signature sizes and due to the nature of its underlying signature scheme, SQIAsignHD is the most compact quantum-safe adaptor signature.

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